JEE (Main) MATHEMATICS SOLVED PAPER

2023
01st Feb. Shift 1

Section A

Q. 1. If $y = y(x)$ is the solution curve of the differential equation $\frac{dy}{dx} + y\tan x = x\sec x, 0 \le x \le \frac{\pi}{3}$, $y(0) = 1$, then $y\left(\frac{\pi}{6}\right)$ is equal to

(1) $\frac{\pi}{12} - \frac{\sqrt{3}}{2}\log_e\left(\frac{2\sqrt{3}}{e}\right)$

(2) $\frac{\pi}{12} - \frac{\sqrt{3}}{2}\log_e\left(\frac{2}{e\sqrt{3}}\right)$

(3) $\frac{\pi}{12} + \frac{\sqrt{3}}{2}\log_e\left(\frac{2}{e\sqrt{3}}\right)$

(4) $\frac{\pi}{12} + \frac{\sqrt{3}}{2}\log_e\left(\frac{2\sqrt{3}}{e}\right)$

Q. 2. Let R be a relation on R, given by
R = $\{(a, b): 3a - 3b + \sqrt{7}$ is an irrational number$\}$.
Then R is
(1) an equivalence relation
(2) reflexive and symmetric but not transitive
(3) reflexive but neither symmetric nor transitive
(4) reflexive and transitive but not symmetric

Q. 3. For a triangle ABC, the value of cos 2A + cos 2B + cos 2C is least. If its inradius is 3 and incentre is M, then which of the following is NOT correct?

(1) perimeter of ΔABC is $18\sqrt{3}$
(2) $\sin 2A + \sin 2B + \sin 2C = \sin A + \sin B + \sin C$
(3) $\overline{MA}.\overline{MB} = -18$
(4) area of ΔABC is $\frac{27\sqrt{3}}{2}$

Q. 4. Let S be the set of all solutions of the equation $\cos^{-1}(2x) - 2\cos^{-1}(\sqrt{1-x^2}) = \pi, x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$.
Then $\sum_{x \in S} 2\sin^{-1}(x^2 - 1)$ is equal to

(1) $\pi - 2\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ **(2)** $\pi - \sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

(3) $\frac{-2\pi}{3}$ **(4)** 0

Q. 5. Let S denote the set of all real values of λ such that the system of equations
$\lambda x + y + z = 1$ $x + \lambda y + z = 1$
$x + y + \lambda z = 1$
is inconsistent, then $\sum_{\lambda \in S} (|\lambda|^2 + |\lambda|)$ is equal to

(1) 4 **(2)** 12 **(3)** 6 **(4)** 2

Q. 6. In a binomial distribution B(n,p), the sum and the product of the mean and the variance are 5 and 6 respectively, then $6(n + p - q)$ is equal to
(1) 52 **(2)** 50 **(3)** 51 **(4)** 53

Q. 7. The combined equation of the two lines $ax + by + c = 0$ and $a'x + b'y + c' = 0$ can be written as $(ax + by + c)(a'x + b'y + c') = 0$.
The equation of the angle bisectors of the lines represented by the equation
$2x^2 + xy - 3y^2 = 0$ is
(1) $x^2 - y^2 - 10xy = 0$ **(2)** $x^2 - y^2 + 10xy = 0$
(3) $3x^2 + 5xy + 2y^2 = 0$ **(4)** $3x^2 + xy - 2y^2 = 0$

Q. 8. The area enclosed by the closed curve C given by the differential equation $\frac{dy}{dx} + \frac{x+a}{y-2} = 0$, $y(1) = 0$ is 4π. Let P and Q be the points of intersection of the curve C and the y-axis. If normals at P and Q on the curve C intersect x-axis at points R and S respectively, then the length of the line segment RS is

(1) 2 **(2)** $\frac{4\sqrt{3}}{3}$ **(3)** $2\sqrt{3}$ **(4)** $\frac{2\sqrt{3}}{3}$

Q. 9. The value of

$\frac{1}{1!50!} + \frac{1}{3!48!} + \frac{1}{5!46!} + \ldots + \frac{1}{49!2!} + \frac{1}{51!1!}$ is:

(1) $\frac{2^{50}}{51!}$ **(2)** $\frac{2^{51}}{50!}$ **(3)** $\frac{2^{50}}{50!}$ **(4)** $\frac{2^{51}}{51!}$

Q. 10. The mean and variance of 5 observations are 5 and 8 respectively. If 3 observations are 1, 3, 5 then the sum of cubes of the remaining two observations is
(1) 1216 **(2)** 1072 **(3)** 1456 **(4)** 1792

Q. 11. The sum to 10 terms of the series

$\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + \ldots$ is

(1) $\frac{55}{111}$ **(2)** $\frac{56}{111}$ **(3)** $\frac{58}{111}$ **(4)** $\frac{59}{111}$

Q. 12. The shortest distance between the lines
$\frac{x-5}{1} = \frac{y-2}{2} = \frac{z-4}{-3}$ and $\frac{x+3}{1} = \frac{y+5}{4} = \frac{z-1}{-5}$ is

(1) $5\sqrt{3}$ **(2)** $7\sqrt{3}$ **(3)** $6\sqrt{3}$ **(4)** $4\sqrt{3}$

Q. 13. $\lim_{n \to \infty}\left[\frac{1}{1+n} + \frac{1}{2+n} + \frac{1}{3+n} + \ldots + \frac{1}{2n}\right]$

(1) $\log_e 2$ **(2)** $\log_e\left(\frac{3}{2}\right)$

(3) $\log_e\left(\frac{2}{3}\right)$ **(4)** 0

Q. 14. Let the image of the point P(2, −1, 3) in the plane $x + 2y - z = 0$ be Q. Then the distance of the plane $3x + 2y + z + 29 = 0$ from the point Q is

(1) $\frac{24\sqrt{2}}{7}$ **(2)** $2\sqrt{14}$ **(3)** $3\sqrt{14}$ **(4)** $\frac{22\sqrt{2}}{7}$

Q. 15. Let $f(x) = 2x + \tan^{-1} x$ and $g(x) = \log_e(\sqrt{1+x^2} + x)$, $x \in [0, 3]$. Then

(1) $\min f'(x) = 1 + \max g'(x)$

(2) $\max f(x) > \max g(x)$

(3) there exist $0 < x_1 < x_2 < 3$ such that $f(x) < g(x), \forall \in (x_1, x_2)$

