2022

CIVIL ENGINEERING (CE) P1

Q. 1. Consider the following expression:

$$z = \sin(y + it) + \cos(y - it)$$

where z, y, and t are variables, and $i = \sqrt{-1}$ is a complex number. The partial differential equation derived from the above expression is

(a) $\frac{\partial^2 z}{\partial t^2} + \frac{\partial^2 z}{\partial y^2} = 0$ **(b)** $\frac{\partial^2 z}{\partial t^2} - \frac{\partial^2 z}{\partial y^2} = 0$

(c) $\frac{\partial z}{\partial t} - i\frac{\partial z}{\partial y} = 0$ **(d)** $\frac{\partial z}{\partial t} + i\frac{\partial z}{\partial y} = 0$

Q. 2. For the equation

$$\frac{d^3y}{dx^3} + x\left(\frac{dy}{dx}\right)^{3/2} + x^2 y = 0$$

the correct description is

(a) an ordinary differential equation of order 3 and degree 2.

(b) an ordinary differential equation of order 3 and degree 3.

(c) an ordinary differential equation of order 2 and degree 3.

(d) an ordinary differential equation of order 3 and degree $\frac{3}{2}$.

Q. 3. The matrix M is defined as

$$M = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$$

and has eigenvalues 5 and − 2. The matrix Q is formed as

$$Q = M^3 - 4M^2 - 2M$$

Which of the following is/are the eigenvalue(s) of matrix Q ?

(a) 15 **(b)** 25

(c) –20 **(d)** –30

Q. 4. Consider the following recursive iteration scheme for different values of variable P with the initial guess $x_1 = 1$:

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{P}{x_n}\right), \qquad n = 1, 2, 3, 4, 5$$

For P = 2, x_5 is obtained to be 1.414, rounded-off to three decimal places. For P = 3, x_5 is obtained to be 1.732, rounded-off to three decimal places.

If P = 10, the numerical value of x_5 is _________ (*round off to three decimal places*).

Q. 5. The Fourier cosine series of a function is given by:

$$f(x) = \sum_{n=0}^{\infty} f_n \cos nx$$

For $f(x) = \cos^4 x$, the numerical value of $(f_4 + f_5)$ is ______ *(round off to three decimal places).*

Q. 6. Let max $\{a, b\}$ denote the maximum of two real numbers a and b.

Which of the following statement(s) is/are TRUE about the function

$f(x) = \max\{3 - x, x - 1\}$?

(a) It is continuous on its domain.

(b) It has a local minimum at $x = 2$.

(c) It has a local maximum at $x = 2$.

(d) It is differentiable on its domain.

Q. 7. Consider the differential equation

$$\frac{dy}{dx} = 4(x + 2) - y$$

For the initial condition $y = 3$ at $x = 1$, the value of y at $x = 1.4$ obtained using Euler's method with a step-size of 0.2 is _________. *(round off to one decimal place)*

Q. 8. A set of observations of independent variable (x) and the corresponding dependent variable (y) is given below.

x	5	2	4	3
y	16	10	13	12

Based on the data, the coefficient a of the linear regression model

$$y = a + bx$$

is estimated as 6.1.

The coefficient b is ________. *(round off to one decimal place)*

Chemical Engineering (CH)

Q. 9. The value of $(1 + i)^{12}$, where $i = \sqrt{-1}$, is

(a) $-64i$ **(b)** $64i$

(c) 64 **(d)** -64

Q. 10. Given matrix $A = \begin{bmatrix} x & 1 & 3 \\ y & 2 & 6 \\ 3 & 5 & 7 \end{bmatrix}$, the ordered pair (x, y) for which det(A) = 0 is

(a) (1,1) **(b)** (1,2)

(c) (2,2) **(d)** (2,1)

Q. 11. Let $f(x) = e^{-|x|}$, where x is real. The value of $\frac{df}{dx}$ at $x = -1$ is

(a) $-e$ **(b)** e

(c) $1/e$ **(d)** $-1/e$

Q. 12. The value of the real variable $x \geq 0$, which maximizes the function $f(x) = x^e e^{-x}$ is

(a) e **(b)** 0

(c) $1/e$ **(d)** 1

Q. 13. The partial differential equation

$$\frac{\partial u}{\partial t} = \left(\frac{1}{\pi^2}\right)\frac{\partial^2 u}{\partial x^2}$$

where, $t \geq 0$ and $x \in [0,1]$, is subjected to the following initial and boundary conditions:

$u(x, 0) = \sin(\pi x)$

$u(0, t) = 0$

$u(1, t) = 0$

The value of t at which

$$\frac{u(0.5, t)}{u(0.5, 0)} = \frac{1}{e}$$ is

(a) 1 **(b)** e

(c) π **(d)** $\frac{1}{e}$

Q. 14. The directional derivative of $f(x, y, z) = 4x^2 + 2y^2 + z^2$ at the point (1, 1, 1) in the direction of the vector $\vec{v} = \hat{i} - \hat{k}$ is ________ *(rounded off to two decimal places).*

Q. 15. Consider a sphere of radius 4, centered at the origin, with outward unit normal $\hat{n}$ on its surface S. The value of the surface integral $\iint_s \left(\frac{2x\hat{i} + 3y\hat{j} + 4z\hat{k}}{4\pi}\right) \cdot \hat{n}\, dA$ is ________

(rounded off to one decimal place).

Q. 16. The $\frac{dy}{dx} = xy^2 + 2y + x - 4.5$ with the initial condition $y\,(x = 0) = 1$ is to be solved using a predictor-corrector approach. Use a predictor based on the implicit Euler's method and a corrector based on the trapezoidal rule of integration, each with a full-step size of 0.5. Considering only positive values of y, the value of y at $x = 0.5$ is ________ *(rounded off to three decimal places).*

CIVIL ENGINEERING (CE) P2

Q. 17. Let y be a non-zero vector of size 2022 × 1. Which of the following statement(s) is/are TRUE?

(a) yy^T is a symmetric matrix.

(b) y^Ty is an eigenvalue of yy^T.

(c) yy^T has a rank of 2022.

(d) yy^T is invertible.

Q. 18. A pair of six-faced dice is rolled thrice. The probability that the sum of the outcomes in each roll equals 4 in exactly two of the three attempts is ______. *(round off to three decimal places)*

Q. 19. Consider the polynomial $f(x) = x^3 - 6x^2 + 11x - 6$ on the domain S given by $1 \leq x \leq 3$. The first and second derivatives are $f'(x)$ and $f''(x)$.

Consider the following statements:

I. The given polynomial is zero at the boundary points $x = 1$ and $x = 3$.

II. There exists one local maxima of $f(x)$ within the domain S.

III. The second derivative $f''(x) > 0$ throughout the domain S.

IV. There exists one local minima of $f(x)$ within the domain S.

The correct option is:

(a) Only statements I, II and III are correct.

(b) Only statements I, II and IV are correct.

(c) Only statements I and IV are correct.

(d) Only statements II and IV are correct.

Q. 20. P and Q are two square matrices of the same order. Which of the following statement(s) is/are correct?

(a) If P and Q are invertible, then $[PQ]^{-1} = Q^{-1}P^{-1}$.

(b) If P and Q are invertible, then $[QP]^{-1} = P^{-1}Q^{-1}$.

(c) If P and Q are invertible, then $[PQ]^{-1} = P^{-1}Q^{-1}$.

(d) If P and Q are not invertible, then $[PQ]^{-1} = Q^{-1}P^{-1}$.

Q. 21. $\int\left(x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots\right)dx$ is equal to

(a) $(1+x)\{\log(1+x)-1\}+\text{constant}$

(b) $\frac{1}{1+x^2}+\text{Constant}$

(c) $-\frac{1}{1-x}+\text{Constant}$

(d) $-\frac{1}{1-x^2}+\text{Constant}$

Q. 22. The function (x, y) satisfies the Laplace equation

$$\nabla^2 f(x,y) = 0$$

on a circular domain of radius $r = 1$ with its center at point P with coordinates $x = 0$, $y = 0$. The value of this function on the circular boundary of this domain is equal to 3. The numerical value of $f(0, 0)$ is:

(a) 0 **(b)** 2

(c) 3 **(d)** 1

ELECTRICAL ENGINEERING (EE)

Q. 23. Consider a 3 × 3 matrix A whose $(i, j)^{th}$ element, $a_{i,j} = (i-j)^3$. Then the matrix A will be

(a) symmetric **(b)** skew-symmetric

(c) unitary **(d)** null

Q. 24. e^A denotes the exponential of a square matrix A. Suppose λ is an eigenvalue and

v is the corresponding eigen-vector of matrix A.

Consider the following two statements:

Statement 1: e^λ is an eigenvalue of e^A.

Statement 2: v is an eigen-vector of e^A.

Which one of the following options is correct?

(a) Statement 1 is true and statement 2 is false.

(b) Statement 1 is false and statement 2 is true.

(c) Both the statements are correct.

(d) Both the statements are false.

Q. 25. Let $f(x)=\int_0^x e^t(t-1)(t-2)dt.$ Then $f(x)$ decreases in the interval

(a) $x \in (1,2)$

(b) $x \in (2,3)$

(c) $x \in (0,1)$

(d) $x \in (0.5,1)$

Q. 26. Consider a matrix $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 4 & -2\\ 0 & 1 & 1\end{bmatrix}$.

The matrix A satisfies the equation $6A^{-1} = A^2 + cA + dI$, where c and d are scalars and I is the identity matrix.

Then $(c + d)$ is equal to

(a) 5 **(b)** 17

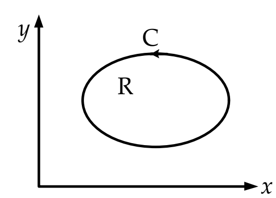
(c) −6 **(d)** 11

Q. 27. Let, $f(x, y, z) = 4x^2 + 7xy + 3xz^2$. The direction in which the function $f(x, y, z)$ increases most rapidly at point P = (1, 0, 2) is

(a) $20\hat{i}+7\hat{j}$ **(b)** $20\hat{i}+7\hat{j}+12\hat{k}$

(c) $20\hat{i}+21\hat{k}$ **(d)** $20\hat{i}$

Q. 28. Let R be a region in the first quadrant of the xy plane enclosed by a closed curve C considered in counter-clockwise direction. Which of the following expressions does not represent the area of the region R?



(a) $\iint_R dxdy$ **(b)** $\oint_C xdy$

(c) $\oint_C ydx$ **(d)** $\frac{1}{2}\oint_C (xdy-ydx)$

Q. 29. Let $\vec{E}(x,y,z) = 2x^2\hat{i} + 5y\hat{j} + 3z\hat{k}$. The value of $\iiint_V (\vec{\nabla}\cdot\vec{E})dV$, where V is the volume enclosed by the unit cube defined by $0 \le x \le 1$, $0 \le y \le 1$, and $0 \le z \le 1$, is

(a) 3 **(b)** 8

(c) 10 **(d)** 5

COMPUTER SCIENCE (CS)

Q. 30. Consider the following two statements with respect to the matrices $A_{m\times n}$, $B_{n\times m}$, $C_{n\times n}$ and $D_{n\times n}$.

Statement 1: tr(AB) = tr(BA)

Statement 2: tr(CD) = tr(DC)

where tr() represents the trace of a matrix. Which one of the following holds?

(a) Statement 1 is correct and Statement 2 is wrong.

(b) Statement 1 is wrong and Statement 2 is correct.

(c) Both Statement 1 and Statement 2 are correct.

(d) Both Statement 1 and Statement 2 are wrong.

Q. 31. The number of arrangements of six identical balls in three identical bins is________.

Q. 32. The value of the following limit is ______________.

$$\lim_{x\to 0^+} \frac{\sqrt{x}}{1-e^{2\sqrt{x}}}$$

Q. 33. Consider solving the following system of simultaneous equations using LU decomposition.

$$\begin{aligned} x_1 + x_2 - 2x_3 &= 4 \\ x_1 + 3x_2 - x_3 &= 7 \\ 2x_1 + x_2 - 5x_3 &= 7 \end{aligned}$$

where L and U are denoted as

$$L = \begin{pmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{pmatrix}, \quad U = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{pmatrix}$$

Which one of the following is the correct combination of values for L_{32}, U_{33}, and x_1?

