JEE (Main) MATHEMATICS SOLVED PAPER

2023
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Section A

Q. 1. Let $y = y(x)$, $y > 0$, be a solution curve of the differential equation $(1 + x^2)\,dy = y\,(x - y)\,dx$. If $y(0) = 1$ and $y(2\sqrt{2}) = \beta$, then

(1) $e^{3\beta^{-1}} = e(5+\sqrt{2})$ **(2)** $e^{3\beta^{-1}} = e(3+2\sqrt{2})$

(3) $e^{\beta^{-1}} = e^{-2}(3+2\sqrt{2})$ **(4)** $e^{\beta^{-1}} = e^{-2}(5+\sqrt{2})$

Q. 2. Let D be the domain of the function $f(x) = \sin^{-1}\left(\log_{3x}\left(\frac{6+2\log_3 x}{-5x}\right)\right)$. If the range of the function $g : D \to R$ defined by $g(x) = x - [x]$, ($[x]$ is the greatest integer function), is (α, β), then $\alpha^2 + \frac{5}{\beta}$ is equal to

(1) 46 **(2)** 135 **(3)** 136 **(4)** 45

Q. 3. Let C be the circle in the complex plane with centre $z_0 = \frac{1}{2}(1+3i)$ and radius $r = 1$. Let $z_1 = 1 + i$ and the complex number z_2 be outside the circle C such that $|z_1 - z_0|\,|z_2 - z_0| = 1$. If z_0, z_1 and z_2 are collinear, then the smaller value of $|z_2|^2$ is equal to

(1) $\frac{7}{2}$ **(2)** $\frac{13}{2}$ **(3)** $\frac{5}{2}$ **(4)** $\frac{3}{2}$

Q. 4. Let $<a_n>$ be a sequence such that $a_1 + a_2 + \ldots + a_n = \frac{n^2+3n}{(n+1)(n+2)}$. If $28\sum_{k=1}^{10}\frac{1}{a_k} = p_1p_2p_3\ldots p_m$, where $p_1, p_2 \ldots\ldots p_m$ are the first m prime numbers, then m is equal to

(1) 8 **(2)** 5 **(3)** 6 **(4)** 7

Q. 5. Let the plane P: $4x - y + z = 10$ be rotated by an angle $\frac{\pi}{2}$ about its line of intersection with the plane $x + y - z = 4$. If α is the distance of the point $(2, 3, -4)$ from the new position of the plane P, then 35α is equal to

(1) 90 **(2)** 105 **(3)** 85 **(4)** 126

Q. 6. Among the two statements

(S1) : $(p \Rightarrow q) \wedge (p \wedge (\sim q))$ is a contradiction and

(S2) : $(p \wedge q) \vee ((\sim p) \wedge q) \vee (p \wedge (\sim q)) \vee ((\sim p) \wedge (\sim q))$ is a tautology

(1) only (S2) is true (2) only (S1) is true

(3) both are true (4) both are false

Q. 7. Two dice A and B are rolled. Let numbers obtained on A and B be α and β respectively. If the variance of $\alpha - \beta$ is $\frac{p}{q}$, where p and q are co-prime, then the sum of the positive divisior of p is equal to

(1) 36 **(2)** 31 **(3)** 48 **(4)** 72

Q. 8. The number of five digit numbers, greater then 40000 and divisible by 5, which can be formed using the digits 0, 1, 3, 5, 7 and 9 without repetition, is equal to

(1) 132 **(2)** 120 **(3)** 72 **(4)** 96

Q. 9. Let α, β be the roots of the quadratic equation $x^2 + \sqrt{6}x + 3 = 0$. Then $\frac{\alpha^{23}+\beta^{23}+\alpha^{14}+\beta^{14}}{\alpha^{15}+\beta^{15}+\alpha^{10}+\beta^{10}}$

(1) 9 **(2)** 729 **(3)** 72 **(4)** 81

Q. 10. If the local maximum value of the function $f(x) = \left(\frac{\sqrt{3}e}{2\sin x}\right)^{\sin^2 x}, x \in \left(0, \frac{\pi}{2}\right)$ is $\frac{k}{e}$, then $\left(\frac{k}{e}\right)^8 + \frac{k^8}{e^5} + k^8$ is equal to

(1) $e^3 + e^6 + e^{11}$ **(2)** $e^5 + e^6 + e^{11}$

(3) $e^3 + e^5 + e^{11}$ **(4)** $e^3 + e^6 + e^{10}$

Q. 11. Let $\lambda \in Z$, $\vec{a} = \lambda\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$. Let $\vec{c}$ be a vector such that $(\vec{a} + \vec{b} + \vec{c}) \times \vec{c} = \vec{0}, \vec{a}.\vec{c} = -17$ and $\vec{b}.\vec{c} = -20$. Then $|\vec{c} \times (\lambda\hat{i} + \hat{j} + \hat{k})|^2$ is equal to

(1) 62 **(2)** 53 **(3)** 49 **(4)** 46

Q. 12. In a triangle ABC, if $\cos A + 2\cos B + \cos C = 2$ and the lengths of the sides opposite to the angles A and C are 3 and 7 respectively, then $\cos A - \cos C$ is equal to

(1) $\frac{5}{7}$ **(2)** $\frac{9}{7}$ **(3)** $\frac{3}{7}$ **(4)** $\frac{10}{7}$

Q. 13. If the point $\left(\alpha, \frac{7\sqrt{3}}{3}\right)$ lies on the curve traced by the mid-points of the line segments of the lines $x\cos\theta + y\sin\theta = 7$, $\theta \in \left(0, \frac{\pi}{2}\right)$ between the co-ordinates axes, then α is equal to

(1) $7\sqrt{3}$ **(2)** -7 **(3)** 7 **(4)** $-7\sqrt{3}$

Q. 14. Let $P\left(\frac{2\sqrt{3}}{\sqrt{7}}, \frac{6}{\sqrt{7}}\right)$, Q, R, and S be four points on the ellipse $9x^2 + 4y^2 = 36$. Let PQ and RS be mutually perpendicular and pass through the origin. If $\frac{1}{(PQ)^2}+\frac{1}{(RS)^2}=\frac{p}{q}$, where p and q are coprime, then $p + q$ is equal to

