JEE (Main) MATHEMATICS SOLVED PAPER

2023
01st Feb. Shift 2

Section A

Q. 1. Let $\alpha x = \exp(x^{\beta}y^{\gamma})$ be the solution of the differential equation $2x^2y\,dy - (1 - xy^2)\,dx = 0$, $x > 0$, $y(2) = \sqrt{\log_e 2}$. Then $\alpha + \beta - \gamma$ equals :

(1) 1 **(2)** –1 **(3)** 3 **(4)** 0

Q. 2. The sum $\sum_{n=1}^{\infty} \frac{2n^2+3n+4}{(2n)!}$ is equal to:

(1) $\frac{13e}{4}+\frac{5}{4e}$ **(2)** $\frac{11e}{2}+\frac{7}{2e}-4$

(3) $\frac{11e}{2}+\frac{7}{2e}$ **(4)** $\frac{13e}{4}+\frac{5}{4e}-4$

Q. 3. Let $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} + 5\hat{k}$ be two vectors. Then which one of the following statements is TRUE ?

(1) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{17}{\sqrt{35}}$ and the direction of the projection vector is same as of $\vec{b}$

(2) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{17}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\vec{b}$

(3) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{-17}{\sqrt{35}}$ and the direction of the projection vector is same as of $\vec{b}$

(4) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{-17}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\vec{b}$

Q. 4. Let $\vec{a} = 2\hat{i} - 7\hat{j} + 5\hat{k}, \vec{b} = \hat{i} + \hat{k}$ and $\vec{c} = \hat{i} + 2\hat{j} - 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$ and $\vec{r}.\vec{b} = 0$, then $|\vec{r}|$ is equal to:

(1) $\frac{11}{5}\sqrt{2}$ **(2)** $\frac{\sqrt{914}}{7}$ **(3)** $\frac{11}{7}\sqrt{2}$ **(4)** $\frac{11}{7}$

Q. 5. Let $f : R - 0, 1 \to R$ be a function such that $f(x) + f\left(\frac{1}{1-x}\right) = 1 + x$. Then $f(2)$ is equal to

(1) $\frac{9}{2}$ **(2)** $\frac{7}{4}$ **(3)** $\frac{9}{4}$ **(4)** $\frac{7}{3}$

Q. 6. Let P(S) denote the power set of S={1, 2, 3, ………, 10}. Define the relations R_1 and R_2 on P(S) as AR_1B if $(A \cap B^c) \cup (B \cap A^c) = \phi$ and AR_2B if $A \cup B^C = B \cup A^C$. $\forall A, B \in P(S)$. Then:

(1) only R_1 is an equivalence relation

(2) only R_2 is an equivalence relation

(3) both R_1 and R_2 are equivalence relations

(4) both R_1 and R_2 are not equivalence relations

Q. 7. The area of the region given by $\{(x, y): xy \le 8, 1 \le y \le x^2\}$ is:

(1) $16\log_e 2 + \frac{7}{3}$ **(2)** $16\log_e 2 - \frac{14}{3}$

(3) $8\log_e 2 - \frac{13}{3}$ **(4)** $8\log_e 2 + \frac{7}{6}$

Q. 8. If $A = \frac{1}{2}\begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$ then:

(1) $A^{30} + A^{25} + A = I$ **(2)** $A^{30} = A^{25}$

(3) $A^{30} + A^{25} - A = I$ **(4)** $A^{30} - A^{25} = 2I$

Q. 9. Which of the following statements is a tautology?

(1) $p \vee (p \wedge q)$

(2) $(p \wedge (p \to q)) \to \sim q$

(3) $(p \wedge q) \to (\sim(p) \to q)$

(4) $p \to (p \wedge (p \to q))$

Q. 10. The sum of the absolute maximum and minimum values of the function $f(x) = |x^2 - 5x + 6| - 3x + 2$ in the itnerval [–1, 3] is equal to:

(1) 12 **(2)** 13 **(3)** 10 **(4)** 24

Q. 11. Let the plane P pass through the intersection of the planes $2x + 3y - z = 2$ and $x + 2y + 3z = 6$ and be perpendicular to the plane $2x + y - z = 0$. If d is the distance of P form the point (–7,1,1,) then d^2 is equal to :

(1) $\frac{250}{83}$ **(2)** $\frac{250}{82}$ **(3)** $\frac{15}{53}$ **(4)** $\frac{25}{83}$

Q. 12. The number of integral values of k, for which one root of the equation $2x^2 - 8x + k = 0$ lies in the interval (1,2) and its other root lies in the interval (2, 3), is :

(1) 3 **(2)** 0 **(3)** 2 **(4)** 1

Q. 13. Let P (x_0, y_0) be the point on the hyperbola $3x^2 - 4y^2 = 36$, which is nearest to the line $3x + 2y = 1$. Then $\sqrt{2}(y_0 - x_0)$ is equal to :

(1) –9 **(2)** –3 **(3)** 3 **(4)** 9

Q. 14. Two dice are thrown independently. Let A be the event that the number appeared on the 1st die is less than the number appeared on the 2nd die, B be the event that the number appeared on the 1st die is even and that on the second die is odd, and C be the event that the number appeared on the 1st die is odd and that on the 2nd is even. Then :

(1) The number of favourable cases of the events A, B and C are 15,6 and 6 respectively

(2) The number of favourable cases of the event $(A \cup B) \cap C$ is 6
(3) B and C are independent
(4) A and B are mutually exclusive

Q. 15. If $y(x) = x^x, x > 0$, then $y''(2) - 2y'(2)$ is equal to:
(1) $4\log_e 2 + 2$ **(2)** $8\log_e 2 - 2$
(3) $4(\log_e 2)^2 + 2$ **(4)** $4(\log_e 2)^2 - 2$

