JEE (Main) MATHEMATICS SOLVED PAPER

2023
10th April Shift 2

Section A

Q. 1. If the coefficients of x and x^2 in $(1 + x)^p (1- x)^q$ are 4 and – 5 respectively, then $2p + 3q$ is equal to
(1) 60 **(2)** 63 **(3)** 66 **(4)** 69

Q. 2. Let A = {2, 3, 4} and B = {8, 9, 12}. Then the number of elements in the relation $R = \{((a_1, b_1), (a_2, b_2)) \in (A \times B, A \times B) : a_1 \text{ divides } b_2 \text{ and } a_2 \text{ divides } b_1\}$ is
(1) 18 **(2)** 24 **(3)** 12 **(4)** 36

Q. 3. Let time image of the point P(1, 2, 6) in the plane passing through the points A(1, 2, 0), B (1, 4, 1) and C (0, 5, 1) be Q (α, β, γ). Then $(\alpha^2 + \beta^2 + \gamma^2)$ is equal to
(1) 70 **(2)** 76 **(3)** 62 **(4)** 65

Q. 4. The statement $\sim[p \vee (\sim (p \wedge q))]$ is equivalent to
(1) $(\sim(p \wedge q)) \wedge q$ **(2)** $\sim (p \vee q)$
(3) $\sim (p \wedge q)$ **(4)** $(p \wedge q) \wedge (\sim p)$

Q. 5. Let $S = \left\{x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) : 9^{1-\tan^2 x} + 9^{\tan^2 x} = 10\right\}$

$b = \sum_{x \in S} \tan^2\left(\frac{x}{3}\right)$, then $\frac{1}{6}(b-14)^2$ is equal to
(1) 16 **(2)** 32 **(3)** 8 **(4)** 64

Q. 6. If the points P and Q are respectively the circumcenter and the orthocentre of a ΔABC, the $\overline{PA} + \overline{PB} + \overline{PC}$ is equal to

(1) $2\overline{QP}$ **(2)** $\overline{PQ}$ **(3)** $2\overline{PQ}$ **(4)** $\overline{PQ}$

Q. 7. Let A be the point (1,2) and B be any point on the curve $x^2 + y^2 = 16$. If the centre of the locus of the point P, which divides the line segment AB in the ratio 3 : 2 is the point C (α, β) then the length of the line segment AC is

(1) $\frac{6\sqrt{5}}{5}$ **(2)** $\frac{2\sqrt{5}}{5}$ **(3)** $\frac{3\sqrt{5}}{5}$ **(4)** $\frac{4\sqrt{5}}{5}$

Q. 8. Let μ be the mean and σ be the standard deviation of the distribution.

x_i	0	1	2	3	4	5
f_i	$k+2$	$2k$	k^2-1	k^2-1	k^2+1	$k-3$

where $\Sigma f_i = 62$. If $[x]$ denotes the greatest integer $\leq x$, then $[\mu^2 + \sigma^2]$ is equal to
(1) 8 **(2)** 7 **(3)** 6 **(4)** 9

Q. 9. If $S_n = 4 + 11 + 21 + 34 + 50 + \ldots$ to n terms, then $\frac{1}{60}(S_{29} - S_9)$ is equal to
(1) 220 **(2)** 227 **(3)** 226 **(4)** 223

Q. 10. Eight persons are to be transported from city A to city B in three cars different makes. If each car can accomodate at most three persons, then the number of ways, in which they can be transported, is
(1) 1120 **(2)** 560 **(3)** 3360 **(4)** 1680

Q. 11. If $A = \frac{1}{5!6!7!}\begin{bmatrix} 5! & 6! & 7! \\ 6! & 7! & 8! \\ 7! & 8! & 9! \end{bmatrix}$, then $|\text{adj}(\text{adj}(2A))|$ is equal to
(1) 2^{16} **(2)** 2^8 **(3)** 2^{12} **(4)** 2^{20}

Q. 12. Let the number $(22)^{2022} + (2022)^{22}$ leave the remainder α when divided by 3 and β when divided by 7. Then $(\alpha^2 + \beta^2)$ is equal to
(1) 13 **(2)** 20 **(3)** 10 **(4)** 5

Q. 13. Let $g(x) = f(x) + f(1-x)$ and $f''(x) > 0, x \in (0,1)$. If g is decreasing in the interval $(0, \alpha)$ and increasing in the interval $(\alpha,1)$, then $\tan^{-1}(2\alpha) + \tan^{-1}\left(\frac{\alpha+1}{\alpha}\right)$ is equal to

(1) $\frac{5\pi}{4}$ **(2)** π **(3)** $\frac{3\pi}{4}$ **(4)** $\tan^{-1}(-2)$

Q. 14. For $\alpha, \beta, \gamma, \delta \in N$, if $\int\left(\left(\frac{x}{e}\right)^{2x} + \left(\frac{e}{x}\right)^{2x}\right)$

$= \frac{1}{\alpha}\left(\frac{x}{e}\right)^{\beta x} - \frac{1}{\gamma}\left(\frac{e}{x}\right)^{\delta x} + C$, where $e = \sum_{n=0}^{\infty}\frac{1}{n!}$ and C is constant of integration, then $\alpha + 2\beta + 3\gamma - 4\delta$ is equal to
(1) 4 **(2)** –4 **(3)** –8 **(4)** 1

Q. 15. Let f be a continuous function satisfying

$\int_0^{t^2}(f(x) + x^2)dx = \frac{4}{3}t^3, \ \forall\ t > 0$. Then $f\left(\frac{\pi^2}{4}\right)$ is equal to

(1) $-\pi^2\left(1 + \frac{\pi^2}{16}\right)$ **(2)** $\pi\left(1 - \frac{\pi^3}{16}\right)$

(3) $-\pi\left(1 + \frac{\pi^3}{16}\right)$ **(4)** $\pi^2\left(1 - \frac{\pi^3}{16}\right)$

Q. 16. Let a dice be rolled n times. Let the probability of getting odd numbers seven times be equal to the probability of getting odd numbers nine times. If the probability of getting even numbers twice is $\frac{k}{2^{15}}$, then k is equal to

(1) 60 **(2)** 30 **(3)** 90 **(4)** 15

Q. 17. Let a circle of radius 4 be concentric to the ellipse $15x^2 + 19y^2 = 285$. Then the common tangents are inclined to the minor axis of the ellipse at the angle.

