JEE (Main) MATHEMATICS SOLVED PAPER

2023
06th April Shift 2

Section A

Q. 1. If $gcd\ (m, n) = 1$ and $1^2 - 2^2 + 3^2 - 4^2 + \ldots.. + (2021)^2 - (2022)^2 + (2023)^2 = 1012\ m^2 n$ then $m^2 - n^2$ is equal to :

(1) 180 **(2)** 220
(3) 200 **(4)** 240

Q. 2. The area bounded by the curves $y = |x - 1| + |x - 2|$ and $y = 3$ is equal to :

(1) 5 **(2)** 4
(3) 6 **(4)** 3

Q. 3. For the system of equations

$x + y + z = 6$
$x + 2y + \alpha z = 10$
$x + 3y + 5z = \beta$, which one of the following is NOT true :

(1) System has a unique solution for $\alpha = 3, \beta \neq 14$.
(2) System has a unique solution for $\alpha = -3, \beta = 14$.
(3) System has no solution for $\alpha = 3, \beta = 24$.
(4) System has infinitely many solutions for $\alpha = 3, \beta = 14$.

Q. 4. Among the statements :

(S1): $(p \Rightarrow q) \vee ((\sim p) \wedge q)$ is a tautology
(S2): $(q \Rightarrow p) \Rightarrow ((\sim p) \wedge q)$ is a contradiction

(1) only (S2) is True
(2) only (S1) is True
(3) neither (S1) and (S2) is True
(4) both (S1) and (S2) are True

Q. 5. $\lim\limits_{n\to\infty}\left\{\left(2^{\frac{1}{2}}-2^{\frac{1}{3}}\right)\left(2^{\frac{1}{2}}-2^{\frac{1}{5}}\right)\ldots\ldots\left(2^{\frac{1}{2}}-2^{\frac{1}{2n-1}}\right)\right\}$ is equal to:

(1) $\frac{1}{\sqrt{2}}$ **(2)** $\sqrt{2}$
(3) 1 **(4)** 0

Q. 6. Let P b a square matrix such that $P^2 = I - P$. For $\alpha, \beta, \gamma, \delta, \in N$, if $P^\alpha + P^\beta = \gamma I - 29P$ and $P^\alpha - P^\beta = \delta I - 13P$, then $\alpha + \beta + \gamma - \delta$ is equal to :

(1) 40 **(2)** 22
(3) 24 **(4)** 18

Q. 7. A plane P contains the line of intersection of the plane $\vec{r}.(\hat{i}+\hat{j}+\hat{k}) = 6$ and $\vec{r}.(2\hat{i}+3\hat{j}+4\hat{k}) = -5$. If P passes through the point (0, 2, –2), then the square of distance of the point (12, 12, 18) from the plane P is :

(1) 620 **(2)** 1240
(3) 310 **(4)** 155

Q. 8. Let $f(x)$ be a function satisfying $f(x) + f(\pi - x) = \pi^2$, $\forall x \in R$. Then $\int_0^\pi f(x)\ \sin x\ dx$ is equal to :

(1) $\frac{\pi^2}{2}$ **(2)** π^2
(3) $2\pi^2$ **(4)** $\frac{\pi^2}{4}$

Q. 9. If the coefficient of x^7 in $\left(ax^2 + \frac{1}{2bx}\right)^{11}$ and x^{-7} in $\left(ax - \frac{1}{3bx^2}\right)^{11}$ are equal, then :

(1) $64\ ab = 343$ **(2)** $32\ ab = 729$
(3) $729\ ab = 32$ **(4)** $243\ ab = 64$

Q. 10. If the tangents at the points P and Q are the circle $x^2 + y^2 - 2x + y = 5$ meet at the point $R\left(\frac{9}{4}, 2\right)$, then the area of the triangle PQR is :

(1) $\frac{5}{4}$ **(2)** $\frac{13}{4}$
(3) $\frac{5}{8}$ **(4)** $\frac{13}{8}$

Q. 11. Three dice are rolled. If the probability of getting different numbers on the three dice is $\frac{p}{q}$, where p and q are co-prime, then $q - p$ is equal to :

(1) 1 **(2)** 2
(3) 4 **(4)** 3

Q. 12. In a group of 100 persons 75 speak English and 40 speak Hindi. Each person speaks at least one of the two languages. If the number of persons, who speak only English is α and the number of persons who speak only Hindi is β, then the eccentricity of the ellipse $25\ (\beta^2 x^2 + \alpha^2 y^2)\ \alpha^2 \beta^2$ is :

(1) $\frac{\sqrt{129}}{12}$ **(2)** $\frac{\sqrt{117}}{12}$
(3) $\frac{\sqrt{119}}{12}$ **(4)** $\frac{3\sqrt{15}}{12}$

