JEE (Main) MATHEMATICS SOLVED PAPER

2023
11^{th} April Shift 2

Section A

Q. 1. The angle of elevation of the top P of a tower from the feet of one person standing due South of the tower is 45° and from the feet of another person standing due west of the tower is 30°. If the height of the tower is 5 meters, then the distance (in meters) between the two persons is equal to:

(1) 10 **(2)** $5\sqrt{5}$ **(3)** $\frac{5}{2}\sqrt{5}$ **(4)** 5

Q. 2. Let a, b, c and d be positive real numbers such that $a + b + c + d = 11$. If the maximum value of $a^5 b^3 c^2 d$ is 3750 β, then the value of β is:

(1) 55 **(2)** 108 **(3)** 90 **(4)** 110

Q. 3. If $f : R \to R$ be a continuous function satisfying

$$\int_0^{\frac{\pi}{2}} f(\sin 2x)\sin x\, dx + \alpha\int_0^{\frac{\pi}{4}} f(\cos 2x)\cos x\, dx = 0,$$

then the value of α is:

(1) $-\sqrt{3}$ **(2)** $\sqrt{3}$ **(3)** $-\sqrt{2}$ **(4)** $\sqrt{2}$

Q. 4. Let f and g be two functions defined by

$$f(x) = \begin{cases} x+1, & x<0 \\ |x-1|, & x \geq 0 \end{cases} \text{ and } g(x) = \begin{cases} x+1, & x<0 \\ 1 & x \geq 0 \end{cases}$$

Then, (gof) (x) is:

(1) continuous everywhere but not differentiable at $x = 1$

(2) continuous everywhere but not differentiable exactly at one point

(3) differentiable everywhere

(4) not continuous at $x = -1$

Q. 5. If the radius of the largest circle with centre (2, 0) inscribed in the ellipse $x^2 + 4y^2 = 36$ is r, then $12r^2$ is equal to:

(1) 69 **(2)** 72 **(3)** 115 **(4)** 92

Q. 6. Let the mean of 6 observations 1, 2, 4, 5 x and y is 5 and their variance be 10. Then, their mean deviation about the mean is equal to:

(1) $\frac{7}{3}$ **(2)** $\frac{10}{3}$ **(3)** $\frac{8}{3}$ **(4)** 3

Q. 7. Let A = {1, 3, 4, 6, 9} and B = {2, 4, 5, 8, 10}. Let R be a relation defined on A×B such that $R = \{((a_1, b_1), (a_2, b_2)) : a_1 \leq b_2 \text{ and } b_1 \leq a_2\}$. Then, the number of elements in the set R is:

(1) 52 **(2)** 160 **(3)** 26 **(4)** 180

Q. 8. Let P be the plane passing through the points (5, 3, 0), (13, 3, –2) and (1, 6, 2). For α ∈ N, if the distances of the points A(3, 4, α) and B(2, α, a) from the plane P are 2 and 3, respectively, then the positive value of a is:

(1) 5 **(2)** 6 **(3)** 4 **(4)** 3

Q. 9. If the letters of the word MATHS are permuted and all possible words so formed are arranged as in a dictionary with serial number, then the serial number of the word THAMS is:

(1) 102 **(2)** 103 **(3)** 101 **(4)** 104

Q. 10. If four distinct points with position vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are coplanar, then $[\vec{a}\vec{b}\vec{c}]$ is equal to:

(1) $[\vec{d}\,\vec{c}\,\vec{a}]+[\vec{b}\,\vec{d}\,\vec{a}]+[\vec{c}\,\vec{d}\,\vec{b}]$

(2) $[\vec{d}\,\vec{b}\,\vec{a}]+[\vec{a}\,\vec{c}\,\vec{b}]+[\vec{d}\,\vec{b}\,\vec{c}]$

(3) $[\vec{a}\,\vec{d}\,\vec{b}]+[\vec{d}\,\vec{c}\,\vec{a}]+[\vec{d}\,\vec{b}\,\vec{c}]$

(4) $[\vec{b}\,\vec{c}\,\vec{d}]+[\vec{d}\,\vec{c}\,\vec{a}]+[\vec{d}\,\vec{b}\,\vec{a}]$

Q. 11. The sum of the coefficients of three consecutive terms in the binomial expansion of $(1 + x)^{n+2}$, which are in the ratio 1 : 3 : 5, is equal to:

(1) 63 **(2)** 92 **(3)** 25 **(4)** 41

Q. 12. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} + \frac{5}{x(x^5+1)}y = \frac{(x^5+1)^2}{x^7}, x > 0$. If $y(1) = 2$, then $y(2)$ is equal to:

(1) $\frac{693}{128}$ **(2)** $\frac{637}{128}$ **(3)** $\frac{697}{128}$ **(4)** $\frac{679}{128}$

Q. 13. The converse of $((\sim p) \wedge q) \Rightarrow r$ is:

(1) $(p \vee (\sim q)) \Rightarrow (\sim r)$ **(2)** $((\sim p) \vee q) \Rightarrow r$

(3) $(\sim r) \Rightarrow ((\sim p) \wedge q$ **(4)** $(\sim r) \Rightarrow p \wedge q$

Q. 14. If the 1011^{th} term from the end in the binominal expansion of $\left(\frac{4x}{5} - \frac{5}{2x}\right)^{2022}$ is 1024 times 1011^{th} term from the beginning, then $|x|$ is equal to:

(1) 8 **(2)** 12 **(3)** $\frac{5}{16}$ **(4)** 15

Q. 15. If the system of linear equations

$7x + 11y + \alpha z = 13$

$5x + 4y + 7z = \beta$

$175x + 194y + 57z = 361$

has infinitely many solutions, then α + β + 2 is equal to :

(1) 3 **(2)** 6 **(3)** 5 **(4)** 4

Q. 16. Let the line passing through the point P (2, –1, 2) and R (5, 3, 4) meet the plane $x - y + z = 4$ at the point T. Then, the distance of the point R from the plane $x + 2y + 3z + 2 = 0$ measured parallel to the line $\frac{x-7}{2} = \frac{y+3}{2} = \frac{z-2}{1}$ is equal to:

(1) 3 **(2)** $\sqrt{61}$ **(3)** $\sqrt{31}$ **(4)** $\sqrt{189}$

Q. 17. Let the function $f : [0, 2] \to R$ be defined as

$$f(x)=\begin{cases} e^{\min(x^2, x-[x])}, & x\in[0,1) \\ e^{[x-\log_e x]}, & x\in[1,2) \end{cases}$$

where $[t]$ denotes the greatest integer less than or equal to t. Then, the value of the integral $\int_0^2 xf(x)dx$ is:

(1) $(e-1)\left(e^2+\frac{1}{2}\right)$ **(2)** $1+\frac{3e}{2}$

(3) $2e-\frac{1}{2}$ **(4)** $2e-1$

Q. 18. For $a \in C$, let $A = \{z \in C : \text{Re}(a+\bar{z}) > \text{Im}(\bar{a}+z)|\}$ and $B=\{z\in C:\text{Re}(a+\bar{z})<\text{Im}(\bar{a}+z)\}$.
The two statements:
(S1) : If Re (a), Im (a) >0, then the set A contains all the real numbers
(S2) : If Re (a), Im (a) < 0, then the set B contains all the real numbers,

(1) only (S1) is true **(2)** both are false
(3) only (S2) is true **(4)** both are true

Q. 19. If $\begin{vmatrix} x+1 & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda^2 \end{vmatrix}=\frac{9}{8}(103x+81)$, then $\lambda, \frac{\lambda}{3}$ are the roots of the equation:

(1) $4x^2-24x-27=0$ **(2)** $4x^2+24x+27=0$
(3) $4x^2-24x+27=0$ **(4)** $4x^2+24x-27=0$

Q. 20. The domain of the function $f(x)=\frac{1}{\sqrt{[x]^2-3[x]-10}}$ is (where $[x]$ denotes the greastest integer less than or equal to x):

(1) $(-\infty,-3]\cup[6,\infty)$ **(2)** $(-\infty,-2]\cup(5,\infty)$

(3) $(-\infty,-3]\cup(5,\infty)$ **(4)** $(-\infty,-2)\cup[6,\infty)$

Section B

Q. 21. If A is the area in the first quadrant enclosed by the curve $C : 2x^2 - y + 1 = 0$, the tangent to C at the point (1, 3) and the line $x + y = 1$, then the value of 60 A is ______.

Q. 22. Let A = {1, 2, 3, 4, 5} and B= {1, 2, 3, 4, 5, 6}. Then, the number of functions $f : A \to B$ satisfying $f(1) + f(2) = f(4) - 1$ is equal to ______.

Q. 23. Let the tangent to the parabola $y^2 = 12x$ at the point $(3, \alpha)$ be perpendicular to the line $2x + 2y = 3$. Then, the square of distance of the point (6, –4) from the normal to the hyperbola $\alpha^2x^2 - 9y^2 = 9\alpha^2$ at its point $(\alpha - 1, \alpha + 2)$ is equal to _____.

Q. 24. For $k \in N$, if the sum of the series $1+\frac{4}{k}+\frac{8}{k^2}+\frac{13}{k^3}+\frac{19}{k^4}+....$ is 10, then the value of k is __________.

Q. 25. Let the line $\ell : x=\frac{1-y}{-2}=\frac{z-3}{\lambda}, \lambda\in R$ meet the plane P : $x + 2y + 3z = 4$ at point (α, β, γ). If the angle between the line ℓ and the plane P is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then $\alpha + 2\beta 6\gamma$ is equal to _____.

Q. 26. The number of points where the curve $f(x) = e^{8x} - e^{6x} - 3e^{4x} - e^{2x} + 1$, $x \in$ cuts x-axis, is equal to __________.

Q. 27. If the line $l_1 : 3y - 2x = 3$ is the angular bisector of the line $l_2 : x - y + 1 = 0$ and $l_3 : \alpha x + \beta y + 17$, then $\alpha^2 + \beta^2 - \alpha - \beta$ is equal to _________.

Q. 28. Let the probability of getting head for a biased coin be $\frac{1}{4}$. It is tossed repeatedly until a head appears. Let N be the number of tosses required. If the probability that the equation $64x^2 + 5Nx + 1 = 0$ has no real root is $\frac{p}{q}$, where p and q are co-prime, then $q - p$ is equal to ______.

Q. 29. Let $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}-\hat{k}$. If $\vec{c}$ is vector such that $\vec{a}.\vec{c}=11, \hat{b}.(\vec{a}\times\vec{c}) = 27$ and $\vec{b}.\vec{c}=-\sqrt{3}|\vec{b}|$, $|\vec{a}\times\vec{c}|^2$ is equal to __________.

Q. 30. Let $\left\{S=\{z\in C-\{i,2i\}:\frac{z^2+8iz-15}{z^2-3iz-2}\in R\right\}$. If $\alpha-\frac{13}{11}i\in S, a\in R-\{0\}$, then $242\alpha^2$ is equal to _________.

Answer Key

Q. No.	Answer	Topic name	Chapter name
1	(1)	Problem based on height an distance	Height and distance
2	(3)	Rrlation b/w a.M., G.M	Sequence and series
3	(3)	Definite integral using properties	Definite integral
4	(2)	Differentiability of a function	Continuity and differentiability
5	(4)	Ellipse and circle	Ellipse

Q. No.	Answer	Topic name	Chapter name
6	(3)	Mean and variance	Statistics
7	(2)	Rrlation	Relation and function
8	(3)	Point and plane	3D
9	(2)	Word problem	Permutation and combination
10	(1)	Scalar triple product	Vector
11	(1)	Properties if ncr	Binomial theorem
12	(1)	Linear differential equation	Differential equation
13	(1)	Compound statement	Mathematical reasoning
14	(3)	General therm	Binomial theorem
15	(4)	Elementary transformation	Matrix
16	(1)	Point and plane	3D
17	(3)	Definite integral using properties	Definite integral
18	(2)	Component of complex number	Complex number
19	(3)	Determinant	Determinant
20	(4)	Domain	Functions
21	[16]	Approximation	Application of derivatives
22	[360]	No. of functions	Functions
23	[116]	Tangent and normal of parabola	Parabola
24	[2]	Method of difference	Sequence and series
25	[11]	Line and plane	3D
26	[2]	Solution of equation	Basics Mathematics
27	[348]	Line and line	3D
28	[27]	Classical approach	Probability
29	[285]	Product of two vectors	Vector
30	[1680]	Locus related problem	Complex number