(a) $L_{32} = 2,\ U_{33} = -\frac{1}{2}, x_1 = -1$

(b) $L_{32} = 2,\ U_{33} = 2, x_1 = -1$

(c) $L_{32} = \frac{1}{2},\ U_{33} = 2, x_1 = 0$

(d) $L_{32} = -\frac{1}{2},\ U_{33} = -\frac{1}{2}, x_1 = 0$

Q. 34. Which of the following is/are the eigenvector(s) for the matrix given below?

$$\begin{pmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{pmatrix}$$

(a) $\begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$ **(b)** $\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$

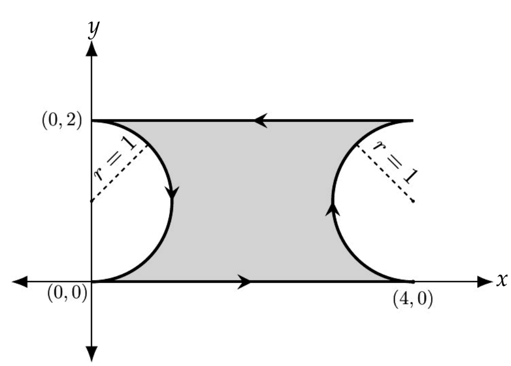
(c) $\begin{pmatrix} -1 \\ 0 \\ 2 \\ 2 \end{pmatrix}$ **(d)** $\begin{pmatrix} 0 \\ 1 \\ -3 \\ 0 \end{pmatrix}$

Electronics Communication (EC)

Q. 35. Consider the two-dimensional vector field $\vec{F}(x,y) = x\vec{i} + y\vec{j}$, where $\vec{i}$ and $\vec{j}$ denote the unit vectors along the x-axis and the y-axis, respectively. A contour C in the xy- plane, as shown in the figure, is composed of two horizontal lines connected at the two ends by two semicircular arcs of unit radius. The contour is traversed in the counter-clockwise sense. The value of the closed path integral

$$\oint_C \vec{F}(x,y)\cdot(dx\vec{i} + dy\vec{j})$$

is _________.



(a) 0 **(b)** 1

(c) 8 + 2π **(d)** −1

Q. 36. Consider a system of linear equations Ax = b, where

$$A = \begin{bmatrix} 1 & -\sqrt{2} & 3 \\ -1 & \sqrt{2} & -3 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

This system of equations admits ________.

(a) a unique solution for x
(b) infinitely many solutions for x
(c) no solutions for x
(d) exactly two solutions for x

Q. 37. Consider the following partial differential equation (PDE)

$$a\frac{\partial^2 f(x,y)}{\partial x^2} + b\frac{\partial^2 f(x,y)}{\partial y^2} = f(x,y),$$

where a and b are distinct positive real numbers. Select the combination(s) of values of the real parameters ξ and η such that $f(x,y) = e^{(\xi x+1)}$ is a solution of thegiven PDE.

(a) $\xi = \frac{1}{\sqrt{2a}}, \eta = \frac{1}{\sqrt{2b}}$

(b) $\xi = \frac{1}{\sqrt{a}}, \eta = 0$

(c) $\xi = 0, \eta = 0$

(d) $\xi = \frac{1}{\sqrt{a}}, \eta = \frac{1}{\sqrt{b}}$

Q. 38. Consider the following wave equation,

$$\frac{\partial^2 f(x,t)}{\partial t^2} = 10000\frac{\partial^2 f(x,t)}{\partial x^2}$$

Which of the given options is/are solution(s) to the given wave equation?

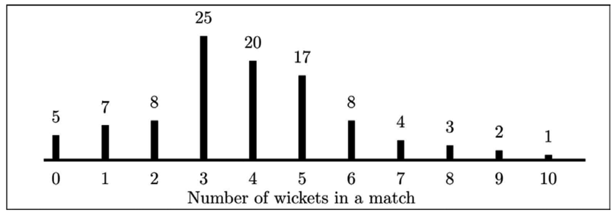
(a) $f(x,t) = e^{-(x-100t)^2} + e^{-(x+100t)^2}$

(b) $f(x,t) = e^{-(x-100t)} + 0.5e^{-(x+1000t)}$

(c) $f(x,t) = e^{-(x-100t)} + \sin(x+100t)$

(d) $f(x,t) = e^{j100\pi(-100x+t)} + e^{j100\pi(100x+t)}$

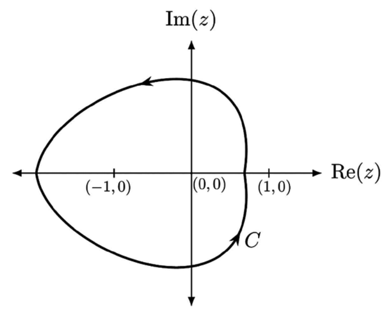
Q. 39. The bar graph shows the frequency of the number of wickets taken in a match by a bowler in her career. For example, in 17 of her matches, the bowler has taken 5 wickets each. The median number of wickets taken by the bowler in a match is ________ *(rounded off to one decimal place)*.



Q. 40. A simple closed path C in the complex plane is shown in the figure. If

$$\oint_C \frac{2^z}{z^2-1}dz = -i\pi A,$$

where $i = \sqrt{-1}$, then the value of A is ______ *(rounded off to two decimal places)*.



Q. 41. The function $f(x) = 8\log_e x - x^2 + 3$ attains its minimum over the interval [1, e] at x = ________.
(Here $\log_e x$ is the natural logarithm of x.)

(a) 2 **(b)** 1
(c) e **(d)** $\frac{1+e}{2}$

Q. 42. Let α, β be two non-zero real numbers and v_1, v_2 be two non-zero real vectors of size 3 × 1. Suppose that v_1 and v_2 satisfy $v_1^T v_2 = 0$, $v_1^T v_1 = 1$, and $v_2^T v_2 = 1$. Let A be the 3 × 3 matrix given by:

$$A = \alpha v_1 v_1^T + \beta v_2 v_2^T +$$

The eigenvalues of A are ________.

(a) 0, α, β **(b)** 0, α + β, α − β
(c) $0, \frac{\alpha+\beta}{2}, \sqrt{\alpha\beta}$ **(d)** $0, 0, \sqrt{\alpha^2+\beta^2}$

Q. 43. Consider the following series:

$$\sum_{n=1}^{\infty} \frac{n^d}{c^n}$$

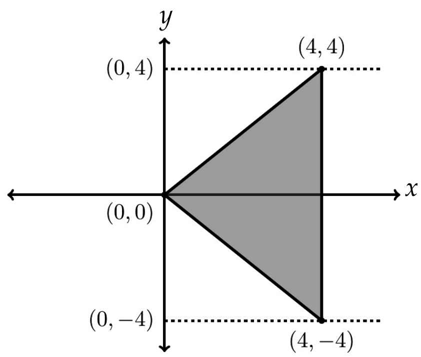
For which of the following combinations of c, d values does this series converge?

(a) $c = 1, d = -1$ **(b)** $c = 2, d = 1$
(c) $c = 0.5, d = -10$ **(d)** $c = 1, d = -2$

Q. 44. The value of the integral

$$\iint_D 3\left(x^2+y^2\right)dxdy,$$

where D is the shaded triangular region shown in the diagram, is _____ *(rounded off to the nearest integer).*



MECHANICAL ENGINEERING (ME) P1

Q. 45. The Fourier series expansion of x^3 in the interval $-1 \le x < 1$ with periodic continuation has

(a) only sine terms
(b) only cosine terms
(c) both sine and cosine terms
(d) only sine terms and a non-zero constant

Q. 46. If $A = \begin{bmatrix} 10 & 2k+5 \\ 3k-3 & k+5 \end{bmatrix}$ is a symmetric matrix, the value of k is __________.

(a) 8 **(b)** 5
(c) -0.4 **(d)** $\dfrac{1+\sqrt{1561}}{12}$

Q. 47. The limit

$$p = \lim_{x\to\pi}\left(\frac{x^2+\alpha x+2\pi^2}{x-\pi+2\sin x}\right)$$

has a finite value for a real α. The value of α and the corresponding limit p are

(a) $\alpha = -3\pi$, and $p = \pi$
(b) $\alpha = -2\pi$, and $p = 2\pi$
(c) $\alpha = \pi$, and $p = \pi$
(d) $\alpha = 2\pi$, and $p = 3\pi$

Q. 48. Solution of $\nabla^2 T = 0$ in a square domain ($0 < x < 1$ and $0 < y < 1$) with boundary conditions: $T(x, 0) = x$; $T(0, y) = y$; $T(x, 1) = 1 + x$; $T(1, y) = 1 + y$ is

(a) $T(x, y) = x - xy + y$
(b) $T(x, y) = x + y$
(c) $T(x, y) = -x + y$
(d) $T(x, y) = x + xy + y$

Q. 49. Given a function $\varphi = \frac{1}{2}\left(x^2+y^2+z^2\right)$ in three-dimensional cartesian space, the value of the surface integral

$$\oiint_S \hat{n}\cdot\nabla\varphi \, dS,$$

where S is the surface of a sphere of unit radius and $\hat{n}$ is the outward unit normal vector on S, is

(a) 4π **(b)** 3π
(c) $\dfrac{4\pi}{3}$ **(d)** 0

Q. 50. The value of the integral

$$\oint\left(\frac{6z}{2z^4-3z^3+7z^2-3z+5}\right)dz$$

evaluated over a counter-clockwise circular contour in the complex plane enclosing only the pole $z = i$, where i is the imaginary unit, is

(a) $(-1 + i)\,\pi$ **(b)** $(1 + i)\,\pi$
(c) $2(1 - i)\,\pi$ **(d)** $(2 + i)\,\pi$

Q. 51. The system of linear equations in real (x, y) given by

$$\begin{pmatrix} x & y \end{pmatrix}\begin{bmatrix} 2 & 5-2\alpha \\ \alpha & 1 \end{bmatrix} = \begin{pmatrix} 0 & 0 \end{pmatrix}$$

involves a real parameter α and has infinitely many non-trivial solutions for special value(s) of α. Which one or more among the following options is/are non-trivial solution(s) of (x, y) for such special value(s) of α?

(a) $x = 2, y = -2$
(b) $x = -1, y = 4$
(c) $x = 1, y = 1$
(d) $x = 4, y = -2$

Q. 52. Let a random variable X follow Poisson distribution such that

Prob(X = 1) = Prob(X = 2).

The value of Prob(X = 3) is __________ *(round off to 2 decimal places).*

Q. 53. Consider two vectors:

$$\vec{a} = 5\hat{i}+7\hat{j}+2\hat{k}$$

$$\vec{a} = 3\hat{i}-\hat{j}+6\hat{k}$$

Magnitude of the component of $\vec{a}$ orthogonal to $\vec{b}$ in the plane containing the vectors $\vec{a}$ and $\vec{b}$ is __________ *(round off to 2 decimal places).*

MECHANICAL ENGINEERING (ME) P2

Q. 54. Given $\int_{-\infty}^{\infty} e^{-x} dx = \sqrt{\pi}$.

If a and b are positive integers, the value of $\int_{-\infty}^{\infty} e^{-a(x+b)^2} dx$ is ________.

(a) $\sqrt{\pi a}$ **(b)** $\sqrt{\frac{\pi}{a}}$

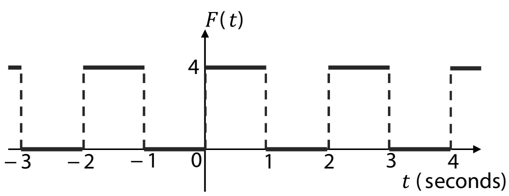
(c) $b\sqrt{\pi a}$ **(d)** $b\sqrt{\frac{\pi}{a}}$

Q. 55. If $f(x) = 2\ln(\sqrt{e^x})$, what is the area bounded by $f(x)$ for the interval [0, 2] on the x-axis?

(a) $\frac{1}{2}$ **(b)** 1

(c) 2 **(d)** 4

Q. 56. F(t) is a periodic square wave function as shown. It takes only two values, 4 and 0, and stays at each of these values for 1 second before changing. What is the constant term in the Fourier series expansion of F(t)?



(a) 1 **(b)** 2

(c) 3 **(d)** 4

Q. 57. Consider a cube of unit edge length and sides parallel to co-ordinate axes, with its centroid at the point (1, 2, 3). The surface integral $\int_A \vec{F} \cdot d\vec{A}$ of a vector field $\vec{F} = 3x\hat{i} + 5y\hat{j} + 6z\hat{k}$ over the entire surface A of the cube is ______.

(a) 14 **(b)** 27

(c) 28 **(d)** 31

Q. 58. Consider the definite integral

$$\int_1^2 \left(4x^2 + 2x + 6\right) dx.$$

Let I_e be the exact value of the integral. If the same integral is estimated using Simpson's rule with 10 equal subintervals, the value is I_s. The percentage error is defined as $e = 100 \times (I_e - I_s)/I_e$. The value of e is

(a) 2.5 **(b)** 3.5

(c) 1.2 **(d)** 0

Q. 59. A polynomial $\varphi(s) = a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0$ of degree $n > 3$ with constant real coefficients $a_n, a_{n-1}, \ldots a_0$ has triple roots at $s = -\sigma$. Which one of the following conditions must be satisfied?

(a) $\varphi(s) = 0$ at all the three values of s satisfying $^3 + \sigma^3 =$

(b) $\varphi(s) = 0, \frac{d\varphi(s)}{ds} = 0$, and $\frac{d^2\varphi(s)}{ds^2} = 0$ at $s = -\sigma$

(c) $\varphi(s) = 0, \frac{d^2\varphi(s)}{ds^2} = 0$, and $\frac{d^4\varphi(s)}{ds^4} = 0$ at $s = -\sigma$

(d) $\varphi(s) = 0$, and $\frac{d^2\varphi(s)}{ds^2} = 0$ at $s = -\sigma$

Q. 60. For the exact differential equation,

$$\frac{du}{dx} = \frac{-xu^2}{2 + x^2 u},$$

which one of the following is the solution?

