JEE (Main) MATHEMATICS SOLVED PAPER

2023
11th April Shift 1

Section A

Q. 1. Let $x_1, x_2, \ldots, x_{100}$ be in an arithmetic progression, with $x_1 = 2$ and their mean equal to 200. If $y_i = i(x_i - i)$, $1 \le i \le 100$, then the mean of $y_1, y_2, \ldots y_{100}$ is:

(1) 10051.50 **(2)** 10100
(3) 10101.50 **(4)** 10049.50

Q. 2. The number of elements in the set $S = \{\theta \in [0, 2\pi] : 3\cos^4\theta - 5\cos^2\theta - 2\sin^6\theta + 2 = 0\}$ is :

(1) 10 **(2)** 9 **(3)** 8 **(4)** 12

Q. 3. The value of the integral

$$\int_{-\log_e 2}^{\log_e 2} e^x\left(\log_e\left(e^x + \sqrt{1+e^{2x}}\right)\right)dx$$ is equal to:

(1) $\log_e\left(\frac{(2+\sqrt{5})^2}{\sqrt{1+\sqrt{5}}}\right) + \frac{\sqrt{5}}{2}$

(2) $\log_e\left(\frac{2(2+\sqrt{5})^2}{\sqrt{1+\sqrt{5}}}\right) - \frac{\sqrt{5}}{2}$

(3) $\log_e\left(\frac{\sqrt{2}(3-\sqrt{5})^2}{\sqrt{1+\sqrt{5}}}\right) + \frac{\sqrt{5}}{2}$

(4) $\log_e\left(\frac{\sqrt{2}(2+\sqrt{5})^2}{\sqrt{1+\sqrt{5}}}\right) - \frac{\sqrt{5}}{2}$

Q. 4. Let $S = \{M = [a_{ij}], a_{ij} \in \{0, 1, 2\}, 1 \le i, j \le 2\}$ be a sample space and $A = \{M \in S : M \text{ is invertible}\}$ be an event. Then P(A) is equal to :

(1) $\frac{16}{27}$ **(2)** $\frac{50}{81}$ **(3)** $\frac{47}{81}$ **(4)** $\frac{49}{81}$

Q. 5. Let $f : [2, 4] \to R$ be a differentiable function such that $(x \log_e x) f'(x) + (\log_e x) f(x) + f(x) \ge 1$, $x \in [2, 4]$ with $f(2) = \frac{1}{2}$ and $f(4) = \frac{1}{4}$. Consider the following two statements:

(A) : $f(x) \le 1$ for all $x \in [2,4]$

(B) : $f(x) \ge \frac{1}{8}$, for all $x \in [2,4]$

Then,

(1) Only statement (B) is true.
(2) Only statement (A) is true.
(3) Neither statement (A) nor statement (B) is true.
(4) Both the statements (A) and (B) are true.

Q. 6. Let A be a 2 × 2 matrix with real entries such that $A' = \alpha A + I$, where $a \in R - \{-1, 1\}$. If $\det(A^2 - A) = 4$, then the sum of all possible values of α is equal to:

(1) 0 **(2)** $\frac{5}{2}$ **(3)** 2 **(4)** $\frac{3}{2}$

Q. 7. The number of integral solutions x of

$$\log_{\left(x+\frac{7}{2}\right)}\left(\frac{x-7}{2x-3}\right)^2 \ge 0$$ is:

(1) 5 **(2)** 7 **(3)** 8 **(4)** 6

Q. 8. For any vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$. with $10|a_i| < 1$, $i = 1, 2, 3$, consider the following statements :

(A) : $\max\{|a_1|, |a_2|, |a_3|\} \le |\vec{a}|$

(B) : $|\vec{a}| \le 3\max\{|a_1|, |a_2|, |a_3|\}$

(1) Only (B) is true
(2) Both (A) and (B) are true
(3) Neither (A) nor (B) is true
(4) Only (A) is true

Q. 9. The number of triplets (x,y,z), where x, y, z are distinct non negative integers satisfying $x + y + z = 15$, is :

(1) 136 **(2)** 114 **(3)** 80 **(4)** 92

Q. 10. Let sets A and B have 5 elements each. Let mean of the elements in sets A and B be 5 and 8 respectively and the variance of the elements in sets A and B be 12 and 20 respectively. A new set C of 10 elements is formed by subtracting 3 from each element of A and adding 2 to each element of B. Then the sum of the mean and variance of the elements of C is __________.

(1) 36 **(2)** 40 **(3)** 32 **(4)** 38

Q. 11. Area of the region $\{(x, y) : x^2 + (y-2)^2 \le 4, x^2 \ge 2y\}$ is:

(1) $\pi + \frac{8}{3}$ **(2)** $2\pi + \frac{16}{3}$ **(3)** $2\pi - \frac{16}{3}$ **(4)** $\pi - \frac{8}{3}$

Q. 12. Let R be a rectangle given by the line $x = 0$, $x = 2$, $y = 0$ and $y = 5$. Let A $(\alpha, 0)$ and B $(0, \beta)$, $\alpha \in [0, 2]$ and $\beta \in [0, 5]$, be such that the line segment AB divides the area of the rectangle R in the ratio 4 : 1. Then, the midpoint of AB lies on a:

(1) straight line **(2)** parabola
(3) circle **(4)** hyperbola

Q. 13. Let $\vec{a}$ be a non-zero vector parallel to the line of intersection of the two planes described by $\hat{i} + \hat{j}, \hat{i} + \hat{k}$ and $\hat{i} - \hat{j}, \hat{i} - \hat{k}$. If θ is the angle between the vector $\vec{a}$ and the vector $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{a}.\vec{b} = 6$, then ordered pair $(\theta, |\vec{a} \times \vec{b}|)$ is equal to:

(1) $\left(\frac{\pi}{3}, 6\right)$ **(2)** $\left(\frac{\pi}{4}, 3\sqrt{6}\right)$

(3) $\left(\frac{\pi}{3}, 3\sqrt{6}\right)$ **(4)** $\left(\frac{\pi}{4}, 6\right)$

Q. 14. Let w_1 be the point obtained by the rotation of $z_1 = 5 + 4i$ about the origin through a right angle in the anticlockwise direction, and w_2 be the point obtained by the rotation of $z_2 = 3 + 5i$ about the origin through a right angle in the clockwise direction. Then, the principal argument of $w_1 - w_2$ is equal to:

(1) $\pi - \tan^{-1}\frac{8}{9}$ **(2)** $-\pi + \tan^{-1}\frac{8}{9}$

(3) $\pi - \tan^{-1}\frac{33}{5}$ **(4)** $-\pi + \tan^{-1}\frac{33}{5}$

Q. 15. Consider ellipse $E_k : kx^2 + k^2y^2 = 1, k = 1, 2, \ldots, 20$. Let C_k be the circle which touches the four chords joining the end points (one on minor axis and another on major axis) of the ellipse E_k. If r_k is the radius of the circle C_k, then the value of $\sum_{k=1}^{20} \frac{1}{r_k^2}$ is:

(1) 3320 **(2)** 3210 **(3)** 3080 **(4)** 2870

Q. 16. If the equation of the plane that contains the point (–2, 3, 5) and is perpendicular to each of the planes $2x + 4y + 5z = 8$ and $3x - 2y + 3z = 5$ is $\alpha x + \beta y + \gamma z + 97 = 0$, then $\alpha + \beta + \gamma =$:

(1) 15 **(2)** 18 **(3)** 17 **(4)** 16

Q. 17. An organisation awarded 48 medals in event 'A', 25 in event 'B' and 18 in event 'C'. If these medals were awarded to total 60 men and only five men got medals in all the three events, then, how many received medals in exactly two of three events ?

(1) 15 **(2)** 9 **(3)** 21 **(4)** 10

Q. 18. Let $y = y(x)$ be a solution curve of the differential equation. $(1 - x^2y^2)dx = ydx + xdy$. If the line $x = 1$ intersects the curve $y = y(x)$ at $y = 2$ and the line $x = 2$ intersects the curve $y = y(x)$ at $y = \alpha$, then the value of α is:

(1) $\frac{1+3e^2}{2(3e^2-1)}$ **(2)** $\frac{1-3e^2}{2(3e^2+1)}$

(3) $\frac{3e^2}{2(3e^2-1)}$ **(4)** $\frac{3e^2}{2(3e^2+1)}$

Q. 19. Let (α, β, γ) be the image of the point P (2, 3, 5) in the plane $2x + y - 3z = 6$. Then, $\alpha + \beta + \gamma$ is equal to:

(1) 5 **(2)** 9 **(3)** 10 **(4)** 12

Q. 20. Let $f(x) = [x^2 - x] + |-x+[x]|$, where $x \in \mathbb{R}$ and $[t]$ denotes the greatest integer less than or equal to t. Then, f is:

(1) not continuous at $x = 0$ and $x = 1$
(2) continuous at $x = 0$ and $x = 1$
(3) continuous at $x = 1$, but not continuous at $x = 0$
(4) continuous at $x = 0$, but not continuous at $x = 1$

Section B

Q. 21. The number of integral terms in the expansion of $\left(3^{\frac{1}{2}} + 5^{\frac{1}{4}}\right)^{680}$ is equal to:

Q. 22. The number of ordered triplets of the truth values of p, q and r such that the truth value of the statement $(p \vee q) \wedge (p \vee r) \Rightarrow (q \vee r)$ is true, is equal to ________.

Q. 23. Let $A = \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}$, where $a, c \in \mathbb{R}$. If $A^3 = A$ and the positive value of a belongs to the interval $(n - 1, n]$, where $n \in \mathbb{N}$, then n is equal to _____.

Q. 24. For $m, n > 0$, let $\alpha(m, n) = \int_0^2 t^m (1+3t)^n dt$. If $11\alpha(10, 6) + 18\alpha(11, 5) = p(14)^6$, then p is equal to ________:

Q. 25. Let $S = 109 + \frac{108}{5} + \frac{107}{5^2} + \ldots + \frac{2}{5^{107}} + \frac{1}{5^{108}}$. Then, the value of $(16S - (25)^{-54})$ is equal to ________.

Q. 26. Let $H_n : \frac{x^2}{1+n} - \frac{y^2}{3+n} = 1, \in \mathbb{N}$. Let k be the smallest even value of n such that the eccentricity of H_k is a rational number. If l is the length of the latus rectum of H_k, then $21\,l$ is equal to ________.

Q. 27. The mean of the coefficients of $x, x^2, \ldots x^7$ in the binomial expansion of $(2 + x)^9$ is ________.

Q. 28. If a and b are the roots of the equation $x^2 - 7x - 1 = 0$, then the value of $\frac{a^{21} + b^{21} + a^{17} + b^{17}}{a^{19} + b^{19}}$ is equal to ________.

Q. 29. In an examination, 5 students have been allotted their seats as per their roll numbers. The number of ways, in which none of the students sits on the allotted seat is ________.

Q. 30. Let a line l pass through the origin and be perpendicular to the lines

$$l_1 : \vec{r} = \hat{i} - 11\hat{j} - 7\hat{k} + \lambda(\hat{i} + 2\hat{j} + 3\hat{k}), \lambda \in \mathbb{R} \text{ and}$$

$$l_2 : \vec{r} = -\hat{i} + \hat{k} + \mu(2\hat{i} + 2\hat{j} + \hat{k}), \mu \in \mathbb{R}$$

If P is the point of intersection of l and l_1, and Q (α, β, γ) is the foot of the perpendicular from P on l_2, then $9(\alpha + \beta + \gamma)$ is equal to ________.

Answer Key

Q. No.	Answer	Topic name	Chapter name
1	(4)	Mean	Statistics
2	(2)	Factorisation method for solving T.E.	Trigonometric equations
3	(4)	Definite integral using properties	Definite integral
4	(2)	Probability involving matrices	Probability
5	(4)	Maxima and minima	Application of derivatives
6	(2)	Determinant	Matrix and determinants
7	(4)	Logarihtmic equation	Basics mathematics
8	(1)	Magnitude of a vector	Vector
9	(2)	No. of integral solution	Permutation and combination
10	(4)	Mean and variance	Statistics
11	(3)	Area between two curves	Area under curves
12	(4)	Locus	Coordinate geometry
13	(4)	Plane	3D
14	(1)	Argument of a complex number	Complex number
15	(3)	Ellipse and circle	Ellipse
16	(1)	Equation of a plane	3D
17	(3)	Application on sets	Sets
18	(1)	Exact differential equation	Differential equation
19	(3)	Image of a point wrt a plane	3D
20	(3)	Continuity of a function	Continuity and differentiability
21	[171]	General term	Binomial theorem
22	[7]	Truth table	Mathematical reasoning
23	[2]	Positive integral power of a matrix	Matrix and determinants
24	[32]	Definite integral using properties	Definite integral
25	[2175]	AGP	Sequence and series
26	[306]	Eccenetricity and latus rectum	Hyperbola
27	[2736]	Properties of binomial cofficients	Binomial theorem
28	[51]	Newtons theorem	Quadratic equation
29	[44]	Dearrangements	Permutation and combination
30	[5]	Point, Line and plane	3D

Solutions

Section A

1. Option (4) is correct.

Given that $x_1 = 2$ and mean = 200 of 100 nos.