(1) $\frac{\pi}{6}$ **(2)** $\frac{\pi}{12}$ **(3)** $\frac{\pi}{3}$ **(4)** $\frac{\pi}{4}$

Q. 18. Let $\vec{a} = 2\hat{i} + 7\hat{j} - \hat{k}, \vec{b} = 3\hat{i} + 5\hat{k}$ and $\vec{c} = \hat{i} - \hat{j} + 2\hat{k}$, Let $\vec{d}$ be a vector which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c}.\vec{d} = 12$. The $(-\hat{i} + \hat{j} - \hat{k}).(\vec{c} \times \vec{d})$ is equal to

(1) 24 **(2)** 42 **(3)** 48 **(4)** 44

Q. 19. Let $S = \left\{z = x + iy : \frac{2z - 3i}{4z + 2i} \text{ is a real number}\right\}$.

Then which of the following is NOT correct ?

(1) $y \in \left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$

(2) $(x, y) = \left(0, -\frac{1}{2}\right)$

(3) $x = 0$

(4) $y + x^2 + y^2 \neq -\frac{1}{4}$

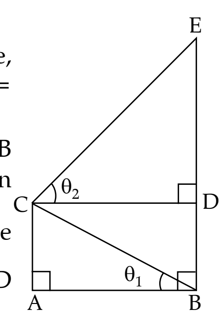
Q. 20. Let the line $\frac{x}{1} = \frac{6-y}{2} = \frac{z+8}{5}$ intersect the lines $\frac{x-5}{4} = \frac{y-7}{3} = \frac{z+2}{1}$ and $\frac{x+3}{6} = \frac{3-y}{3} = \frac{z-6}{1}$ at the points A and B respectively. Then the distance of the mid-point of the line segment AB from the plane $2x - 2y + z = 14$ is

(1) 3 **(2)** $\frac{10}{3}$ **(3)** 4 **(4)** $\frac{11}{3}$

Section B

Q. 21. The sum of all the four-digit numbers that can be formed using all the digits 2, 1, 2, 3 is equal to _______.

Q. 22. In the figure, $\theta_1 + \theta_2 = \frac{\pi}{2}$ and $\sqrt{3}$ (BE) = 4 (AB). If the area of ΔCAB is $2\sqrt{3} - 3$ unit^2, when $\frac{\theta_2}{\theta_1}$ is the largest, then the perimeter (in unit) of ΔCED is equal to _______.



Q. 23. Let the tangent at any point P on a curve passing through the points (1,1) and $\left(\frac{1}{10}, 100\right)$, intersect positive x-axis and y-axis at the points A and B respectively . If PA : PB =1 : k and $y = y(x)$ is the solution of the differential equation $e^{\frac{dy}{dx}} = kx + \frac{k}{2}, y(0) = k$, then 4$y$ then 4y(1) – 5$\log e^3$ is equal to_______.

Q. 24. Suppose a_1, a_2, 2, a_3, a_4 be in an arithemetico-geometric progression. If the common ratio of the corresponding geometric progression is 2 and the sum of all 5 terms of the arithmetico-geometric progression is $\frac{49}{2}$, then a_4 is equal to_______.

Q. 25. If the area of the region $\{(x, y) : |x^2 - 2| \leq x\}$ is A, then 6A + $16\sqrt{2}$ is equal to _____.

Q. 26. Let the foot of perpendicular from the point A (4, 3, 1) on the plane P : $x - y + 2z + 3 = 0$ be N. If B(5, α, β), $\alpha, \beta \in Z$ is a point on plane P such that the area of the triangle ABN is $3\sqrt{2}$, then $\alpha^2 + \beta^2 + \alpha\beta$ is equal to _______.

Q. 27. Let S be the set of values of λ, for which the system of equations

$6\lambda x - 3y + 3z = 4\lambda^2$,

$2x + 6\lambda y + 4z = 1$,

$3x + 2y + 3\lambda z = \lambda$ has no solution. Then $12\sum_{1 \in S} |\lambda|$ is equal to _______.

Q. 28. If the domain of the function $f(x) = \sec^{-1}\left(\frac{2x}{5x+3}\right)$ is $[\alpha, \beta) \cup (\gamma, \delta]$, then $|3\alpha + 10(\beta + \gamma) + 21\delta|$ is equal to _____.

Q. 29. Let the quadratic curve passing through the point (–1,0) and touching the line $y = x$ at (1,1) be $y = f(x)$. Then the x-intercept of the normal to the curve at the point $(\alpha, \alpha + 1)$ in the first quadrant is _______.

Q. 30. Let the equations of two adjacent sides of a parallelogram ABCD be $2x - 3y = -23$ and $5x + 4y = 23$. If the equation of its one diagonal AC is $3x + 7y = 23$ and the distance of A from the other diagonal is d, then 50 d^2 is equal to _______.