Solutions

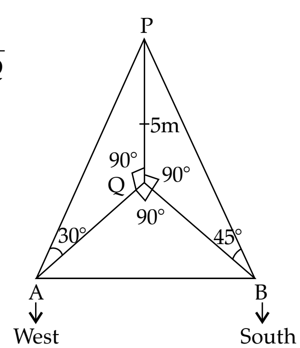
Section A

1. Option (1) is correct.

In ΔPQA

$$\tan 30° = \frac{PQ}{AQ} \Rightarrow \frac{1}{\sqrt{3}} = \frac{5}{AQ}$$

$$\Rightarrow AQ = 5\sqrt{3}$$



In ΔPQB

$$\tan 45° = \frac{PQ}{BQ} \Rightarrow 1 = \frac{5}{BQ}$$

$\Rightarrow BQ = 5$

Now, in ΔABQ

$AB^2 = AQ^2 + BQ^2$

$AB^2 = (5\sqrt{3})^2 + (5)^2 = 75 + 25 = 100$

$\Rightarrow AB = 10$

Hence, distance between the two persons in 10 m.

2. Option (3) is correct.

Given that $a + b + c + d = 11$

Now to find the maximum value of $a^5b^3c^2d$,

Since, a is repeated 5 times, b is 3 times, c is 2 times and d is one time.

Therefore by using A.M. ≥ G.M., we get

$$\frac{\frac{a}{5}+\frac{a}{5}+\frac{a}{5}+\frac{a}{5}+\frac{a}{5}+\frac{b}{3}+\frac{b}{3}+\frac{b}{3}+\frac{c}{2}+\frac{c}{2}+\frac{d}{1}}{11} \geq \left(\left(\frac{a}{5}\right)^5\left(\frac{b}{3}\right)^3\left(\frac{c}{2}\right)^2 d\right)^{\frac{1}{11}}$$

$$\Rightarrow \left(\frac{a+b+c+d}{11}\right)^{11} \geq \left(\frac{a}{5}\right)^5\left(\frac{b}{3}\right)^3\left(\frac{c}{2}\right)^2 d$$

$$\Rightarrow \left(\frac{11}{11}\right)^{11} \geq \frac{a^5b^3c^2d}{5^5 3^3 2^2}$$

$$\Rightarrow 1 \geq \frac{3750\beta}{337500} = \frac{\beta}{90} \Rightarrow \beta \leq 90$$

3. Option (3) is correct.

Given that $f : \mathrm{R} \to \mathrm{R}$ and

$$\int_0^{\frac{\pi}{2}} f(\sin 2x)\sin x dx + \alpha \int_0^{\frac{\pi}{4}} f(\cos 2x)\cos x\, dx = 0$$

$$\Rightarrow \int_0^{\frac{\pi}{4}} f(\sin 2x)\sin x\, dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} f(\sin 2x)\sin x\, dx + \alpha \int_0^{\frac{\pi}{4}} f(\cos 2x)\cos x\, dx = 0$$

$$\Rightarrow \int_0^{\frac{\pi}{4}} f(\cos 2x)\sin\left(\frac{\pi}{4} - x\right) dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} f(\sin 2x)\sin x\, dx + \alpha \int_0^{\frac{\pi}{4}} f(\cos 2x)\cos x dx = 0$$

Putting $x = t + \frac{\pi}{4}; dx = dt$

$$\Rightarrow \int_0^{\frac{\pi}{4}} f(\cos 2x)\sin\left(\frac{\pi}{4} - x\right) dx + \int_0^{\frac{\pi}{4}} f(\cos 2t)\sin\left(t + \frac{\pi}{4}\right) dx + \alpha \int_0^{\frac{\pi}{4}} f(\cos 2x)\cos dx = 0$$

$$\Rightarrow \int_0^{\frac{\pi}{4}} f(\cos 2x)\sin\left(\frac{\pi}{a} - x\right) dx + \int_0^{\frac{\pi}{4}} f(\cos 2x)\sin\left(\frac{\pi}{4} + x\right) dx + \alpha \int_0^{\frac{\pi}{4}} f(\cos 2x)\cos x\, dx = 0$$

$$\Rightarrow \int_0^{\frac{\pi}{4}} f(\cos 2x)\left\{\sin\left(\frac{\pi}{4} - x\right) + \sin\left(\frac{\pi}{4} + x\right) + \alpha\cos x\right\} dx = 0$$

$$\Rightarrow \int_0^{\frac{\pi}{4}} f(\cos 2x)\left\{\sqrt{2}\cos x + \alpha\cos x\right\} dx = 0$$

$$\Rightarrow (\sqrt{2} + \alpha) \times \int_0^{\frac{\pi}{4}} \cos x . f(\cos 2x) dx = 0$$

$$\Rightarrow \sqrt{2} + \alpha = 0 \qquad \left\{\because f(\cos 2x)\cos x \text{ is} \neq 0 \text{ in } \left(0, \frac{\pi}{4}\right)\right\}$$

$$\Rightarrow \alpha = -\sqrt{2}$$

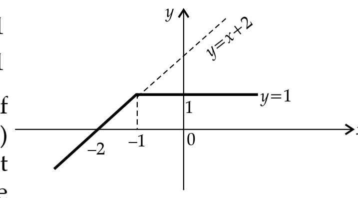
4. Option (2) is correct.

Given that $f(x) = \begin{cases} x+1, & x < 0 \\ |x-1|, & x \geq 0 \end{cases} = \begin{cases} x+1, & x < 0 \\ 1-x, & 0 \leq x < 1 \\ x-1, & x \geq 1 \end{cases}$

and $g(x) = \begin{cases} x+1, & x < 0 \\ 1, & x \geq 0 \end{cases}$

$$\Rightarrow g(f(x)) = \begin{cases} f(x)+1, & f(x) < 0 \\ 1, & f(x) \geq 0 \end{cases} = \begin{cases} x+1+1, & x+1 < 0 \\ 1, & x+1 \geq 0 \end{cases}$$

$$y = \begin{cases} x+2, & x < -1 \\ 1, & x \geq -1 \end{cases}$$

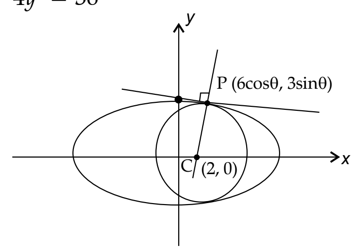


From the graph, if is clear that $g(f(x))$ is continuous but not differentiable at one point.

