JEE Advanced (2023)

PAPER 2

Mathematics

General Instructions:

SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e., the question is unanswered);
Negative Mark : –1 In all other cases.

Q. 1. Let $f : [1, \infty) \to \mathbb{R}$ be a differentiable function such that $f(1) = \frac{1}{3}$ and $3\int_1^x f(t)dt = x\,f(x) - \frac{x^3}{3}, x \in [1, \infty)$. Let e denote the base of the natural logarithm. Then the value of $f(e)$ is

(A) $\frac{e^2+4}{3}$ (B) $\frac{\log_e 4 + e}{3}$

(C) $\frac{4e^2}{3}$ (D) $\frac{e^2-4}{3}$

Q. 2. Consider an experiment of tossing a coin repeatedly until the outcomes of two consecutive tosses are same. If the probability of a random toss resulting in heads is $\frac{1}{3}$, then the probability that the experiment stops with head is

(A) $\frac{1}{3}$ (B) $\frac{5}{21}$

(C) $\frac{4}{21}$ (D) $\frac{2}{7}$

Q. 3. For any $y \in \mathbb{R}$, let $\cot^{-1}(y) \in (0, \pi)$ and $\tan^{-1}(y) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then the sum of all the solutions of the equation $\tan^{-1}\left(\frac{6y}{9-y^2}\right) + \cot^{-1}\left(\frac{9-y^2}{6y}\right) = \frac{2\pi}{3}$ for $0 < |y| < 3$, is equal to

(A) $2\sqrt{3} - 3$ (B) $3 - 2\sqrt{3}$

(C) $4\sqrt{3} - 6$ (D) $6 - 4\sqrt{3}$

Q. 4. Let the position vectors of the point P, Q, R and S be $\vec{a} = \hat{i} + 2\hat{j} - 5\hat{k}, \vec{b} = 3\hat{i} + 6\hat{j} + 3\hat{k}$, $\vec{c} = \frac{17}{5}\hat{i} + \frac{16}{5}\hat{j} + 7\hat{k}$ and $\vec{d} = 2\hat{i} + \hat{j} + \hat{k}$, respectively. Then which of the following statements is true?

(A) The points P, Q, R and S are NOT coplanar

(B) $\frac{\vec{b} + 2\vec{d}}{3}$ is the position vector of a point which divides PR internally in the ratio 5 : 4

(C) $\frac{\vec{b} + 2\vec{d}}{3}$ is the position vector of a point which divides PR externally in the ratio 5 : 4

(D) The square of the magnitude of the vector $\vec{b} \times \vec{d}$ is 95

General Instructions:

SECTION 2 (Maximum Marks: 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;

Zero Marks : 0 If unanswered;

Negative Marks : –2 In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option(s) (i.e., the question is unanswered) will get 0 marks and
  choosing any other option(s) will get –2 marks.

Q. 5. Let $M = (a_{ij})$, $i, j \in \{1, 2, 3\}$, be the 3×3 matrix such that $a_{ij} = 1$ if $j + 1$ is divisible by i, otherwise $a_{ij} = 0$. Then which of the following statements is (are) true?

(A) M is invertible

(B) There exists a non-zero column matrix $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ such that $M\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} -a_1 \\ -a_2 \\ -a_3 \end{pmatrix}$

(C) The set $\{X \in \mathbb{R}^3 : MX = 0\} \neq \{0\}$, where $0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

(D) The matrix $(M - 2I)$ is invertible, where I is the 3×3 identify matrix

Q. 6. Let $f : (0, 1) \to \mathbb{R}$ be the function defined as $f(x) = [4x]\left(x - \frac{1}{4}\right)^2\left(x - \frac{1}{2}\right)$, where $[x]$ denotes the greatest integer less than or equal to x. Then which of the following statements is (are) true?