(a) $u^2 + 2x^2$ = constant

(b) $xu^2 + u$ = constant

(c) $\frac{1}{2}x^2u^2 + 2u$ = constant

(d) $\frac{1}{2}ux^2 + 2x$ = constant

Q. 61. A manufacturing unit produces two products P1 and P2. For each piece of P1 and P2, the table below provides quantities of materials M1, M2, and M3 required, and also the profit earned. The maximum quantity available per day for M1, M2 and M3 is also provided. The maximum possible profit per day is ₹__________.

	M1	M2	M3	Profit per piece (₹)
P1	2	2	0	150
P2	3	1	2	100
Maximum quantity available per day	70	50	40	

(a) 5000 **(b)** 4000

(c) 3000 **(d)** 6000

Q. 62. A is a 3×5 real matrix of rank 2. For the set of homogeneous equations AX = 0, where 0 is a zero vector and X is a vector of unknown variables, which of the following is/are true?

(a) The given set of equations will have a unique solution.

(b) The given set of equations will be satisfied by a zero vector of appropriate size.

(c) The given set of equations will have infinitely many solutions.

(d) The given set of equations will have many but a finite number of solutions

Q. 63. If the sum and product of eigenvalues of a 2 × 2 real matrix $\begin{bmatrix} 3 & p \\ p & q \end{bmatrix}$ are 4 and −1 respectively, then $|p|$ is ______ (*in integer*).

Q. 64. Given $z = x + iy$, $i = \sqrt{-1}$. C is a circle of radius 2 with the centre at the origin.

If the contour C is traversed anticlockwise, then the value of the integral $\frac{1}{2\pi}\int_C \frac{1}{(z-i)(z+4i)}dz$ is ______ *(round off to one decimal place).*

Answer Key

Q. No.	Answer	Topic Name	Chapter Name
1	(a)	Formulation of PDE	Partial Differential Equation
2	(a)	Order and Degree	Differential Equation
3	(a, c)	Eigen Value	Matrix
4	[3.162]	Iteration Method	Numerical Analysis
5	[0.125]	Coefficient Value	Fourier Series
6	(a, b)	Local Minima	Maxima and Minima
7	[6.4]	Euler's Method	Numerical Analysis
8	[1.9]	Straight Line	Curve Fitting
9	(d)	Complex Number	Complex Number
10	(b)	Matrix	Matrix
11	(a)	Differentiation	Calculus
12	(a)	Maxima and Minima	Calculus
13	[1]	Laplace Transform	Integral Transform
14	[4.24]	Directional Derivative	Vector Differentiation
15	[192]	Gauss Divergence Theorem	Vector Integration
16	[0.875]	Euler's Method, Trapezoidal Rule	Numerical Analysis
17	(a, b)	Eigenvalues	Linear Algebra
18	[0.019]	Probability	Probability and Statistics
19	(b)	Local Maxima and Minima	Calculus
20	(a, b)	Matrix Algebra	Linear Algebra
21	(a)	Indefinite Integrals	Calculus
22	(c)	Laplace Equation	Partial Differential Equation
23	(b)	Matrix Algebra	Linear Algebra
24	(c)	Eigenvalues and Eigenvectors	Linear Algebra
25	(a)	Increasing Decreasing Function	Calculus
26	(a)	Eigenvalues And Eigenvectors	Linear Algebra
27	(b)	Vector Calculus	Calculus
28	(c)	Vector Calculus	Calculus
29	(c)	Vector Calculus	Calculus

30	(c)	Matrix Algebra	Linear Algebra
31	[7]	Probability	Probability And Statistics
32	[–0.5]	Limits	Calculus
33	(d)	LU Decomposition	Linear Algebra
34	(a, c, d)	Eigenvalues And Eigenvectors	Linear Algebra
35	(a)	Vector Calculus	Calculus
36	(c)	Solution of Linear Equations	Linear Algebra
37	(a, b)	Partial Differential Equations	Partial Differential Equations
38	(a, c)	Wave Equations	Partial Differential Equations
39	[4]	Median	Probability And Statistics
40	[0.5]	Cauchy's Integral Formula	Complex Analysis
41	(b)	Absolute Maximum And Minimum	Calculus
42	(a)	Eigenvalues And Eigenvectors	Linear Algebra
43	(b, d)	Convergence Tests	Complex Analysis
44	[512]	Multiple Integrals	Calculus
45	(a)	Fourier Series for Even and Odd Function	Fourier Series
46	(a)	Symmetric Matrix	Matrix
47	(a)	Limits	Limits and Continuity
48	(b)	Boundary Value Problems	Partial Differential Equations
49	(a)	Gauss Divergence Theorem	Application of Vectors Calculus
50	(a)	Residues Theorem	Complex Number
51	(a, b)	Solution of Linear Equations	Matrix
52	[0.18]	Poisson Distribution	Probability Distributions
53	[8.32]	Orthogonal Vectors	Vectors
54	(b)	Beta and Gamma Functions	Definite Integral
55	(c)	Area Bounded	Definite Integral
56	(b)	Value of Constant	Fourier Series
57	(a)	Application of Vectors	Vector Calculus
58	(b)	Error and Approximation	Numerical Analysis
59	(b)	Maxima and Minima	Differential Calculus
60	(c)	Exact Differential Equation	Differential Equation
61	(b)	Graphical Method	Linear Programming
62	(b, c)	Homogeneous Equations	Matrix
63	[2]	Eigenvalues and Eigenvectors	Matrix
64	[0.2]	Residues Theorem	Complex Number

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ANSWERS WITH EXPLANATIONS

1. Option (a) is correct.

$z = \sin(y + it) + \cos(y - it)$ …(i)

$z = f(y, t)$ then Differentiating partially with respect to t both sides,

$$\frac{\partial z}{\partial t} = i\cos(y+it) + i\sin(y-it)$$

Again differentiate

$$\frac{\partial^2 z}{\partial t^2} = -i^2\sin(y+it) - i^2\cos(y-it)$$

$$= \sin(y + it) + \cos(y - it) \quad \text{…(ii)}$$

Partial differentiate with respect to y both sides,

$$\frac{\partial z}{\partial y} = \cos(y+it) - \sin(y-it)$$

Again differentiate partially

$$\frac{\partial^2 z}{\partial y^2} = -\sin(y+it) - \cos(y-it)$$

$$= -[\sin(y+it) + \cos(y-it)] \quad \text{…(iii)}$$

From (ii) and (iii), we have

$$\frac{\partial^2 z}{\partial y^2} = -\frac{\partial^2 z}{\partial t^2}$$

$$\therefore \quad \frac{\partial^2 z}{\partial y^2} + \frac{\partial^2 z}{\partial t^2} = 0$$

2. Option (a) is correct.

It is an ordinary differential equation.

$$\frac{d^3y}{dx^3} + x^2y = -x\left(\frac{dy}{dx}\right)^{\frac{3}{2}}$$

As squaring both sides, we get

$$\left(\frac{d^3y}{dx^3} + x^2y\right)^2 = x^2\left(\frac{dy}{dx}\right)^3$$

$$\left(\frac{d^3y}{dx^3}\right)^2 + x^4y^2 + 2.x^2y\frac{d^3y}{dx^3} = x^2\left(\frac{dy}{dx}\right)^3$$

Order is 3 and degree is 2

3. Options (a, c) are correct.

Given, $M = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$

Then, $M^2 = M \times M$

$$= \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 9 \\ 12 & 16 \end{bmatrix}$$

$\therefore \quad M^3 = M^2 \times M$

$$= \begin{bmatrix} 13 & 9 \\ 12 & 16 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 49 & 57 \\ 76 & 68 \end{bmatrix}$$

$$Q = M^3 - 4M^2 - 2M$$

$$= \begin{bmatrix} 49 & 57 \\ 76 & 68 \end{bmatrix} - \begin{bmatrix} 52 & 36 \\ 48 & 64 \end{bmatrix} - \begin{bmatrix} 2 & 6 \\ 8 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 49 & 57 \\ 76 & 68 \end{bmatrix} - \begin{bmatrix} 54 & 42 \\ 56 & 68 \end{bmatrix} = \begin{bmatrix} -5 & 15 \\ 20 & 0 \end{bmatrix}$$

Eigenvalue $= |A - \lambda I| = 0 \quad \Rightarrow \begin{vmatrix} -5-\lambda & 15 \\ 20 & -\lambda \end{vmatrix} =$

$\Rightarrow \quad (-5-\lambda)(-\lambda) - 15 \times 20 = 0$

$\Rightarrow \quad 5\lambda + \lambda^2 - 300 = 0$

$\Rightarrow \quad \lambda^2 + 5\lambda - 300 = 0$

$\Rightarrow \quad \lambda^2 + 20\lambda - 15\lambda - 300 = 0$

$\Rightarrow \quad \lambda(\lambda + 20) - 15(\lambda + 20) = 0$

$\Rightarrow \quad (\lambda - 15)(\lambda + 20) = 0 \quad \Rightarrow \lambda = 15, -20$

Another method

Given $\lambda_1 = 5 \quad \lambda_2 = -2$

$$Q = M^3 - 4M^2 - 2M$$

$$Q\lambda_1 = (5)^3 - 4(5)^2 - 2 \times 5$$

$= 125 - 100 - 10 = 15$

$$Q\lambda_2 = (-2)^3 - 4(-2)^2 - 2\times(-2)$$

$= -8 - 16 + 4 = -20$

Correct ans (a) and (c)

4. Correct answer is [3.162].

Given, $x_1 = 1$ Initial Value

If P = 10

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{10}{x_n}\right) \quad \ldots\text{(i)}$$

Put $n = 1$

$$x_2 = \frac{1}{2}\left(x_1 + \frac{10}{x_1}\right) = \frac{1}{2}\left(1+\frac{10}{1}\right) = \frac{11}{2} = 5.5$$

$n = 2$

$$x_3 = \frac{1}{2}\left(x_2 + \frac{10}{x_2}\right) = \frac{1}{2}\left(5.5+\frac{10}{5.5}\right) = 3.6590$$

$n = 3$

$$x_4 = \frac{1}{2}\left(x_3 + \frac{10}{x_3}\right) = \frac{1}{2}\left(3.659+\frac{10}{3.659}\right) = 3.1959$$

$n = 4$

$$x_5 = \frac{1}{2}\left(x_4 + \frac{10}{x_4}\right) = \frac{1}{2}\left(3.196+\frac{10}{3.196}\right) = 3.162$$

Hence, at P = 10 the numerical value of $x_5 = 3.162$

Another method

When $x \to \infty$ $\quad x_{n+1} = x_n = \infty$

$$\therefore \quad \infty = \frac{1}{2}\left(\infty + \frac{P}{\infty}\right)$$

$$\Rightarrow \quad 2\infty = \frac{(\infty^2 + P)}{\infty}$$

$$\Rightarrow \quad 2\infty^2 = \infty^2 + P$$

$$\therefore \quad \infty^2 - P = 0 \quad \Rightarrow \infty^2 = P \quad \Rightarrow \quad \infty = \sqrt{P}$$

At P = 10

$$\infty = \sqrt{10} = 3.162$$

5. Correct answer is [0.125].

$$f(x) = \cos^4 x = \left[\cos^2 x\right]^2 = \left[\frac{1+\cos 2x}{2}\right]^2$$

$$= \frac{1}{4}\left[1+\cos^2 2x + 2\cos 2x\right]$$

$$= \frac{1}{4}\left[1+\frac{1+\cos 4x}{2} + 2\cos 2x\right]$$

$$= \frac{1}{4}+\left[\frac{1+\cos 4x}{8} + \frac{1}{2}\cos 2x\right]$$

$$= \frac{1}{4}+\frac{1}{8}+\frac{\cos 4x}{8} + \frac{1}{2}\cos 2x$$

$$= \frac{3}{8}+\frac{\cos 4x}{8} + \frac{1}{2}\cos 2x$$

$$f(x) = \frac{3}{8} + 0\cos x + \frac{1}{2}\cos 2x + 0\cos 3x + \frac{1}{8}\cos 4x + 0\cos 5x +$$

Coefficient of

$$f(4) + f(5) = \frac{1}{8} + 0 = \frac{1}{8} = 0.125$$

6. Options (a, b) are correct.

For finding intersection point

$3 - x = x - 1$

At $x = 2$

so, $3 - x$ and $x - 1$ intersects at $x = 2$

$$f(x) = \begin{cases} 3 - x & x < 2 \\ x - 1 & x > 2 \\ 1 & x = 2 \end{cases}$$

Check for continuity

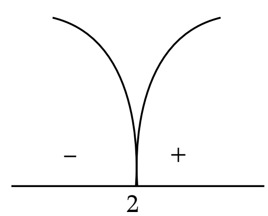
$$f(2^-) = f(2^+) = f(2)$$

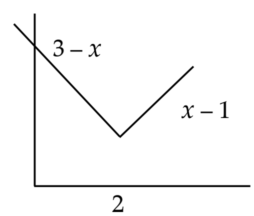
It is continuous in its domain

Check for differentiability

$$f(x) = \begin{cases} -1 & x < 2 \\ 1 & x > 2 \end{cases}$$

It is not differentiable. For maxima and minima





It has local minima at $x = 2$

7. Correct answer is [6.4].

We have by Euler's method

$$y_{n+1} = y_{n+h} f(x_n, y_n) \quad \ldots\text{(i)}$$

Put $n = 0$

$y_1 = y_0 + hf(x_0, y_0)$

At $x_1 = x_0 + a$

$= 1 + 0.2 = 1.2$

$y_1 = 3 + 0.2 f(1, 3) = 3 + 0.2\,[4(1+2)–3]$

$y_1 = 3 + 0.2\,[9] = 3 + 1.8 = 4.8$

Put $n = 1$

$y_2 = y_1 + hf(x_1, y_1)$ …(ii)