5. Option (4) is correct.

Given ellipse is $x^2 + 4y^2 = 36$

$$\Rightarrow \frac{x^2}{6^2} + \frac{y^2}{3^2} = 1$$



Any point on this ellipse is (6 cosθ, 3 sinθ) and eqn. of normal at this point is

$ax \sec\theta - by \operatorname{cosec}\theta = a^2 - b^2$

$\Rightarrow 6x \sec\theta - 3y \operatorname{cosec}\theta = 36 - 9 = 27$

If the circle touches the ellipse, then this normal must pass through the centre of circle i.e. (2, 0)

so $6 \times 2 \sec\theta - 0 = 27$

$$\Rightarrow \sec\theta = \frac{27}{12} = \frac{9}{4}$$

or $\cos\theta = \frac{4}{9}$ and $\sin\theta = \sqrt{1 - \frac{16}{81}} = \frac{\sqrt{65}}{9}$

So p (6 cosθ, 3 sinθ) $\equiv \left(\frac{6 \times 4}{9}, \frac{3 \times \sqrt{65}}{9}\right) \equiv \left(\frac{8}{3}, \frac{\sqrt{65}}{3}\right)$

and $CP^2 = r^2 = \left(2 - \frac{8}{3}\right)^2 + \left(0 - \frac{\sqrt{65}}{3}\right)^2$

$$= \frac{4}{9} + \frac{65}{9} = \frac{65}{9} \text{ and } 12r^2 = \frac{12 \times 69}{9} = 4 \times 23$$

$\Rightarrow 12r^2 = 92$

6. Option (3) is correct.

Given that mean = 5, variance = 10

So $\frac{1+2+4+5+x+y}{6} = 5$

$\Rightarrow x + y + 12 = 30$ or $x + y = 18$ ...(i)

Also, variance = 10

$$\Rightarrow \frac{1+4+16+25+x^2+y^2}{6} - 5^2 = 10$$

$\Rightarrow 46 + x^2 + y^2 - 6 \times 25 = 60$

$\Rightarrow x^2 + y^2 = 164$ ...(ii)

On solving equations (i) & (ii), we get

$x = 8, y = 10$

Hence, mean deviation about mean

$$= \frac{|1-5| + |2-5| + |4-5| + |5-5| + |8-5| + |10-5|}{6}$$

$$= \frac{4+3+1+0+3+5}{6} = \frac{16}{6} = \frac{8}{3}$$

7. Option (2) is correct.

Given that A = {1, 3, 4, 6, 9} & B = {2, 4, 5, 8, 10} and

$R = \{(a_1, b_1), (a_2, b_2)) : a_1 \leq b_2 \text{ and } b_1 \leq a_2\}$

Here, if $a_1 = 1, 3, 4, 6, 9$, then b_2 can take 5, 4, 4, 2, 1 choices, respectively. Also if $b_1 = 2, 4, 5, 8, 10$ then a_2 can take 4, 3, 2, 1, choices respectively.
Hence, total number of required relations are
$= (5 + 4 + 4 + 2 + 1) \times (4 + 3 + 2 + 1)$
$= 16 \times 10 = 160$

8. Option (3) is correct.

Eqn. of the plane passing through the points (5, 3, 0), (13, 3, –2) and (1, 6, 2) is given by

$$\begin{vmatrix} x-5 & y-3 & z-0 \\ 8 & 0 & -2 \\ 4 & -3 & -2 \end{vmatrix} = 0$$

or $3x - 4y + 12z = 3$ ...(i)

Now, distance of this plane from A (3, 4, α) & B(2, α, a) are 2 and 3, respectively, then

$$\frac{|9-16+12\alpha-3|}{\sqrt{9+16+144}} = 2 \text{ and } \frac{|6-4\alpha+12a-3|}{\sqrt{9+16+144}} = 3$$

$\Rightarrow |12\alpha - 10| = 26$
$\Rightarrow 12\alpha = 10 \pm 26$
$\Rightarrow \alpha = 3 \left(\alpha = \frac{-16}{12} \text{ rejected as } \alpha \in N\right)$

and $\frac{|12a+3-4\alpha|}{13} = 3$

$\Rightarrow |12a + 3 - 4 \times 3| = 3 \times 13$
$12a - 9 = \pm 39$
$\Rightarrow 12a = 9 + 39 = 48 \Rightarrow a = 4$

9. Option (2) is correct.

Given word is MATHS
If we start with A, then total words = 4! = 24
If we start with H, then total words = 4! = 24
If we start with M, then total words = 4! = 24
If we start with S, then total words = 4! = 24
If we start with TA, then total words = 3! = 6
Then, the next word is THAMS which is the required word.
Hence, the total serial number of the word THAMS is $24 \times 4 + 6 + 1 = 103$

10. Option (1) is correct.

Since $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are coplanar that $(\vec{a}-\vec{d}), (\vec{b}-\vec{d})$ and $(\vec{c}-\vec{d})$ must be coplanar.

So $[(\vec{a}-\vec{d}), (\vec{b}-\vec{d}), (\vec{c}-\vec{d})] = 0$

$\Rightarrow (\vec{a}-\vec{d}).((\vec{b}-\vec{d}) \times (\vec{c}-\vec{d})) = 0$

$\Rightarrow (\vec{a}-\vec{d}).(\vec{b}\times\vec{c} - \vec{b}\times\vec{d} - \vec{d}\times\vec{c}) = 0$

$\Rightarrow [\vec{a}\ \vec{b}\ \vec{c}] - [\vec{a}\ \vec{b}\ \vec{d}] - [\vec{a}\ \vec{d}\ \vec{c}] - [\vec{d}\ \vec{b}\ \vec{c}] = 0$

or $[\vec{a}\ \vec{b}\ \vec{c}] = [\vec{d}\ \vec{c}\ \vec{a}] + [\vec{b}\ \vec{d}\ \vec{a}] + [\vec{c}\ \vec{d}\ \vec{b}]$