(A) The function f is discontinuous exactly at one point in $(0, 1)$

(B) There is exactly one point in $(0, 1)$ at which the function f is continuous but NOT differentiable

(C) The function f is NOT differentiable at more than three points in $(0, 1)$

(D) The minimum value of the function f is $-\frac{1}{512}$

Q. 7. Let S be the set of all twice differentiable function f from $\mathbb{R}$ to $\mathbb{R}$ such that $\frac{d^2 f}{dx^2}(x) > 0$ for all $x \in (-1, 1)$. For $f \in S$, let X_f be the number of points $x \in (-1, 1)$ for which $f(x) = x$. Then which of the following statements is (are) true?

(A) There exists a function $f \in S$ such that $X_f = 0$

(B) For every function $f \in S$, we have $X_f \leq 2$

(C) There exists a function $f \in S$ such that $X_f = 2$

(D) There does NOT exist any function f in S such that $X_f = 1$

General Instructions:

SECTION 3 (Maximum Marks: 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 If **ONLY** the correct integer is entered;

  Zero Marks : 0 In all other cases.

Q. 8. For $x \in \mathbb{R}$, let $\tan^{-1}(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then the minimum value of the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \int_0^{x\tan^{-1}x} \frac{e^{(t-\cos t)}}{1 + t^{2023}}\, dt$ is

Q. 9. For $x \in \mathbb{R}$, let $y(x)$ be a solution of the differential equation $(x^2 - 5)\frac{dy}{dx} - 2xy = -2x(x^2 - 5)^2$ such that $y(2) = 7$.

Then the maximum value of the function $y(x)$ is

Q. 10. Let X be the set of all five digit numbers formed using 1, 2, 2, 2, 4, 4, 0. For example, 22240 is in X while 02244 and 44422 are not in X. Suppose that each element of X has an equal chance of being chosen. Let P be the conditional probability that an element chosen at random is a multiple of 20 given that it is a multiple of 5. Then the value of $38p$ is equal to

Q. 11. Let $A_1, A_2, A_3, ..., A_8$ be the vertices of a regular octagon that lie on a circle of radius 2. Let P be a point on the circle and let PA_i denote the distance between the points P and A_i for $i = 1, 2, ..., 8$. If P varies over the circle, then the maximum value of the product $PA_1 \cdot PA_2 \cdots PA_8$, is

Q. 12. Let

$$R = \left\{ \begin{pmatrix} a & 3 & b \\ c & 2 & d \\ 0 & 5 & 0 \end{pmatrix} : a, b, c, d \in \{0, 3, 5, 7, 11, 13, 17, 19\} \right\}.$$

Then the number of invertible matrices in R is

Q. 13. Let C_1 be the circle of radius 1 with center at the origin. Let C_2 be the circle of radius r with centre at the point $A = (4, 1)$, where $1 < r < 3$. Two distinct common tangents PQ and ST of C_1 and C_2 are drawn. The tangent PQ touches C_1 at P and C_2 at Q. The tangent ST touches C_1 at S and C_2 at T. Mid points of the line segments PQ and ST are joined to form a line which meets the x-axis at a point B. If $AB = \sqrt{5}$, then the value of r^2 is

***General Instructions*:**

SECTION 4 (Maximum Marks: 12)

- This section contains **TWO (02)** paragraphs.
- Based on each paragraph, there are **TWO (02)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If ONLY the correct numerical value is entered in the designated place;
  Zero Marks : 0 In all other cases.

PARAGRAPH "I"

Consider an obtuse angled triangle ABC in which the difference between the largest and the smallest angle is $\frac{\pi}{2}$ and whose sides are in arithmetic progression. Suppose that the vertices of this triangle lie on a circle of radius 1.

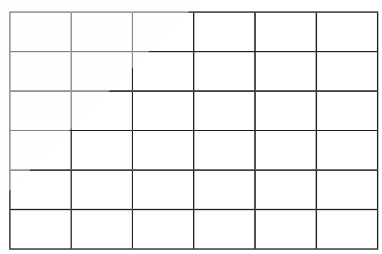
(There are two question based on PARAGRAPH "I", the question given below is one of them)

Q. 14 Let a be the area of the triangle ABC. Then the value of $(64a)^2$ is

Q. 15. Then the inradius of the triangle ABC is

PARAGRAPH "II"

Consider the 6×6 square in the figure. Let $A_1, A_2, ..., A_{49}$ be the points of intersection (dots in the picture) in some order. We say that A_i and A_j are friends if they are adjacent along a row or along a column. Assume that each points A_i has an equal chance of being chosen.