Answer Key

Q. No.	Answer	Topic name	Chapter name
1	(2)	Binomial theorem for any index	Binomial theorem
2	(4)	No of relation	Relation and function
3	(4)	Point and plane	3D
4	(4)	Compound statement	Mathematical reasoning
5	(2)	Trigonometric series	Trigonometry
6	(4)	Algebra on vector	Vector
7	(3)	Equation of circle	Circle
8	(1)	Mean and variance	Statistics
9	(4)	Method of difference	Sequence and series
10	(4)	Division into groups	Permutation and combination
11	(1)	Adjoint of a matrix	Matrix and determinants
12	(4)	Divisibility problem	Binomial theorem
13	(4)	Maxima and minima	Application of derivatives
14	(1)	Inegration using substitutions	Indefinit integral
15	(2)	Lebnitz rule	Definite integral
16	(1)	Binomial distribution	Probability
17	(1)	Circle and ellipse	Ellipse
18	(4)	Product of two vectors	Vector
19	(3)	Algebra of complex number	Complex number
20	(3)	Point and line	3D
21	[26664]	Sum of numbers	Permutation and combination
22	[6]	Perimeter of triangle	Properties of triangle
23	[5]	Geometrical problem	Differential equations
24	[16]	AGP	Sequence and series
25	[27]	Area b/w two curves	Area under curves
26	[7]	Point and line	3D
27	[24]	Solving system of linear equations	Matrix and determinants
28	[24]	Domain	Inverse trigonometric function
29	[11]	Tangent and normal	Application of derivatives
30	[529]	Point and line	Straight lines

Solutions

Section A

1. Option (2) is correct.

We have $(1 + x)^p (1 - x)^q$

$$= \left(1 + px + \frac{p(p-1)}{2}x^2 + ...\right)\left(1 - qx + \frac{q(q-1)}{2}x^2 - ...\right)$$

Coefficient of x is 4

$\Rightarrow p - q = 4$ ...(i)

and coefficient of x^2 is -5

$$\Rightarrow \frac{p(p-1)}{2} - pq + \frac{q(q-1)}{2} = -5$$

or $p^2 - p - 2pq + q^2 - q + 10 = 0$ ...(ii)

on solving equation (i) and (ii), we get

$p = 15$ and $q = 11$

so $2p + 3q = 30 + 33 = 63$

2. Option (4) is correct.

Given that A = {2, 3, 4} and B = {8, 9, 12}

Since, each element of A have two choices that $\forall a \in A$ and $b \in B$ such that a divides b therefore, a_1 divides b_2 have $3 \times 2 = 6$ elements and a_2 divides b_1 have $3 \times 2 = 6$ elements.

Hence, total number of element in R are $6 \times 6 = 36$ elements.

3. Option (4) is correct.

Equation of plane passing through the points A (1, 2, 0), B (1, 4, 1) and C(0, 5, 1) is

$$\begin{vmatrix} x-1 & y-2 & z-0 \\ 0 & 2 & 1 \\ -1 & 3 & 1 \end{vmatrix} = 0$$

$\Rightarrow x + y - 2z = 3$

Now Q (α, β, γ) is the image of the point P(1, 2, 6) in the plane $x + y - 2z - 3 = 0$

$$\Rightarrow \frac{\alpha-1}{1} = \frac{\beta-2}{1} = \frac{\gamma-6}{-2} = \frac{-(1+2-12-3)}{1^2+1^2+(-2)^2}$$

$$= \frac{(-2)(-12)}{6} = 4$$

$\Rightarrow \alpha = 5, \beta = 6, \gamma = -2$

and $\alpha^2 + \beta^2 + \gamma^2 = 25 + 36 + 4 = 65$

4. Option (4) is correct.

$\sim [p \vee (\sim p \wedge q))]$

$\Rightarrow \sim p \wedge ((p \wedge q)$ $\qquad [\because \sim(p \wedge q) = \sim p \vee \sim q]$

5. Option (2) is correct.

Let $9^{\tan^2 x} = t$

Now $9^{1-\tan^2 x} + 9^{\tan^2 x} = 10$

$\Rightarrow 9 \times \frac{1}{t} + t = 10 \Rightarrow t = 1$ or $t = 9$

$\Rightarrow 9^{\tan^2 x} = 9^0$ or $9^{\tan^2 x} = 9^1$

$\Rightarrow \tan^2 x = 0$ or $\tan^2 x = 1$

$\Rightarrow x = 0$ or $\pm\frac{\pi}{4}$

Also $b = \sum_{x \in S} \tan^2\left(\frac{x}{3}\right) = \tan^2 0 + \tan^2\left(\frac{\pi}{12}\right) + \tan^2\left(\frac{-\pi}{12}\right)$

$b = 0 + 2\tan^2\left(\frac{\pi}{12}\right)$

$b = 2(2-\sqrt{3})^2 = 2(4+3-4\sqrt{3})$

$b = 14 - 8\sqrt{3} \Rightarrow b - 14 = -8\sqrt{3}$

and $\frac{1}{6}(b-14)^2 = \frac{1}{6} \times 64 \times 3 = 32$

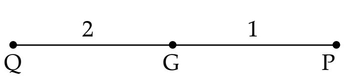
6. Option (4) is correct.

Given that P is circumcentre and Q is orthocentre of ΔABC.

Let P be at origin and G be the centroid of the ΔABC, Since, centroid divides the orthocentre and circumcentre in 2 : 1 then

$$\overrightarrow{PG} = \frac{2\overrightarrow{PP} + 1\overrightarrow{PQ}}{2+1} = \frac{\overrightarrow{O} + \overrightarrow{PQ}}{3} = \frac{\overrightarrow{PQ}}{3}$$

$\Rightarrow \overrightarrow{PQ} = 3\overrightarrow{PG}$



$$= 3\left(\frac{\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC}}{3}\right)$$

Hence, $\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} = \overrightarrow{PQ}$

7. Option (3) is correct.

Any point on the circle $x^2 + y^2 = 4^2$ is B $(4\cos\theta, 4\sin\theta)$ and A (1, 2)

Let P be (h, k) which divides AB in 3 : 2

So

$$h = \frac{12\cos\theta + 2}{3+2} \text{ and } k = \frac{12\sin\theta + 2\times 2}{3+2}$$