11. Option (1) is correct.

Given that 3 consecutive terms are in the ratio 1 : 3 : 5. So

$^{n+2}C_{r-1} : {}^{n+2}C_r : {}^{n+2}C_{r+1} = 1 : 3 : 5$

$$\Rightarrow \frac{^{n+2}C_{r-1}}{^{n+2}C_r} = \frac{1}{3} \text{ and } \frac{^{n+2}C_r}{^{n+2}C_{r+1}} = \frac{3}{5}$$

$$\Rightarrow \frac{r}{n+2-r+1} = \frac{1}{3} \text{ and } \frac{r+1}{n+2-(r+1)+1} = \frac{3}{5}$$

$\Rightarrow 3r = n - r + 3$ and $5r + 5 = 3n - 3r + 6$
or $4r = n + 3$ ...(i)
and $8r = 3n + 1$ ...(ii)
On solving eqn. (i) and (ii), we get
$r = 2$ and $n = 5$
Hence, sum of terms $= {}^7C_1 + {}^7C_2 + {}^7C_3$
$= 7 + 21 + 35 = 63$

12. Option (1) is correct.

Given that

$$\frac{dy}{dx} + \frac{5}{x(x^5+1)}y = \frac{(x^5+1)^2}{x^7}, x > 0$$

which is a linear differential eqn.

$$\text{I.F.} = e^{\int \frac{5}{x(x^5+1)}dx} = e^{\int \frac{5}{x^6\left(1+\frac{1}{x^5}\right)}dx}$$

Let $t = 1 + \frac{1}{x^5}$

$$dt = -\frac{5}{x^6}dx \text{ or } -dt = \frac{5dx}{x^6}$$

$$\text{I.F.} = e^{-\int \frac{1}{t}dt} = e^{-\log t} = \frac{1}{t} = \frac{x^5}{1+x^5}$$

Solution of differential eqn. is given by

$$y \times \frac{x^5}{1+x^5} = \int \frac{(x^5+1)^2}{x^7} \times \frac{x^5}{(1+x^5)} dx + c$$

$$= \int \frac{x^5+1}{x^2}dx + c = \int (x^3 + x^{-2})dx + c$$

$$\Rightarrow \frac{yx^5}{1+x^5} = \frac{x^4}{4} - \frac{1}{x} + c$$

Putting $x = 1, y = 2$, we get

$$\Rightarrow \frac{2}{1+1} = \frac{1}{4} - 1 + C \Rightarrow C = \frac{7}{4}$$

$$\Rightarrow \frac{yx^5}{1+x^5} = \frac{x^4}{4} - \frac{1}{x} + \frac{7}{4}$$

Putting $x = 2$

$$\Rightarrow \frac{y \times 32}{1+32} = \frac{16}{4} - \frac{1}{2} + \frac{7}{4}$$

$$\rightarrow y = \frac{33}{32}\left[\frac{16+7-2}{4}\right] = \frac{33 \times 21}{32 \times 4}$$

$$\Rightarrow y = \frac{693}{128}$$

13. Option (1) is correct.

Converse of $((\sim p) \wedge q) \rightarrow r$ is

$\sim ((\sim p) \wedge q) \rightarrow \sim r$

$\Rightarrow (\sim(\sim p)) \vee (\sim q) \to \sim r$

$\Rightarrow p \vee (\sim q) \to \sim r$

14. Option (3) is correct.

Given that, in exp of $\left(\frac{4x}{5}-\frac{5}{2x}\right)^{2022}$

$T'_{1011} = 1024 \times T_{1011}$

$$\Rightarrow {}^{2022}C_{1010}\left(\frac{-5}{2x}\right)^{2022-1010}\left(\frac{4x}{5}\right)^{1010} = 2^{10} \times {}^{2022}C_{1010}\left(\frac{-5}{2x}\right)^{1010}\left(\frac{4x}{5}\right)^{1012}$$

$$\Rightarrow \left(\frac{-5}{2x}\right)^{2} = 2^{10} \times \left(\frac{4x}{5}\right)^{2}$$

$$\Rightarrow x^4 = \frac{5^4}{2^{16}} = \frac{5^4}{(2^4)^4} \text{ or } |x| = \frac{5}{16}$$

15. Option (4) is correct.

The augmented matrix of the given system of equations can be written as

$$[A : B] = \begin{bmatrix} 175 & 194 & 57 & : & 361 \\ 7 & 11 & \alpha & : & 13 \\ 5 & 4 & 7 & : & \beta \end{bmatrix}$$

Now, $R_2 \to 25R_2 - R_1$ and $R_3 \to 35\,R_3 - R_1$, we get

$$[A : B] \sim \begin{bmatrix} 175 & 194 & 57 & : & 361 \\ 0 & 81 & 25\alpha-57 & : & -36 \\ 0 & -54 & 188 & : & 35\beta-361 \end{bmatrix}$$

Again applying $R_2 \leftrightarrow R_3$

$$[A : B] \sim \begin{bmatrix} 175 & 194 & 57 & : & 361 \\ 0 & -54 & 188 & : & 35\beta-361 \\ 0 & 81 & 25\alpha-57 & : & -36 \end{bmatrix}$$

Applying $R_3 \to 54\,R_3 + 81R_2$, we get

$$[A : B] \sim \begin{bmatrix} 175 & 194 & 57 & : & 361 \\ 0 & -54 & 188 & : & 35\beta-361 \\ 0 & 0 & 1350\alpha+12150 & : & 2835\beta-2305 \end{bmatrix}$$

For infinite solutions

$1350\,\alpha + 12150 = 0$ and $2835\beta - 2305 = 0$

$\Rightarrow \alpha = -9$ and $\beta = 11$

$\Rightarrow \alpha + \beta + 2 = -9 + 11 + 2 = 4$

16. Option (1) is correct.

Eqn. of line passing through P (2, –1, 2) and Q (5, 3, 4) is

$$\frac{x-5}{3} = \frac{y-3}{4} = \frac{z-4}{2} = \lambda(\text{say})$$

$\Rightarrow R\,(3\lambda + 5, 4\lambda + 3, 2\lambda + 4)$

This point lies on $x - y + z = 4$

So, $3\lambda + 5 - 4\lambda - 3 + 2\lambda + 4 = 4$

$\Rightarrow \lambda = -2$

So R (–1, –5, 0)

