JEE (Main) MATHEMATICS SOLVED PAPER

2023
06^{th} April Shift 1

Section A

Q. 1. The straight lines l_1 and l_2 pass through the origin and trisect the line segment of the line L : $9x + 5y = 45$ between the axes. If m_1 and m_2 are the slopes of the lines l_1 and l_2, then the point of intersection of the line $y = (m_1 + m_2)x$ with L lies on.

(1) $6x + y = 10$ **(2)** $6x - y = 15$

(3) $y - 2x = 5$ **(4)** $y - x = 5$

Q. 2. Let the position vectors of the points A, B, C and D be $5\hat{i}+5\hat{j}+2\lambda\hat{k}, \hat{i}+2\hat{j}+3\hat{k}, -2\hat{i}+\lambda\hat{j}+4\hat{k}$ and $-\hat{i}+5\hat{j}+6\hat{k}$. Let the set S = $\{\lambda \in R$: the points A, B, C and D are coplanar}. $\sum_{\lambda\in S}(\lambda+2)^2$ is equal to:

(1) $\frac{37}{2}$ **(2)** 13 **(3)** 25 **(4)** 41

Q. 3. Let $I(x)=\int\frac{x^2(x\sec^2 x+\tan x)}{(x\tan x+1)^2}dx$. If I(0) = 0, then $I\left(\frac{\pi}{4}\right)$ is equal to :

(1) $\log_e\frac{(\pi+4)^2}{16}+\frac{\pi^2}{4(\pi+4)}$

(2) $\log_e\frac{(\pi+4)^2}{32}-\frac{\pi^2}{4(\pi+4)}$

(3) $\log_e\frac{(\pi+4)^2}{16}-\frac{\pi^2}{4(\pi+4)}$

(4) $\log_e\frac{(\pi+4)^2}{32}+\frac{\pi^2}{4(\pi+4)}$

Q. 4. The sum of the first 20 terms of the series 5 + 11 + 19 + 29 + 41 + ….. is :

(1) 3450 **(2)** 3420 **(3)** 3520 **(4)** 3250

Q. 5. A pair of dice is thrown 5 times. For each throw, a total of 5 is considered a success. If probability of at least 4 successes is $\frac{k}{3^{11}}$, then k is equal to :

(1) 164 **(2)** 123 **(3)** 82 **(4)** 75

Q. 6. Let A = $[a_{ij}]_{2\times2}$, where $a_{ij} \neq 0$ for all i, j and $A^2 = I$. Let a be the sum of all diagonal elements of A and $b = |A|$. Then $3a^2 + 4b^2$ is equal to :

(1) 14 **(2)** 4 **(3)** 3 **(4)** 7

Q. 7. Let $a_1, a_2, a_3, \ldots, a_n$ be n positive consecutive terms of an arithmetic progression. If $d > 0$ is its common difference, then :

$$\lim_{n\to\infty}\sqrt{\frac{d}{n}}\left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+\ldots+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}\right)$$ **is**

(1) $\frac{1}{\sqrt{d}}$ **(2)** 1 **(3)** $\sqrt{d}$ **(4)** 0

Q. 8. If ${}^{2n}C_3 : {}^{n}C_3 : 10 : 1$, then the ratio $(n^2 + 3n) : (n^2 - 3n + 4)$ is :

(1) 27 : 11 **(2)** 35 : 16 **(3)** 2 : 1 **(4)** 65 : 37

Q. 9. Let A = $\{x \in R : [x + 3] + [x + 4] \leq 3\}$;

$$B=\left\{x\in R:3^x\left(\sum_{x-3}^{\infty}\frac{3}{10^r}\right)<3^{-3x}\right\},$$

where $[t]$ denotes greatest integer function. Then,

(1) $A \subset B, A \neq B$ **(2)** $A \cap B = \phi$

(3) A = B **(4)** $B \subset C, A \neq B$

Q. 10. One vertex of a rectangular parallelepiped is at the origin O and the lengths of its edges along x, y and z axes are 3, 4 and 5 units respectively. Let P be the vertex (3, 4, 5). Then the shortest distance between the diagonal OP and an edge parallel to z axis, not passing through O or P is :

(1) $\frac{12}{5\sqrt{5}}$ **(2)** $12\sqrt{5}$ **(3)** $\frac{12}{5}$ **(4)** $\frac{12}{\sqrt{5}}$

