JEE (Main) MATHEMATICS SOLVED PAPER

2023
10^{th} April Shift 1

Section A

Q. 1. An arc PQ of a circle subtends a right angle at its centre O. The mid point of the arc PQ is R. $\overrightarrow{OP} = \vec{u}, \overrightarrow{OR} = \vec{v}$ and $\overrightarrow{OQ} = \alpha\vec{u} + \beta\vec{v}$, then α, β^2 are the roots of the equation

(1) $3x^2 - 2x - 1 = 0$ **(2)** $3x^2 + 2x - 1 = 0$
(3) $x^2 - x - 2 = 0$ **(4)** $x^2 + x - 2 = 0$

Q. 2. A square piece of tin of side 30 cm is to be made into a box without top by cutting a square from each corner and folding up the flaps to form a box. If the volume of the box is maximum, then its surface area (in cm^2) is equal to :

(1) 800 **(2)** 1025 **(3)** 900 **(4)** 675

Q. 3. Let O be the origin and the position vector of the point P be $-\hat{i} - 2\hat{j} + 3\hat{k}$. . If the position vectors of the A, B and C are $2\hat{i} + \hat{j} - 3\hat{k}$, $-2\hat{i} + 4\hat{j} - 2\hat{k}$ and $-4\hat{i} + 2\hat{j} - \hat{k}$ respectively, then the projection of the vector $\overrightarrow{OP}$ on a vector perpendicular to the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is :

(1) $\frac{10}{3}$ **(2)** $\frac{8}{3}$ **(3)** $\frac{7}{3}$ **(4)** 3

Q. 4. If A is a 3 × 3 matrix and $|A| = 2$, then $|3 \text{ adj}(|3A|A^2)|$ is equal to :

(1) $3^{12}.6^{10}$ **(2)** $3^{11}.6^{10}$ **(3)** $3^{12}.6^{11}$ **(4)** $3^{10}.6^{11}$

Q. 5. Let two vertices of a triangle ABC be (2, 4, 6) and (0, –2, –5), and its centroid be (2, 1, –1). If the image of the third vertex in the plane $x + 2y + 4z = 11$ is (α, β, γ), then $\alpha\beta + \beta\gamma + \gamma\alpha$ is equal to :

(1) 76 **(2)** 74 **(3)** 70 **(4)** 72

Q. 6. The negation of the statement:
$(p \vee q) \wedge (q \vee (\sim r))$ is

(1) $((\sim p) \vee r)) \wedge (\sim q)$
(2) $((\sim p) \vee (\sim q)) \wedge (\sim r)$
(3) $((\sim p) \vee (\sim q)) \vee (\sim r)$
(4) $(p \vee r) \wedge (\sim q)$

Q. 7. The shortest distance between the lines $\frac{x+2}{1} = \frac{y}{-2} = \frac{z-5}{2}$ and $\frac{x-4}{1} = \frac{y-1}{2} = \frac{z+3}{0}$ is:

(1) 8 **(2)** 7 **(3)** 6 **(4)** 9

Q. 8. If the coefficient of x^7 in $\left(ax - \frac{1}{bx^2}\right)^{13}$ and the coefficient of x^{-5} in $\left(ax + \frac{1}{bx^2}\right)^{13}$ are equal, then a^4b^4 is equal to :

(1) 22 **(2)** 44 **(3)** 11 **(4)** 33

Q. 9. A line segment AB of length λ moves such that the points A and B remain on the periphery of a circle of radius λ. Then the locus of the point, that divides the line segment AB in the ratio 2 : 3, is a circle of radius :

(1) $\frac{2}{3}\lambda$ **(2)** $\frac{\sqrt{19}}{7}\lambda$ **(3)** $\frac{3}{5}\lambda$ **(4)** $\frac{\sqrt{19}}{5}\lambda$

Q. 10. For the system of linear equations
$2x - y + 3z = 5$
$3x + 2y - z = 7$
$4x + 5y + \alpha z = \beta$,
which of the following is NOT correct ?

(1) The system in inconsistent for $\alpha = -5$ and $\beta = 8$
(2) The system has infinitely many solutions for $\alpha = -6$ and $\beta = 9$
(3) The system has a unique solution for $\alpha \neq -5$ and $\beta = 8$
(4) The system has infinitely many solutions for $\alpha = -5$ and $\beta = 9$

Q. 11. Let the first term a and the common ratio r of a geometric progression be positive integers. If the sum of squares of its first three is 33033, then the sum of these terms is equal to :

(1) 210 **(2)** 220 **(3)** 231 **(4)** 241

Q. 12. Let P be the point of intersection of the line $\frac{x+3}{3} = \frac{y+2}{1} = \frac{1-z}{2}$ and the plane $x + y + z = 2$. If the distance of the point P from the plane $3x - 4y + 12z = 32$ is q, then q and $2q$ are the roots of the equation :

(1) $x^2 + 18x - 72 = 0$ **(2)** $x^2 + 18x + 72 = 0$
(3) $x^2 - 18x - 72 = 0$ **(4)** $x^2 - 18x + 72 = 0$

Q. 13. Let f be a differentiable function such that $x^2 f(x) - x = 4\int_0^x tf(t)dt$, $f(1) = \frac{2}{3}$. Then $18 f(3)$ is equal to:

(1) 180 **(2)** 150 **(3)** 210 **(4)** 160

Q. 14. Let N denote the sum of the numbers obtained when two dice are rolled. If the probability that $2^N < N!$ is $\frac{m}{n}$. where m and n are coprime, then $4m - 3n$ equal to :

(1) 12 **(2)** 8 **(3)** 10 **(4)** 6

Q. 15. If $I(x) = \int e^{\sin^2 x} (\cos x \sin 2x - \sin x)\, dx$ and $I(0) = 1$, then $I\left(\frac{\pi}{3}\right)$ is equal to:

(1) $e^{\frac{3}{4}}$ **(2)** $-e^{\frac{3}{4}}$ **(3)** $\frac{1}{2}e^{\frac{3}{4}}$ **(4)** $-\frac{1}{2}e^{\frac{3}{4}}$