Now, eqn. of line passing through R(–1, 5, 0) and parallel to line whose d.r.'s are (2, 2, 1) is

$$\frac{x+1}{2} = \frac{y+5}{2} = \frac{z-0}{1} = \mu(\text{say})$$

$\Rightarrow (x, y, z) \equiv (2\mu - 1, 2\mu - 5, \mu)$ which lies on $x + 2y + 3z + 2 = 0$

So $2\mu - 1 + 4\mu - 10 + 3\mu + 2 = 0 \Rightarrow 9\mu - 9 = 0$

$\Rightarrow \mu = 1$

So, point on the plane is (1, –3, 1)

Hence, the required distance is

$$= \sqrt{(1+1)^2 + (-3+5)^2 + (1-0)^2} = 3 \text{ units.}$$

17. Option (3) is correct.

Given that $f(x) = \begin{cases} e^{\min\{x^2,\, x-[x]\}}, & 0 \le x < 1 \\ e^{[x-\log_e x]}, & 1 \le x < 2 \end{cases}$

$$\Rightarrow f(x) = \begin{cases} e^{x^2}, & 0 \le x < 1 \\ e, & 1 \le x < 2 \end{cases}$$

Now, $\int_0^2 xf(x)dx = \int_0^1 xf(x)dx + \int_1^2 xf(x)dx$

$$= \int_0^1 xe^{x^2}dx + \int_1^2 x\,e\,dx$$

Let $t = x^2$

$$\frac{dt}{2} = x\,dx$$

$$\int_0^2 xf(x)dx = \int_0^1 \frac{1}{2}e^t dt + e\left[\frac{x^2}{2}\right]_1^2$$

$$= \frac{1}{2}\left[e^t\right]_0^1 + e\left[2 - \frac{1}{2}\right]$$

$$= \frac{1}{2}(e-1) + \frac{3}{2}e = \frac{e}{2} - \frac{1}{2} + \frac{3e}{2} = 2e - \frac{1}{2}$$

18. Option (2) is correct.

Given that A = $\{z \in C : \text{Re}\,(a+\bar{z}) > I_m(\bar{a}+z)\}$ and

B = $\{x \in C : \text{Re}\,(a+\bar{z}) < I_m(\bar{a}+z)\}$

Let $z = x + iy$ and $a = p + iq$

Then, Re $(a+\bar{z}) > I_m(\bar{a}+z)$

$\Rightarrow$ Re $(p + iq + x - iy) > I_m(p - iq + x + iy)$

$\Rightarrow p + x > y - q$

which is not true for all real values of p, q, x, y

$\Rightarrow S_1$ is false

Again, Re $(a+\bar{z}) < I_m(\bar{a}+z)$

$\Rightarrow p + x < y - q$

which is also not true for all real values of p, q, x, y

$\Rightarrow S_2$ is false.

19. Option (3) is correct.

Given that $\begin{vmatrix} x+1 & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda^2 \end{vmatrix} = \frac{9}{8}(103x + 81)$

Putting $x = 0$, we get

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda^2 \end{vmatrix} = \frac{9}{8}(0+81)$$

$$\Rightarrow \lambda^3 = \left(\frac{9}{2}\right)^3$$

$\Rightarrow \lambda = \frac{9}{2}$ and $\frac{\lambda}{3} = \frac{3}{2}$

So, the required quadratic equation is
or $4x^2 - 24x + 27 = 0$

20. Option (4) is correct.

Given that $f(x) = \frac{1}{\sqrt{[x]^2 - 3[x] - 10}}$

Now, $f(x)$ is defined, when

$[x]^2 - 3[x] - 10 > 0$

$\Rightarrow ([x] + 2)([x] - 5) > 0$

$\Rightarrow [x] < -2$ or $[x] > 5$

$\Rightarrow x < -2$ or $x \geq 6$

or $x \in (-\infty, -2) \cup [6, \infty)$

Section B

21. Correct answer is [16].

Given that $y = 2x^2 + 1$ which is parabola

differentiating : $\frac{dy}{dx} = 4x$

$\left.\frac{dy}{dx}\right|_{(1,3)} = 4$

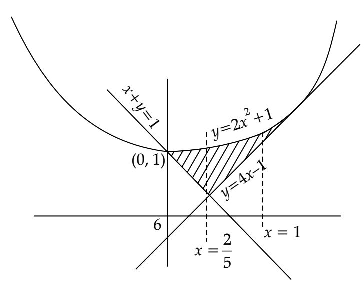
$\Rightarrow$ Eqn. of tangent at (1, 3) is $y - 3 = 4(x - 1)$

$\Rightarrow y = 4x - 1$ ...(i)

Solving line (i) with $x + y = 1$ ...(ii) we get

$x = \frac{2}{5}$ and $y = \frac{3}{5}$

which can be represented as



So, the required area is

$$A = \int_0^1 y_{\text{Parabola}}\, dx - \int_0^{2/5} y_{\text{Line (ii)}}\, dx - \int_{2/5}^{1} y_{\text{Line (i)}}\, dx$$

$$= \int_0^1 (2x^2 + 1)dx - \int_0^{\frac{2}{5}} (1 - x)dx - \int_{\frac{2}{5}}^{1} (4x - 1)dx$$

$$= \left[\frac{2x^3}{3} + x\right]_0^1 - \left[x - \frac{x^2}{2}\right]_0^{\frac{2}{5}} - \left[\frac{4x^2}{2} - x\right]_{\frac{2}{5}}^{1}$$

and $15A = 4$
or $60A = 16$

22. Correct answer is [360].

$f(1) + f(2) + 1 = f(4) \leq 6$
$f(1) + f(2) \leq 5$

Case (i) $f(1) = 1 \Rightarrow f(2) = 1, 2, 3, 4 \Rightarrow 4$ choice
Case (ii) $f(1) = 2 \Rightarrow f(2) = 1, 2, 3 \Rightarrow 3$ choice
Case (iii) $f(1) = 3 \Rightarrow f(2) = 1, 2 \Rightarrow 2$ choice
Case (iv) $f(1) = 4 \Rightarrow f(2) = 1 \Rightarrow 1$ choice
$f(3)$ and $f(5)$ both have 6 choice
Number of functions $= (4 + 3 + 2 + 1) \times 6 \times 6 = 360$

