JEE (Main) MATHEMATICS SOLVED PAPER

2023
08^{th} April Shift 1

General Instructions :

(i) *There are 30 questions in this section.*
(ii) *Section A consists of 20 Multiple choice questions and Section B consists of 10 Numerical value type questions. In Section B, candidates have to attempt any five questions out of 10.*
(iii) *There will be only one correct choice in the given four choices in Section A. For each question for Section A, 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice questions and zero mark will be awarded for not attempted questions.*
(iv) *For Section B questions, 4 marks will be awarded for correct answer and zero for unattempted and incorrect answer.*
(v) *Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.*
(vi) *All calculations/ written work should be done in the rough sheet which is provided with Question Paper.*

Section A

Q. 1. The area of the region $\{(x, y): x^2 \le y \le 8 - x^2, y \le 7\}$ is
(A) 24 **(B)** 21
(C) 20 **(D)** 18

Q. 2. Let $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$. . If P^T

$Q^{2007} P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $2a + b - 3c - 4d$ equal to

(A) 2004 **(B)** 2007
(C) 2005 **(D)** 2006

Q. 3. Negation of $(p \to q) \to (q \to p)$ is
(A) $(-q) \wedge p$ **(B)** $p \vee (\sim q)$
(C) $(\sim p) \vee q$ **(D)** $q \wedge (\sim p)$

Q. 4. Let C(α, β) be the circumcenter of the triangle formed by the lines
$4x + 3y = 69$,
$4y - 3x = 17$ and
$x + 7y = 61$.
Then $(\alpha - \beta)^2 + \alpha + \beta$ is equal to
(A) 18 **(B)** 15
(C) 16 **(D)** 17

Q. 5. Let α, β, γ, be the three roots of the equation $x^3 + bx + c = 0$. If $\beta\gamma = 1 = -\alpha$, then $b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3$ is equal to

(A) $\frac{155}{8}$ **(B)** 21

(C) 19 **(D)** $\frac{169}{8}$

Q. 6. Let the number of elements in sets A and B be five and two respectively. Then the number of subsets of A × B each having at least 3 and at most 6 elements is:
(A) 752 **(B)** 772
(C) 782 **(D)** 792

Q. 7. If the coefficients of three consecutive terms in the expansion of $(1 + x)^n$ are in the ratio 1 : 5 : 20, then the coefficient of the fourth term is
(A) 5481 **(B)** 3654
(C) 2436 **(D)** 1817

Q. 8. Let R be the focus of the parabola $y^2 = 20x$ and the line $y = mx + c$ intersect the parabola at two points P and Q.

Let the point G(10, 10) be the centroid of the triangle PQR. If $c - m = 6$, then $(PQ)^2$ is
(A) 325 **(B)** 346
(C) 296 **(D)** 317

Q. 9. Let $S_K = \frac{1 + 2 + ... + K}{K}$ and $\sum_{j=1}^{n} S_j^2 = \frac{n}{A}(Bn^2 + Cn + D)$, where A, B, C, D ∈ N and A has least value. Then
(A) A + B is divisible by D
(B) A + B = 5 (D –C)
(C) A + C + D is not divisible by B
(D) A + B + D is divisible by 5

Q. 10. The shortest distance between the lines

$\frac{x-4}{4} = \frac{y+2}{5} = \frac{z+3}{3}$ and $\frac{x-1}{3} = \frac{y-3}{4} = \frac{z-4}{2}$ is

(A) $2\sqrt{6}$ **(B)** $3\sqrt{6}$

(C) $6\sqrt{3}$ **(D)** $6\sqrt{2}$

Q. 11. The number of arrangements of the letters of the word "INDEPENDENCE" in which all the vowels always occur together is.

(A) 16800 **(B)** 14800
(C) 18000 **(D)** 33600

Q. 12. If the points with position vectors $\alpha\hat{i}+10\hat{j}+13\hat{k}$, $6\hat{i}+11\hat{j}+11\hat{k}, \frac{9}{2}\hat{i}+\beta\hat{j}-8\hat{k}$ are collinear, then $(19\alpha - 6\beta)^2$ is equal to

(A) 49 **(B)** 36
(C) 25 **(D)** 16

Q. 13. In a bolt factory, machines A, B and C manufacture respectively 20%, 30% and 50% of the total bolts. Of their output 3, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random form the product. If the bolt drawn is found the defective, then the probability that it is manufactured by the machine C is.

(A) $\frac{5}{14}$ **(B)** $\frac{3}{7}$
(C) $\frac{9}{28}$ **(D)** $\frac{2}{7}$

Q. 14. If for $z = \alpha + i\beta$, $|z+2| = z + 4(1+i)$, then $\alpha + \beta$ and $\alpha\beta$ are the roots of the equation

(A) $x^2 + 3x - 4 = 0$ **(B)** $x^2 + 7x + 12 = 0$
(C) $x^2 + x - 12 = 0$ **(D)** $x^2 + 2x - 3 = 0$

Q. 15. $\lim_{x\to 0}\left(\left(\frac{(1-\cos^2(3x))}{\cos^3(4x)}\right)\left(\frac{\sin^3(4x)}{(\log_e(2x+1))^5}\right)\right)$ is equal to __________.

(A) 24 **(B)** 9
(C) 18 **(D)** 15

Q. 16. The number of ways, in which 5 girls and 7 boys can be seated at a round table so that no two girls sit together, is

(A) $7\,(720)^2$ **(B)** 720
(C) $7\,(360)^2$ **(D)** $126\,(5!)^2$

Q. 17. Let $f(x) = \frac{\sin x + \cos x - \sqrt{2}}{\sin x - \cos x}, x \in [0, \pi] - \left\{\frac{\pi}{4}\right\}$. Then $f\left(\frac{7\pi}{12}\right)f''\left(\frac{7\pi}{12}\right)$ is equal to

(A) $\frac{-2}{3}$ **(B)** $\frac{2}{9}$
(C) $\frac{-1}{3\sqrt{3}}$ **(D)** $\frac{2}{3\sqrt{3}}$

Q. 18. If the eqation of the plane containing the line $x + 2y + 3z - 4 = 0 = 2x + y - z + 5$ and perpendicular to the plane $\vec{r} = (\hat{i}-\hat{j}) + \lambda(\hat{i}+\hat{j}+\hat{k}) + \mu(\hat{i}-2\hat{j}+3\hat{k})$ is $ax + by + cz = 4$, then $(a - b + c)$ is equal to

