JEE (Main) MATHEMATICS SOLVED PAPER

2023
13^{th} April Shift 1

Section A

Q. 1. $\int_0^{\infty} \frac{6}{e^{3x}+6e^{2x}+11e^{x}+6}dx$

(1) $\log_e\left(\frac{32}{27}\right)$ (2) $\log_e\left(\frac{256}{81}\right)$

(3) $\log_e\left(\frac{512}{81}\right)$ (4) $\log_e\left(\frac{64}{27}\right)$

Q. 2. Among

$(S_1): \lim_{n\to\infty} \frac{1}{n^2}(2+4+6+....+2n)=1$

$(S_2): \lim_{n\to\infty} \frac{1}{n^{16}}(1^{15}+2^{15}+3^{15}+....+n^{15})=\frac{1}{16}$

(1) Only (S_1) is true.
(2) Both (S_1) and (S_2) are true.
(3) Both (S_1) and (S_2) are false.
(4) Only (S_2) is true.

Q. 3. The number of symmetric matrices of order 3, with all the entries from the set {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, is

(1) 10^9 (2) 10^6 (3) 9^{10} (4) 6^{10}

Q. 4. Let $\vec{a}=\hat{i}+4\hat{j}+2\hat{k}, \vec{b}=3\hat{i}-2\hat{j}+7\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$. If a vector $\vec{d}$ satisfies $\vec{d}\times\vec{b}=\vec{c}\times\vec{b}$ and $\vec{d}.\vec{a}=24$, then $|\vec{d}|^2$ is equal to:

(1) 323 (2) 423 (3) 413 (4) 313

Q. 5. A coin is biased so that the head is 3 times as likely to occur as tail. This coin is tossed until a head or three tails occur. If X denotes the number of tosses of the coin, then the mean of X is:

(1) $\frac{21}{16}$ (2) $\frac{15}{16}$ (3) $\frac{81}{64}$ (4) $\frac{37}{16}$

Q. 6. $\max_{0\le x<\pi}\left\{x-2\sin x\cos x+\frac{1}{3}\sin 3x\right\}=$

(1) 0 (2) π

(3) $\frac{5\pi+2+3\sqrt{3}}{6}$ (4) $\frac{\pi+2-3\sqrt{3}}{6}$

Q. 7. The set of all $a \in R$ for which the equation $x|x-1| + |x+2| + a = 0$ has exactly one real root is:

(1) $(-\infty,-3)$ (2) $(-\infty,\infty)$
(3) $(-6,\infty)$ (4) $(-6,-3)$

Q. 8. Let PQ be a focal chord of the parabola $y^2 = 36x$ of length 100, making an acute angle with the positive x-axis. Let the ordinate of P be positive and M be the point on the line segment PQ such that PM : MQ = 3 : 1. Then, which of the following points does NOT lie on the line passing through M and perpendicular to the line PQ?

(1) (3, 33) (2) (6, 29)
(3) (–6, 45) (4) (–3, 43)

Q. 9. For the system of linear equations

$2x + 4y + 2az = b$ $x + 2y + 3z = 4$
$2x - 5y + 2z = 8$

which of the following is NOT correct?

(1) It has infinitely many solutions if $a = 3, b = 8$.
(2) It has unique solution if $a = b = 8$.
(3) It has unique solution if $a = b = 6$.
(4) It has infinitely many solutions if $a = 3, b = 6$.

Q. 10. Let $s_1, s_2, s_3, ..., s_{10}$, respectively be the sum to 12 terms of 10 A.P. s whose first terms are 1, 2, 3, ..., 10 and the common differences are 1, 3, 5, ..., 19, respectively. Then, $\sum_{i=1}^{10} s_i$.