Q. 11. If the equation of the plane passing through the line of intersection of the planes $2x - y + z = 3$, $4x - 3y + 5z + 9 = 0$ and parallel to the line $\frac{x+1}{-2}=\frac{y+3}{4}=\frac{z-2}{5}$ is $ax + by + cz + 6 = 0$, then $a + b + c$ is equal to :

(1) 15 **(2)** 14 **(3)** 13 **(4)** 12

Q. 12. If the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^n$ is $\sqrt{6}:1$, then the third term from the beginning is :

(1) $30\sqrt{2}$ **(2)** $60\sqrt{2}$ **(3)** $30\sqrt{3}$ **(4)** $60\sqrt{3}$

Q. 13. The sum of all the roots of the equation $|x^2 - 8x + 15| - 2x + 7 = 0$ is :

(1) $11-\sqrt{3}$ **(2)** $9-\sqrt{3}$

(3) $9+\sqrt{3}$ **(4)** $11+\sqrt{3}$

Q. 14. From the top A of a vertical wall AB of height 30 m, the angles of depression of the top P and bottom Q of a vertical tower PQ are 15° and 60° respectively, B and Q are on the same horizontal level. If C is a point on AB such that CB = PQ, then the area (in m^2) of the quadrilateral BCPQ is equal to :

(1) $200(3-\sqrt{3})$ **(2)** $300(\sqrt{3}+1)$

(3) $300(\sqrt{3}-1)$ **(4)** $600(\sqrt{3}-1)$

Q. 15. Let $\vec{a}=2\hat{i}+3\hat{j}+4\hat{k}, \vec{b}=\hat{i}-2\hat{j}-2\hat{k}$ and $\vec{c}=-\hat{i}+4\hat{j}+3\hat{k}$. If $\vec{d}$ is a vector perpendicular to both $\vec{b}$ and $\vec{c}$, and $\vec{a}.\vec{d}=18$, then $[\vec{a}\times\vec{d}]^2$ is equal to :

(1) 760 **(2)** 640 **(3)** 720 **(4)** 680

Q. 16. If $2x^y + 3y^x = 20$, then $\frac{dy}{dx}$ at (2, 2) is equal to :

(1) $-\left(\frac{3+\log_e 8}{2+\log_e 4}\right)$ **(2)** $-\left(\frac{2+\log_e 8}{3+\log_e 4}\right)$

(3) $-\left(\frac{3+\log_e 4}{2+\log_e 8}\right)$ **(4)** $-\left(\frac{3+\log_e 16}{4+\log_e 8}\right)$

Q. 17. If the system of equations

$x + y + az = b$

$2x + 5y + 2z = 6$

$x + 2y + 3z = 3$

has infinitely many solutions, then $2a + 3b$ is equal to :

(1) 28 **(2)** 20 **(3)** 25 **(4)** 23

Q. 18. Statement $(P \Rightarrow Q) \wedge (R \Rightarrow Q)$ is logically equivalent to:

(1) $(P \vee R) \Rightarrow Q$ **(2)** $(P \Rightarrow R) \vee (Q \Rightarrow R)$

(3) $(P \Rightarrow R) \wedge (Q \Rightarrow R)$ **(4)** $(P \wedge R) \Rightarrow Q$

Q. 19. Let $5f(x)+4f\left(\frac{1}{x}\right)=\frac{1}{x}+3, x>0$. Then $18\int_1^2 f(x)dx$ is equal to :

(1) $10\log_e 2 - 6$ **(2)** $10\log_e 2 + 6$

(3) $5\log_e 2 - 3$ **(4)** $5\log_e 2 + 3$

Q. 20. The mean and variance of a set of 15 numbers are 12 and 14 respectively. The mean and variance of another set of 15 numbers are 14 and σ^2 respectively. If the variance of all the 30 numbers in the two sets is 13, then σ^2 is equal to :

(1) 12 **(2)** 10 **(3)** 11 **(4)** 9

Section B

Q. 21. Let the tangents to the curve $x^2 + 2x - 4y + 9 = 0$ at the point P(1, 3) on it meet the y-axis at A. Let the line passing through P and parallel to the line $x - 3y = 6$ meet the parabola $y^2 = 4x$ at B. If B lies on the line $2x - 3y = 8$, then $(AB)^2$ is equal to ______.