(A) 22 **(B)** 24
(C) 20 **(D)** 18

Q. 19. Let $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$. If $|\text{adj (adj (adj } 2A))|$ $= (16)^n$, then n is equal to

(A) 8 **(B)** 9
(C) 12 **(D)** 10

Q. 20. Let $I(x) = \int \frac{(x+1)}{x(1+xe^x)^2}dx, x > 0.$ $\lim_{x\to\infty} I(x) = 0$, then I(1) is equal to

(A) $\frac{e+1}{e+2} - \log_e(e+1)$ **(B)** $\frac{e+2}{e+1} + \log_e(e+1)$
(C) $\frac{e+2}{e+1} - \log_e(e+1)$ **(D)** $\frac{e+1}{e+2} + \log_e(e+1)$

Section B

Q. 21. Let A = {0, 3, 4, 6, 7, 8, 9, 10} and R be the relation defined on A such that R =$(x, y) \in$ A × A: $x - y$ is odd positive integer or $x - y = 2$}. The minimum number of elements that must be added to the relation R, so that it is a symmetric relation, is equal to ______.

Q. 22. Let [t] denote the greatest integer $\leq t$, If the constant term in the expansion of $\left(3x^2 - \frac{1}{2x^5}\right)^7$ is α, then $[\alpha]$ is equal to _____.

Q. 23. Let λ_1, λ_2 be the values of λ for which the points $\left(\frac{5}{2}, 1, \lambda\right)$ and (–2, 0, 1) are at equal distance from the plane $2x + 3y - 6z + 7 = 0$. If $\lambda_1 > \lambda_2$, then the distance of the point $(\lambda_1 - \lambda_2, \lambda_2, \lambda_1)$ from the line $\frac{x-5}{1} = \frac{y-1}{2} = \frac{z+7}{2}$ is

Q. 24. If the solution curve of the differential equation $(y - 2\log_e x)dx + (x\log_e x^2)\,dy = 0$, $x > 1$ passes through the points $\left(e, \frac{4}{3}\right)$ and (e^4, α), then α is equal to ______.

Q. 25. Let $\vec{a} = 6\hat{i}+9\hat{j}+12\hat{k}, \vec{b} = \alpha\hat{i}+11\hat{j}-2\hat{k}$ and $\vec{c}$ be vectors such that $\vec{a}\times\vec{c} = \vec{a}\times\vec{b}$. If $\vec{a}.\vec{c} = -12$, $\vec{c}.(\hat{i}-2\hat{j}+\hat{k}) = 5$, then $\vec{c}(\hat{i}+\hat{j}+\hat{k})$ is equal to ________.

Q. 26. The largest natural number n such that $3n$ divides 66! is ______.

Q. 27. If a_0 is the greatest term in the sequence $a_n = \frac{n^3}{n^4 + 147}, n = 1, 2, 3, ...,$ then a is equal to ________________.

Q. 28. Let the mean and variance of 8 numbers x, y, 10, 12, 6, 12, 4, 8 be 9 and 9.25 respectively. If $x > y$, then $3x - 2y$ is equal to _________.

Q. 29. Consider a circle $C_1 : x^2 + y^2 - 4x - 2y = \alpha - 5$. Let its mirror image in the line $y = 2x + 1$ be another circle $C_2 : 5x^2 + 5y^2 - 10fx - 10gy + 36 = 0$. Let r be the radius of C_2. Then $\alpha + r$ is equal to _________.

Q. 30. Let $[t]$ denote the greatest integer $\le t$. Then $\frac{2}{\pi}\int_{\pi/6}^{5\pi/6} (8\,[\text{cosec } x] - 5\,[\cot x])\, dx$ is equal to _______.

Answer Key

Q. No.	Answer	Topic Name	Chapter Name
1	C	Area between the curves	Integral Calculus
2	C	Algebra of matrices	Matrices
3	D	Negation of a statement	Mathematical Reasoning
4	D	Circumcentre	Straight line
5	C	Cube root of unity	Cubic Equation
6	D	r things out of n things	Permutation and Combination
7	B	Coefficient of a term	Binomial theorem
8	A	Parabola	Conic Section
9	A	Sum of n terms	Sequences and series
10	B	Shortest distance	Three dimensional geometry
11	A	Number of ways	Permutation and Combination
12	B	Collinearity	Vector algebra
13	A	Conditional probability	Probability
14	B	Roots of equation	Complex numbers
15	C	Limits of trigonometry	Limits
16	D	Number of ways	Permutation and Combination
17	B	Higher order derivatives	Differentiability
18	A	Equation of plane	Three dimensional geometry
19	D	Adjoint	Matrices and Determinants
20	C	Indefinite Integral	Integral Calculus
21	[19]	Symmetric relation	Relation and Function
22	[1275]	General term	Binomial theorem
23	[9]	Plane	Three dimensional geometry
24	[3]	Linear Differential Equation	Differential equation
25	[11]	Algebra of vectors	Vector algebra
26	[31]	Remainder theorem	Binomial theorem
27	[5]	Maxima/Minima	Application of derivatives
28	[25]	Mean, Variance	Statistics
29	[2]	Circle	Conic Section
30	[14]	Definite Integral	Integral Calculus

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Solutions

Section A

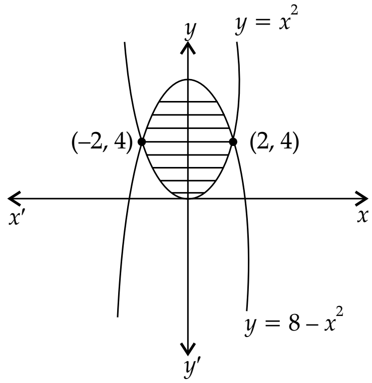
1. Option (C) is correct.

The given curves are

$x^2 \le y,\ y \le 8 - x^2;\ y \le 7$

On solving, we get

$x^2 = 8 - x^2$



$\Rightarrow x^2 = 4$

$\Rightarrow x = \pm 2$

$$\text{So, area} = 2\left[\int_0^4 \sqrt{y}\,dy + \int_4^7 \sqrt{8-y}\,dy\right]$$

$$= 2\left\{\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4 + \left[\frac{-(8-y)^{\frac{3}{2}}}{\frac{3}{2}}\right]_4^7\right\}$$

$$= 2\times\frac{2}{3}\left\{[4^{3/2} - 0] + (-(1)^{3/2} + (4)^{3/2})\right\}$$

$$= \frac{4}{3}\{8 - 1 + 8\} = \frac{4}{3}\times 15 = 20 \text{ sq. units}$$

HINT:

Draw the graph of both curves, then find the bounded region and proceed.

