JEE (Main) MATHEMATICS SOLVED PAPER

2021
24^{th} February Shift 1

Time : 1 Hour **Total Marks : 100**

General Instructions :

1. *In Chemistry Section, there are 30 Questions (Q. no. 1 to 30).*
2. *In Chemistry, Section A consists of 20 multiple choice questions & Section B consists of 10 numerical value type questions. In Section B, candidates have to attempt any five questions out of 10.*
3. *There will be only one correct choice in the given four choices in Section A. For each question for Section A, 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice questions and zero mark will be awarded for not attempted question.*
4. *For Section B questions, 4 marks will be awarded for correct answer and zero for unattempted and incorrect answer.*
5. *Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.*
6. *All calculations / written work should be done in the rough sheet is provided with Question Paper.*

Mathematics

Section A

Q. 1. The locus of the mid-point of the line segment joining the focus of the parabola $y^2=4ax$ to a moving point of the parabola, is another parabola whose directrix is:.

(1) $x = a$ **(2)** $x = 0$

(3) $x = -\frac{a}{2}$ **(4)** $x = \frac{a}{2}$

Q. 2. A scientific committee is to be formed from 6 Indians and 8 foreigners, which includes at least 2 Indians and double the number of foreigners as Indians. Then the number of ways, the committee can be formed is:

(1) 560 **(2)** 1050

(3) 1625 **(4)** 575

Q. 3. The equation of the plane passing through the point (1, 2, –3) and perpendicular to the planes

$3x + y - 2z = 5$ and $2x - 5y - z = 7$, is:

(1) $3x - 10y - 2z + 11 = 0$

(2) $6x - 5y - 2z - 2 = 0$

(3) $11x + y + 17z + 38 = 0$

(4) $6x - 5y + 2z + 10 = 0$

Q. 4. A man is walking on a straight line. The arithmetic mean of the reciprocals of the intercepts of this line on the coordinate axes is $\frac{1}{4}$. Three stones A, B and C are placed at the points (1, 1), (2, 2) and (4, 4) respectively. Then which of these stones is/are on the path of the man?

(1) B only **(2)** A only

(3) All the three **(4)** C only

Q. 5. The statement among the following that is a tautology is:

(1) $A \wedge (A \vee B)$ **(2)** $B \rightarrow [A \wedge (A \rightarrow B)]$

(3) $A \vee (A \wedge B)$ **(4)** $[A \wedge (A \rightarrow B)] \rightarrow B$

Q. 6. Let $f : R \rightarrow R$ be defined as $f(x) = 2x - 1$ and $g : R - \{1\} \rightarrow R$ be defined as $g(x) = \frac{x - \frac{1}{2}}{x - 1}$.

Then the composition function $f(g(x))$ is :

(1) Both one-one and onto

(2) Onto but not one-one

(3) Neither one-one nor onto

(4) One-one but not onto

Q. 7. If $f : R \to R$ is a function defined by

$f(x) = [x-1] \cos\left(\frac{2x-1}{2}\right)\pi$, where [] denotes the greatest integer function, then f is :

(1) discontinuous only at $x = 1$

(2) discontinuous at all integral values of x except at $x = 1$

(3) continuous only at $x = 1$

(4) continuous for every real x

Q. 8. The function

$$f(x) = \frac{4x^3 - 3x^2}{6} - 2\sin x + (2x-1)\cos x :$$

(1) increases in $\left[\frac{1}{2}, \infty\right)$

(2) decreases $\left(-\infty, \frac{1}{2}\right]$

(3) increases in $(-\infty, \infty)$

(4) decreases $\left[\frac{1}{2}, \infty\right)$

Q. 9. The distance of the point (1, 1, 9) from the point of intersection of the line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ and the plane

$x + y + z = 17$ is:

(1) $\sqrt{38}$ **(2)** $19\sqrt{2}$

(3) $2\sqrt{19}$ **(4)** 38

Q. 10. $\lim_{x \to 0^+} \frac{\int_0^{x^2} \left(\sin\sqrt{t}\right) dt}{x^3}$ is equal to :

(1) $\frac{2}{3}$ **(2)** 0

(3) $\frac{1}{15}$ **(4)** $\frac{3}{2}$

Q. 11. Two vertical poles are 150 m apart and the height of one is three times that of the other. If from the middle point of the line joining their feet, an observer finds the angles of elevation of their tops to be complementary, then the height of the shorter pole (in meters) is:

(1) 25 **(2)** $20\sqrt{3}$

(3) 30 **(4)** $25\sqrt{3}$

Q. 12. If the tangent to the curve $y = x^3$ at the point $P(t, t^3)$ meets the curve again at Q, then the ordinate of the point which divides PQ internally in the ratio 1 : 2 is :

(1) $-2t^3$ **(2)** $-t^3$

(3) 0 **(4)** $2t^3$

Q. 13. The area (in sq. units) of the part of the circle $x^2 + y^2 = 36$, which is outside the parabola $y^2 = 9x$, is :

(1) $24\pi + 3\sqrt{3}$ **(2)** $12\pi + 3\sqrt{3}$

(3) $12\pi - 3\sqrt{3}$ **(4)** $24\pi - 3\sqrt{3}$

Q. 14. If $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx = a \sin^{-1}\left(\frac{\sin x + \cos x}{b}\right) + c$,

where c is a constant of integration, then the ordered pair (a, b) is equal to :

(1) (1, –3) **(2)** (1, 3)

(3) (–1, 3) **(4)** (3, 1)

Q. 15. The population $P = P(t)$ at time 't' of a certain species follows the differential equation $\frac{dP}{dt} = 0.5P - 450$. If $P(0) = 850$, then the time at which population becomes zero is :

(1) $\frac{1}{2}\log_e 18$ **(2)** $2\log_e 18$

(3) $\log_e 9$ **(4)** $\log_e 18$

Q. 16. The value of $-{}^{15}C_1 + 2.{}^{15}C_2 - 3.{}^{15}C_3 + ... - 15.{}^{15}C_{15} + {}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + ... + {}^{14}C_{11}$ is:

(1) 2^{14} **(2)** $2^{13} - 13$

(3) $2^{16} - 1$ **(4)** $2^{13} - 14$

Q. 17. An ordinary dice is rolled for a certain number of times. If the probability of getting an odd number 2 times is equal to the probability of getting an even number 3 times, then the probability of getting an odd number for odd number of times is :

(1) $\frac{3}{16}$ **(2)** $\frac{1}{2}$

(3) $\frac{5}{16}$ **(4)** $\frac{1}{32}$

Q. 18. Let p and q be two positive numbers such that $p + q = 2$ and $p^4 + q^4 = 272$. Then p and q are roots of the equation :

(1) $x^2 - 2x + 2 = 0$ **(2)** $x^2 - 2x + 8 = 0$

(3) $x^2 - 2x + 136 = 0$ **(4)** $x^2 - 2x + 16 = 0$

Q. 19. If $e^{(\cos^2 x + \cos^4 x + \cos^6 x + \ldots\infty)\log_e 2}$ satisfies the equation $t^2 - 9t + 8 = 0$, then the value of $\frac{2\sin x}{\sin x + \sqrt{3}\cos x}\left(0 < x < \frac{\pi}{2}\right)$ is :

(1) $\frac{3}{2}$ **(2)** $2\sqrt{3}$

(3) $\frac{1}{2}$ **(4)** $\sqrt{3}$

Q. 20. The system of linear equations

$3x - 2y - kz = 10$

$2x - 4y - 2z = 6$

$x + 2y - z = 5m$

is inconsistent if :

(1) $k = 3, m = \frac{4}{5}$ **(2)** $k \neq 3, m \in R$

(3) $k \neq 3, m \neq \frac{4}{5}$ **(4)** $k = 3, m \neq \frac{4}{5}$

Section B

Q. 21. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in R$. Suppose $Q = [q_{ij}]$ is a matrix satisfying $PQ = kI_3$ for some non-zero $k \in R$. If $q_{23} = -\frac{k}{8}$ and $|Q| = \frac{k^2}{2}$, then $\alpha^2 + k^2$ is equal to ______.

Q. 22. Let B_i (i = 1, 2, 3) be three independent events in a sample space. The probability that only B_1 occur is α, only B_2 occurs is β and only B_3 occurs is γ. Let p be the probability that none of the events B_i occurs and these 4 probabilities satisfy the equations $(\alpha - 2\beta)p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$ (All the probabilities are assumed to lie in the interval (0, 1)). Then $\frac{P(B_1)}{P(B_3)}$ is equal to :

Q. 23. The minimum value of α for which the equation $\frac{4}{\sin x} + \frac{1}{1 - \sin x} = \alpha$ has at least one solution in $\left(0, \frac{\pi}{2}\right)$ is ______.

Q. 24. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord of another circle 'C' whose center is at (2, 1), then its radius is ______.

Q. 25. $\lim_{n \to \infty} \tan\left\{\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{1 + r + r^2}\right)\right\}$ is equal to __.

Q. 26. If $\int_{-a}^{a} (|x| + |x - 2|)\, dx = 22$, $(a > 2)$ and $[x]$ denotes the greatest integer $\leq x$, then $\int_{a}^{-a} (x + [x])dx$ is equal to _____.