$$\Rightarrow \frac{\frac{100}{2}[2\times 2+(100-1)d]}{100} = 200$$

$\Rightarrow 4 + 99d = 400 \Rightarrow d = 4$

So $x_i = x_1 + (i-1)d = 2 + (i-1)4$

$\Rightarrow x_i = 4i - 2$ and $y_i = i\,(x_i - i) = i\,(4i - 2 - i)$

$\Rightarrow y_i = 3i^2 - 2i$

$$\sum_{i=1}^{100} y_i = 3\Sigma i^2 - 2\Sigma i$$

$$= \frac{3(100)(101)(201)}{6} - \frac{2\times(100)(101)}{2}$$

$= 50 \times 101 \times 201 - 101 \times 100 = 1004950$

Hence, the mean of $y_1, y_2, \ldots y_{100} = \frac{\Sigma y_i}{100}$

$$= \frac{1004950}{100} = 10049.50$$

2. Option (2) is correct.

Given that

$3\cos^4\theta - 5\cos^2\theta - 2\sin^6\theta + 2 = 0$ and $\theta \in [0, 2\pi]$

$\Rightarrow 3\cos^4\theta - 3\cos^2\theta - 2\cos^2\theta - 2\sin^6\theta + 2 = 0$

$\Rightarrow 3\cos^2\theta\,(\cos^2\theta - 1) - 2 + 2\sin^2\theta - 2\sin^6\theta + 2 = 0$

$\Rightarrow -3\cos^2\theta\sin^2\theta + 2\sin^2\theta\,(1 - \sin^4\theta) = 0$

$\Rightarrow -3\sin^2\theta\cos^2\theta + 2\sin^2\theta\cos^2\theta\,(1 + \sin^2\theta) = 0$

$\Rightarrow \sin^2\theta \cos^2\theta\,(2 + 2\sin^2\theta - 3) = 0$

$\Rightarrow \sin^2\theta \cos^2\theta\,(2\sin^2\theta - 1) = 0$

If $\sin^2\theta = 0$, then $\theta = nx$ i.e. $\{0, \pi, 2\pi\}$

If $\cos^2\theta = 0$, then $\theta = nx + \frac{\pi}{2}$ i.e., $\left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$

If $2\sin^2\theta - 1 = 0$, then $\sin^2\theta = \left(\frac{1}{\sqrt{2}}\right)^2 = \sin^2\frac{\pi}{4}$

$\theta = n\pi \pm \frac{\pi}{4}$ i.e. $\left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$

Hence, there are total 9 solutions possible in $[0, 2\pi]$

3. Option (4) is correct.

Let $I = \int_{-\log_e 2}^{\log_e 2} e^x\left(\log_e\left(e^x + \sqrt{1+e^{2x}}\right)\right)dx$

putting $t = e^x$, $dt = e^x dx$

If $x = \log_e 2$, then $t = 2$

if $x = -\log_e 2$ then $t = \frac{1}{2}$

So, $I = \int_{\frac{1}{2}}^{2} \underset{\text{II}}{1}.\underset{\text{I}}{\log_e\left(t+\sqrt{1+t^2}\right)}dt$

$$\Rightarrow I = \left[\log_e(t+\sqrt{1+t^2}).t\right]_{\frac{1}{2}}^{2} - \int_{\frac{1}{2}}^{2}\frac{1}{t+\sqrt{1+t^2}} \times \left(1+\frac{2t}{2\sqrt{1+t^2}}\right)t\,dt$$

$$= 2\log(2+\sqrt{5}) - \frac{1}{2}\log\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right) - \int_{\frac{1}{2}}^{2}\left(\frac{t}{\sqrt{1+t^2}}\right)dt$$

$$I = \log\left(\frac{(2+\sqrt{5})^2}{\left(\frac{1+\sqrt{5}}{2}\right)^{\frac{1}{2}}}\right) - I_1, \text{ where } I_1 = \int_{\frac{1}{2}}^{2}\frac{t}{\sqrt{1+t^2}}dt$$

Now, putting $u^2 = 1 + t^2$, in I_1

We have $2u\,du = 2t\,dt$

$t = \frac{1}{2} \Rightarrow u = \frac{\sqrt{5}}{2}$ and $t = 2 \Rightarrow u = \sqrt{5}$

$$I_1 = \int_{\sqrt{5}/2}^{\sqrt{5}}\frac{u\,du}{u} = [u]_{\sqrt{5}/2}^{\sqrt{5}} = \frac{\sqrt{5}}{2}$$

$$\text{Hence, } I = \log\left(\frac{(2+\sqrt{5})^2}{\left(\frac{1+\sqrt{5}}{2}\right)^{\frac{1}{2}}}\right) - \frac{\sqrt{5}}{2}$$

4. Option (2) is correct.

Given that $M = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22}\end{bmatrix}$ where $a_{ij} \in \{0, 1, 2\}$

and $1 \le i, j \le 2$

Here, every element has 3 choices

So, $n(S) = 3 \times 3 \times 3 \times 3 = 81$

Also, M is invertible if $a_{11}a_{22} - a_{21}a_{22} \neq 0$

So, M is non invertible if

$a_{11}a_{12} = a_{21}a_{12} = 0 \Rightarrow (3 \times 3 - 2 \times 2)^2 = 25$

and $a_{11}a_{22} = a_{21}a_{12} = 1 \Rightarrow 1 \times 1 = 1$

and $a_{12}a_{22} = a_{21}a_{12} = 2 \Rightarrow 2 \times 2 = 4$

and $a_{11}a_{22} = a_{21}a_{12} = 4 \Rightarrow 1 \times 1 = 1$

Hence, $P(\bar{A}) = \frac{25+1+4+1}{81} = \frac{31}{81}$

and $P(A) = \frac{50}{81}$

5. Option (4) is correct.

Given that

$x(\log_e x)f'(x) + (\log_e x)f(x) + f(x) \ge 1, x \in [2, 4]$

or $x\log x\frac{dy}{dx} + (\log x + 1)y \ge 1$

$\frac{d}{dx}(y.x\log x) \ge 1 = \frac{d}{dx}(x)$

$\Rightarrow \frac{d}{dx}(y.x\log x - x) \ge 0, x \in [2,4]$

Therefore, $y.x\log x - x = g(x)$ (say) is increasing in $[2, 4]$

or $g(x) = x\log x\,f(x) - x$ is increasing in $[2, 4]$

$g(2) = 2\log 2\,f(2) - 2 = \log 2 - 2$

and $g(4) = 4\log 4\,f(4) - 4 = \log 4 - 4$

Since, $g(x)$ is increasing

Therefore, $g(2) \le g(x) \le g(4)$

$\Rightarrow \log_e 2 - 2 \le x\log_e x\,f(x) - x \le \log 4 - 4$

$\Rightarrow \log_e 2 - 2 \le x\log_e x\,f(x) - x \le 2(\log_e 2 - 2)$

$$\Rightarrow \frac{x+\log_e 2 - 2}{x\log_e x} \le f(x) \le \frac{2(\log_e 2 - 2) + x}{x\log_e x}$$