2. Option (C) is correct.

$$\text{Here, P} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, \text{A} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\text{Here, PP}^T = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{4}+\frac{1}{4} & -\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{4} \\ \frac{-\sqrt{3}}{4}+\frac{\sqrt{3}}{4} & \frac{1}{4}+\frac{3}{4} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$| |\ P^TP = I$

$\because Q = PAP^T$

$\Rightarrow Q^{2007} = (PAP^T)\,(PAP^T)$......... 2007 time

$= PA^{2007}P^T$

$$\text{As, A} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1+0 & 1+1 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$A^{2007} = \begin{bmatrix} 1 & 2007 \\ 0 & 1 \end{bmatrix}$$

$$\text{Hence, } P^TQ^{2007}P = A^{2007} = \begin{bmatrix} 1 & 2007 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 2007 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow a = 1, b = 2007, c = 0, d = 1$

$\therefore 2a + b - 3c - 4d = 2\,(1) + 2007 - 3(0) - 4(1)$

$= 2 + 2007 - 4 = 2005$

HINT:

Transpose the given matrix and multiply the matrices to solve further.

3. Option (D) is correct.

Given: $(p \to q) \to (q \to p)$

Negation of above statement is

$\sim[(p \to q) \to (q \to p)]$

$\equiv \sim [\sim p \to q \wedge q \to p]$

$\equiv\ p \to q \wedge \sim q \to p$

$\equiv \sim p \vee q \wedge q \wedge \sim p]$

$\equiv q \wedge (\sim p)$

HINT:

The negation of a statement is the opposite of the given mathematical statement.

4. Option (D) is correct.

We have,

$4x + 3y = 69$ ...(i)

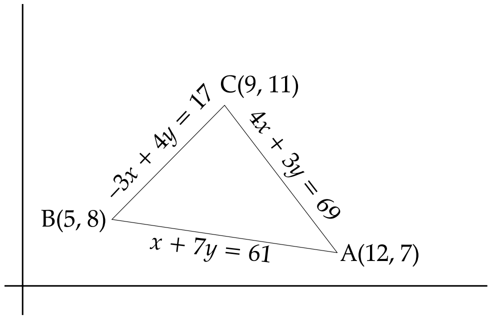
$4y - 3x = 17$ ...(ii)

$x + 7y = 61$ ...(iii)

On solving (i) and (iii), we get

$x = 12$, and $y = 7$

So, $A \equiv (12, 7)$



On solving (ii) and (iii), we get

$x = 5$ and $y = 8$

So, B ≡ (5, 8)

Hence, circumcentre $\equiv \left(\frac{12+5}{2}, \frac{7+8}{2}\right)$

$\equiv \left(\frac{17}{2}, \frac{15}{2}\right)$

$\therefore \alpha = \frac{17}{2}, \beta = \frac{15}{2}$

$\therefore (\alpha - \beta)^2 + (\alpha + \beta) = \left(\frac{17}{2} - \frac{15}{2}\right)^2 + \left(\frac{17}{2} + \frac{15}{2}\right)$

$= (1)^2 + (16) = 17$

HINT:

Circumcentre of a right triangle is the midpoint of hypotenuse of the triangle.

5. Option (C) is correct.

Given cubic equation is :

$x^3 + bx + c = 0$

$\because \alpha, \beta, \gamma$ are the roots of above equation.

And $\beta\gamma = 1 = -\alpha$

So, product of roots $= -c$

$\Rightarrow \alpha\beta\gamma = -c$

$\Rightarrow (-1)(1) = -c$

$\Rightarrow c = 1$

Since, $\alpha = -1$ is the root. So,

$\Rightarrow -1 - b + c = 0$

$\Rightarrow c - b = 1$

$\Rightarrow 1 - b = 1 \Rightarrow b = 0$

The given equation becomes $x^3 + 1 = 0$

So, roots are $-1, -\omega, -\omega^2$

$\therefore b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3$

$= 0 + 2 - 3(-1)^3 - 6(-\omega)^3 - 8(-\omega^2)^3$

$= 2 + 3 + 6\omega^3 + 8\omega^6$

$= 5 + 6 + 8 = 19$

HINT:

For a cubic equation, $ax^3 + bx^2 + cx + d = 0$

Sum of roots $= \frac{-b}{a}$,

Product of roots taken two at a time $= \frac{c}{a}$

Product of roots $= \frac{-d}{a}$

6. Option (D) is correct.

Since, $n(A) = 5$, $n(B) = 2$

$\Rightarrow n(A \times B) = n(A) \times n(B)$

$= 5 \times 2 = 10$

So, number of subsets having 3 elements $= {}^{10}C_3$

Number of subsets having 4 elements $= {}^{10}C_4$

Number of subsets having 5 elements $= {}^{10}C_5$

Number of subsets having 6 elements $= {}^{10}C_6$

$\therefore$ No. of subsets $= {}^{10}C_3 + {}^{10}C_4 + {}^{10}C_5 + {}^{10}C_6$

$= 120 + 210 + 252 + 210 = 792$

HINT:

No of subsets having r elements out of total n elements $= {}^nC_r$

7. Option (B) is correct.

Given: ${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1}$

$= 1 : 5 : 20$

$\Rightarrow \frac{n!}{(r-1)!(n-r+1)!} \times \frac{r!(n-r)!}{n!} = \frac{1}{5}$

$\Rightarrow \frac{r}{(n-r+1)} = \frac{1}{5}$

$\Rightarrow 5r = n - r + 1$

$\Rightarrow n = 6r - 1$ ...(i)

Also, $\frac{n}{r!(n-r)!} \times \frac{(r+1)!(n-r-1)!}{n!} = \frac{5}{20} = \frac{1}{20}$

$\Rightarrow \frac{(r+1)}{(n-r)} = \frac{1}{4}$

$\Rightarrow 4r + 4 = n - r$

$\Rightarrow n = 5r + 4$ ...(ii)

From (i) and (ii), we get

$6r - 1 = 5r + 4$

$\Rightarrow r = 5$

So, $n = 5(5) + 4 = 29$

So, coefficient of 4th terms $= {}^nC_3 = {}^{29}C_3$

$= \frac{29!}{3!26!} = \frac{29 \times 28 \times 27}{3 \times 2} = 3654$

HINT:

In the expansion of $(a + b)^n$, the general term is $T_{r+1} = {}^nC_r (a)^{n-r} b^r$