(4) there exists $\hat{x} \in [0,3]$ such that $f'(\hat{x}) < g'(\hat{x})$

Q. 16. If the orthocentre of the triangle, whose vertices are (1, 2) (2, 3) and (3, 1) is (α, β), then the quadratic equation whose roots are α + 4β and 4α + β, is

(1) $x^2 - 20x + 99 = 0$ **(2)** $x^2 - 19x + 90 = 0$

(3) $x^2 - 22x + 120 = 0$ **(4)** $x^2 - 18x + 80 = 0$

Q. 17. Let $S = \{x : x \in R \text{ and } (\sqrt{3}+\sqrt{2})^{x^2-4} + (\sqrt{3}-\sqrt{2})^{x^2-4} = 10\}$

Then $n(S)$ is equal to

(1) 4 **(2)** 0 **(3)** 6 **(4)** 2

Q. 18. If the center and radius of the circle $\left|\frac{z-2}{z-3}\right| = 2$ are respectively (α, β) and γ. then 3 (α + β + γ) is equal to

(1) 11 **(2)** 12 **(3)** 9 **(4)** 10

Q. 19. Let $f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & \sin 2x \\ \sin^2 x & 1+\cos^2 x & \sin 2x \\ \sin^2 x & \cos^2 x & 1+\sin 2x \end{vmatrix}$, $x \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$. If α and β respectively are the maximum and the minimum values of f, then

(1) $\alpha^2 + \beta^2 = \frac{9}{2}$ **(2)** $\beta^2 - 2\sqrt{\alpha} = \frac{19}{4}$

(3) $\alpha^2 - \beta^2 = 4\sqrt{3}$ **(4)** $\beta^2 + 2\sqrt{\alpha} = \frac{19}{4}$

Q. 20. The negation of the expression $q \vee ((\sim q) \wedge p)$ is equivalent to

(1) $(\sim p) \vee (\sim q)$ **(2)** $p \wedge (\sim q)$

(3) $(\sim p) \vee q$ **(4)** $(\sim p) \wedge (\sim q)$

Section B

Q. 21. Let $\vec{v} = \alpha\hat{i} + 2\hat{j} - 3\hat{k}, \vec{w} = 2\alpha\hat{i} + \hat{j} - \hat{k}$ and $\vec{u}$ be a vector such that $|\vec{u}| = \alpha > 0$. If the minimum value of the scalar triple product $|\vec{u}\,\vec{v}\,\vec{w}|$ is $-\alpha\sqrt{3401}$. and $|\vec{u}.\hat{i}|^2 = \frac{m}{n}$ where m and n are coprime natural numbers, then $m + m$ is equal to

Q. 22. The number of words, with or without meaning, that can be formed using all the letters of the word ASSASSINATION so that the vowels occur together, is

Q. 23. The remainder, when $19^{200} + 23^{200}$ is divided by 49 , is

Q. 24. The number of 3-digit numbers, that are divisible by either 2 or 3 but not divisible by 7 , is

Q. 25. Let $f : R \to R$ be a differentiable function such that $f'(x) + f(x) = \int_0^2 f(t)dt$. If $f(0) = e^{-2}$, then $2f(0) - f(2)$ is equal to

Q. 26. If $f(x) = x^2 + g'(1)\,x + g''(2)$ and $g(x) = f(1)\,x^2 + xf'(x) + f''(x)$, then the value of $f(4) - g(4)$ is equal to

Q. 27. Let A be the area bounded by the curve $y = x|x - 3|$, the x-axis and the ordinates $x = -1$ and $x = 2$. Then 12A is equal to

Q. 28. If $\int_0^1 (x^{21} + x^{14} + x^7)(2x^{14} + 3x^7 + 6)^{1/7}dx = \frac{1}{l}(11)^{\frac{m}{n}}$ where $l, m, n \in N$, m and n are coprime then $l + m + n$ is equal to

Q. 29. Let $a_1 = 8, a_2, a_3, \ldots, a_n$ be an A.P. If the sum of its first four terms is 50 and the sum of its last four terms is 170 , then the product of its middle two terms is

Q. 30. A (2, 6, 2), B (–4, 0, λ), C (2, 3, –1) and D(4, 5, 0), | λ | ≤ 5 are the vertices of a quadrilateral ABCD. If its area is 18 square units, then 5 – 6λ is equal to

Answer Key

Q. No.	Answer	Topic Name	Chapter Name
1	(2)	Linear differential equation	Differential equations
2	(3)	Equivalence relation	Set, relations and functions
3	(4)	Triangles	Trigonometry
4	(3)	Inverse trigonometric function	Trigonometry
5	(3)	System of equations	Matrices and determinant
6	(1)	Mean and variance	Statistics and probability
7	(1)	Angle bisector	Coordinate geometry

Q. No.	Answer	Topic Name	Chapter Name
8	(2)	Separable variable form	Differential equations
9	(1)	Summation of series	Binomial theorem
10	(2)	Mean	Statistics and probability
11	(1)	Special series	Sequence and series
12	(3)	Shortest distance	Three dimensional geometry
13	(1)	Limit	Limit, continuity and differentiability
14	(3)	Image of a point	Three dimensional geometry
15	(2)	Monotonicity	Limit, continuity and differentiability
16	(1)	Slope	Coordinate geometry
17	(1)	Solution of a quadratic equation	Complex number and quadratic equation
18	(2)	Solution	Complex number and quadratic equation
19	(2)	Expansion	Matrices and determinant
20	(4)	Negation of a statement	Mathematical reasoning
21	[3501]	Dot and cross product	Vector algebra
22	[50400]	Number of ways	Permutation and combination
23	[29]	Remainder theorem	Binomial theorem
24	[514]	Operations on set	Set, relations and functions
25	[1]	Leibnitz rule	Integral calculus
26	[14]	Higher order derivative	Limit, continuity and differentiability
27	[62]	Area under the curve	Integral calculus
28	[63]	General rule of integration	Integral calculus
29	[754]	Sum of n terms	Sequence and series
30	[11]	Area of quadrilateral	Vector algebra