Q. 16. Let
$$S = \left\{x \in : 0 < x < 1 \text{ and } 2\tan^{-1}\left(\frac{1-x}{1+x}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right\}$$
If $n(S)$ denotes the number of elements in S then :
(1) $n(S) = 2$ and only one element in S is less then $\frac{1}{2}$
(2) $n(S) = 1$ and the element in S is more then $\frac{1}{2}$
(3) $n(S) = 0$
(4) $n(S)=1$ and the element in S is less than $\frac{1}{2}$

Q. 17. The value of the integral $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x+\frac{\pi}{4}}{2-\cos 2x}dx$ is :
(1) $\frac{\pi^2}{12\sqrt{3}}$ **(2)** $\frac{\pi^2}{6\sqrt{3}}$ **(3)** $\frac{\pi^2}{6}$ **(4)** $\frac{\pi^2}{3\sqrt{3}}$

Q. 18. For the system of linear equations $\alpha x + y + z = 1$, $x + \alpha y + z = 1$, $x + y + \alpha z = \beta$, which one of the following statements is NOT correct ?
(1) It has infinitely many solutions if $\alpha = 2$ and $\beta = -1$
(2) It has no solution if $\alpha = -2$ and $\beta = 1$
(3) $x + y + z = \frac{3}{4}$ if $\alpha = 2$ and $\beta = 1$
(4) It has infinitely many solutions if $\alpha = 1$ and $\beta = 1$

Q. 19. Let $9 = x_1 < x_2 < ..... < x_7$, ... x_7 be in an A.P. with common difference d. If the standard deviation of $x_1, x_2 ......, x_7$ is 4 and mean is $\bar{x}$, then $\bar{x} + x_6$ is equal to:
(1) $2\left(9+\frac{8}{\sqrt{7}}\right)$ **(2)** $18\left(1+\frac{1}{\sqrt{3}}\right)$
(3) 25 **(4)** 34

Q. 20. Let a, b be two real numbers such that $ab < 0$. If the complex number $\frac{1+ai}{b+i}$ is of unit modulus and $a + ib$ lies on the circle $|z - 1| = |2z|$, then a possible value of $\frac{1+[a]}{4b}$, where $[t]$ is greatest integer function, is :
(1) $-\frac{1}{2}$ **(2)** -1
(3) 1 **(4)** $\frac{-(1+\sqrt{7})}{4}$

Section B

Q. 21. Let $\alpha x + \beta y + \gamma z = 1$ be the equation of a plane through the point (3, –2, 5)and perpendicular to the line joning the points (1, 2, 3) and (–2, 3, 5). Then the value of $\alpha\beta y$ is equal to

Q. 22. If the term without x in the expansion of $\left(x^{\frac{2}{3}} + \frac{a}{x^3}\right)^{22}$ is 7315, then $|\alpha|$ is equal to

Q. 23. If the x- intercept of a focal chord of the parabola $y^2 = 8x + 4y + 4$ is 3 , then the length of this chord is equal to

Q. 24. Let the sixth term in the binomial expansion of $\left(\sqrt{2^{\log_2(10-3^x)}} + \sqrt[5]{2^{x-2\log_2 3}}\right)^m$, in the increasing powers of $2^{(x-2)\log_2 3}$, be 21. If the binomial coefficients of the second, third and fourth terms in the expansion are respectively the first, third and fifth terms of A.P., then the sum of the squares of all possible values of x is

Q. 25. The point of intersection C of the plane $8x + y + 2z = 0$ and the line joining the point A(–3, –6, 1) and B(2, 4, –3) divides the line segment AB internally in the ratio k:. If a, b, c ($|a|, |b|, |c|$) are coprime are the direction ratios of the perpendicular form the point C on the line $\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}$, then $|a + b + c|$ is equal to

Q. 26. The line $x = 8$ is the directrix of the ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with the corresponding focus (2, 0). If the tangent to E at the point P in the first quadrant passes through the point $(0, 4\sqrt{3})$ and intersects that x-axis at Q then $(3PQ)^2$ equal to

Q. 27. The total number of six digit numbers, formed using the digits 4, 5, 9 only and divisible by 6 , is

Q. 28. Number of integral solutions to the equation $x + y + z = 21$, where $x \geq 1, y \geq 3, z \geq 4$, is equal to

Q. 29. The sum of the common terms of the following three arithmetic progressions.
3, 7, 11, 15,…..,399
2, 5, 8, 11,……,359 and
2, 7, 12, 17,……, 197
is equal to

Q. 30. If
$$\int \frac{5^{\cos x}(1+\cos x\cos 3x+\cos^2 x+\cos^3 x\cos 3x)dx}{1+5^{\cos x}}$$
$= \frac{k\pi}{16}$, then k is equal to

Answer Key

Q. No.	Answer	Topic Name	Chapter Name
1	(1)	Linear differential equation	Differential equations
2	(4)	Special series	Sequence and series
3	(3)	Projection of a vector	Vector algebra
4	(3)	Magnitude of a vector	Vector algebra
5	(3)	Value of a function	Set, relations and functions
6	(3)	Equivalence relation	Set, relations and functions
7	(2)	Area between the curves	Integral calculus
8	(3)	Multiplication of matrix	Matrices and determinant
9	(3)	Tautology	Mathematical reasoning
10	(4)	Maxima, minima	Limit, continuity and differentiability
11	(1)	Intersection of two planes	Three dimensional geometry
12	(4)	Shortest distance	Limit, continuity and differentiability
13	(1)	Extreme values	Limit, continuity and differentiability
14	(2)	Hyperbola	Coordinate geometry
15	(4)	Derivative	Limit, continuity and differentiability
16	(1)	Inverse trigonometry	Trigonometry
17	(2)	Definite integrals	Integral calculus
18	(4)	System of equations	Matrices and determinant
19	(4)	Variance	Statistics and probability
20	(4)	Modulus	Complex number and quadratic equations
21	[6]	Equation of plane	Three dimensional geometry
22	[1]	General term	Binomial theorem
23	[16]	Parabola	Coordinate geometry
24	[4]	General term	Binomial theorem
25	[10]	Section formula	Three dimensional geometry
26	[39]	Ellipse	Coordinate geometry
27	[81]	Number of ways	Permutation and combinations
28	[105]	Combination	Permutation and combinations
29	[321]	A.P., G.P.	Sequence and series
30	[13]	Definite integrals	Integral calculus