Now, for $x \in [2, 4]$

$$\frac{2(\log 2 - 2)}{x\log_e x} + \frac{1}{\log x} < \frac{2(\log 2 - 2)}{2\log_e 2} + \frac{1}{\log_e 2}$$

$$= 1 - \frac{1}{\log_e 2} < 1$$

$\Rightarrow f(x) \le 1\ \forall\ x \in [2,4]$

$$\text{and } \frac{\log 2 - 2}{x\log_e x} + \frac{1}{\log_e x} \ge \frac{\log_e 2 - 2}{4\log_e 4} + \frac{1}{\log_e 4} = \frac{1}{8} + \frac{1}{2\log_e 2} \ge \frac{1}{8}$$

$\Rightarrow f(x) \ge \frac{1}{8}\ \forall\ x \in [2,4]$

Hence, both the statements are true.

6. Option (2) is correct.

Given that $A' = \alpha A + I$

$\Rightarrow (A')' = (\alpha A + I)'$

$\Rightarrow A = \alpha A' + I$

$\Rightarrow A = \alpha(\alpha A + I) + I$

$\Rightarrow A = \alpha^2 A + \alpha I + I$

$\Rightarrow A(1-\alpha^2) = I(\alpha + 1)$

$\Rightarrow A = \dfrac{I}{1-\alpha}$

$\Rightarrow |A| = \left|\dfrac{I}{1-\alpha}\right| = \dfrac{1}{(1-\alpha)^2}|I| = \dfrac{1}{(1-\alpha)^2}$

Also, $|A^2 - A| = 4 \Rightarrow |A| \,.\, |A - I| = 4$

$\Rightarrow |A| . \left|\dfrac{I}{1-\alpha} - I\right| = 4 \quad \Rightarrow |A| . \left|\dfrac{\alpha}{1-\alpha} I\right| = 4$

$\Rightarrow |A| . \left(\dfrac{\alpha}{1-\alpha}\right)^2 |I| = 4$

$\Rightarrow \dfrac{1}{(1-\alpha)^2} \times \dfrac{\alpha^2}{(1-\alpha)^2} \times 1 = 4$

$\Rightarrow 2(1-\alpha)^2 = \pm \alpha$

Case I: $2(1-\alpha)^2 = \alpha$

$\Rightarrow 2 + 2\alpha^2 - 4\alpha = \alpha$

$\Rightarrow 2\alpha^2 - 5\alpha + 2 = 0$

$\Rightarrow$ Sum of roots $= \dfrac{5}{2}$

Case II: $2(1-\alpha)^2 = -\alpha$

$\Rightarrow 2 + 2\alpha^2 - 4\alpha = -\alpha$

$\Rightarrow 2\alpha^2 - 3\alpha + 2 = 0$

Imaginary roots.

7. Option (4) is correct.

Given that $\log_{\left(x+\frac{7}{2}\right)}\left(\dfrac{x-7}{2x-3}\right)^2 \geq 0$ ...(i)

$\Rightarrow x + \dfrac{7}{2} > 0, x + \dfrac{7}{2} \neq 1$ and $\left(\dfrac{x-7}{2x-3}\right)^2 > 0$

$\Rightarrow x > -\dfrac{7}{2}, x \neq \dfrac{-5}{2}$ and $x \neq 7$ and $x \neq \dfrac{3}{2}$

Now, from eqn. (i)

$\log_{\left(x+\frac{7}{2}\right)}\left(\dfrac{x-7}{2x-3}\right)^2 \geq 0$

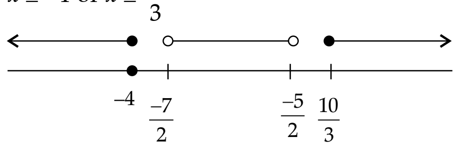
Case I: If $0 < x + \dfrac{7}{2} < 1$ or $\dfrac{-7}{2} < x < \dfrac{-5}{2}$

then $\left(\dfrac{x-7}{2x-3}\right)^2 \leq \left(x + \dfrac{7}{2}\right)^0 \Rightarrow \left(\dfrac{x-7}{2x-3}\right)^2 - 1 \leq 0$

or $\left(\dfrac{x-7+2x-3}{2x-3}\right)\left(\dfrac{x-7-2x+3}{2x-3}\right) \leq 0$

$\Rightarrow \left(\dfrac{3x-10}{2x-3}\right)\left(\dfrac{-x-4}{2x-3}\right) \leq 0$

$\Rightarrow x \leq -4$ or $x \geq \dfrac{10}{3}$



No common solution.

Case II:

If $x + \dfrac{7}{2} > 1$ or $x > \dfrac{-5}{2}$

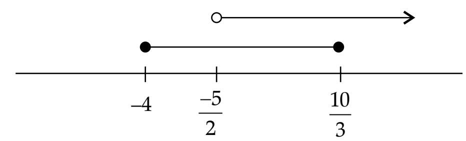
then $\left(\dfrac{x-7}{2x-3}\right)^2 \geq \left(x + \dfrac{7}{2}\right)^0 \Rightarrow \left(\dfrac{x-7}{2x-3}\right)^2 \geq 1$

$\Rightarrow (x-7)^2 - (2x-3)^2 \geq 0$

$\Rightarrow (x - 7 - 2x + 3)(x - 7 + 2x - 3) \geq 0$

$\Rightarrow (-x-4)(3x-10) \geq 0 \; (x+4)(3x-10) \leq 0$

$\Rightarrow -4 \leq x \leq \dfrac{10}{3}$



So $x \in \left(\dfrac{-5}{2}, \dfrac{10}{3}\right] - \left\{\dfrac{3}{2}\right\}$

Hence, integral values of x are $\{-2, -1, 0, 1, 2, 3\}$

i.e. no. of integral values of x are 6.