Solutions

Section A

1. Option (2) is correct.

The given differential equation is :

$$\frac{dy}{dx} + y\tan x = x\sec x, 0 \le x \le \frac{\pi}{3}$$

As the above differential equation is linear i.e.,

$$\frac{dy}{dx} + P(x)y = Q(x)$$

So, integrating factor (I.F.) = $e^{\int P(x)dx}$

$$= e^{\int \tan x\, dx} = e^{\ln \sec x} = \sec x$$

Now, the solution is :

y × I.F. = $\int Q(x) \times \text{I.F.}\,dx$

$$\Rightarrow y \times \sec x = \int x\sec^2 x\,dx$$

$$= x\int \sec^2 dx - \int\left(\frac{d}{dx}(x)\int \sec^2 x\,dx\right)dx$$

$$= x\tan x - \int \tan x\,dx$$

$\Rightarrow y \sec x = x \tan x - \ln \sec x + C$ ...(i)

Using initial condition, $y(0) = 1$, we get

(1) sec0 = 0 – ln sec 0 + C

$\Rightarrow 1 = -\ln 1 + C \Rightarrow C = 1$

Now, put C = 1 in equation (i) we get

$y \sec x = x \tan x - \ln \sec x + 1$ ...(i)

Put $x = \frac{\pi}{6}$ in equation (ii) we get

$$y \times \sec\frac{\pi}{6} = \frac{\pi}{6} \times \tan\frac{\pi}{6} - \ln\sec\frac{\pi}{6} + 1$$

$$\Rightarrow y \times \frac{2}{\sqrt{3}} = \frac{\pi}{6} \times \frac{1}{\sqrt{3}} - \ln\left(\frac{2}{\sqrt{3}}\right) + 1$$

$$\Rightarrow y = \frac{\pi}{12} - \frac{\sqrt{3}}{2}\ln\left(\frac{2}{\sqrt{3}}\right) + \frac{\sqrt{3}}{2}\ln e$$

$$= \frac{\pi}{12} - \frac{\sqrt{3}}{2}\left(\ln\left(\frac{2}{e\sqrt{3}}\right)\right) \qquad \left(\ln a - \ln b = \ln\frac{a}{b}\right)$$

2. Option (3) is correct.

The given relation is

R = {(a, b) : $3a - 3b + \sqrt{7}$ is an irrational number}

(i) Reflexive: Let $(a, a) \in R$

So, $3a - 3a + \sqrt{7} = \sqrt{7}$, Which is an irrational number

$\therefore$ R is reflexive

(ii) Symmetric: Let $(a, b) \in R$

Let $a = \frac{\sqrt{7}}{3}$ and $b = 0$

So, $3a - 3b + \sqrt{7} = 3\left(\frac{\sqrt{7}}{3}\right) - 3(0) + \sqrt{7}$

$= 2\sqrt{7}$, an irrational number

Now, $3b - 3a + \sqrt{7} = 3(0) - 3\left(\frac{\sqrt{7}}{3}\right) + \sqrt{7}$

= 0, *a* rational number

So, $(b, a) \notin R$

Hence, R is not symmetric

(iii) Transitive:

Let $(a, b) = \left(\frac{\sqrt{7}}{3}, 1\right) \in R$ and $(b, c) = \left(1, \frac{2\sqrt{7}}{3}\right) \in R$

So, $3a - 3c + \sqrt{7} = 3\left(\frac{\sqrt{7}}{3}\right) - 3\left(\frac{2\sqrt{7}}{3}\right) + \sqrt{7}$

$= \sqrt{7} - 2\sqrt{7} + \sqrt{7} = 0$, a rational number.

So, $(a, c) \notin R$

Hence, R is not transitive

HINT:

An equivalence relation is the relation which is reflexive, symmetric and transitive.

3. Option (4) is correct.

Since $\cos 2A + \cos 2B + \cos 3C = \frac{-3}{2}$

$\Rightarrow A = B = C = 60°$

As, inradius = $\frac{\text{Area of } \Delta ABC}{\text{Semi-perimeter of } \Delta ABC}$

$\Rightarrow 3 = \frac{\frac{\sqrt{3}}{4}(a^2)}{\frac{3a}{2}} = \frac{a}{2\sqrt{3}}$

(because triangle ABC is equilateral triangle)

$\Rightarrow a = 6\sqrt{3}$

Hence, perimeter of triangle = $3 \times 6\sqrt{3} = 18\sqrt{3}$

Area of triangle = $\frac{\sqrt{3}}{4} \times (6\sqrt{3})^2 = \frac{108\sqrt{3}}{4} = 27\sqrt{3}$

HINT:

Inradius of a circle = $\frac{\text{Area of triangle}}{\text{Semi-perimeter of triangle}}$

4. Option (3) is correct.

We have

$\cos^{-1}(2x) - 2\cos^{-1}(\sqrt{1-x^2}) = \pi, x \in \left[\frac{-1}{2}, \frac{1}{2}\right]$

$\Rightarrow \cos^{-1}(2x) - [\cos^{-1}(2(1 - x^2) - 1)] = \pi$

$\Rightarrow \cos^{-1}(2x) - [\cos^{-1}(-2x^2 + 1)] = \pi$

$\Rightarrow -\cos^{-1}[-2x^2 + 1] = \pi - \cos^{-1} 2x$

Applying cos on both sides, we get

$\Rightarrow 1 - 2x^2 = -2x \Rightarrow 2x^2 - 2x - 1 = 0$

$\Rightarrow x = \frac{2 \pm \sqrt{4+8}}{4} = \frac{2 \pm 2\sqrt{3}}{4} = \frac{1 \pm \sqrt{3}}{2}$

As $x \in \left[\frac{-1}{2}, \frac{1}{2}\right] \Rightarrow x = \frac{1-\sqrt{3}}{2}$

$\therefore \sum 2\sin^{-1}[x^2 - 1] = \sum 2\sin^{-1}\left[\frac{-\sqrt{3}}{2}\right] = \frac{-2\pi}{3}$

5. Option (3) is correct.

The given system of equation is inconsistent

So, $\Delta = 0$

$$\Rightarrow \begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$

Apply $R_3 \to R_3$ 2 $- R$

$$\begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 0 & 1-\lambda & \lambda - 1 \end{vmatrix} = 0$$