At $x_2 = x_1 + h$

$= 1.2 + 0.2$

$= 1.4$

$y_2 = 4.8 + 0.2 f(1.2, 4.8)$

$= 4.8 + 0.2[4(1.2+2) – 4.8]$

$= 4.8 + 0.2\,[4\times3.2 – 4.8]$

$= 4.8 + 0.2\,[12.8 – 4.8]$

$= 4.8 + 0.2 \times 8 = 4.8 + 1.6 = 6.4$

Hence, $y = 6.4$ at $x = 1.4$

8. Correct answer is [1.9].

$y = a + bx$ straight line …(i)

Normal equation of straight line

$\sum y = an + b\Sigma x$ (where $n = 4$)…(ii)

$\sum xy = a\Sigma x + b\Sigma x^2$ …(iii)

Given $a = 6.1$

x	y	x^2	xy
5	16	25	80
2	10	04	20
4	13	16	52
3	12	9	36
$\Sigma x = 14$	$\Sigma y = 51$	$\Sigma x^2 = 54$	$\Sigma xy = 188$

$51 = 4a + 14b$ …(ii)

$188 = 14a + 54b$ …(iii)

$\because$ $a = 6.1$, from (ii)

$51 = 4 \times 6.1 + 14b$

$\Rightarrow$ $51 = 24.4 + 14b$

$\Rightarrow$ $14b = 51 – 24.4 = 26.6$

$\therefore$ $b = \dfrac{26.6}{14} = 1.9$

9. Option (d) is correct.

$(1+i)^{12} = [(1+i)^2]^6 = [(1+i^2+2i)]^6$

$= [1-1+2i]^6$ $[i^2 = -1]$

$= (2i)^6 = 2^6 i^6$

$= 2^6\left(i^2\right)^3 = 2^6\left(-1\right)^3 = -2^6 = -64$

10. Option (b) is correct.

$$|A| = 0 \Rightarrow \begin{vmatrix} x & 1 & 3 \\ y & 2 & 6 \\ 3 & 5 & 7 \end{vmatrix} = 0$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 5R_1$

$$\begin{vmatrix} x & 1 & 3 \\ y-2x & 0 & 0 \\ 3-5x & 0 & -8 \end{vmatrix} = 0$$

$\Rightarrow$ $x(0-0) - 1[-8(y-2x) - 0] + 3(0-0) = 0$

$\Rightarrow$ $8(y-2x) = 0$

$\Rightarrow$ $8y – 16x = 0$

$\Rightarrow$ $y = 2x$

$\therefore$ $y = 2x = t$

$y = t = 2x = t$

$\because$ $\dfrac{y}{2} = x = t$ (say)

$\therefore$ $\dfrac{y}{2} = t$ $\Rightarrow$ $y = 2t$ and $x = t$

Ordered pair is $(x, y) = (1, 2)$

11. Option (a) is correct.

$f(x) = e^{-|x|} = e^{-x}$, x is real value

$$\left[\because\ |x| = \begin{cases} -x & x < 0 \\ x & x \ge 0 \end{cases}\right]$$

$\therefore$ Differentiate with respect to x

$\Rightarrow \dfrac{df}{dx} = -e^{-x}$

At $x = -1$ put $\left(\dfrac{dt}{dx}\right)_{at\,x=-1} = -1e^{+1} = -e$

12. Option (a) is correct.

$f(x) = x^e e^{-x}$

The necessary condition for maxima and maxima

$\dfrac{df}{dx} = 0$ ie $f'(x) = 0$ …(i)

$\Rightarrow \dfrac{df}{dx} = f'(x) = x^e\cdot\left(-e^{-x}\right) + e^{-x}\cdot ex^{e-1}$

$\therefore f'(x) = 0$ $\Rightarrow$ $\dfrac{-x^e}{e^x} + \dfrac{ex^e}{e^x x} = 0$

$$\Rightarrow \frac{-x^e}{e^x} = \frac{-ex^e}{e^x x} \Rightarrow 1 = \frac{e}{x}$$

$\therefore \quad x = e$

13. Correct answer is [1].

Given partial differential equation

$$\frac{\partial u}{\partial t} = \frac{1}{\pi^2}\frac{\partial^2 u}{\partial x^2} \qquad \ldots(i)$$

When $u = u\,(x, t)$

We have by Laplace transform

$L\{f(t)\} = F(S)$

$L\{u(x, t)\} = \bar{u}\,(x, s) = \bar{u}$

Taking Laplace Transform both sides

$$L\left\{\frac{\partial u}{\partial t}\right\} = \frac{1}{\pi^2} L\left\{\frac{\partial^2 u}{\partial x^2}\right\}$$

$$s\bar{u}(x, s) - u(x, 0) = \frac{1}{\pi^2}\frac{\partial^2 \bar{u}}{\partial x^2}$$

$$\frac{1}{\pi^2}\frac{d^2\bar{u}}{dx^2} - s\bar{u} = -u(x,\, 0) = -\sin \pi x$$

$$\therefore \frac{d^2\bar{u}}{dx^2} - \pi^2 s\bar{u} = -\pi \sin \pi x \qquad \ldots(ii)$$

This is second order ordinary differential equations.

C.S = C.F. + P.I

$(D^2 - \pi^2 s)\bar{u} = -\pi \sin \pi x$

$$C.F = c_1 e^{\pi\sqrt{s}x} + c_2 e^{-\pi\sqrt{s}x}$$

$$P.I = \frac{-\pi \sin \pi x}{D^2 - \pi^2 s} \quad D^2 = -a^2 = -\pi^2$$

$$= \frac{-\pi \sin \pi x}{-\pi^2 - \pi^2 s} = \frac{1}{\pi}\frac{\sin \pi x}{(s+1)}$$

$$\bar{u} = C \cdot F + P \cdot I = c_1 e^{\pi\sqrt{s}} + c_2 e^{-\pi\sqrt{s}} + \frac{1}{\pi}\frac{\sin \pi x}{(s+1)} \qquad \ldots(iii)$$

To evaluate c_1, c_2 we are given that

given condition $u(0, t) = 0,\ u(1, t) = 0.$

$\Rightarrow L\{u(0, t)\} = 0 \quad and \quad L\{u(1, t)\} = 0$

$\Rightarrow \bar{u}(0, s) = 0 \quad and \quad \bar{u}(1, s) = 0$

$$\bar{u}(x, s) = c_1 e^{\pi\sqrt{s}x} + c_2 e^{-\pi\sqrt{s}x} + \frac{1}{\pi}\frac{\sin \pi x}{(s+1)} \qquad \ldots(iii)$$

Put $x = 0 \qquad \bar{u}(0, s) = 0$

$$0 = c_1 e^{\pi\sqrt{s}\times 0} + c_2 e^{-\pi\sqrt{s}\times 0} \qquad \ldots(iv)$$

$$c_1 + c_2 = 0$$

Put $x = 1 \qquad \bar{u}(1, s) = 0$

$$0 = c_1 e^{\pi\sqrt{s}} + c_2 e^{-\pi\sqrt{s}} + \frac{1}{\pi}\frac{\sin \pi}{(s+1)}$$

$$\therefore\ c_1 e^{\pi\sqrt{s}} + c_2 e^{-\pi\sqrt{s}} = 0 \qquad \ldots(v)$$

From (iv) and (v)

$$c_1 = c_2 = 0$$

$$\bar{u}(x \cdot s) = \frac{1}{\pi}\frac{\sin \pi x}{(s+1)}$$

Taking L^{-1} both sides

$$L^{-1}\left\{\bar{u}(x \cdot s)\right\} = \frac{1}{\pi}\sin \pi x\, L^{-1}\left\{\frac{1}{s+1}\right\}$$

$$u(x \cdot t) = \frac{1}{\pi}\sin \pi x e^{-t}$$

Given that

$$\frac{u\left(\frac{1}{2}, t\right)}{u\left(\frac{1}{2}, 0\right)} = \frac{1}{e}$$

$$\Rightarrow \frac{\frac{1}{\pi}\sin\frac{\pi}{2}e^{-t}}{\frac{1}{\pi}\sin\frac{\pi}{2}e^{0}} = \frac{1}{e}$$

$$\Rightarrow \frac{e^{-t}}{1} = \frac{1}{e}$$

$$\Rightarrow \frac{1}{e^t} = \frac{1}{e^1}$$

$\Rightarrow\ t = 1$

14. Correct answer is [4.24].

We have by the directional derivative of f at the point (1, 1, 1) in the direction of $\bar{v} = \hat{i} - \hat{k}$ is

$$\frac{df}{ds} = \text{grad } f \cdot \hat{v} \qquad \text{...(i)}$$

$$\text{grad } f = \nabla f = \left(\frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}\right)$$

grad $f = \left(8x\hat{i} + 4y\hat{j} + 2z\hat{k}\right)$ at point (1, 1, 1)

$(\text{grad } f)_{(1,1,1)} = 8\hat{i} + 4\hat{j} + 2\hat{k}$

Direction vector $\hat{v} = \hat{i} - 0\hat{j} - \hat{k}$

$$\therefore \quad \hat{v} = \frac{\bar{v}}{|\bar{v}|} = \frac{\hat{i} - 0\hat{j} - \hat{k}}{\sqrt{1+1}} = \frac{\hat{i} - \hat{k}}{\sqrt{2}} \quad \ldots(ii)$$

Directional derivative of f

$$\frac{df}{ds} = \left(8\hat{i} + 4\hat{j} + 2\hat{k}\right) \cdot \left(\frac{\hat{i} - \hat{k}}{\sqrt{2}}\right) = \frac{8-2}{\sqrt{2}} = \frac{6}{\sqrt{2}}$$

$$\therefore \quad \frac{df}{ds} = \frac{6}{\sqrt{2}} = 3\sqrt{2} = 4.24$$

15. Correct answer is [192].

$$\iint_s \frac{(2x\hat{i} + 3y\hat{j} + 4z\hat{k})}{4\pi} \hat{n} ds$$

We have by Gauss Divergence Theorem, Relation between surface volume integral and volume integral

$$\iint_s \vec{F} \cdot \hat{n} ds = \iiint_v div\vec{F}\, dv \quad \ldots(i)$$

where $\vec{F} = \dfrac{2x\hat{i} + 3y\hat{j} + 4z\hat{k}}{4\pi}$

$$= \iiint_v \nabla \cdot \vec{F}\, dv = \frac{1}{4\pi}\iiint_v \left(\frac{\partial}{\partial z}(2x) + \frac{\partial}{\partial y}(3y) + \frac{\partial}{\partial z}(4z)\right) dv$$

$$= \frac{1}{4\pi}\iiint_v (2+3+4)\, dv$$

$$= \frac{1}{4\pi} \times 9 \times \iiint_v dv = \frac{9}{4\pi} \times \text{ volume of sphere.}$$

$$= \frac{9}{4\pi} \times \frac{4}{3}\pi(4)^3 = 64 \times 3 = 192$$

Hence, $\iint_s \left(\dfrac{2x\hat{i} + 3y\hat{j} + 4z\hat{k}}{4\pi}\right) \hat{n}\, ds = 192$

16. Correct answer is [0.857].

Given differential equation

$$\frac{dy}{dx} = xy^2 + 2y + x - 4.5$$

Initial condition $y(0) = 1$, $h = 0.5$

Given $x_1 = x_0 + h = 0 + 0.5 = 0.5$

By Implicit Euler Method

$$y_1 = y_0 + h\, f(x_1, y_1)$$

$$\int_0^{0.5} dy = \int_0^{0.5} (xy^2 + 2y + x - 4.5)\, dx$$

$$y(0.5) - y(0) = \int_0^{0.5} (xy^2 + 2y + x - 4.5)\, dx$$

x	0	0.5
$f(x)$	–2.5	2

By Trapezoidal Rule

$$y(0.5) = y(0) + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1)\right]$$

$$= 1 + \frac{0.5}{2}[-2.5 + 2]$$

$$= 1 + \frac{0.5}{2} \times (-0.5)$$

$$= 1 - \frac{0.25}{2} = \frac{7}{8} = 0.875$$

17. Option (a, b) are correct.

$$\text{Let } y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_{2022} \end{bmatrix}_{2022\times 1}$$

$$\text{Then } y^T = \begin{bmatrix} y_1 & y_2 & y_3 \cdots & y_{2022} \end{bmatrix}_{1\times 2022}$$

$$yy^T = \begin{bmatrix} y_1^2 & y_1y_2 & y_1y_3 & \cdots & y_1y_{2022} \\ y_2y_1 & y_2^2 & y_2y_3 & \cdots & y_2y_{2022} \\ y_3y_1 & \ddots & & & \\ \vdots & & \ddots & & \\ y_{2022}y_1 & & & \ddots & y_{2022}^2 \end{bmatrix}$$

For $yy^T = [z_{ij}]_{2022\times 2022}$

Clearly $z_{ij} = z_{ji}$ for all i,j of the matrix so yy^T is symmetric matrix

Option (a) – True

Now $y^T y = [y_1^2 + y_2^2 + y_3^2 + \ldots + y^2{}_{2022}]_{1\times 1}$ …(i)

Let λ be eigen value of yy^T. Then

$(yy^T)\, x = \lambda\, x$

Then $y^T y(y^T x) = \lambda(y^T x)$ (pre multiplying by y^T)

$(y^T y)u = \lambda u$ for $y^T x = u$

So eigenvalue of yy^T is same as $y^T y$ and from (i) as $y^T y$ is (1×1) the matrix is scalar and eigen value for itself.