Q. 13. If the solution curve $f(x, y) = 0$ of the differential equation $(1 + \log_e x)\frac{dx}{dy} - x\log_e x = e^y$, $x > 0$, passes through the points (1, 0) and $(\alpha, 2)$, then α^α is equal to :

(1) $e^{\sqrt{2}e^2}$ **(2)** e^{e^2}
(3) $e^{2e^{\sqrt{2}}}$ **(4)** e^{2e^2}

Q. 14. Let the sets A and B denote the domain and range respectively of the function $f(x)=\frac{1}{\sqrt{[x]-x}}$, where $[x]$ denotes the smallest integer greater than or equal to x. Then among the statements :

(S1) : $A \cap B = (1, \infty) - N$ and (S2) : $A \cup B = (1, \infty)$

(1) only (S1) is true
(2) neither (S1) nor (S2) is true
(3) only (S2) is true
(4) both (S1) and (S2) are true

Q. 15. Let $a \neq b$ be two-zero real numbers. Then the number of elements in the set $X = \{z \in C : \text{Re}(az^2 + bz) = a \text{ and } \text{Re}(bz^2 + az) = b\}$ is equal to :

(1) 0 **(2)** 2
(3) 1 **(4)** None of these

Q. 16. The sum of all values of α, for which the points whose position vectors are $\hat{i}-2\hat{j}+3\hat{k}$, $2\hat{i}-3\hat{j}+4\hat{k}$, $(\alpha+1)\hat{i}+2k$ and $9\hat{i}+(\alpha-8)\hat{j}+6\hat{k}$ are coplanar, is equal to:

(1) –2 **(2)** 2
(3) 6 **(4)** 4

Q. 17. Let the line L pass through the point (0, 1, 2), intersect the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and be parallel to the plane $2x + y - 3z = 4$. Then the distance of the point P (1, –9, 2) from the line L is:

(1) 9 **(2)** $\sqrt{54}$
(3) $\sqrt{69}$ **(4)** $\sqrt{74}$

Q. 18. All the letters of the word PUBLIC are written in all possible orders and these words are written as in a dictionary with serial numbers. Then the serial number of the word PUBLIC is :

(1) 580 **(2)** 578
(3) 576 **(4)** 582

Q. 19. Let the vectors $\vec{a},\vec{b},\vec{c}$ represent three coterminous edges of a parallelepiped of volume V. Then the volume of the parallelepiped, whose coterminous edges are represented by $\vec{a},\vec{b}+\vec{c}$ and $\vec{a}+2\vec{b}+3\vec{c}$ is equal to :

(1) 2 V **(2)** 6 V
(3) 3 V **(4)** V

Q. 20. Among the statements :

(S1) : $2023^{2022} - 1999^{2022}$ is divisible by 8

(S2) : $13(13)^n - 11n - 13$ is divisible by 144 for infinitely many $n \in N$

(1) only (S2) is correct
(2) only (S1) is correct
(3) both (S1) and (S2) are incorrect
(4) both (S1) and (S2) are correct

Section B

Q. 21. The value of tan 9° – tan 27° – tan 63° + tan 81° is ________ :

Q. 22. If $(20)^{19} + 2(21)(20)^{18} + 3(21)^2 (20)^{17} + \ldots + 20(21)^{19} = k(20)^{19}$, then k is equal to _____ :

Q. 23. Let the eccentricity of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is reciprocal to that of the hyperbola $2x^2 - 2y^2 = 1$. If the ellipse intersects the hyperbola at right angles, then square of length of the latus-rectum of the ellipse is _____:

Q. 24. For $\alpha, \beta, z \in C$ and $\lambda > 1$, if $\sqrt{\lambda-1}$ is the radius of the circle $|z-\alpha|^2 + |z-\beta|^2 = 2\lambda$, then $|\alpha-\beta|$ is equal to _____:

Q. 25. Let a curve $y = f(x)$, $x \in (0, \infty)$ pass through the points $P\left(1,\frac{3}{2}\right)$ and $Q\left(a,\frac{1}{2}\right)$. If the tangent at any point R $(b, f(b))$ to the given curve cuts the y-axis at the points S$(0, c)$ such that $bc = 3$, then $(PQ)^2$ is equal to _____:

Q. 26. If the lines $\frac{x-1}{2}=\frac{2-y}{-3}=\frac{z-3}{\alpha}$ and $\frac{x-4}{5}=\frac{y-1}{2}=\frac{z}{\beta}$ intersect, then the magnitude of the minimum value of $8\alpha\beta$ is _____:

Q. 27. Let $f(x)=\frac{x}{1+x^{n\frac{1}{n}}}, x \in R-\{-1\}, n \in N, n > 2$ If $f^n(x) = n$ (*fofof*..... upto n times) (x), then $\lim_{n\to\infty}\int_0^1 x^{n-2}(f^n(x))dx$ is equal to _____:

Q. 28. If the mean and variance of the frequency distribution.

x_i	2	4	6	8	10	12	14	16
f_i	4	4	α	15	8	β	4	5

are 9 and 15.08 respectively, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is _____:

Q. 29. The number of points, where the curve $y = x^5 - 20x^3 + 50x + 2$ crosses the x-axis is ____:

Q. 30. The number of 4-letter words, with or without meaning, each consisting of 2 vowels and 2 consonants, which can be formed from the letters of the word UNIVERSE without repetition is __________:

Answer Key

Q. No.	Answer	Topic name	Chapter name
1	(4)	Special series	Sequence and series
2	(2)	Coplanarity	Function
3	(1)	Solution of system of linear equations	Determinants
4	(4)	Truth table	Mathematical reasoning
5	(4)	Sandwich theorem	Limits
6	(3)	Matrix polynomial	Matrix
7	(1)	Distance between planes	3D
8	(2)	Properties of definite integral	Definite integral
9	(3)	General term	Binomial theorem
10	(3)	Chord of contact	Circle
11	(3)	Probability	Probability
12	(3)	Eccentricity	Ellipse and sets
13	(4)	Linear differential equation	Differential equation
14	(2)	Domain	Function
15	(4)	Component of complex number	Complex number
16	(2)	Coplanarity of points	Vector
17	(4)	Line and plane	3D
18	(4)	Word problem	Permutations and combination
19	(4)	Scalar triple product	Vector
20	(4)	Divisibility problem	Binomial theorem
21	[4]	Double angle formula	Trigonometry ratio and identities
22	[400]	Method of difference	Sequence and series
23	[2]	Equation of hyperbola and ellipse	Hyperbola, ellipse
24	[2]	Modulus of complex number	Complex number
25	[5]	Tangent and normal	Application of derivative
26	[18]	Coplanarity of liines	3D
27	[0]	Limits of composite function	Limits
28	[5]	Mean and variance	Statistics
29	[5]	Nature of roots	Application of derivative
30	[432]	Restricted permutations	Permutations and combination

Solutions

Section A

1. Option (4) is correct.

Given $gcd\ (m, n) = 1$ and

$\Rightarrow 1^2 - 2^2 + 3^2 - 4^2 + ... + (2021)^2 - (2022)^2 + (2023)^2 = 1012\ m^2n$

$\Rightarrow 1^2 - 2^2 + 3^2 - 4^2 + ... + (2021)^2 - (2022)^2 + (2023)^2 = 1012\ m^2n$

$\Rightarrow (1-2)(1+2) + (3-4)(3+4) + ... (2021-2022)(2021+2022) + (2023)^2 = (1012)\ m^2n$

$\Rightarrow (-1)(1+2) + (-1)(3+4) + ... + (-1)(2021+2022) + (2023^2) = (1012)\ m^2n$

$\Rightarrow (-1)\ [1 + 2 + 3 + 4 + ... + 2022] + (2023)^2 = 1012\ m^2n$

$$\Rightarrow (-1)\left[\frac{(2022).(2022+1)}{2}\right] + (2023)^2 = (1012)m^2n$$

$$\Rightarrow (-1)\left[\frac{(2022)(2023)}{2}\right] + (2023)^2 = (1012)m^2n$$

$\Rightarrow (2023)\ [2023 - 1011] = (1012)\ m^2n$

$\Rightarrow m^2n = 2023$

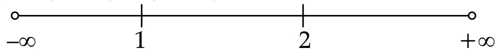
$\Rightarrow m^2n\ (17)^2 \times 7$ compare both side

$\Rightarrow m = 17, n = 7$

$m^2 + n^2 = (17)^2 - 7^2 = 289 - 49 = 240.$

2. **Option (2) is correct.**

$y = |x-1| + |x-2|$



(i) $-\infty < x < 1$

$y = -x + 1 - x + 2 = -2x + 3$

(ii) $1 \le x < 2$

$y\ (x-1) - (x-2) = x - 1 - x + 2 = 1$

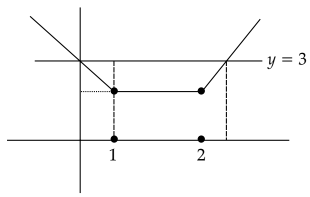
(iii) $2 \le x$

$y = x - 1 + x - 2 = 2x - 3$

$$y = |x-1| + |x-2| = \begin{cases} -2x+3, & -\infty < x < 1 \\ 1 & 1 \le x < 2 \\ 2x-3, & 2 \le x < \infty \end{cases}$$

and $y = 3$

Draw the graph



Area $= \frac{1}{2}[1+3] \times 2 = 4$

3. **Option (1) is correct.**

For unique solution $\Delta \ne 0$ for infinite many solution

$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\Delta_1 = \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & \alpha \\ \beta & 3 & 5 \end{vmatrix} = 6(10-3\alpha) - (50-\alpha\beta) + (30-2\beta)$$