(1) 137 **(2)** 143 **(3)** 157 **(4)** 147

Q. 15. Let the lines $l_1:\frac{x+5}{3}=\frac{y+4}{1}=\frac{z-\alpha}{-2}$ $l_2: 3x + 2y + z - 2 = 0 = x - 3y + 2z - 13$ be coplanar. If the point P(a, b, c) on l_1 is nearest to the point Q(–4, –3, 2), then $|a| + |b| + |c|$ is equal to

(1) 10 **(2)** 8 **(3)** 12 **(4)** 14

Q. 16. If $\frac{1}{n+1}{}^nC_n+\frac{1}{n}{}^nC_{n-1}+...+\frac{1}{2}{}^nC_1+{}^nC_0=\frac{1023}{10}$ then n is equal to

(1) 7 **(2)** 9 **(3)** 6 **(4)** 8

Q. 17. The sum, of the coefficients of the first 50 terms in the binomial expansion of $(1 - x)^{100}$, is equal to

(1) $-{}^{101}C_{50}$ **(2)** ${}^{99}C_{49}$ **(3)** ${}^{100}C_{50}$ **(4)** $-{}^{99}C_{49}$

Q. 18. Let a, b, c be three distinct real numbers, none equal to one. If the vectors $a\hat{i}+\hat{j}+\hat{k}, \hat{i}+b\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+c\hat{k}$ are coplanar, then $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}$ is equal to

(1) 1 **(2)** 2 **(3)** –2 **(4)** –1

Q. 19. Let $A=\begin{bmatrix}1 & \frac{1}{51}\\ 0 & 1\end{bmatrix}$. If $B=\begin{bmatrix}1 & 2\\ -1 & -1\end{bmatrix}A\begin{bmatrix}-1 & -2\\ 1 & 1\end{bmatrix}$, then the sum of all the elements of the matrix $\sum_{n=2}^{50}B^n$ is equal to

(1) 50 **(2)** 75 **(3)** 125 **(4)** 100

Q. 20. The area of the region enclosed by the curve $y = x^3$ and its tangent at the point (–1, –1) is

(1) $\frac{23}{4}$ **(2)** $\frac{19}{4}$ **(3)** $\frac{31}{4}$ **(4)** $\frac{27}{4}$

Section B

Q. 21. Let the positive numbers a_1, a_2, a_3, a_4 and a_5 be in a G.P. Let their mean and variance be $\frac{31}{10}$ and $\frac{m}{n}$ respectively, where m and n are co-prime. If the mean of their reciprocal is $\frac{31}{40}$ and $a_3 + a_4 + a_5 =$ 14, then $m + n$ is equal to _____.

Q. 22. Let $D_k=\begin{vmatrix}1 & 2k & 2k-1\\ n & n^2+n+2 & n^2\\ n & n^2+n & n^2+n+2\end{vmatrix}$. If $\sum_{k=1}^{a}D_k=$ 96, then n is equal to

Q. 23. If $\int_{-0.15}^{0.15}|100x^2-1|dx=\frac{k}{3000}$. then k is equal to _____.

Q. 24. Two circles in the first quadrant of radii r_1 and r_2 touch the coordinate axes. Each of them cuts off an intercept of 2 units with the line $x + y = 2$. Then $r_1^2+r_2^2-r_1r_2$ is equal to _____.

Q. 25. A fair $n(n > 1)$ faces die is rolled repeatedly until a number less than n appears. If the mean of the number of tosses required is $\frac{n}{9}$, then n is equal to ______.

Q. 26. Let the digits a, b, c be in A.P. Nine-digit numbers are to be formed using each of these three digits thrice such that three consecutive digits are in A.P. at least once. How many such numbers can be formed?

Q. 27. Let the plane $x + 3y - 2z + 6 = 0$ meet the co-ordinate axes at the points A, B, C. If the orthocenter of the triangle ABC is $\left(\alpha,\beta,\frac{6}{7}\right)$, then $98(\alpha + \beta)^2$ is equal to _____.

Q. 28. The number of the relations, on the set {1, 2, 3} containing (1, 2) and (2, 3), which are reflexive and transitive but not symmetric, is _______.

Q. 29. Let $[x]$ be the greatest integer $\le x$. Then the number of points in the interval (–2, 1), where the function $f(x)=|[x]|+\sqrt{x-[x]}$ is discontinuous, is _______.

Q. 30. Let $I(x)=\int\sqrt{\frac{x+7}{x}}dx$ and $I(9) = 12 + 7\log_e 7$. If $I(1) = \alpha+7\log_e(1+2\sqrt{2})$, then α^4 is equal to _________.

Answer Key

Q. No.	Answer	Topic name	Chapter name
1	(2)	Linear differential equation	Differential equation
2	(2)	Domain	Inverse trigonometric function
3	(3)	Geometrical problem	Complex number
4	(3)	Telescopic series	Sequence and series
5	(4)	Family of planes	3D
6	(3)	Truth table	Mathematical reasoning

Q. No.	Answer	Topic name	Chapter name
7	(3)	Binomial distribution	Probability
8	(2)	Restricted permutations	Permutations and combination
9	(4)	Demovires theorem	Complex number
10	(1)	Logarithmic differentiation	Differentiation
11	(4)	Product of two vectors	Vector
12	(4)	Cosine rules	Properties of triangle
13	(3)	Locus	Straight lines
14	(3)	Equation of ellipse	Ellipse
15	(1)	Family of planes	3D
16	(2)	Properties of binomial cofficients	Binomial theorem
17	(4)	Properties of binomial cofficients	Binomial theorem
18	(1)	Properties of determinants	Determinanants
19	(4)	Positive integral power of a matrix	Matrix
20	(4)	Area b/w two curves	Area under the curves
21	[211]	Geometric progression	Sequence and series
22	[6]	Determinant using properties	Determinant
23	[575]	Definite integral using properties	Definite integral
24	[7]	Chord of circle	Circle
25	[10]	Special series	Sequence and series
26	[1260]	Restricted permutations	Permutations and combination
27	[288]	Triangle	3D
28	[2]	Type of relation	Relation and function
29	[2]	Continuity of a function	Continuity and differentiability
30	[64]	Special integral	Indefinite integral