Solutions

Section A

1. Option (1) is correct.

The given d.e. is :

$$2x^2y\frac{dy}{dx}-(1-xy^2)=0$$

$$\Rightarrow 2y\frac{dy}{dx}-\frac{1}{x^2}+\frac{y^2}{x}=0$$

Let $y^2 = u \Rightarrow 2y\dfrac{dy}{dx}=\dfrac{du}{dx}$

So, $\dfrac{du}{dx}+\dfrac{u}{x}=\dfrac{1}{x^2}$ (linear d.e)

I.F. $= e^{\int\frac{1}{x}dx} = e^{\ln x} = x$

$\therefore$ The solution is

$u \times x = \displaystyle\int\frac{1}{x^2}\times x\,dx$

$\Rightarrow ux = \log_e x + C \Rightarrow y^2x = \log_e x + C$

Now, using $y(2) = \sqrt{\log_e 2}$, we get

$2\log_e 2 = \log_e 2 + C \Rightarrow C = \log_e 2$

$\therefore$ The solution is :

$y^2x = \ln x + \ln 2 = \ln 2x \Rightarrow 2x = \exp(xy^2)$

On comparing with $\alpha x = \exp(x^\beta y^\gamma)$, we get

$\alpha = 2, \beta = 1, \gamma = 2$

$\therefore \alpha + \beta - \gamma = 2 + 1 - 2 = 1$

HINT:

Substitute $y^2 = u$ and make a linear differential equation.

2. Option (4) is correct.

$$\sum_{n=1}^{\infty} \frac{2n^2+3n+4}{(2n)!}$$

Let $t = 2n \Rightarrow n = \frac{t}{2}$

$$\Rightarrow \sum_{t\to\text{even}} \frac{2\left[\frac{t^2}{4}\right]+3\left[\frac{t}{2}\right]+4}{2t!}$$

$$\Rightarrow \sum_{t\to\text{even}} \frac{\frac{t^2}{2}+\frac{3t}{2}+4}{t!}$$

$$= \frac{1}{2}\sum_{t\to\text{even}} \frac{t^2+3t+8}{t!}$$

$$= \frac{1}{2}\left\{\sum_{t\to\text{even}} \frac{t+1-1}{(t-1)!}+\frac{3}{(t-1)!}+\frac{8}{t!}\right\}$$

$$= \frac{1}{2}\left\{\sum_{t\to\text{even}} \frac{1}{(t-2)!}+\frac{1}{(t-1)!}+\frac{3}{(t-1)!}+\frac{8}{t!}\right\}$$

$$= \frac{1}{2}\left\{\frac{e+\frac{1}{e}}{2}+4\left[\frac{e-\frac{1}{e}}{2}\right]+8\left[\frac{e+\frac{1}{e}}{2}\right]-8\right\}$$

$$= \frac{1}{4}\left\{e+\frac{1}{e}+4e-\frac{4}{e}+8e+\frac{8}{e}-16\right\}$$

$$= \frac{1}{4}\left\{13e+\frac{5}{e}-16\right\}$$

$$= \frac{13e}{4}+\frac{5}{4e}-4$$

3. Option (3) is correct.

$\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$

$\vec{b} = \hat{i} + 3\hat{j} + 5\hat{k}$

Projection of $\vec{a}$ or $\vec{b} = \frac{(5)(1)+(-1)(3)+(-3)(5)}{\sqrt{1+9+25}}$

$$= \frac{5-3-15}{\sqrt{35}} = \frac{-13}{\sqrt{35}}$$

4. Option (3) is correct.

Given $\vec{a} = 2\hat{i} - 7\hat{j} + 5\hat{k}$

$\vec{b} = \hat{i} + \hat{k}; \vec{c} = \hat{i} + 2\hat{j} - 3\hat{k}$

Since, $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$

$\Rightarrow \vec{r} = \vec{c} + \lambda\vec{a}$

$\Rightarrow \vec{r}.\vec{b} = \vec{c}.\vec{b} + \lambda\vec{a}.\vec{b}$

$\Rightarrow (1 + 0 - 3) + \lambda(2 + 0 + 5) = 0$

$\Rightarrow -2 + 7\lambda = 0 \Rightarrow \lambda = \frac{2}{7}$

$\therefore \vec{r} = (\hat{i} + 2\hat{j} - 3\hat{k}) + \frac{2}{7}(2\hat{i} - 7\hat{j} + 5\hat{k})$

$= \frac{11}{7}\hat{i} + 0\hat{j} + \frac{11}{7}\hat{k} = \frac{11}{7}(\hat{i} + \hat{k})$

$\Rightarrow |\vec{r}| = \frac{11}{7}\sqrt{1+1} = \frac{11}{7}\sqrt{2}$

5. Option (3) is correct.

Since, $f(x) + f\left(\frac{1}{1-x}\right) = 1 + x$

Let $x = 2 \Rightarrow f(2) + f(-1) = 3$ ...(A)

Let $x = -1 \Rightarrow f(-1) + f\left(\frac{1}{2}\right) = 0$ ...(B)

Let $x = \frac{1}{2} \Rightarrow f\left(\frac{1}{2}\right) + f(2) = \frac{3}{2}$ ...(C)