(1) 7260 (2) 7380 (3) 7220 (4) 7360

Q. 11. For the differentiable function $f : R - \{0\} \to R$. Let $3f(x) + 2f\left(\frac{1}{x}\right)=\frac{1}{x}-10$, then $\left|f(3)+f'\left(\frac{1}{4}\right)\right|$ is equal to

(1) 13 (2) $\frac{29}{5}$ (3) $\frac{33}{5}$ (4) 7

Q. 12. The negation of the statement $((A \wedge (B \vee C)) \Rightarrow (A \vee B)) \Rightarrow A$ is:

(1) equivalent to $B \vee \sim C$
(2) a fallacy
(3) equivalent to $\sim C$
(4) equivalent to $\sim A$

Q. 13. Let the tangent and normal at the point $(3\sqrt{3},1)$ on the ellipse $\frac{x^2}{36}+\frac{y^2}{4}=1$ meet the y-axis at the points A and B, respectively. Let the circle C be drawn taking AB as a diameter and the line $x=2\sqrt{5}$ intersect C at the points P and Q. If the tangents at the points P and Q on the circle intersect at the point (α, β), then $\alpha^2 - \beta^2$ is equal to:

(1) $\frac{304}{5}$ (2) 60 (3) $\frac{314}{5}$ (4) 61

Q. 14. The distance of the point (–1, 2, 3) from the plane $\vec{r}.(\hat{i}-2\hat{j}+3\hat{k})=10$ parallel to the line of the shortest distance between the lines $\vec{r}=(\hat{i}-\hat{j})+\lambda(2\hat{i}+\hat{k})$ and $\vec{r}=(2\hat{i}-\hat{j})+\mu(\hat{i}-\hat{j}+\hat{k})$ is:

(1) $2\sqrt{5}$ (2) $3\sqrt{5}$ (3) $3\sqrt{6}$ (4) $2\sqrt{6}$

Q. 15. Let $B=\begin{bmatrix}1 & 3 & \alpha\\ 1 & 2 & 3\\ \alpha & \alpha & 4\end{bmatrix}, \alpha > 2$ be the adjoint of a matrix A and $|A| = 2$, then $[\alpha \;\; -2\alpha \;\; \alpha]B\begin{bmatrix}\alpha\\ -2\alpha\\ \alpha\end{bmatrix}$ is equal to:

(1) 16 **(2)** 32 **(3)** 0 **(4)** –16

Q. 16. For $x \in R$, two real valued functions $f(x)$ and $g(x)$ are such that, $g(x) = \sqrt{x}+1$ and $fog(x) = x + 3 - \sqrt{x}$. Then, $f(0)$ is equal to:

(1) 5 **(2)** 0 **(3)** –3 **(4)** 1

Q. 17. Let the equation of the plane passing through the line of intersection of the planes $x + 2y + az = 2$ and $x - y + z = 3$ be $5x - 11y + bz = 6a - 1$. For $c \in Z$, if the distance of this plane from the point $(a, -c, c)$ is $\frac{2}{\sqrt{a}}$, then $\frac{a+b}{c}$ is equal to:

(1) –4 **(2)** 2 **(3)** –2 **(4)** 4

Q. 18. Fractional part of the number is $\frac{4^{2022}}{15}$ is equal to:

(1) $\frac{4}{15}$ **(2)** $\frac{8}{15}$ **(3)** $\frac{1}{15}$ **(4)** $\frac{14}{15}$

Q. 19. Let $y = y_1(x)$ and $y = y_2(x)$ be the solution curves of the differential equation $\frac{dy}{dx} = y + 7 \quad y+7$ with initial conditions $y_1(0) = 0$ and $y_2(0) = 1$ respectively. Then the curves $y = y_2(x)$ and $y = y_2(x)$ intersect at

(1) no point

(2) infinite number of points

(3) one point

(4) two points

Q. 20. The area of the region enclosed by the curve $f(x) = \max\{\sin x, \cos x\}, -\pi \le x \le \pi$ and the x-axis is

(1) $2\sqrt{2}(\sqrt{2}+1)$ **(2)** $4(\sqrt{2})$

(3) 4 **(4)** $2(\sqrt{2}+1)$

Section B

Q. 21. The sum to 20 terms of the series $2.2^2 - 3^2 + 2.4^2 - 5^2 + 2.6^2 - ....$ is equal to _______.

Q. 22. Let the mean of the data

x	1	3	5	7	9
Frequency (f)	4	24	28	α	8

be 5. If m and σ^2 are respectively the mean deviation about the mean and the variance of the data, then $\frac{3\alpha}{m+\sigma^2}$ is equal to _______.