(There are two question based on PARAGRAPH "II", the question given below is one of them)

Q. 16. Let p_i be the probability that a randomly chosen point has i many friends, $i = 0, 1, 2, 3, 4$. Let X be a random variable such that for $i = 0, 1, 2, 3, 4$, the probability $P(X = i) = p_i$. Then the value of $7E(X)$ is

Q. 17. Two distinct points are chosen randomly out of the points $A_1, A_2, ..., A_{49}$. Let p be the probability that they are friends. Then the value of $7\,p$ is

ANSWER KEY

Q.No.	Answer key	Topic's name	Chapter's name
Section-I			
1	(C)	Linear differential equation	Differential equation
2	(B)	Conditional probability	Probability
3	(C)	Solution of Equation	Inverse Trigonometric function
4	(B)	Product of vectors and its Application	Vector
Section-II			
5	(B, C)	Solution of system of linear equations	Matrix and determinants
6	(A, B)	Maxima and Minima	Application of derivatives
7	(A, B, C)	Concavity of curve	Application of derivatives
Section-III			
8	0	Leibnitz theorem & Maxima, Minima	Application of derivatives
9	16	Linear differential equation	Differential equation
10	31	Probability based on permutation & combination	Probability
11	512	Demovire's theorem and triangular inequality	Complex number
12	3780	Permutation involving in matrix	Matrix
13	2	Radical axis and its properties	Circle
Section-IV			
14	1008	Area of triangle	Properties of triangle
15	0.25	Inradius	Properties of triangle
16	24	Binomial distribution	Probability
17	0.5	Conditional Probability	Probability

JEE Advanced (2023)

PAPER 2

ANSWERS WITH EXPLANATIONS

Mathematics

1. Correct option is (C).

$$3\int_1^x f(t)dt = xf(x) - \frac{x^2}{3} \quad x \in (1, \infty)$$

Using Leibnitz rule,

$$3f(x) = x f'(x) + f(x) - x^2$$

$$\Rightarrow xf'(x) - 2f(x) - x^2 = 0$$

$$\Rightarrow f'(x) - \frac{2}{x}f(x) - x = 0$$

$$\Rightarrow \frac{dy}{dx} - \frac{2}{x}y = x$$

Linear Differential Equation in x

Integrating Factor $= e^{-\int \frac{2}{x}dx} = e^{-2\ln x}$

$$= \frac{1}{x^2}$$

Now $\quad y.\frac{1}{x^2} = \int x.\frac{1}{x^2}dx$

$$= \ln x + C$$

$$\Rightarrow \quad \frac{1}{3} = 0 + C \quad \left[\because f(1) = \frac{1}{3}\right]$$

$$\Rightarrow \quad C = 3$$

$$\Rightarrow \quad y = x^2 \ln x + \frac{x^2}{3}$$

$$f(e) = e^2 + \frac{e^2}{3}$$

$$f(e) = \frac{4e^2}{3}$$

2. Correct option is (B).

$$P(H) = \frac{1}{3} \quad P(T) = \frac{2}{3}$$

Tossing coin is repeatedly this process end with last two head in out come.

$\Rightarrow$ Lets Experiment end with trial : (Two trial) or (Three trial) or (Four trial) or (Five trial) or (Six trial) so, on ....

i.e., (HH) or (THH), (HTHH), (THTHH) (HTHTHH) .....