Adding A, B & C, we get

$$2\left[f(2) + f(-1) + f\left(\frac{1}{2}\right)\right] = 3 + \frac{3}{2}$$

$$f(2) + 0 = \frac{1}{2}\left[\frac{9}{2}\right] = \frac{9}{4}$$

$$\Rightarrow f(2) = \frac{9}{4}$$

HINT:

Use $x \to \frac{1}{1-x}$ and $x \to \frac{x-1}{x}$ and then put $x = 2$

6. Option (3) is correct.

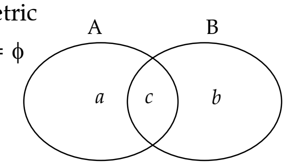
S = {1, 2, 3, ...10}

$AR_1B \Rightarrow (A \cap B') \cup (A' \cap B) = \phi$

$\therefore R_1$ is reflexive & symmetric

Also, $(A \cap B') \cup (A' \cap B) = \phi$

$\Rightarrow A = B$

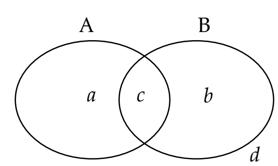


$(B \cap C') \cup (B' \cap C) = \phi$

$\Rightarrow B = C$

$\therefore A = C$

So, R_1 is equivalence.



$R_2 = A \cup B' = A' \cup B$

$\therefore R_2$ is reflexive and symmetric

$(A \cup B') = (A' \cup B)$

$\Rightarrow \{a, c, d\} = \{b, c, d\}$

$\Rightarrow \{a\} = \{b\}$

$\Rightarrow A = B$

Similarly,

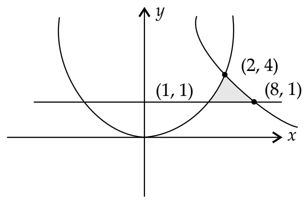
$(B \cup C') = (B' \cup C)$

$\Rightarrow B = C$

$\therefore A = C$

So, R_2 is an equivalence relation.

7. Option (2) is correct.



Required area $= \int_1^2 (x^2-1)dx + \int_2^8 \left(\frac{8}{x}-1\right)dx$

$= \left[\frac{x^3}{3}-x\right]_1^2 + \left[8\ln x - x\right]_2^8$

$= \left[\left(\frac{8}{3}-2\right)-\left(\frac{1}{3}-1\right)\right]+\left[(8\ln 8-8)-8\ln 2+2\right]$

$= \left[\frac{2}{3}+\frac{2}{3}\right]+[24\ln 2-8-8\ln 2+2]$

$\left[\frac{4}{3}+16\ln 2-6\right] = 16\ln 2-\frac{14}{3}$

8. Option (3) is correct.

$A = \frac{1}{2}\begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$

$= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ for $\theta = \frac{\pi}{3}$

$\Rightarrow A^2 = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

and $A^3 = \begin{bmatrix} \cos 3\theta & \sin 3\theta \\ -\sin 3\theta & \cos 3\theta \end{bmatrix}$

$A^{25} = \begin{bmatrix} \cos 25\theta & \sin 25\theta \\ -\sin 25\theta & \cos 25\theta \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{-\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix} = A$

$A^{30} = \begin{bmatrix} \cos 30\theta & \sin 30\theta \\ -\sin 30\theta & \cos 30\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$\therefore A^{30} + A^{25} - A = I + A - A = I$

HINT:

Using the multiplication of matrix, find the general value of A^n.

9. Option (3) is correct.

$(p \wedge q) \to (\sim p \to q)$

$\equiv \sim(p \wedge q) \vee (p \vee q)$

$\equiv (\sim p \vee \sim q) \vee (p \vee q)$

$\equiv (\sim p \vee p) \vee (\sim q \vee q)$

$\equiv T \vee T \equiv T$

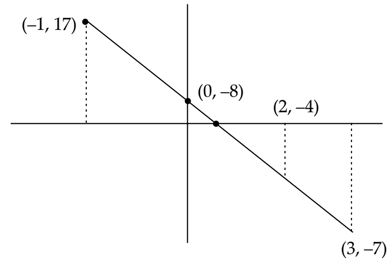
10. Option (4) is correct.

$f(x) = |x^2 - 5x + 6| - 3x + 2 \qquad x \in [-1, 3]$

$= |(x-2)(x-3)| - 3x + 2 \qquad x \in [-1, 3]$

$= \begin{cases} x^2-5x+6-3x+2 & -1 \le x \le 2 \\ -(x^2-5x+6)-3x+2 & x \in (2,3) \end{cases}$

$= \begin{cases} x^2-8x+8 & -1 \le x \le 2 \\ -x^2+2x-4 & 2 < x < 3 \end{cases}$



$\therefore$ Maximum value $= |f(-1)| = 17$

Minimum value $= |f(3)| = 7$

$\therefore$ Sum $= 17 + 7 = 24$

11. Option (1) is correct.

The two planes are

$2x + 3y - z = 2$

and $x + 2y + 3z = 6$

Let the general equation of plane through the intersection of above plane is :

$(2x + 3y - z - 2) + \lambda(x + 2y + 3z - 6) = 0$

$\Rightarrow x(2 + \lambda) + y(3 + 2\lambda) + z(-1 + 3\lambda) - 2 - 6\lambda = 0 \qquad ...(i)$

Since plane in equation (i) is perpendicular to the plane $2x + y - z = 0$

$\therefore (2)(2 + \lambda) + (1)(3 + 2\lambda) + (-1)(-1 + 3\lambda) = 0$

$\Rightarrow 4 + 2\lambda + 3 + 2\lambda + 1 - 3\lambda = 0$

$\Rightarrow \lambda = -8$

So, required equation of plane is

$x(2-8) + y(3-16) + z(-1-24) - 2 + 48 = 0$

$\Rightarrow -6x - 13y - 25z + 46 = 0$

Now, distance of above plane from the point$(-7, 1, 1)$ is :

$$d = \left|\frac{(-7)(-6)+(1)(-13)+(1)(-25)+46}{\sqrt{(-6)^2+(-13)^2+(-25)^2}}\right|$$