23. Correct answer is [116].

Given that $(3, \alpha)$ lies on $y^2 = 12x$
$\Rightarrow \alpha^2 = 36 \Rightarrow \alpha = \pm 6$

and $2y\frac{dy}{dx} = 12 \Rightarrow \frac{dy}{dx} = \frac{6}{y}$

or $\left.\frac{dy}{dx}\right|_{\alpha=\pm 6} = \frac{6}{\pm 6} = \pm 1$ = slope of tangent

But slope of the given line $2x + 2y = 3$ is $= -1$
So, slope of tangent m = –1 is rejected.
Also, $\alpha^2 x^2 - 9y^2 = 9\alpha^2$

$\Rightarrow \frac{x^2}{9} - \frac{y^2}{\alpha^2} = 1$

and Normal at $(\alpha - 1, \alpha + 2)$ i.e. $(6 - 1, 6 + 2) \equiv (5, 8)$ is

$\frac{9x}{5} + \frac{36y}{8} = 45$

$\Rightarrow 2x + 5y = 50$

Now, distance of (6, –4) from $2x + 5y - 50 = 0$ is

$$d = \left|\frac{2 \times 6 + 5(-4) - 50}{\sqrt{4 + 25}}\right| = \left|\frac{12 - 20 - 50}{\sqrt{29}}\right| = \frac{58}{\sqrt{29}}$$

$$d^2 = \frac{58 \times 58}{29} = 2 \times 58 = 116$$

24. Correct answer is [2].

Given that, the sum of the given series is 10, so

$$10 = 1 + \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \frac{19}{k^4} + ...\infty$$

On multiplying by $\frac{1}{k}$, we get

$$\frac{10}{k} = \frac{1}{k} + \frac{4}{k^2} + \frac{8}{k^3} + \frac{13}{k^4} + ...\infty$$

__________ on subtracting

$$10 - \frac{10}{k} = 1 + \frac{3}{k} + \frac{4}{k^2} + \frac{5}{k^3} + \frac{6}{k^4} + ...\infty$$

$$\text{or } \left(9 - \frac{10}{k}\right) = \frac{3}{k} + \frac{4}{k^2} + \frac{5}{k^3} + \frac{6}{k^4} + ...\infty$$

Again multiplying by $\frac{1}{k}$, we get

$$\left(9 - \frac{10}{k}\right)\frac{1}{k} = \frac{3}{k^2} + \frac{4}{k^3} + \frac{5}{k^4} + ...\infty$$

____________________ again subtracting

$$\left(9 - \frac{10}{k}\right)\left(1 - \frac{1}{k}\right) = \frac{3}{k} + \frac{1}{k^2} + \frac{1}{k^3} + \frac{1}{k^4} + ...\infty$$

$$\frac{(9k - 10)(k - 1)}{k^2} = \frac{3}{k} + \frac{\frac{1}{k^2}}{1 - \frac{1}{k}} = \frac{3}{k} + \frac{1}{k(k - 1)}$$

$$\frac{(9k-10)(k-1)}{k^2} = \frac{3k-3+1}{k(k-1)} = \frac{3k-2}{k(k-1)}$$

$(9k - 10)\ (k - 1)^2 = k(3k - 2)$

$(9k - 10)\ (k^2 - 2k + 1) = 3k^2 - 2k$

$9k^3 - 18k^2 + 9k - 10k^2 + 20k - 10 - 3k^2 + 2k = 0$

$9k^3 - 31k^2 + 31k = -10 = 0$

$k = 2$ satisfies the above cubic

Hence, $k = 2$.

25. Correct answer is [11].

Given that the angle between the line

$\frac{x}{1} = \frac{y-1}{2} = \frac{z-3}{\lambda}$ and plane $x + 2y + 3z = 4$ is

$$\cos^{-1}\left(\sqrt{\frac{5}{14}}\right) \text{ or } \sin^{-1}\left(\frac{3}{\sqrt{14}}\right)$$

$$= \sin^{-1}\left|\frac{1+4+3\lambda}{\sqrt{1+4+\lambda^2}.\sqrt{1+4+9}}\right|$$

$$\Rightarrow \frac{3}{\sqrt{14}} = \frac{3\lambda+5}{\sqrt{\lambda^2+5}.\sqrt{14}}$$

$\Rightarrow 9\lambda^2 + 45 = 9\lambda^2 + 25 + 30\lambda$

$\Rightarrow 20 = 30\lambda$ or $\lambda = \frac{2}{3}$

So, $l : \frac{x}{1} = \frac{y-1}{2} = \frac{z-3}{\frac{2}{3}} = t$ (say)

$$\Rightarrow (x, y, z) \equiv \left(t, 2t+1, \frac{2t}{3}+3\right)$$

If this point lies on plane $x + 2y + 3z = 4$, then $t + 4t + 2 + 2t + 9 = 4$

$\Rightarrow 7t = -7 \Rightarrow t = -1$

$$\therefore \left(-1, -1, \frac{7}{3}\right) \equiv (\alpha, \beta, \gamma)$$

So, $\alpha + 2\beta + 6\gamma = -1 - 2 + \frac{7}{3} \times 6 = 11$

26. Correct answer is [2].

To find the POI of $y = f(x)$ with x-axis,

Putting $f(x) = 0$, we get

$e^{8x} - e^{6x} - 3e^{4x} - e^{2x} + 1 = 0$

$\Rightarrow e^{4x} - e^{2x} - 3 - e^{-2x} + e^{-4x} = 0$ (dividing by e^{4x})

$\Rightarrow (e^{2x} + e^{-2x})^2 - 2 - (e^x + e^{-x})^2 + 2 - 3 = 0$

$\Rightarrow ((e^x + e^{-x}) - 2)^2 - (e^x + e^{-x})^2 - 3 = 0$

Let $t = (e^x + e^{-x})^2$ then

$(t - 2)^2 - t - 3 = 0$

$\Rightarrow t^2 + 4 - 4t - t - 3 = 0$

$\Rightarrow t^2 - 5t + 1 = 0$

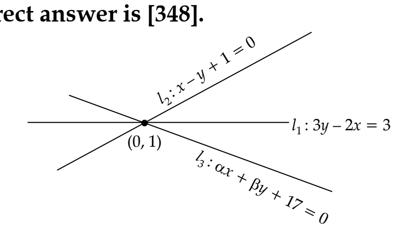
$$t = \frac{5 \pm \sqrt{25-4}}{2} = \frac{5 \pm \sqrt{21}}{2}$$

$(e^x + e^{-x})^2 = \frac{5+\sqrt{21}}{2}$ (neglecting minus sign)

$$e^x + \frac{1}{e^x} = \pm\sqrt{\frac{5+\sqrt{21}}{2}}$$

$\Rightarrow x$ has two real values.