Q. 27. Let three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ be such that $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}, \vec{a}.\vec{c} = 7$ and $\vec{b}$ is perpendicular to $\vec{c}$ where $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{k}$, then the value of $2|\vec{a} + \vec{b} + \vec{c}|^2$ is _______.

Q. 28. Let A = $\{n \in N : n$ is a 3-digit number$\}$

B = $\{9k + 2 : k \in N\}$

and $C = \{9k + \ell : k \in N\}$ for some ℓ $(0 < \ell < 9)$

If the sum of all the elements of the set $A \cap (B \cup C)$ is 274×400, then ℓ is equal to _______.

Q. 29. If the least and the largest real values of α, for which the equation $z + \alpha\,|z - 1| + 2i = 0$ $\left(z \in C \text{ and } i = \sqrt{-1}\right)$ has a solution, are p and q respectively; then $4(p^2 + q^2)$ is equal to ___.

Q. 30. Let M be any 3×3 matrix with entries from the set $\{0, 1, 2\}$. The maximum number of such matrices, for which the sum of diagonal elements of $M^T M$ is seven, is _____.

Answer Key

Q. No.	Answer	Topic Name	Chapter Name
1	2	Basics of Parabola	Parabola
2	3	Combinations	Permutation and Combination
3	3	Equation of Plane	3D Geometry
4	1	Double Intercept Form of Line	Straight Line
5	4	Tautology	Mathematical Reasoning
6	4	Composition, Mapping of Functions	Functions
7	4	Continuity of Functions	Continuity
8	1	Monotonicity of a Function	Application of Derivative
9	1	Line and Plane	3D Geometry
10	1	Limit of a Function	Limits
11	4	Heights and Distances	Heights and Distances
12	1	Equation of Tangents	Application of Derivative
13	4	Area Under Curve	Area Under Curve
14	2	Indefinite Integration	Indefinite Integration
15	2	Variable Separable Form	Differential Equation
16	4	Series Involving Binomial Coefficients	Binomial Theorem
17	2	Bernoulli's Trials	Probability
18	4	Quadratic Equation	Quadratic Equation
19	3	Infinite G.P., Quadratic Equation	Trigonometry
20	4	Consistency of System of Equations	Determinants and Matrices
21	15	Matrices	Matrices and Determinant
22	6	Independent Events, Venn Diagram	Probability
23	9	Minima and Maxima	Application of Derivative
24	3	Distance Between Points	Circle
25	1	Series Involving Inverse Trigonometric Functions	Inverse Trigonometric Functions
26	3	Definite Integration of Piecewise Defined Functions	Integration
27	75	Vector Triple Product	Vector
28	5	Sum of A.P.	Sequence and Progression
29	10	Solution of Equations Involving Complex Numbers	Complex Number
30	540	Grouping of Objects	Permutation and Combination

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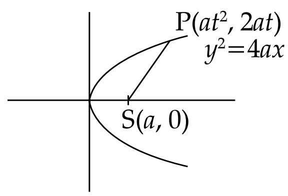
ANSWERS WITH EXPLANATIONS

Mathematics

Section A

1. **Option (2) is correct.**

Given: Parabola $y^2 = 4ax$



Focus of parabola, $y^2 = 4ax$ is $S(a, 0)$.

Consider a moving point $P(at^2, 2at)$ on the parabola.

Let midpoint of PS is Q (h, k)

Using section formula,

$$h = \frac{a+at^2}{2}, k = \frac{0+2at}{2} = at$$

$$\Rightarrow \quad t^2 = \frac{2h-a}{a} \text{ and } t = \frac{k}{a}$$

$$\therefore \quad \frac{k^2}{a^2} = \frac{2h-a}{a}$$

$$\Rightarrow \quad k^2 = 2a\left(h - \frac{a}{2}\right)$$

Replace h by x and k by y to find locus of (h, k).

$$\therefore \text{ Locus is } \quad y^2 = 2a\left(x - \frac{a}{2}\right)$$

Let $y = Y$ and $x - \frac{a}{2} = X$

$\therefore$ Locus become $Y^2 = 2a X$

Whose directrix is $X = \frac{-a}{2}$

$$\Rightarrow \quad x - \frac{a}{2} = \frac{-a}{2}$$

$$\Rightarrow \quad x = 0$$

Hence, equation of directrix is $x = 0$.

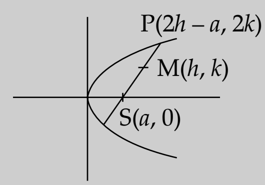
Hint:

(i) Consider the variable point on parabola in parametric form.

(ii) Find the midpoint of given line segment and eliminate the parameter to find the locus.

(iii) Compare with equation of standard parabola to find equation of directrix.

Shortcut Method:



$$4k^2 = 4a(2h - a)$$

$$\Rightarrow \quad y^2 = 2a\left(x - \frac{a}{2}\right)$$

Directrix: $x - \frac{a}{2} = \frac{-a}{2}$

$$\Rightarrow \quad x = 0$$

2. **Option (3) is correct.**

Given: A scientific committee is to be formed from 6 Indians and 8 foreigners, which includes at least 2 Indians and double the number of foreigners as Indians.

There are multiple cases to satisfy given criteria.

Committee can consists of (2I, 4F), or (3I, 6F) or (4I, 8F) where I represents Indian and F represents foreigners.

3 Indians from 6 Indians can be selected in ${}^6C_3$ ways

6 foreigners out of 8 can be selected in $^8C_6$ ways.

Similarly, we can find the number of ways in all other cases.

So, total number of ways in which committee can be formed = $^6C_2 \times {}^8C_4 + {}^6C_3 \times {}^8C_6 + {}^6C_4 \times {}^8C_8$

$= 15 \times 70 + 20 \times 28 + 15 \times 1$

$= 1050 + 560 + 15 = 1625$

Hint:

(i) No. of ways to select r persons out of n persons = ^nC_r.

(ii) Add the number of ways possible in each of the cases.

Shortcut Method:

Required number of ways

$= {}^6C_2 \times {}^8C_4 + {}^6C_3 \times {}^8C_6 + {}^6C_4 \times {}^8C_8$

$= 15 \times 70 + 20 \times 28 + 15 \times 1$

$= 1050 + 560 + 15$

$= 1625$

3. **Option (3) is correct.**

Given: A plane is passing through point (1, 2, – 3) and perpendicular to the planes

$3x + y - 2z = 5$ and $2x - 5y - z = 7$.

Plane $P_1 : 3x + y - 2z = 5$

Plane $P_2 : 2x - 5y - z = 7$.

Normal to P_1 has direction ratios (3, 1, – 2)

Direction ratios of normal to P_2 are (2, – 5, – 1).

Let $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$

and $\vec{b} = 2\hat{i} - 5\hat{j} - \hat{k}$

Direction ratios of required plane's normal is parallel to $\vec{a} \times \vec{b}$ *i.e.*,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 2 & -5 & -1 \end{vmatrix}$$

$$= \hat{i}(-1-10) - \hat{j}(-3+4) + \hat{k}(-15-2)$$

$$= -11\hat{i} - \hat{j} - 17\hat{k}$$

So, Direction ratios of normal to required plane are (–11, –1, –17)

Equation of plane passing through (x_1, y_1, z_1) and direction ratios of normal are (a, b, c) is $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

So, required plane is

$$-11(x-1) - 1(y-2) - 17(z+3) = 0$$

$$\Rightarrow \quad -11x + 11 - y + 2 - 17z - 51 = 0$$

$$\Rightarrow \quad 11x + y + 17z + 38 = 0$$

Hint:

(i) Coefficients of x, y, z in equations of planes are direction ratios of normal.

(ii) Normal vector is $a\hat{i} + b\hat{j} + c\hat{k}$ if direction ratios are a, b, c

(iii) Find the cross product of both normal vectors to find the normal vector of the plane perpendicular to both.

(iv) Finally, write the equation of required plane in the form $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$ where a, b, c are direction ratios of normal to the plane and plane passes through (x_1, y_1, z_1).

Shortcut Method:

$$\vec{n} = (3\hat{i} + \hat{j} - 2\hat{k}) \times (2\hat{i} - 5\hat{j} - \hat{k})$$

$$= -11\hat{i} - \hat{j} - 17\hat{k}$$

$\therefore$ Equation of plane is

$11(x - 1) + (y - 2) + 17(z + 3) = 0$

$\Rightarrow \quad 11x + y + 17z + 38 = 0$

4. **Option (1) is correct.**

Given: A man is walking on straight line. The arithmetic mean of the reciprocals of the intercepts of this line on the coordinate areas is $\frac{1}{4}$. Three stones A, B and C are placed at the points (1, 1), (2, 2) and (4, 4) respectively.

Equation of straight line having intercepts a and b is

$$\frac{x}{a} + \frac{y}{b} = 1$$

This line passes through (h, k)

$$\therefore \quad \frac{h}{a} + \frac{k}{b} = 1 \quad \text{...(i)}$$

A.M. of reciprocals of intercepts

$$\frac{\frac{1}{a}+\frac{1}{b}}{2} = \frac{1}{4}$$

$$\Rightarrow \quad \frac{1}{a}+\frac{1}{b} = \frac{1}{2} \quad \text{...(ii)}$$

Compare (i) and (ii)

$$h = 2, k = 2$$

The line passes through (2, 2).

So, only stone B is on the path of man.