Q. 16. $96\cos\frac{\pi}{33}\cos\frac{2\pi}{33}\cos\frac{4\pi}{33}\cos\frac{8\pi}{33}\cos\frac{16\pi}{33}$

(1) 4 **(2)** 2 **(3)** 3 **(4)** 1

Q. 17. Let the complex number $z = x + iy$ be such that $\frac{2z-3i}{2z+i}$ is purely imaginary. If $x + y^2 = 0$, then $y^4 + y^2 - y$ is equal to:

(1) $\frac{3}{2}$ **(2)** $\frac{2}{3}$ **(3)** $\frac{4}{3}$ **(4)** $\frac{3}{4}$

Q. 18. If $f(x) = \frac{(\tan 1°)x + \log_e(123)}{x\log_e(1234) - (\tan 1°)}, x > 0$ then the least value of $f(f(x)) + f\left(f\left(\frac{4}{x}\right)\right)$ is:

(1) 2 **(2)** 4 **(3)** 8 **(4)** 0

Q. 19. The slope of tangent at any point (x, y) on a curve $y = y(x)$ is $\frac{x^2+y^2}{2xy}, x > 0$. If $y(2) = 0$, then a value of $y(8)$ is :

(1) $4\sqrt{3}$ **(2)** $-4\sqrt{2}$ **(3)** $-2\sqrt{3}$ **(4)** $2\sqrt{3}$

Q. 20. Let the ellipse $E : x^2 + 9y^2 = 9$ intersect the positive x-and y-axis at the points A and B respectively. Let the major axis of E be a diameter of the circle C. Let the line passing through A and B meet the circle C at the point P. If the area of the triangle with vertices A, P and the origin O is $\frac{m}{n}$, where m and n are coprime, then $m - n$ is equal to :

(1) 16 **(2)** 15 **(3)** 18 **(4)** 17

Section B

Q. 21. Some couples participated in a mixed doubles badminton tournament. If the number of matches played, so that no couple in a match, is 840, then the total numbers of persons, who participated in the tournament, is ____.

Q. 22. The number of elements in the set $\{n \in Z : |n^2 - 10n + 19| < 6\}$ is _____.

Q. 23. The number of permutations of the digits 1, 2, 3, …., 7 without repetition, which neither contain the string 153 nor the string 2467, is _____.

Q. 24. Let $f: (-2, 2) \to R$ be defined by

$$f(x) = \begin{cases} x[x], & -2 < x < 0 \\ (x-1)[x], & 0 \le x < 2 \end{cases}$$

where $[x]$ denotes the greatest integer function. If m and n respectively are the number of points in $(-2, 2)$ at which $y = |f(x)|$ is not continuous and not differentiable, then $m + n$ is equal to ______.

Q. 25. Let a common tangent to the curves $y^2 = 4x$ and $(x - 4)^2 + y^2 = 16$ touch the curves at the points P and Q. Then $(PQ)^2$ is equal to ____ :

Q. 26. If the mean of the frequency distribution

Class	0-10	10-20	20-30	30-40	40-50
Frequency	2	3	x	5	4

is 28, then its variance is _____.

Q. 27. The coefficient of x^7 in $(1 - x + 2x^3)^{10}$ is _________.

Q. 28. Let $y = p(x)$ be the parabola passing through the points $(-1, 0)$, $(0, 1)$ and $(1, 0)$. If the area of the region $\{(x, y): (x + 1)^2 + (y - 1)^2 \le p(x)\}$ is A, then $12(\pi - 4A)$ is equal to _____:

Q. 29. Let a, b, c be three distinct positive real numbers such that $(2a)^{\log_e a} = (bc)^{\log_e b}$ and $b^{\log_e 2} = a^{\log_e c}$. Then $6a + 5bc$ is equal to _____.

Q. 30. The sum of all those terms, of the arithmetic progression 3, 8, 13, …., 373, which are not divisible by 3, is equal to _____.

Answer Key

Q. No.	Answer	Topic name	Chapter name
1	(3)	Operation on vectors	Vector
2	(1)	Maxima and minima	Application of derivatives
3	(4)	Projection of a vector	Vector
4	(2)	Adjoint of a matrix	Matrix
5	(2)	Point and plane	3D
6	(1)	Compound statement	Mathematical reasoning
7	(4)	Shortest distance b/w lines	3D
8	(1)	General therm	Binomial theorem
9	(4)	Cosine rule	Properties of triangle
10	(4)	Elementary transformation	Matrix
11	(3)	Geometric progression	Sequence and series
12	(4)	Point and plane	3D
13	(4)	Linear differential equation	Differential equation
14	(2)	Binomial distribution	Probability

Q. No.	Answer	Topic name	Chapter name
15	(3)	Integration by parts	Indefinite integral
16	(3)	Cosine series	Trigonometry
17	(4)	Locus related problem	Complex number
18	(2)	Relation b/w A.M. & G.M.	Sequence and series
19	(1)	Homogeneous differential equation	Differential equation
20	(4)	Circle and ellipse	Ellipse
21	[16]	Combination	Permutation and combination
22	[9]	Modulus	Function
23	[4898]	Permutation	Permutation and combination
24	[4]	Differentiability	Continuity and differentiability
25	[32]	Circle and parabola	Parabola
26	[151]	Mean and variance	Statistics
27	[960]	Multinomial theorem	Binomial theorem
28	[16]	Area b/w two curves	Area under curves
29	[11]	Logarithmic indices	Basic mathematics
30	[9525]	Arithmetic progression	Sequence and series

Solutions

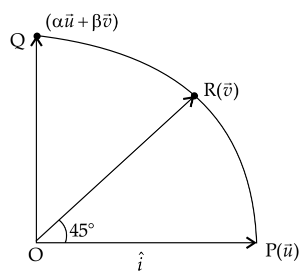
Section A

1. Option (3) is correct.

Since arc PQ subtends right angle at center, So it can be taken as

$\overrightarrow{OP} = \vec{u} = \hat{i}$ and

$\overrightarrow{OQ} = \alpha\vec{u} + \beta\vec{v} = \hat{j}$



Also R is mid point of arc PQ

So $\overrightarrow{OR} = \vec{v} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} \Rightarrow \vec{v} = \frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$

$\Rightarrow \sqrt{2}\,\vec{v} = \vec{u} + \alpha\vec{u} + \beta\vec{v}$ $\quad (\because \vec{u} = \hat{i}, \alpha\vec{u}+\beta\vec{v} = \hat{j})$

$\Rightarrow \sqrt{2}\,\vec{v} = (1+\alpha)\vec{u} + \beta\vec{v}$

On comparing, we get

$1 + \alpha = 0$ and $\beta = \sqrt{2}$

$\Rightarrow \alpha = -1$ and $\beta^2 = 2$

so α and β^2 are the roots of

$x^2 - (\alpha + \beta^2)x + \alpha \times \beta^2 = 0$

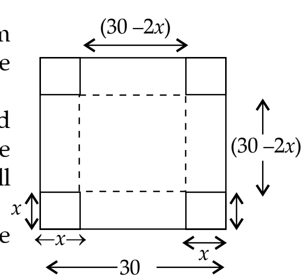
$\Rightarrow x^2 - x - 2 = 0$

2. Option (1) is correct.

Let the square of x cm length to be cut off for the maximum volume.