$= 40 - 18\alpha + \alpha\beta - 2\beta$

$$\Delta_2 = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 10 & \alpha \\ 1 & \beta & 5 \end{vmatrix} = (50-\alpha\beta) - 6(5-\alpha) + (\beta - 10)$$

$= 10 + 6\alpha + \beta - \alpha\beta$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 1 & 3 & \beta \end{vmatrix} = (2\beta - 30) - (\beta - 10) + 6(1) \ = \beta - 14$$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & \alpha \\ 1 & 3 & 5 \end{vmatrix} = 1(10-3\alpha) - (5-\alpha) + (3-2) \ = 6 - 2\alpha$$

For inifinite solution $\Delta = 0, \Delta_1 = \Delta_2 = \Delta_3 = 0$

$\alpha = 3, \beta = 14$

For unique solution $\alpha \ne 3$

4. **Option (4) is correct.**

P	Q	$\sim p$	$\sim p \wedge q$	$p \Rightarrow q$	$(p \Rightarrow q) \vee (\sim p \wedge q)$
T	T	F	F	T	T
T	F	F	F	F	F
F	T	T	T	T	T
F	F	T	F	T	T

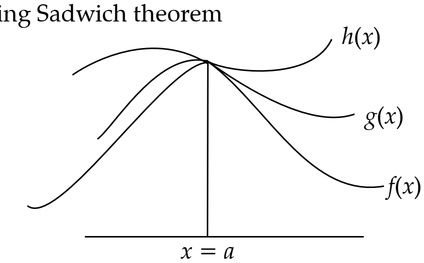
So,

P	Q	$q \Rightarrow p$	$\sim p$	$(\sim p) \wedge q$	$(q \Rightarrow p) \Rightarrow (\sim p \wedge q)$
T	T	T	F	F	F
T	F	T	F	F	F
F	T	F	T	T	T
F	F	T	T	F	F

5. **Option (4) is correct.**

Let $p = \lim\limits_{n \to \infty} \left(2^{\frac{1}{2}} - 2^{\frac{1}{3}}\right)\left(2^{\frac{1}{2}} - 2^{\frac{1}{5}}\right) \ldots \left(2^{\frac{1}{2}} - 2^{\frac{1}{2n+1}}\right)$

Using Sadwich theorem



$f(x) \le g(x) \le h(x)$

$\lim\limits_{x \to a} f(x) \le \lim\limits_{x \to a} g(x) \le \lim\limits_{x \to a} h(x)$

$L = \lim\limits_{x \to a} g(x) \le L$

Let $2^{\frac{1}{2}} - 2^{\frac{1}{3}} \to$ smallest

$2^{\frac{1}{2}} - 2^{\frac{1}{2n+1}} \to$ largest

$$\left(2^{\frac{1}{2}} - 2^{\frac{1}{3}}\right) \le p \le \left(2^{\frac{1}{2}} - 2^{\frac{1}{2n+1}}\right)^n$$

$$\lim_{n \to \infty} \left(2^{\frac{1}{2}} - 2^{\frac{1}{3}}\right)^n \le p \le \lim_{n \to \infty} \left(2^{\frac{1}{2}} - 2^{\frac{1}{2n+1}}\right)^n$$

$0 = p$

6. **Option (3) is correct.**

$P^2 = I - P$ ...(1)

$P^\alpha + P^\beta = \gamma I - 29P$ ...(2)

$P^\alpha - P^\beta = \delta I - 13P$ ...(3)

By (1)

$P^2 = I - P$

$\Rightarrow P^4 = (I - P)^2$

$\Rightarrow P^4 = I + P^2 - 2P$ $[\because P^2 = I - P]$

$\Rightarrow P^8 = (2I - 3P)^2$

$\Rightarrow P^8 = 4I^2 + 9P^2 - 12P$

$\Rightarrow P^8 = 13I - 21P$ ...(1)

$P^6 = P^4 . P^2 = (2I - 3P)(I - P)$

$P^6 = 5I - 8P$ ...(2)

(1) + (2) and (1) – (2)

$P^8 + P^6 = 18I - 29P \quad P^8 - P^6 = 8I - 13P$

Compare eqn. (2) and (3)