$\Rightarrow 5h - 2 = 12\cos\theta$ and $5k - 4 = 12\sin\theta$

On squaring and adding, we get

$(5h-2)^2 + (5k-4)^2 = 144$

Locus of (h, k) is $(5x-2)^2 + (5y-4)^2 = 12^2$

which represents a circle of centre $\left(\frac{2}{5}, \frac{4}{5}\right) \equiv C(\alpha, \beta)$

So $AC = \sqrt{\left(1-\frac{2}{5}\right)^2 + \left(2-\frac{4}{5}\right)^2} = \sqrt{\frac{9}{25} + \frac{36}{25}} = \frac{3\sqrt{5}}{5}$

8. Option (1) is correct.

Given that $\Sigma f_i = 62$

$\Rightarrow k + 2 + 2k + k^2 - 1 + k^2 - 1 + k^2 + 1 + k - 3 = 62$

$\Rightarrow (3k + 16)(k - 4) = 0 \Rightarrow k = 4$

$\left(\because k = \frac{-16}{3} \text{ is not possible}\right)$

x_i	f_i	$f_i x_i$	$f_i x_i^2$
0	6	0	0
1	8	8	8
2	15	30	60
3	15	45	135
4	17	68	272
5	1	5	25
Total	62	156	500

$$\mu = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{156}{62}$$

$$\sigma^2 = \frac{1}{N}\Sigma f_i x_i^2 - \left(\frac{\Sigma f_i x_i}{\Sigma f_i}\right)^2$$

$$= \frac{500}{62} - \left(\frac{156}{62}\right)^2$$

Now $[\mu^2 + \sigma^2] = \left[\left(\frac{156}{62}\right)^2 + \frac{500}{62} - \left(\frac{156}{62}\right)^2\right]$

$$= \left[\frac{500}{62}\right] = 8$$

9. Option (4) is correct.

Given that

$S_n = 4 + 11 + 21 + 24 + 50 + \ldots + T_n$

$S_n = \quad 4 + 11 + 21 + 34 + \; + T_{n-1} + T_n$

$- \quad - \quad - \quad - \quad - \quad - \quad -$

$0 = 4 + 7 + 10 + 13 + 16 + \ldots. (T_n - T_{n-2}) - T_n$

$\Rightarrow T_n = 4 + 7 + 10 + 13 + 16 + \ldots.$ to n terms

$\Rightarrow T_n = \frac{n}{2}[2 \times 4 + (n-1)3]$

$T_n = \frac{3}{2}n^2 + \frac{5}{2}n$

So $S_n = \Sigma T_n = \frac{3}{2}\Sigma n^2 + \frac{5}{2}\Sigma n$

$$\Rightarrow S_n = \frac{3}{2} \times \frac{n(n+1)(2n+1)}{6} + \frac{5}{2} \times \frac{n(n+1)}{2}$$

$\Rightarrow S_n = \frac{n(n+1)}{4}(2n+1+5) = \frac{n(n+1)(n+3)}{2}$

Hence, $\frac{1}{60}(S_{29} - S_9) = \frac{1}{60} \times \frac{1}{2}(29 \times 30 \times 32 - 9 \times 10 \times 12)$

$= 223$

10. Option (4) is correct.

Total persons are 8, so they can be transported as in group of 3, 3 and 2.

So, total no. of way $= \frac{8!}{3! \times 3! \times 2!} \times 3! \times \frac{1}{2}$

$= \frac{8 \times 7 \times 6 \times 5 \times 4}{2 \times 2} = 1680$

11. Option (1) is correct.

Given that

$$A = \frac{1}{5!\,6!\,7!}\begin{bmatrix} 5! & 6! & 7! \\ 6! & 7! & 8! \\ 7! & 8! & 9! \end{bmatrix}$$

$$\Rightarrow |A| = \frac{1}{5!\,6!\,7!}\begin{vmatrix} 5! & 6! & 7! \\ 6! & 7! & 8! \\ 7! & 8! & 9! \end{vmatrix}$$

$$\Rightarrow |A| = \frac{1}{5!\,6!\,7!} \times 5!\,6!\,7!\begin{vmatrix} 1 & 6 & 7\times 6 \\ 1 & 7 & 8\times 7 \\ 1 & 8 & 9\times 8 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 6 & 42 \\ 1 & 7 & 56 \\ 1 & 8 & 72 \end{vmatrix} = 2$$

$\Rightarrow |A| = 2$

Hence $|\text{adj}(\text{adj}(2A))| = |2A|^{(n-1)^2} = |2A|^4$

$= (2^3|A|)^4 = (2^3 \times 2)^4 = 2^{16}$

12. Option (4) is correct.

Given that $(22)^{2022} + (2022)^{22}$

Since, α is remainder when divided by 3

So, $(22)^{2022} + (2022)^{22} = (21+1)^{2022} + (2022)^{21}$

divisible by 3

$= 3k + 1 + 0$ where $k \in$ Integer

$\Rightarrow \alpha = 1$

Also, β is remainder when divided by 7

So, $(22)^{2022} + (2022)^{22} = (21+1)^{2022} - (2023 - 1^{22})$

2023 is divisible by 7

$= 7k + 1 - (7q - 1)$

$= 7(k - q) + 2$ where $(k - q) \in$ Integer

$\Rightarrow \beta = 2$ So $\alpha^2 + \beta^2 = 1 + 4 = 5$

13. Option (4) is correct.

Given that $g(x) = f(x) + f(1-x)$

and $f''(x) > 0, \forall x \in (0,1)$

So $g'(x) = f'(x) - f'(1-x)$

and $g''(x) = f''(x) + f''(1-x) > 0\ \forall x \in (0,1)$

Also for maxima or minima, putting $g'(x) = 0$

$\Rightarrow f'(x) = f'(1-x) \Rightarrow x = 1 - x$

or $x = \frac{1}{2}$

and $g''(x) > 0\ \forall x \in (0,1)$

$\Rightarrow g$ is concave up and also $\alpha = \frac{1}{2}$

Hence, $\tan^{-1}(2\alpha) + \tan^{-1}\left(\frac{\alpha+1}{\alpha}\right)$

$= \tan^{-1}(1) + \tan^{-1}(3)$

$= \tan^{-1}\left(\frac{4}{1-3}\right) = \tan^{-1}(-2)$

(**Note:** There is some error in the question, that's why answer is not matching)