Solutions

Section A

1. Option (2) is correct.

Given that

$(1 + x)^2 dy = y(x - y)\,dx,\ y(0) = 1$

$$\Rightarrow \frac{dy}{dx} = \frac{xy}{1+x^2} - \frac{y^2}{1+x^2}$$

$$\text{or } \frac{dy}{dx} + \left(\frac{-x}{1+x^2}\right)y = \left(\frac{-1}{1+x^2}\right)y^2$$

$$\text{or } \frac{1}{y^2}\frac{dy}{dx} + \left(\frac{-x}{1+x^2}\right)\frac{1}{y} = \frac{-1}{1+x^2}$$

$$\text{putting } \frac{1}{y} = t \Rightarrow \frac{1}{y^2}\frac{dy}{dx} = \frac{-dt}{dx}$$

$$\Rightarrow \frac{-dt}{dx} - \frac{x}{1+x^2}\times t = \frac{-1}{1+x^2}$$

$$\text{or } \frac{dt}{dx} + \left(\frac{x}{1+x^2}\right)t = \frac{1}{1+x^2}$$

which is LDE, where I.F. $= e^{\int \frac{x}{1+x^2}dx}$

$$\text{or I.F.} = e^{\frac{1}{2}\log(1+x^2)} = (1+x^2)^{\frac{1}{2}}$$

and solution of eqn. (i) is given by-

$$t \times \sqrt{1+x^2} = \int \frac{1}{1+x^2} \times \sqrt{1+x^2}\,dx + C$$

$$= \log(x + \sqrt{1+x^2}) + C$$

Putting $x = 0$, $y = 1$, we get

$1 = \log(1) + C \Rightarrow C = 1$

$$\text{So } \frac{\sqrt{1+x^2}}{y} = \log(x + \sqrt{1+x^2}) + 1$$

Putting $x = 2\sqrt{2}$ and $y = \beta$, we get

$$\frac{3}{\beta} = \log(2\sqrt{2} + 3) + 1 = \log(e(3 + 2\sqrt{2}))$$

$$\text{or } e^{3\beta^{-1}} = e(3 + 2\sqrt{2})$$

2. Option (2) is correct.

Given that D be the domain of

$$f(x)=\sin^{-1}\left(\log_{3x}\left(\frac{6+2\log_3 x}{-5x}\right)\right)$$

so $\dfrac{6+2\log_3 x}{-5x}>0, 3x>0, 3x\neq 1,$

$$-1\leq \log_{3x}\left(\frac{6+2\log_e x}{-5x}\right)\leq 1$$

$\Rightarrow x>0, x\neq \dfrac{1}{3}\Rightarrow 6+2\log_3 x<0$

or $x<3^{-3}=\dfrac{1}{27}$

and $\log_{3x}\left(\dfrac{6+2\log_3 x}{-5x}\right)\leq 1\Rightarrow \dfrac{6+2\log_3 x}{-5x}\geq 3x$

$$\left(\because x<\frac{1}{27}\right)$$

or $6+2\log_3 x\leq -15x^2$

or $15x^2 + 2\log_3 x + 6 \leq 0$

also $\log_{3x}\left(\dfrac{6+2\log_3 x}{-5x}\right)\geq -1\Rightarrow \dfrac{6+2\log_3 x}{-5x}\leq \dfrac{1}{3x}$

$\Rightarrow 6+2\log_3 x\geq \dfrac{-5}{3}$

$\Rightarrow 2\log_3 x\geq \dfrac{-23}{3}$ or $x\geq 3^{\frac{-23}{6}}$

So, concluding the above inequalities, we get

$$x\in\left[3^{\frac{-23}{6}},\frac{1}{27}\right)$$

and range of $g(x) = x - [x] = \{x\}$ depends on D as (α, β)

so $\alpha=3^{\frac{-23}{6}}$ and $\beta=\dfrac{1}{27}$

$$\alpha^2+\frac{5}{\beta}=3^{\frac{-23}{3}}+\frac{5}{\frac{1}{27}}=\frac{1}{3^{23/3}}+135$$

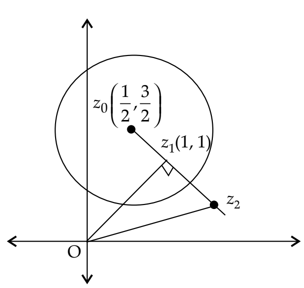
Which is just greater than 135.

3. Option (3) is correct.

Given that $z_0=\dfrac{1+3i}{2}$ is centre and $r = 1$ is radius of circle C and $z_1 = 1 + i$

Now $|z_1-z_0| = \left|1+i-\dfrac{1}{2}-\dfrac{3}{2}i\right|=\left|\dfrac{1}{2}-\dfrac{1}{2}i\right|=\sqrt{\dfrac{1}{4}+\dfrac{1}{4}}=\dfrac{1}{\sqrt{2}}$

$\Rightarrow |z_1-z_0|=\dfrac{1}{\sqrt{2}}$



so $|z_1-z_0|\ |z_2-z_0| = 1$

$\Rightarrow |z_2-z_0|=\sqrt{2}$

Now $z_1z_2 = z_0z_2 - z_0z_1$

or $|z_1 - z_2| = |z_0 - z_1| - |z_0 - z_1|$

$$=\sqrt{2}-\frac{1}{\sqrt{2}}$$

and in ΔOz_1z_2

$|z_2|^2 = |z_1 - 0|^2 + |z_1 - z_2|^2$

$$=(\sqrt{2})^2+\left(\sqrt{2}-\frac{1}{\sqrt{2}}\right)^2=2+2+\frac{1}{2}-2=\frac{5}{2}$$

Hence, value of $|z_2|^2=\dfrac{5}{2}$

4. Option (3) is correct.

Given that

$$a_1 + a_2 + ... + a_n = \frac{n^2+3n}{(n+1)(n+2)}=S_n$$

So, $a_n = S_n - S_{n-1} = \dfrac{n^2+3n}{(n+1)(n+2)}=-\dfrac{(n-1)^2+3(n-1)}{n(n+1)}$

$$=\frac{1}{n(n+1)(n+2)}[n^3+3n^2-(n+2)(n^2-2n+1)$$
$$-3(n-1)(n+2)$$

$$=\frac{n^3+3n^2-n^3+2n^2-n-2n^2+4n-2-3n^2-3n+6}{n(n+1)(n+2)}$$

$$a_n=\frac{4}{n(n+1)(n+2)}$$

Now $28\displaystyle\sum_{k=1}^{10}\frac{1}{a_k}=\frac{28}{4}\sum_{k=1}^{10}k(k+1)(k+2)$

$$=\frac{7}{4}\sum_{k=1}^{10}\ [k(k+1)(k+2)(k+3)-(k-1)k(k+1)(k+2)]$$

$$=\frac{7}{4}[10\times 11\times 12\times 13]=2\times 3\times 5\times 7\times 11\times 13$$