So, the required probability is given by:

P = (HH) + (THH) + (HTHH) + (THTHH) (HTHTHH) + ... ∞

$$P = \left(\frac{1}{3}\right)^2 + \frac{2}{3}\left(\frac{1}{3}\right)^2 + \frac{2}{3}\left(\frac{1}{3}\right)^3 + \left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^3 + \left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^4 + \ldots\infty$$

$$= \left(\left(\frac{1}{3}\right)^2 + \frac{2}{3}\left(\frac{1}{3}\right)^3 + \left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^4 + \ldots\infty\right) + \left(\frac{2}{3}\left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^3 + \ldots\infty\right)$$

$$= \frac{\left(\frac{1}{3}\right)^2}{1-\frac{2}{9}} + \frac{\frac{2}{3}\times\frac{1}{9}}{1-\frac{2}{3}\times\frac{1}{3}}$$

$$= \frac{1}{7} + \frac{2}{21} = \frac{5}{21}$$

3. Correct option is (C).

$$\tan^{-1}\left(\frac{6y}{9-y^2}\right) + \cot^{-1}\left(\frac{9-y^2}{6y}\right) = \frac{2\pi}{3} \quad \ldots(i)$$

where $0 < |y| < 3$

$$\cot^{-1}\left(\frac{1}{x}\right) = \begin{cases} \tan^{-1}x & x > 0 \\ \pi + \tan^{-1}x & x < 0 \end{cases}$$

Case-I When $0 < y < 3$

$$\tan^{-1}\frac{6y}{9-y^2} + \tan^{-1}\frac{6y}{9-y^2} = \frac{2\pi}{3}$$

$$\Rightarrow \quad \tan^{-1}\frac{6y}{9-y^2} = \frac{\pi}{3}$$

$$\Rightarrow \quad \frac{6y}{9-y^2} = \sqrt{3}$$

$$\Rightarrow \quad 6y = 9\sqrt{3} - \sqrt{3}y^2$$

$$\Rightarrow \quad \sqrt{3}y^2 + 6y - 9\sqrt{3} = 0$$

$$\Rightarrow \quad \sqrt{3}y^2 + 9y - 3y - 9\sqrt{3} = 0$$

$$\Rightarrow \quad \sqrt{3}y(y + 3\sqrt{3}) - 3(y + 3\sqrt{3}) = 0$$

$$(\sqrt{3}y - 3)(y + 3\sqrt{3}) = 0$$

So, the value satisfied is $y = \sqrt{3}$

Case II: When $-3 < y < 0$

$$\tan^{-1}\left(\frac{6y}{9-y^2}\right) + \pi + \tan^{-1}\frac{6y}{9-y^2} = \frac{2\pi}{3}$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{6y}{9-y^2}\right) = \frac{-\pi}{6}$$

$$\frac{6y}{9-y^2} = -\frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad 6\sqrt{3}y = y^2 - 9$$

$$\Rightarrow \quad y^2 - 6\sqrt{3}y - 9 = 0$$

$$\Rightarrow \quad y = \frac{6\sqrt{3} \pm \sqrt{108 + 36}}{2}$$

$$= \frac{6\sqrt{3} \pm 12}{2} = 3\sqrt{3} \pm 6$$

So, the value satisfied is $y = 3\sqrt{3} - 6$

Hence, the sum of solutions

$$3\sqrt{3} - 6 + \sqrt{3} = 4\sqrt{3} - 6$$

4. Correct option is (B).

$$P(\vec{a}) = \hat{i} + 2\hat{j} - 5\hat{k}$$

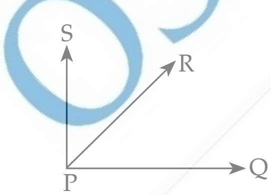
$$Q(\vec{b}) = 3\hat{i} + 6\hat{j} + 3\hat{k}$$

$$R(\vec{c}) = \frac{17}{5}\hat{i} + \frac{16}{5}\hat{j} + 7\hat{k}$$

$$S(\vec{d}) = 2\hat{i} + \hat{j} + \hat{k}$$

From option

(A)

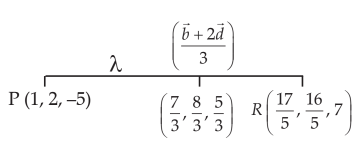


$[\overline{PQ}, \overline{PR}, \overline{PS}] \rightarrow$ S.T.P

$$\begin{vmatrix} 2 & 4 & 6 \\ \frac{12}{5} & \frac{6}{5} & 12 \\ 1 & -1 & 6 \end{vmatrix} = 0$$

Hence P, Q, R, S are coplanar.