$\alpha = 8, \beta = 6, \gamma = 18, \delta = 8$

$\alpha + \beta + \gamma - \delta = 32 - 8 = 24.$

7. Option (1) is correct.

Given plane $\vec{r}.(\hat{i}+\hat{j}+\hat{k})=6$ and

$\vec{r}.(2\hat{i}+3\hat{j}+4\hat{k})=-5$

Equation of plane passing through both plane

$P_1 \rightarrow (x\hat{i}+y\hat{j}+2\hat{k})(\hat{i}+\hat{j}+\hat{k}) = 6$

$P_1 = x + y + z = 6$

$P_2 \rightarrow (x\hat{i}+y\hat{j}+z\hat{k}).(2\hat{i}+3\hat{j}+4\hat{k})=-5$

$P_2 \rightarrow = 2x + 3y + 4z = -5$

$P_1 + \lambda P_2 = 0$

$\Rightarrow (x + y + z - 6) + \lambda (2x + 3y + 4z + 5) = 0$

passes through (0, 2, –2)

$\Rightarrow (0 + 2 - 2 - 6) + \lambda(2 \times 0 + 3 \times 2 + 4 \times (-2) + 5) = 0$

$\Rightarrow \lambda = 2$

Equation of plane

$5x + 7y + 9z + 4 = 0$

Distance (12, 12, 18) is

$$d=\left|\frac{5\times 12+7\times 12+9\times 18+4}{\sqrt{5^2+7^2+9^2}}\right|$$

$$d=\frac{310}{\sqrt{155}}$$

$$\Rightarrow d^2=620$$

8. Option (2) is correct.

Given $f(x) + f(\pi - x) = \pi^2$

Use property of defintie integer

$$I=\int_0^{\pi} f(\pi-x)\sin(\pi-x)dx$$

Add (1) + (2)

$$2I=\int_0^{\pi}[f(x)+f(\pi-x)]\sin x\,dx$$

$$2I=\int_0^{\pi}\pi^2\sin x\,dx \qquad \left[\because \int \sin x\,dx=-\cos x+C\right]$$

$I = \pi^2$

9. Option (3) is correct.

Given $\left(ax^2+\frac{1}{2bx}\right)^{11}$

$$T_{r+1}={}^{11}C_r\,.\,(ax^2)^{11-r}+\left(\frac{1}{2bx}\right)^r$$

and $\left(ax-\frac{1}{3bx}\right)^{11}$

$$T_{r+1}={}^{11}C_r\,(ax)^{11-r}\left(\frac{-1}{3bx}\right)^r$$

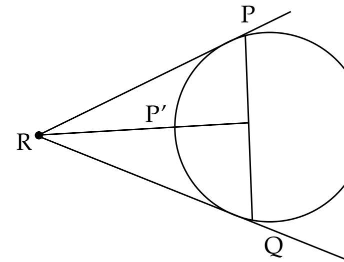
According to question

$${}^{11}C_5\,(a)^6\left(\frac{1}{2^5b^5}\right)={}^{11}C_6\,\frac{a^5}{3^6b^6}$$

$$\Rightarrow ab=\frac{2^5}{3^6}$$

$\Rightarrow 729ab = 32$

10. Option (3) is correct.



Equation of chrod of contact is T = 0

$$\frac{9}{4}x+2y-\left(x+\frac{9}{4}\right)+\frac{1}{2}(y+2)-5=0$$

$\Rightarrow x + 2y = 5$

Area $=\frac{1}{2}\times\frac{\sqrt{5}}{4}\times\sqrt{5} \;=\frac{5}{8}$

Eqn. of chord of contact

$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

PQ $= 2\sqrt{r^2-p}=\sqrt{5}$

11. Option (3) is correct.

Favourable outcomes $=\frac{{}^6C_3\times 3!}{6\times 6\times 6}=\frac{(20)\times 6}{6\times 6\times 6}=\frac{5}{9}=\frac{p}{q}$