Q. 22. Let the point $(p, p + 1)$ lie inside the region $E = \{(x, y) : 3 - x \le y \le \sqrt{9-x^2}, 0 \le x \le 3\}$. If the set of all values of p is the interval (a, b), then $b^2 + b - a^2$ is equal to _____.

Q. 23. Let $y = y(x)$ be a solution of the differential $(x \cos x)dy + (xy \sin x + y \cos x - 1)\, dx = 0, 0 < x < \frac{\pi}{2}$. If $\frac{\pi}{3}y\left(\frac{\pi}{3}\right)=\sqrt{3}$, then $\left|\frac{\pi}{6}y''\left(\frac{\pi}{6}\right)+2y'\left(\frac{\pi}{6}\right)\right|$ is equal to ____.

Q. 24. Let $a \in \mathbb{Z}$ and $[t]$ be the greatest integer $\le t$. Then the number of points, where the function $f(x) = [a + 13 \sin x], x \in (0, \pi)$ is not differentiable, is _____.

Q. 25. If the area of the region $S = \{(x, y) : 2y - y^2 \le x^2 \le 2y, x \ge y\}$ is equal to $\frac{n+2}{n+1}=\frac{\pi}{n-1}$, then the natural number n is equal to _____.

Q. 26. The number of ways of giving 20 distinct oranges to 3 children such that each child gets at least one orange is _____.

Q. 27. Let the image of the point P (1, 2, 3) in the plane $2x - y + z = 9$ be Q. If the coordinates of the point R are (6, 10, 7). then the square of the area of the triangle PQR is ____.

Q. 28. A circle passing through the point P(α, β) in the first quadrant touches the two coordinate axes at the points A and B. The point P is above the line AB. The point Q on the line segment AB is the foot of perpendicular from P on AB. If PQ is equal to 11 units, then value of αβ is ____.

Q. 29. The coefficient of x^{18} in the expansion of $\left(x^4-\frac{1}{x^3}\right)^{15}$ is _______.

Q. 30. Let A = {1, 2, 3, 4, …., 10} and B = {0, 1, 2, 3, 4}. The number of elements in the relation $R = \{(a, b) \in A \times A: 2(a-b)^2 + 3(a-b) \in B\}$ is _____.

Answer Key

Q. No.	Answer	Topic name	Chapter name
1	(4)	Equation of line	Straight lines
2	(4)	Coplanarity	Vector
3	(2)	Integration by parts	Indefinite Integral
4	(3)	Method of difference	Sequence and Series
5	(2)	Binomial Distribution	Probability
6	(2)	Determinants value of matrix	Matrix and Determinants
7	(2)	Arithmetic Progression	Sequence and series limit
8	(3)	Properties of NCR	Binomial Theorem
9	(3)	GP	Function, Sequence and Series
10	(3)	Shortest distance between skew lines	3 D
11	(3)	Family of planes	3 D
12	(4)	General Term	Binomial theorem
13	(3)	Modulus	Function
14	(4)	Practical problem	Height and Distance
15	(3)	Scalar triple product	Vector
16	(2)	Logarithmic differentiation	Differentiation
17	(4)	Solution of system of linear equations	Determinants
18	(1)	Duality	Mathematical reasoning
19	(1)	Definite integral	Definite Integral
20	(2)	Mean and variance	Statistics
21	[92]	Tangent of a circle	Circle
22	[3]	Inequality in two variable	Circle
23	[2]	Linear differential Equation	Differential equation
24	[25]	Points of non differentiability	Continuity and Differentiability
25	[5]	Area between two curves	Area under curves
26	[3483838676]	Distribution of objects	Permutation and combinations
27	[594]	Image of a point wrt a plane	3 D
28	[121]	Line and Circle	Circle
29	[5005]	General term	Binomial Theorem
30	[18]	Relation	Relations

Solutions

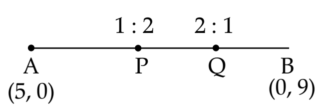
Section A

1. Option (4) is correct.

$L = 9x + 5y = 45$ ...(1)