$= p_1 \times p_2 \times p_3 \times .... \times p_m$

$\Rightarrow m = 6$

5. Option (4) is correct.

Given that P : $4x - y + z = 10$ is rotated by the angle 90° about line of intersection with plane $x + y - z = 40$, So by using the concept of family of planes, we have

$(4x - y + z - 10) + \lambda(x + y - z - 40) = 0$

or $(4 + \lambda)\ x + (-1 + \lambda)\ y + (1 - \lambda)\ z - 10 - 40\ \lambda = 0$

this plane is $\perp$ to plane $4x - y + z = 10$

so $4\ (4 + \lambda) - 1\ (-1 + \lambda) + 1\ (1 - \lambda) = 0$

$16 + 4\ \lambda + 1 - \lambda + 1 - \lambda = 0$

$2\lambda = -18 \Rightarrow \lambda = -9$

So, the new plane is

$-5x - 10y + 10z - 10 + 36 = 0$

$\Rightarrow x + 2y - 2z = \dfrac{26}{5}$

and $\alpha=\dfrac{\left|2+6+8-\frac{26}{5}\right|}{\sqrt{1+4+4}}=\dfrac{54}{5\times 3}$

$\Rightarrow 35\ \alpha = 126$

6. Option (3) is correct.

Given that $S_1 : (p \Rightarrow q) \wedge (p \wedge (\sim q))$

p	q	$p \Rightarrow q$	$\sim q$	$p \wedge (\sim q)$	S_1
T	T	T	F	F	F
T	F	F	T	T	F
F	T	T	F	F	F
F	F	T	T	F	F

$\Rightarrow S_1$ is a contradiction

again S_2: $(p \wedge q) \vee ((\sim p) \wedge q) \vee (p \wedge (\sim q)) \vee ((\sim p) \wedge (\sim q))$

p	q	$\sim p$	$\sim q$	$p \wedge q$	$p \wedge (\sim q)$	$(\sim p) \wedge q$	$(\sim p) \wedge (\sim q)$	S_2
T	T	F	F	T	F	F	F	T
T	F	F	T	F	T	F	F	T
F	T	T	F	F	F	T	F	T
F	F	T	T	F	F	F	T	T

$\Rightarrow S_2$ is a tautology.

7. **Option (3) is correct.**

Given that α & β are the numbers represent on A and B respectively.

So $\alpha - \beta$ can have the value 0, ± 1, ± 2, ± 3, ± 4, ± 5

If $(\alpha - \beta) = 0$ then there are 6 possible cases i.e., (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6). So P $(x = 0) = \frac{6}{36}$

Similarly if $\alpha - \beta = \pm 1$ then for both cases $P(x = \pm 1) = \frac{5}{36}$

$$P(X=\pm 2)=\frac{4}{36}, P(X=\pm 3)=\frac{3}{36}, P(X=\pm 4)=\frac{2}{36}$$

and $P(X=\pm 5)=\frac{1}{36}$

Now variance $= \frac{p}{q} \Rightarrow \sum x^2 P(x)=\frac{p}{q}$

$$\Rightarrow 0\times\frac{6}{36}+2\left[\frac{1^2\times 5}{36}+\frac{2^2\times 4}{36}+\frac{3^2\times 3}{36}+\frac{4^2\times 2}{36}+\frac{5^3\times 1}{36}\right] = \frac{p}{q}$$

$$\Rightarrow \frac{p}{q}=\frac{35}{6}$$

$\Rightarrow p = 35$ and divisor of 35 are 1, 5, 7, 35

Hence, sum of divisors are 1 + 5 + 7 + 35 = 48

8. **Option (2) is correct.**

Given numbers are 0, 1, 3, 5, 7, 9

To make numbers greater than 40000

if zero at unit place than $\underline{3} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 72$

if five at unit place than $\underline{2} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 48$

Hence, total number = 72 + 48 = 120

9. **Option (4) is correct.**

Given that $x^2+\sqrt{6}x+3=0$

$$\Rightarrow x=\frac{-\sqrt{6}\pm\sqrt{6-4\times 3}}{2\times 1}=\frac{-\sqrt{6}\pm\sqrt{-6}}{2}=\frac{\sqrt{-6}\pm i\sqrt{6}}{2}$$

$$=\frac{\sqrt{6}}{2}(-1\pm i)=\sqrt{3}\left(\frac{-1}{\sqrt{2}}\pm\frac{i}{\sqrt{2}}\right)$$

$$=\sqrt{3}\left(\cos\frac{3\pi}{4}\pm i\sin\frac{3\pi}{4}\right)=\sqrt{3}e^{\pm 3i\frac{\pi}{4}}$$

$$\therefore \alpha=\sqrt{3}\,e^{3i\frac{\pi}{4}} \text{ and } \beta=\sqrt{3}e^{-3i\frac{\pi}{4}}$$

Now $\dfrac{\alpha^{23}+\beta^{23}+\alpha^{14}+\beta^{14}}{\alpha^{15}+\beta^{15}+\alpha^{10}+\beta^{10}} =$

$$\frac{(\sqrt{3})^{23}\left\{2\cos\frac{69\pi}{4}\right\}+(\sqrt{3})^{14}\left\{2\cos\frac{42\pi}{4}\right\}}{(\sqrt{3})^{15}\left\{2\cos\frac{45\pi}{4}\right\}+(\sqrt{3})^{10}\left\{2\cos\frac{30\pi}{4}\right\}}$$

$$=\frac{(\sqrt{3})^{23}\left\{2\left(-\frac{1}{2}\right)\right\}+(\sqrt{3})^{14}\{0\}}{(\sqrt{3})^{15}\left\{2\left(-\frac{1}{\sqrt{2}}\right)\right\}+(\sqrt{3})^{10}\{0\}}=(\sqrt{3})^8=(3)^4=81$$