Q. 23. Let α be the constant term in the binomial expansion of $\left(\sqrt{x} - \frac{6}{x^{\frac{3}{2}}}\right)^n$, $n \le 15$. If the sum of the coefficients of the remaining terms in the expansion is 649 and the coefficient of x^{-n} is λa, then λ is equal to ________.

Q. 24. Let $\omega = z\bar{z} + k_1 z + k_2 iz + \lambda(1 + i)$, $k_1, k_2 \in R$. Let $Re(\omega) = 0$ be the circle C of radius 1 in the first quadrant touching the line $y = 1$ and the y-axis. If the curve $Im(\omega) = 0$ intersects C at A and B, then $30(AB)^2$ is equal to ______.

Q. 25. Let $\vec{a} = 3\hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = 2\hat{i} - 3\hat{j} + 3\hat{k}$. If $\vec{b}$ is a vector such that $\vec{a} = \vec{b} \times \vec{c}$ and $|\vec{b}|^2 = 50$, then $|72 - |\vec{b} + \vec{c}|^2|$ is equal to ____.

Q. 26. Let m_1, and m_2 be the slopes of the tangents drawn from the point P(4,1) to the hyperbola H: $\frac{y^2}{25} - \frac{x^2}{16} = 1$. If Q is the point from which the tangents drawn to H have slopes $|m_1|$ and $|m_2|$ and they make positive intercepts α and β on the x-axis, then $\frac{(PQ)^2}{\alpha\beta}$ is equal to _______.

Q. 27. Let the image of the point $\left(\frac{5}{3}, \frac{5}{3}, \frac{8}{3}\right)$ in the plane $x - 2y + z - 2 = 0$ be P. If the distance of the point Q(6, –2,α), $\alpha > 0$, from P is 13, then α is equal to ______.

Q. 28. Let for $x \in R$, $S_0(x) = x$, $S_k(x) = C_k x + k\int_0^{\pi} S_{k-1}(t)dt$ where $C_0 = 1$, $C_k = 1 - \int_0^1 S_{k-1}(x)dx$, $k = 1, 2, 3, ...$ Then, $S_2(3) + 6C_3$ is equal to _________.

Q. 29. If S = $\left\{x \in R : \sin^{-1}\left(\frac{x+1}{\sqrt{x^2+2x+2}}\right) - \sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right) = \frac{\pi}{4}\right\}$, then is equal to _________.

Q. 30. The number of seven digit positive integers formed using the digits 1, 2, 3 and 4 only and sum of the digits equal to 12 is _________.

Answer Key

Q. No.	Answer	Topic name	Chapter name
1	(1)	Definite integral using indefinite	Definite integral
2	(2)	Limit as sum of series	Limits
3	(2)	Transpose of a matrix	Matrix
4	(3)	Product of two vector	Vector
5	(1)	Binomial distribution	Probability
6	(3)	Maxima and minima	Application of derivative
7	(2)	Increasing and decreasing function	Application of derivative
8	(4)	Chord of parabola	Parabola
9	(1)	Solution of system of linear equations	Matrix
10	(1)	Arithmetic progression	Sequence and series
11	(1)	Differentiation	Function and its differentiation
12	(4)	Compound statement	Mathematical reasoning
13	(1)	Tangent and normal of ellipse	Ellipse
14	(4)	Line and plane	3D
15	(4)	Adjoint of a matrix	Matrix
16	(1)	Comosite function	Function
17	(1)	Family of planes	3D
18	(3)	Divisibility problem	Binomial theorem
19	(1)	Linear differential equation	Differential equation
20	(2)	Area under simple curves	Area under curves
21	[1310]	Method of difference	Sequence and series
22	[8]	Mean and variance	Statistics
23	[36]	General term	Binomial theorem
24	[24]	Geometrical properties of complex number	Complex number
25	[66]	Product of two vector	Vector
26	[8]	Tangent	Hyperbola
27	[15]	Image of a point wrt a plane	3D
28	[18]	Definite integral using indefinite	Definite integral
29	[0]	Equation involving itf	Inverse trigonometric function
30	[413]	Restricted permutations	Permutations and combination

Solutions

Section A

1. Option (1) is correct.

Let $I = \int_0^\infty \frac{6}{e^{3x}+6e^{2x}+11e^x+6}dx$

$I = \int_0^\infty \frac{6\,dx}{(e^x+1)(e^x+2)(e^x+3)}$ (on factorising the Dr)