14. Option (1) is correct.

Let $I = \int\left(\left(\frac{x}{e}\right)^{2x} + \left(\frac{e}{x}\right)^{2x}\right)\log_e x\,dx$

$\Rightarrow I = \int\left(\left(\frac{e^{\log_e x}}{e}\right)^{2x} + \left(\frac{e}{e^{\log_e x}}\right)^{2x}\right)\log_e x\,dx$

$\Rightarrow I = \int\left(e^{2(x\log x - x)} + e^{-2(x\log x - x)}\right)\log_e x\,dx$

Let $t = x\log x - x \Rightarrow dt = \log_e x\,dx$

So $I = \int\left(e^{2t} + e^{-2t}\right)dt = \frac{e^{2t}}{2} - \frac{e^{-2t}}{2} + C$

$I = \frac{1}{2}\left(\frac{x}{e}\right)^{2x} - \frac{1}{2}\left(\frac{e}{x}\right)^{2x} + C$

Hence $\alpha = 2, \beta = 2, \gamma = 2, \delta = 2$

and $\alpha + 2\beta + 3\gamma - 4\delta = 2 + 4 + 6 - 8 = 4$

15. Option (2) is correct.

Given that

$\int_0^{t^2}(f(x) + x^2)dx = \frac{4}{3}t^3, \forall\, t > 0$

On differentiating using Leibnitz rule, we get

$\left(f(t^2) + t^4\right) \times 2t = \frac{4}{3} \times 3t^2$

$\Rightarrow f(t^2) + t^4 = 2t$

$\Rightarrow f(t^2) = 2t - t^4$

If $t = \frac{\pi}{2}$

$f\left(\frac{\pi^2}{4}\right) = \pi - \frac{\pi^4}{16} = \pi\left(1 - \frac{\pi^3}{16}\right)$

16. Option (1) is correct.

Given that

P (odd number seven times) = P (Odd number nine times)

$\Rightarrow {}^nC_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^{n-7} = {}^nC_9\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^{n-9}$

$\Rightarrow {}^nC_7 = {}^nC_9$

$\Rightarrow n = 7 + 9 = 16$

Hence, P (Even number twice) $= \frac{k}{2^{15}}$

$\Rightarrow {}^{16}C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{16-2} = \frac{k}{2^{15}}$

$\Rightarrow \frac{16 \times 15}{2} \times \frac{1}{2^{16}} = \frac{k}{2^{15}} \Rightarrow k = 60$

17. Option (1) is correct.

Given ellipse is $15x^2 + 19y^2 = 285$

$$\Rightarrow \frac{x^2}{19}+\frac{y^2}{15}=1$$

$\Rightarrow a^2 = 19$ & $b^2 = 15$

So any tangent to this ellipse, is given by

$$y = mx \pm \sqrt{a^2m^2+b^2} = mx \pm \sqrt{19m^2+15}$$

or $mx - y \pm \sqrt{19m^2+15} = 0$

If this line is also a tangent to circle of radius 4 whose centre at (0, 0), then

$$\left|\frac{\pm\sqrt{19m^2+15}}{\sqrt{m^2+1}}\right| = 4$$

$\Rightarrow 19m^2 + 15 = 16m^2 + 16$

$$\Rightarrow m = \pm\frac{1}{\sqrt{3}}$$

$$\text{or } \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}$$

18. Option (4) is correct.

If $\vec{d}$ is $\perp$ to both $\vec{a}$ and $\vec{b}$ then

$$\vec{d} = \lambda(\vec{a}\times\vec{b}) = \lambda\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 7 & -1\\ 3 & 0 & 5\end{vmatrix} = (35\hat{i} - 13\hat{j} - 21\hat{k})\lambda$$

but $\vec{c}.\vec{d} = 12 \Rightarrow \lambda(35\times1 + 13\times1 - 21\times2) = 12$

$\Rightarrow \lambda\,(6) = 12$ or $\lambda = 2$

$\vec{\lambda} = 2(35\hat{i} - 13\hat{j} - 21\hat{k})$

and $(-\hat{i} - +\hat{j} - \hat{k}).(\vec{c}\times\vec{d})$

$$= \begin{vmatrix}-1 & 1 & -1\\ 1 & -1 & 2\\ 70 & -26 & -42\end{vmatrix}$$

$= -94 + 182 - 44 = 44$

19. Option (3) is correct.

Given that $z = x + iy$

$$\text{then } \frac{2z-3i}{4z+2i} = \frac{2(x+iy)-3i}{4(x+iy)+2i}$$

$$= \frac{2x+i(2y-3)}{4x+i(4y+2)}\times\frac{4x-i(4y+2)}{4x-i(4y+2)}$$

$$= \frac{8x^2+(2y-3)(4y+2)}{(4x)^2+(4y+2)^2} + i\left(\frac{4x(2y-3)-2x(4y+2)}{(4x)^2+(4y+2)^2}\right)$$

$$\text{Since, } \frac{2z-3i}{4z+2i} \text{ is Real} \Rightarrow \text{Img}\left(\frac{2z-3i}{4z+2i}\right) = 0$$

$\Rightarrow 4x\,(2y - 3) - 2x\,(4y + 2) = 0$

$\Rightarrow 2x\,(4y - 6 - 4y - 2) = 0$

$\Rightarrow 2x\,(-8) = 0 \Rightarrow x = 0$

20. Option (3) is correct.

Given that

$$\frac{x-5}{4} = \frac{y-7}{3} = \frac{z+2}{1} = \lambda \text{ (say)} \qquad \text{...(i)}$$