(B)



$$\Rightarrow \quad \frac{\frac{17\lambda}{5} + 1}{1 + \lambda} = \frac{7}{3}$$

$$\Rightarrow \quad \frac{(17\lambda + 5)}{1 + \lambda} = \frac{35}{3}$$

$$\Rightarrow \quad 15\lambda + 15 = 35 + 35\lambda$$

$$\Rightarrow \quad 16\lambda = 20$$

$$\Rightarrow \quad \lambda = \frac{5}{4}$$

Hence option (B) is correct.

(D) $$|\vec{b} \times \vec{d}|^2 = |\vec{b}|^2 |\vec{d}|^2 - (\vec{b} . \vec{d})^2$$

$$= 54 \times 6 - 225$$

$$= 324 - 225$$

$$= 99$$

5. Correct options are (B and C).

$$M = [a_{ij}]\ i, j \in \{1, 2, 3\}$$

$$a_{ij} = \begin{cases} 1 & \text{if } j + 1 \text{ divisible by } i \\ 0 & \text{other wise} \end{cases}$$

$$M = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$|M| = 0$$

$\Rightarrow$ M^{-1} not exist

$$M\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} -a_1 \\ -a_2 \\ -a_3 \end{pmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} -a_1 \\ -a_2 \\ -a_3 \end{pmatrix}$$

$$\Rightarrow \quad a_1 + a_2 + a_3 = -a_1 \quad \text{...(i)}$$

$$a_1 + a_3 = -a_2 \quad \text{...(ii)}$$

$$\Rightarrow \quad a_2 + a_3 = 0 \quad \text{...(iii)}$$

From (i) and (iii),

$$a_1 = 0$$

From (ii)

$$a_2 + a_3 = 0$$

Hence, there exist infinite many solution for a_2, and a_2

$$MX = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad x + y + z = 0 \quad \text{...(iv)}$$

$$\Rightarrow \quad x + z = 0 \quad \text{...(v)}$$

$$\Rightarrow \quad y = 0 \quad \text{...(vi)}$$

From (iv) and (v)

and $M - 2I = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$$= \begin{bmatrix} -1 & 1 & 1 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{bmatrix}$$

$|M - 2I| = 0$

Hence, $(M - 2I)^{-1}$ does not exist

6. Correct options are (A and B).

Given $f : (0, 1) \to R$

$$f(x) = [4x]\left(x - \frac{1}{4}\right)^2\left(x - \frac{1}{2}\right) \quad ...(i)$$

when $x \in (0, 1) \Rightarrow 4x \in (0, 4)$

$x : 0 - 1$

$4x : 0 - 1 - 2 - 3 - 4$

$x : 0 - \frac{1}{4} - \frac{1}{2} - \frac{3}{4} - 1$

From (*i*)

$$f(x) = \begin{cases} 0 & 0 < x < \frac{1}{4} \\ \left(x - \frac{1}{4}\right)^2\left(x - \frac{1}{2}\right) & \frac{1}{4} \le x < \frac{1}{2} \\ 2\left(x - \frac{1}{4}\right)^2\left(x - \frac{1}{2}\right) & \frac{1}{2} \le x < \frac{3}{4} \\ 3\left(x - \frac{1}{4}\right)^2\left(x - \frac{1}{2}\right) & \frac{3}{4} \le x < 1 \end{cases}$$

Check continuity and differentiability at $x = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$