Hence $y^T y$ is one of the eigen value of yy^T, and other eigen values are zero for yy^T

Option (b) – TRUE

Also rank of yy^T = rank of y = rank of y^T

= 1 $\because$ y is a non-zero column matrix

Option (c) – False

As rank of yy^T is less than 2022, it is a singular matrix which makes it non-invertible.

Option (d) – False

18. Correct answer is [0.019].

Experiment: pair of six faced dice is rolled thrice

$n(S) = 6 \times 6 = 36$

Event: sum of the outcome is 4

$E = \{(1, 3), (3, 1) (2, 2)\}$

$n(E) = 3$

For success

$$p = P(E) = \frac{n(E)}{n(S)}$$

$$= \frac{3}{36} = \frac{1}{12}$$

For failure

$$q = 1 - p = 1 - \frac{1}{12} = \frac{11}{12}$$

P(exactly two of the three attempts the sum is 4)

$$= {}^3C_2 p^2 q$$

$$= 3\left(\frac{1}{12}\right)^2 \frac{11}{12}$$

$$= 0.019$$

19. Option (b) is correct.

For $f(x) = x^3 - 6x^2 + 11x - 6$

$$f'(x) = 3x^2 - 12x + 11$$

$$f''(x) = 6x - 12$$

Here $f(1) = 1^3 - 6(1)^2 + 11(1) - 6$

$f(1) = 0$

$f(3) = 3^3 - 6(3)^2 + 11(3) - 6$

$f(3) = 0$

so, $x = 1$, $x = 3$ are zeros of $f(x)$

Statement I - True

For stationary point

$$f'(x) = 0$$

$$3x \quad - 12x + 11 = 0$$

$$\Rightarrow x = \frac{12 \pm \sqrt{12}}{2(3)}$$

$x = 2 \pm \frac{1}{\sqrt{3}}$ both lies within [1, 3]

$$f''\left(2 + \frac{1}{\sqrt{3}}\right) = 6\left(2 + \frac{1}{\sqrt{3}}\right) - 12$$

$$f''\left(2 + \frac{1}{\sqrt{3}}\right) > 0$$

So $x = 2 + \frac{1}{\sqrt{3}}$ is a minima

Statement (IV) true

$$f''\left(2 - \frac{1}{\sqrt{3}}\right) = 6\left(2 - \frac{1}{\sqrt{3}}\right) - 12$$

$$f''\left(2 - \frac{1}{\sqrt{3}}\right) < 0$$

So $x = 2 - \frac{1}{\sqrt{3}}$ is a maxima

Statement (II) is true

Also for $f''(x) = 6x - 12$

For $f''(x) = 0$

$\Rightarrow$ $6x - 12 = 0$

$\Rightarrow$ $x = 2$

$\therefore$ $f''(x) < 0$ for $x < 2$

$f''(x) > 0$ for $x > 2$

Statement (III) is false

20. Options (a, b) are correct.

If P and Q are two non-singular or invertible matrix, then by matrix inverse property

$$(PQ)^{-1} = Q^{-1}P^{-1}$$

And $(QP)^{-1} = P^{-1}Q^{-1}$

Also for singular matrices P and Q, PQ will also be singular

Option (d) is false

21. Option (a) is correct.

As $\log(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$

$$I = \int\left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\right) dx$$

$$= \int \log(1+x)\,dx$$

For $1 + x = u$

$dx = du$

$$\therefore \quad I = \int \log u \, du$$

$$= \log u \cdot \int du - \int\left(\frac{1}{u}\left(\int du\right)\right) du \qquad \text{by parts}$$

$$= \log u \cdot \int du - \int\left(\frac{1}{u}\left(\int du\right)\right) du \qquad \text{by parts}$$

$$= \log u \cdot u - \int \frac{1}{u} \cdot u \, du$$

$$= u \log u - \int du$$

$$= u \log u - u$$

$$= u(\log u - 1)$$

$$I = (1 + x)\{\log(1 + x) - 1\}$$

(Substituting back $u=1+x$)

22. Option (c) is correct.

For $f(x, y)$ satisfying Laplace equation

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$$

Or in polar coordinates

$$f_{rr} + \frac{1}{r} f_r + \frac{1}{r^2} f_{\theta\theta} = 0$$

For the boundary condition with $r = 3$, $0 \le \theta \le 2\pi$

$f(x, y) = 3$

This is possible only if $f(x, y)$ is a constant function.

So, $f(x, y) = 3$

And $f(0, 0) = 3$

23. Option (b) is correct.

Given $A_{3\times3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ with the condition

$$a_{ij} = (i-j)^3$$

$$\therefore \quad A = \begin{bmatrix} (1-1)^3 & (1-2)^3 & (1-3)^3 \\ (2-1)^3 & (2-2)^3 & (2-3)^3 \\ (3-1)^3 & (3-2)^3 & (3-3)^3 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & (-1)^3 & (-2)^3 \\ 1^3 & 0 & (-1)^3 \\ 2^3 & 1^3 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & -1 & -8 \\ 1 & 0 & -1 \\ 8 & 1 & 0 \end{bmatrix}$$

With $a_{ij} = -a_{ji}$ for all values of i & j in $A_{3\times3}$ it is a skew-symmetric matrix.

24. Option (c) is correct.

For exponential of square matrix A

$$e^A = 1 + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \cdots$$

Now with λ eigenvalue v eigenvector for A

$$Av = \lambda v$$

Then $A^2 v = A(A\, v) = A \cdot \lambda v = \lambda \cdot Av = \lambda(\lambda v) = \lambda^2 v$

Similarly $A^n v = \lambda^n v$

$$\text{So, } e^A v = v + Av + \frac{A^2}{2!} v + \frac{A^3}{3!} v + \cdots$$

$$= v + \lambda v + \frac{\lambda^2}{2!} v + \frac{A^3}{3!} v + \cdots$$

$$e^A v = e^\lambda \cdot v \qquad \text{...(i)}$$

As we know if v is an eigenvector for an eigenvalue & then $e^{At} v = e^{\lambda t} v$

Hence from (i) e^λ is eigenvalue and v is eigenvector for e^A.

25. Option (a) is correct.

Let $g(t) = e^t (t-1)(t-2)$,

Clearly $g(t)$ is continuous in open interval.

By second fundamental theorem of calculus

For $f(x) = \int_0^x g(t)\, dt$

We get $f'(x) = g(x)$

$$\therefore \quad f'(x) = e^x (x-1)(x-2)$$

Now for $f(x)$ to decrease in an interval

$$f'(x) < 0$$

$$i.e \quad e^x (x-1)(x-2) < 0$$

$$\Rightarrow \quad (x-1)(x-2) < 0 \qquad \because e^x > 0 \text{ for all } x.$$

It's clear for $(x - 1)(x - 2) < 0$

$$x \in (1, 2)$$

So, $f(x)$ decreases in interval (1, 2).

26. Option (a) is correct.

For given matrix A, the characteristic equation is given

$|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 4-\lambda & -2 \\ 0 & 1 & 1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)\begin{vmatrix} 4-\lambda & -2 \\ 1 & 1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)\{(4-\lambda)(1-\lambda)-(-2)(1)\} = 0$$

$$\Rightarrow (1-\lambda)\{\lambda^2 - 5\lambda + 4 + 2\} = 0$$

$$\Rightarrow (1-\lambda)(\lambda^2 - 5\lambda + 6) = 0$$

$$\Rightarrow \lambda^2 - 5\lambda + 6 - \lambda^3 + 5\lambda^2 - 6\lambda = 0$$

$$\Rightarrow \lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$$

So, by Clayley-Hamilton Theorem, we have

$$A^3 + 6A^2 + 11A - 6 = 0$$

$$\Rightarrow A^3 + 6A^2 + 11A = 6 \quad \text{...(i)}$$

As it is given

$$6A^{-1} = A^2 + cA + dI$$

$$\Rightarrow 6 = A^3 + cA^2 + dA \quad \text{...(ii)}$$

[On multiplying by matrix A on both sides.]

Comparing (i) & (ii)

$c = -6, d = 11$

$\therefore \ c + d = -6 + 11 = 5$

27. Option (b) is correct.

As we know gradient of a scalar field function, points in the direction of greatest increase of function. This direction is steepest ascent.

$$\nabla f = \frac{df}{dx}\hat{i} + \frac{df}{dy}\hat{j} + \frac{df}{dz}\hat{k}$$

$$\nabla f = (8x + 7y + 3z^2)\hat{i} + 7x\hat{j} + 6xz\hat{k}$$

Putting (1, 0, 2)

$$(\nabla f)_p = \left(8\times1 + 7\times0 + 3\times2^2\right)\hat{i} + 7\times1\hat{j} + 6\times1\times2\hat{k}$$

$$(\nabla f)_p = 20\hat{i} + 7\hat{j} + 12\hat{k}$$

So, $(20\hat{i} + 7\hat{j} + 12\hat{k})$ is the direction in which $f(x, y, z)$ increases most rapidly at P (1, 0, 2).

28. Option (c) is correct.

As we know $\iint_R f(x,y)\,dA$ for $f(x,y) = 1$ becomes

$\iint_R dx\,dy \qquad \because dA = dx\,dy$ in cartesian plane

= Area of the bounded region R.

Option (a) is true

As per Green's theorem

$$\oint_C P\,dx + Q\,dy = \iint_R \left(\frac{dQ}{dx} - \frac{dP}{dy}\right) dA$$

For $\oint_C x\,dy$ with $P = 0,\ Q = x$

$$\oint_C x\,dy = \iint_R \left(\frac{dx}{dx} - \frac{d0}{dy}\right) dA = \iint_R dA$$

= Area of the bounded region R.

Option (b) is true

For $\oint_C y\,dx$ with P = y, Q = 0

$$\oint_C y\,dx = \iint_R \left(\frac{d0}{dx} - \frac{dy}{dy}\right) dA = -\iint_R dA$$

= –(Area of the bounded region R)

Option (c) is false

For $\frac{1}{2}\oint_C (x\,dy - y\,dx)$ with P = $-y$, Q = x

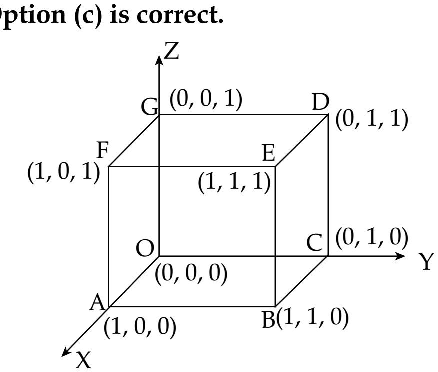
$$\frac{1}{2}\oint_C (x\,dy - y\,dx) = \frac{1}{2}\iint_R \left(\frac{dx}{dx} - \frac{d(-y)}{dy}\right) dA$$

$$= \frac{1}{2}\iint_R (1-(-1))\,dA = \iint_R dA$$

= Area of the bounded region R.

Option (d) is true

29. Option (c) is correct.



For $\vec{E} = 2x^2\hat{i} + 5y\hat{j} + 3z\hat{k}$

$$\nabla\vec{E} = \frac{d2x^2}{dx} + \frac{d5y}{dy} + \frac{d3z}{dz}$$

$= 4x + 5 + 3$

$\therefore \nabla\vec{E} = 4x + 8$

$$\iiint_V \nabla\cdot\vec{E}\,dv = \int_{z=0}^{1}\int_{y=0}^{1}\int_{x=0}^{1}(4x+8)\,dx\,dy\,dz$$

$$= \int_{z=0}^{1}\int_{y=0}^{1}\left(\left[2x^2+8x\right]\Big|_0^1\right)dy\,dz$$

$$= \int_{z=0}^{1}\int_{y=0}^{1}10\,dy\,dz$$

$$= 10\int_{z=0}^{1}\left([y]\big|_0^1\right)dz$$

$$= 10\int_{z=0}^{1}dz$$

$$= 10\ [z]\big|_0^1$$

$$= 10$$

so, $\iiint_v \left(\nabla\cdot\vec{E}\right)dv = 10$

30. Option (c) is correct.

For any two real or complex matrices

$P_{m\times n}$, $Q_{n\times m}$

Trace (PQ) = Trace (QP)

On this basis tr (AB) = tr (BA)

tr (CD) = tr (DC)

So both the statements are correct.

31. Correct answer is [7].

Here all six balls are identical and all three bins are also identical

So total number of arrangements is only about grouping these six balls in different count. This could be

(6, 0, 0), (5, 1, 0), (4, 1, 1), (4, 2, 0), (3, 3, 0), (3, 1, 2), (2, 2, 2),

So possibly 7 different arrangements.