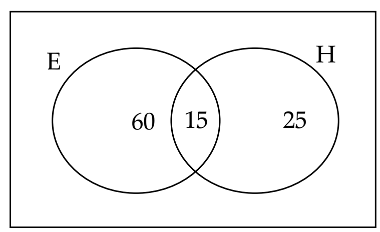
$P = 5, q = 9$

12. Option (3) is correct.

$n(A\cup B) = n(A) + n(B) - n(A\cap B)$

$n(A\cap B) = 75 + 40 - 100$

$n(A\cap B) = 15$



Only English = 60 $\quad\alpha = 60$

Only Hindi = 25 $\quad\beta = 25$

$\Rightarrow 25(\beta^2x^2 + \alpha^2y^2) = \alpha^2\beta^2$

$$\Rightarrow \frac{25x^2}{\alpha^2}+\frac{25y^2}{\beta^2}=1$$

$$\Rightarrow \frac{25x^2}{(60)^2}+\frac{25y^2}{(25)^2}=1$$

$$\Rightarrow e^2=1-\left[\frac{25\times 25}{(60)^2}\right]$$

$$\Rightarrow e^2=\frac{(60)^2-(25)^2}{(60)^2}$$

$$\Rightarrow e=\frac{\sqrt{119}}{12}$$

13. Option (4) is correct.

$(1+\ln x)\frac{dx}{dy} - x\log_e x = e^y$

Let $x \ln x = t$

$(1+\ln x)\frac{dx}{dy} = \frac{dt}{dy}$

$\frac{dt}{dy} - t = e^y$

Here $p = -1$, $Q = e^y$

I.F. $= e^{\int P dy} = e^{\int -1 dy} = e^{-y}$

t (I.F.) $= \int Q.(\text{I.F.})dy$

$t(e^{-y}) = \int e^{-y}.e^y dy$

$(x \ln x)e^{-y} = y + C$ passes $(1, 0) \Rightarrow C = 0$

passes through $(\alpha, 2)$

$\alpha^a = e^{2e^2}$

14. Option (2) is correct.

$f(x) = \frac{1}{\sqrt{[x]-x}}$

domain $[x] - x > 0 \Rightarrow [x] > x$

$\Rightarrow x \in \phi$

Range $\rightarrow \phi$

Neither S_1 nor S_2 is true.

15. Option (4) is correct.

We know that $z + \bar{z} = 2\text{Re}(z)$

$\therefore\ (az^2 + bz) + (a\bar{z}^2 + b\bar{z}) = 2a$

$\Rightarrow\ a\left(z^2 + \bar{z}^2\right) + b\left(z + \bar{z}\right)2a$ ...(i)

Add $(bz^2 + az) + \left(b\bar{z}^2 + a\bar{z}\right) = 2b$

$\Rightarrow\ b\left(z^2 + \bar{z}^2\right) + a\left(z + \bar{z}\right) = 2b$ ...(ii)

From (i) × b – (ii) × a

$(b^2 - a^2)\left(z + \bar{z}\right) = 0$

$z + \bar{z} = 0$ $(\because\ a \neq b)$

From (i) × b – (ii) × a

$(a^2 - b^2)\left(z^2 + \bar{z}^2\right) = 2(a^2 - b^2)$ $[a^2 \neq b^2]$

$z^2 + \bar{z}^2 = 2 \Rightarrow \left(z + \bar{z}\right)^2 - 2z\bar{z} = 2$

$\Rightarrow\ z\bar{z} = -1 \Rightarrow 1 + 1^2 = 1$ (No solution)

But when $a = -b$

$\text{Re}(az^2 - az) = a$

Put $z = x + iy$

$\therefore\ x^2 - x - 1 = y^2$

For any real value of y there two values of x, hence infinite complex number are possible.

16. Option (2) is correct.

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$

$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA}$

are coplanar if

$[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}] = 0$

$$\Rightarrow \begin{vmatrix} 1 & -1 & 1 \\ \alpha & 2 & -1 \\ 8 & \alpha-6 & 3 \end{vmatrix} = 0$$

$\Rightarrow \alpha = 4, -2$

Sum of all values of $\alpha = 2$

17. Option (4) is correct.

$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$

$(x, y, z) \rightarrow (2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$

Plane $2x + y - 3z = 4$

$\vec{r}.\vec{n} = p$

$\Rightarrow \vec{r}.(2\hat{i} + \hat{j} - 3\hat{k}) = 4$

$\overrightarrow{PQ}.\vec{n} = 0$

$\Rightarrow (2\lambda + 1) \times 2 + (3\lambda + 1) \times 1 + (4\lambda + 3) \times (-3) = 0$

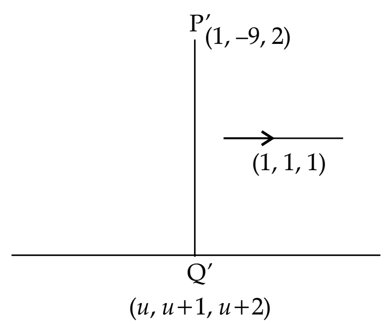
$\Rightarrow \lambda = 0$

Q $(2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$

Q $(1, 2, 3)$

Equation of line $\frac{x-0}{1} = \frac{y-1}{1} = \frac{z-2}{1} = \mu$

distance of the line term $(1, -9, 2)$



$\overrightarrow{PQ}.(\hat{i} + \hat{j} + \hat{k}) = 0$

$u = 3$

$Q' = (-3, -2, -1)$

$PQ' = \sqrt{16 + 49 + 9} = \sqrt{74}$

18. Option (4) is correct.

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$4 \times 9! + 4 \times 4! + 0 \times 3! + 2 \times 2! + 1 \times 1! + 1$