Clearly $f(x)$ is discontinuous at $x = \frac{3}{4}$ and continuous at $x = \frac{1}{4}, \frac{1}{2}$

also $$f'(x) = \begin{cases} 0 & 0 < x < \frac{1}{4} \\ \left(x - \frac{1}{4}\right)\left(3x - \frac{5}{4}\right) & \frac{1}{4} < x < \frac{1}{2} \\ 2\left(x - \frac{1}{4}\right)\left(3x - \frac{5}{4}\right) & \frac{1}{2} < x < \frac{3}{4} \\ 3\left(x - \frac{1}{4}\right)\left(3x - \frac{5}{4}\right) & \frac{3}{4} < x < 1 \end{cases}$$

at $x = \frac{1}{4}$ function is continuous and differentiable

at $x = \frac{1}{2}$ function is continuous but not differentiable

For maxima and minima

Put $f'(x) = 0$

$$x = \frac{1}{4}, \frac{5}{12}$$

Clearly $f(x)$ give minimum value

at $x = \frac{5}{12}$

$$f_{\min} = f\left(\frac{5}{12}\right) = \frac{-1}{432}$$

7. Correct options are (A, B and C).

$$\frac{d^2f}{dx^2} > 0$$

$\Rightarrow$ $y = f(x)$ concave upward in $(1, 1)$

Graph: $y = f(x)$ in $(-1, 1)$

The line $y = x$ cut above goopn either in 0, 1 or 2 point

So, the options A, B, C are correct.

8. Correct answer is [0].

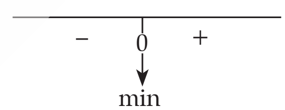
$$f(x) = \int_0^{x\tan^{-1}x} \frac{e^{t - \cos t}}{1 + t^{2023}}\,dt$$

$$f'(x) = \frac{e^{x\tan^{-1}x - \cos(x\tan^{-1}x)}}{1 + (x\tan^{-1}x)^{2023}}\left(\frac{x}{1 - x^2} + \tan^{-1}x\right)$$

For max/min put $f'(x) = 0$

$\Rightarrow$ $\frac{x}{1 + x^2} + \tan^{-1}x = 0$

$\Rightarrow$ $\boxed{x = 0}$



$\boxed{f(0) = 0}$

9. Correct answer is [16].

$$(x^2 - 5)\frac{dy}{dx} - 2xy = -2x\,(x^2 - 5)^2$$

$\Rightarrow$ $\frac{dy}{dx} - \frac{2x}{(x^2 - 5)}y = -2x(x^2 - 5)$

$$\text{I.F.} = e^{-\int \frac{2x}{x^2 - 5}dx} = \frac{1}{(x^2 - 5)}$$

Now $y \cdot \frac{1}{(x^2 - 5)} = -\int 2x\,dx$

$= -x^2 + c$

$\Rightarrow$ $y = c(x^2 - 5) - x^2(x^2 - 5)$

$y(2) = 7$

$\Rightarrow$ $7 = -c + 4$

$\Rightarrow$ $\boxed{c = -3}$

So, $y = (x^2 - 5)(-x^2 - 3)$ ...(1)

$$\frac{dy}{dx} = (x^2 - 5)(-2x) + (-x^2 - 3)(2x)$$
$$= 2x(-x + 5 - x^2 - 3)$$
$$= 2x(-2x^2 + 2)$$

For maxima and minima, put $\frac{dy}{dx} = 0$

$\Rightarrow$ $x = 0, \pm 1$

– -1 + 0 – 1 +

-1 ↓ max, 1 ↓ max

From (1)

$$y_{max} = 16$$

10. Correct answer is [31].

A = Number of elements in x which is multiple of 5

1	2	2	2	0	$\rightarrow \frac{4!}{3!} = 4$
1	4	2	2	0	$\rightarrow \frac{4!}{2!} = 12$
1	2	4	4	0	$\rightarrow \frac{4!}{2!} = 12$
4	2	2	2	0	$\rightarrow \frac{4!}{3!} = 4$
2	2	4	4	0	$\rightarrow \frac{4!}{2!2!} = 6$

$n(A) = 4 + 12 + 12 + 4 + 6 = 38$

B = Number of elements in x which is multiple of 20

2	2	2	1	0	$\rightarrow 1$
4	2	2	1	0	$\rightarrow \frac{3!}{2!} = 3$
2	4	4	1	0	$\rightarrow \frac{3!}{2!} = 3$

So, number of element in x which is multiple of $20 = n(B)$

$$= (4 - 1) + (12 - 3) + (12 - 3) + 4 + 6$$
$$= 31$$

$\Rightarrow$ $P\left(\frac{B}{A}\right) = \frac{n(A \cup B)}{n(A)} = \frac{31}{38} = P$

$\Rightarrow$ $\boxed{38P = 31}$

11. Correct answer is [512].

Let $z = 2(1)^{1/8}$ $[\because \ |z| = 2]$

$\Rightarrow$ $z = 2, 2x, 2x^2, 2x^3, \ldots 2x^7$ are root.