8. Option (A) is correct.

$y^2 = 20x$, $y = mx + c$

Put value of x

$y^2 = 20\left(\frac{y-c}{m}\right)$

$\Rightarrow y^2 - \frac{20}{m}y + \frac{20}{m}c = 0$ ...(i)

Since, centroid = (10, 10)

So, $\frac{y_1 + y_2 + 0}{3} = 10$

$\Rightarrow y_1 + y_2 = 30$

From (1),

Sum of roots $= \frac{20}{m} = 30 \Rightarrow m = \frac{2}{3}$

Also, $c - m = 6 \Rightarrow c = 6 + \frac{2}{3} = \frac{20}{3}$

Now, the equation is :

$$y^2 - \frac{20}{2} \times 3y + \frac{20}{2} \times 3 \times \frac{20}{3} = 0$$

$\Rightarrow y^2 - 30y + 200 = 0$

$\Rightarrow y^2 - 20y - 10y + 200 = 0$

$\Rightarrow (y - 20)(y - 10) = 0$

$\Rightarrow y = 10, 20 \Rightarrow x = 5, x = 20$

$\therefore$ P $\equiv$ (5, 10), Q $\equiv$ (20, 20)

So, $(PQ)^2 = (20 - 5)^2 + (20 - 10)^2$

$= 225 + 100 = 325$

HINT:

Centroid of a triangle having vertices (a, b), (c, d) & (e, f) is $\left(\frac{a+c+e}{3}, \frac{b+d+f}{3}\right)$

9. Option (A) is correct.

$\because S_k = \frac{1+2+...+k}{k}$

$= \frac{k(k+1)}{2k} = \frac{k+1}{2}$

$\Rightarrow S_k^2 = \left(\frac{k+1}{2}\right)^2 = \frac{k^2+1+2k}{4}$

$\Rightarrow \sum_{j=1}^{n} S_j^2 = \frac{1}{4}\left[\sum_{j=1}^{n} k^2 + \sum_{j=1}^{n} 1 + 2\sum_{j=1}^{n} k\right]$

$= \frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6} + n + \frac{2n(n+1)}{2}\right]$

$= \frac{n}{4}\left[\frac{(n+1)(2n+1)}{6} + 1 + n + 1\right]$

$= \frac{n}{24}[2n^2 + 3n + 1 + 6 + 6n + 6]$

$= \frac{n}{24}[2n^2 + 9n + 13]$

On comparing, we get

A = 24, B = 2, C = 9, D = 13

(1) A + B = 24 + 2 = 26, divisible by 13

(2) A + B = 26

5 (D – C) = 5 (13 – 9) = 20

$\therefore 26 \neq 20$

(3) A + C + D = 46, which is divisible by 2

(4) A + B + D = 39, which is not divisible by 5.

HINT:

$1 + 2 + ... + n = \frac{n(n+1)}{2}$

$1^2 + 2^2 + ... n^2 = \frac{n(n+1)(2n+1)}{6}$

10. Option (B) is correct.

The given lines are :

$\frac{x-4}{4} = \frac{y+2}{5} = \frac{z+3}{3}$ and $\frac{x-1}{3} = \frac{y-3}{4} = \frac{z-4}{2}$

So, $\vec{b}_1 = 4\hat{i} + 5\hat{j} + 3\hat{k}$

$\vec{b}_2 = 3\hat{i} + 4\hat{j} + 2\hat{k}$

$\vec{a}_1 = 4\hat{i} - 2\hat{j} - 3\hat{k}, \vec{a}_2 = \hat{i} + 3\hat{j} + 4\hat{k}$

$$\therefore \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 5 & 3 \\ 3 & 4 & 2 \end{vmatrix}$$

$= (10 - 12)\hat{i} - (8 - 9)\hat{j} + (16 - 15)\hat{k}$

$= -2\hat{i} + \hat{j} + \hat{k}$

Shortest distance, $d = \left|\frac{(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$

$= \left|\frac{(3\hat{i} - 5\hat{j} - 7\hat{k}).(-2\hat{i} + \hat{j} + \hat{k})}{\sqrt{4+1+1}}\right|$

$= \left|\frac{-6-5-7}{\sqrt{6}}\right| = \frac{18}{\sqrt{6}} = 3\sqrt{6}$ units

HINT:

Shortest distance between two lines is:

$$d = \left|\frac{(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$$

11. Option (A) is correct.

In the given word,

vowels are : I, E, E, E, E

Consonants are : N, D, P, N, D, N, C

So, number of words = $\frac{8!}{3!2!} \times \frac{5!}{4!}$

$= \frac{8 \times 7 \times 6 \times 5 \times 4}{2} \times 5 = 16800$

HINT:

Out of n objects, if r things are same, so number of ways = $\frac{n!}{r!}$

12. Option (B) is correct.

Given: Points with position vectors

$\alpha\hat{i} + 10\hat{j} + 13\hat{k}, 6\hat{i} + 11\hat{j} + 11\hat{k}$

and $\frac{9}{2}\hat{i} + \beta\hat{j} - 8\hat{k}$ are collinear.

So, $\frac{\alpha - 6}{6 - \frac{9}{2}} = \frac{10 - 11}{11 - \beta} = \frac{13 - 11}{11 + 8}$

$\Rightarrow \frac{2(\alpha - 6)}{3} = \frac{-1}{11 - \beta} = \frac{2}{19}$

$\Rightarrow \frac{2}{3}(\alpha - 6) = \frac{2}{19}$

$\Rightarrow 19\alpha - 114 = 3 \Rightarrow 19\alpha = 117$

$\Rightarrow \alpha = \frac{117}{19}$

And, $\frac{-1}{11-\beta} = \frac{2}{19}$

$\Rightarrow -19 = 22 - 2\beta$

$\Rightarrow 2\beta = 41$

$\Rightarrow \beta = \frac{41}{2}$

$\therefore (19\alpha - 6\beta)^2 = \left(19 \times \frac{117}{19} - \frac{6 \times 41}{2}\right)^2$

$= (117 - 123)^2 = 36$

HINT:

If point $(\alpha_1, \beta_1, \gamma_1)$, $(\alpha_2, \beta_2, \gamma_2)$, $(\alpha_3, \beta_3, \gamma_1)$ are collinear, then $\frac{\alpha_1 - \alpha_2}{\alpha_2 - \alpha_3} = \frac{\beta_1 - \beta_2}{\beta_2 - \beta_3} = \frac{\gamma_1 - \gamma_2}{\gamma_2 - \gamma_3}$.