$P_x = \dfrac{2\times5+1\times0}{1+2} = \dfrac{10}{3}$

$P_y = \dfrac{0\times2+9\times1}{2+1} = \dfrac{9}{3} = 3$

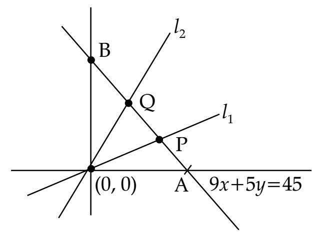


$P \to \left(\dfrac{10}{3}, 3\right)$

Similarly $Q : \left(\dfrac{5}{3}, 6\right)$

Now $m_1 = \dfrac{3-0}{\frac{10}{3}-0} = \dfrac{9}{10}$

$m_2 = \dfrac{6-0}{\frac{5}{3}-0} = \dfrac{18}{5}$



Now line $L_1 : y = (m_1 + m_2)\,x$

$\Rightarrow y = \dfrac{9}{2}x \Rightarrow 2y = 9x$

$9x - 2y = 0$ ...(2)

By (1), (2) solving we get

$x = \dfrac{10}{7}, y = \dfrac{45}{7}$

Which satisfy $y - x = 5$.

2. Option (4) is correct.

A, B, C, D are coplanar

$\Rightarrow [\overline{AB}\ \overline{AC}\ \overline{AD}] = 0$

$$\Rightarrow \begin{vmatrix} -4 & -3 & 3-2\lambda \\ -7 & \lambda-5 & 4-2\lambda \\ -6 & 0 & 6-2\lambda \end{vmatrix} = 0$$

$\Rightarrow \lambda^2 - 5\lambda + 6 = 0 \Rightarrow \lambda = 3, 2$

$$\sum_{\lambda \in S} (\lambda+2)^2 = 16 + 25 = 41$$

3. Option (2) is correct.

$$I(X) = \int \frac{x^2(x\sec^2 x + \tan x)dx}{(x\tan x + 1)^2}$$

Let $x \tan x + 1 = t$

$$I(X) = x^2\left(\frac{-1}{x\tan x + 1}\right) + \int \frac{2x}{x\tan x + 1}dx$$

$$I(X) = x^2\left(\frac{-1}{x\tan x + 1}\right) + 2\ln|x\sin x + \cos x| + C$$

$I(0) = 0 \Rightarrow C = 0$

$$I\left(\frac{\pi}{4}\right) = \log_e \frac{(\pi+4)^2}{16} - \frac{\pi^2}{4(\pi+4)}$$

4. Option (3) is correct.

$S_n = 5 + 11 + 19 + 29 + 41 + \ldots + T_n$

$S_n = \quad 5 + 11 + 19 + 29 + \ldots + T_{n-1} + T_n$

$0 = 5 + 6 + 8 + 10 + 12 + \ldots - T_n$

$$T_n = 5 + \left(\frac{n-1}{2}\right)[2 \times 6 + (n-2) \times 2]$$

$T_n = n^2 + 3n + 1$

$S_n = \Sigma T_n = \Sigma n^2 + 3n + 1$

$S_{20} = 3520$

5. Option (2) is correct.

n (total 5) = {(1, 4), (2, 3), (3, 2), (4, 1)} = 4

$$P\text{ (success)} = \frac{4}{36} = \frac{1}{9}$$

P (at least 4 success) = P (4 success) + P (5 success)

$$= {}^5C_4\left(\frac{1}{9}\right)^4 \times \frac{8}{9} + {}^5C_5\left(\frac{1}{9}\right)^5$$

$$= \frac{K}{3^{11}} = \frac{123}{31^{11}}$$

K = 123

6. Option (2) is correct.

$A^2 = I \Rightarrow |A|^2 = I \Rightarrow |A| = \pm 1 = b$

$$\text{Let } A = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} = I$$

$$\Rightarrow A^2 = \begin{bmatrix} \alpha^2 + \beta\gamma & \alpha\beta + \beta\delta \\ \alpha\gamma + \gamma\delta & \gamma\beta + \delta^2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\alpha\beta + \beta\delta = 0 \Rightarrow (\alpha + \delta)\beta = 0 \Rightarrow \alpha + \delta = 0 = a$