Hint:

(i) Double intercept form of line with intercepts a and b is

$$\frac{x}{a}+\frac{y}{b} = 1$$

(ii) A.M. of reciprocals of intercepts of this line $= \frac{\frac{1}{a}+\frac{1}{b}}{2}$

Shortcut Method:

Equation: $\frac{x}{a}+\frac{y}{b} = 1$

$$\text{A.M.} = \frac{\frac{1}{a}+\frac{1}{b}}{2} = \frac{1}{4}$$

$$\Rightarrow \quad \frac{1}{a}+\frac{1}{b} = \frac{1}{2}$$

$$(x, y) \equiv (2, 2)$$

Stone B is in path of man.

5. Option (4) is correct.

Tautology is the case where each value is true.

In option (1),

$A \wedge (A \vee B) \equiv T$ or F

In option (2),

$\Rightarrow \quad B \rightarrow [A \wedge (A \rightarrow B)]$

$\equiv B \rightarrow [A \wedge (\sim A \vee B)]$

$\equiv B \rightarrow F$

$\equiv T$ or F

In option (3),

$\Rightarrow \quad A \vee (A \wedge B) \equiv T$ or F

In option (4),

$[A \wedge (A \rightarrow B)] \rightarrow B$

$\Rightarrow \quad A \wedge (\sim A \vee B) \rightarrow B$

$= [(A \wedge \sim A) \vee (A \wedge B)] \rightarrow B$

$= (A \wedge B) \rightarrow B$

$= (\sim A \vee \sim B) \vee B$

$= T$

Hint:

(i) $A \rightarrow B \equiv \sim A \vee B$

(ii) $A \wedge \sim A \equiv F$

(iii) $A \vee \sim A \equiv T$

6. Option (4) is correct.

Given: $f(x) = 2x - 1$ and $g: R - \{1\} \rightarrow R$

$$g(x) = \frac{x-\frac{1}{2}}{x-1}$$

$$f(g(x)) = 2g(x) - 1 = \frac{2\left(x-\frac{1}{2}\right)}{x-1} - 1$$

$$= \frac{2x-1-x+1}{x-1}$$

$$= \frac{x}{x-1}$$

$$= \frac{x-1+1}{x-1}$$

$$= \frac{x-1}{x-1}+\frac{1}{x-1}$$

$$y = f(g(x)) = 1+\frac{1}{x-1}$$

Differentiate both sides w.r.t. x

$$\frac{d}{dx} f(g(x)) = 0 - \frac{1}{(x-1)^2}$$

$$\Rightarrow \quad \frac{d}{dx} f(g(x)) = -\frac{1}{(x-1)^2}$$

which is always negative.

So, $f(g(x))$ is strictly decreasing function.

Hence, $f(g(x))$ is one-one function.

To check for onto function:

Let $\quad y = 1+\frac{1}{x-1}$

$$\Rightarrow \quad y - 1 = \frac{1}{x-1}$$

$$\Rightarrow \quad x - 1 = \frac{1}{y-1}$$

$$\Rightarrow \quad x = 1+\frac{1}{y-1}, y \neq 1$$

$\therefore \quad x$ is defined for all $y \in R - \{1\} \Rightarrow y = 1$ has no pre-image

Hence, $f(g(x))$ is into function.

Hint:

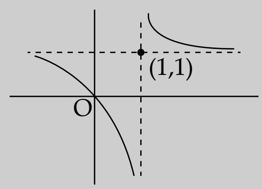
(i) If a function is strictly increasing or strictly decreasing in entire domain, then function is one-one.

(ii) If range of a function is equal to co-domain then function is onto.

(iii) If range of a function is completely a subset of co-domain, then function is into.

Shortcut Method:

$$f(g(x)) = 2g(x) - 1 = \frac{x}{x-1} = 1 + \frac{1}{x-1}$$



So, function is one-one and into.

7. Option (4) is correct.

Given: $f(x) = [x-1]\cos\left(\frac{2x-1}{2}\right)\pi$

$[x-1]$ is discontinuous at all points where $x-1$ is an integer.

$\cos\left(\frac{2x-1}{2}\right)\pi$ is always continuous.

So, check whether $f(x)$ is discontinuous at $x = n$, $n \in I$

L.H.L $= \lim_{x\to n^-} f(x)$

$= \lim_{x\to n^-} [x-1]\cos\left(\frac{2x-1}{2}\right)\pi$

$= \lim_{h\to 0} [n-h-1]\cos\left(\frac{2n-2h-1}{2}\right)\pi$

$= (n-1)\cos(2n-1)\frac{\pi}{2} = 0$

$\left(\because \cos x \text{ is equal to zero when } x \text{ is odd integral multiple of } \frac{\pi}{2}\right)$

R.H.L. $= \lim_{x\to n^+} f(x)$

$= \lim_{x\to n^+} [x-1]\cos\left(\frac{2x-1}{2}\right)\pi$

$= \lim_{h\to 0} [n+h-1]\cos\left(\frac{2n+2h-1}{2}\right)\pi$

$= (n-1)\cos(2n-1)\frac{\pi}{2}$

$= 0$

$f(n) = [n-1]\cos(2n-1)\frac{\pi}{2}$

$= 0$

So, $f(n^-) = f(n^+) = f(n)\ \forall n \in I$

Hence, $f(x)$ is continuous for every real x.

Hint:

(i) $[x-1]$ is discontinuous for all $x \in I$.

(ii) $\cos(2n-1)\frac{\pi}{2} = 0$

Shortcut Method:

Check the options

$\cos(2x-1)\frac{\pi}{2}$ is zero in surrounding of 1

So, $f(x)$ is continuous at $x = 1$

Similarly, case arise for all integers.

So, only correct option is (4).

8. Option (1) is correct.

Given:

$$f(x) = \frac{4x^3 - 3x^2}{6} - 2\sin x + (2x-1)\cos x$$

Now,

$f'(x) = (2x^2 - x) - 2\cos x + 2\cos x - \sin x(2x-1)$

$= (2x-1)(x - \sin x)$

for $x > 0$, $x - \sin x > 0$

and, $x < 0$, $x - \sin x < 0$

for $x \in (-\infty, 0] \cup \left[\frac{1}{2}, \infty\right)$, $f'(x) \geq 0$

for $x \in \left[0, \frac{1}{2}\right]$, $f'(x) \leq 0$

$\Rightarrow f(x)$ increases in $\left[\frac{1}{2}, \infty\right)$.

9. Option (1) is correct.

Given :

Point is P(1, 1, 9)

Line is $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$

Plane is $x + y + z = 17$

$$\frac{x-3}{1}=\frac{y-4}{2}=\frac{z-5}{2}=k \text{ (say)}$$

$\Rightarrow$ $x = k + 3, y = 2k + 4, z = 2k + 5$

$(k + 3, 2k + 4, 2k + 5)$

Points (x, y, z) lies on plane $x + y + z = 17$

$\Rightarrow$ $k + 3 + 2k + 4 + 2k + 5 = 17$

$\Rightarrow$ $5k = 5$

$\Rightarrow$ $k = 1$

Point of intersection of line and plane is

Q $(k + 3, 2k + 4, 2k + 5)$ *i.e.*

Q $(4, 6, 7)$

Required distance = PQ

$$=\sqrt{(4-1)^2+(6-1)^2+(9-7)^2}$$

(using distance formula)

$$=\sqrt{3^2+5^2+2^2}$$

$$=\sqrt{38}$$

Hint:

(i) Consider a general point on line.

(ii) Substitute the point in plane to find value of parameter and later find the point.

(iii) Use distance formula *i.e.*

Distance between $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

Shortcut Method:

$x = k + 3, y = 2k + 4, z = 2k + 5$

lies on plane, $x + y + z = 17$

$\therefore$ $k + 3 + 2k + 4 + 2k + 5 = 17$

$\Rightarrow$ $5k = 5$

$\Rightarrow$ $k = 1$

Required distance between points (1, 1, 9) and (4, 6, 7):

$$=\sqrt{(4-1)^2+(6-1)^2+(9-7)^2}$$

$$=\sqrt{38}$$

10. Option (1) is correct.

Given: $\lim\limits_{x\to 0^+}\dfrac{\int_0^{x^2}(\sin\sqrt{t})dt}{x^3}$

Apply Leibniz's rule:

$$\frac{d}{dx}\int_{f(x)}^{g(x)}u(t)\,dt = u(g(x)).\,g'(x)-u(f(x)).f'(x)$$

$$\therefore \quad \frac{d}{dx}\int_0^{x^2}\sin\sqrt{t}dt=\sin\left(\sqrt{x^2}\right).2x-\sin(0).0$$

$$=\sin(|x|).2x$$

Apply L' Hospital's rule in

$$\lim_{x\to 0^+}\frac{\int_0^{x^2}(\sin\sqrt{t})dt}{x^3}$$

$$=\lim_{x\to 0^+}\frac{\frac{d}{dx}\int_0^{x^2}(\sin\sqrt{t})\,dt}{\frac{d}{dx}(x^3)}$$

$$=\lim_{x\to 0^+}\frac{2x.\sin(|x|)}{3x^2}$$

$$=\frac{2}{3}\lim_{x\to 0^+}\frac{\sin|x|}{x}$$

$$=\frac{2}{3}\times 1=\frac{2}{3} \qquad \left(\because \lim_{x\to 0}\frac{\sin x}{x}=1\right)$$

Hint:

(i) Leibniz's rule:

$$\frac{d}{dx}\int_{f(x)}^{g(x)}u(t)\,dt=u\,(g(x)).\,g'(x)-u(f(x)).f'(x)$$

(ii) L' Hospital's rule :

If $f(x)$ and $g(x)$ are differentiable then

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}=\lim_{x\to a}\frac{f''(x)}{g''(x)}$$

This rule is applied till $\frac{0}{0}$ or $\frac{\infty}{\infty}$ indeterminate form is not removed.