13. Option (A) is correct.

Given: $P(A) = \frac{20}{100} = \frac{2}{10}$

$P(B) = \frac{30}{100} = \frac{3}{10}$; $P(C) = \frac{50}{100} = \frac{5}{10}$

Let E → Event that the bolt is defective.

So, $P(E/A) = \frac{3}{100}, P\left(\frac{E}{B}\right) = \frac{4}{100}, P\left(\frac{E}{C}\right) = \frac{2}{100}$

So, P(C/E)

$$= \frac{P\left(\frac{E}{C}\right) \times P(C)}{P\left(\frac{E}{A}\right) \times P(A) + P\left(\frac{E}{B}\right) \times P(B) + P\left(\frac{E}{C}\right) \times P(C)}$$

$$= \frac{\frac{5}{10} \times \frac{2}{100}}{\frac{3}{100} \times \frac{2}{10} + \frac{4}{100} \times \frac{3}{10} + \frac{2}{100} \times \frac{5}{10}}$$

$$= \frac{10}{6+12+10} = \frac{10}{28} = \frac{5}{14}$$

HINT:

Conditional probability P(C/E)

$$= \frac{P\left(\frac{E}{C}\right) \times P(C)}{P\left(\frac{E}{A}\right) \times P(A) + P\left(\frac{E}{B}\right) \times P(B) + P\left(\frac{E}{C}\right) \times P(C)}$$

14. Option (B) is correct.

Given: $|z + 2| = z + 4(1 + i)$

Also, $z = \alpha + i\beta$

$\therefore |z + 2| = |\alpha + i\beta + 2| = (\alpha + i\beta) + 4 + 4i$

$\Rightarrow |(\alpha + 2) + i\beta| = (\alpha + 4) + i(\beta + 4)$

$\Rightarrow \sqrt{(\alpha+2)^2 + \beta^2} = (\alpha + 4) + i(\beta + 4)$

$\Rightarrow \beta + 4 = 0 \Rightarrow \beta = -4$

Now, $(\alpha + 2)^2 + \beta^2 = (\alpha + 4)^2$

$\Rightarrow \alpha^2 + 4 + 4\alpha + \beta^2 = \alpha^2 + 16 + 8\alpha$

$\Rightarrow 4 + 4\alpha + 16 = 16 + 8\alpha$

$\Rightarrow 4\alpha = 4 \Rightarrow \alpha = 1$

So, $\alpha + \beta = -3$ and $\alpha\beta = -4$

$\therefore$ Required equation is

$x^2 - (-3 - 4)x + (-3)(-4) = 0$

$\Rightarrow x^2 + 7x + 12 = 0$

15. Option (C) is correct.

$$\lim_{x\to 0}\left[\left(\frac{1-\cos^2(3x)}{\cos^3(4x)}\right)\left(\frac{\sin^3(4x)}{(\log_e(2x+1))^5}\right)\right]$$

$$= \lim_{x\to 0}\left[\frac{1-\cos^2(3x)}{9x^2} \times \frac{9x^2}{\cos^3(4x)}\right] \times \frac{\frac{\sin^3 4x}{(4x)^3} \times 64x^3}{\left[\frac{\log_e(2x+1)}{2x}\right]^5 \times (2x)^5}$$

$$= \left[\frac{1 \times 9 \times 1}{(1)}\right] \times \left[\frac{1 \times 64}{1 \times 32}\right]$$

$= 9 \times 2 = 18$

HINT:

$$\lim_{x\to 0}\frac{\sin x}{x} = 1$$

$$\lim_{x\to 0}\frac{\sin x}{x} = 1$$

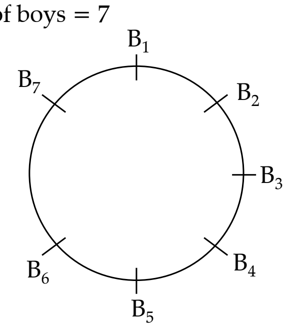
$$\lim_{x\to 0}\frac{\ln(1+x)}{x} = 1$$

16. Option (D) is correct.

We have,

Number of girls = 5

Number of boys = 7



So, number of ways of arranging boys around the table = 6! and 5 girls can be arranged in 7 gaps in ${}^7P_5$ ways

$\therefore$ Required no. of ways = $6! \times {}^7P_5$

$= 126 \times (5!)^2$

17. Option (B) is correct.

$$f(x) = \frac{\sin x + \cos x - \sqrt{2}}{\sin x - \cos x}$$

$$=\frac{\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x-1}{\frac{1}{\sqrt{2}}\sin x-\frac{1}{\sqrt{2}}\cos x}$$

$$=\frac{\cos\left(x-\frac{\pi}{4}\right)-1}{\sin\left(x-\frac{\pi}{4}\right)}$$

$$=\frac{-2\sin^2\left(\frac{x}{2}-\frac{\pi}{8}\right)}{2\sin\left(\frac{x}{2}-\frac{\pi}{8}\right)\cos\left(\frac{x}{2}-\frac{\pi}{8}\right)}$$

$$\Rightarrow f(x)=-\tan\left(\frac{x}{2}-\frac{\pi}{8}\right)$$

$$\Rightarrow f'(x)=-\frac{1}{2}\sec^2\left(\frac{x}{2}-\frac{\pi}{8}\right)$$

$$\Rightarrow f''(x)=-\frac{1}{2}.2\sec\left(\frac{x}{2}-\frac{\pi}{8}\right).\sec\left(\frac{x}{2}-\frac{\pi}{8}\right)\tan\left(\frac{x}{2}-\frac{\pi}{8}\right)\times\frac{1}{2}$$

$$=-\frac{1}{2}\sec^2\left(\frac{x}{2}-\frac{\pi}{8}\right).\tan\left(\frac{x}{2}-\frac{\pi}{8}\right)$$

Now, $f\left(\frac{7\pi}{12}\right)f''\left(\frac{7\pi}{12}\right)$

$$=-\tan\left(\frac{7\pi}{24}-\frac{\pi}{8}\right)\times\frac{-1}{2}\sec^2\left(\frac{7\pi}{24}-\frac{\pi}{8}\right)\times\tan\left(\frac{7\pi}{24}-\frac{\pi}{8}\right)$$

$$=\frac{1}{2}\tan^2\left(\frac{\pi}{6}\right)\times\sec^2\frac{\pi}{6}$$

$$=\frac{1}{2}\times\frac{1}{3}\times\frac{4}{3}=\frac{2}{9}$$

HINT:

$$\frac{d}{dx}(\tan x)=\sec^2 x$$

$$\frac{d}{dx}(\sec^2 x)=2\sec^2 x.\tan x$$

18. Option (A) is correct.

Equation of plane P containing the given lines is

$(x + 2y + 3z - 4) + \lambda(2x + y - z + 5) = 0$

$\Rightarrow (1 + 2\lambda)\, x + (2 + \lambda)y + (3 - \lambda)\, z + (-4 + 5\,\lambda) = 0$