$(\alpha + \delta)\gamma = 0 \Rightarrow \beta\gamma + \delta^2 = 0$

Now $3a^2 + 4b^2 = 3(0)^2 + 4(1) = 4$

7. Option (2) is correct.

$$\lim_{n\to\infty} \sqrt{\frac{d}{n}}\left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \ldots + \frac{1}{\sqrt{a_{n+1}}+\sqrt{a_n}}\right)$$

$$\Rightarrow \lim_{n\to\infty} \sqrt{\frac{d}{n}}\left(\frac{\sqrt{a_1}-\sqrt{a_2}}{a_1 - a_2} + \frac{\sqrt{a_2}-\sqrt{a_3}}{a_2 - a_3} + \ldots + \frac{\sqrt{a_{n-1}}-\sqrt{a_n}}{\sqrt{a_n}+\sqrt{a_n}}\right)$$

$$\Rightarrow \lim_{n\to\infty} \sqrt{\frac{d}{n}}\left(\frac{\sqrt{a_n}-\sqrt{a_1}}{d}\right)$$

$$\Rightarrow \lim_{n\to\infty} \frac{1}{\sqrt{n}}\left(\frac{\sqrt{a+(n-1)d}-\sqrt{a_1}}{\sqrt{d}}\right)$$

$= 1$

8. Option (3) is correct.

$$\frac{{}^{2n}C_3}{{}^nC_3} = 10 \Rightarrow \frac{2n!(n-3)!}{(2n-3)!n!} = 10 \qquad \left[\because {}^nC_r = \frac{n!}{r(n-r)!}\right]$$

$$\Rightarrow \frac{4(2n-1)}{n-2} = 10 \Rightarrow n = 8$$

$$\text{Now } \frac{n^2 + 8n}{n^2 - 3n + 4} = \frac{64 + 24}{64 - 24 + 4} = \frac{2}{1}$$

9. Option (3) is correct.

$A = [x \in R : [x + 3] + [x + 4] \le 3]$

$\Rightarrow 2[x] + 7 \le 3$

$\Rightarrow 2[x] \le -4$

$\Rightarrow [x] \le -2 \Rightarrow x < -1$

$$B = \left\{x \in R : 3^x\left(\sum_{r=1}^{\infty}\left(\frac{3}{10^r}\right)^{x-3} < 3^{-3x}\right)\right\}$$

$$= 3^{2x-3}\left(\frac{\frac{1}{10}}{1 - \frac{1}{10}}\right)^{x-3} < 3^{-3x}$$

$\Rightarrow 3^{6-2x} < 3^{3-5x}$

$\Rightarrow 6 - 2x < 3 - 5x$

$\Rightarrow x < -1$, A = B

10. Option (3) is correct.

Equation of OP is

$$\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$$

$a_1 = (0, 0, 0), a_2 = (3, 0, 5)\ b_1 = (3, 4, 5), b_2 = (0, 0, 1)$

Equation of edge parallel to z axis

$$\frac{x-3}{0} = \frac{y-0}{0} = \frac{z-5}{1}$$

$$\text{Shortest distance } = \frac{3 \times 4}{|4\hat{i} - 3\hat{j}|} = \frac{12}{5}$$

11. Option (3) is correct.

Equation of plane intersection of two plane

$P_1 + \lambda P_2 = 0$

$P_1 : 2x - y + z = 3$, and $P_2 : 4x - 3y + 5z + 9 = 0$

$\Rightarrow (2 + 4\lambda)x - (1 + 3\lambda)y + (1 + 5\lambda)z = (3 - 9\lambda)$ ...(i)

Since given plane parallel to

$$\frac{x+1}{-2} = \frac{y+3}{+4} = \frac{z-2}{5}$$

$\because \vec{b}.\vec{n} = 0$

$\Rightarrow -2(2 + 4\lambda) - 4(1 + 3\lambda) + 5(1 + 5\lambda) = 0$

$\Rightarrow \lambda = +\frac{3}{5}$

then plane

$P : 11x - 7y + 10z + 6 = 0$

$\Rightarrow a = 11, b = -7, c = 10,$

$a + b + c = 11 - 7 + 10 = 14$

12. Option (4) is correct.

$T_{r+1} = {}^nC_r x^{n-r} a^r$

$$\frac{T_5}{T'_5} = \frac{{}^nC_4 \times \left((2)^{\frac{1}{4}}\right)^{n-4} \left(\frac{1}{3^{1/4}}\right)^4}{{}^nC_4 \left(\frac{1}{3^{1/4}}\right)^{n-4} \left(2^{1/4}\right)^4} = \sqrt{6}$$

$$\Rightarrow 2^{\frac{n-8}{4}} \cdot \left(3^{\frac{1}{4}}\right)^{-4-4+n} = \sqrt{6}$$