(iii) $\lim\limits_{x\to 0}\dfrac{\sin x}{x}=1$

Shortcut Method:

$$\lim_{x\to 0^+}\frac{\int_0^{x^2}(\sin\sqrt{t})dt}{x^3}=\lim_{x\to 0^+}\frac{(\sin(|x|)).2x}{3x^2}$$

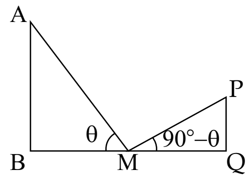
$$=\lim_{x\to 0^+}\left(\frac{\sin x}{x}\right)\times\frac{2}{3}=\frac{2}{3}$$

11. Option (4) is correct.

Given: Let AB and PQ are poles 150m apart.

Height of AB is 3 times that of PQ.

Let midpoint of BQ is M.



Let $\angle AMB = \theta$

$\therefore \quad \angle PMQ = 90° - \theta$

($\because$ Both angles are complementary)

Let AB = h, PQ = $\frac{h}{3}$, BM = x, QM = x

In $\Delta AMB, \tan\theta = \frac{h}{x}$ ...(i)

In $\Delta PMQ, \tan(90° - \theta) = \frac{\frac{h}{3}}{x}$

$\Rightarrow \quad \cot\theta = \frac{h}{3x}$ ...(ii)

Multiply equation (i) and equation (ii)

$$\frac{h}{x} \times \frac{h}{3x} = 1$$

$\Rightarrow \quad h^2 = 3x^2$

$\Rightarrow \quad h = \sqrt{3}x$

Given, $2x = 150 \quad \Rightarrow x = 75$

$\therefore \quad h = 75\sqrt{3}$

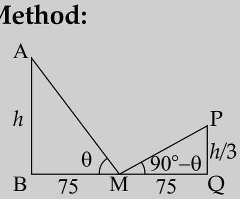
Height of shorter pole $= \frac{h}{3} = 25\sqrt{3}$ m

Hint:

(i) $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$

(ii) Complementary angles are angles whose sum is 90°.

Shortcut Method:



$$\tan\theta \times \cot\theta = \frac{h}{75} \times \frac{h}{3 \times 75}$$

$\Rightarrow \quad h = 75\sqrt{3}$

$\Rightarrow \quad \frac{h}{3} = 25\sqrt{3}\text{m}$

$\text{PQ} = 25\sqrt{3}\text{m}$

12. Option (1) is correct.

Given: Tangent to the curve $y = x^3$ at the point P(t, t^3) meets the curve again at Q

Curve is $y = x^3$ ...(i)

Differentiate both sides w.r.t. x

$$\frac{dy}{dx} = 3x^2$$

$\Rightarrow \quad \left.\frac{dy}{dx}\right|_{(t,t^3)} = 3t^2$ = Slope of tangent at P

Equation of straight line having slope m and passing through (x_1, y_1) is

$$y - y_1 = m(x - x_1)$$

Here, $\quad m = 3t^2, (x_1, y_1) \equiv (t, t^3)$

$\therefore$ Equation of tangent at P(t, t^3) is

$(y - t^3) = 3t^2(x - t)$ ...(ii)

Putting value of y from equation (i), we get

$x^3 - t^3 = 3t^2(x - t)$

$\Rightarrow \quad x^2 + xt + t^2 = 3t^2$

$\Rightarrow \quad x^2 + xt - 2t^2 = 0$

$\Rightarrow \quad (x - t)(x + 2t) = 0$

$\Rightarrow \quad x_1 = t, x_2 = -2t,$

From, $\quad y = x^3$

$y_1 = t^3, y_2 = -8t^3$

$\therefore$ Point Q is $(-2t, -8t^3)$.

Use section formula, to find R which divides PQ internally in the ratio 1 : 2

$$\therefore R \equiv \left(\frac{1 \times (-2t) + 2t}{1+2}, \frac{1 \times (-8t^3) + 2t^3}{1+2}\right) \equiv (0, -2t^3)$$

So, ordinate of required point is $-2t^3$.

Hint:

(i) Equation of straight line passing through (x_1, y_1) and slope m is $(y - y_1) = m(x - x_1)$

(ii) Coordinates of point which divides (x_1, y_1) and (x_2, y_2) in the ratio $m:n$ is

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right).$$

Shortcut Method:

Equation of tangent at P(t, t^3)

$(y - t^3) = 3t^2(x - t)$ (i)

Solve the above equation with $y = x^3$

$x = -2t, y = -8t^3$

Ordinate of required point

$$= \frac{2t^3 + (-8t^3)}{3} = -2t^3.$$

13. Option (4) is correct.

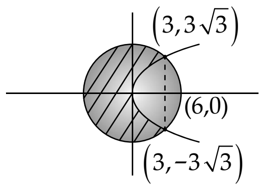
Given: Circle $x^2 + y^2 = 36$

Parabola $y^2 = 9x$

Solve equations to find point of intersection.

$$x^2 + 9x - 36 = 0$$

$\Rightarrow$ $x = -12, 3$



Required Area

$$= \pi r^2 - 2\left[\int_0^3 \sqrt{9x}\,dx + \int_3^6 \sqrt{36 - x^2}\,dx\right]$$

$$= 36\pi - 12\sqrt{3} - 2\left(\frac{x}{2}\sqrt{36 - x^2} + 18\sin^{-1}\left(\frac{x}{6}\right)\right)_3^6$$

$$= 36\pi - 12\sqrt{3} - 2\left(9\pi - \left(\frac{9\sqrt{3}}{2} + 3\pi\right)\right)$$

$= 24\pi - 3\sqrt{3}$ sq. units

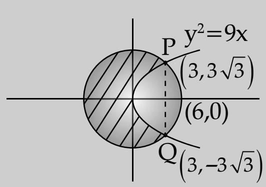
Hint:

(i) $\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$

(ii) $\int \sqrt{x}\,dx = \frac{2}{3}x^{\frac{3}{2}} + c$

Shortcut Method:

Required area



$$= \pi r^2 - 2\left[\int_0^3 \sqrt{9x}\,dx + \int_3^6 \sqrt{36 - x^2}\,dx\right]$$

$$= 36\pi - 12\sqrt{3} - 2\left(9\pi - \left(\frac{9\sqrt{3}}{2} + 3\pi\right)\right)$$

$= 24\pi - 3\sqrt{3}$ sq.units.

14. Option (2) is correct.

Given :

$$I = \int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}}\,dx = a\sin^{-1}\left(\frac{\sin x + \cos x}{b}\right) + c$$

Let, $\sin x + \cos x = t$ ....(i)

Squaring both sides, we get

$(\sin x + \cos x)^2 = t^2$

$\Rightarrow$ $\sin^2 x + \cos^2 x + 2\sin x \cos x = t^2$

$\Rightarrow$ $1 + 2\sin x\cos x = t^2$ ($\because \sin^2 x + \cos^2 x = 1$)

$\Rightarrow$ $\sin 2x = t^2 - 1$ ($\because 2\sin x\cos x = \sin 2x$)

Differentiate equation (i) both sides w.r.t. x

$(\cos x - \sin x)dx = dt$ ....(ii)

Putting value we get,

$$I = \int \frac{dt}{\sqrt{9 - t^2}} = \sin^{-1}\left(\frac{t}{3}\right) + c$$

$$\left(\because \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c\right)$$

$$= \sin^{-1}\left(\frac{\sin x + \cos x}{3}\right) + c$$

Compare with $a\sin^{-1}\left(\frac{\sin x + \cos x}{b}\right) + c$

$\therefore$ $a = 1$ and $b = 3$

Ordered pair $(a, b) = (1, 3)$

Hint:

(i) Put $\sin x + \cos x = t$ in integral of given form.