32. Correct answer is [–0.5].

For $x\to 0^+$ function takes $\frac{0}{1-1}$ *i.e* $\frac{0}{0}$ form

So applying L Hospital rule

$$\lim_{x\to 0^+}\frac{\frac{1}{2}\cdot\frac{1}{\sqrt{x}}}{0-2\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{x}}e^{2\sqrt{x}}} = \lim_{x\to 0^+}\frac{-1}{2}\cdot e^{-2\sqrt{x}}$$

$$= \frac{-1}{2} = -0.5$$

33. Option (d) is correct.

For given set of simultaneous equation

Ax = B

$$\text{For } A = \begin{bmatrix}1 & 1 & -2\\ 1 & 3 & -1\\ 2 & 1 & -5\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ l_{21} & 1 & 0\\ l_{31} & l_{32} & 1\end{bmatrix}\begin{bmatrix}u_{11} & u_{12} & u_{13}\\ 0 & u_{22} & u_{23}\\ 0 & 0 & u_{33}\end{bmatrix}$$

By Guess elimination method for A

$R_2 \to R_2 - 1R_1 \qquad \because l_{21} = 1$

$$\begin{bmatrix}1 & 1 & -2\\ 0 & 2 & 1\\ 2 & 1 & -5\end{bmatrix}$$

$R_3 \to R_3 - 2R_1 \qquad \because l_{31} = 2$

$$\begin{bmatrix}1 & 1 & -2\\ 0 & 2 & 1\\ 0 & -1 & -1\end{bmatrix}$$

$R_3 \to R_3 - \left(\frac{-1}{2}\right)R_2 \qquad \because l_{32} = -1$

$$\begin{bmatrix}1 & 1 & -2\\ 0 & 2 & -1\\ 0 & 0 & -\frac{1}{2}\end{bmatrix}$$

$$\text{So } U = \begin{bmatrix}1 & 1 & -2\\ 0 & 2 & 1\\ 0 & 0 & \frac{-1}{2}\end{bmatrix}$$

$$\text{Similarly, } L = \begin{bmatrix}1 & 0 & 0\\ 1 & 1 & 0\\ 2 & \frac{-1}{2} & 1\end{bmatrix}$$

So $u_{33} = \frac{-1}{2}$ and $l_{32} = \frac{-1}{2}$

As Ax = B

$\Rightarrow$ LU $x = b$

$\Rightarrow$ L$y = b$ for U$x = y$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & \frac{-1}{2} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \\ 7 \end{bmatrix}$$

By forward substitution

$y_1 = 4, y_2 = 3, \; y_3 = \frac{1}{2}$

Again

$$\begin{bmatrix} 1 & 1 & -2 \\ 0 & 2 & 1 \\ 0 & 0 & \frac{-1}{2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ \frac{1}{2} \end{bmatrix}$$

By back substitution

$x_3 = -1$

$x_2 = 2$

$x_1 = 0$

So $l_{32} = \frac{-1}{2}, u_{33} = \frac{-1}{2}, x_1 = 0$

34. Options (a, c, d) are correct.

Let the given matrix be A, then we know

$Ax = \lambda x$ where $\lambda =$ eigenvalue and

x = eigenvector

Option (a)

$$Ax = \begin{bmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix} = 1 \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$\therefore \; Ax = \lambda x$ with $\lambda = 1$

Option (b)

$$Ax = \begin{bmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$$

$= \begin{bmatrix} -7 \\ -5 \\ 12 \\ 25 \end{bmatrix}$ is not equal to λx for any real value

Option (c)

$$Ax = \begin{bmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 2 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} -3 \\ 0 \\ 6 \\ 6 \end{bmatrix} = 3 \begin{bmatrix} -1 \\ 0 \\ 2 \\ 2 \end{bmatrix}$$

$\therefore \; Ax = \lambda x$ with $\lambda = 3$

Option (d)

$$Ax = \begin{bmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ -3 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 3 \\ -9 \\ 0 \end{bmatrix} = 3 \begin{bmatrix} 0 \\ 1 \\ -3 \\ 0 \end{bmatrix}$$

$\therefore \; Ax = \lambda x$ with $\lambda = 3$

35. Option (a) is correct.

For the given $\vec{F}(x, y) = x\hat{i} + y\hat{j}$

$P = x, Q = y$

By Green's theorem

$$\oint_C \vec{F}(x, y)(dx\hat{i} + dy\hat{j}) = \iint_R \left(\frac{dQ}{dx} - \frac{dP}{dy} \right) dx\, dy$$

$$= \iint_R \left(\frac{dy}{dx} - \frac{dx}{dy} \right) dx\, dy$$

$$= \iint_R 0 \; dxdy = 0$$

Alternative:

For $\vec{F}(x,y) = x\hat{i} + y\hat{j}$

$\vec{F}(x,y) = \frac{x^2}{2} + \frac{y^2}{2}$ could be potential fuction

So that $\vec{F} = \frac{df}{dx}\hat{i} + \frac{df}{dy}\hat{j}$, which makes the field conservative

$$\oint_C \vec{F}(dx\hat{i} + dy\hat{j}) = 0$$

36. Option (c) is correct.

For system of linear equation Ax = b

[A|b] will be

$$\left[\begin{array}{ccc:c} 1 & -\sqrt{2} & 3 & 1 \\ -1 & \sqrt{2} & -3 & 3 \end{array}\right]$$

For rank of A $\because$ Row 1 = –Row 2

The rows are linearly dependent.

$\therefore$ rank A = 1

For rank of [A|b], second order minor

$$\begin{vmatrix} 1 & 1 \\ -1 & 3 \end{vmatrix} = 3\times 1 - (-1)1 = 4 \neq 0$$

$\therefore$ rank of [A|b] = 2

So, we have rank of A $\neq$ rank of [A|b]

Hence system of linear equation has no solution for x.

37. Options (a, b) are correct.

For given $f(x,y) = e^{\xi x + \eta y}$

$$\frac{\partial f}{\partial x} = \xi \cdot e^{(\xi + \eta y)}$$

$$\frac{\partial^2 f}{\partial x^2} = \xi^2 \cdot e^{(\xi x + \eta y)}$$

Also, $\frac{\partial f}{\partial y} = \eta e^{(\xi x + \eta y)}$

$$\frac{\partial^2 f}{\partial y^2} = \eta^2 e^{(\xi x + \eta y)}$$

For given PDE

$$a\frac{\partial^2 f(x,y)}{\partial x^2} + b\frac{\partial^2 f(x,y)}{\partial y^2} = f(x,y)$$

$$\Rightarrow \quad a\,\xi^2 e^{(\xi x + \eta y)} + b\,\eta^2 e^{(\xi x + \eta y)} = e^{(\xi x + \eta y)}$$

$$\Rightarrow \quad a\,\xi^2 + b\eta^2 = 1 \qquad \because e^{\xi x + \eta y} \neq 0$$

This is a general equation of ellipse.

$$\frac{\xi^2}{\left(\frac{1}{\sqrt{a}}\right)^2} + \frac{\eta^2}{\left(\frac{1}{\sqrt{b}}\right)^2} = 1$$

Both option (a) and (b) satisfies this equation.

38. Options (a, c) are correct.

Option (a) : $f(x,t) = e^{-(x-100t)^2} + e^{-(x+100t)^2}$

$$\frac{\partial^2 f(x,t)}{\partial x^2} = e^{-x^2+200tx-10000t^2}(-2x+200t)^2$$
$$-2e^{-x^2+200tx-10000t^2} + e^{-x^2-200tx-10000t^2}$$
$$(-2x-200t)^2 - 2e^{-x^2-200tx-10000t^2}$$

$$\frac{\partial^2 f(x,t)}{\partial t^2} = e^{-x^2+200xt-10000t^2}(-20000t+200x)^2$$
$$-20000e^{-x^2+200xt-10000t^2}$$
$$+e^{-x^2-200xt-10000t^2}(-20000t-200x)^2$$
$$-20000e^{-x^2-200xt-10000t^2}$$

Substituting the values of $\frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial^2 f}{\partial t^2}$ in given equation, we get

$$\frac{\partial^2 f(x,t)}{\partial t^2} = 10000\frac{\partial^2 f(x,t)}{\partial x^2}$$

Option (a) is satisfied

Option (b): $f(x,t) = e^{-(x-100t)} + 0.5e^{-(x+1000t)}$

$$\frac{\partial^2 f(x,t)}{\partial x^2} = e^{-x+100t} + 0.5e^{-x-1000t}$$

$$\frac{\partial^2 f(x,t)}{\partial x^2} = 10000e^{-x+100t} + 500000e^{-x-1000t}$$

Substituting the values of $\frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial^2 f}{\partial t^2}$ in given equation, we get

$$\frac{\partial^2 f(x,t)}{\partial t^2} \neq 10000\frac{\partial^2 f(x,t)}{\partial x^2}$$

$\therefore$ Option (b) is not satisfied

Option (c): $f(x,t) = e^{-(x-100t)} + \sin(x+100t)$

$$\frac{\partial^2 f(x,t)}{\partial t^2} = e^{-x+100t} - \sin(x+100t)$$

$$\frac{\partial^2 f(x,t)}{\partial t^2} = 10000e^{-x+100t} - 10000\sin(x+100t)$$

Substituting the values of $\frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial^2 f}{\partial t^2}$ in given equation, we get

$$\frac{\partial^2 f(x,t)}{\partial t^2} = 1000\frac{\partial^2 f(x,t)}{\partial x^2}$$

Option (c) is satisfied

Option (d): $f(x,t) = e^{j100\pi(-100x+t)} + e^{j100\pi(100x+t)}$

$$\frac{\partial^2 f(x,t)}{\partial x^2} = 1000\pi^2 j^{200} e^{\pi_j 100(-100x+t)} + 1000\pi^2 j^{200} e^{\pi_j 100(100x+t)}$$

$$\frac{\partial^2 f(x,t)}{\partial t^2} = \pi^2 j^{200} e^{\pi_j 100(-100x+t)} + \pi^2 j^{200} e^{\pi_j 100(100x+t)}$$

Substituting the values of $\frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial^2 f}{\partial t^2}$ in given equation, we get

$$\frac{\partial^2 f(x,t)}{\partial t^2} \neq 10000\ \frac{\partial^2 f(x,t)}{\partial x^2}$$

$\therefore$ Option (d) is not satisfied.

39. Correct answer is [4].

Total number of matches played is given by sum of the frequency

$$N = \sum f$$

$$= 5+7+8+25+20+17+8+4+3+2+1$$

N = 100 {even value}

No. of wickets	Frequency	Cumulative Frequency
0	5	5
1	7	12
2	8	20
3	25	45
4	20	65
5	17	82
6	8	90
7	4	94
8	3	97
9	2	99
10	1	100

For N being even

$$\text{Median } = \frac{1}{2}\left\{\frac{N^{th}}{2}\text{term} + \left(\frac{N}{2}+1\right)^{th}\text{ term}\right\}$$

$$= \frac{1}{2}(50^{th}\text{ term} + 51^{st}\text{ term})$$

$$= \frac{1}{2}\{4+4\}$$

$$= 4$$

40. Correct answer is [0.5].

For $g(z) = \frac{2^z}{z^2-1}$, we have poles at $z = 1, z = -1$

Here $z = 1$ lies outsides the contour and $z = -1$ lies inside the contour.

So, $f(z) = \frac{2^z}{z-1}$ is analytical in C and $z_0 = -1$ lies within C.

By Cauchy's Integral formula

$$\oint \frac{f(z)}{z-z_0}dz = 2\pi i\, f(z_0)$$

$$\Rightarrow \oint \frac{\frac{2^z}{z-1}}{z-(-1)}dz = 2\pi i\, f(-1)$$

$$= 2\pi i\left(\frac{2^{-1}}{-1-1}\right)$$

$$= 2\pi i \cdot \frac{1}{2}\cdot\frac{1}{-2}$$

$$= -\frac{\pi i}{2}$$

Given $\oint \frac{2^z}{z^2-1}dz = -i\ \pi\ A$

$$\therefore\quad -i\ \pi\ A = -i\ \pi \cdot \frac{1}{2}$$

Hence, $A = \frac{1}{2} = 0.5$

41. Option (b) is correct. logex

For $f(x) = 8\log_e x - x^2 + 3$

$$f'(x) = \frac{8}{x} - 2x + 0,$$

$f'(x)$ is not defined at $x = 0$, which is not in $[1,e]$

For $f'(x) = 0$,

$$\frac{8}{x} - 2x = 0$$

$\Rightarrow\ 8 = 2x^2$

$\Rightarrow\ 4 = x^2$

or $x = \pm 2$

$x = -2$ is not in $[1, e)$, $x = 2$ is in $[1, e]$

Checking $f(x)$ at end points and critical points with $[1, e]$

$f(1) = 8 \log_e 1 - 1^2 + 3 = 2$

$f(e) = 8 \log_e e - e^2 + 3 = 3.61$

$f(2) = 8 \log_e 2 - 2^2 + 3 = 4.54$

Here, $f(x) = 2$ is minimum at $x = 1$.