Now, plane P is perpendicular to plane P'

$$\vec{r}=(\hat{i}-\hat{j})+\lambda(\hat{i}+\hat{j}+\hat{k})+\mu(\hat{i}-2\hat{j}+3\hat{k})$$

So, normal to plane P' is

$$\vec{n}=(\hat{i}+\hat{j}+\hat{k})\times(\hat{i}-2\hat{j}+3\hat{k})$$

$$\Rightarrow\vec{n}=5\hat{i}-2\hat{j}-3\hat{k}$$

$\therefore$ P and P' are perpendicular

$\therefore 5\,(1 + 2\lambda) - 2(2 + \lambda) - 3\,(3 - \lambda) = 0$

$\Rightarrow 5 + 10\lambda - 4 - 2\lambda - 9 + 3\lambda = 0$

$\Rightarrow 11\lambda = 8 \Rightarrow \lambda=\frac{8}{11}$

$$\therefore \text{P}:\left(1+\frac{16}{11}\right)x+\left(2+\frac{8}{11}\right)y+\left(3-\frac{8}{11}\right)z+\left(5\times\frac{8}{11}-4\right)=0$$

i.e., $27x + 30y + 25z = 4$

which is same as $ax + by + cz = 4$

$\therefore a = 27, b = 30$ and $c = 25$

$\Rightarrow a - b + c = 27 - 30 + 25 = 22$

HINT:

When two planes are perpendicular, then dot product of their normals is zero.

19. Option (D) is correct.

We have,

$$|\text{A}| = \begin{vmatrix}2 & 1 & 0\\1 & 2 & -1\\0 & -1 & 2\end{vmatrix}=2(4-1)-1(2-0)+0$$

$= 6 - 2 = 4$

So, $|2\text{A}| = 2^3\,|\text{A}| = 8 \times 4 = 32$

Now, $|\text{adj (adj (adj 2A))}| = |2\text{A}|^{(n-1)^3}$

$=(32)^{2^3}=32^8$

$\Rightarrow 16^n = (32)^8 = 2^8 \times 16^8$

$\Rightarrow 16^n = 16^{2+8} \Rightarrow n = 10$

HINT:

(1) $|k\text{A}| = k^n\,|\text{A}|$

(2) $|\text{adj A}| = |\text{A}|^{n-1}$

20. Option (C) is correct.

$$\text{I}=\int\frac{x+1}{x(1+xe^x)^2}dx$$

Put $1 + xe^x = t \Rightarrow xe^x = t - 1$

$\Rightarrow (xe^x + e^x)\, dx = dt$

$\Rightarrow e^x\,(x + 1)\, dx = dt$

$$\therefore I=\int\frac{dt}{e^x.x\,t^2}=\int\frac{dt}{(t-1)t^2}$$

Let $\frac{1}{t^2(t-1)}=\frac{\text{A}}{(t-1)}+\frac{\text{B}t+\text{C}}{t^2}$

$\Rightarrow 1 = \text{A}t^2 + (\text{B}t + \text{C})\,(t - 1)$

Comparing coefficients of t^2, t and constant terms, we get

A + B = 0, C – B = 0, –C = 1

On solving above equations, we get

C = –1, =B, A = 1

$$\therefore\ \text{I}=\int\frac{1}{t-1}dt+\int\frac{-t-1}{t^2}dt$$

$$=\int\frac{1}{t-1}dt-\int\frac{1}{t}dt-\int\frac{1}{t^2}dt$$

$$=\log|t-1|-\log|t|+\frac{1}{t}+\text{C}$$

$$\Rightarrow \text{I}=\log|x\,e^x|-\log|1+x\,e^x|+\frac{1}{1+xe^x}+c$$

$$= \log\left|\frac{xe^x}{1+xe^x}\right| + \frac{1}{1+xe^x} + C$$

Now, $\lim_{x\to\infty} I(x) = 0$

$$\Rightarrow \lim_{x\to\infty}\left\{\log\left|\frac{xe^x}{1+xe^x}\right| + \frac{1}{1+xe^x} + C\right\} = 0$$

$$\Rightarrow \lim_{x\to\infty}\left\{\log\left(\frac{e^x}{\frac{1}{x}+e^x}\right) + \frac{\frac{1}{x}}{\frac{1}{x}+e^x} + C\right\}$$

$\Rightarrow 0 + 0 + C = 0 \Rightarrow C = 0$

$$\therefore I(x) = \log\left|\frac{xe^x}{1+xe^x}\right| + \frac{1}{1+xe^x}$$

$$\Rightarrow I(1) = \log\left|\frac{e}{1+e}\right| + \frac{1}{1+e} = 1 - \log(1+e) + \frac{1}{1+e}$$

$$= \frac{2+e}{1+e} - \log|1+e|$$

HINT:

(1) $\lim_{x\to\infty} \frac{1}{x} = 0$

(2) $\log 1 = 0$

21. The correct answer is (19).

We have, A = {0, 3, 4, 6, 7, 8, 9, 10}

Case I: $x - y$ is odd, if one is odd and one is even and $x > y$.

$\therefore$ Possibilites are {(3, 0), (4, 3), (6, 3), (7, 6), (7, 4), (7, 0), (8, 7), (8, 3), (9, 8), (9, 6), (9, 4), (9, 0), (10, 9), (10, 7), (10, 3)}

No. of cases = 15

Case II: $x - y = 2$

$\therefore$ Possibilities are {(6, 4), (8, 6), (9, 7), (10, 8)}

$\therefore$ No. of cases = 4

So, minimum ordered pair to be added = 15 + 4 = 19

HINT:

Any relations said to be symmetric if $(a, b) \in R$ $\Rightarrow (b, a) \in R$

22. The correct answer is (1275).

Let T_{r+1} be the constant term.

$$T_{r+1} = {}^7C_r (3x^2)^{7-r}\left(\frac{-1}{2x^5}\right)^r$$

For constant term, power of x should be zero.