$= \left|\frac{42-13-25+46}{\sqrt{36+169+625}}\right| = \left|\frac{50}{\sqrt{830}}\right|$ units

So, $d^2 = \frac{2500}{830} = \frac{250}{83}$ units

12. Option (4) is correct.

The given equation is

$f(x) = 2x^2 - 8x + k = 0$

Hence, $f(1) = 2(1)^2 - 8(1) + k > 0$

$\Rightarrow k > 6$

$f(2) < 0$

$\Rightarrow 2(2)^2 - 8(2) + k < 0$

$\Rightarrow 8 - 16 + k < 0$

$\Rightarrow k < 8$

$f(3) > 0$

$\Rightarrow 2(3)^2 - 8(3) + k > 0$
$\Rightarrow 18 - 24 + k > 0$
$\Rightarrow k > 6$
So, $k \in (6, 8)$
$\therefore$ The integral value of k is 7.

HINT:
Find the value of function at the extreme points.

13. Option (1) is correct.
The given hyperbola is :
$3x^2 - 4y^2 = 36$
Equation of line is
$3x + 2y = 1$

$$\Rightarrow 2y = 1 - 3x \Rightarrow y = \frac{-3}{2}x + \frac{1}{2}$$

So, $m = \frac{-3}{2}$

and $m = \frac{3\sec\theta}{\sqrt{12}\tan\theta}$

$$\therefore \frac{3/\cos\theta}{\sqrt{12}\frac{\sin\theta}{\cos\theta}} = \frac{-3}{2}$$

$$\Rightarrow \frac{3}{\sqrt{12}\sin\theta} = \frac{-3}{2} \Rightarrow \sin\theta = \frac{-1}{\sqrt{3}}$$

$$\therefore (x_0, y_0) \equiv (\sqrt{12}\sec\theta, 3\tan\theta)$$

$$= \left(\sqrt{12} \times \frac{\sqrt{3}}{\sqrt{2}}, \frac{-3\times 1}{\sqrt{2}}\right) = \left(\frac{6}{\sqrt{2}}, \frac{-3}{\sqrt{2}}\right)$$

$$\therefore \sqrt{2}(y_0 - x_0) = \sqrt{2}\left[\frac{-9}{\sqrt{2}}\right] = -9$$

HINT:
Find the slope of given curves and compare them.

14. Option (2) is correct.
Here,
A = { (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 5), (4, 6), (5, 6)}
$\therefore n(A) = 15$
B = {(2, 1), (2, 3), (2, 5), (4, 1), (4, 3), (4, 5), (6, 1), (6, 3), (6, 5)}
$\therefore n(B) = 9$
C = {(1, 2), (1, 4), (1, 6), (3, 2), (3, 4), (3, 6), 5, 2, (5, 4), (5, 6), }
$\therefore n(c) = 9$
(1) False, $n(A) = 15$, $n(B) = 9 \neq 6$, $n(c) = 9 \neq 6$
(2) $(A \cap C)$ = {(1, 2), (1, 4), (1, 6), (3, 4), (3, 6), (5, 6)}
$\therefore n(A \cap C) = 6$
$(B \cap C) = \phi$
$\therefore n(B \cap C) = 0$
and $n(A \cap B \cap C) = 0$
$\therefore n((A \cup B) \cap C) = 6$
(3) $P(B) = \frac{9}{36} = \frac{1}{4}$, $P(C) = \frac{9}{36} = \frac{1}{4}$, $P(B \cap C) = 0$
As, $P(B) . P(C) \neq P(B \cap C)$
So, B and C are not independent.
(4) As, $A \cap B = \{(4, 5)\} \neq \phi$
So, A and B are not exclusive events.

HINT:
$n((A \cup B) \cap C)) = n(A \cap C) + n(B \cap C) - n(A \cap B \cap C)$

15. Option (4) is correct.
Given $y(x) = x^x$, $x > 0$
Taking ln both sides,
$\Rightarrow \ln y = x \ln x$
In differentiating, we get

$$\frac{1}{y} \times y' = x \times \frac{1}{x} + \ln x$$

$\Rightarrow y' = y[1 + \ln x] = x^x[1 + \ln x]$
Put $x = 2$,
$\Rightarrow y'(2) = 2^2[1 + \ln 2]$
$= 4[1 + \ln 2]$ ...(A)

Now, $y'' = y'(1 + \ln x) + y\left[0 + \frac{1}{x}\right]$

$$= y'[1 + \ln x] + \frac{y}{x}$$

$$\therefore y''(2) = y'(2)[1 + \ln 2] + \frac{4}{2}$$

$= 4(1 + \ln 2)^2 + 2 = 4[1 + (\ln 2)^2 + 2\ln 2] + 2$
$= 4 + 4(\ln 2)^2 + 8\ln 2 + 2$
$= 4(\ln 2)^2 + 8\ln 2 + 6$
So, $y''(2) - 2y'(2) = 4(\ln 2)^2 + 3\ln 2 + 6 - 8 - \ln 2$
$= 4(\ln 2)^2 - 2$

16. Option (1) is correct.
Let $x = \tan\theta$; $0 < \theta < \frac{\pi}{4}$

$$\therefore 2\tan^{-1}\left(\frac{1-x}{1+x}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

$$= 2\tan^{-1}\left[\frac{1-\tan\theta}{1+\tan\theta}\right] = \cos^{-1}\left[\frac{1-\tan^2\theta}{1+\tan^2\theta}\right]$$

$$= 2\tan^{-1}\left(\tan\left(\frac{\pi}{4} - \theta\right)\right) = \cos^{-1}(\cos 2\theta)$$

$$\Rightarrow 2\left(\frac{\pi}{4} - \theta\right) = 2\theta \Rightarrow \frac{\pi}{4} - \theta = \theta$$

$$\Rightarrow 2\theta = \frac{\pi}{4} \Rightarrow \theta = \frac{\pi}{8} \Rightarrow x = \tan\frac{\pi}{8}$$

$$= \sqrt{2} - 1 < \frac{1}{2}$$

17. Option (2) is correct.

$$\text{Let I} = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x + \frac{\pi}{4}}{2 - \cos 2x} dx$$