$\Rightarrow n = 10$

$$T_3 = {}^{10}C_2 \left(2^{\frac{1}{4}}\right)^8 \left(\frac{1}{3^{\frac{1}{4}}}\right)^2$$

$T_3 = 60\sqrt{3}$

13. Option (3) is correct.

$|x^2 - 8x + 15| = 2x - 7$

take '+'

$x^2 - 8x + 15 = 2x - 7$

$\Rightarrow x^2 - 10x + 22 = 0$

$x_1 = 5 + \sqrt{3}, x_2 = 5 - \sqrt{3}$ (reject)

take '–'

$x^2 - 8x + 15 = 7 - 2x$

$x^2 - 6x + 8 = 0$

$x_3 = 4, x_4 = 2$ (reject)

Sum of roots $= 5 + \sqrt{3} + 4 = 9 + \sqrt{3}$

14. Option (4) is correct.

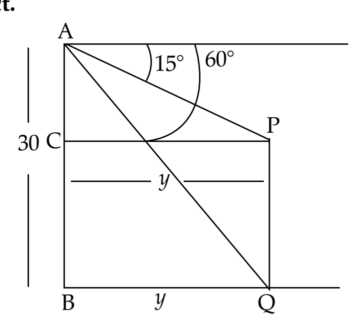
$\frac{AB}{BQ} = \tan 60°$

$\Rightarrow BQ = 10\sqrt{3} = y$

In ΔACP

$\frac{AC}{CP} = \tan 15°$

$\Rightarrow x = 60 - 20\sqrt{3}$



Area $= xy$

$= (60 - 20\sqrt{3}) \times 10\sqrt{3} = 600(\sqrt{3} - 1)$

15. Option (3) is correct.

$d = \lambda(\vec{b} \times \vec{c})$ $\quad \left[\because \vec{a}.\vec{d} = 18\right]$

$\vec{a}.\vec{d} = \lambda[\vec{a}.(\vec{b} \times \vec{c})] \Rightarrow 18 = \lambda[\vec{a}\ \vec{b}\ \vec{c}]$

$$\Rightarrow 18 = \lambda \begin{vmatrix} 2 & 3 & 4 \\ 1 & -2 & -2 \\ -1 & 4 & 3 \end{vmatrix}$$

$\Rightarrow 18 = 9\lambda \Rightarrow \lambda = 2$

$\Rightarrow \vec{d} = 2(2\hat{i} - \hat{j} + 2\hat{k})$

$|\vec{a} \times \vec{d}|^2 = a^2 d^2 - (\vec{a}.\vec{d})^2$

$= 29 \times 36 - (18)^2 = 18(58 - 18) = 18 \times 40 = 720$

16. Option (2) is correct.

$2x^y + 3y^x = 20$

$$\Rightarrow 2x^y\left(y.\frac{1}{x} + \ln x \frac{dy}{dx}\right) + 3y^x\left(x\frac{1}{y}\frac{dy}{dx} + \ln y.1\right) = 0$$

but (2, 2)

$$\Rightarrow 2.2^2\left(1 + \ln 2.\frac{dy}{dx}\right) + 3.4\left(1\frac{dy}{dx} + \ln 2\right) = 0$$

$$\Rightarrow \frac{dy}{dx}[8\ln 2 + 12] + 8 + 12\ln 2 = 0$$

$$\Rightarrow \frac{dy}{dx} = -\left[\frac{2 + \ln 8}{3 + \ln 4}\right]$$

17. Option (4) is correct.

for infinite many solutions

$\Delta = \Delta x = \Delta y = \Delta z = 0$

$$\text{then } \Delta = \begin{vmatrix} 1 & 1 & a \\ 2 & 5 & 2 \\ 1 & 2 & 3 \end{vmatrix} = 0$$

$11 - 4 - a = 0 \Rightarrow a = 7$

$$\Delta_z = \begin{vmatrix} 1 & 1 & b \\ 2 & 5 & 6 \\ 1 & 2 & 3 \end{vmatrix} = 0 \Rightarrow 3 - 0 - b = 0 \Rightarrow b = 3$$