So, the length and breadth of the box will be $(30 - 2x)$ and height will be x cm.

If V be the volume of the box, So



$V = l \times b \times h$

$\Rightarrow V = x(30 - 2x)^2$

$$\frac{dV}{dx} = x \times 2(30-2x)(-2) + (30-2x)^2$$

$= (30 - 2x)\,[-4x + 30 - 2x]$

$$\frac{dV}{dx} = (30-2x)(30-6x)$$

For maxima or minima, putting $\frac{dV}{dx} = 0$

$\Rightarrow x = 15$ or $x = 5$

If $x = 15$ then $V = 0$ which is not possible and if $x = 5$, then

$$\frac{d^2V}{dx^2} = (30-2x)(-6) + (30-6x)(-2)$$

$= -180 + 12x - 60 + 12x = 24x - 240$

$$\left.\frac{d^2V}{dx^2}\right|_{x=5} = 120 - 240 = -120 < 0$$

$\Rightarrow$ V is maximum, when x is 5

$\Rightarrow$ therefore $l = b = 30 - 2x = 30 - 2 \times 5 = 20$

and $h = 5$

$\Rightarrow$ Surface area (without top) $= 2\,(lb + bh + hl) - lb$

$= 2\,(20 \times 20 + 20 \times 5 + 5 \times 20) - 20 \times 20 = 800$ cm^2

3. Option (4) is correct.

Given that $\overrightarrow{OP} = -\hat{i} - 2\hat{j} + 3\hat{k}, \overrightarrow{OA} = -2\hat{i} + \hat{j} - 3\hat{k}$

$\overrightarrow{OB} = 2\hat{i} + 4\hat{j} - 2\hat{k}$, and $\overrightarrow{OC} = -4\hat{i} + 2\hat{j} - \hat{k}$

Now $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 4\hat{i} + 3\hat{j} + \hat{k}$

and $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = -2\hat{i} + \hat{j} + 2\hat{k}$

and $\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i}&\hat{j}&k\\4&3&1\\-2&1&2\end{vmatrix}=5\hat{i}-10\hat{j}+10\hat{k}=\vec{a}$ (say)

Since $\overrightarrow{AB}\times\overrightarrow{AC}=\vec{a}$, which is perpendicular to both $\overrightarrow{AB}$ and $\overrightarrow{AC}$ therefore, projection of $\overrightarrow{OP}$ on $\vec{a}$ is given by $\dfrac{\overrightarrow{OP}.\vec{a}}{|\vec{a}|}$

$$\frac{\overrightarrow{OP}.\vec{a}}{|\vec{a}|}=\frac{(-\hat{i}-2\hat{j}+3\hat{k}).(5\hat{i}-10\hat{j}+10\hat{k})}{\sqrt{(5)^2+(-10)^2+(10)^2}}$$

$$=\frac{-5+20+30}{\sqrt{25+100+100}}=\frac{45}{15}=3$$

4. Option (2) is correct.

Given that A is a matrix of order 3 × 3 and $|A| = 2$

$\Rightarrow |3A| = 3^3|A|$ $\quad (\because |kA| = k^n|A|)$

$= 3^3 \times 2$

Now adj $(|3A|A^2)$ = adj $(3^3 \times 2\,A^2)$

$= (3^3 \times 2)^{3-1}(\text{adj } A)^2$

$= 3^6 \times 2^2(\text{adj } A)^2$

and $|3 \text{ adj } (|3A|\,A^2)| = |3 \times 3^6 \times 2^2(\text{adj } A)^2|$

$= (3^7 \times 2^2)^3\,|\text{adj } A|^2$

$= (3^7 \times 2^2)^3\,(|A|^2)^2$

$= 3^{21} \times 2^6 \times (2^2)^2$

$= 3^{11} \times 6^{10}$

5. Option (2) is correct.

Given that A(2, 4, 6), B (0, –2, –5) and centroid G(2, 1, –1)

Let C (x_1, y_1, z_1)

Since G is centroid of ΔABC

therefore

$$(2, 1, -1) \equiv \left(\frac{2+0+x_1}{3}, \frac{4-2+y_1}{3}, \frac{6-5+z_1}{3}\right)$$

$$\Rightarrow \frac{2+x_1}{3}=2, \frac{2+y_1}{3}=1, \frac{1+z_1}{3}=-1$$

$\Rightarrow x_1 = 4, y_1 = 1, z_1 = -4$

$\Rightarrow$ C (4, 1, –4)

Also given that the image of C (4, 1, –4) in the plane $x + 2y + 4z = 11$ is (α, β, γ)

$$\Rightarrow \frac{\alpha-4}{1}=\frac{\beta-1}{2}=\frac{\gamma+4}{4}=\frac{-2(4+2\times1+4(-4)-11)}{1^2+2^2+4^2}$$

$$=\frac{-2(-21)}{21}=2$$

$\Rightarrow \alpha = 6, \beta = 5, \gamma = 4$

and $\alpha\beta + \beta\gamma + \gamma\alpha$

$= 6 \times 5 + 5 \times 4 + 4 \times 6$

$= 30 + 20 + 24 = 74$

6. Option (1) is correct.

Negation of $(p \vee q)\ (q \vee (\sim r))$

$= \sim [(p \vee q) \wedge (q \vee (\sim r))]$

$= \sim (p \vee q) \vee (\sim(q \vee (\sim r))$

$= (\sim p \wedge \sim q) \vee (\sim q \wedge r)$

$= (\sim p \wedge \sim q) \vee (r \wedge \sim q)$

$= (\sim p \vee r) \wedge (\sim q)$

7. Option (4) is correct.

By using the formula of shortest distance

$$d=\frac{\begin{vmatrix}a_2-a_1&b_2-b_1&c_2-c_1\\l_1&m_1&n_1\\l_2&m_2&n_2\end{vmatrix}}{\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\l_1&m_1&n_1\\l_2&m_2&n_2\end{vmatrix}}$$