i.e., $14 - 2r - 5r = 0$

$\Rightarrow 14 = 7r \Rightarrow r = 2$

Now, constant term = α

$$\Rightarrow {}^7C_2 (3)^5 \left(\frac{-1}{2}\right)^2 = \alpha$$

$$\Rightarrow 21 \times 243 \times \frac{1}{4} = \alpha$$

$\Rightarrow [\alpha] = [1275.75] = 1275$

HINT:

Let $(a + b)^n$, then $T_{r+1} = {}^nC_r\, a^{n-r} . b^r$

23. The correct answer is (9).

Since $\left(\frac{5}{2}, 1, \lambda\right)$ and (–2, 0, 1) are equidistant from plane $2x + 3y - 6z + 7 = 0$

$$\therefore \left|\frac{2\left(\frac{5}{2}\right) + 3(1) - 6(\lambda) + 7}{\sqrt{2^2+3^2+6^2}}\right| = \left|\frac{2(-2)+3(0)-6(1)+7}{\sqrt{2^2+3^2+6^2}}\right|$$

$\Rightarrow |5 + 3 - 6\lambda + 7| = |-4 - 6 + 7|$

$\Rightarrow |15 - 6\lambda| = |-3|$

$\Rightarrow 15 - 6\lambda = \pm 3$

$\Rightarrow 15 - 6\lambda = 3$ or $15 - 6\lambda = -3$

$\Rightarrow 6\lambda = 12$ or $6\lambda = 18$

$\Rightarrow \lambda = 2$ or $\lambda = 3$

$\because \lambda_1 > \lambda_2$

$\therefore \lambda_1 = 3$ and $\lambda_2 = 2$

So, point will be (1, 2, 3)

Let $M_0 = (1, 2, 3)$

M_1 is the point through which line passes i.e, (5, 1, –7)

and $\vec{s} = \hat{i} + 2\hat{j} + 2\hat{k}$

$\therefore \overrightarrow{M_0M_1} = 4\hat{i} - \hat{j} - 10\hat{k}$

Now, required distance $= \left|\frac{\overrightarrow{M_0M_1} \times \vec{s}}{|\vec{s}|}\right|$

$$= \frac{|(4\hat{i} - \hat{j} - 10\hat{k}) \times (\hat{i} + 2\hat{j} + 2\hat{k})|}{\sqrt{1+4+4}}$$

$$= \frac{|18\hat{i} - 18\hat{j} + 9\hat{k}|}{3} = 9$$

HINT:

Distance of a point (a, b, c) from a plane $px + qy + rz + s = 0$ is $\frac{|ap + bq + cr + s|}{\sqrt{p^2+q^2+r^2}}$

24. The correct answer is (3).

The given differential equation is,

$(y - 2\log x)\,dx + (x \log x^2)\,dy = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{(2\log x - y)}{2x\log x}$$

$$\Rightarrow \frac{dy}{dx} + \frac{y}{2x\log x} = \frac{1}{x}$$

It is a linear differential equation.

$$\therefore \text{I.F.} = e^{\int \frac{1}{2x\log x}dx}$$

Put $\log x = t \Rightarrow \frac{1}{x}dx = dt$

$$\therefore \text{I.F.} = e^{\int \frac{1}{2t}dt} = e^{\log(t)^{\frac{1}{2}}} = \sqrt{t} = \sqrt{\log x}$$

So, required solution is,

$$y\sqrt{\log x} = \int \frac{\sqrt{\log x}}{x}dx$$

$$\log x = v \Rightarrow \frac{1}{x}dx = dv$$

$$\Rightarrow y\sqrt{\log x} = \int \sqrt{v}\,dv + C$$

$$\Rightarrow y\sqrt{\log x} = \frac{2v^{3/2}}{3} + C$$

$$\Rightarrow y\sqrt{\log x} = \frac{2}{3}(\log x)^{3/2} + C$$

Now, this curve passes through $\left(e, \frac{4}{3}\right)$ and (e^4, α)

$$\therefore \frac{4}{3}\sqrt{\log e} = \frac{2}{3}(\log e)^{3/2} + C$$

$$\Rightarrow C = \frac{4}{3} - \frac{2}{3} = \frac{2}{3}$$

Also, $\alpha\sqrt{\log e^4} = \frac{2}{3}(\log e^4)^{3/2} + \frac{2}{3}$

$$\Rightarrow 2\alpha = \frac{2}{3} \times (4)^{3/2} + \frac{2}{3} = \frac{16}{3} + \frac{2}{3} = \frac{18}{3}$$

$\Rightarrow \alpha = 3$

HINT:

Reduce the given differential equation to linear differential equation and find its solution.

25. The correct answer is (11).

Let $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

Now, $\vec{a}.\vec{c} = -12$

$\Rightarrow 6C_1 + 9C_2 + 12C_3 = -12$ ...(i)

Also, $\vec{c}.(\hat{i} - 2\hat{j} + \hat{k}) = 5$

$\Rightarrow C_1 - 2C_2 + C_3 = 5$ ...(ii)

Now, $\vec{a} \times \vec{c} = \vec{a} \times \vec{b}$

$\Rightarrow \vec{a} \times (\vec{c} - \vec{b}) = 0$

$\Rightarrow \vec{a}$ is parallel to $(\vec{c} - \vec{b})$

$\Rightarrow \vec{a} = \lambda(\vec{c} - \vec{b})$

$$\Rightarrow 6\hat{i} + 9\hat{j} + 12\hat{k} = \lambda(c_1 - \alpha)\hat{i} + \lambda(c_2 - 11)\hat{j} + \lambda(c_3 + 2)\hat{k}$$

On comparing, we get

$$c_1 = \frac{6}{\lambda} + \alpha, c_2 = \frac{9}{\lambda} + 11, c_3 = \frac{12}{\lambda} - 2$$

Put there values in (ii), we get

$$\frac{6}{\lambda} + \alpha - \frac{18}{\lambda} - 22 + \frac{12}{\lambda} - 2 = 5$$

$\Rightarrow \alpha = 29$

From (i) and values of C_1, C_2, C_3, and α we have

$$6\left(\frac{6}{\lambda} + 29\right) + 9\left(\frac{9}{\lambda} + 11\right) + 12\left(\frac{12}{\lambda} - 2\right) = -12$$

$$\Rightarrow \frac{261}{\lambda} = -261 \Rightarrow \lambda = -1$$

So, $C_1 = 23$, $C_2 = 2$, $C_3 = -14$

$$\therefore \vec{c}.(\hat{i} + \hat{j} + \hat{k}) = (23\hat{i} + 2\hat{j} + -14\hat{k}).(\hat{i} + \hat{j} + \hat{k})$$