Hence $2a + 3b = 23$

18. Option (1) is correct.

We know that

$P \Rightarrow Q \equiv \sim p \vee Q$

$\Rightarrow (\sim p \vee Q) \wedge (\sim R \vee Q)$

$\Rightarrow (\sim p \wedge \sim R) \vee Q$

$\Rightarrow \sim (P \vee R) \vee Q$

$\Rightarrow (P \vee R) \Rightarrow Q$

19. Option (1) is correct.

$$5f(x) + 4f\left(\frac{1}{x}\right) = \frac{1}{x} + 3 \quad ...(1)$$

$x \to \frac{1}{x}$

$$5f\left(\frac{1}{x}\right) + 4f(x) = x + 3 \quad ...(2)$$

(1) × 5 – (2) × 4

$$\Rightarrow f(x) = \frac{5}{9x} - \frac{4x}{9} + \frac{1}{3}$$

$$\Rightarrow 18\int_1^2 f(x) = 18\left[\frac{5}{9}\ln x - \frac{4}{9}\frac{x^2}{2} + \frac{1x}{3}\right]_1^2$$

$\Rightarrow 10 \ln 2 - 6$

20. Option (2) is correct.

Combine variance $= \dfrac{n_1\sigma^2 + n_2\sigma^2}{n_1 + n_2} + \dfrac{n_1 n_2(m_1 - m_2)^2}{(n_1 + n_2)^2}$

$$13 = \frac{15.14 + 15.\sigma^2}{30} + \frac{15 \times 15(12-14)^2}{30 \times 30}$$

$\sigma^2 = 10$

Section B

21. Correct answer is [92].

C: $x^2 + 2x - 4y + 9 = 0$

C: $(x + 1)^2 = 4(y - 2)$

Tangent at p (1, 3)

$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + C = 0$

$\Rightarrow x . 1 + (x + 1) - 2(y + 3) + 9 = 0$

$\Rightarrow x - y + 2 = 0$

A : (0, 2)

Line is parallel to $x - 3y = 6$ passes (1, 3) is $x - 3y + 8 = 0$

Meet parabola $y^2 = 4x$

$\Rightarrow y^2 = 4(3y - 8)$

$\Rightarrow (y - 8)(y - 4) = 0$

Point of intersection are (4, 4) are (16, 8) lies on $2x - 3y = 8$

Hence A (0, 2)

B (16, 8)

$(AB)^2 = 256 + 36 = 292$

22. Correct answer is [3].

$3 - x < y \le \sqrt{9 - x^2}; 0 \le x \le 3$

$L(A) > 0 \Rightarrow p + p + 1 - 3 > 0 \Rightarrow p > 1$

$S(A) < 0 \Rightarrow p + 1 - \sqrt{9 - p^2} < 0$

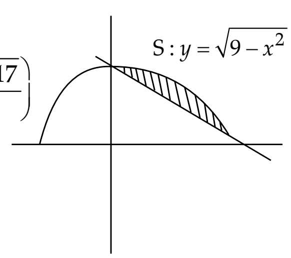
$\Rightarrow p + 2p + 1 < 9 - p^2$

$\Rightarrow 2p^2 + 2p - 8 < 0$

$\Rightarrow p^2 + p - 4 < 0$

$\Rightarrow p \in \left(\dfrac{-1-\sqrt{17}}{2}, \dfrac{-1+\sqrt{17}}{2}\right)$

$= p \in \left(1, \dfrac{-1+\sqrt{17}}{2}\right)$



$b^2 + b - a^2 = 4 - 1 = 3$

23. Correct answer is [2].

$(x \cos x)\, dy + (xy \sin x + y \cos x - 1)\, dx$

$= 0, 0 < x < \dfrac{\pi}{2}$

$$\Rightarrow \frac{dy}{dx} + \left(\frac{x\sin x + \cos x}{x\cos x}\right)y = \frac{1}{x\cos x}$$

$$\text{I.F.} = e^{\int \frac{x\sin x + \cos x}{x\cos x}dx}$$

I.F. $= x \sec x$

General solution y (IF) $= \int Q(\text{I.F.})dx$

$$y.x\sec x = \int \frac{x\sec x}{x\cos x}dx = \tan x + C$$

Since $y\left(\dfrac{\pi}{3}\right) = \dfrac{3\sqrt{3}}{\pi}$ Hence $C = \sqrt{3}$