$\Rightarrow x = 4\lambda + 5, y = 3\lambda + 7, z = \lambda - 2$

$$\text{and } \frac{x+3}{6} = \frac{3-y}{3} = \frac{z-6}{1} = \mu \text{ (say)} \qquad \text{...(ii)}$$

$\Rightarrow x = 6\mu - 3, y = 3 - 3\mu, z = \mu + 6$

$$\text{and } \frac{x}{1} = \frac{6-y}{2} = \frac{z+8}{5} \qquad \text{...(iii)}$$

Since, A is the POI of line (i) and line (iii)

$$\text{So } \frac{4\lambda+5}{1} = \frac{6-(3\lambda+7)}{2}$$

$\Rightarrow 8\lambda + 10 = 6 - 3\lambda - 7$

or $11\lambda = -11 \Rightarrow \lambda = -1$

So A (1, 4, –3)

Similarly B is the P.O.I of line (ii) and line (iii)

$$\text{So } \frac{6\mu-3}{1} = \frac{6-(3-3\mu)}{2}$$

$\Rightarrow 12\mu - 6 = 3 + 3\mu$

$\Rightarrow 9\mu = 9 \Rightarrow \mu = 1$

So B (3, 0, 7) and mid point of AB is (2, 2, 2)

Now $\perp$ distance of $2x - 2y + z = 14$ from (2, 2, 2) is

$$= \frac{|4-4+2-14|}{\sqrt{4+4+1}} = \frac{12}{3} = 4$$

Section B

21. Correct answer is [26664].

If 1 at unit place then total no. are $\dfrac{3!}{2!} = 3$

If 2 at unit place then total no. are $3! = 6$

If 3 at unit place then total no. are $\dfrac{3!}{2!} = 3$

So sum of digits at unit place is $3 \times 1 + 6 \times 2 + 3 \times 3 = 24$

Hence, required sum $= 24 \times 1000 + 24 \times 100 + 24 \times 10 + 24 \times 1$

$= 24 \times (1000 + 100 + 10 + 1)$

$= 24 \times 1111 = 26664$

22. Correct answer is [6].

In the given figure, let AB = CD = x then DE = $x \tan\theta_2$, and AC = $x \tan\theta_1$

Also given that $\theta_1 + \theta_2 = \dfrac{\pi}{2}$

$\sqrt{3}\text{BE} = 4\text{AB}$

and ar (ΔCAB) $= 2\sqrt{3} - 3$

$$\frac{1}{2}\times \text{AB}\times\text{AC} = 2\sqrt{3}-3$$

$$\frac{1}{2}\times x\times x\tan\theta_1 = 2\sqrt{3}-3 \qquad \text{...(i)}$$

Now $\sqrt{3}\text{BE} = 4\text{AB}$

$\Rightarrow \sqrt{3}(x\tan\theta_2 + \text{BD}) = 4\text{AB}$

$\Rightarrow \sqrt{3}(x\tan\theta_2 + x\tan\theta_1) = 4\text{AB}$

$\Rightarrow \sqrt{3}\,x(\tan\theta_1 + \tan\theta_2) = 4x$

$$\Rightarrow \tan\theta_1 + \tan\left(\frac{\pi}{2} - \theta_1\right) = \frac{4}{\sqrt{3}}$$

$$\Rightarrow \tan\theta_1 + \cot\theta_1 = \frac{4}{\sqrt{3}} = \sqrt{3} + \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan\theta_1 = \sqrt{3} \text{ or } \theta_1 = \frac{\pi}{3} \quad \text{and } \theta_2 = \frac{\pi}{6}$$

$$\text{or } \theta_1 = \frac{\pi}{6} \text{ and } \theta_2 = \frac{\pi}{3}$$

$\because \frac{\theta_2}{\theta_1}$ is largest

$$\therefore \theta_1 = \frac{\pi}{6} \text{ and } \theta_2 = \frac{\pi}{3}$$

So from eqn. (i)

$$\frac{1}{2}x^2 \times \frac{1}{\sqrt{3}} = 2\sqrt{3} - 3$$

$$\Rightarrow x^2 = 12 - 6\sqrt{3} = \left(3 - \sqrt{3}\right)^2$$

$$\Rightarrow x = 3 - \sqrt{3}$$

Hence, perimeter of ΔCED = CE + ED + CD

$= x\sec\theta_2 + x\tan\theta_2 + x$

$$= x\left(\sec\frac{\pi}{3} + \tan\frac{\pi}{3} + 1\right)$$

$$= (3 - \sqrt{3})(2 + \sqrt{3} + 1)$$

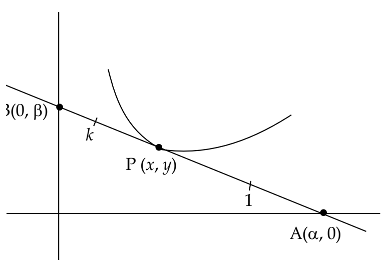
$$= (3 - \sqrt{3})(3 + \sqrt{3})$$

= 9 – 3 = 6 units.

23. Correct answer is [5].

Let the equation of tangent to the curve at (x, y).