$= 4 \times 120 + 4 \times 24 + 0 + 4 + 1 + 1$

$= 582$

19. Option (4) is correct.

Volume of parallelopiped

$[\vec{a}\ \vec{b}\ \vec{c}]$

$v_1 = [\vec{a}\ \vec{b} + \vec{c}, \vec{a} + 2\vec{b} + 3\vec{c}]$

$$v_1 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 2 & 3 \end{vmatrix} [\vec{a}\ \vec{b}\ \vec{c}]$$

$v_1 = (3 - 2)$ V

20. Option (4) is correct.

$x^n - y^n = (x - y)[x^{n-1} + x^{n-2}y + x^{n-3}y^2 + ... + y^{n-1}]$

$(S_1) \rightarrow (2023)^{2022} - (1999)^{2022}$

$\rightarrow (2023) - (1999) = 24$ is divisible by 8'

$(S_2) \rightarrow (13)(1 + 12)^n - 11n - 13$

$= 13[1 + {}^nC_1(12) + {}^nC_2(12)^2 + ...] - 11n - 13$

$\Rightarrow 145n + 13.\,{}^nC_2(12)^2 + 13\,{}^nC_3(12)^3 + ...$

Section B

21. Correct answer is [4].

$(\tan 9° + \cot 9°) - (\tan 27° + \cot 27°)$

$$\Rightarrow \left(\frac{\sin^2 9° + \cos^2 9°}{\sin 9° \cos 9°}\right) - \left(\frac{\sin^2 27° + \cos^2 27°}{\sin 27° \cos 27°}\right)$$

$$= \frac{2}{2\sin 9° \cos 9°} - \frac{2}{2\sin 27° \cos 27°}$$

$$\Rightarrow \frac{2}{\sin 18°} - \frac{2}{\sin 54°} \qquad [\because \sin 2A = 2\sin A \cos A]$$

$$\Rightarrow \frac{2}{\frac{\sqrt{5}-1}{4}} - \frac{2}{\frac{\sqrt{5}+1}{4}}$$

$$\because \sin 18 = \frac{\sqrt{5}-1}{4}$$

$$\because \sin 54° = \frac{\sqrt{5}+1}{4}$$

22. Correct answer is [400].

Let $S = (20)^{19} + 2(21)(20)^{18} + 3 \times (21)^2 \times (20)^{17} + ... + (20)(21)^{19}$

$\frac{21}{20}S = (21)(20)^{18} + 2 \times (21)^9 (20)^{17} + ... + (21)^{19}$

Subtract

$$S\left(\frac{-1}{20}\right) = (21)^{20} - (20)^{20} - (21)^{20}$$

$k = (20)^2 = 400$

23. Correct answer is [2].

$$E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$H = x^2 - y^2 = \frac{1}{2} \Rightarrow e' = \sqrt{2}$$

$$e = \frac{1}{\sqrt{2}} \Rightarrow e^2 = \frac{1}{2}$$

$$\Rightarrow e^2 = 1 - \frac{b^2}{a^2} = \frac{1}{2} \Rightarrow \frac{b^2}{a^2} = \frac{1}{2}$$

$\Rightarrow a^2 = 2b^2$

E and H are right angle

they are confocal

focus of hyperbola = Focus of ellipse

$$\left(\pm\frac{1}{\sqrt{2}}.\sqrt{2}, 0\right) = \left(\pm\frac{a}{\sqrt{2}}, 0\right)$$

$a = \sqrt{2}$

$\because a^2 = 2b^2 \Rightarrow b^2 = 1$

Length of L.R. $= \frac{2b^2}{a} = \frac{2 \times 1}{\sqrt{2}} = \sqrt{2}$

Square of L.R = 2

24. Correct answer is [2].

$|z - z_1|^2 + |z - z_2|^2 = |z_1 - z_2|^2$

Let $z_1 = \alpha$, $z_2 = \beta$

$\Rightarrow |\alpha - \beta|^2 = 2\lambda$

$\Rightarrow |\alpha - \beta| = \sqrt{2\lambda}$

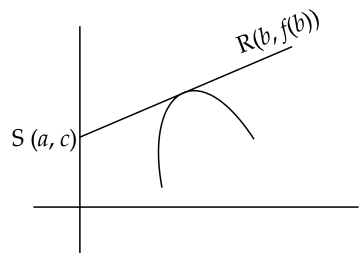
$\Rightarrow 2r = \sqrt{2\lambda} \qquad \because r = \sqrt{\lambda - 1}$