(ii) $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c$

Shortcut Method:

Put $\sin x + \cos x = t \Rightarrow \sin 2x = t^2 - 1$

$\Rightarrow$ $(\cos x - \sin x)dx = dt$

$\therefore$ $I = \int \frac{dt}{\sqrt{9 - t^2}} = \sin^{-1}\left(\frac{t}{3}\right) + c$

$$= \sin^{-1}\left(\frac{\sin x + \cos x}{3}\right) + c$$

$\Rightarrow$ $a = 1$ and $b = 3$

15. Option (2) is correct.

Given:

Population (P) = P (t) at time 't' of a certain species follows the differential equation.

$$\frac{dP}{dt} = 0.5\,P - 450 = \frac{P}{2} - \frac{900}{2}$$

$$\Rightarrow \quad \frac{dP}{dt} = \frac{P-900}{2}$$

It is in variable-separable form

$$\therefore \quad \frac{dP}{P-900} = \frac{1}{2}dt$$

Integrate both sides

P = 850 at $t = 0$

P = 0 at $t = t$

$$\int_{850}^{0} \frac{dP}{P-900} = \frac{1}{2}\int_{0}^{t} dt$$

$$\Rightarrow \quad \ln|P-900|\Big|_{850}^{0} = \frac{1}{2}[t]_0^t \quad \left(\because \int \frac{dx}{x} = ln|x| + c, \ \int dx = x + c\right)$$

$$\Rightarrow \quad \ln 900 - \ln 50 = \frac{1}{2}t$$

$$\Rightarrow \quad \ln\left(\frac{900}{50}\right) = \frac{t}{2}$$

$$\left[\text{Using } \ln x - \ln y = \ln\frac{x}{y}\right]$$

$$\Rightarrow \quad \ln 18 = \frac{t}{2}$$

$$\Rightarrow \quad t = 2\ln 18$$

Hint:

(i) $\frac{dy}{dx} = \frac{f(x)}{g(y)}$ is variable separable form that can be solved as $\int g(y)dy = \int f(x)dx$

(ii) $\int \frac{dx}{x} = ln|x| + c$

(iii) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$ where $n \neq -1$.

Shortcut Method:

$$\frac{dP}{dt} = \frac{P-900}{2}$$

$$\Rightarrow \quad \int_{850}^{0} \frac{dP}{P-900} = \frac{1}{2}\int_{0}^{t} dt$$

$$\Rightarrow \quad \ln\left(\frac{900}{50}\right) = \frac{t}{2}$$

$$\Rightarrow \quad t = 2\ln 18$$

16. Option (4) is correct.

Given:

$$S = -{}^{15}C_1 + 2.{}^{15}C_2 - 3.{}^{15}C_3 + \ldots\ldots - 15.{}^{15}C_{15} + {}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + \ldots\ldots + {}^{14}C_{11}$$

Let $S_1 = -{}^{15}C_1 + 2.{}^{15}C_2 - \ldots\ldots - 15.{}^{15}C_{15}$

$$= \sum_{r=1}^{15}(-1)^r.r.{}^{15}C_r$$

$$= 15\sum_{r=1}^{15}(-1)^r.{}^{14}C_{r-1}$$

$$= 15(-{}^{14}C_0 + {}^{14}C_1 - \ldots\ldots - {}^{14}C_{14})$$

$$= 15(0) = 0.$$

$$(\because -{}^{n}C_0 + {}^{n}C_1 - \ldots\ldots - {}^{n}C_n = 0)$$

$$S_2 = {}^{14}C_1 + {}^{14}C_3 + \ldots. + {}^{14}C_{11}$$

$$= ({}^{14}C_1 + {}^{14}C_3 + \ldots.. + {}^{14}C_{11} + {}^{14}C_{13}) - {}^{14}C_{13}$$

Since, ${}^{n}C_1 + {}^{n}C_3 + {}^{n}C_5 + \ldots\ldots = 2^{n-1}$

$\therefore$ $S_2 = 2^{13} - 14$

Now, $S = S_1 + S_2 = 0 + 2^{13} - 14$

$\therefore$ $S = 2^{13} - 14$

Hint:

(i) $(1+x)^n = {}^{n}C_0 + {}^{n}C_1 x + {}^{n}C_2 x^2 + \ldots\ldots + {}^{n}C_n x^n$

(ii) Put $x = -1$

$0 = {}^{n}C_0 - {}^{n}C_1 + {}^{n}C_2 - \ldots\ldots + {}^{n}C_n (-1)^n$

(iii) ${}^{n}C_0 + {}^{n}C_2 + {}^{n}C_4 + \ldots\ldots = 2^{n-1}$

(iv) ${}^{n}C_1 + {}^{n}C_3 + {}^{n}C_5 + \ldots\ldots = 2^{n-1}$.

Shortcut Method:

$$S = (-{}^{15}C_1 + 2.{}^{15}C_2 - 3.{}^{15}C_3 + \ldots\ldots - 15.{}^{15}C_{15}) + ({}^{14}C_1 + {}^{14}C_3 + \ldots\ldots + {}^{14}C_{11})$$

$$= 15\sum_{r=1}^{15}(-1)^r.{}^{14}C_{r-1} + ({}^{14}C_1 + {}^{14}C_3 +$$
$$\ldots\ldots + {}^{14}C_{13}) - {}^{14}C_{13}$$
$$= 15(0) + 2^{13} - 14$$
$$= 2^{13} - 14$$

17. Option (2) is correct.

Given: An ordinary dice is rolled for a certain number of times. Probability of getting an odd number 2 times is equal to the probability of getting an even number 3 times.

Probability of r successes in n trials

$$= {}^nC_r p^r q^{n-r}$$

where p = Probability of success

$q = 1 - p$ = Probability of failure

p = Probability of getting odd number

$$= \frac{3}{6} = \frac{1}{2}$$

q = Probability of getting even number

$$= 1 - \frac{1}{2} = \frac{1}{2}$$

P (getting odd number twice) = P (getting even number thrice)

$$\Rightarrow \quad {}^nC_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{n-2} = {}^nC_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{n-3}$$

$$\Rightarrow \quad {}^nC_2 = {}^nC_3$$

$$\Rightarrow \quad n = 2 + 3$$

$$\Rightarrow \quad n = 5$$

P(getting odd number for odd number of times)

= P(odd no. 1 times) + P(odd no. 3 times) + P(odd no. 5 times)

$$= {}^5C_1\left(\frac{1}{2}\right)^5 + {}^5C_3\left(\frac{1}{2}\right)^5 + {}^5C_5\left(\frac{1}{2}\right)^5$$

$$= \frac{16}{2^5} = \frac{1}{2}.$$

Hint:

(i) P(r successes) = ${}^nC_r p^r q^{n-r}$

where p = probability of success

q = probability of failure

(ii) P(getting odd no.)

= P(getting 1, 3 or 5)

$$= \frac{3}{6} = \frac{1}{2}$$

(iii) P(getting even no.) = P(getting 2, 4 or 6)

$$= \frac{3}{6} = \frac{1}{2}.$$

Shortcut Method:

P(odd no. twice) = P(even no. thrice)

$$\Rightarrow \quad {}^nC_2\left(\frac{1}{2}\right)^n = {}^nC_3\left(\frac{1}{2}\right)^n$$

$$\Rightarrow \quad n = 5$$

P(odd successes) = P(1) + P(3) + P(5)

$$= ({}^5C_1 + {}^5C_3 + {}^5C_5).\left(\frac{1}{2}\right)^5$$

$$= \frac{2^4}{2^5} = \frac{1}{2}$$

18. Option (4) is correct.

Given: p and q are two positive numbers such that $p + q = 2$ and $p^4 + q^4 = 272$

$$(p + q)^2 = 2^2$$
$$\Rightarrow \quad p^2 + q^2 + 2pq = 4$$
$$p^2 + q^2 = 4 - 2pq$$

Squaring both sides,

$$(p^2 + q^2)^2 = (4 - 2pq)^2$$
$$\Rightarrow \quad p^4 + q^4 + 2p^2q^2 = 16 + 4p^2q^2 - 16pq$$
$$\Rightarrow \quad 272 - 2p^2q^2 = 16 - 16pq$$
$$(\because p^4 + q^4 = 272)$$
$$\Rightarrow \quad p^2q^2 - 8pq - 128 = 0$$
$$\Rightarrow \quad (pq)^2 - 8pq - 128 = 0$$
$$pq = \frac{8 \pm 24}{2} = 16 \text{ or } -8$$
$$\Rightarrow \quad pq = 16$$

Now, quadratic equation whose roots are p, q is

$$x^2 - (p + q)x + pq = 0$$
$$\Rightarrow \quad x^2 - 2x + 16 = 0$$

Hint:

(i) $(p + q)^2 = p^2 + q^2 + 2pq$

(ii) $p^4 + q^4 = (p^2 + q^2)^2 - 2p^2q^2$

$= ((p + q)^2 - 2pq)^2 - 2p^2q^2$

Shortcut Method:

$$p^4 + q^4 = (p^2 + q^2)^2 - 2p^2q^2$$
$$= ((p + q)^2 - 2pq)^2 - 2p^2q^2$$
$$\Rightarrow \quad (pq)^2 - 8pq - 128 = 0$$
$$\Rightarrow \quad pq = 16 \quad (\because p, q \text{ are positive})$$

Now, quadratic eq^n whose roots are p, q

$$x^2 - (p + q)x + pq = 0$$
$$\Rightarrow \quad x^2 - 2x + 16 = 0$$

19. Option (3) is correct.

Given: $e^{(\cos^2 x+\cos^4 x+\cos^6 x+.......)\log_e 2}$ satisfies the equation $t^2 - 9t + 8 = 0$.

Sum of G.P. with first term = a, common ratio = r for infinite terms = $\frac{a}{1-r}$.