Whose slope is $\frac{dy}{dx}$ is



$$Y - y = \frac{dy}{dx}(X - x)$$

Putting Y = 0 $\Rightarrow X = x - \frac{y}{\left(\frac{dy}{dx}\right)}$ t

$$\Rightarrow \alpha = x - \frac{y}{\left(\frac{dy}{dx}\right)}$$

and putting $X = 0 \Rightarrow Y = y - x\frac{dy}{dx}$

$$\Rightarrow \beta = y - x\frac{dy}{dx}$$

$\because$ P divides AB in 1 : k

$$x = \frac{k\alpha + 0}{k+1} \text{ and } y = \frac{k \times 0 + \beta}{k+1}$$

$$\Rightarrow x(k+1) = k\left(x - \frac{y}{\frac{dy}{dx}}\right)$$

$$\Rightarrow xk + x = xk - \frac{yk}{\frac{dy}{dx}} \Rightarrow x\frac{dy}{dx} = -yk$$

$$\text{or } \int\frac{dy}{y} = -k \times \int\frac{1}{x}dx$$

$\Rightarrow \log y = -k\log x + \log C$

or $\log y \times x^k = \log C$

$\Rightarrow yx^k = C$

putting $x = 1, y = 1 \Rightarrow c = 1$

so $yx^k = 1$

Putting $x = \frac{1}{10}, y = 100 \Rightarrow 100 \times \left(\frac{1}{100}\right)^k = 1 \Rightarrow k = 2$

so $yx^2 = 1$ or $y = \frac{1}{x^2}$

Now $e^{\frac{dy}{dx}} = kx + \frac{k}{2}$

$$\Rightarrow \frac{dy}{dx} = \log_e\left(kx + \frac{k}{2}\right) = \log_e(2x+1)$$

On integrating

$$y = \int 1.\log_e(2x+1)dx$$

$$= x\log_e(2x+1) - \int\frac{1 \times 2}{2x+1} \times x\,dx$$

$$= x\log_e(2x+1) - \int 1 - \frac{1}{2x+1}dx$$

$$y = x\log_e(2x+1) - x + \frac{1}{2}\log_e(2x+1) + c$$

Put $x = 0, y = k = 2 \Rightarrow c = 2$

putting $x = 1$

$y(1) = \log_e 3 - 1 + \frac{1}{2}\log_e 3 + 2 = \frac{3}{2}\log_e 3 - 1 + 2$

$\Rightarrow 4y(1) = 6\log_e 3 + 4$

or $4y(1) - 5\log_e 3 = \log_e 3 + 4$

24. Correct answer is [16].

Since, common ratio of A.G.P. is 2

therefore A.G.P. can be taken as

$$\frac{(a-2d)}{4}, \frac{(c-d)}{2}, a, 2(a+d), 4(a+2d)$$

or $a_1, a_2, 2, a_3, a_4$ (Given)

$\Rightarrow a = 2$

also sum of thes A.G.P. is $\frac{49}{2}$

$$\Rightarrow \frac{2-2d}{4} + \frac{2-d}{2} + 2 + 2(2+d) + 4(2+2d) = \frac{49}{2}$$

$$\Rightarrow \frac{1}{4}[2 - 2d + 4 - 2d + 8 + 16 + 8d + 32 + 32d] = \frac{49}{2}$$

$\Rightarrow 36d + 62 = 98$

$\Rightarrow 36d = 36 \Rightarrow d = 1$

Hence, $a_4 = 4(a + 2d) = 4(2 + 2 \times 1) = 16$

25. Correct answer is [27].

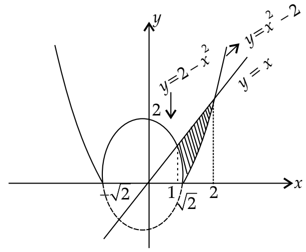
Given that

$\{(x, y) : |x^2 - 2| \le x\}$

Let $y = |x^2 - 2|$

and $y = x$

which can be plotted as



By using the graph, we have

$$A = \int_1^{\sqrt{2}} [x - (2 - x^2)]\, dx + \int_{\sqrt{2}}^{2} [x - (x^2 - 2)]\, dx$$

$$= \left[\frac{x^2}{2} - 2x + \frac{x^3}{3}\right]_1^{\sqrt{2}} + \left[\frac{x^2}{2} - \frac{x^3}{3} + 2x\right]_{\sqrt{2}}^{2}$$

$$= \left(\frac{2}{2} - 2\sqrt{2} + \frac{2\sqrt{2}}{2}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right) + \left(\frac{4}{2} - \frac{8}{3} + 4\right) -$$
$$\left(\frac{2}{2} - \frac{2\sqrt{2}}{3} + 2\sqrt{2}\right)$$

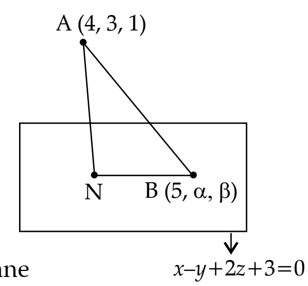
$$= 1 - 2\sqrt{2} + \frac{2\sqrt{2}}{3} + \frac{3}{2} - \frac{1}{3} + 6 - \frac{8}{3} - 1 + \frac{2\sqrt{2}}{3} - 2\sqrt{2}$$

$$A = -\frac{8\sqrt{2}}{3} + \frac{9}{2}$$

$\Rightarrow 6A = -16\sqrt{2} + 27$ or $6A + 16\sqrt{2} = 27$

26. Correct answer is [7].

Given that A(4, 3, 1)



So $AN = \dfrac{|4 - 3 + 2 + 3|}{\sqrt{1 + 1 + 4}}$

$= \dfrac{6}{\sqrt{6}} = \sqrt{6}$

and B(5, α, β) lies on plane

so $5 - \alpha + 2\beta + 3 = 0$

$\Rightarrow \alpha - 2\beta = 8$ ...(i)

Since N is foot of $\perp$ from A to given plane,

So $\dfrac{x - 4}{1} = \dfrac{y - 3}{-1} = \dfrac{z - 1}{2} = \dfrac{-(4 - 3 + 2 + 3)}{1 + 1 + 4}$

$= -1$

$\Rightarrow x = 3, y = 4, z = -1$

so N (3, 4, –1)

Also, ar $(\Delta ABN) = 3\sqrt{2}$

$\Rightarrow \dfrac{1}{2} \times AN \times BN = 3\sqrt{2}$

$\Rightarrow \dfrac{1}{2} \times \sqrt{6} \times BN = 3\sqrt{2} \Rightarrow BN = 2\sqrt{3}$

or $BN^2 = 12$

$\Rightarrow (5 - 3)^2 + (\alpha - 4)^2 + (\beta + 1)^2 = 12$

$4 + (2\beta + 4)^2 + (\beta + 1)^2 = 12$ (using eqn (i))