$\Rightarrow$ $(z^8 - 2^8) = (z - 2)(z - 2x)(z - 2x^2) \ldots (z - 2x^7)$

Using triangular in equalities

$$|z^8 - 2^8| = |z - 2|\,|z - 2x^2|\,|z - 2x^3| \ldots |z - 2x^7|$$
$$\leq |z^8| + |-2^8|$$
$$\leq 2^8 + 2^8$$
$$\leq 2^9$$

Max $PA_1 . PA_2 \cdot PA_3 \ldots PA_8 = 2^9$

12. Correct answer is [3780].

$$R = \begin{bmatrix} a & 3 & b \\ c & 2 & d \\ 0 & 5 & 0 \end{bmatrix}$$

$a, b, c, d, \in \{0, 3, 5, 7, 11, 13, 17, 19\}$

Number of invertible matrices = (Total matrices) – (Non Invertible matrices)

Total matrices = a, b, c, d → 8, 8, 8, 8

$$= 8 \times 8 \times 8 \times 8 = 8^4 = 4096$$

For Non-invertible matrices,

$$|R| = 0$$
$$|R| = -5(ad - bc) = 0$$

$\Rightarrow$ $\boxed{ad = bc = 0}$

Both side zero / Both side same but not zero

Cases when both side are zero.

(i) All four a, b, c, d are zero.

$ad = bc = 0$ 1 ways

(ii) Three zero and one different digit used for a, b, c, d.

$\Rightarrow$ $ad = bc$

Select three from four a, b, c, d & assign them zero.

i.e., ${}^4C_3 \times 1 \times 7 = 28$ ways

(iii) Two zero and two different digits

i.e., $ad = bc$

$\Downarrow$ $\Downarrow$

${}^2C_1 \times 1 \times 7$ ${}^2C_1 \times 1 \times 7$

Hence $2 \times 7 \times 2 \times 7 = 196$ ways

Case II: When both side are same but non zero number.

$$ad = bc \neq 0$$

(i) All four a, b, c, d are same.

i.e., $ad = bc$ (7 ways)

(ii) Two alike & two alike of another.

$ad = bc$

$^7C_1 \times {}^6C_1 \times 2! = 84$ ways

Total number of non invertible matrices are

$= 1 + 28 + 196 + 7 + 84$

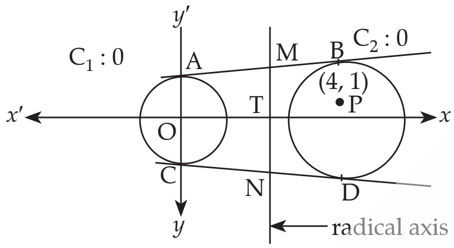
$= 316$

Hence number of invertible matric

$= 8^4 - 316$

$= 3780$

13. Correct answer is [2].



Equation of radical axis : $C_1 - C_2 = 0$

$$\Rightarrow \quad 8x + 2y - 18 + r^2 = 0$$

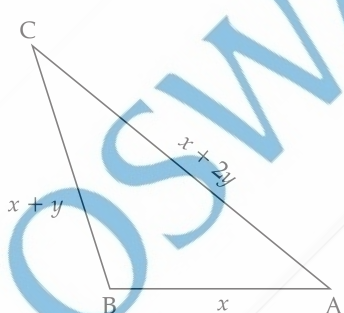
$$T\left(\frac{18 - r^2}{8}, 0\right)$$

$$AT = \sqrt{5} \text{ [given]}$$

$$\Rightarrow \quad \left(\frac{18 - r^2}{8} - 4\right) + (0 - 1)^2 = 5$$

$$r^2 = 2$$

Paragraph I



Let B be greatest angle and C be small angle. Each side of triangle is mention in figure.