Let $\frac{6}{(e^x+1)(e^x+2)(e^x+3)} = \frac{A}{e^x+1}+\frac{B}{e^x+2}+\frac{C}{e^x+3}$

On solving, we get A= 3, B = –6, C = 3

so $I = \int_0^\infty \frac{3}{e^x+1}dx - \int_0^\infty \frac{6}{e^x+2}dx + \int_0^\infty \frac{3}{e^x+3}dx$

$= 3\int_0^\infty \frac{e^{-x}}{1+e^{-x}}dx - 6\int_0^\infty \frac{e^{-x}dx}{1+2e^{-x}} + 3\int_0^\infty \frac{e^{-x}}{1+3e^{-x}}dx$

$= -3\left[\log\left(1+e^{-x}\right)\right]_0^\infty + 6\times\frac{1}{2}\left[\log\left(1+2e^{-x}\right)\right]_0^\infty - 3\times\frac{1}{3}\left[\log(1+3e^{-x})\right]_0^\infty$

= –3 (0 – log 2) + 3 (0 – log 3) – (0 – log 4)
= 3 log 2 – 3 log 3 + log 4

$= \log\frac{2^3\times 4}{3^3} = \log\frac{32}{27}$

2. Option (2) is correct.

$S_1 : \underset{n\to\infty}{\text{Lt}} \frac{1}{n^2}(2+4+6+....+2n)$

$= \underset{n\to\infty}{\text{Lt}} \frac{2}{n^2} \times \frac{n(n+1)}{2} = \underset{n\to\infty}{\text{Lt}} \left(1+\frac{1}{n}\right) = 1$

and $S_2 : \underset{n\to\infty}{\text{Lt}} \frac{1}{n^{16}}\left(1^{15}+2^{15}+3^{15}+....+n^{15}\right)$

$= \underset{n\to\infty}{\text{Lt}} \frac{1}{n^{16}} \sum_{r=1}^{15} r^{15} = \underset{n\to\infty}{\text{Lt}} \frac{1}{n} \sum_{r=1}^{15} \left(\frac{r}{n}\right)^{15}$

$= \int_0^1 x^{15} dx = \left[\frac{x^{16}}{16}\right]_0^1 = \frac{1}{16}$

Hence, both S_1 & S_2 are true.

3. Option (2) is correct.

If A is symmetric matrix, then $a_{ij} = a_{ji}$

or $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

then there can be 6 different entries out of {0, 1, 2, 3, ... 9}

Hence, no. of symmetric matrices are 10^6.

4. Option (3) is correct.

Given that $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$ and $\vec{d} \times \vec{b} = \vec{c} \times \vec{b}, \vec{d}.\vec{a} = 24$

$\Rightarrow (\vec{d} - \vec{c}) \times \vec{b} = 0$

$\vec{d} - \vec{c}$ and $\vec{b}$ are parallel

So $\vec{d} - \vec{c} = \lambda\, \vec{b}$ or $\vec{d} = \vec{c} + \lambda\vec{b}$

$\vec{d}.\vec{a} = 24 \Rightarrow (\vec{c} + \lambda\vec{b}).\vec{a} = 24$

$\Rightarrow \vec{c}.\vec{a} + \lambda\vec{a}.\vec{b} = 24$

$(2 - 4 + 8) + \lambda\,(3 - 8 + 14) = 24$

$6 + \lambda\,(9) = 24$ or $\lambda = 2$

$\Rightarrow\ \vec{d} = \vec{c} + 2\vec{b}$

$= 2\hat{i} - \hat{j} + 4\hat{k} + 6\hat{i} - 4\hat{j} + 14\hat{k}$

$\Rightarrow \vec{d} = 8\hat{i} - 5\hat{j} + 18\hat{k}$

$|\vec{d}|^2 = 64 + 25 + 324 = 413$

5. Option (1) is correct.

Given that $P(H) = \frac{3}{4}$ and $P(T) = \frac{1}{4}$

IF X denotes the number of tosses of the coin, then

$X = 1 \Rightarrow P(X = 1) = \frac{3}{4}$

$X = 2 \Rightarrow P(X = 2) = \frac{1}{4} \times \frac{3}{4} = \frac{3}{16}$

$X = 3 \Rightarrow P(X = 3)$

$= \left(\frac{1}{4}\right)^3 + \left(\frac{1}{4}\right)^2 \times \frac{3}{4} = \frac{1}{64} + \frac{3}{64} = \frac{4}{64} = \frac{1}{16}$