10. **Option (1) is correct.**

Given that $y=f(x)=\left(\frac{\sqrt{3}e}{2\sin x}\right)^{\sin^2 x}, x\in\left(0,\frac{\pi}{2}\right)$

$$\Rightarrow \log y=\sin^2 x\log\left(\frac{\sqrt{3}e}{2\sin x}\right)$$

$$\frac{1}{y}\frac{dy}{dx}=2\sin x\cos x\log\left(\frac{\sqrt{3}e}{2\sin x}\right)+\sin^2 x\times\frac{2\sin x}{\sqrt{3}e}\times\frac{\sqrt{3}e}{2}(-\operatorname{cosec}x\cot x)$$

$$\frac{1}{y}\frac{dy}{dx}=\sin x\cos x\left[2\log\left(\frac{\sqrt{3}e}{2\sin x}\right)-1\right]=0$$

(for maxima or minima)

$$\Rightarrow 2\log\left(\frac{\sqrt{3}e}{2\sin x}\right)=1$$

or $\log\dfrac{3e}{4\sin^2 x}=1\Rightarrow\dfrac{3e}{4\sin^2 x}=e$

$$\Rightarrow \sin x=\frac{\sqrt{3}}{2}$$

so, minimum value of $f(x)$ is $\left(\dfrac{\sqrt{3}e}{2\times\frac{\sqrt{3}}{2}}\right)^{\frac{3}{4}}=e^{\frac{3}{8}}$

so $e^{\frac{3}{8}}=\dfrac{k}{e}$ (given)

$\Rightarrow k^8 = e^{11}$

and $\left(\dfrac{k}{e}\right)^8+\dfrac{k^8}{e^5}+k^8=\dfrac{e^{11}}{e^8}+\dfrac{e^{11}}{e^5}+e^{11}$

$= e^3 + e^6 + e^{11}$

11. **Option (4) is correct.**

$\vec{a}=\lambda\hat{i}+\hat{j}-\hat{k}, \vec{b}=3\hat{i}-\hat{j}+2\hat{k}$

$(\vec{a}+\vec{b}+\vec{c})\times\vec{c}=\vec{0}, \vec{a}.\vec{c}=-17, \vec{b}.\vec{c}=-20$

$\Rightarrow (\vec{a}+\vec{b})\times\vec{c}+\vec{c}\times\vec{c}=0$

$\Rightarrow (\vec{a}+\vec{b})\times\vec{c}=0$

$\Rightarrow \vec{a}+\vec{b}$ is parallel to $\vec{c}$

so $\vec{c}=\mu(\vec{a}+\vec{b})$

$\vec{c}=\mu((\lambda+3)\hat{i}+\hat{k})$

$\vec{c}=\mu(\lambda+3)\hat{i}+\mu\hat{k}$

$\vec{b}.\vec{c}=-20 \Rightarrow 3\mu(\lambda+3)+2\mu=-20$ ...(i)

and $\vec{a}.\vec{c}=-17 \Rightarrow \lambda\mu(\lambda+3)-\lambda=-17$ ...(ii)

from (i) & (ii) $\lambda = 3$ and $\mu = -1$

$\Rightarrow \vec{c}=-6\hat{i}-\hat{k}$

Now $\vec{c}\times(\lambda\hat{i}+\hat{j}+\hat{k})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -6 & 0 & -1\\ 3 & 1 & 1\end{vmatrix}=\hat{i}+3\hat{j}-6\hat{k}$

and $\left|\vec{c}\times(\lambda\hat{i}+\hat{j}+\hat{k})\right|^2=(1+9+36)=46$

12. Option (4) is correct.

Given that cosA + 2 cosB + cosC = 2

$\Rightarrow$ cosA + cosC = 2 (1 – cosB)

$\Rightarrow 2\cos\left(\frac{A+C}{2}\right).\cos\left(\frac{A-C}{2}\right)=2\times 2\sin^2\frac{B}{2}$

$\Rightarrow 2\sin\frac{B}{2}.\cos\left(\frac{A-C}{2}\right)=4\sin^2\frac{B}{2}$

$\Rightarrow \cos\left(\frac{A-C}{2}\right)=2\sin\frac{B}{2}$

$\Rightarrow 2\cos\frac{B}{2}\cos\left(\frac{A-C}{2}\right)=4\sin\frac{B}{2}\cos\frac{B}{2}$

$\Rightarrow 2\sin\left(\frac{A+C}{2}\right)\cos\left(\frac{A-C}{2}\right)=2\sin B$

$\Rightarrow$ sinA + sinC = 2 sinB

$\Rightarrow ka + kc = 2 \times kb$

$\Rightarrow a + c = 2b \Rightarrow 3 + 7 = 2b \Rightarrow b = 5$

Now $\cos A-\cos C=\frac{b^2+c^2-a^2}{2bc}-\frac{a^2+b^2-c^2}{2ab}$

$=\frac{25+49-9}{2\times5\times7}-\frac{9+25-49}{2\times3\times5}$

$=\frac{65}{70}+\frac{15}{30}=\frac{13}{14}+\frac{1}{2}=\frac{20}{14}$

$\cos A-\cos C=\frac{10}{7}$

13. Option (3) is correct.

Given that $x\cos\theta + y\sin\theta = 7$

or $\frac{x}{\frac{7}{\cos\theta}}+\frac{y}{\frac{7}{\sin\theta}}=1$

Let (h, k) be the mid point between the axis, then

$h=\frac{7}{2\cos\theta}, k=\frac{7}{2\sin\theta}$

But $\left(\alpha,\frac{7\sqrt{3}}{3}\right)$ lies on curve, then $\frac{7}{2\sin\theta}=\frac{7\sqrt{3}}{3}$

$\Rightarrow \sin\theta=\frac{\sqrt{3}}{2}\Rightarrow\theta=\frac{\pi}{3}$

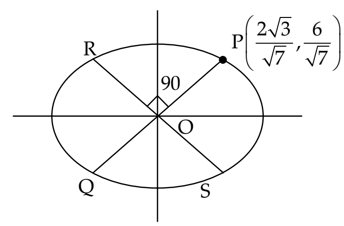
$\alpha=\frac{7}{2\cos\theta}=\frac{7}{2\cos\frac{\pi}{3}}=\frac{7}{2\times\frac{1}{2}}=7$