We get

$$d=\frac{\begin{vmatrix}4+2&1-0&-3-5\\1&-2&2\\1&2&0\end{vmatrix}}{\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&-2&2\\1&2&0\end{vmatrix}}=\frac{\left\|\begin{matrix}6&1&-8\\1&-2&2\\1&2&0\end{matrix}\right\|}{\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&-2&2\\1&2&0\end{vmatrix}}$$

$$\Rightarrow d=\frac{|6(0-4)-1(0-2)-8(2+2)|}{|\hat{i}(0-4)-\hat{j}(0-2)+\hat{k}(2+2)|}$$

$$=\frac{|-54|}{\sqrt{16+4+16}}=\frac{|-54|}{6}$$

$$\Rightarrow d=\frac{54}{6}=9$$

8. Option (1) is correct.

In the expansion of $\left(ax-\dfrac{1}{bx^2}\right)^{13}$

$$T_{r+1}={}^{13}C_r(ax)^{13-r}\left(-\frac{1}{bx^2}\right)^r$$

$$T_{r+1}={}^{13}C_r(a)^{13-r}\left(-\frac{1}{b}\right)^r(x)^{13-3r}$$

for the coefficient of x^7, putting $13 - 3r = 7$

$\Rightarrow r = 2$

So $T_{2+1}={}^{13}C_2(a)^{13-2}\left(-\dfrac{1}{b}\right)^2(x)^{13-6}$

$$={}^{13}C_2(a)^{11}\times\frac{1}{b^2}x^7$$

$\Rightarrow$ Coefficient of x^7 in $\left(ax-\dfrac{1}{bx^2}\right)^{13}$ is ${}^{13}C_2\dfrac{a^{11}}{b^2}$

Similarly, in the expansion of $\left(ax+\dfrac{1}{bx^2}\right)^{13}$

We have

$$T_{r+1}={}^{13}C_r(ax)^{13-r}\left(\frac{1}{bx^2}\right)^r$$

$$={}^{13}C_r(a)^{13-r}\left(\frac{1}{b}\right)^r x^{13-3r}$$

for the coefficient of x^{-5}, putting $13 - 3r = -5$

$\Rightarrow r = 6$

So $T_{6+1}={}^{13}C_6(a)^7\left(\dfrac{1}{b^6}\right)x^{-5}$

$\Rightarrow$ Coefficient of x^{-5} in $\left(ax+\dfrac{1}{bx^2}\right)^{13}$ is ${}^{13}C_6\dfrac{a^7}{b^6}$

Now according to the question

$${}^{13}C_2 \frac{a^{11}}{b^2} = {}^{13}C_6 \frac{a^7}{b^6}$$

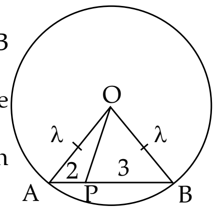
$$\Rightarrow a^4 b^4 = \frac{{}^{13}C_6}{{}^{13}C_2} = \frac{13!\,2!\,11!}{6!\,7!\,13!} = 22$$

9. Option (4) is correct.

Given that length of segment AB is λ and radius of circle is also λ.

Therefore ΔOAB must be equilateral.

Also in P divides AB is 2 : 3 then



$AP = \frac{2\lambda}{5}$

Now in ΔOAP,

by using cosine rule, we get

$$\cos A = \frac{OA^2 + AP^2 - OP^2}{2 \times OA \times AP}$$

$$\Rightarrow \cos 60° = \frac{\lambda^2 + \left(\frac{2\lambda}{5}\right)^2 - OP^2}{2 \times \lambda \times \frac{2\lambda}{5}}$$

$$\Rightarrow \frac{1}{2} = \frac{\lambda^2 + \frac{4}{25}\lambda^2 - OP^2}{\frac{4}{5}\lambda^2}$$

$$\Rightarrow \frac{2}{5}\lambda^2 = \frac{29}{25}\lambda^2 - OP^2$$

$$\Rightarrow OP^2 = \frac{19}{25}\lambda^2 \quad \text{or} \quad OP = \frac{\sqrt{19}}{5}\lambda$$

Hence, the locus of point P will be a circle of radius $\frac{\sqrt{19}}{5}\lambda$ units.

10. Option (4) is correct.

Since, Augmented matrix of given system of equation can be written as

$$[A : B] = \begin{bmatrix} 2 & -1 & 3 & : & 5 \\ 3 & 2 & -1 & : & 7 \\ 4 & 5 & \alpha & : & \beta \end{bmatrix}$$

Applying $R_1 \to R_1/2$

$$[A : B] \sim \begin{bmatrix} 1 & \frac{-1}{2} & \frac{3}{2} & : & \frac{5}{2} \\ 3 & 2 & -1 & : & 7 \\ 4 & 5 & \alpha & : & \beta \end{bmatrix}$$

Applying $R_2 \to R_2 - 3R_1$ and $R_3 \to R_2 - 4R_1$

$$\Rightarrow [A : B] \sim \begin{bmatrix} 1 & \frac{-1}{2} & \frac{3}{2} & : & \frac{5}{2} \\ 0 & \frac{7}{2} & \frac{-11}{2} & : & \frac{-1}{2} \\ 0 & 7 & \alpha - 6 & : & \beta - 10 \end{bmatrix}$$

Applying $R_3 \to R_3 - 2R_2$

$$\Rightarrow [A : B] \sim \begin{bmatrix} 1 & \frac{-1}{2} & \frac{3}{2} & : & \frac{5}{2} \\ 0 & \frac{7}{2} & \frac{-11}{2} & : & \frac{-1}{2} \\ 0 & 0 & \alpha + 5 & : & \beta - 9 \end{bmatrix}$$