In case of $\cos^2 x + \cos^4 x + \cos^6 x......$

$$a = \cos^2 x,\ r = \frac{\cos^4 x}{\cos^2 x} = \cos^2 x$$

$$\cos^2 x + \cos^4 x + \cos^6 x...... = \frac{\cos^2 x}{1-\cos^2 x} = \frac{\cos^2 x}{\sin^2 x} = \cot^2 x$$

$$e^{(\cos^2 x+\cos^4 x+.......)\log_e 2} = e^{\cot^2 x.\log_e 2} = e^{\log_e 2^{\cot^2 x}}$$

$(\because\ m\log x = \log x^m)$

$$= 2^{\cot^2 x}$$

$\left(\because a^{\log a^x} = x\right)$

$2^{\cot^2 x}$ satisfies $t^2 - 9t + 8 = 0$

$\Rightarrow\quad t^2 - t - 8t + 8 = 0$

$\Rightarrow\quad t(t-1) - 8(t-1) = 0$

$\Rightarrow\quad t = 1$ or 8

$\Rightarrow$ Either, $2^{\cot^2 x} = 1$ or $2^{\cot^2 x} = 8$

$\Rightarrow\quad 2^{\cot^2 x} = 2^0$ or $2^{\cot^2 x} = 2^3$

$\Rightarrow\quad \cot^2 x = 0$ or $\cot^2 x = 3$

$\because\quad 0 < x < \frac{\pi}{2}$

$\therefore\quad \cot x = \sqrt{3} \quad\Rightarrow\quad x = \frac{\pi}{6}$

Now, $$\frac{2\sin x}{\sin x+\sqrt{3}\cos x} = \frac{2}{1+\sqrt{3}\times\cot\frac{\pi}{6}} = \frac{1}{2}$$

Hint:

(i) Sum of infinite terms of G.P. with first term = a and common ratio = r is $\frac{a}{1-r}$

(ii) $m\log_a x = \log_a x^m \quad (a>0, a\neq 1, x^m>0, x>0)$

(iii) $a^{\log_a x} = x \qquad (a>0, a\neq 1)$

Shortcut Method:

$$e^{(\cos^2 x+\cos^4 x+\cos^6 x+.......)\log_e 2}$$

$$= e^{\log_e 2^{\cot^2 x}}$$

$$= 2^{\cot^2 x}$$

$t^2 - 9t + 8 = 0 \Rightarrow t = 1$ or 8

$0 < x < \frac{\pi}{2}$

$\cot x = \sqrt{3}$

$$\Rightarrow \frac{2\sin x}{\sin x+\sqrt{3}\cos x} = \frac{2}{1+\sqrt{3}\cot x} = \frac{2}{4} = \frac{1}{2}$$

20. Option (4) is correct.

Given: System of linear equations

$3x - 2y - kz = 10$
$2x - 4y - 2z = 6$
$x + 2y - z = 5m$

is inconsistent.

Form determinant D with coefficients of x, y, z.
D_x is formed by replacing coefficients of x with constant terms.
D_y is formed by replacing coefficients of y with constant terms.
D_z is formed by replacing coefficients of z with constant terms.

For, equation to be inconsistent,

$$D = \begin{vmatrix} 3 & -2 & -k \\ 2 & -4 & -2 \\ 1 & 2 & -1 \end{vmatrix} = 0$$

$\Rightarrow\quad 3(4+4) + 2(-2+2) - k(4+4) = 0$

$\Rightarrow\quad k = 3$

Also, at least one of D_x, D_y, D_z must be non-zero.

$$D_x = \begin{vmatrix} 10 & -2 & -3 \\ 6 & -4 & -2 \\ 5m & 2 & -1 \end{vmatrix} \neq 0$$

$\Rightarrow\quad 10(4+4) + 2(-6+10m) - 3(12+20m) \neq 0$

$\Rightarrow\quad 80 - 12 + 20m - 36 - 60m \neq 0$

$\Rightarrow\quad 40m \neq 32$

$\Rightarrow\quad m \neq \frac{4}{5}$

$$D_y = \begin{vmatrix} 3 & 10 & -3 \\ 2 & 6 & -2 \\ 1 & 5m & -1 \end{vmatrix}$$

$$= -\begin{vmatrix} 3 & 10 & 3 \\ 2 & 6 & 2 \\ 1 & 5m & 1 \end{vmatrix} = 0$$

($\because$ Two columns of D_y are same)

$$D_z = \begin{vmatrix} 3 & -2 & 10 \\ 2 & -4 & 6 \\ 1 & 2 & 5m \end{vmatrix} \neq 0$$

$\Rightarrow$ $3(-20m - 12) + 2(10m - 6) + 10(4 + 4)$
$-40m + 32 \neq 0$

$\Rightarrow$ $m \neq \frac{4}{5}$

Hint:
(i) D is formed by coefficients of x, y, z in rows or columns of determinant.
(ii) Replace coefficients of x by constant terms in D to form D_x.
(iii) Similarly, D_y and D_z are formed by replacing coefficients of y and z by constant terms respectively.
(iv) For the system to be inconsistent D = 0 and at least one of D_x, D_y or D_z is non-zero.

Shortcut Method:

$3x - 2y - kz = 10$ ...(i)
$2x - 4y - 2z = 6$
$\Rightarrow$ $x - 2y - z = 3$ ...(ii)
$x + 2y - z = 5m$ ...(iii)

From (2), $z = x - 2y - 3$
(1) $\Rightarrow$ $3x - 2y - k(x - 2y - 3) = 10$
$\Rightarrow$ $(3 - k)x + (2k - 2)y + 3k - 10 = 0$ ...(iv)
(3) $\Rightarrow$ $x + 2y - x + 2y + 3 = 5m$
$\Rightarrow$ $4y = 5m - 3$
$\Rightarrow$ $4y - 5m + 3 = 0$ ...(v)

For eq^{n}s to be inconsistent,

$$\frac{3-k}{0} = \frac{2k-2}{4} \neq \frac{3k-10}{-5m+3}$$

$\Rightarrow$ $k = 3, -5m + 3 \neq 3k - 10$
$\Rightarrow$ $-5m + 3 \neq -1$
$\Rightarrow$ $m \neq \frac{4}{5}$

Section B

21. Correct answer is [15].

Given:

$$P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}, \alpha \in R$$

$PQ = k\,I_3$ for some non-zero $k \in R$
where $Q = [q_{ij}]$

$$q_{23} = \frac{-k}{8} \text{ and } |Q| = \frac{k^2}{2}$$

$PQ = kI$

Multiply both sides by P^{-1}

$P^{-1}PQ = kP^{-1}I$
$P^{-1}P = I, P^{-1}I = P^{-1}$

$\therefore$ $Q = kP^{-1}$

$\Rightarrow$ $\frac{1}{k}Q = P^{-1}$

$$Q = k\frac{1}{|P|}(\text{adj}(P)).I$$

$q_{23} = \frac{-k}{8} \Rightarrow 2^{nd}$ row, 3^{rd} column element in P^{-1} is $\frac{-1}{8}$

Also $|P| = 3(0 + 5\alpha) + 1(0 - 3\alpha)\ 2(-10)$
$= 15\alpha - 3\alpha + 20$
$= 12\alpha + 20$

$$\Rightarrow \frac{1}{20 + 12\alpha}(\text{cofactor of } P_{32}) = \frac{-1}{8}$$

$$= \frac{1}{20 + 12\alpha}(-(3\alpha + 4)) = \frac{-1}{8}$$

$\Rightarrow$ $2(3\alpha + 4) = 5 + 3\alpha \Rightarrow \alpha = -1$

and $\left|\frac{1}{k}Q\right| = |P^{-1}| = \frac{1}{20 + 12\alpha} = \frac{1}{8}$ $\left\{\because Q = \frac{k^2}{2}\right\}$

$$\frac{1}{k^3}|Q| = \frac{1}{8} \Rightarrow \frac{k^2}{2k^3} = \frac{1}{8} \Rightarrow k = 4$$

Now, $\alpha^2 + k^2$
$= (-1)^2 + (4)^2$
$= 15$

Hint:
(1) $P^{-1}P = I, PI = P$
(2) $P^{-1} = \frac{1}{|P|}\text{adj}(P)$
(3) $|kA| = k^n|A|$ when n is order of matrix A.

22. Correct answer is [6].

Given: B_1, B_2, B_3 are three independent events in a sample space.

P (only B_1 occurs) = α

P (only B_2 occurs) = β

P (only B_3 occurs) = γ

P (none of events B_i occurs) = p

$(\alpha - 2\beta)\, p = \alpha\beta$

Let x, y, z be probability of B_1, B_2, B_3 respectively.