42. Option (a) is correct.

Let $v_1 = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$, $\quad v_2 = \begin{bmatrix} d \\ e \\ f \end{bmatrix}$

$$v_1^{\mathrm{T}} \cdot v_2 = \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} d \\ e \\ f \end{bmatrix} = ad + be + cf$$

$\because \quad v_1^{\mathrm{T}} v_2 = 0 \quad$ (given)

$\therefore \quad ad + be + cf = 0 \quad$ ...(i)

Now, $v_2^{\mathrm{T}} \cdot v_1 = \begin{bmatrix} d & e & f \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = da + eb + fc$

$\therefore \quad v_2^{\mathrm{T}} v_1 = 0 \quad$ [from (i)]

Given, $\mathrm{A} = \alpha\, v_1 \cdot v_1^{\mathrm{T}} + \beta v_2 \cdot v_2^{\mathrm{T}} \quad$ ...(ii)

Then $\quad v_1 = \alpha v\ v_1 + \beta v_2 v_2\ v_1$

(post multiplication by v_1)

$\Rightarrow \quad \mathrm{A}v_1 = \alpha v_1 (v_1^{\mathrm{T}} v_1) + \beta v_2 (v_2^{\mathrm{T}} v_1)$

$\Rightarrow \quad \mathrm{A}v_1 = \alpha v_1 \quad \{v_1^T v_1 = 1,\ v_2^T v_1 = 0\}$

So α is an eigenvalue with v_1 as eigenvector.

Again $\mathrm{A} = \alpha\, v_1 v_1^{\mathrm{T}} + \beta v_2 \cdot v_2^{\mathrm{T}}$

$\mathrm{A}v_2 = \alpha\, v_1 v_1^{\mathrm{T}} v_2 + \beta v_2 \cdot v_2^{\mathrm{T}} \cdot v_2$

(Post multiplication by v_2)

$\mathrm{A}v_2 = \alpha\, v_1 (v_1^{\mathrm{T}} v_2) + \beta v_2 (v_2^{\mathrm{T}} v_2)$

$\mathrm{A}v_2 = \beta v_2$

So, β is an eigenvalue with v_2 as eigenvector.

Also A will be a singular matrix so, zero could be an eigenvalue.

43. Options (b, d) are correct.

For series $\sum_{n=1}^{\infty} \frac{n^d}{c^n}$

(a) $c = 1, d = -1$

Series $\sum_{n=1}^{\infty} \frac{n^{-1}}{1^n} = \sum_{n=1}^{\infty} \frac{1}{n}$

is of the form $\sum_{n=1}^{\infty} \frac{1}{n^p}$ with $p \leq 1$, therefore series diverges.

(b) $c = 2, d = 1$

Series $\sum_{n=1}^{\infty} \frac{n}{2^n}$

Ratio test

$$p = \lim_{n\to\infty} \left| \frac{a^{n+1}}{a^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(n+1)}{2^{n+1}} \cdot \frac{2^n}{n} \right|$$

$$= \frac{1}{2} \lim_{n\to\infty} \left| \frac{n+1}{n} \right|$$

$p = \frac{1}{2}$ with $0 \leq p < 1$, the series converges.

(c) $c = 0.5, d = -10$

Series $\sum_{n=1}^{\infty} \frac{n^{-10}}{0 \cdot 5^n} = \sum_{n=1}^{\infty} \frac{2^n}{n^{10}}$

Ratio test

$$p = \lim_{n\to\infty} \left| \frac{a^{n+1}}{a^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{2^{n+1}}{(n+1)^{10}} \cdot \frac{n^{10}}{2^n} \right|$$

$$= 2 \lim_{n\to\infty} \left| \left(\frac{n}{n+1} \right)^{10} \right|$$

$\therefore \quad p = 2$

With $p = 2$, the series diverges.

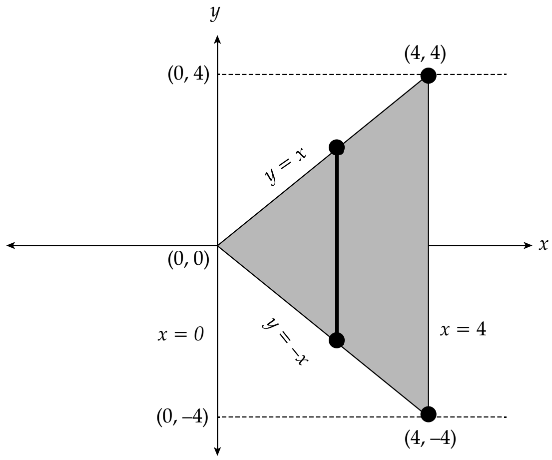
(d) $c = 1, d = -2$

Series $\sum_{n=1}^{\infty} \frac{n^{-2}}{1^n} = \sum_{n=1}^{\infty} \frac{1}{n^2}$

is of the form $\sum_{n=1}^{\infty} \frac{n^{-2}}{n^p}$ with $p > 1$, therefore series converges.

44. Correct answer is [512].

For the given triangular region



$$I = 3\int_{x=0}^{x=4}\int_{y=-x}^{y=x}(x^2+y^2)\,dy\,dx$$

$$= 3\int_{x=0}^{4}\left\{\left[x^2y+\frac{y^3}{3}\right]_{-x}^{x}\right\}dx$$

$$= 3\int_{0}^{4}\left[x^3+\frac{x^3}{3}-\left(-x^3-\frac{x^3}{3}\right)\right]dx$$

$$= 3\int_0^4\frac{8x^3}{3}dx$$

$$= 2x^4\Big|_0^4$$

$$= 512$$

45. Option (a) is correct.

Given $f(x) = x^3$ (odd function) $-1 \le x < 1$

$f(-x) = (-x)^3 = -x^3 = -f(x)$

$f(-x) = -f(x)$

We have by Fourier series for odd function

$\boxed{a_0 = 0}$ $\boxed{a_n = 0}$ $\boxed{b_n = ?}$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin m\pi x \qquad \ldots\text{(i)}$$

$$\because \left[\begin{array}{l}\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx \\ \qquad \text{if } f(x) \text{ is even function} \\ \int_{-a}^{a} f(x)\,dx = 0 \\ \qquad \text{if } f(x) \text{ is odd function}\end{array}\right.$$

Only sine terms are present.

46. Option (a) is correct.

Symmetric Matrix:

If a square matrix A is said to be Symmetric Matrix

If $\boxed{A^T = A}$ $\boxed{A^T = \text{Transpose of A}}$

Given $A = \begin{bmatrix} 10 & 2k+5 \\ 3k-3 & k+5 \end{bmatrix}$

$\Rightarrow A^T = \begin{bmatrix} 10 & 3k-3 \\ 2k+5 & k+5 \end{bmatrix}$

From equation (i)

$$A^T = A$$

$$\begin{bmatrix} 10 & 3k-3 \\ 2k+5 & k+5 \end{bmatrix} = \begin{bmatrix} 10 & 2k+5 \\ 3k-3 & k+5 \end{bmatrix}$$

Equating elements of both matrices, we get

$3k - 3 = 2k + 5$

$3k - 2k = 5 + 3$

$k = 8$

47. Option (a) is correct.

Given $p = \lim_{x\to\pi}\left[\frac{x^2+\alpha x+2\pi^2}{x-\pi+2\sin x}\right]$

For $x = \pi$, the denominator becomes zero, thus for p to have a finite value. The numerator must also be zero for $x = \pi$

At $x = \pi$ $\quad \pi^2 + \alpha\pi + 2\pi^2 = 0$

$\Rightarrow \alpha\pi + 3\pi^2 = 0$

$\Rightarrow \alpha\pi = -3\pi^2 \quad \therefore \alpha = -3\pi$

Now,

$$p = \lim_{x\to\pi}\left[\frac{x^2-3\pi x+2\pi^2}{x-\pi+2\sin x}\right]$$

$\left[\frac{0}{0}\right]$ Indeterminant form

By L'Hospital Rule.

$$p = \lim_{x\to\pi}\left[\frac{2x-3\pi}{1+2\cos x}\right] = \frac{2\pi-3\pi}{1+2\cos\pi} = \frac{-\pi}{1-2} = \pi$$

Thus, $p = \pi$

48. Option (b) is correct.

Option (a) $T(x,1) = x - x + 1 = 1 \ne 1 + x$

Option (b) $T(x,y) = x + y$

$T(x,1) = x + 1$ satisfy

Option (c) $T(x,y) = -x + y$

$T(x,1) = -x+1$ not satisfy

Option (d) $T(x,y) = x + xy + y$

$T(x,1) = x + x + 1 = 2x + 1 \neq 1 + x$ not satisfy

49. Option (a) is correct.

Given, $\varphi = \frac{1}{2}(x^2 + y^2 + z^2)$

$$\overline{\nabla}\varphi = \frac{\partial\varphi}{\partial x}\hat{i} + \frac{\partial\varphi}{\partial y}\hat{j} + \frac{\partial\varphi}{\partial z}\hat{k} = \frac{1}{2}\left(2x\hat{i} + 2y\hat{j} + 2z\hat{k}\right)$$

$$\overline{\nabla}\varphi = x\hat{i} + y\hat{j} + z\hat{k} = \vec{F}$$

We have by Gauss Divergence Theorem

$$\oiint_S \hat{n}\cdot\overline{\nabla}\varphi\, dS = \iiint_V \text{div}.\vec{F}\, dV = \iiint_V \vec{\nabla}.\vec{F}\, dV$$

$$\Rightarrow \iiint_V \vec{\nabla}.\vec{\nabla}\varphi\, dV = \iiint_V \vec{\nabla}.(x\hat{i} + y\hat{j} + z\hat{k})\, dV$$

$\Rightarrow \iiint_V (1+1+1)\, dV = 3\iiint_V 1\, dV = 3\times$ Volume of sphere

$$\Rightarrow 3\times\frac{4}{3}\pi(1)^3 = 4\pi$$

Here V is the closed region bounded by the surface S. S is the surface area of sphere of unit radius.

50. Option (a) is correct.

Let $f(z) = 2z^4 - 3z^3 + 7z^2 - 3z + 5$

Here, $f(i) = 2i^4 - 3i^3 + 7i^2 - 3i + 5$

$= 2(1) - 3(-i) + 7(-1) - 3i + 5$

$= 2 + 3i - 7 - 3i + 5$

$= 0$

$\therefore$ $z = i$ is a singular point

Thus, $\oint \frac{6z}{2z^4 - 3z^3 + 7z^2 - 3z + 5}\, dz$

$$= 2\pi i\left[\left|\text{ residue }\right|_{z=i}\right]$$

where $\text{Residue}|_{z=i} = \lim_{z\to i}(z-i)f(z)$

$$= \lim_{z\to i}(z-i)\frac{6z}{2z^4 - 3z^3 + 7z^2 - 3z + 5}$$

$$\Rightarrow \lim_{z\to i}\frac{6z^2 - 6zi}{2z^4 - 3z^3 + 7z^2 - 3z + 5}\left[\frac{0}{0}\text{form}\right].$$

By L'Hospital Rule

$$\Rightarrow \lim_{z\to i}\left[\frac{12z - 6i}{8z^3 - 9z^2 + 14z - 3}\right] = \frac{12i - 6i}{8i^3 - 9i^2 + 14i - 3}$$

$$\Rightarrow \frac{6i}{8(-i) - 9(-1) + 14i - 3} = \frac{6i}{-8i + 9 + 14i - 3}$$

$$= \frac{6i}{6 + 6i} = \frac{i}{i+1}$$

$$\therefore \oint\frac{6z\, dz}{2z^4 - 3z^3 + 7z^2 - 3z + 5}$$

$$= 2\pi i\left(\frac{i}{i+1}\right) = \frac{2\pi(-1)}{(i+1)} = \frac{-2\pi(i-1)}{(i+1)\times(i-1)}$$

$$\Rightarrow \frac{-2\pi(i-1)}{i^2 - 1} = \frac{-2\pi(i-1)}{-1-1}$$

$$= -\frac{2\pi(i-1)}{-2} = \pi(i-1)$$

51. Options (a, b) are correct.

For Non-Trivial solution, $|A| = 0$

$$\Rightarrow \begin{vmatrix} 2 & 5-2\alpha \\ \alpha & 1 \end{vmatrix} = 0 \Rightarrow 2 - \alpha(5 - 2\alpha) = 0$$

$$\Rightarrow 2 - 5\alpha + 2\alpha^2 = 0 \Rightarrow 2\alpha^2 - 5\alpha + 2 = 0$$

$$\Rightarrow 2\alpha^2 - 4\alpha - \alpha + 2 = 0 \Rightarrow 2\alpha(\alpha - 2) - 1(\alpha - 2) = 0$$

$$\Rightarrow (\alpha - 2)(2\alpha - 1) = 0 \quad \therefore \alpha = 2,\ \alpha = \frac{1}{2}$$

For $\alpha = 2$, $A = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$

Thus, $[x\ \ y]\begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix} = [0, 0]$

$\Rightarrow 2x + 2y = 0 \Rightarrow x + y = 0 \Rightarrow x = -y$

Option (a) satisfy this condition.