8. Option (1) is correct.

Given that $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

Since, a_1, a_2, a_3 are fixed number

so we can assume as $|a_1| \leq |a_2| \leq |a_3|$

Now, $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

$\Rightarrow |\vec{a}|^2 = a_1^2 + a_2^2 + a_3^2 \geq a_3^2$

$\Rightarrow |\vec{a}| \geq |a_3|$ which is maximum of $|a_1|, |a_2|, |a_3|$

so $|\vec{a}| \geq \max\{|a_1|, |a_2|, |a_3|\} \Rightarrow$ A is true

Again, $|\vec{a}|^2 = a_1^2 + a_2^2 + a_3^2 \leq a_3^2 + a_3^2 + a_3^2 = 3a_3^2$

$\Rightarrow |\vec{a}| \leq \sqrt{3}|a_3|$ and $|a_3|$ is maximum of $|a_1|, |a_2|$ & $|a_{13}|$

so $|\vec{a}| \leq \sqrt{3} \max\{|a_1|, |a_2|, |a_3|\}$

or $|\vec{a}| \leq 3\max\{|a_1|, |a_2|, |a_3|\} \Rightarrow$ B is true.

9. Option (2) is correct.

Given that

$x + y + z = 15$

By using ${}^{n+r-1}C_{r-1}$, we get total no. of solutions

i.e. ${}^{15+3-1}C_{3-1} = {}^{17}C_2 = \dfrac{17 \times 16}{2} = 136$

Now, to find all distinct solutions.

Let $x = y \neq z$

then, $2x + z = 15 \Rightarrow z = 15 - 2x$

$\Rightarrow x$ can be $\{0, 1, 2, 3, 4, 6, 7\}$

and $x \neq 5$ as $x = 5$ gives $z = 5$

i.e. 7×3 solutions are possible.

Again, if $x = y = z$ i.e. all 3 are equal, then there is one solution possible.

Hence, total number of distinct solutions are $136 - 3 \times 7 - 1 = 136 - 22 = 114$

10. Option (4) is correct.

Let A $\{a_1, a_2, a_3, a_4, a_5\}$

and B = $\{b_1, b_2, b_3, b_4, b_5\}$

Mean (A) = 5 and Mean (B) = 8

$\Rightarrow \frac{\Sigma a_i}{5} = 5$ and $\frac{\Sigma b_i}{5} = 8$

$\Rightarrow \Sigma a_i = 25$ and $\Sigma b_i = 40$

Also, Var (A) = 12 and Var (B) = 20

$\Rightarrow \frac{1}{5}\sum a_i^2 - (\text{Mean(A)})^2 = 12$

and $\frac{1}{5}\sum b_i^2 - (\text{Mean(B)})^2 = 20$

$\frac{1}{5}\sum a_i^2 - 25 = 12$ and $\frac{1}{5}\sum b_i^2 - 64 = 20$

$\sum a_i^2 = 185$ and $\sum b_i^2 = 420$

Now, let C = $\{C_1, C_2, ... C_{10}\}$

Mean (C) = $\frac{\Sigma(a_i - 3) + \Sigma(b_i + 2)}{10}$

$= \frac{\sum a_i - 15 + \sum b_i + 10}{10} = \frac{25 + 40 - 5}{10}$

Mean (c) = 6

and Var (c) = $\frac{1}{10}\sum C_i^2 - (\text{Mean(c)})^2$

$= \frac{1}{10}\left(\sum(a_i - 3)^2 + \sum(b_i + 2)^2\right) - (6)^2$

$= \frac{1}{10}\left(\sum(a_i{}^2 + 9 \times 5 - 6\sum a_i + \sum b_i^2 + 4 \times 5 + 4\sum b_i\right) - 36$

$= \frac{1}{10}(185 + 45 - 6 \times 25 + 420 + 20 + 4 \times 40) - 36$

$= \frac{1}{10}(680) - 36 = 68 - 36 = 32$

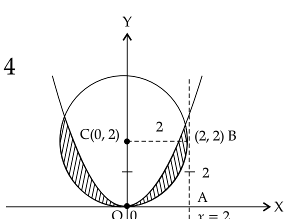
Hence, Mean + Variance = 6 + 32 = 38

11. Option (3) is correct.

$\{(x, y) : x^2 + (y - 2)^2 \le 4, x^2 \ge 2y\}$

Here, $x^2 + y - 2)^2 \le 4$ represents the interior part of the circle and $x^2 \ge 2y$ represents the exterior part of the parabola.

Which can be drawn as-



On solving $x^2 + (y - 2)^2 = 4$

and $x^2 = 2y$, we get,

$2y + y^2 + 4 - 4y = 4$

$y^2 - 2y = 0$

$\Rightarrow y = 0, 2$ and $x = 0, \pm 2$

Hence, required area

= –2 [Area of square OABC – Area of sector of circle $-\int_0^2 y$ parabola]

$= -2\left[2 \times 2 - \frac{\pi \times 2^2}{4} - \int_2^2 \frac{x^2}{2}dx\right]$

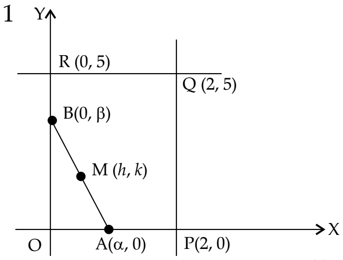
$= -2\left[4 - \frac{4\pi}{4} - \left[\frac{x^3}{6}\right]_0^2\right]$

$= -2\left[4 - \frac{4\pi}{4} - \frac{8}{6}\right] = 2\pi + \frac{8}{3} - 8 = 2\pi - \frac{16}{3}$

12. Option (4) is correct.

Given that $x = 0$, $x = 2$, $y = 0$, $y = 5$ is a rectangle and $\alpha \in [0, 2]$, $\beta \in [0, 5]$

and $\frac{\text{ar(ABRQPA)}}{\text{ar(OABO)}} = \frac{4}{1}$



$\frac{10 - \frac{1}{2}\alpha\beta}{\frac{1}{2}\alpha\beta} = 4$

$\frac{20}{\alpha\beta} - 1 = 4$

$\Rightarrow \alpha\beta = 4$ ...(i)

But M is the mid point of AB

then $h = \frac{\alpha}{2}$ and $k = \frac{\beta}{2}$

$\Rightarrow \alpha = 2h$ and $\beta = 2k$

from eqn. (i),

$2h \times 2k = 4 \Rightarrow hk = 1$

Hence, locus of (h, k) is $xy = 1$, which is a rectangular hyperbola.