$\Rightarrow \quad x(1-y)(1-z) = \alpha$ ...(i)

$y(1-x)(1-z) = \beta$ ...(ii)

$z(1-x)(1-y) = \gamma$ ...(iii)

$(1-x)(1-y)(1-z) = p$ ...(iv)

Since, $(\alpha - 2\beta)p = \alpha\beta$

$\Rightarrow \quad x(1-y)(1-z)$
$\quad - 2y(1-x)(1-z)\,[(1-x)(1-y)(1-z)]$
$\quad = xy(1-x)(1-y)(1-z)^2$

$\Rightarrow \quad x - xy - 2y + 2xy = xy$

$\Rightarrow \quad x = 2y$ ...(v)

Similarly, $(\beta - 3r)p = 2\beta r$

$\Rightarrow \quad y = 3z$ ...(vi)

From (v) and (vi)

$$x = 6z$$

$$\Rightarrow \quad \frac{x}{z} = 6$$

$$\Rightarrow \quad \frac{P(B_1)}{P(B_3)} = 6$$

Hint:

(i) $P(X) = p$

$P(\overline{X}) = 1 - p.$

(ii) If A, B, C are independent events

P (only A occurs)

$= P(A\overline{B}\overline{C}) = P(A).P(\overline{B}).P(\overline{C}).$

(iii) Similarly, for P(B) and P(C)

(iv) P(none occurs) $= P(\overline{A}).P(\overline{B}).P(\overline{C})$

Shortcut Method:

$P(B_1) = x,\ P(B_2) = y,\ P(B_3) = z$

$x(1-y)(1-z) = \alpha$

$y(1-x)(1-z) = \beta$

$z(1-x)(1-y) = r$

$(1-x)(1-y)(1-z) = p$

$(\alpha - 2\beta)p = \alpha\beta$

$\Rightarrow \quad x = 2y$ ...(i)

$(\beta - 3r)p = 2\beta r$

$\Rightarrow \quad y = 3z$ ...(ii)

From (i) and (ii)

$$x = 6z$$

$$\Rightarrow \quad \frac{x}{z} = 6$$

$$\Rightarrow \quad \frac{P(B_1)}{P(B_3)} = 6$$

23. Correct answer is [9].

$$f(x) = \frac{4}{\sin x} + \frac{1}{1 - \sin x}$$

Let $\sin x = t$

$$0 < x < \frac{\pi}{2}$$

$$\Rightarrow \quad 0 < t < 1$$

$$f(t) = \frac{4}{t} + \frac{1}{1-t}$$

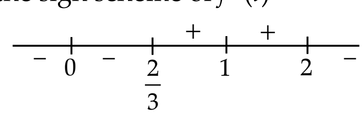
$$f'(t) = \frac{-4}{t^2} + \frac{1}{(1-t)^2}$$

$$= \frac{t^2 - 4(1-t)^2}{t^2(1-t)^2}$$

$$= \frac{(t - 2(1-t))(t + 2(1-t))}{t^2(1-t)^2}$$

$$f'(t) = \frac{(3t-2)(2-t)}{t^2(1-t)^2}$$

Find the sign scheme of $f'(t)$



Minimum of f occurs at the point where f' change its sign from negative to positive.

f is minimum at $t = \frac{2}{3}$

$$\therefore \quad \alpha_{min} = f\left(\frac{2}{3}\right) = \frac{4}{\frac{2}{3}} + \frac{1}{1 - \frac{2}{3}} = 6 + 3 = 9$$

Hint:

(i) Find sign scheme of $f'(t)$.

(ii) f is minimum at the point where f' changes its sign from negative to positive.

(iii) Substitute that value in function to find minimum value.

Shortcut Method:

$$f(x) = \frac{4}{\sin x} + \frac{1}{1-\sin x}$$

$$f(t) = \frac{4}{t} + \frac{1}{1-t}$$

$$f'(t) = \frac{-4}{t^2} + \frac{1}{(1-t)^2} = 0$$

$$t = 2 \text{ or } \frac{2}{3}$$

but $t = \sin x \neq 2$

$\therefore$ $t = \frac{2}{3}$

$$f''\left(\frac{2}{3}\right) > 0$$

f is minimum at $t = \frac{2}{3}$

$$\therefore \quad \alpha_{min} = f\left(\frac{2}{3}\right) = \frac{4}{\frac{2}{3}} + \frac{1}{1-\frac{2}{3}} = 9$$

24. Correct answer is [3].

Given: One of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord of another circle C whose center is at (2, 1).

Compare given equation of circle with

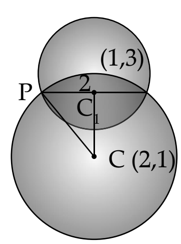
$x^2 + y^2 + 2gx + 2fy + c = 0$

$g = -1, f = -3, c = 6$

Center $\equiv C_1(-g, -f) \equiv C_1(1, 3)$

Radius $= \sqrt{g^2 + f^2 - c}$

$r_1 = \sqrt{(-1)^2 + (-3)^2 - 6} = 2$



Let center of another circle is C

Midpoint of chord coincide with center C_1

Distance between C and C_1

$$= \sqrt{(2-1)^2 + (1-3)^2}$$

$$= \sqrt{1^2 + (-2)^2}$$

$$CC_1 = \sqrt{5}$$

$$PC^2 = CC_1^2 + r_1^2$$

$$= \left(\sqrt{5}\right)^2 + 2^2$$

$$= 5 + 4 = 9$$

$$PC = \sqrt{9} = 3$$

Hence, required radius = 3

Hint:

(i) Perpendicular from the center to the chord bisects the chord

(ii) Draw the figure and apply Pythagoras theorem to find the radius of circle C.

25. Correct answer is [1].

Given: $\lim_{n\to\infty} \tan\left\{\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{1+r+r^2}\right)\right\}$

Let $T_r = \tan^{-1}\left(\frac{1}{1+r+r^2}\right)$

$$= \tan^{-1}\left(\frac{1}{1+r(r+1)}\right)$$

1 can be written as $(r + 1) - r$

$$T_r = \tan^{-1}\left(\frac{(r+1)-r}{1+(r+1)r}\right)$$

$$\left(\because \tan^{-1}\left(\frac{x-y}{1+xy}\right) = \tan^{-1}x - \tan^{-1}y\right)$$

$T_r = \tan^{-1}(r + 1) - \tan^{-1}(r)$

$T_1 = \tan^{-1}(2) - \tan^{-1}(1)$

$T_2 = \tan^{-1}(3) - \tan^{-1}(2)$

$T_3 = \tan^{-1}(4) - \tan^{-1}(3)$

$\vdots$

$T_n = \tan^{-1}(n + 1) - \tan^{-1}(n)$

Adding all, we get

$$\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{1+r+r^2}\right) = \tan^{-1}(n+1) - \tan^{-1}(1) \quad \text{...(i)}$$

$$= \tan^{-1}\left(\frac{(n+1)-1}{1+n+1}\right)$$

$$= \tan^{-1}\left(\frac{n}{n+2}\right)$$

$$= \tan^{-1}\left(\frac{1}{1+\frac{2}{n}}\right)$$

$$\lim_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\left(\frac{1}{1+r+r^2}\right) = \tan^{-1}(1) = \frac{\pi}{4}$$

$$\lim_{n\to\infty}\tan\left\{\sum_{r=1}^{n}\tan^{-1}\left(\frac{1}{1+r+r^2}\right)\right\}$$

$$= \tan\frac{\pi}{4} = 1$$

Hint:

(i) $\tan^{-1}(x) - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$

(ii) $\tan(\tan^{-1}x) = x, \forall\, x \in R$

Shortcut Method:

$$\sum_{r=1}^{n}\tan^{-1}\left(\frac{1}{1+r+r^2}\right)$$

$$= \sum_{r=1}^{n}\left(\tan^{-1}(r+1) - \tan^{-1}(r)\right)$$

$$= \tan^{-1}(n+1) - \tan^{-1}(1)$$

$$= \tan^{-1}\left(\frac{n}{n+2}\right)$$

$$\lim_{n\to\infty}\tan\left(\sum_{r=1}^{n}\tan^{-1}\left(\frac{1}{1+r+r^2}\right)\right)$$

$$= \lim_{n\to\infty}\frac{n}{n+2} = 1$$

26. Correct answer is [3].

Given: $\int_{-a}^{a}(|x|+|x-2|)\,dx = 22 \quad (a>2)$

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$|x-2| = \begin{cases} x-2; & x \geq 2 \\ -(x-2); & x < 2 \end{cases}$$

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx \quad \text{if } f(-x) = f(x)$$

$$\therefore \quad \int_{-a}^{a}(|x|+|x-2|)\,dx$$

$$= \int_{-a}^{a}|x|\,dx + \int_{-a}^{a}|x-2|\,dx$$

$$= 2\int_{0}^{a}x\,dx + \int_{-a}^{2}(2-x)\,dx + \int_{2}^{a}(x-2)\,dx$$

$$= 2\left(\frac{x^2}{2}\right)_0^a + \left(2x - \frac{x^2}{2}\right)_{-a}^{2} + \left(\frac{x^2}{2} - 2x\right)_2^a$$

$$= \frac{2a^2}{2} + \left(2a + \frac{a^2}{2}\right) + \frac{a^2}{2} - 2a + 4 - 2 - 2 + 4$$

$= 2a^2 + 4$

$2a^2 + 4 = 22$ (Given)

$\Rightarrow \quad a^2 = 9$

$\Rightarrow \quad a = 3 \qquad (\because a > 2)$

$$I = \int_{a}^{-a}(x+[x])\,dx$$

$$= \int_{3}^{-3}(x+[x])\,dx$$

$$\because \quad \int_{a}^{b} f(x)\,dx = -\int_{b}^{a} f(x)\,dx$$

$$\therefore \quad I = -\int_{-3}^{3}x\,dx - \int_{-3}^{3}[x]\,dx$$

$$\int_{-a}^{a} f(x)\,dx = 0 \text{ if } f(-x) = -f(x)$$

$$\therefore \quad I' = \int_{-3}^{3}x\,dx = 0$$

$$\therefore \quad I = 0 - \int_{-3}^{3}[x]\,dx$$

$$= \left[0 - \int_{-3}^{-2}(-3)\,dx + \int_{-2}^{-1}(-2)\,dx + \int_{-1}^{0}(-1)\,dx + \int_{0}^{1}0.dx + \int_{1}^{2}1\,dx + \int_{2}^{3}2\,dx\right]$$