$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left[\frac{-x + \frac{\pi}{4}}{2 - \cos 2x}\right] dx$$

On adding, we get

$$2I = \int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \frac{\pi}{2(2-\cos 2x)}dx$$

$$\Rightarrow 2I = 2\int_0^{\frac{\pi}{4}} \frac{\pi dx}{2\left[2-\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right)\right]}$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \frac{\pi}{2}\left\{\frac{1+\tan^2 x}{2+2\tan^2 x-1+\tan^2 x}\right\}dx$$

$$= \frac{\pi}{2}\int_0^{\frac{\pi}{4}} \left[\frac{1+\tan^2 x}{1+3\tan^2 x}\right]dx$$

Let $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

$\Rightarrow (1 + \tan^2 x)\, dx = dx$

$$\Rightarrow I = \frac{\pi}{2}\int_0^1 \frac{dt}{1+3t^2} = \frac{\pi}{2}\left[\frac{\tan^{-1}\sqrt{3}t}{\sqrt{3}}\right]_0^1$$

$$= \frac{\pi}{2}\left[\frac{\tan^{-1}\sqrt{3}}{\sqrt{3}} - \frac{\tan^{-1}0}{\sqrt{3}}\right]$$

$$= \frac{\pi}{2}\left(\frac{\pi}{3\sqrt{3}} - 0\right) = \frac{\pi^2}{6\sqrt{3}}$$

18. Option (4) is correct.

$$\therefore \Delta = \begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0$$

$\Rightarrow \alpha(\alpha^2 - 1) - 1(\alpha - 1) + (1 - \alpha) = 0$

$\Rightarrow \alpha^3 - \alpha - \alpha + 1 + 1 - \alpha = 0$

$\Rightarrow \alpha^3 - 3\alpha + 2 = 0 \Rightarrow 2, 1, -2$

(1) If $\alpha = 2 \Rightarrow \Delta \neq 0$

$\therefore$ Unique solution exist

(2) If $\alpha = -2$, $\beta = 1$

$\Rightarrow \Delta = 0$ and $\Delta_n \neq 0$

$\therefore$ No solution exist

(3) If $\alpha = 2$, $\beta = 1$

$\therefore 2x + y + z = 1$

$x + 2y + z = 1$

$x + y + 2z = 1$

On adding, we get

$4(x + y + z) = 3$

$\Rightarrow x + y + z = \frac{3}{4}$

(4) For $\alpha = 1$, $\beta = 1$, it has inifinite solutions.

19. Option (4) is correct.

$$\bar{x} = \frac{\sum_{i=1}^{n} X_i}{7} = \frac{\frac{7}{2}[2a+(7-1)d]}{7}$$

$$= \frac{2}{2}(a+3d) = a+3d = x_4$$

variance = 16

$$\therefore \sum_{i=1}^{7}\frac{(x_i-\bar{x})^2}{7} = 16 = \sum_{i=1}^{7}\frac{(x_i-x_4)^2}{7} = 16$$

$$\Rightarrow \frac{9d^2+4d^2+d^2+0+d^2+4d^2+9a^2}{7} = 16$$

$\Rightarrow 28d^2 = 112 \Rightarrow d^2 = 4 \Rightarrow d = 2$

$\therefore \bar{x} + x_6 = x_4 + x_6$

$= 9 + 3(2) + 9 + 5(2) = 9 + 6 + 9 + 10 = 34$

HINT:

Use the mean and variance to find the common difference.
Also, variance = $(SD)^2$

20. Option (4) is correct.

Since, $|1 + ai| = |b + i|$

$\Rightarrow a^2 + 1 = b^2 + 1 \Rightarrow a^2 = b^2$

And $|a + ib - 1| = |2a + 2bi||$

$\Rightarrow a^2 + 1 - 2a + b^2 = 4a^2 + 4b^2$

$\Rightarrow 3a^2 + 3b^2 + 2a - 1 = 0$

$\Rightarrow 6a^2 + 2a - 1$ $\qquad (\because a^2 = b^2)$

$$\Rightarrow a = \frac{-2\pm\sqrt{4+24}}{12}$$

$$= \frac{-2\pm 2\sqrt{7}}{12} = \frac{-1\pm\sqrt{7}}{6}$$

$$\therefore (a,b) \equiv \left(\frac{-1+\sqrt{7}}{6}, \frac{1-\sqrt{7}}{6}\right) \text{ or } \left(\frac{-1-\sqrt{7}}{6}, \frac{1+\sqrt{7}}{6}\right)$$

$\Rightarrow [a] = 0$

$$\frac{1+[a]}{4b} = 0 \text{ or } \frac{-\left(1+\sqrt{7}\right)}{4}$$

Section B

21. Correct answer is [6].

So, equation of plane is

$a(x - 3) + b(y + 2) + c(z - 5) = 0$

Now direction ratio of plane

$= \langle 1 + 2, 2 - 3, 3 - 5\rangle \equiv \langle 3, -1, -2\rangle$

$\therefore$ Equation of plane is:

$3(x - 3) - 1(y + 2) - 2(z - 5) = 0$

$\Rightarrow 3x - 9 - y - 2 - 2z + 10 = 0$

$\Rightarrow 3x - y - 2z = 1$

On comparing, we get

$\alpha = 3$, $\beta = -1$, $\gamma = -2$

$\therefore \alpha\beta\gamma = (3)(-1)(-2) = 6$

22. Correct answer is [1].

$$T_{r+1} = {}^{22}C_r \left((x)^{\frac{2}{3}}\right)^{22-r}\left(\frac{\alpha}{x^3}\right)^r$$

$$= {}^{22}C_r\, x^{\frac{2(22-r)}{3}-3r}\alpha^r$$

$\because$ Term is independent of x

So, $\frac{2}{3}(22-r)-3r=0$

$\Rightarrow 44-2r-9r=0 \Rightarrow 11r=44 \Rightarrow r=4$

$\therefore T_5 = {}^{22}C_4\,\alpha^4 = 7315$

$7315\ \alpha^4 = 7315$

$\Rightarrow \alpha^4 = 1 \Rightarrow \alpha = 1$

23. Correct answer is [16].

$y^2 = 8x + 4y + 4$

$\Rightarrow y^2 + 4 - 4y = 8x + 4 + 4$

$\Rightarrow (y-2)^2 = 8(x+1)$

$\therefore a = 2,\ X = x+1,\ Y = y-2$

$\Rightarrow$ Focus $(1, 2)$

Now, equation of chord will be $(y-2) = m(x-1)$

Since, above line passes through $(3, 0)$

$\therefore (0-2) = m(3-1)$

$\Rightarrow 2m = -2 \Rightarrow m = -1$

$\Rightarrow$ Equation of chord is $y - 2 = -x + 1$

$\Rightarrow x + y = 3$

Using (1)

$(3-x)^2 = 8x + 4(3-x) + 4$

$\Rightarrow 9 + x^2 - 6x = 8x + 12 - 4x + 4$

$\Rightarrow x^2 - 10x - 7 = 0$

$\Rightarrow x = \frac{10 \pm \sqrt{100+28}}{2} = \frac{10 \pm 8\sqrt{2}}{2}$

$= 5 \pm 4\sqrt{2} \quad \Rightarrow \quad y = -2 \pm 4\sqrt{2}$

$\therefore$ Length of focal chord $= \sqrt{4+32+4+32} = 16$

HINT:
X intercept means when chord will cut the x-axis

24. Correct answer is [4].

$T_6 = T_{5+1} = {}^mC_5\,(10-3^x)^{\frac{m-5}{2}} \times (3)^{x-2} = 21$

$\because {}^mC_1, {}^mC_2, {}^mC_3 \rightarrow$ A.P.

$\Rightarrow {}^mC_2 = \frac{{}^mC_1 + {}^mC_3}{2}$

$\Rightarrow \frac{2m!}{2!(m-2)!} = m + \frac{m!}{3!(m-3)!}$

$\Rightarrow m(m-1) = m + \frac{m(m-1)(m-2)}{6}$

$\Rightarrow 6(m-1) = 6 + (m-1)(m-2)$

$\Rightarrow 6m - 6 = 6 + m^2 - 3m + 2$

$\Rightarrow m^2 - 9m + 14 = 0$

$\Rightarrow m^2 - 7m - 2m + 14 = 0$

$\Rightarrow m(m-7) - 2(m-7) = 0$

$\Rightarrow (m-2)(m-7) = 0$

$m = 2, 7$

So, $m = 7$ $\qquad m = 2$ (rejected)

$\therefore T_6 = {}^7C_5(10-3^x)^{\frac{7-5}{2}} \times 3^{x-2} = 21$

$\Rightarrow 21(10-3^x) \times 3^{x-2} = 21$

$\Rightarrow (10-3^x)\,3^{x-2} = 1 \Rightarrow 10.3^x - (3^x)^2 = 9$

$\Rightarrow (3^x)^2 - 10.3^x + 9 = 0$

Let $3^x = t$

$\Rightarrow t^2 - 10t + 8 = 0 \Rightarrow (t-9)(t-1) = 0$

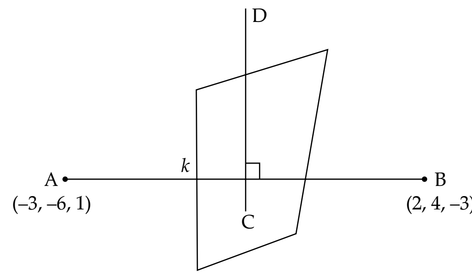
$\Rightarrow t = 1, 9$

So, $3^x = 1 = 3^0,\ 3^x = 9 = 3^2$

$\Rightarrow x = 0, 2$

So, required value $= 0 + 4 = 4$

25. Correct answer is [10].



$\therefore C \equiv \left(\frac{2k-3}{k+1}, \frac{4k-6}{k+1}, \frac{-3k+1}{k+1}\right)$

$\because$ The above point lies on $8x + y + 2z = 0$

$\Rightarrow 8\left(\frac{2k-3}{k+1}\right) + \left(\frac{4k-6}{k+1}\right) + 2\left(\frac{-3k+1}{k+1}\right) = 0$

$\Rightarrow 16k - 24 + 4k - 6 - 6k + 2 = 0$

$\Rightarrow 14k = 28 \Rightarrow k = 2$

So, $C \equiv \left(\frac{1}{3}, \frac{2}{3}, \frac{-5}{3}\right)$

Now given line :

$\frac{x-1}{-1} = \frac{y+4}{2} = \frac{z+2}{3} = t$

$\Rightarrow x = 1 - t,\ y = 2t - 4,\ z = 3t - 2$

For the condition of perpendicularity

$(-1)\left(1 - t - \frac{1}{3}\right) + (2)\left(2t - 4 - \frac{2}{3}\right) + (3)\left(3t - 2 + \frac{5}{3}\right) = 0$

$\Rightarrow t - \frac{2}{3} + 4t - \frac{28}{3} + 9t - 1 = 0$

$\Rightarrow 14t = 11 \Rightarrow t = \frac{11}{14}$

$\therefore \overrightarrow{CD} = \left\langle \frac{-5}{42}, \frac{-130}{42}, \frac{85}{42} \right\rangle$

$\therefore |a + b + c| = |-1 - 26 + 17| = 10$

HINT:
Use section formula to find the point which lies on the line joining the point A & B.