13. Option (4) is correct.

Let $\vec{n}$ be the normal vector to the plane $\hat{i} + \hat{j}, \hat{i} + \hat{k}$

then $\vec{n}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = \hat{i} - \hat{j} - \hat{k}$

and $\vec{n}_2$ be the normal vector to plane $\hat{i} - \hat{j}, \hat{i} - \hat{k}$ then

$\vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{vmatrix} = \hat{i} + \hat{j} + \hat{k}$

So $\vec{a}$ can be taken as $\vec{a} = \lambda\,|\vec{n}_1 \times \vec{n}_2|$

$\Rightarrow \vec{a} = \lambda\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -1 \\ 1 & 1 & 1 \end{vmatrix} = \lambda(-2\hat{j} + 2\hat{k})$

Also given that $\vec{a}.\vec{b} = 6$, where $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$

$\Rightarrow \lambda\,(4 + 2) = 6 \Rightarrow \lambda = 1$

So $\vec{a} = -2\hat{j} + 2\hat{k}$

Hence, θ is the angle between $\vec{a}$ and $\vec{b}$ then

$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} = \frac{6}{\sqrt{4+4} \times \sqrt{4+4+1}} = \frac{6}{2\sqrt{2} \times 3} = \frac{1}{\sqrt{2}}$

$\Rightarrow \cos\theta = \cos\frac{\pi}{4} \Rightarrow \theta = \frac{\pi}{4}$

and $|\vec{a} \times \vec{b}| = \left\|\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -2 & 2 \\ 2 & -2 & 1 \end{matrix}\right\| = |2\hat{i} + 4\hat{j} + 4\hat{k}| = 6$

Hence, required pair is $\left(\frac{\pi}{4}, 6\right)$

14. Option (1) is correct.

Here, $w_1 = z_1 x_i$

$= (5 + 4i) \times i = -4 + 5i$

and $w_2 = z_2(-i) = (3 + 5i) \times (-i) = 5 - 3i$

then $w_1 - w_2 = -4 + 5i - 5 + 3i = -9 + 8i \in 2^{nd}$ Q

Therefore, principal argument of $w_1 - w_2$

$= \pi - \tan^{-1}\left(\frac{8}{9}\right)$

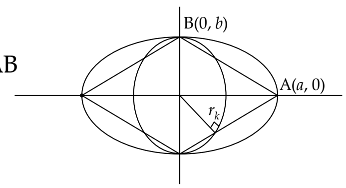
15. Option (3) is correct.

Given that $kx^2 + k^2y^2 = 1$, $k \in [1, 20]$ and $k \in I^4$

$$\Rightarrow \frac{x^2}{\left(\frac{1}{\sqrt{k}}\right)^2} + \frac{y^2}{\frac{1}{k^2}} = 1$$

On comparing with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ we get,

$a = \frac{1}{\sqrt{k}}$ and $b = \frac{1}{k}$



So eqn. of tangent AB of circle is $\frac{x}{a} + \frac{y}{b} = 1$

or $bx + ay = ab$

Now, applying the condition of tangency we get,

$$\frac{ab}{\sqrt{a^2 + b^2}} = \gamma_k$$

or $\gamma_k^2 = \frac{a^2b^2}{a^2 + b^2} \Rightarrow \frac{1}{\gamma_k^2} = \frac{1}{a^2} + \frac{1}{b^2}$

$$\Rightarrow \frac{1}{r_k^2} = k + k^2$$

and $\sum_{k=1}^{20} \frac{1}{\gamma_k^2} = \sum_{k=1}^{20} k + \sum_{k=1}^{20} k^2 = \frac{20 \times 21}{2} + \frac{20 \times 21 \times 41}{6}$

$= 210 + 2870 = 3080$

16. Option (1) is correct.

Given that the plane $\alpha x + \beta y + \gamma z + 97 = 0$ is perpendicular to both the planes

$3x - 2y + 3z = 5$ and $2x + 4y + 5z = 8$ then

$3\alpha - 2\beta + 3\gamma = 0$ ...(i)

and $2\alpha + 4\beta + 5\gamma = 0$ ...(ii)

Solving these two with cross multiplication method we get

$$\frac{\alpha}{\begin{vmatrix} -2 & 3 \\ 4 & 5 \end{vmatrix}} = \frac{\beta}{\begin{vmatrix} 3 & 3 \\ 5 & 2 \end{vmatrix}} = \frac{\gamma}{\begin{vmatrix} 3 & -2 \\ 2 & 4 \end{vmatrix}}$$

$$\Rightarrow \frac{\alpha}{-22} = \frac{\beta}{-9} = \frac{\gamma}{16} = k$$

$\Rightarrow \alpha = -22k, \beta = -9k, \gamma = 16k$

But the point $(-2, 3, 5)$ lies on $\alpha x + \beta y + \gamma z + 97 = 0$

then $-2\alpha + 3\beta + 5\gamma + 97 = 0$

$\Rightarrow 44k - 27k + 80k + 97 = 0$

$\Rightarrow 97k = -97 \Rightarrow k = -1$

so $\alpha = 22, \beta = 9, \gamma = -16$

and $\alpha + \beta + \gamma = 22 + 9 - 16 = 15$

17. Option (3) is correct.

Given that $n(A) = 48$, $n(B) = 25$, $n(C) = 18$

$n(A \cup B \cup C) = 60$ and $n(A \cap B \cap C) = 5$

Using $n(A \cup B \cup C) = n(A) + n(A) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

$\Rightarrow 60 = 48 + 25 + 18 - [n(A \cap B) + n(B \cap C) + n(C \cap A] + 5$

$\Rightarrow n(A \cap B) + n(B \cap C) + n(C \cap A)$

$= 48 + 25 + 18 + 5 - 60 = 36$

So, the number of men who received exactly 2 medals are $n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$

$= 36 - 3 \times 5 = 21$

18. Option (1) is correct.

Given that

$(1 - x^2y^2)\, dx = ydx + xdy = d(x\, y)$

$$\Rightarrow \int dx = \int \frac{d(xy)}{1 - (xy)^2}$$

$$\Rightarrow x = \frac{1}{2}\log\left|\frac{1 + xy}{1 - xy}\right| + C$$

But given that $x = 1, y = 2$

then $1 = \frac{1}{2}\log\left|\frac{1 + 2}{1 - 2}\right| + C$

$$\Rightarrow C = 1 - \frac{1}{2}\log 3$$

On putting $x = 2$ and $y = \alpha$, we get

$$2 = \frac{1}{2}\log\left|\frac{1 + 2\alpha}{1 - 2\alpha}\right| + 1 - \frac{1}{2}\log 3$$

$$\Rightarrow 1 + \frac{1}{2}\log 3 = \frac{1}{2}\log\left|\frac{1 + 2\alpha}{1 - 2\alpha}\right|$$

$$\Rightarrow \log\left|\frac{1 + 2\alpha}{1 - 2\alpha}\right| = 2 + \log 3 = \log_e e^2 + \log_e 3 = \log_e 3e^2$$

$$\Rightarrow \left|\frac{1 + 2\alpha}{1 - 2\alpha}\right| = 3e^2 \text{ and } \frac{1 + 2\alpha}{1 - 2\alpha} = \pm 3e^2$$