$= 23 + 2 - 14 = 11$

HINT:

$$\vec{a} \times \vec{c} = \vec{a} \times \vec{b} \Rightarrow \vec{a} \,||\, (\vec{c} - \vec{b}) \Rightarrow a = \lambda(\vec{c} - \vec{b})$$

26. The correct answer is (31).

We have,

$$\left[\frac{66}{3}\right] = 22$$

$$\left[\frac{66}{3^2}\right] = 7$$

$$\left[\frac{66}{3^3}\right] = 2$$

Highest powers of 3 is greater than 66. So, their g.i.f. is always 0

$\therefore$ Required natural number = 22 + 7 + 2 = 31

27. The correct answer is (5).

Let $y = \dfrac{x^3}{x^4 + 147}$

$$\Rightarrow \frac{dy}{dx} = \frac{(x^4 + 147) \times 3x^2 - x^3(4x^3)}{(x^4 + 147)^2}$$

$$= \frac{3x^6 + 441x^2 - 4x^6}{(x^4 + 147)^2} = \frac{441x^2 - x^6}{(x^4 + 147)^2}$$

For maxima/minima, put $\frac{dy}{dx} = 0$

$\Rightarrow 441\,x^2 - x^6 = 0 \Rightarrow x^4 = 441$

$\Rightarrow x = \pm\sqrt{21}, \ \pm\sqrt{21}i$

Now, by descrates rule on number line we have

+	−	+	−
	$-\sqrt{21}$	0	$\sqrt{21}$

Since sign changes from negative to positive at 0.

$\therefore$ Maximum value of is at $x = \sqrt{21} = 4.58$

Now, 4 < 4.5 < 5

$$\therefore y \text{ at } x = 4 = \frac{64}{403} = 0.159$$

$$y \text{ at } x = 5 = \frac{125}{772} = 0.162$$

So, y is maximum at $x = 5$

$\therefore \alpha = 5$

HINT:

For maximum value, find $\frac{dy}{dx}$ and then observe the change in signs using decrates rule.

28. The correct answer is (25).

x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$

x	$x-9$	$(x-9)^2$
y	$y-9$	$(y-9)^2$
10	1	1
12	3	9
6	–3	9
12	3	9
4	–5	25
8	–1	1
$x+y+92$		$(x-9)^2+(y-9)^2+54$

Now, mean $(\bar{x}) = 9$

$$\Rightarrow \frac{x+y+52}{8} = 9$$

$\Rightarrow x + y = 20$ ...(i)

Also, variance = 9.25

$$\Rightarrow \frac{(x-9)^2+(y-9)^2+54}{8} = 9.25$$

$\Rightarrow x^2 + y^2 + 81 + 81 - 2 \times 9(x + y) = 20$

$\Rightarrow x^2 + y^2 - 18 \times 20 = -142$

$\Rightarrow x^2 + y^2 = 218$

$\Rightarrow x^2 + (20 - x)^2 = 218$

$\Rightarrow x^2 + 400 + x^2 - 40x = 218$

$\Rightarrow 2x^2 - 40x + 182 = 0$

$$\Rightarrow x = \frac{40 \pm 12}{4}$$

$\Rightarrow x = 13$ or $x = 7 \Rightarrow y = 7$ or $y = 13$

But $x > y$

$\therefore x = 13$ and $y = 7$

So, $3x - 2y = 39 - 14 = 25$

HINT:

(1) Mean $= \frac{\Sigma x_i}{n}$

(2) Variance $= \frac{\Sigma(x_i - \bar{x})^2}{n}$

29. The correct answer is (2).

We have,

$C_1 : x^2 + y^2 - 4x - 2y = \alpha - 5$

$C_1 : (x-2)^2 + (y-1)^5 - 5 = \alpha - 5$

$C_1 : (x-2)^2 + (y-1)^2 = (\sqrt{\alpha})^2$

So, centre and radius of C_1 are (2, 1) and $\sqrt{\alpha}$ respectively

Now, image of (2, 1) along the line $y = 2x + 1$ is,

$$\frac{x-2}{2} = \frac{y-1}{-1} = \frac{-2(4-1+1)}{2^2+(-1)^2}$$

$$\Rightarrow \frac{x-2}{2} = \frac{y-1}{-1} = \frac{-8}{5}$$

$$\Rightarrow x = \frac{-6}{5} \text{ and } y = \frac{13}{5}$$

Now, $\left(\frac{-6}{5}, \frac{13}{5}\right)$ will be the centre of C_2

$$\therefore f = \frac{6}{5} \text{ and } g = \frac{-13}{5}$$

Now, radius of $C_2 = r = \sqrt{f^2 + g^2 - \frac{36}{5}}$

$$\Rightarrow r = \sqrt{\frac{36}{25} + \frac{169}{25} - \frac{36}{5}} = 1$$

$\because r = 1$ so, $\alpha = 1$

$\therefore \alpha + r = 1 + 1 = 2$

HINT:

Image of a point (x_1, y_1) w.r.t. $ax + by + c = 0$ is (x, y), then

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{-2(ax_1+by_1+c)}{(a^2+b^2)}$$

30. The correct answer is (14).

$$\text{Let I} = \frac{2}{\pi}\int_{\frac{\pi}{6}}^{5\frac{\pi}{6}} \{8[\operatorname{cosec} x] - 5[\cot x]\}dx$$

$$= \frac{2}{\pi}\left[8\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} [\operatorname{cosec} x]dx - 5\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} [\cot x]dx\right]$$

$$= \frac{2}{\pi}\left[8\int_{\pi/6}^{5\pi/6} dx - 5\left\{\int_{\pi/6}^{\pi/4} dx + \int_{\pi/4}^{\pi/2} 0.dx + \int_{\pi/2}^{3\pi/4} (-1)dx + \int_{3\pi/4}^{5\pi/6} (-2)dx\right\}\right]$$

$$= \frac{2}{\pi}\left[8 \times \left(\frac{5\pi}{6} - \frac{\pi}{6}\right) - 5\left\{\left(\frac{\pi}{4} - \frac{\pi}{6}\right) - \left(\frac{3\pi}{4} - \frac{\pi}{2}\right)\right\} - 2\left(\frac{5\pi}{6} - \frac{3\pi}{4}\right)\right]$$

$$= \frac{2}{\pi}\left[\frac{16\pi}{3} + \frac{5\pi}{3}\right] = 14$$

HINT:

Check the graph of [cosec x] and [cot x].