So, the given system of equation have

(i) unique solution, if

Rank of A = Rank of [A : B] = 3

$\Rightarrow \alpha \neq -5$ and $\beta \in R$

(ii) Infinite solution, if

Rank of A = Rank of [A : B] < 3 i.e., 2

$\Rightarrow \alpha = -5$ and $\beta = 9$

(iii) Inconsistent, if

Rank of A < Rank of [A : B]

$\Rightarrow \alpha = -5$ and $\beta \neq 9$

11. Option (3) is correct.

Let the 3 terms of G.P are a, ar, ar^2 then according to the question, we have

$(a)^2 + (ar)^2 + (ar^2)^2 = 33033$

$\Rightarrow a^2 (1 + r^2 + r^4) = 11^2 \times (3 \times 7 \times 13)$

Since, a and r are positive integers, so by comparing

We have $a = 11$ and $1 + r^2 + r^4 = 3 \times 7 \times 13$

$\Rightarrow r^4 + r^2 - 272 = 0$

$\Rightarrow (r^2 - 16)(r^2 + 17) = 0$

$\Rightarrow r^2 = 16$ or $r^2 = -17$ (Not possible)

$\Rightarrow r = 4$ ($\because$ r is positive integer)

So, the numbers are $11, 11 \times 4, 11 \times 4^2$

or 11, 44, 176

and sum of these numbers = 11 + 44 + 176 = 231

12. Option (4) is correct.

To find point P, let

$$\frac{x+3}{3} = \frac{y+2}{1} = \frac{1-z}{2} = k \text{ (say)}$$

$\Rightarrow x = 3k - 3, y = k - 2, z = 1 - 2k$

So P $(3k - 3, k - 2, 1 - 2k)$ lie on $x + y + z = 2$

$\Rightarrow 3k - 3 + k - 2 + 1 - 2k = 2$

$\Rightarrow 2k = 6$

or $k = 3$

So point P will be P $(3 \times 3 - 3, 3 - 2, 1 - 2 \times 3)$

or P (6, 1, –5)

Also it is given that the distance of point P (6, 1, –5) from the plane $3x - 4y + 12z = 32$ is q

$$\Rightarrow q = \frac{|3 \times 6 - 4 \times 1 - 12 \times 5 - 32|}{\sqrt{(3)^2 + (-4)^2 + (12)^{12}}}$$

$$= \frac{|18 - 4 - 60 - 32|}{\sqrt{9 + 16 + 144}} = \frac{78}{13}$$

$\Rightarrow q = 6$ and $2q = 12$

So the required equation having roots q and $2q$ is $x^2 - (q + 2q)x + q \times 2q = 0$

$\Rightarrow x^2 - 18x + 72 = 0$

13. Option (4) is correct.

$x^2 f(x) - x = 4\int_0^x t\,f(t)\,dt$

Differentiating with respect to x, we get

$x^2 f'(x) + 2x f(x) - 1 = 4x f(x) \times 1 - 0$

$\Rightarrow x^2 f'(x) - 2x f(x) - 1 = 0$

or $f'(x) - \frac{2}{x} f(x) = \frac{1}{x^2}$

or $\frac{dy}{dx} - \frac{2}{x} y = \frac{1}{x^2}$ (as $y = f(x)$ and $\frac{dy}{dx} = f'(x)$)

which is a linear differential equation where

$P = \frac{-2}{x}, Q = \frac{1}{x^2}$

$\Rightarrow$ I.F. $= e^{\int P\,dx} = e^{\int \frac{-2}{x} dx} = e^{-2\log x} = \frac{1}{x^2}$

So, the required solution is

$y \times$ I.F. $= \int Q \times \text{I.F}\,dx + C$

or $y \times \frac{1}{x^2} = \int \frac{1}{x^2} \times \frac{1}{x^2} dx + C$

$\frac{y}{x^2} = \int \frac{1}{x^4} dx + C$

$\frac{y}{x^2} = -\frac{1}{3x^3} + C$

or $f(x) = -\frac{1}{3x} + Cx^2$

but $f(1) = \frac{2}{3}$

$\Rightarrow \frac{2}{3} = -\frac{1}{3} + C \times 1^2 \Rightarrow C = 1$

so $f(x) = -\frac{1}{3x} + x^2$

and $f(3) = -\frac{1}{9} + 9 = +\frac{80}{9}$

$\Rightarrow 18 f(3) = \frac{18 \times 80}{9} = 160$

14. Option (2) is correct.

Since, we know that $2^N < N!$ is satisfied only when N ≥ 4

therefore, required probability can be written as-

$P(N \geq 4) = 1 - P(N < 4)$ ...(i)

But, it is given that N denotes the sum of numbers obtained when two dice are rolled,

So when N = 1 then it is not possible

When N = 2 then only the case is possible i.e., (1, 1)

$\Rightarrow P(N = 2) = \frac{1}{36}$

When N = 3, then two cases are possible i.e., (1, 2), (2, 1)

$\Rightarrow P(N = 3) = \frac{2}{36}$

So P (N < 4) = P (N = 1) + P (N = 2) + P (N = 3)

$= 0 + \frac{1}{36} + \frac{2}{36} = \frac{3}{36} = \frac{1}{12}$

Now from equation (i), we have

$P(N \geq 4) = 1 - P(N < 4)$

$= 1 - \frac{1}{12}$

$P(N \geq 4) = \frac{11}{12} = \frac{m}{n}$ (given)

$\Rightarrow m = 11$ and $n = 12$

and $4m - 3n = 4 \times 11 - 3 \times 12$

$= 44 - 36 = 8$

15. Option (3) is correct.

$I = \int e^{\sin^2 x}(\cos x \sin 2x - \sin x)dx$

$I = \int \underbrace{e^{\sin^2 x} \sin 2x}_{II} \underset{I}{\cos x}dx - \int e^{\sin^2 x} \sin x\,dx$

$I = \cos \times \int e^{\sin^2 x} \sin 2x dx - \int((-\sin x).$

$\int e^{\sin^2 x} \sin 2x dx)dx - \int e^{\sin^2 x} \sin x\,dx + C$

Let $t = \sin^2 x$

$\Rightarrow dt = 2\sin x \cos x\,dx = \sin 2x\,dx$

So I $= \cos \int e^t dt + \int (\sin x \int e^t dt)dx -$

$\int e^{\sin^2 x} \sin x\,dx + C$

$\Rightarrow I = \cos e^t + \int \sin x\,e^t dx - \int e^{\sin^2 x} \sin x\,dx + C$