14. Option (3) is correct.

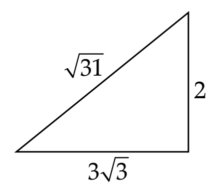
Given that $9x^2 + 4y^2 = 36$

$\Rightarrow \frac{x^2}{2^2}+\frac{y^2}{3^2}=1$ and any point on the ellipse can be taken as R (2cosθ, 3 sinθ)

Since, OP ⊥ OR



$\therefore \frac{3\sin\theta}{2\cos\theta}\times\frac{\frac{6}{\sqrt{7}}}{\frac{2\sqrt{3}}{\sqrt{7}}}=-1$



$\Rightarrow \tan\theta=\frac{-2}{3\sqrt{3}}$

$\Rightarrow R\left(2\times\frac{3\sqrt{3}}{\sqrt{31}},\frac{-3\times2}{\sqrt{31}}\right)$

So $\frac{1}{(PQ)^2}+\frac{1}{(RS)^2}=\frac{1}{4}\left(\frac{1}{(OP)^2}+\frac{1}{(OR)^2}\right)$

$=\frac{1}{4}\left(\frac{1}{\frac{12}{7}+\frac{36}{7}}+\frac{1}{\frac{108}{31}+\frac{36}{31}}\right)=\frac{1}{4}\left(\frac{7}{48}+\frac{31}{144}\right)$

$=\frac{13}{144}=\frac{p}{q}$

$\Rightarrow p = 13, q = 144$ and $p + q = 157$

15. Option (1) is correct.

Given that $l_2: 3x + 2y + z - 2 = 0 = x - 3y + 2z - 13$

Eqn. of family of plane is

$(3x + 2y + z - 2) + \lambda(x - 3y + 2z - 13) = 0$

If line $l_1: \frac{x+5}{3}=\frac{y+4}{1}=\frac{z-\alpha}{-2}$ is coplanar with above plane, then

3 (3+ λ) + (2 – 3 λ) – 2(1 + 2λ) = 0

9 + 3λ + 2 – 3λ – 2 – 4λ = 0

$4\lambda = 9 \Rightarrow \lambda=\frac{9}{4}$

also point (–5, –4, α) lies on the above plane

So $(-15-8+\alpha-2)+\frac{9}{4}(-5+12+2\alpha-13)=0$

$\Rightarrow -100 + 4\alpha - 54 + 18\alpha = 0$

$\Rightarrow \alpha = 7$

So $l_2: \frac{x+5}{3}=\frac{y+4}{1}=\frac{z-7}{-2}=k$

then P $(a, b, c) \equiv (3k-5, k-4, -2k+7)$

If P is nearest to Q $(-4, -3, 2)$

the PQ $\perp l_1$

$\Rightarrow 3(3k-1) + 1(k-1) - 2(-2k+5) = 0$

$\Rightarrow 9k - 3 + k - 1 + 4k - 10 = 0$

$\Rightarrow 14k - 14 = 0$

$\Rightarrow k = 1$

$\therefore$ P $(a, b, c) \equiv (-2, -3, 5)$

Hence, $|a| + |b| + |c| = 4 + 3 + 5 = 10$

16. Option (2) is correct.

Given that

$$\frac{1}{n+1}{}^nC_n+\frac{1}{n}{}^nC_{n-1}+....+\frac{1}{2}{}^nC_1+{}^nC_0=\frac{1023}{10}$$

$$\sum_{r=0}^{n}\frac{{}^nC_r}{r+1}=\frac{1023}{10}$$

$$\sum_{r=0}^{n}\frac{(r+1)}{(n+1)}\frac{{}^{n+1}C_{r+1}}{(r+1)}=\frac{1023}{10}$$

$$\Rightarrow \frac{1}{n+1}\sum_{r=0}^{n}{}^{n+1}C_{r+1}=\frac{1023}{10}$$

$$\Rightarrow \frac{1}{n+1}(2^{n+1}-1)=\frac{1023}{10}$$

$$\Rightarrow \frac{2^{n+1}-1}{n+1}=\frac{1024-1}{10}=\frac{2^{10}-1}{10}$$

On comparing, $n + 1 = 10 \Rightarrow n = 9$

17. Option (4) is correct.

Sum of the coefficients of the first 50 terms in expansion of $(1-x)^{100}$ is

$$=\frac{-1}{2}\times{}^{100}C_{50}=-\frac{1}{2}\times\frac{100!}{50!\times 50!}$$

$$=-\frac{1}{2}\times\frac{100\times 99!}{50\times 49!\times 50!}=-{}^{99}C_{49}$$

18. Option (1) is correct.

If $a\hat{i}+\hat{j}+\hat{k}, \hat{i}+b\hat{j}+\hat{k}\ \&\ \hat{i}+\hat{j}+c\hat{k}$ are coplanar, then

$$\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix}=0$$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$, we get

$$\begin{vmatrix} a & 1-a & 1-a \\ 1 & b-1 & 0 \\ 1 & 0 & c-1 \end{vmatrix}=0$$

$$\frac{a}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=0$$

$$\Rightarrow -1+\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=0$$

$$\Rightarrow \frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=1$$

19. Option (4) is correct.

Given that $A=\begin{bmatrix} 1 & \frac{1}{51} \\ 0 & 1 \end{bmatrix} \Rightarrow A^n=\begin{bmatrix} 1 & \frac{n}{51} \\ 0 & 1 \end{bmatrix}$

and $B=\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}A\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$

Here $=\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}\times\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}=I$

So $B^n=\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}A^n\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$

$$B^n=\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & \frac{n}{51} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$$

$$\Rightarrow B^n=\begin{bmatrix} 1 & \frac{n}{51}+2 \\ -1 & \frac{-n}{51}-1 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1+\frac{n}{51}+2 & -2+\frac{n}{51}+2 \\ 1-\frac{n}{51}-1 & 2-\frac{n}{51}-1 \end{bmatrix}$$

$$B^n=\begin{bmatrix} 1+\frac{n}{51} & \frac{n}{51} \\ \frac{-n}{51} & 1-\frac{n}{51} \end{bmatrix}$$

$$\sum_{n=1}^{50}B^n=\begin{bmatrix} \Sigma\left(1+\frac{n}{51}\right) & \Sigma\left(\frac{n}{51}\right) \\ \Sigma\left(-\frac{n}{51}\right) & \Sigma\left(1-\frac{n}{51}\right) \end{bmatrix}$$