$\Rightarrow 2\sqrt{\lambda - 1} = \sqrt{2\lambda}$

$\lambda = 2$

$\Rightarrow |\alpha - \beta| = 2$

25. Correct answer is [5].



Equation of tangent $y - f(b) = f'(b)(x - b)$

Which passes through $(0, c)$

$\Rightarrow c - f(b) = f'(b)(-b)$

$$\Rightarrow \frac{3}{b} - f(b) = f'(b)(-b)$$

$$\Rightarrow \frac{bf'(b) - f(b)}{b^2} = \frac{-3}{b^3}$$

$$\Rightarrow d\left(\frac{f(b)}{b}\right) = \frac{-3}{b^3} = \frac{f(b)}{b} = \frac{3}{2b^3} + \lambda$$

passes through $\left(1, \frac{3}{2}\right)$

$$\Rightarrow \frac{3}{2} = \frac{3}{2} + \lambda \Rightarrow \lambda = 0$$

$$f(b) = \frac{3}{2b}$$

$f(a)=\frac{1}{2}\Rightarrow b=3$

$\Rightarrow C=1\Rightarrow Q\left(3,\frac{1}{2}\right)$

$PQ^2 = 2^2 + 1^2 = 5$

26. Correct answer is [18].

$\frac{x-1}{2}=\frac{2-y}{-3}=\frac{z-3}{\alpha}$

$\frac{x-4}{5}=\frac{y-1}{2}=\frac{z}{\beta}$

Coplanar condition

$$=\begin{vmatrix}2 & 3 & \alpha\\ 5 & 2 & \beta\\ -3 & 1 & 3\end{vmatrix}=0$$

$\Rightarrow \alpha - \beta = 3 \Rightarrow \alpha = \beta + 3$

Given expression

$$=8\left(\beta^2+3\beta+\frac{9}{4}\frac{-9}{4}\right)=8\left(\beta+\frac{3}{2}\right)^2-18$$

So magnitude of minimum value = 18

27. Correct answer is [0].

Let $f(x) = \frac{x}{[1+(x^n)]^{1/n}}$, $x \in P - \{-1\}, n \in N, n > 2$

If $f^n(x) = n\,(f_0 f_0 f_0$ .... upto n times) (x)

$$\lim_{n\to\infty}\int_0^1 x^{n-2}(f^n(x))\,dx$$

$$f(f(x))=\frac{1}{(1+2x^n)^{\frac{1}{n}}}$$

$$f(f(x))=\frac{x}{(1+3x^n)^{\frac{1}{n}}}$$

Similarly

$$f^n(x)=\frac{x}{(1+nx^n)^{\frac{1}{n}}}$$

$$\lim_{n\to\infty}\int\frac{x^{n-2}.xdx}{(1+nx^n)^{\frac{1}{n}}}=\lim_{n\to\infty}\int\frac{x^{n-1}dx}{(1+nx^n)^{\frac{1}{n}}}$$

Now $1 + nx^n = t$

$$x^{n-1}=\frac{dt}{n^2}$$

$$\Rightarrow \lim_{n\to\infty}\frac{1}{n^2}\left[\frac{t^{1-\frac{1}{n}}}{t-\frac{1}{n}}\right]^{1+n}$$

Let $n=\frac{1}{n}$

$$=\lim_{n\to 0}\frac{\left(1+\frac{1}{n}\right)^{1-n}-1}{\frac{1}{n}\left(\frac{1-n}{n}\right)}=0$$

28. Correct answer is [5].

$N = \Sigma f_i = 40 + \alpha + \beta$

$\Sigma f_i x_i = 360 + 6\alpha + 12\beta$

Mean $(\bar{x})=\frac{\Sigma f_i x_i}{\Sigma f_i}=9$

$\Rightarrow \alpha = \beta$

$$\sigma^2=\frac{\Sigma f_i x_i^2}{\Sigma f_i}-\left(\frac{\Sigma f_i x_i}{\Sigma f_i}\right)^2$$

$\sigma^2 = 15.08$

$\alpha = 5$

$\alpha^2 + \beta^2 - \alpha\beta \Rightarrow \alpha^2 = 25$

$\Rightarrow \alpha = 5$

29. Correct answer is [5].

$y = x^5 - 20x^3 + 50x + 2$

$\frac{dy}{dx}=5(x^4-12x^2+10)$

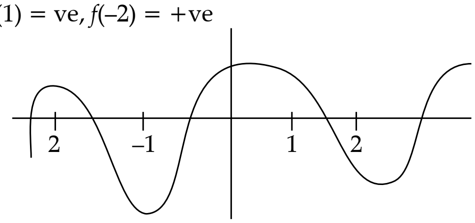
$\frac{dy}{dx}=0\Rightarrow x^4-12x^2+10=0$

$\Rightarrow x^2 = 11.1, 0.9$

$\Rightarrow x \approx \pm 3.3, \pm 0.95$

$f(0) = 2, f(1) = +ve, f(2) = -ve$

$f(1) = ve, f(-2) = +ve$



Number of point the curve cut the axis = 5

30. Correct answer is [432].

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Vowel	Consonant
E E	N V
I U	R S

2 vowel different 2 consonant different

$^3C_2 \times {^4C_2} \times 4! = 432$

❑❑