$\therefore$ Mean $= 1 \times \frac{3}{4} + 2 \times \frac{3}{16} + 3 \times \frac{1}{16}$

$= \frac{3}{4} + \frac{6}{16} + \frac{3}{16} = \frac{21}{16}$

6. Option (3) is correct.

Let $f(x) = x - 2 \sin x \cos x + \frac{1}{3}\sin 3x$

$= x - \sin 2x + \frac{1}{3}\sin 3x$

$\Rightarrow f'(x) = 1 - 2\cos 2x + \frac{1}{3} \times 3\cos 3x$

$f'(x) = 1 - 2\cos 2x + \cos 3x$

For maxima or minima, putting $f'(x) = 0$

$1 - 2\cos 2x + \cos 3x = 0$

$1 - 2\,(2\cos^2 x - 1) + (4\cos^3 x - 3\cos x) = 0$

$\Rightarrow 1 - 2\,(2\cos^2 x - 1) + 4\cos^3 x - 3\cos x = 0$

$= 1 - 4\cos^2 x + 2 + 4\cos^3 x - 3\cos x = 0$

$\Rightarrow 4\cos^3 x - 4\cos^2 x - 3\cos x + 3 = 0$

$\Rightarrow 4\cos^2 x\,(\cos x - 1) - 3\,(\cos x - 1) = 0$

$\cos x = 1, \cos x = \frac{\sqrt{3}}{4} = \pm\frac{\sqrt{3}}{2}$

$\Rightarrow \cos x = \frac{\pm\sqrt{3}}{2} = \cos\frac{\pi}{6}$ or $\cos\frac{5\pi}{6}$

$\Rightarrow x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

Also, $f''(x) = 0 + 4\sin 2x - 3\sin 3x$

and $f''\left(\frac{5\pi}{6}\right) = 4\sin\frac{5\pi}{3} - 3\sin\frac{5\pi}{2} < 0$

$\Rightarrow x = \frac{5\pi}{6}$ is a point of maxima

and $f\left(\frac{5\pi}{6}\right) = \frac{5\pi}{6} - \sin\left(\frac{2\times 5\pi}{6}\right) + \frac{1}{3}\sin\left(\frac{3\times 5\pi}{6}\right)$

$= \frac{5\pi}{6} - \sin\frac{5\pi}{3} + \frac{1}{3}\sin\frac{5\pi}{2} = \frac{5\pi}{6} + \frac{\sqrt{3}}{2} + \frac{1}{3}$

7. Option (2) is correct.

Given that $a \in R$ and $x\,|x - 1| + |x + 2| + a = 0$

Let $y = x\,|x - 1| + |x + 2|$ and $y = -a$

So $y = \begin{cases} -x(x-1) - (x+2), & x < -2 \\ -x(x-1) + (x+2), & -2 \le x < 1 \\ x(x-1) + (x+2), & x \ge 1 \end{cases}$

$= \begin{cases} -x^2 - 2, & x < -2 \\ -x^2 + 2x + 2, & -2 \le x < 1 \\ x^2 + 2, & x \ge 1 \end{cases}$

$f(-2^-) = -6, f(-2^+) = -6, f(-2) = -6 \Rightarrow$ continuous at -2

and $f(1^-) = 3, f(1^+) = 3, f(1) = 3 \Rightarrow$ continuous at 1

Also $f'(x) = \begin{cases} -2x, & x < -2 \\ -2x + 2, & -2 < x < 1 \\ 2x, & x < 1 \end{cases} \Rightarrow f'(x) > 0\ \forall\ x \in R$

So $f(x)$ is continous and strictly increasing $\forall\ x \in R$

and $y = -a$ is a st. line || to x-axis

Hence, $x\,|x - 1| + |x + 2| + a = 0$ has exactly one real solution $\forall\ a \in R$