Expanding along C_1, we get

$\lambda(\lambda(\lambda - 1) - (1 - \lambda)) - 1(1(\lambda - 1) - (1 - \lambda)) + 0 = 0$

$\Rightarrow \lambda(\lambda^2 - \lambda - 1 + \lambda) - 1(\lambda - 1 - 1 + \lambda) = 0$

$\Rightarrow \lambda(\lambda^2 - 1) - 2(\lambda - 1) = 0$

$\Rightarrow (\lambda - 1)[\lambda(\lambda + 1) - 2] = 0$

$\Rightarrow (\lambda - 1)(\lambda^2 + \lambda - 2) = 0$

$\Rightarrow (\lambda - 1)(\lambda - 1)(\lambda + 2) = 0$

$\Rightarrow \lambda = 1, -2$

For $\lambda = -2$ ($\because \lambda = 1$ is rejected)

$\sum_{\lambda \in S}(|\lambda|^2 + |\lambda|) = 4 + 2 = 6$

6. Option (1) is correct.

Here,

mean = np

Variance = npq

So, $np + npq = 5$

$\Rightarrow np(1 + q) = 5$ ...(i)

And $(np)(npq) = 6$ ...(ii)

$\Rightarrow n^2p^2q = 6$ ...(ii)

So, we have $\frac{n^2p^2(1+q)^2}{n^2p^2q} = \frac{25}{6}$

$\Rightarrow 6(1 + q)^2 = 25q \Rightarrow 6 + 6q^2 + 12q = 25q$

$\Rightarrow 6q^2 - 13q + 6 = 0 \Rightarrow 6q^2 - 9q - 4q + 6 = 0$

$\Rightarrow 3q(2q - 3) - 2(2q - 3) = 0$

$\Rightarrow (3q - 2)(2q - 3) = 0$

$\therefore q = \frac{2}{3}, \frac{3}{2}$ (rejected)

Hence, $p = 1 - q = 1 - \frac{2}{3} = \frac{1}{3}$

Using (i), we get

$n \times \frac{1}{3}\left(1 + \frac{2}{3}\right) = 5 \Rightarrow n \times \frac{5}{9} = 5 \Rightarrow n = 9$

So, $6(n + p - q) = 6\left(9 + \frac{1}{3} - \frac{2}{3}\right)$

$= 6\left(9 - \frac{1}{3}\right) = 6 \times \frac{26}{3} = 52$

7. Option (1) is correct.

The given equation of lines is : $2x^2 + xy - 3y^2 = 0$

$\therefore$ Equation of angle bisector is:

$\frac{x^2 - y^2}{xy} = \frac{2+3}{\frac{1}{2}}$ [Here, $a = 2, b = -3, h = \frac{1}{2}$]

$\Rightarrow x^2 - y^2 = 10xy \Rightarrow x^2 - y^2 - 10xy = 0$

8. Option (2) is correct.

$\because \frac{dy}{dx} + \frac{x+a}{y-2} = 0, y(1) = 0$

$\Rightarrow \int (y-2)dy = -\int dx(x+a)$

$\Rightarrow \frac{y^2}{2} - 2y = -\frac{x^2}{2} - ax + C$

$\Rightarrow \frac{y^2 - 4y}{2} = \frac{-x^2 - 2ax + 2c}{2}$

$\Rightarrow y^2 - 4y + x^2 + 2ax - c_1 = 0$ ...(i)

Using initial condition, we get

$\Rightarrow 0 - 0 + 1 + 2a - c_1 = 0$

$\Rightarrow c_1 = 1 + 2a$

$\therefore x^2 + y^2 + 2ax - 4y = 1 + 2a$ [Using (i)]

The above equation is of circle of area 4π

$\Rightarrow$ Radius = 2

$\Rightarrow r^2 = (-a)^2 + 2^2 + 1 + 2a = 4$

$\Rightarrow a = -1$

Hence equation of circle is

$x^2 + y^2 - 2x - y = -1$

$\Rightarrow (x-1)^2 + (y-2)^2 = 4$

For intersection point, put $x = 0$

$\Rightarrow (y-2)^2 = 4 - 1 = 3 \Rightarrow y - 2 = \pm\sqrt{3}$

$\therefore (0, \pm\sqrt{3}+2) \Rightarrow P(0, \sqrt{3}+2)\ Q(0, -\sqrt{3}+2)$

So, equation of line passing the P & (1, 2) is

$\left(y - \left(\sqrt{3}+2\right)\right) = -\sqrt{3}(x)$

$\therefore$ The above line will cut x-axis at $y = 0$

$\therefore R \equiv \left(\frac{\sqrt{3}+2}{\sqrt{3}}, 0\right)$

Now, equation of line passing the Q & (1, 2) is

$(y - (-\sqrt{3}+2)) = \sqrt{3}x$

The above line will cut x-axis at $y = 0$

$\therefore S \equiv \left(\frac{\sqrt{3}-2}{\sqrt{3}}, 0\right)$

Hence, RS $= \sqrt{\left(\frac{\sqrt{3}-2}{\sqrt{3}} - \frac{\sqrt{3}+2}{\sqrt{3}}\right)^2 + 0}$

$= \sqrt{\left(\frac{\sqrt{3}-2-\sqrt{3}-2}{\sqrt{3}}\right)^2} = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$

HINT:

A separable differential equation can be solved by separating the variables and differential of same type on same side and then integrating.

9. Option (1) is correct.

Let E $= \frac{1}{1!50!} + \frac{1}{3!48!} + \frac{1}{5!46!} + \ldots + \frac{1}{49!2!} + \frac{1}{51!1!}$

Multiply and divide by 51!, we get

E $= \frac{1}{51!}\left[\frac{51!}{50!1!} + \frac{51!}{3!48!} + \ldots + 1\right]$

$= \frac{1}{51!}\,[{}^{51}C_{50} + {}^{51}C_{48} + \ldots + {}^{51}C_2 + {}^{51}C_9]$

$= \frac{1}{51!} \times 2^{51-1}$ $\left[\because 2^{n-1} = {}^nC_0 + {}^nC_2 + \ldots + {}^nC_{n-1}\right]$

$= \frac{2^{50}}{51!}$

10. Option (2) is correct.

Let missing observations be p & q

Given, mean = 5

$\Rightarrow \frac{1+3+5+p+q}{5} = 5$

$\Rightarrow 9 + p + q = 25$

$\Rightarrow p + q = 16$ ...(i)