So, sum of elements $=\sum_{n=1}^{50}\left[1+\frac{n}{51}+\frac{n}{51}-\frac{n}{51}+1-\frac{n}{51}\right]$

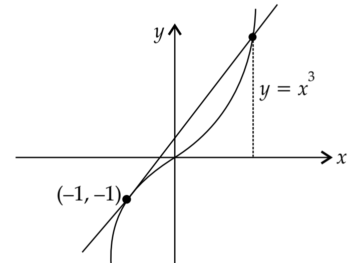
$$=\sum_{n=1}^{50}(2)=50\times 2=100$$

20. Option (4) is correct.

$y = x^3$ ...(i)

$\frac{dy}{dx}=3x^2$

$\left.\frac{dy}{dx}\right|_{(-1,-1)}=3$



Eqn of tangent is

$y + 1 = 3(x + 1)$

or $y = 3x + 2$ ...(ii)

Solving (i) & (ii), we get $x = 2$

So required area A =

$$\int_{-1}^{2}(y_{\text{Upper curve}} - y_{\text{Lower curve}})dx$$

$$\Rightarrow \text{A}=\int_{-1}^{2}(3x+2-x^3)dx = \left[\frac{3x^2}{2}+2x-\frac{x^4}{4}\right]_{-1}^{2}$$

$$=\left[(6+4-4)-\left(\frac{3}{2}-2-\frac{1}{4}\right)\right] = 6+\frac{3}{4}=\frac{27}{4}$$

Section B

21. Correct answer is [211].

Let the numbers are $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$ then mean $= \frac{31}{10}$

$$\Rightarrow \frac{1}{5}\left[\frac{a}{r^2}+\frac{a}{r}+a+ar+ar^2\right]=\frac{31}{10}$$

$$\Rightarrow \frac{a}{r^2}[1+r+r^2+r^3+r^4]=\frac{31}{10}\times 5 \quad \text{...(i)}$$

and mean of reciprocals $= \frac{31}{40}$

$$\Rightarrow \frac{1}{5}\left[\frac{r^2}{a}+\frac{r}{a}+\frac{1}{a}+\frac{1}{ar}+\frac{1}{ar^2}\right]=\frac{31}{40}$$

$$\Rightarrow \frac{1}{ar^2}[r^4+r^3+r^2+r+1]=\frac{31}{40}\times 5 \quad \text{...(ii)}$$

from (i) & (ii)

$$\frac{a\times ar^2}{r^2}=\frac{31\times 5}{10}\times\frac{40}{31\times 5}=4$$

$\Rightarrow a^2 = 4 \Rightarrow a = 2$

also $a_3 + a_4 + a_5 = 14$

$\Rightarrow a + ar + ar^2 = 14 \Rightarrow r = 2$

So the numbers are $\frac{1}{2}, 1, 2, 4, 8$

and variance $= \frac{\sum x^2}{\text{N}}\left(\frac{\sum x}{\text{N}}\right)^2$

$$=\frac{\left(\frac{1}{4}+1+4+16+64\right)}{5}-\left(\frac{31}{10}\right)^2$$

$$=\frac{341}{20}-\frac{961}{100}=\frac{744}{100}$$

$$\Rightarrow \text{Variance } = \frac{186}{25}=\frac{m}{n}$$

$\Rightarrow m = 186, n = 25$ and $m + n = 211$

22. Correct answer is [6].

$$\text{D}_k = \begin{vmatrix} 1 & 2k & 2k-1 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$$

$$\sum_{k=1}^{n}\text{D}_k = \begin{vmatrix} \Sigma 1 & 2\Sigma k & 2\Sigma k - \Sigma 1 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix} = 96$$

$$\Rightarrow \begin{vmatrix} n & n(n+1) & n(n+1)-n \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix} = 96$$

$\text{R}_2 \to \text{R}_2 - \text{R}_1$ & $\text{R}_3 \to \text{R}_3 - \text{R}_1$

$$\Rightarrow \begin{vmatrix} n & n^2+n & n^2 \\ 0 & 2 & 0 \\ 0 & 0 & n+2 \end{vmatrix} = 96$$

$\Rightarrow n = 6$

23. Correct answer is [575].

$$\int_{-0.15}^{0.15}|100x^2-1|dx=\frac{k}{3000}$$

$$\Rightarrow \int_{0}^{0.15}|100x^2-1|\,dx=\frac{k}{3000}$$

$$\Rightarrow 2\left[\int_{0}^{0.1}(1-100x^2)dx+\int_{0.1}^{0.15}(100x^2-1)dx\right]=\frac{k}{3000}$$

$$\Rightarrow \left[x-\frac{100x^3}{3}\right]_0^{0.1}+\left[\frac{100x^3}{3}-x\right]_{0.1}^{0.15}=\frac{k}{6000}$$

$$\Rightarrow 0.1-\frac{0.1}{3}+\frac{0.3375}{3}-0.15-\frac{0.1}{3}+0.1=\frac{k}{6000}$$

$$\Rightarrow 0.2-\frac{0.2}{3}-0.15+0.1125=\frac{k}{6000}$$

$$\Rightarrow 0.2000-\frac{0.2}{3}-0.1500+0.1125=\frac{k}{6000}$$

$$\Rightarrow \left(0.1625-\frac{0.2}{3}\right)\times 6000 = k$$

$\Rightarrow k = 575$

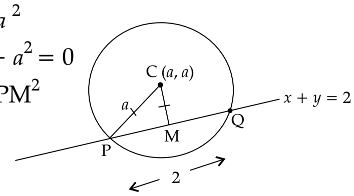
24. Correct answer is [7].