8. Option (4) is correct.

Given that $y^2 = 36x \Rightarrow a = 9$

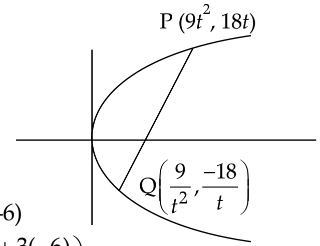
and length of focal chord = 100

$\Rightarrow a\left(t + \frac{1}{t}\right)^2 = 100$

$\Rightarrow 9\left(t+\frac{1}{t}\right)^2 = 100$

or $t+\frac{1}{t} = \frac{10}{3} = 3+\frac{1}{3}$

$\Rightarrow t = 3$



$\Rightarrow$ P(81, 54) and Q (1, –6)

$M\left(\frac{81\times1+3\times1}{3+1}, \frac{54\times1+3(-6)}{3+1}\right)$

$\Rightarrow M\left(\frac{84}{4}, \frac{36}{4}\right) \equiv M\ (21, 9)$

Slope of line PQ $= \frac{54+6}{81-1} = \frac{60}{80} = \frac{3}{4}$

So, slope of line $\perp$ to $PQ = \frac{-4}{3}$

and eqn. of line passing through M is

$y-9 = \frac{-4}{3}(x-21)$

$\Rightarrow 4x + 3y = 111$

Here, $4(-3) + 3 \times 45 = -12 + 129 = 117 \neq 111$

Hence, (–3, 43) does not lie on the line.

9. Option (1) is correct.

Given system of eqn can be written as

$$[A : B] = \begin{bmatrix} 1 & 2 & 3 & : & 4 \\ 2 & -5 & 2 & : & 8 \\ 2 & 4 & 2a & : & b \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - R_1$

$$\Rightarrow [A : B] \sim \begin{bmatrix} 1 & 2 & 3 & : & 4 \\ 0 & -9 & -4 & : & 8 \\ 0 & 0 & 2a-6 & : & b-8 \end{bmatrix}$$

Now, for unique solution $2a - 6 \neq 0$ and $b - 8 \in R$

$\Rightarrow a \neq 3$ and $b \in R$

and for infinite solutions $2a - 6 = 0$ and $b - 8 = 0$

$\Rightarrow a = 3$ and $b = 8$

10. Option (1) is correct.

Here, first terms are 1, 2, 3, ...10

Common differences are 1, 3, 5, ..., 19

No. of terms in each sequence are 12

Then, $S_k = \frac{12}{2}[2\times k + (12-1)(2k-1)]$

$= 6\ [2k + 22k - 11] = 6\ [24k - 11]$

$S_k = 144k - 66$

Hence, $\sum_{i=1}^{10} S_i = \sum_{i=1}^{10}(144k-66)$

$= 144 \times \frac{10\times11}{2} - 66\times10$

$= 72 \times 11 \times 10 - 660$

$= 7920 - 660 = 7260$

11. Option (1) is correct.

Given that $f : R - \{0\} \to R$

and $3f(x) + 2f\left(\frac{1}{x}\right) = \frac{1}{x} - 10$ ...(1)

Replacing x by $\frac{1}{x}$

$3f\left(\frac{1}{x}\right) + 2f(x) = x - 10$ ...(2)

Applying 3 × eqn. (1) – 2 × eqn. (2), we get

$5f(x) = \frac{3}{x} - 30 - 2x + 20$

$\Rightarrow f(x) = \frac{1}{5}\left(\frac{3}{x} - 2x - 10\right)$ and $f'(x) = \frac{1}{5}\left(\frac{-3}{x^2} - 2\right)$

So $\left|f(3) + f'\left(\frac{1}{4}\right)\right| = \left|\frac{1}{5}(1-6-10) + \frac{1}{5}(-48-2)\right|$

$= |-3 - 10| = 13$

12. Option (4) is correct.

Since, we know that $p \Rightarrow q = \sim p \vee q$

So $(A \wedge (B \vee C)) \Rightarrow (A \vee B)) \Rightarrow A$

$= (\sim (A \wedge (B \vee C)) \vee (A \vee B)) \Rightarrow A$

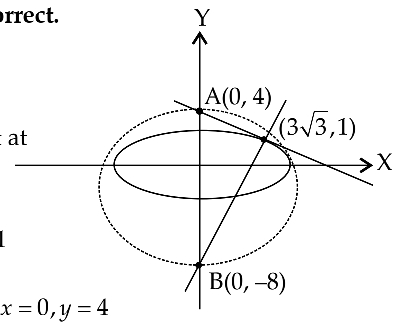
$(A \wedge (B \vee C) \wedge \sim (A \vee B) \vee A$