Given $\quad B - C = \frac{\pi}{2}$

$$\Rightarrow \quad B = \frac{\pi}{2} + C$$

$$A + B + C = \pi$$

$$\Rightarrow \quad A = \frac{\pi}{2} - 2C$$

Again AB, BC, CA are in AP

$$2BC = AB + AC$$

$$\Rightarrow \quad 4R \sin A = 2R \sin B + 2R \sin C$$

$$\Rightarrow \quad 2 \sin A = \sin B + \sin C$$

$$\Rightarrow \quad 2 \sin\left(\frac{\pi}{2} - 2C\right) = \sin\left(\frac{\pi}{2} + 2C\right) + \sin C$$

$$\Rightarrow \quad 2 \cos 2C = \cos C + \sin C$$

$$\Rightarrow \quad \cos C - \sin C = \frac{1}{2}$$

Squaring both side we get

$$\Rightarrow \quad 1 - \sin 2C = \frac{1}{4}$$

$$\Rightarrow \quad \sin 2C = \frac{3}{4}$$

14. Correct answer is [1008].

Area of $\Delta ABC = \dfrac{AB \cdot BC \cdot AC}{4R}$

$$\Rightarrow \quad a = \frac{8 \sin A \cdot \sin B \sin C}{4}$$

$$= 2 \sin\left(\frac{\pi}{2} - 2C\right) \sin\left(\frac{\pi}{2} + C\right) \sin C$$

$$= 2 \cos 2C \cdot \cos C \cdot \sin C$$

$$= \cos 2C \cdot \sin 2C$$

$$= \sqrt{1 - \sin^2 2C} \cdot \sin 2C$$

$$= \sqrt{1 - \frac{9}{6}} \cdot \times \frac{3}{4}$$

$$\Rightarrow \quad a = \frac{3\sqrt{7}}{16}$$

$$(64\, a)^2 = 1008$$

15. Correct answer is [0.25].

In radius $\quad r = \dfrac{\Delta}{S} = \left[\dfrac{a}{2R(\sin A + \sin B + \sin C)}\right]$

$$r = \frac{a}{\sin\left(\frac{\pi}{2} - 2C\right) \sin\left(\frac{\pi}{2} + C\right) + \sin C}$$

$$= \frac{a}{\cos 2C + \cos C + \sin C}$$

$$= \frac{a}{\cos 2C + \sqrt{1 + \sin 2C}}$$

$$= \frac{\frac{3\sqrt{7}}{16}}{\sqrt{\frac{7}{4}} + \sqrt{\frac{7}{2}}} = \frac{1}{4}$$

$$\Rightarrow \quad r = \frac{1}{4} = 0.25$$

$$\Rightarrow \quad r = 0.25$$

16. Correct answer is [24].

$$P(x = 0) = 0$$

$$P(x = 3) = \frac{20}{49}$$

$$P(x = 1) = 0$$

$$P(x = 4) = 1 - \frac{24}{49}$$

$$P(x = 2) = \frac{4}{49}$$

$$= \frac{25}{49}$$

We have

$$E(X_i) = \sum_{i=0}^{4} i\,P(x = i)$$

$$= 0\,.\,P(x = 0) + 1\,P(x = 1) + 2(x = 2) + 3P(x = 3) + 4P(x = 4)$$

$$= 0 + 0 + 2\frac{4}{49} + 3\cdot\frac{20}{49} + 4\cdot\frac{25}{49}$$

$$= \frac{8 + 60 + 100}{49} = \frac{168}{49} = \frac{24}{7}$$

$$7E(X_i) = 24$$

17. Correct answer is [0.5].

$$P = \frac{6\times 7 + 6\times 7}{^{49}C_2} = \frac{2\times 6\times 7}{\frac{49\times 48}{2}}$$

$$P = \frac{1}{14}$$

$$7P = \frac{1}{2} = 0.5$$

$$7P = 0.5$$