Also, variance = 8

$\Rightarrow \frac{1^2 + 3^2 + 5^2 + p^2 + q^2}{5} - 25 = 8$

$\Rightarrow 1 + 9 + 25 + p^2 + q^2 = 33 \times 5$

$\Rightarrow p^2 + q^2 = 165 - 35 = 130$ ...(ii)

As, we know that

$(p+q)^2 = p^2 + q^2 + 2pq$

$\Rightarrow 16^2 = 130 + 2pq$ (Using (i) & (ii))

$\Rightarrow 2pq = 256 - 130 = 126$

$\Rightarrow pq = 63$ ...(iii)

Now, sum of cubes of p & q $= p^3 + q^3$

$\Rightarrow (p+q)^3 - 3pq(p+q)$

$\Rightarrow 16^3 - 3 \times 63\,(16) = 4096 - 3024 = 1072$

11. Option (1) is correct.

We have, $\sum_{r=1}^{10} \frac{r}{1 + r^2 + r^4}$

$= \frac{1}{2}\left\{\sum_{r=1}^{10}\left[\frac{1}{r^2 - r + 1} - \frac{1}{r^2 + r + 1}\right]\right\}$

$$=\frac{1}{2}\left\{\left[\frac{1}{1}-\frac{1}{3}\right]+\left[\frac{1}{3}-\frac{1}{7}\right]+\ldots+\left[\frac{1}{91}-\frac{1}{111}\right]\right\}$$

$$=\frac{1}{2}\left\{1-\frac{1}{111}\right\}=\frac{1}{2}\times\frac{110}{111}=\frac{55}{111}$$

12. Option (3) is correct.

The given lines are :

$$L_1: \frac{x-5}{1}=\frac{y-2}{2}=\frac{z-4}{-3}$$

$$L_2: \frac{x+3}{1}=\frac{y+5}{4}=\frac{z-1}{-5}$$

For L_1: $\vec{a}_1=5\hat{i}+2\hat{j}+4\hat{k};\vec{r}_1=\hat{i}+2\hat{j}-3\hat{k}$

For L_2: $\vec{a}_2=-3\hat{i}-5\hat{j}+\hat{k};\vec{r}_2=\hat{i}+4\hat{j}-5\hat{k}$

Hence, the vector product of $\vec{r}_1$ & $\vec{r}_2$ is :

$$\vec{r}_1\times\vec{r}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & -3\\ 1 & 4 & -5\end{vmatrix}$$

Expanding along R_1, we get

$$\vec{r}_1\times\vec{r}_2=\hat{i}(-10+12)-\hat{j}(-5+3)+\hat{k}(4-2)$$

$$=2\hat{i}+2\hat{j}+2\hat{k}$$

Hence, the shortest distance between the given lines is

$$d=\left|\frac{(\vec{r}_1\times\vec{r}_2).(\vec{a}_1-\vec{a}_2)}{|\vec{r}_1\times\vec{r}_2|}\right|$$

$$=\left|\frac{(2\hat{i}+2\hat{j}+2\hat{k}).(8\hat{i}+7\hat{j}+3\hat{k})}{\sqrt{2^2+2^2+2^2}}\right|$$

$$=\left|\frac{16+14+6}{\sqrt{12}}\right|=\frac{36}{2\sqrt{3}}=6\sqrt{3}$$

13. Option (1) is correct.

$$\lim_{n\to\infty}\left[\frac{1}{1+n}+\frac{1}{2+n}+\frac{1}{3+n}+\ldots+\frac{1}{2n}\right]$$

$$=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{r+n}$$

Taking n common in the denominator,

$$=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n\left[\frac{r}{n}+1\right]}=\int_0^1\frac{1}{1+x}dx=[\ln(1+x)]_0^1$$

$$=[\ln(2)-\ln 1]=\ln 2=\log_e 2$$

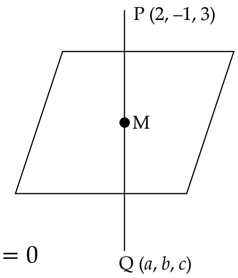
HINT:

Use the fact :

$$\sum_{r=1}^{n}\frac{1}{n\left(\frac{r}{n}+1\right)}=\int_0^1\frac{dx}{1+x}$$

14. Option (3) is correct.

Let the point Q be (a, b, c)



Hence, equation of line PM is

$$\frac{x-2}{1}=\frac{y+1}{2}=\frac{z-3}{-1}=\lambda$$

$\Rightarrow x=\lambda+2, y=2\lambda-1,$
$z=-\lambda+3$

Hence, $(\lambda+2, 2\lambda-1, -\lambda+3)$ lie on the plane, So,

$(\lambda+2)+2(2\lambda-1)-(-\lambda+3)=0$

$\Rightarrow \lambda+2+4\lambda-2+\lambda-3=0$

$$\Rightarrow 6\lambda=3\Rightarrow\lambda=\frac{1}{2}$$

$$\because \text{Point } M\equiv\left(\frac{1}{2}+2,2\left(\frac{1}{2}\right)-1,-\frac{1}{2}+3\right)$$

$$\equiv\left(\frac{5}{2},0,\frac{5}{2}\right)$$

Now, as M is mid point of PQ.

$$\text{So, }\frac{2+a}{2}=\frac{5}{2};\frac{-1+b}{2}=0;\frac{3+c}{2}=\frac{5}{2}$$

$\Rightarrow a=5-2;\ b=0+1;\ c=5-3$
$=3 \quad =1 \quad =2$

$\therefore Q\equiv(3, 1, 2)$

So, required distance,

$$=\left|\frac{3(3)+2(1)+2+29}{\sqrt{3^2+1^2+2^2}}\right|$$

$$=\left|\frac{9+2+2+29}{\sqrt{14}}\right|=\frac{42}{\sqrt{14}}\times\frac{\sqrt{14}}{\sqrt{14}}$$

$=3\sqrt{14}$ units

15. Option (2) is correct.

Given $f(x)=2x+\tan^{-1}x$

$$\Rightarrow f'(x)=2+\frac{1}{1+x^2}>0 \qquad \text{for } x\in[0, 3]$$

So, $f(x)$ is increasing in [0, 3]