$\Rightarrow I = \cos e^{\sin^2 x} + \int \sin x\,e^{\sin^2 x} dx - \int e^{\sin^2 x} \sin dx + C$

$\Rightarrow I = \cos x.e^{\sin^2 x} + C$

but I(0) = 1

$\Rightarrow 1 = \cos 0 \times e^{\sin^2 0} + C$

$\Rightarrow 1 = 1 \times e^0 + C \Rightarrow C = 0$

so I $= \cos x.e^{\sin^{2x}}$

and $I\left(\frac{\pi}{3}\right) = \cos\frac{\pi}{3} e^{\sin^2\left(\frac{\pi}{3}\right)}$

$I\left(\frac{\pi}{3}\right) = \frac{1}{2} \times e^{\frac{3}{4}}$

16. Option (3) is correct.

Since, cosA. cos2A. $\cos 2^2$A ...... $\cos 2^{n-1} A = \frac{\sin(2^n A)}{2^n \sin A}$

therefore,

$96 \cos\frac{\pi}{33} \cos\frac{2\pi}{33} \cos\frac{4\pi}{33} \cos\frac{8\pi}{33} \cos\frac{16\pi}{33}$

$= 96 \cos\frac{\pi}{33} \cos 2\left(\frac{\pi}{33}\right) \cos 2^2\left(\frac{\pi}{33}\right) \cos 2^3\left(\frac{\pi}{33}\right) \cos 2^4\left(\frac{\pi}{33}\right)$

$= 96 \times \frac{\sin\left(2^5\left(\frac{\pi}{33}\right)\right)}{2^5 \sin\left(\frac{\pi}{33}\right)}$

$= \frac{96 \sin\left(\frac{32\pi}{33}\right)}{32 \sin\left(\frac{\pi}{33}\right)}$

$= \frac{\overset{3}{\cancel{96}} \sin\left(\pi - \frac{\pi}{33}\right)}{\cancel{32} \sin\left(\frac{\pi}{33}\right)} = \frac{3 \sin\left(\frac{\pi}{33}\right)}{\sin\left(\frac{\pi}{33}\right)} = 3$

17. Option (4) is correct.

If $z = x + iy$, then

$$\frac{2z-3i}{2z+i}=\frac{2x+2iy-3i}{2x+2iy+i}$$

$$\frac{2x+i(2y+3)}{2x+i(2y+1)}\times\frac{2x-i(2y+1)}{2x-i(2y+1)}$$

$$\frac{2z-3i}{2z+i}=\left(\frac{4x^2+(2y-3)(2y+1)}{4x^2+(2y+1)^2}\right)+ i\left(\frac{2x(2y-3)-2x(2y+1)}{4x^2+(2y+1)^2}\right)$$

Now $\frac{2z-3i}{2z+i}$ is purely imaginary if

$$\frac{4x^2+(2y-3)(2y+1)}{4x^2+(2y+1)^2}=0$$

$\Rightarrow 4x^2+4y^2-4y-3=0$

but $x+y^2=0$ is given

so $x=-y^2$ and $x^2=y^4$

$\Rightarrow 4y^4+4y^2-4y-3=0$

or $y^4+y^2-y=\frac{3}{4}$

18. Option (2) is correct.

Since, tan i, $\log_e 123$ and $\log_e 1234$ are constants, so let

$a=\tan 1^\circ$, $b=\log_e 123$ and $c=\log_e 1234$

So $f(x)=\frac{(\tan 1^\circ)x+\log_e 123}{x\log_e(1234)-(\tan 1^\circ)}=\frac{ax+b}{cx-a}$

Now $f(f(x))=\frac{a\left(\frac{ax+b}{cx-a}\right)+b}{c\left(\frac{ax+b}{cx-a}\right)-a}=\frac{a^2x+ab+bcx-ab}{acx+bc-acx+a^2}$

$=\frac{x(a^2+bc)}{(a^2+bc)}=x$

and $f\left(f\left(\frac{4}{x}\right)\right)=\frac{4}{x}$

So using A.M. ≥ G.M.

$$\frac{f(f(x))+f\left(f\left(\frac{4}{x}\right)\right)}{2}\geq\sqrt{f(f(x))\times f\left(f\left(\frac{4}{x}\right)\right)}$$

$$\Rightarrow f(f(x))+f\left(f\left(\frac{4}{x}\right)\right)\geq 2\sqrt{x\times\frac{4}{x}}=4$$

Hence least value of $f(f(x))+f\left(f\left(\frac{4}{x}\right)\right)$ is 4

19. Option (1) is correct.

Given that, slope of tangent at any point is $\frac{x^2+y^2}{2xy}$

$\Rightarrow \frac{dy}{dx}=\frac{x^2+y^2}{2xy}$

which is homogeneous differential eqn.,

so putting $y=vx$ and $\frac{dy}{dx}=v+x\frac{dv}{dx}$, we get

$$v+x\frac{dv}{dx}=\frac{x^2+v^2x^2}{2x\times vx}=\frac{1+v^2}{2v}$$

$$x\frac{dv}{dx}=\frac{1+v^2}{2v}-v=\frac{1+v^2-2v^2}{2v}$$

or $\int\frac{2v\,dv}{1-v^2}=\int\frac{dx}{x}$

$-\log(1-v^2)=\log x+\log C$

or $\frac{1}{1-v^2}=Cx$

or $\frac{x^2}{x^2-y^2}=Cx$

or $x=c(x^2-y^2)$

but, it is given that $y(2)=0$

$\Rightarrow 2=C(4-0)\Rightarrow C=\frac{1}{2}$

so $x=\frac{1}{2}(x^2-y^2)$ or $2x=x^2-y^2$

or $y=\sqrt{x^2-2x}$

Putting $x=8$

$y(8)=\sqrt{8^2-2\times 8}=\sqrt{64-16}=\sqrt{48}\Rightarrow y(8)=4\sqrt{3}$

20. Option (4) is correct.

Given that $x^2+9y^2=9$

$\Rightarrow \frac{x^2}{3^2}+\frac{y^2}{1^2}=1$

which represent an ellipse whose major axis is 3 and minor axis is of length l.