$= 0 - [-\{3(-2+3)\,2(-1+2) + (0+1)\} + (0+1+2)]$

$= -[-3-2-1+3]$

$= 3$

Hint:

(i) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

(ii) $\int_{-a}^{a} f(x)dx = 0$ if $f(-x) = -f(x)$

(iii) $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$ if $f(-x) = f(x)$

Shortcut Method:

$$\int_{-a}^{0}(-2x+2)dx + \int_0^2 (x+2-x)dx + \int_2^a (2x-2)dx$$

$\Rightarrow \quad x^2 - 2x\Big|_o^{-a} + 2x\Big|_0^2 + x^2 - 2x\Big|_2^a = 22$

$\Rightarrow \quad 2a^2 = 18$

$\Rightarrow \quad a = 3$

$$\int_{-3}^{-3}\left(x+[x]dx\right) = -\int_{-3}^{3}[x]dx$$

$= -(-3-2-1+0+1+2)$

$= 3$

27. Correct answer is [75].

Given:

$\vec{c}$ is coplanar with $\vec{a}\times\vec{b}$

$\Rightarrow \quad \vec{c} = x\vec{a} + y\vec{b}$

$\because \ \vec{b}.\vec{c} = 0 \Rightarrow x(\vec{a}.\vec{b}) + y(\vec{b}.\vec{b}) = 0 \Rightarrow -x + 5y = 0$ (i)

$\vec{c}.\vec{a} = 7 \Rightarrow x(\vec{a}.\vec{a}) + y(\vec{a}.\vec{b}) = 7 \Rightarrow 3x - y = 7$

$\Rightarrow \quad y = \frac{1}{2}, x = \frac{5}{2}$

$\therefore \quad \vec{c} = \frac{5\vec{a}+\vec{b}}{2}$

$\Rightarrow \quad 2(\vec{a}+\vec{b}+\vec{c}) = 2\vec{a}+2\vec{b}+5\vec{a}+\vec{b} = 7\vec{a}+3\vec{b}$

Squaring above equation both the sides,

$\Rightarrow \quad 4(\vec{a}+\vec{b}+\vec{c})^2 = 49(\vec{a}.\vec{a}) + 9(\vec{b}.\vec{b}) + 42(\vec{a}.\vec{b})$

$\Rightarrow \quad = 49(3) + 9(5) - 42(1) = 150$

$2(\vec{a}+\vec{b}+\vec{c})^2 = 75$

Hint:

(i) Vector coplanar with $\vec{b}$ and $\vec{c}$ and is perpendicular to $\vec{a}$ is $\vec{d} = \vec{a}\times(\vec{b}\times\vec{c})$

(ii) Use given conditions to find $\vec{c}$ and substitute to find required value.

Shortcut Method:

$\vec{c} = k\left(\vec{b}\times(\vec{a}\times\vec{b})\right)$

$= k\left(5(-\hat{i}+\hat{j}+\hat{k}) + 2\hat{i} + \hat{k}\right)$

$= k\left(-3\hat{i}+5\hat{j}+6\hat{k}\right)$

$\vec{c}.\vec{a} = 7 \quad \Rightarrow \quad k = \frac{1}{2}$

$2|\vec{a}+\vec{b}+\vec{c}|^2$

$= 2\left(\frac{1}{4} + \frac{49}{4} + 25\right)$

$= 25 + 50$

$= 75$

28. Correct answer is [5].

Given: A = {$n \in$ N : n is a 3-digit number)

B = {$9k + 2$; $k \in$ N}

C = {$9k + l$; $k \in$ N} for some l $(0 < l < 9)$

3-digit numbers of the form $9k + 2$ are {101, 110, 992}

It forms an A.P. with

First term = a = 101, common difference, d = 9 and last term = 992

$\therefore \quad$ Sum $(S_1) = \frac{100}{2}\{101 + 992\}$

$= 50 \times 1093$

$S_1 = 54650$

Given, $\quad 274 \times 400 = S_1 + S_2$

$\Rightarrow \quad 274 \times 400 = 50 \times 1093 + S_2$

$S_2 = 109600 - 54650$

or, $\quad S_2 = 54950$

Now, $\quad 54950 = \frac{100}{2}\left[(99+l) + (990+l)\right]$

$1099 = 2l + 1089$

$\Rightarrow \quad l = 5$

Hint:

(i) Put $k = 1, 2, ...$ to find numbers of form $9k + 2$

(ii) Find 3-digit number among them.

(iii) No. of term in an

$$\text{A.P.} = \frac{\text{Last term} - \text{first term}}{\text{Common diff.}} + 1$$

(iv) Sum of n terms of A.P $= \frac{n}{2}[a + l]$ where a = first term, l = last term and n is no. of terms.

Shortcut Method:

$$274 \times 400 = \frac{100}{2}[101 + 992] + S_2$$

$$S_2 = 109600 - 54650$$

$$= \frac{100}{2}\left[(99+l)+(990+l)\right] = 54950$$

$$1099 = 2l + 1089$$

$\Rightarrow$ $l = 5$

29. Correct answer is [10].

Given:

$z+\alpha|z-1|+2i=0$ $\left(z\in \mathbb{C} \text{ and } i=\sqrt{-1}\right)$
has a solution.

Put $z = x + iy$

$$(x+iy)+\alpha|x+iy-1|+2i=0$$

$\because$ Modulus of $z = x + iy$ is $|z| = \sqrt{x^2+y^2}$

$\therefore$ $(x + iy) + \alpha\sqrt{(x-1)^2+y^2} + 2i = 0$

Compare real and imaginary parts,

$\therefore$ $y + 2 = 0$ and $x+\alpha\sqrt{(x-1)^2+y^2} = 0$

$\Rightarrow$ $y = -2$ and $x+\alpha\sqrt{(x-1)^2+4} = 0$

$\Rightarrow$ $x^2 = \alpha^2(x-1)^2 + 4\alpha^2$

$\Rightarrow$ $\alpha^2 = \dfrac{x^2}{x^2-2x+5}$

$\Rightarrow$ $x^2(\alpha^2-1) - 2x\alpha^2 + 5\alpha^2 = 0$

is a quadratic in x whose roots are real.

$\therefore$ $D \geq 0$

$\Rightarrow$ $(2\alpha^2)^2 - 4(\alpha^2-1)(5\alpha^2) \geq 0$

$\Rightarrow$ $4\alpha^4 - 4(\alpha^2-1)(5\alpha^2) \geq 0$

$\Rightarrow$ $\alpha^2(-4\alpha^2+5) \geq 0$

$\Rightarrow$ $\alpha^2\left(\alpha^2 - \frac{5}{4}\right) \leq 0$

$$\alpha^2 \in \left[0, \frac{5}{4}\right]$$

$$\alpha \in \left[-\frac{\sqrt{5}}{2}, \frac{\sqrt{5}}{2}\right]$$

$$p = -\frac{\sqrt{5}}{2}, q = \frac{\sqrt{5}}{2}$$

$$4(p^2 + q^2) = 4\left(\frac{5}{4}+\frac{5}{4}\right)$$

$$= 10$$

Hint:

(i) Put $z = x + iy$ in given equation

(ii) Compare real and imaginary parts both sides.

(iii) For the roots to be real.
$D \geq 0$

(iv) Find minimum and maximum value of α to find required value.

Shortcut Method:

$$\alpha = \frac{-z-2i}{|z-1|}$$

$$z = x + iy$$

$$\alpha = \frac{-x-(y+2)i}{|z-1|}$$ α is real

$$\alpha = \frac{-x}{\sqrt{(x-1)^2+y^2}}$$

$$\alpha^2 = \frac{x^2}{x^2-2x+5}$$ $\therefore$ $y = -2$

$\Rightarrow$ $x^2(\alpha^2-1) - 2x\alpha^2 + 5\alpha^2 = 0$

$x \in \mathbb{R}$, $D \geq 0$

$\Rightarrow$ $\alpha^2\left(\alpha^2 - \frac{5}{4}\right) \leq 0$

$$\alpha \in \left[-\frac{\sqrt{5}}{2}, \frac{\sqrt{5}}{2}\right]$$

$$4(p^2 + q^2) = 4\left(\frac{5}{4}+\frac{5}{4}\right) = 10$$

30. Correct answer is [540].

Given: M is any 3 × 3 matrix with entries from set {0, 1, 2}.

Let $$M = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$M^T = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

$$M^TM = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

Sum of diagonal elements = 7

$\Rightarrow$ $a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 7$

Case I : Seven (1' *s*) and two (0' *s*) can be used

$\therefore$ No. of ways = ${}^9C_2 = 36$ (Selection of 2 zero out of 7(1' *s*) and 2 (0' *s*) or selection of 7(1' *s*) from 7(1' *s*) and 2 (0' *s*))

Similarly,

Case II : One (2' *s*) and 3(1' *s*) and (0' *s*)

$\therefore$ ${}^9C_1 \times {}^8C_3 \times {}^5C_5 = 504$

$\therefore$ Total = 36 + 504 = 540

Hint:

(i) Consider *a* 3 × 3 matrix

(ii) Find the sum of diagonal elements of product of matrices M^T and M where M^T is formed by interchanging rows and columns of M.

❑❑❑