$\Rightarrow 4\beta^2 + 16 + 16\beta + \beta^2 + 1 + 2\beta = 8$

$\Rightarrow 5\beta^2 + 18\beta + 9 = 0$

$\Rightarrow (5\beta + 3)(\beta + 3) = 0$

$\Rightarrow \beta = -3$ and $\alpha = 2$

also $\alpha^2 + \beta^2 + \alpha\beta = 9 + 4 - 6 = 7$

27. Correct answer is [24].

Given that S be the set of values of λ for which given system of equations has no solution

Therefore for the given set of equations

$$\Delta = \begin{vmatrix} 6\lambda & -3 & 3 \\ 2 & 6\lambda & 4 \\ 3 & 2 & 3\lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda = 1, \frac{-1}{3}, \frac{-2}{3}$$

Also for each values of $\lambda = 1, \dfrac{-1}{3}, \dfrac{-2}{3}$, we have

$$\begin{vmatrix} 6\lambda & -3 & 4\lambda^2 \\ 2 & 6\lambda & 1 \\ 3 & 2 & \lambda \end{vmatrix} \ne 0$$

which implies that, for each values of λ, the given system of equations has no solution

Therefore $S \in \left\{1, \dfrac{-1}{3}, \dfrac{-2}{3}\right\}$ and $12\sum\limits_{\lambda \in S} |\lambda|$

$$= 12\left(|1| + \left|\frac{-1}{3}\right| + \left|\frac{-2}{3}\right|\right)$$

$$= 12\left(1 + \frac{1}{3} + \frac{2}{3}\right) = 12\left(\frac{6}{3}\right) = 24$$

28. Correct answer is [24].

Given that $f(x) = \sec^{-1}\left(\dfrac{2x}{5x + 3}\right)$

Since, the domain for $\sec^{-1} x$ is $|x| \ge 1$

Therefore $\left|\dfrac{2x}{5x + 3}\right| \ge 1$

$\Rightarrow |2x| \ge |5x + 3|$

or $(5x + 3)^2 - (2x)^2 \le 0$

$\Rightarrow (x + 1)(7x + 3) \le 0$

$\Rightarrow x \in \left[-1, \dfrac{-3}{7}\right]$

but $5x + 3 \ne 0 \Rightarrow x \ne \dfrac{-3}{5}$

Hence $x \in \left[-1, \dfrac{-3}{5}\right) \cup \left(\dfrac{-3}{5}, \dfrac{-3}{7}\right]$

and on comparing, we get

$\alpha = -1, \beta = \dfrac{-3}{5}, \gamma = \dfrac{-3}{5}$ and $\delta = \dfrac{-3}{7}$

then $|3\alpha + 10(\beta + \gamma) + 21\delta|$

$$= \left|-3 + 10\left(\frac{-3}{5} - \frac{3}{5}\right) + 21\left(\frac{-3}{7}\right)\right|$$

$= |-3 - 12 - 9| = 24$

29. Correct answer is [11].

Let quadratic equation be $y = ax^2 + bx + c$

which passing through (–1, 0) and (1, 1)

$\Rightarrow a - b + c = 0$ ...(i)

and $a + b + c = 1$ ...(ii)

Also slope of tangent at (1, 1) is 1

so $\frac{dy}{dx} = 2ax + b \Rightarrow \left.\frac{dy}{dx}\right|_{(1,1)} = 2a + b = 1$

$\Rightarrow 2a + b = 1$ ...(iii)

Solving (i), (ii) and (iii), we get

$a = \frac{1}{4}, b = \frac{1}{2}, c = \frac{1}{4}$

so $y = \frac{1}{4}x^2 + \frac{1}{2}x + \frac{1}{4}$

Since, $(\alpha, \alpha + 1)$ lies on this curve, therefore

$\alpha + 1 = \frac{1}{4}\alpha^2 + \frac{1}{2}\alpha + \frac{1}{4}$

$\Rightarrow 4\alpha + 4 = \alpha^2 + 2\alpha + 1$

$\Rightarrow \alpha = 3$ ($\alpha = -1$ is rejected as $(\alpha, \alpha + 1) \in$ IQ)

Now $\frac{dy}{dx} = 2ax + b$

$\left.\frac{dy}{dx}\right|_{(3,4)} = 2 \times \frac{1}{4} \times 3 + \frac{1}{2} = 2$

$\Rightarrow$ Slope of normal $= \frac{-1}{2}$

and equation of normal

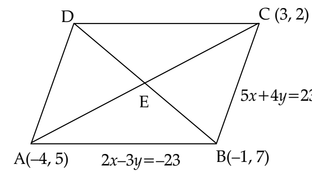
$y - 4 = -\frac{1}{2}(x - 3)$

Putting $y = 0 \Rightarrow -4 = -\frac{1}{2}(x - 3)$

$\Rightarrow x = 11$

Hence, x-intercept is 11.

30. Correct answer is [529].



On solving the equation of diagonal with the equation of sides, we get

A(–4, 5) and C (3, 2)

also by solving the two sides, we get B(–1, 7) and mid point of AC is $E\left(\frac{-1}{2}, \frac{7}{2}\right)$

Now equation of diagonal BD is

$$y - 7 = \frac{7 - \frac{7}{2}}{-1 + \frac{1}{2}}(x + 1) = \frac{\frac{7}{2}}{-\frac{1}{2}}(x + 1)$$

$-y + 7 = 7x + 7$ or $7x + y = 0$

So, $d = \frac{|7 \times (-4) + 5|}{\sqrt{49 + 1}} = \frac{23}{\sqrt{50}}$

and $50d^2 = 23 \times 23 = 529$

❑❑