$\therefore f(0)=0$ and $f(3)=6+\tan^{-1}3$

Also, $g(x)=\log_e(\sqrt{1+x^2}+x)$

$$\Rightarrow g'(x)=\frac{1}{\sqrt{1+x^2}+x}\times\left[\frac{1\times 2x}{2\sqrt{1+x^2}}+1\right]$$

$$=\frac{1}{x+\sqrt{1+x^2}}\left[\frac{x+\sqrt{1+x^2}}{\sqrt{1+x^2}}\right]=\frac{1}{\sqrt{1+x^2}}$$

$\therefore g'(x)>0$ for $x\in[0, 3]$

$\therefore g(x)$ is increasing in [0, 3]

$$g(0)=\log_e(\sqrt{1+0}+0)=\log_e(1)=0$$

$$g(3)=\log_e(\sqrt{1+9}+3)=\log_e(\sqrt{10}+3)$$

HINT:

Find the first derivaties and use the conditions to check whether $f(x)$ & $g(x)$ are increasing or not.

16. Option (2) is correct.

Slope of line (BH) × Slope of line (AC) = –1

$\Rightarrow \left(\frac{\beta-3}{\alpha-2}\right)\times\left(\frac{1-2}{3-1}\right)=-1$

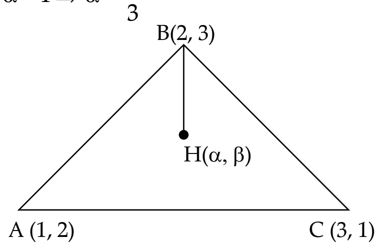
$\Rightarrow (\beta - 3) = (\alpha - 2)\,(2)$

$\Rightarrow \beta - 3 = 2\alpha - 4 \Rightarrow \beta = 2\alpha - 1$

Also, slope of line (AD) × slope of line (BC) = –1

$\Rightarrow \left(\frac{\beta-2}{\alpha-1}\right)\times\left(\frac{1-3}{3-2}\right)=-1 \Rightarrow \beta-2=(\alpha-1)\frac{1}{2}$

$\Rightarrow 2(2\alpha-1)-2=\alpha-1 \Rightarrow \alpha=\frac{5}{3}$



$\therefore \beta = 2\left(\frac{5}{3}\right)-1$

$=\frac{10-3}{3}=\frac{7}{3}$

$\therefore H \equiv \left(\frac{5}{3},\frac{7}{3}\right)$

Sum $= \alpha+4\beta=\frac{33}{3}=11$

Sum $= 4\alpha+\beta=\frac{27}{3}=9$

∴ Required equation is

$x^2 - (11 + 9)\,x + (11 \times 9) = 0$

$\Rightarrow x^2 - 20x + 99 = 0$

17. Option (1) is correct.

$S = \left\{x : x \in R \,\&\, (\sqrt{3}+\sqrt{2})^{x^2-4} + (\sqrt{3}-\sqrt{2})^{x^2-4} = 10\right\}$

Let $(\sqrt{3}+\sqrt{2})^{x^2-4} = a$ ...(i)

$\Rightarrow (\sqrt{3}-\sqrt{2})^{x^2-4} = \frac{1}{a}$ ...(ii)

So, $t+\frac{1}{t}=10$

$\Rightarrow \frac{t^2+1}{t}=10 \Rightarrow t^2-10t+1=0$

$\Rightarrow t=\frac{10\pm\sqrt{100-4}}{2}=\frac{10\pm4\sqrt{6}}{2}$

$=5\pm2\sqrt{6}=\left((3+2)\pm2\sqrt{6}\right)$

$=(\sqrt{3}\pm\sqrt{2})^2$

Case (i): $(\sqrt{3}+\sqrt{2})^{x^2-4}=(\sqrt{3}+\sqrt{2})^2$

$\Rightarrow x^2 - 4 = 2 \Rightarrow x^2 = 6 \Rightarrow x=\pm\sqrt{6}$

Case (ii): $(\sqrt{3}+\sqrt{2})^{x^2-4}=(\sqrt{3}-\sqrt{2})^2$

$=(\sqrt{3}+\sqrt{2})^{-2}$

$\Rightarrow x^2 - 4 = -2 \Rightarrow x^2 = 2 \Rightarrow x=\pm\sqrt{2}$

$\therefore n(S) = 4$

18. Option (2) is correct.

Let $z = x + iy$

$\Rightarrow |x + iy - 2| = 2\,|x + iy - 3|$

$\Rightarrow (x - 2)^2 + y^2 = 4\,\{(x - 3)^2 + y^2\}$

$\Rightarrow x^2 + 4 - 4x + y^2 = 4x^2 + 36 - 24x + 4y^2$

$\Rightarrow 3x^2 - 20x + 3y^2 + 32 = 0$

$\Rightarrow x^2+y^2-2\left(\frac{10}{3}\right)x+\frac{32}{3}=0$

$\therefore$ Radius $=\sqrt{\left(\frac{10}{3}\right)^2-\frac{32}{3}}=\sqrt{\frac{100}{9}-\frac{32}{3}}$

$=\sqrt{\frac{100-96}{9}}=\frac{2}{3}=\gamma$

and centre $\equiv \left(\frac{10}{3},0\right)\equiv(\alpha,\beta)$

So, $3(\alpha+\beta+\gamma)=3\left(\frac{10}{3}+0+\frac{2}{3}\right)$

$=3\left[\frac{12}{3}\right]=12$

19. Option (2) is correct.

$f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & \sin 2x \\ \sin^2 x & 1+\cos^2 x & \sin 2x \\ \sin^2 x & \cos^2 x & 1+\sin 2x \end{vmatrix}$

Applying $R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$

$f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & \sin 2x \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$

Applying $R_3 \to R_3 - R_2$

$f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & \sin 2x \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{vmatrix}$

Expanding along C_1, we get

$= (1 + \sin^2 x)\,\{1 - 0\} + 1\,\{\cos^2 x + \sin 2x\}$

$= 1 + \sin^2 x + \cos^2 x + \sin 2x = 2 + \sin 2x$

As $x\in\left[\frac{\pi}{6},\frac{\pi}{3}\right]\Rightarrow 2x\in\left[\frac{\pi}{3},\frac{2\pi}{3}\right]$

$\Rightarrow \frac{\sqrt{3}}{2}<\sin 2x\le 1 \Rightarrow 2+\frac{\sqrt{3}}{2}\le 2+\sin 2x<3$

Maximum value $(\alpha) = 2 + 1 = 3$

Minimum value $(\beta) = 2+\frac{\sqrt{3}}{2}$

Hence, $\beta^2-2\sqrt{\alpha}=\left(2+\frac{\sqrt{3}}{2}\right)^2-2(\sqrt{3})$

$=4+\frac{3}{4}+2\sqrt{3}-2\sqrt{3}=\frac{19}{4}$

HINT:

Solve the given function by expanding the determinant using row or column operations.