Now, $\frac{1 + 2\alpha}{1 - 2\alpha} = 3e^2$ and $\frac{1 + 2\alpha}{1 - 2\alpha} = -3e^2$

$$\Rightarrow \alpha = \frac{3e^2 - 1}{2(3e^2 + 1)} \qquad \Rightarrow \alpha = \frac{3e^2 + 1}{2(3e^2 - 1)}$$

19. Option (3) is correct.

Given that (α, β, γ) is the image of the point P (2, 3, 5) with respect to the plane $2x + y - 3z = 6$

so $\frac{\alpha - 2}{2} = \frac{\beta - 3}{1} = \frac{\gamma - 5}{-3} = \frac{-2\,(4 + 3 - 15 - 6)}{4 + 1 + 9}$

$$= \frac{-1}{7}(-14) = 2$$

$\Rightarrow \alpha = 6, \beta = 5, \gamma = -1$

and $\alpha + \beta + \gamma = 6 + 5 - 1 = 10$

20. Option (3) is correct.

Given that

$f(x) = [x^2 - x] + |-x + [x]|$

Hence, we have to check the continuity of $f(x)$ at 0 and 1.

Now to check continuity of $f(x)$ at $x = 0$

$f(x) = [x(x-1)] + |-x + [x]|$

$\Rightarrow f(0^-) = \lim_{h\to 0}[-h(-h-1)] + |+h+[-h]|$

$= 0 + |0-1| = 1$ and $f(0) = 0$

since $f(0^-) \neq f(0)$, therefore $f(x)$ is not continuous at $x = 0$

Similarly to check at $x = 1$

$f(1^-) = \lim_{h\to 0}[(1-h)(-h)] + |-1+h+[1-h]| = -1+|-1| = 0$

$f(1^+) = \lim_{h\to 0}[(1+h)h] + |-1-h+[1+h]| = 0+0 = 0$

and $f(1) = 0$

Since $f(1^-) = f(1^+) = f(1)$, therefore $f(x)$ is continuous at $x = 1$

Section B

21. Correct answer is [171].

Given that $\left(3^{\frac{1}{2}} + 5^{\frac{1}{4}}\right)^{680}$

$T_{r+1} = {}^{680}C_r\left(3^{\frac{1}{2}}\right)^{680-r}\left(5^{\frac{1}{4}}\right)^r$

$= {}^{680}C_r(3)^{340-\frac{r}{2}}(5)^{\frac{r}{4}}$

Now to find the no. of integral terms–

$340 - \frac{r}{2}$ and $\frac{r}{4}$ must be integers

So r must be a multiple of 4.

$\Rightarrow$ possible values of r are 0, 4, 8, 12, ..., 680 which is an A.P.

So applying $T_n = a + (n-1)d$, we get

$680 = 0 + (n-1) \times 4$

$\Rightarrow n = 170 + 1 = 171$

22. Correct answer is [7].

p	q	r	$p\vee q$	$p\vee r$	$(p\vee q) \wedge(p\vee r)$	$q\vee r$	$(p\vee q) \wedge p\vee r) \to (q\vee r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	T	T	T	T
T	F	F	T	T	T	F	F
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	F	T	F	T	T
F	F	F	F	F	F	F	T

In the last column, there is only one false and seven true.

Hence, total no. of order of True are 7.

23. Correct answer is [2].

Given that $A = \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}$ and $A^3 = A$

Then $A^2 = A \times A =$

$$\begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix} = \begin{bmatrix} a+2 & 2c & 3 \\ 3 & a+3c & 2a \\ ac & 1 & 2+3c \end{bmatrix}$$

and $A^3 = A^2 \times A = \begin{bmatrix} a+2 & 2c & 3 \\ 3 & a+3c & 2a \\ ac & 1 & 2+3c \end{bmatrix}\begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix} = \begin{bmatrix} 2ac+3 & a+2+3c & 2a+4+6c \\ a^2+3ac+2a & 3+2ac & 6+3a+9c \\ a+2+3c & ac+2c+3c^2 & 2ac+3 \end{bmatrix}$$

On comparing the corresponding elements,

$2ac + 3 = 0$ ...(i)

$a + 2 + 3c = 1$ ...(ii)

Using eqn. (i) and (ii) we get,

$a + 3\left(\frac{-3}{2a}\right) = -1$

$\Rightarrow 2a^2 + 2a - 9 = 0$

$\Rightarrow a = \frac{-2 \pm \sqrt{4 + 4\times 2\times 9}}{2\times 2} = \frac{-2\pm 2\sqrt{19}}{4} \Rightarrow a = \frac{\pm\sqrt{19}-1}{2}$

Here, the positive value of $a = \frac{\sqrt{19}-1}{2} = \frac{4.35-1}{2} = 1.67$

which lies between 1 and 2

Hence, $n = 2$

24. Correct answer is [32].

Given that $m, n > 0$

and $\int_0^2 t^m(1+3t)^n\,dt = \alpha(m,n)$

Now, $11\alpha(10, 6) + 18\alpha(11, 5) = p(14)^6$

$\Rightarrow 11\int_0^2 \underset{\text{II}}{t^{10}}\underset{\text{I}}{(1+3t)^6}\,dt + 18\int_0^2 (1+3t)^5 t^{11}dt = (14)^6 p$

$\Rightarrow 11\left\{\left[(1+3t)^6 \times \frac{t^{11}}{11}\right]_0^2 - \int_0^2 6(1+3t)^5 \times 3 \times \frac{t^{11}}{11}dt\right\} +$

$18\int_0^2 t^{11}(1+3t)^5 dt = 14^6 p$

$\Rightarrow 11\left\{\frac{7^6 \times 2^{11}}{11} - \frac{18}{11}\int_0^2 t^{11}(1+3t)^5 dt\right\} +$

$18\int_0^2 t^{11}(1+3t)^5 dt = 14^6 p$

$\Rightarrow 7^6 \times 2^{11} = 14^6 p$

$\Rightarrow 7^6 \times 2^6 \times 2^5 = 14^6 p \Rightarrow p = 32$

25. Correct answer is [2175].

Given that

$S = 109 + \frac{108}{5} + \frac{107}{5^2} + \frac{105}{5^3} + ... + \frac{2}{5^{107}} + \frac{1}{5^{108}}$ ...(i)

which is an A.G.P. so multiplying both sides by $\frac{1}{5}$, we get

$\frac{S}{5} = \frac{109}{5} + \frac{108}{5^2} + \frac{107}{5^3} + ... + \frac{3}{5^{107}} + \frac{2}{5^{108}} + \frac{1}{5^{109}}$ ...(ii)