26. Correct answer is [39].

Given ellipse is :

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\because$ Directric $= 8$

$\Rightarrow \frac{a}{e} = 8\ \&\ ae = 8$

$$\Rightarrow 8e = \frac{2}{e} \Rightarrow e^2 = \frac{1}{4} \Rightarrow e = \frac{1}{2} \Rightarrow a = 4$$

Also, $b^2 = a^2(1 - e^2)$

$$= 16\left(1 - \frac{1}{4}\right) = 12$$

$$\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1 \Rightarrow \frac{x\cos\theta}{4} + \frac{y\sin\theta}{2\sqrt{3}} = 1$$

$$\therefore \sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$$

Hence, $P \equiv (2\sqrt{3}, \sqrt{3})$ & $\theta \equiv \left(\frac{8}{\sqrt{3}}, 0\right)$

$$(PQ)^2 = \left(\frac{8}{\sqrt{3}} - 2\sqrt{3}\right)^2 + (0 - \sqrt{3})^2$$

$$= \left(\frac{2}{\sqrt{3}}\right)^2 + 3 = \frac{4}{3} + 3 = \frac{13}{3}$$

$$\therefore (3PQ)^2 = 9 \times \frac{13}{3} = 39$$

27. Correct answer is [81].

Case I: $4 \to 6$ times

No. of ways = 1

Case II: $4 \to 5$ times

No. of ways = $\frac{5!}{3!} = 20$

Case III: $4 \to 3$ times

$5 \to 3$ times

No. of ways = $\frac{5!}{2!3!} = 10$

Case IV: $4 \to 3$ times

$9 \to 3$ times

No. of ways = $\frac{5!}{2!3!} = 10$

Case V: $4 \to 2$ times

$5 \to 2$ times

$9 \to 2$ times

No. of ways = $\frac{5!}{2!2!} = 30$

Case VI: $4 \to 1$ time

$5 \to 1$ time

$9 \to 4$ times

No. of ways = $\frac{5!}{4!} = 5$

Case VI: $4 \to 1$ time

$5 \to 1$ time

$9 \to 1$ time

No. of ways = $\frac{5!}{4!} = 5$

$\therefore$ Total no of ways = 1 + 20 + 10 + 10 + 30 + 5 + 5 = 81

28. Correct answer is [105]

Required no. of solution = ${}^{15}C_2$

$$= \frac{15!}{2!13!} = \frac{15 \times 14}{2} = 105$$

29. Correct answer is [321].

A.P. $\to$ 3, 7, 11, 15, ..., 399

Common differrence $(d_1) = 7 - 3 = 4$

A.P. $\to$ 2, 5, 8, 11, ..., 359

Common difference $(d_2) = 5 - 2 = 3$

A.P. $\to$ 2, 7, 12, 17, ..., 197

Common difference $(d_3) = 7 - 2 = 5$

Now, LCM (4, 3, 5) = 60

$\therefore$ Common terms are 47, 107, 167

$\therefore$ Required sum = 47 + 107 + 167 = 321

30. Correct answer is [13].

Let

$$I = \int_0^{\pi} \frac{5^{\cos x}(1 + \cos x\cos 3x + \cos^2 x + \cos^3 x\cos 3x)}{1 + 5^{\cos x}} dx$$

$$\Rightarrow I = \int_0^{\pi} \frac{5^{-\cos x}(1 + \cos x\cos 3x + \cos^2 x + \cos^3 x\cos 3x)dx}{1 + 5^{-\cos x}}$$

$$\Rightarrow 2I = \int_0^{\pi} [1 + \cos x\cos 3x + \cos^2 x + \cos^3 x\cos 3x]dx$$

$$\Rightarrow I = \int_0^{\pi} (1 + \cos x\cos 3x + \cos^2 x + \cos^3 x\cos 3x)dx$$

$$= \int_0^{\frac{\pi}{2}} \left[1 + \frac{1}{2}(\cos 4x + \cos 2x) + \frac{1}{2}(\cos 2x + 1) + \frac{1}{4}(\cos 3x + 3\cos x)\cos 3x\right]dx$$

$$= \int_0^{\frac{\pi}{2}} 1 + \frac{1}{2}(\cos 4x + \cos 2x) + \frac{1}{2}(\cos 2x + 1) + \frac{1}{4}\left[\frac{1}{2}(1 + \cos 6x) + \frac{3}{2}(\cos 4x + \cos 2x)\right]dx$$

$$= \int_0^{\frac{\pi}{2}} \left[\cos 4x\left(\frac{1}{2} + \frac{3}{8}\right) + \cos 2x\left(\frac{1}{2} + \frac{1}{2} + \frac{3}{8}\right) + \cos 6x\left(\frac{1}{8}\right) + \left(\frac{1}{2} + 1 + \frac{1}{8}\right)\right]dx$$

$$= \int_0^{\frac{\pi}{2}} \left[\frac{7}{8}\cos 4x + \frac{11}{8}\cos 2x + \frac{1}{8}\cos 6x + \frac{13}{8}\right]dx$$

$$= \left[\frac{7}{8} \times \frac{\sin 4x}{4} + \frac{11}{8} \times \frac{\sin 2x}{2} + \frac{1}{8} \times \frac{\sin 6x}{6} + \frac{13x}{8}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{7}{32}[(\sin\theta\pi - \sin\theta)] + \frac{11}{16}[\sin\pi - \sin\theta] + \frac{1}{48}[\sin 3\pi - \sin 0] + \frac{13}{8}\left[\frac{\pi - 0}{2}\right]$$

$$= \frac{13\pi}{6} \Rightarrow k = 13$$

❑❑