20. Option (4) is correct.

Negation of $q \vee ((\sim q) \wedge p)$

$= \sim [q \vee ((\sim q) \wedge p)]$

$= \sim q \wedge \sim ((-q) \wedge p)$

$= \sim q \wedge (q \vee \sim p)$

$= (\sim q \wedge q) \vee (\sim q \wedge \sim p)$

$= F \vee (\sim q \wedge \sim p) = (\sim q) \wedge (\sim p)$

HINT:

Negation of p is $\sim p$.

Section B

21. Correct answer is [3501].

Here, $\vec{v} = \alpha\hat{i} + 2\hat{j} - 3\hat{k}$

$\vec{w} = 2\alpha\hat{i} + \hat{j} - \hat{k}$

$$\therefore \vec{v} \times \vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 2 & -3 \\ 2\alpha & 1 & -1 \end{vmatrix} = \hat{i}(-2+3) - \hat{j}(-\alpha+6\alpha) + \hat{k}(\alpha - 4\alpha)$$

$= \hat{i} - (5\alpha)\hat{j} - (3\alpha)\hat{k}$

Now, $\vec{u}.(\vec{v} \times \vec{w}) = |\vec{u}||\vec{v} \times \vec{w}|\cos\theta$

$\therefore \cos\theta = -1$ $\quad (\because [\vec{u}\ \vec{v}\ \vec{w}]$ is least$)$

$\Rightarrow -\alpha\sqrt{1+34\alpha^2} = -\alpha\sqrt{3401}$

$\Rightarrow 1 + 34\alpha^2 = 3401 \Rightarrow 34\alpha^2 = 3400$

$\Rightarrow \alpha = 10$

Now, $\vec{u} = \lambda(\vec{v} \times \vec{w}) = \lambda(\hat{i} - 5\alpha\hat{j} - 3\alpha\hat{k})$

$\Rightarrow |\lambda|\sqrt{3401} = 10 \quad \Rightarrow |\lambda| = \frac{10}{\sqrt{3401}} \Rightarrow \lambda = \pm\frac{10}{\sqrt{3401}}$

$\therefore \vec{u} = \pm\frac{10}{\sqrt{3401}}(\hat{i} - 5\hat{j} - 30\hat{k})$

$\Rightarrow |\vec{u}.\hat{i}|^2 = \frac{10}{\sqrt{3401}} \times \frac{10}{\sqrt{3401}} = \frac{100}{3401} = \frac{m}{n}$

$\therefore m + n = 100 + 3401 = 3501$

22. Correct answer is [50400].

Vowels $\rightarrow$ A, A, A, I, I, O $\rightarrow$ 6

Consonants $\rightarrow$ S, S, S, S, N, N, T $\rightarrow$ 7

Since vowels are together, so number of words

$= \left(\frac{8!}{4!2!}\right) \times \left(\frac{6!}{3!2!}\right)$

$= \left(\frac{8 \times 7 \times 6 \times 5 \times 4!}{4!2!}\right) \times \left(\frac{6 \times 5 \times 4 \times 3!}{3! \times 2!}\right)$

$= 840 \times 60 = 50400$

23. Option [29] is correct.

$19^{200} + 23^{200} = (21-2)^{200} + (21+2)^{200}$

$= 2\left\{\binom{200}{0}21^{200} + \binom{200}{2}21^{198}2^2 + \ldots + \binom{200}{200}2^{200}\right\}$

$= 49q + 2^{201} = 49q + 8^{67} = 49q + (7+1)^{67}$

$= 49q + \left[\binom{67}{0}7^{67} + \binom{67}{1}7^{66} + \ldots + \binom{67}{65}7^2 + \binom{67}{66}7 + \binom{67}{67}\right]$

$= 49q + 49(m) + 67 \times 7 + 1$

$\because$ Remainder $= \frac{67 \times 7 + 1}{49}$ i.e., 29

24. Correct answer is [514].

No. divisible by 2 = 450

No. divisible by 3 = 300

No. divisible by 7 = 128

No. divisible by 2 and 7 = 64

No. divisible by 3 & 7 = 43

No. divisible by 2 & 3 = 150

No. divisible by 2, 3 and 7 = 21

$\therefore$ Required number

$= (450 + 300 - 150) + (-64 - 43) + 21 = 600 - 86 = 514$

25. Correct answer is [A].

Let $\int_0^2 f(t)dt = \mu$

$\Rightarrow f'(x) + f(x) = \mu$

The solution $f(x)\ e^{\int dx} = \int \mu.e^{\int dx}dx$

$f(x).\ e^x = \int \mu e^x dx = \mu e^x + C$

Use $f(0) = e^{-2}$

$\Rightarrow e^{-2}(1) = \mu(1) + C \quad \Rightarrow C = e^{-2} - \mu$

$\therefore$ Solution is $f(x) = e^{-x}(\mu e^x + e^{-2} - \mu)$

$= \mu + e^{-x}(e^{-2} - \mu)$

So, $\mu = \int_0^2 f(t)dt = \int_0^2 \mu + (e^{-2} - \mu)e^{-t}dt$

$\Rightarrow \mu = \left[\mu t + \frac{(e^{-2} - \mu)e^{-t}}{(-1)}\right]_0^2$

$= [2\mu - (e^{-2} - \mu)e^{-2}] - [0 - (e^{-2} - \mu)e^{-0}]$

$\mu = e^{-4} - e^{-2}/e^{-2} \Rightarrow \mu = e^{-2} - 1$

So, $2f(0) - f(2) = 2e^{-2} - 2e^{-2} + 1 = 1$

26. Correct answer is [14].

$\because f(x) = x^2 + g'(1)x + g''(2)$ ...(i)