On subtracting eqn. (ii) from eqn (i) we get,

$$S-\frac{S}{5}=109+\left(-\frac{1}{5}-\frac{1}{5^2}-\frac{1}{5^3}-....-\frac{1}{5^{107}}-\frac{1}{5^{108}}\right)-\frac{1}{5^{109}}$$

$$\frac{4S}{5}=109-\left(\frac{1}{5}+\frac{1}{5^2}+\frac{1}{5^3}+....+\frac{1}{5^{109}}\right)$$

$$=109-\frac{\frac{1}{5}\left(1-\frac{1}{5^{109}}\right)}{1-\frac{1}{5}}$$

$$\frac{4S}{S}=109-\frac{\frac{1}{5}}{\frac{4}{5}}\left(1-\frac{1}{5^{109}}\right)=109-\frac{1}{4}+\frac{1}{4\times 5^{109}}$$

$$\Rightarrow S=\frac{5\times 109}{4}-\frac{5}{4\times 4}+\frac{5}{4\times 4\times 5^{109}}$$

$\Rightarrow 16S = 20 \times 109 - 5 + 5^{-108}$

or $16S - (25)^{-54} = 2180 - 5 = 2175$

26. Correct answer is [306].

Given that

$$H_n:\frac{x^2}{1+n}-\frac{y^2}{3+n}=1, n\in N$$

$\Rightarrow a^2 = 1 + n$ and $b^2 = 3 + n$

and eccentricity e is given by

$$e=\sqrt{1+\frac{b^2}{a^2}}\ \Rightarrow e=\sqrt{1+\frac{3+n}{1+n}}=\sqrt{\frac{2n+4}{n+1}}$$

By putting different values of n, we see that $n = 48$ is the smallest even value for which

$$e=\sqrt{\frac{2\times 48+4}{48+1}}=\sqrt{\frac{100}{49}}=\frac{10}{7}\in \text{ Rational number}$$

So, $e=\frac{10}{7}$

$a^2 = 1 + n = 49$ & $b^2 = 3 + n = 51$

Now, length of L.R. $=\frac{2b^2}{a}=\frac{2\times 51}{7}$

$$\Rightarrow l=\frac{102}{7}\ \text{ and } 21l=21\times\frac{102}{7}=306$$

27. Correct answer is [2736].

Given that : $(2 + x)^9$

$T_{r+1} = {}^9C_r (2)^{9-r} x^r$

So, the coefficient of $x^r = 2^{9-r} \times {}^9C_r$

Now, the mean of coefficient of $x, x^2, ... x^7$ is

$$=\frac{2^8\times{}^9C_1+2^7\times{}^9C_2+...+2^2\times{}^9C_7}{7}$$

$$=\frac{(1+2)^9-{}^9C_0 2^9-{}^9C_8\times 2^1-{}^9C_9\times 2^0}{7}$$

$$=\frac{1}{7}\left\{3^9-1\times 2^9-9\times 2-1\times 1\right\}=\frac{1}{7}\{19152\}=2736$$

28. Correct answer is [51].

Given that a and b are the roots of $x^2 - 7x - 1 = 0$

So, by using Newton's theorem, we get

$S_{n+2} - 7S_{n+1} - S_n = 0$

On putting $n = 19$, 18 and 17 we get,

$S_{21} - 7S_{20} - S_{19} = 0$

$S_{20} - 7S_{19} - S_{18} = 0$

$S_{19} - 7S_{18} - S_{17} = 0$

Now

$$\frac{a^{21}+b^{21}+a^{17}+b^{17}}{a^{19}+b^{19}}=\frac{S_{21}+S_{17}}{S_{19}}$$

$$=\frac{S_{21}+(S_{19}-7S_{18})}{S_{19}}=\frac{50S_{19}+(S_{21}-7S_{20})}{S_{19}}$$

$$=51\times\frac{S_{19}}{S_{19}}=51$$

29. Correct answer is [44].

Given that there are 5 students out of which no. of the students sits on their alloted seat.

So, total number of ways one D_5

and $D_5 = 5!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right)$

$$=5!\left(1-1+\frac{1}{2}-\frac{1}{6}+\frac{1}{24}-\frac{1}{120}\right)$$

$= 60 - 20 + 5 - 1 = 44$

30. Correct answer is [5].

Let d.r.'s of line l be (a, b, c).

Now, line l is $\perp$ to both

$l_1:\vec{r}=\hat{i}-11\hat{j}-7\hat{k}+\lambda(\hat{i}+2\hat{j}+3\hat{k})$ and

$l_2:\vec{r}=-\hat{i}+\hat{k}+\mu(2\hat{i}+2\hat{j}+\hat{k})$

then $a + 2b + 3c = 0$ ...(i)

and $2a + 2b + c = 0$ ...(ii)

Solving the eqn. (i) and (ii) using cross multiplication, method

$$\frac{a}{\begin{matrix}2 & 3\\ 2 & 1\end{matrix}}=\frac{b}{\begin{matrix}3 & 1\\ 1 & 2\end{matrix}}=\frac{c}{\begin{matrix}1 & 2\\ 2 & 2\end{matrix}}$$

$$\frac{a}{-4}=\frac{b}{5}=\frac{c}{-2}=k$$

$\Rightarrow a = -4k, b = 5k, c = -2k$

So, eqn. of line l, passing through (0, 0, 0) can be written as $l:(0\hat{i}+0\hat{j}+0\hat{k})+(-4\hat{i}+5\hat{j}-2\hat{k})k=0$

or $l:(-4\hat{i}+5\hat{j}-2\hat{k})\,k=\vec{r}$

Since P is the point of intersection of l and l_1,

Therefore

$-4k = 1 + \lambda, 5k = -11 + 2\lambda, -2k = -7 + 3\lambda$

$\Rightarrow P(4, -5, 2)$

Also, let $Q(-1 + 2\mu, 2\mu, 1 + \mu)$ be on l_2

then $\overline{PQ}.(2\hat{i}+2\hat{j}+\hat{k})=0$

$\Rightarrow (-5 + 2\mu)2 + 2(2\mu + 5) + 1(-1 + \mu) = 0$

$\Rightarrow 9\mu = 1 \ \Rightarrow \mu=\frac{1}{9}$

$$\text{So } Q\left(-1+\frac{2}{9},\frac{2}{9},1+\frac{1}{9}\right)\equiv Q\left(\frac{-7}{9},\frac{2}{9},\frac{10}{9}\right)\equiv Q(\alpha,\beta,\gamma)$$

$$\text{and } 9(\alpha+\beta+\gamma)=9\left(\frac{-7}{9}+\frac{2}{9}+\frac{10}{9}\right)=9\left(\frac{5}{9}\right)=5$$

❑❑