So, the eqn. of circle is $x^2+y^2=3^2$

or $x^2+y^2=9$ ...(i)

and eqn. of line through A(3, 0), B (0, 1) is $\frac{x}{3}+\frac{y}{1}=1$

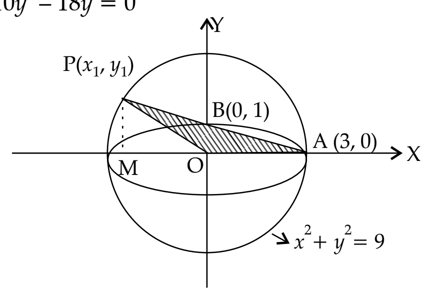
or $x=3(1-y)$

Now to find point P (x_1, y_1), putting the value of $x=3(1-y)$ in eqn. (i), we get

$9(1-y)^2+y^2=9$

$9+9y^2-18y+y^2=9$

$\Rightarrow 10y^2-18y=0$



$\Rightarrow y=\frac{9}{5}$ ($\because$ $y=0$ is not possible)

and $x=3(1-y)=3\left(1-\frac{9}{5}\right)=3\left(\frac{-4}{5}\right)$

$\Rightarrow x=\frac{-12}{5}$

So, $P(x_1, y_1)\equiv P\left(\frac{-12}{5},\frac{9}{5}\right)$

Area of ΔOAP = $\frac{1}{2}$ × Base × Height

$=\frac{1}{2}\times OA\times PM=\frac{1}{2}\times 3\times\frac{9}{5}=\frac{27}{10}=\frac{m}{n}$

$\Rightarrow m=27$ and $n=10$

and $m-n=17$

Section B

21. Correct answer is [16].

Let number of complex be n

So according to the question:

$^nC_2 \times {}^{n-2}C_2 \times 2 = 840$

$\Rightarrow \frac{n(n-1)}{2} \times \frac{(n-2)(n-3)}{2} \times 2 = 840$

or $n(n-1)(n-2)(n-3) = 840 \times 2 = 8 \times 7 \times 6 \times 5$

On comparing, we get $n = 8$

and total number of persons = $2n = 16$

22. Correct answer is [9].

Given that $\{n \in Z : |n^2 - 10n + 19| < 6\}$

$\Rightarrow |n^2 - 10n + 19| < 6 \ \forall \ n \in Z$

or $-6 < n^2 - 10n + 19 < 6$

$\Rightarrow n^2 - 10n + 19 > -6$ and $n^2 - 10n + 19 < 6$

$\Rightarrow n^2 - 10n + 25 > 0$ and $n^2 - 10n + 13 < 0$

$\Rightarrow (n-5)^2 > 0$ and $n \in z$

where α, β are the roots of $n^2 - 10n + 13 = 0$

$\Rightarrow n = \frac{10 \pm \sqrt{100-52}}{2} = \frac{10 \pm 4\sqrt{3}}{2}$

$\Rightarrow \alpha = 5 - 2\sqrt{3}$ and $\beta = 5 + 2\sqrt{3}$

$\Rightarrow n \in Z - \{5\}$ and $n \in (5 - 2\sqrt{3}, 5 + 2\sqrt{3})$

$\Rightarrow n \in Z - \{5\}$ and $n \in (-1.52, 8.46)$

$n \in \{-1, 0, 1, 2, ..., 8\}$

So, common integers are : –1, 0, 1, 2, 3, 4, 6, 7, 8

$\Rightarrow$ Hence, no. of elements is set are 9

23. Correct answer is [4898].

Given number are 1, 2, 3, 4, 5, 6, 7

So, total number of permutation are 7! = 5040.

Now total number of permutation having string (153) = (153), 2, 4, 6, 7

$\Rightarrow n(153) = 5! = 120$

and total number of permutation having string (2467) = (2467), 1, 3, 5

$\Rightarrow n(2467) = 4! = 24$

also total permutation having string (153) and (2467) $n(153 \cap 2467) = 2! = 2$

Now $n(153 \cup 2467) = 120 + 24 - 2 = 142$

So, the required no. of permtuations are

$n(\overline{153} \cap \overline{2467}) = n(\overline{153 \cup 2467})$

= Total – $n(153 \cup 2467)$

= 5040 – 142 = 4898

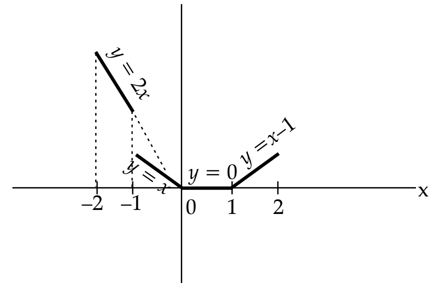
24. Correct answer is [4].

$f : (-2, 2) \to R$

$$f(x) = \begin{cases} x[x], & -2 < x < 0 \\ (x-1)[x], & 0 \le x < 2 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} x(-2), -2 < x < -1 \\ x(-1), -1 \le x < 0 \\ (x-1) \times 0, 0 \le x < 1 \\ (x-1) \times 1, 1 \le x < 2 \end{cases} = \begin{cases} -2x, -2 < x < -1 \\ -x, -1 \le x < 0 \\ 0, -0 \le x < 1 \\ x-1, 1 \le x < 2 \end{cases}$$

which can be plotted as



from the graph, its clear that $f(x)$ is discontinous at 1 point and non differentiable at 3 points

So $m = 1$ and $n = 3$

and $m + n = 4$

25. Correct answer is [32].

Given parabola is $y^2 = 4 \times 1x$

where $a = 1$,

whose tangent is $y = mx + \frac{a}{m}$ i.e. $y = mx + \frac{1}{m}$ ...(i)

and point of contact is $Q\left(\frac{a}{m^2}, \frac{2a}{m}\right)$ i.e., $Q\left(\frac{1}{m^2}, \frac{2}{m}\right)$

Since, eqn. (i) is also a tangent of circle $(x-4)^2 + y^2 = 16$.