Let the centre at (a, a) and radius is a of the circle then

$(x-a)^2 + (y-a)^2 = a^2$

$x^2 + y^2 - 2ax - 2ay + a^2 = 0$

And $\text{CP}^2 = \text{CM}^2 + \text{PM}^2$



$$a^2 = \left|\frac{a+a-2}{\sqrt{2}}\right|^2 + 1^2$$

$$\Rightarrow a^2 - 1 = \left(\frac{2(a-1)}{\sqrt{2}}\right)^2 = 2(a-1)^2$$

$\Rightarrow a^2 - 1 = 2a^2 + 2 - 4a$

$\Rightarrow a^2 - 4a + 3 = 0$

which is quadratic of radius

$\Rightarrow r_1 + r_2 = 4, r_1 \times r_2 = 3$

and $r_1^2 + r_2^2 - r_1r_2 = (r_1+r_2)^2 - 3r_1r_2$

$= 16 - 3 \times 3 = 7$

25. Correct answer is [10].

According to given condition, we have

$$\text{Mean} = 1\times\frac{n-1}{n}+2\times\frac{1}{n}\left(\frac{n-1}{n}\right)+3\times\left(\frac{1}{n}\right)^2\left(\frac{n-1}{n}\right)+....$$

$$\Rightarrow \frac{n}{9}=\frac{n-1}{n}\left\{1+\frac{2}{n}+\frac{3}{n^2}+....\right\}$$

$$\Rightarrow \frac{n}{9} = \frac{n-1}{n}\left(1-\frac{1}{n}\right)^{-2} = \frac{n-1}{n}\left(\frac{n-1}{n}\right)^{-2}$$

$$= \frac{n-1}{n} \times \frac{n^2}{(n-1)^2}$$

$$\Rightarrow \frac{n}{9} = \frac{n}{n-1} \Rightarrow n-1 = 9$$

or $n = 10$

26. Correct answer is [1260].

If a, b, c are in A.P then c, b, a also must be in A.P.

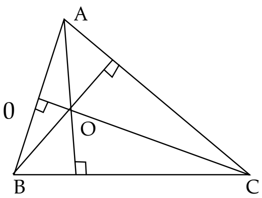
Now to find required number of numbers - abc .....

We take abc as a group, so total numbers are

$$\frac{{}^7C_1 \times 2 \times 6!}{2! \times 2! \times 2!} = \frac{7 \times 2 \times 6 \times 120}{2 \times 2 \times 2} = 7 \times 3 \times 60$$

$= 1260$

27. Correct answer is [288].



Given that $x + 3y - 2z + 6 = 0$

or $\frac{x}{-6} + \frac{y}{-2} + \frac{y}{3} = 1$

$\Rightarrow$ A(–6, 0, 0) B (0, –2, 0), C (0, 0, 3)

if O is orthocentre i.e.,

$0\left(\alpha, \beta, \frac{6}{7}\right)$ then

$$\Rightarrow (\alpha+6)(0-0) + (\beta-0)(0+2) + \left(\frac{6}{7}\right)(3-0) = 0$$

$$\Rightarrow 2\beta + \frac{18}{7} = 0 \Rightarrow \beta = -\frac{9}{7}$$

and BO $\perp$ AC

$$\Rightarrow (\alpha-0)(0+6) + (\beta+2)(0-0) + \left(\frac{6}{7}-0\right)(3-0) = 0$$

$$\Rightarrow 6\alpha + \frac{18}{7} = 0 \Rightarrow \alpha = \frac{-3}{7}$$

$$\therefore 98(\alpha+\beta)^2 = 98\left(\frac{-9}{7} - \frac{3}{7}\right)^2 = \frac{98 \times 144}{49} = 288$$

28. Correct answer is [2].

Given that A = {1, 2, 3}

for reflexive, R must contain (1, 1), (2, 2), (3, 3)

for Transitive, (1, 2) & (2, 3) $\in$ R then (1, 3) must be in R for not symmetric (2, 1) & (3, 2) $\notin$R

As A × A = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)}

Hence, there is only two possibilities of relations which contain (3, 1) and not contain (3, 1).

29. Correct answer is [2].

Given that

$f(x) = |[x]| + \sqrt{x-[x]}$ and $x \in (-2, 1)$

or $f(x) = |[x]| + \sqrt{\{x\}}$

$$\text{or } f(x) = \begin{cases} 2+\sqrt{x+2}, & -2 \le x, < -1 \\ 1+\sqrt{x+1}, & -1 \le x < 0 \\ 0+\sqrt{x}, & 0 \le x < 1 \end{cases}$$

Now $f(-1^-) = 2+\sqrt{1} = 3$

$f(-1) = 1 + 0 = 1$

$\Rightarrow$ Discontinous at $x = -1$

and $f(0^-) = 1 + \sqrt{1} = 2$

$f(0) = 0 + \sqrt{0} = 0$

$\Rightarrow$ Discontinous at $x = 0$

Hence $f(x)$ is discontinuous at two points.

30. Correct answer is [64].

Given that I = $\int \sqrt{\frac{x+7}{x}}\,dx$

Putting $x = t^2$, $dx = 2t\ dt$, we get

$$I = \int \sqrt{\frac{t^2+7}{t^2}} \times 2t\,dt = 2\int \sqrt{t^2+(\sqrt{7})^2}\,dt$$

$$\Rightarrow I = 2\left[\frac{t}{2}\sqrt{t^2+7} + \frac{7}{2}\log\left|t+\sqrt{t^2+7}\right|\right] + C$$

$$\Rightarrow I = \sqrt{x}\sqrt{x+7} + 7\log\left|\sqrt{x}+\sqrt{x+7}\right| + C$$

but I (9) = $12 + 7\log_e 7$

$\Rightarrow 12 + 7\log_e 7 = 3 \times 4 + 7\log|3+4| + C$

$\Rightarrow 12 + 7\log 7 = 12 + 7\log 7 + C$

$\Rightarrow C = 0$

So $I = \sqrt{x}\sqrt{x+7} + 7\log\left|\sqrt{x}+\sqrt{x+7}\right|$

Putting $x = 1$ and I = $\alpha + 7\log_e(1+2\sqrt{2})$ we get

$$\alpha + 7\log_e(1+2\sqrt{2}) = 2\sqrt{2} + 7\log|1+2\sqrt{2}|$$

$$\Rightarrow \alpha = 2\sqrt{2} \text{ and } \alpha^4 = 64$$

❑❑