Hence $\left|\dfrac{\pi}{6}y''\left(\dfrac{\pi}{6}\right) + y'\left(\dfrac{\pi}{6}\right)\right| = |-2| = 2$

24. Correct answer is [25].

$f(x) = [a + 13 \sin x]$

$f(x) = a + [13 \sin x]$ in $(0, \pi)$

$x \in (0, \pi)$

$\Rightarrow 0 < 13 \sin x \le 13$

$\Rightarrow [13 \sin x] = \{0, 1, 2, 3 \ldots\ldots 12, 13\}$

Total point of not differentiable = 25

25. Correct answer is [5].

$x^2 + y^2 - 2y \ge 0$ ...(1)

$x^2 - 2y \le 0$ ...(2)

$x \ge y$ ...(3)

Hence required area

$$= \frac{1}{2} \times 2 \times 2 - \int_0^2 \frac{x^2}{2}dx - \left(\frac{\pi}{4} - \frac{1}{2}\right)$$

$\Rightarrow n = 5$

26. Correct answer is [3483838676].

Number of ways = Total – (One child receive no orange + two child receive no orange)

$= 3^{20} - ({}^3C_1, (2^{20} - 2) + {}^3C_2\, 1^{20})$

= 3483838676

27. Correct answer is [594].

Let Q (α, β, γ) be the image of point p about the plane $2x - y + z = 9$

$$\Rightarrow \frac{\alpha - 1}{2} = \frac{\beta - 2}{-1} = \frac{\gamma - 3}{1} = \frac{-2(2 - 2 + 3 - 9)}{6}$$

$\Rightarrow \alpha = 5, \beta = 0, \gamma = 5$

area of ΔPQR is $\dfrac{1}{2}|\overrightarrow{PQ} \times \overrightarrow{PR}|$

$= |-12\hat{i} - 3\hat{j} + 21\hat{k}|$

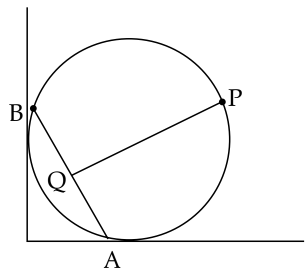
$= \sqrt{594}$

Square of the area = 594

28. Correct answer is [121].

Let the equation of the circle

$(x - a)^2 + (y - a)^2 = a^2$



Which passes through P (α, β)

then

$(\alpha - a)^2 + (\beta - a)^2 = a^2$

$\Rightarrow \alpha^2 + \beta^2 - 2\alpha a - 2\beta a + a^2 = 0$

Here equation of AB is $x + y = a$

Let Q (α, β′) be foot of the perpendicular of p on AB

$$\frac{\alpha' - \alpha}{1} = \frac{\beta' - \beta}{1} = \frac{-(\alpha + \beta - a)}{2}$$

$$PQ^2 = (\alpha' - \alpha)^2 + (\beta' - \beta)^2 = \frac{1}{4}(\alpha + \beta - a)^2 + \frac{1}{4}(\alpha + \beta - a)^2$$

$$\Rightarrow 121 = \frac{1}{2}(\alpha + \beta - a)^2$$

29. Correct answer is [5005].

Given $\left(x^4 - \frac{1}{x^3}\right)^{15}$

$$T_{r+1} = {}^{15}C_r (x^4)^{15-r}\left(\frac{-1}{x^3}\right)^r$$

$\Rightarrow 60 - 7r = 18$

$\Rightarrow r = 6$

Coefficient of $x^{18} = {}^{15}C_6 = 5005$

30. Correct answer is [18].

$A = \{1, 2, 3, ... 10\}$

$B = \{0, 1, 2, 3, 4\}$

$R = \{(a, b) \in A \times A : 2(a-b)^2 + 3(a-b) \in B\}$

Now $2(a-b)^2 + 3(a-b)$

$= (a-b)[2(a-b) + 3]$

$\Rightarrow a = b$ or $a - b = -2$

when $a = b \Rightarrow$ 10 order pair

when $a - b = -2 \Rightarrow$ 8 order pair

Total = 18

❑❑