So,

$$\frac{\left|m \times 4 + \frac{1}{m}\right|}{\sqrt{m^2+1}} = 4$$

$$\Rightarrow \left|\frac{4m^2+1}{m}\right| = 4\sqrt{m^2+1}$$

$\Rightarrow 16m^4 + 1 + 8m^2 = 16m^4 + 16m^2$

$\Rightarrow 8m^2 = 1$ or $m = \frac{1}{2\sqrt{2}}$

So $Q(8, 4\sqrt{2})$

and PQ = $\sqrt{S_1} \Rightarrow (PQ)^2 = S_1$

$= (8-4)^2 + (4\sqrt{2})^2 - 16 = 32$

26. Correct answer is [151].

Given that mean is $\bar{x} = 28$

Class	f_i	x_i	$f_i x_i$	x_i^2	$f_x x_i^2$
0-10	2	5	10	25	50
10-20	3	15	45	225	675
20-30	x	25	$25x$	625	3750
30-40	5	35	175	1225	6125
40-50	4	45	180	2025	8100
$\Sigma f_i = 14 + x$			$\Sigma f_i x_i = 410 + 25x$		18700

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$28 = \frac{410 + 25x}{14 + x}$$

$\Rightarrow x = 6$

and variance = $\frac{1}{N}\Sigma f_i x_i^2 - (\bar{x})^2$

$= \frac{1}{20} \times 18700 - (28)^2$

$= 935 - 784 = 151$

27. Correct answer is [960].

Let the general term in the expansion of $(1 - x + 2x^3)^{10}$ is given by

$T_n = \frac{10!}{a!b!c!}(1)^a(-x)^b(2x^3)^c$

or $T_n = \frac{10!}{a!b!c!}(-1)^b \times 2^c \times x^{b+3c}$

where $a + b + c = 10$ ....(i)

and $b + 3c = 7$ ...(ii)

a	b	c
3	7	0
5	4	1
7	1	2

So, the coefficient of x^7 is

$= \frac{10!}{3!7!0!} \times (-1)^7 \times 2^0 + \frac{10!}{5!4!1!} \times (-1)^4 \times 2^1 + \frac{10!}{7!1!2!} \times (-1)^1 \times 2^2$

$= \frac{10 \times 9 \times 8}{6}(-1) + \frac{10 \times 9 \times 8 \times 7 \times 6 \times 2}{4 \times 3 \times 2 \times 1} + \frac{10 \times 9 \times 8}{2 \times 1}(-1) \times 4$

$= -120 + 2520 - 1440 = 960$

28. Correct answer is [16].

Let the equation of parabols is

$x^2 = -4a(y - 1)$ which pass through (1, 0)

$\Rightarrow 1 = -4a(a - 1) \Rightarrow a = \frac{1}{4}$

so $x^2 = -4 \times \frac{1}{4}(y - 1)$

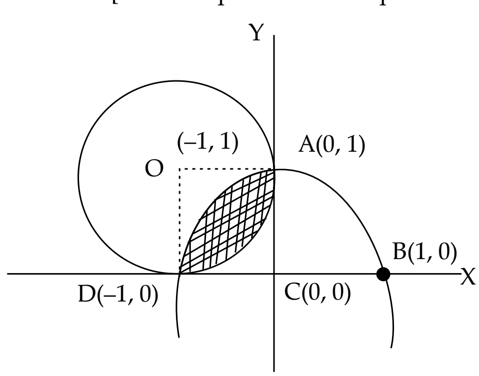
or $x^2 = -(y - 1)$ ...(i)

and $\{(x, y) : (x + 1)^2 + (y - 1)^2 \le 1, y \le p(x)\}$

$\Rightarrow (x + 1)^2 + (y - 1)^2 \le 1$ represents interior part of the circle.

So required area is

$A = \int_{-1}^{0} y_{\text{parabola}}\,dx -$ [Area of square – area of quarter of circle]



$A = \int_{-1}^{0}(1 - x^2)dx - \left[1 \times 1 - \frac{\pi \times 1^2}{4}\right]$

$= \left[x - \frac{x^3}{3}\right]_{-1}^{0} - 1 + \frac{\pi}{4}$

$A = 0 - \left(-1 + \frac{1}{3}\right) - 1 + \frac{\pi}{4}$

$A = \frac{\pi}{4} - \frac{1}{3}$

or $12A = 3\pi - 4$

or $48A = 12\pi - 16$

or $12(\pi - 4A) = 16$

29. Correct answer is [11].

Given that $(2a)^{\log_e a} = (bc)^{\log_e b}$

$\Rightarrow \log_e a(\log_e 2 + \log_e a) = \log_e b(\log_e b + \log_e c)$ ...(i)

and $b^{\log_e 2} = a^{\log_e c}$

$\Rightarrow \log_e 2 \times \log_e b = \log_e c \,.\, \log_e a$

$\Rightarrow \log_e 2 = \frac{\log_e c . \log_e a}{\log_e b}$

Putting in equation (i), we get

$\log_e a(\log_e c . \log_e a + \log_e a . \log_e b)$

$= (\log_e b)^2(\log_e b + \log_e c)$

$(\log_e a)^2(\log_e b + \log_e c) - (\log_e b)^2(\log_e b + \log_e c) = 0$

$\Rightarrow \log_e bc\,\{(\log_e a)^2 - (\log_e b)^2\} = 0$

$\Rightarrow \log_e bc = 0$ and $\log_e a = \log_e b$

$\Rightarrow bc = 1$ and $ab = 1$

If $bc = 1$ then $(2a)^{\log_e a} = (bc)^{\log_e b} = (1)^{\log_e b} = 1$

$\Rightarrow (2a)^{\log_e a} = 1$

$\Rightarrow a = 1$ or $a = \frac{1}{2}$

Now if $a = 1$ and $bc = 1$ then $6a + 5bc = 11$

and if $a = \frac{1}{2}$ and $bc = 1$ then $6a + 5bc = 8$

30. Correct answer is (9525).

Given A.P. is 3, 8, 13, ..., 373

Using $T_n = a + (n - 1)d$, we get

$373 = 3 + (n - 1)5$

$\Rightarrow n = 75$

Sum of complete AP $= \frac{n}{2}(a + l) = \frac{75}{2}(3 + 373)$

$= 14100$

Now the numbers divisible by 3 are 3, 18, 33, ..., 363

Using $T_n = a + (k - 1)d$, we get

$363 = 3 + (k - 1)15$

$\Rightarrow k = 25$

So sum of this A.P. $= \frac{k}{2}(a + l) = \frac{25}{2}(3 + 363) = 4575$

Hence, the required sum is $14100 - 4575 = 9525$

❑❑