On differentiating, we get

$f'(x) = 2x + g'(1)$ ...(ii)

$\Rightarrow f''(x) = 2$

And $g(x) = f(1)x^2 + xf'(x) + f''(x)$

$= f(1)x^2 + x(2x + g'(1)) + 2$

$= f(1)x^2 + 2x^2 + g'(1)x + 2$ ...(iii)

$\Rightarrow g'(x) = 2xf(1) + 4x + g'(1)$ ...(iv)

$\Rightarrow g''(x) = 2f(1) + 4$ ...(v)

At $x = 1$ eqn (iv)

$g'(1) = 2f(1) + 4 + g'(1)$

$\Rightarrow 2f(1) = -4$

$\Rightarrow f(1) = -2 \Rightarrow g''(x) = 0$

Also, $g''(2) = 2(-2) + 4 = 0$

Using (i), $f(1) = 1 + g'(1) + g''(2)$

$\Rightarrow -2 = 1 + g'(1) = 0 \Rightarrow g'(1) = -3$

$\therefore f(x) = x^2 - 3x \Rightarrow f(4) - g(4)$

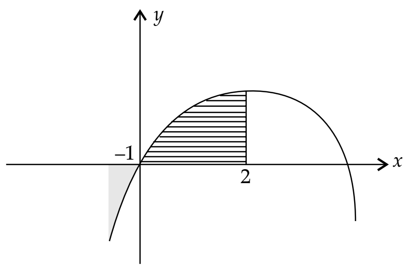
$= (16 - 12) - (-12 + 2) = 14$

$g(x) = -3x + 2$

HINT:

Find the derivatives of the given differential equation and hence find the required value.

27. Correct answer is [62].



$y = x\,|x - 3| = x(3 - x)$; as $-1 \le x \le 2$

$\therefore$ Required area (A) $= \left|\int_{-1}^{0}(3x - x^2)dx\right| + \int_{0}^{2}(3x - x^2)dx$

$$= \left|\left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_{-1}^{0}\right| + \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_{0}^{2}$$

$$= \left|(0-0) - \left(\frac{3}{2} + \frac{1}{3}\right)\right| + \left[\left(6 - \frac{8}{3}\right) - (0)\right]$$

$$= \left|-\left(\frac{9+2}{6}\right)\right| + \left[\frac{18-8}{3}\right] = \frac{11}{6} + \frac{10}{3} = \frac{11+22}{6} = \frac{31}{6}$$

So, $12A = 12 \times \frac{31}{6} = 62$

28. Correct answer is [63].

Let $I = \int_0^1 (x^{21} + x^{14} + x^7)(2x^{14} + 3x^7 + 6)^{\frac{1}{7}} dx$

$$= \int_0^1 x(x^{20} + x^{13} + x^6)(2x^{14} + 3x^7 + 6)^{\frac{1}{7}} dx$$

$$= \int_0^1 (x^{20} + x^{13} + x^6)(2x^{21} + 3x^{14} + 6x^7)^{\frac{1}{7}} dx$$

Let $2x^{21} + 3x^{14} + 6x^7 = u$

$\Rightarrow (42x^{20} + 42x^{13} + 42x^6)\,dx = du$

$\Rightarrow (x^{20} + x^{13} + x^6)dx = \frac{du}{42}$

$x = 0 \Rightarrow u = 0$

$x = 1 \Rightarrow u = 2 + 3 + 6 = 11$

$$\therefore x = \int_0^{11} \frac{u^{1/7}}{42} du = \frac{1}{42}\left[\frac{u^{8/7}}{\frac{8}{7}}\right]_0^{11}$$

$$= \frac{1}{42}\left[\frac{7}{8}u^{\frac{8}{7}}\right]_0^{11} = \frac{1}{48}(11^{\frac{8}{7}}) = \frac{1}{l}(11)^{\frac{m}{n}}$$

$\Rightarrow l = 48, m = 8, n = 7$

$\therefore l + m + n = 48 + 8 + 7 = 63$

29. Correct answer is [7.54].

$S_4 = \frac{4}{2}[2a_1 + 3d]$

$\Rightarrow 50 = 2\,[16 + 3d]$ $[\because a_1 = 8]$

$\Rightarrow 25 = 16 + 3d \Rightarrow 3d = 9 \Rightarrow d = 3$

Also, sum of last 4 terms = 170

$\frac{4}{2}[2[8 + (n-4)d] + 3d] = 170$

$\Rightarrow 2\,[2\,(8 + (n - 4)\,3) + 9] = 170$

$\Rightarrow n - 4 = 10 \Rightarrow n = 14$

$\therefore T_7 \times T_8 = (8 + 6d) \times (8 + 7d)$

$= (8 + 18) \times (8 + 21) = 26 \times 29 = 754$

30. Correct answer is [11].

Here, $\overline{AC} = (2-2)\hat{i} + (3-6)\hat{j} + (-1-2)\hat{k}$

$= 0\hat{i} - 3\hat{j} - 3\hat{k}$

$\overline{BD} = (4+4)\hat{i} + (5-0)\hat{j} + (0-\lambda)\hat{k}$

$= 8\hat{i} + 5\hat{j} - \lambda\hat{k}$

$\therefore$ Area $= \frac{1}{2}(\overline{AC} \times \overline{BD})$

$$18 = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -3 & -3 \\ 8 & 5 & -\lambda \end{vmatrix}$$

$36 = |(3\lambda + 15)\hat{i} - (0 + 24)\hat{j} + (0 + 24)\hat{k}|$

$= |(3\lambda + 15))\hat{i} - 24\hat{j} + 24\hat{k}|$

$\Rightarrow 36^2 = 9\lambda^2 + 225 + 90\lambda + 576 + 576$

$\Rightarrow 9\lambda^2 + 90\lambda + 81 = 0 \Rightarrow \lambda^2 + 10\lambda + 9 = 0$

$\Rightarrow \lambda^2 + 9\lambda + \lambda + 9 = 0 \Rightarrow (\lambda + 9)(\lambda + 1) = 0$

$\therefore \lambda = -1$ $\because |\lambda| \le = 5$

So, $5 - 6\lambda = 5 - 6(-1) = 5 + 6 = 11$

❑❑