27. Correct answer is [348].



Point of intersection of l_1 and l_2 is given by (0, 1) which lies on l_3

$\Rightarrow \alpha \times 0 + \beta \times 1 + 17 = 0 \Rightarrow \beta = -17$

Also, any point on l_2 is (1, 2) whose image w.r.t., l_1 is given by $\frac{x-1}{2} = \frac{y-2}{-3} = \frac{-2(2-6+3)}{4+9} = \frac{2}{13}$

$\Rightarrow x = 1 + \frac{4}{13} = \frac{17}{13}$ and $y = 2 - \frac{6}{13} = \frac{20}{13}$

which lies on $l_3 : \alpha x - 17y + 17 = 0$

$\Rightarrow \alpha \times \frac{17}{13} - 17 \times \frac{20}{13} + 17 = 0$

$\Rightarrow \alpha - 20 + 13 = 0$

$\Rightarrow \alpha = 7$

Hence, $\alpha^2 + \beta^2 - \alpha - \beta = 49 + 289 - 7 + 17$

$= 348$

28. Correct answer is [27].

Given quadratic $64x^2 + 5Nx + 1 = 0$ has no real roots, so $D < 0$

$\Rightarrow 25N^2 - 4 \times 64 \times 1 < 0$

$\Rightarrow N^2 - \left(\frac{16}{5}\right)^2 < 0$

$\Rightarrow \frac{-16}{5} < N < \frac{16}{5}$ and $N \in$ Natural no.

$\Rightarrow N = 1, 2, 3$

$\therefore$ Required probability $= \frac{p}{q}$

$$\Rightarrow \frac{1}{4} + \frac{3}{4} \times \frac{1}{4} + \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} = \frac{p}{q}$$

$$\Rightarrow \frac{16+12+9}{64} = \frac{p}{q} = \frac{37}{64}$$

$\Rightarrow p = 37, q = 64$

and $q - p = 27$

29. Correct answer is [285].

Given that $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = \hat{i} + \hat{j} - \hat{k}$

$\Rightarrow |\vec{a}| = \sqrt{14}, |\vec{b}| = \sqrt{3}, \vec{a}.\vec{c} = 11$

$\vec{b}.\vec{c} = -\sqrt{3}\,|\vec{b}| = -3$, and $\vec{a}.\vec{b} = 0$

Let θ be the angle between $\vec{b}$ and $\vec{a} \times \vec{c}$

then $\vec{b} \times (\vec{a} \times \vec{c}) = (\vec{b}.\vec{c})\vec{a} - (\vec{b}.\vec{a})\vec{c}$

$= -3\vec{a} - 0\vec{c}$

$\vec{b} \times (\vec{a} \times \vec{c}) = -3\vec{a}$

$\Rightarrow |\vec{b}|\,|\vec{a} \times \vec{c}| \sin\theta = |-3\vec{a}|$

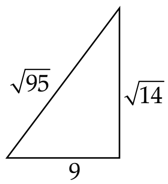
$\Rightarrow \sqrt{3}\,|\vec{a} \times \vec{c}| \sin\theta = 3 \times \sqrt{14}$

and $|\vec{b}|.|\vec{a} \times \vec{c}| \cos\theta = 27$

$\Rightarrow \sqrt{3}\,|\vec{a} \times \vec{c}| \cos\theta = 27$

On dividing $\tan\theta = \frac{\sqrt{14}}{9}$

$\Rightarrow \sin\theta = \frac{\sqrt{14}}{\sqrt{95}}$



and $\sqrt{3}\,|\vec{a} \times \vec{c}| \times \frac{\sqrt{14}}{\sqrt{95}} = 3\sqrt{14}$

$\Rightarrow |\vec{a} \times \vec{c}| = \frac{3\sqrt{14} \times \sqrt{95}}{\sqrt{3} \times \sqrt{14}} = \sqrt{3 \times 95}$

and $|\vec{a} \times \vec{c}|^2 = 285$

30. Correct answer is [1680].

Given that $\frac{z^2 + 8iz - 15}{z^2 - 3iz - 2} \in R$

$\Rightarrow \frac{(z^2 - 3iz - 2) + (11iz - 13)}{z^2 - 3iz - 2} \in R$

$\Rightarrow 1 + \frac{11iz - 13}{z^2 - 3iz - 2} \in R$ ...(i)

but $\alpha - \frac{13}{11}i \in S$

or $z = \alpha - \frac{13i}{11} = \alpha + \frac{13}{11i}$

$\Rightarrow 11iz - 13 = i\alpha$

So eqn. (i) becomes

$1 + \frac{i\alpha}{z^2 - 3iz - 2} \in R$

$\Rightarrow z^2 - 3iz - 2 \in$ Img.

Let $z = x + iy$

$\Rightarrow x^2 - y^2 - 2ixy - 3ix + 3y - 2 \in$ Img

$\Rightarrow \text{Re}\,(x^2 - y^2 + 3y - 2 - i\,(3x + 2xy)) = 0$

$\Rightarrow x^2 - y^2 + 3y - 2 = 0$

$\Rightarrow x^2 = y^2 - 3y + 2$

$\Rightarrow x^2 = (y - 1)\,(y - 2)$

$\therefore z = \alpha - \frac{13}{11}i$

Putting $x = \alpha,\ y = \frac{-13}{11}$

$\alpha^2 = \left(\frac{-13}{11} - 11\right)\left(\frac{-13}{11} - 2\right)$

$\alpha^2 = \frac{24 \times 35}{121}$

$\Rightarrow 242\alpha^2 = 48 \times 35 = 1680$

❑❑