For $\alpha = \frac{1}{2}$ $A = \begin{bmatrix} 2 & 4 \\ \frac{1}{2} & 1 \end{bmatrix}$

Thus, $[x\ \ y]\begin{bmatrix} 2 & 4 \\ \frac{1}{2} & 1 \end{bmatrix} = [0, 0]$

$2x + \frac{1}{2}y = 0$ and $4x + 1y = 0$

$\Rightarrow x = \frac{-1}{4}y$

Thus, $y = -4x$

52. Correct answer is [0.18].

Given that P (X = 1) = P (X = 2)

We have by poison distribution

$p(r) = \frac{e^{-m}\cdot m^r}{r!}$ where $r = 0, 1, 2, 3\ldots$

P (X = 1) = P (X = 2)

$$\Rightarrow \frac{e^{-m}m}{1!} = \frac{e^{-m}m^2}{2!}$$

$$\Rightarrow \frac{m}{1} = \frac{m^2}{2} \Rightarrow m^2 = 2m$$

$$\Rightarrow m^2 - 2m = 0 \Rightarrow m(m-2) = 0 \Rightarrow m = 0, 2$$

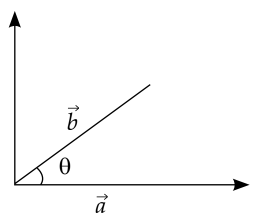
Thus, $m = 2$ (Because m cannot be zero)

$$P(X=3) = \frac{e^{-2}(2)^3}{3!} = \frac{8}{6}e^{-2} = 0.18$$

53. Correct answer is [8.32].

Given $\vec{a} = 5\hat{i} + 7\hat{j} + 2\hat{k}$

$\vec{b} = 3\hat{i} - \hat{j} + 6\hat{k}$



Magnitude of $\vec{a}$ orthogonal to $\vec{b}$ in place of $\vec{a}$ and $\vec{b}$ $= |\vec{a}|\,|\sin\theta|$

We have

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|} = \frac{(5\hat{i}+7\hat{j}+2\hat{k})\cdot(3\hat{i}-\hat{j}+6\hat{k})}{\sqrt{5^2+7^2+2^2}\cdot\sqrt{3^2+(-1)^2+6^2}}$$

$$\cos\theta = \frac{5\times3+7\times-1+2\times6}{\sqrt{78}\cdot\sqrt{46}} = \frac{20}{\sqrt{78}\cdot\sqrt{46}}$$

$$\cos\theta = \frac{20}{\sqrt{78}\cdot\sqrt{46}} \Rightarrow \theta = 70.495°$$

$$\left||\vec{a}|\sin\theta\right| = \left|\sqrt{78}\sin(70.495°)\right|$$

$$= 8.32$$

54. Option (b) is correct.

Given that $\int_{-\infty}^{\infty} e^{-x^2}dx = \sqrt{\pi}$ ...(i)

Find the value of $\int_{-\infty}^{\infty} e^{-a(x+b)^2}dx$

we have $\int_{-\infty}^{\infty} e^{[\sqrt{a}(x+b)]^2}dx$

let $\sqrt{a}(x+b) = t$

$$\Rightarrow \sqrt{a}\ dx = dt$$

$$\therefore\ dx = \frac{dt}{\sqrt{a}}$$

Thus, $\int_{-\infty}^{\infty} e^{-t^2}\frac{dt}{\sqrt{a}} \Rightarrow \frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} e^{-t^2}dt$ from equation (i)

$$= \frac{1}{\sqrt{a}}\sqrt{\pi} = \frac{\sqrt{\pi}}{\sqrt{a}}$$

$$\therefore \int_{-\infty}^{\infty} e^{-a(x+b)^2}dx = \sqrt{\frac{\pi}{a}}$$

55. Option (c) is correct.

Given, $f(x) = 2\log(\sqrt{e^x}) = \frac{1}{2}\times 2\log(e^x)$

$f(x) = x\log_e e = x$ $\quad\because \boxed{\log_e e = 1}$

We have, area bounded by $f(x)$ with in the interval [0, 2] on x-axis.

Therefore, $\int_0^2 f(x)\,dx = \int_0^2 x\,dx = \left[\frac{x^2}{2}\right]_0^2$

$$= \frac{1}{2}[2^2 - 0^2] = \frac{1}{2}\times 4 = 2$$

56. Option (b) is correct.

We have Fourier Series Expansion

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \text{ cosnt} + b_n \text{ sinnt} \quad ...(i)$$

Constant term in Fourier Series is $\frac{a_0}{2}$.

where $a_0 = \frac{1}{l}\int_0^2 f(t)\,dt = \frac{1}{1}\int_0^2 f(t)\,dt$

where $l = 1$ and period [0, 2]

$a_0 = \frac{1}{1}\int_0^2 f(t)\,dt = 1 \times$ Area under the curve with in [0, 2]

$\Rightarrow\ 1 \times 4 = 4$

Constant Term $= \frac{a_0}{2} = \frac{4}{2} = 2$

57. Option (a) is correct.

Given, $\vec{F} = 3x\hat{i} + 5y\hat{j} + 6z\hat{k}$

We have by Gauss's Divergence Theorem,

Relation between surface integral and volume integral

$$\iint_S \vec{F}\cdot d\vec{A} = \iiint_V \text{div } \vec{F}\cdot d\vec{V} \quad ...(i)$$

$$\text{div } \vec{F} = \vec{\nabla}\cdot\vec{F} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot(3x\hat{i}+5y\hat{j}+6z\hat{k})$$

$$= \frac{\partial}{\partial x}(3x) + \frac{\partial}{\partial y}(5y) + \frac{\partial}{\partial z}(6z)\ \left(\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1\right)$$

$$\text{div }\vec{F} = 3 + 5 + 6 = 14$$

Here V is the closed region bounded by the surface S.

$$\int_A \vec{F}\cdot d\vec{A} = \int_A \vec{F}\cdot\hat{n}\,dA = \iiint_v 14\,dV \quad \boxed{\overrightarrow{dA} = \hat{n}\,dA}$$

$\Rightarrow$ 14 × Volume of cube

$\Rightarrow$ $14 \times (1)^3 = 14$ [volume of a cube = a^3]

58. Option (b) is correct.

Here the function $f(x) = 4x^2 + 2x + 6$ is a polynomial of degree 2 and the simpson's 1/3rd Rule also uses a second degree polynomial for approximation.

The value of I_e and I_s will be same.

i.e $I_e = I_s$

Then we have

Percentage Error e =

$$\frac{\text{Exact Values}(I_e) - \text{Estimated Value }(I_s)}{\text{Exact Values}(I_e)} \times 100$$

Thus, $e = \left(\frac{I_e - I_e}{I_e}\right) \times 100 = 0$

59. Option (b) is correct.

Since $\varphi(s)$ has triple roots at $s = -\sigma$, $(s + \sigma)^3$ will be one of its factors and $\varphi(-\sigma) = 0$

There will be an inflection point on the curve of $\varphi(s)$ (Just like in the case of x^3), hence, the first and second order derivative at $s = -\sigma$ will be zero.

60. Option (c) is correct.

Given Differential Equation

$$\frac{du}{dx} = \frac{-xu^2}{2 + x^2u}$$

It can be written as

$$xu^2dx + (2 + x^2u)\,du = 0 \quad \text{...(i)}$$

within is in the from

$$Mdx + Ndu = 0 \quad \text{...(ii)}$$

where $M = f_1(x, u)$ and $N = f_2(x, u)$

Comparing $M = xu^2$ and $N = 2 + x^2u$

We check the condition of exactness

$$\frac{\partial M}{\partial u} = \frac{\partial N}{\partial x}$$

$$\frac{\partial M}{\partial u} = 2xu \quad \text{...(iii)}$$

and $\frac{\partial N}{\partial x} = 0 + 2xu$ ...(iv)

From (iii) and (iv), we have $\frac{\partial M}{\partial u} = \frac{\partial N}{\partial x}$ which is exact differential equation.

Solution $\int_{\substack{\text{Take } u \text{ as} \\ \text{constant}}} M\,dx + \int_{\substack{\text{Take only } x \text{ terms} \\ \text{not contain } x}} N\,du = C$

$$\Rightarrow \int xu^2dx + \int 2du = c \Rightarrow u^2\frac{x^2}{2} + 2u = c$$

61. Option (b) is correct.

Formulate given as an Linear Programming problem

Profit Maximize $z = 150x_1 + 100x_2$ ...(i)

$2x_1 + 3x_2 \le 70$ ...(ii)

$2x_1 + x_2 \le 50$ ...(iii)

$2x_2 \le 40$ ...(iv)

All $x_1, x_2 \ge 0$ (Non Negative Quantity)

By Graphical Method

Convert all inequalities in equalities

$2x_1 + 3x_2 = 70$ ...(ii)

$2x_1 + x_2 = 50$ ...(iii)

and $2x_1 = 40$ ...(iv)

$x_1, x_2 = 0$

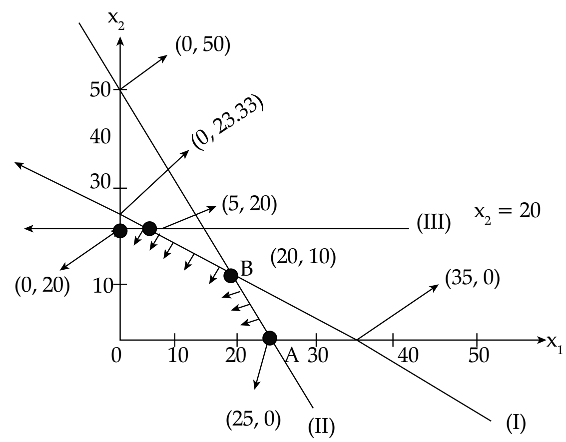
From equation (ii) $2x_1 + 3x_2 = 70$

x_1	0	35
x_2	23.33	0

From equation (iii) $2x_1 + x_2 = 50$

x_1	0	25
x_2	50	0

From equation (iv) $2x_2 = 40$



Coordinate of B is the intersecting point of line I and II

$$\begin{array}{r} 2x_1 + 3x_2 = 70 \\ 2x_1 + x_2 = 50 \\ - \quad - \quad - \\ \hline 2x_2 = 20 \end{array}$$

$\Rightarrow \quad x_2 = 10$

Therefore, $2x_1 + x_2 = 50$

$\Rightarrow \quad 2x_1 + 10 = 50$

$\therefore \quad 2x_1 = 40$

$\Rightarrow \quad x_1 = 20$

Coordinate of point B (20, 10)

Put $x_2 = 20$ in equation

$2x_1 + 3x_2 = 70$, we get

$2x_1 + 60 = 70$

$\Rightarrow 2x_1 = 10$

Thus, $x_1 = 5$

Hence, C (5, 20)

Bounded Regain OABCDO

Maximize $z = 150x_1 + 100x_2$

At A (25, 0), $z = 150 \times 25 + 100 \times 0 =$ ₹3750

At B (20, 10), $z = 150 \times 20 + 100 \times 10 =$ ₹4000

At C (5, 20), $z = 150 \times 5 + 100 \times 20 =$ ₹2750

At D (0, 10), $z = 150 \times 0 + 100 \times 20 =$ ₹2000

80 profit maximum per day is ₹4000

62. Options (b, c) are correct.

Given $\rho(A) = 2$

In homogeneous equation $AX = 0 \quad \therefore B = 0$

where A is the Coefficient matrix.

Homogeneous equations have a solution.

Because $\rho(A) = \rho(C) = 2 <$ No of unknowns

Infinite many solutions including a zero vector of appropriate size.

63. Correct answer is [2].

We have by the properties of eigenvalues

1. The sums of the eigenvalues of Matrix = Trace of the Matrix

$\Rightarrow \quad 4 = 3 + q$

$\therefore \quad q = 1$

2. The product of the eigenvalues of the Matrix A = The Determinant of A

$\Rightarrow \quad -1 = \begin{vmatrix} 3 & p \\ p & q \end{vmatrix}$

$\Rightarrow \quad 3q - p^2 = -1$

$\because \quad q = 1$, then $3 - p^2 = -1$

$\therefore \quad p^2 = 3 + 1 = 4$

$\Rightarrow \quad p^2 = \pm 2$

Thus, $|p| = 2$

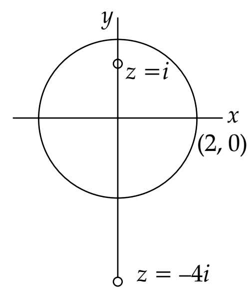
64. Correct answer is [0.2].

Given, $\frac{1}{2\pi}\int_c \frac{1}{(z-i)(z+4i)}dz$ ...(i)

Poles : Taking denominator of $f(z)$ equal to two

$(z-i)(z+4i) = 0 \therefore z = i, -4i$

and C is the circle of Radius 2. $z = +i$ will lies inside the circle.



We have by Cauchy's Residue Theorem,

$\int_C f(z)dz = 2\pi i$ [Residues $f(z)]_{z=i}$

where Residues$[f(z)]_{z=i} = \lim_{z\to i}(z-i)f(z)$

$$\lim_{z\to i}(z-i)\times\frac{1}{(z-i)(z+4i)} = \lim_{z\to i}\left(\frac{1}{z+4i}\right) = \frac{1}{i+4i} = \frac{1}{5i}$$

$$\therefore \quad \frac{1}{2\pi}\int_C \frac{1}{(z-i)(z+4i)}dz$$

$$= \frac{2\pi i}{2\pi}\left(\frac{1}{5i}\right) = \frac{2\pi}{5\times 2\pi} = \frac{1}{5} = 0.2.$$