$= (f \vee A) = A$

Hence, Negation of $((A \wedge (B \vee C)) \Rightarrow (A \vee B)) \Rightarrow A$ is $\sim A$

13. Option (1) is correct.

$\frac{x^2}{36} + \frac{y^2}{4} = 1$



Eqn. of tangent at $(3\sqrt{3},1)$

$\frac{3\sqrt{3}x}{36} + \frac{y\times1}{4} = 1$

$\frac{x}{4\sqrt{3}} + \frac{y}{4} = 1 \Rightarrow x = 0, y = 4$

Eqn. of normal at $(3\sqrt{3},1)$

$\frac{x}{4} - \frac{y}{4\sqrt{3}} = \frac{2}{\sqrt{3}} \Rightarrow x = 0, y = -8$

Eqn. of circle having A(0, 4), B(0, –8) as diameter is

$x^2 + (y + 4)(y - 8) = 0$

$x^2 + y^2 - 4y - 32 = 0$

$hx + ky + 2(y + k) - 32 = 0$

$\Rightarrow k = -2$

$hx + 2k - 32 = 0$

$hx = 36$

$\alpha = h = \frac{36}{2\sqrt{5}}$ & $\beta = k = -2$

So, $\alpha^2 - \beta^2 = \frac{324}{5} - 4 = \frac{304}{5}$

14. Option (4) is correct.

Given that lines $\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} + \hat{k})$ ...(1)

and $\vec{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} - \hat{j} + \hat{k})$ ...(2)

Now, direction of line of the shortest distance of line (1) and (2) is given by

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ 1 & -1 & 1 \end{vmatrix} = \hat{i} - \hat{j} - 2\hat{k}$$

So, eqn. of line passing through (–1, 2, 3) whose d.r's are (1, –1, –2) is given by

$\frac{x+1}{1} = \frac{y-2}{-1} = \frac{z-3}{-2} = \lambda$

$\Rightarrow (x, y, z) \equiv (\lambda - 1, -\lambda + 2, -2\lambda + 3)$

If this point lies on plane $\vec{r}.(\hat{i} - 2\hat{j} + 3\hat{k}) = 10$

Then, $\lambda - 1 - 2(-\lambda + 2) + 3(-2\lambda + 3) = 10$

$\Rightarrow \lambda - 1 + 2\lambda - 4 - 6\lambda + 9 = 10$

$\Rightarrow 3\lambda = -6 \Rightarrow \lambda = -2$

So, point on plane is $(-3, 4, 7)$

and required distance is $\sqrt{(-1+3)^2 + (2-4)^2 + (3-7)^2}$

$= \sqrt{4+4+16} = 2\sqrt{6}$

15. Option (4) is correct.

Given that B = adjA

and $|B| = |\text{adj}A| = |A|^2 = 4$

$$\Rightarrow \begin{vmatrix} 1 & 3 & \alpha \\ 1 & 2 & 3 \\ \alpha & \alpha & 4 \end{vmatrix} = 4$$

$\Rightarrow \alpha = 2$ or $\alpha = 4$

But $\alpha > 2$ (Given)

So, $\alpha = 4$ and $[\alpha \;\; -2\alpha \;\; \alpha]B\begin{bmatrix} \alpha \\ -2\alpha \\ \alpha \end{bmatrix}$

$$= [4 \;\; -8 \;\; 4]\begin{bmatrix} 1 & 3 & 4 \\ 1 & 2 & 3 \\ 4 & 4 & 4 \end{bmatrix}\begin{bmatrix} 4 \\ -8 \\ 4 \end{bmatrix}$$

$$= [12 \;\; 12 \;\; 8]\begin{bmatrix} 4 \\ -8 \\ 4 \end{bmatrix} = [-16]$$

16. Option (1) is correct.

Given that $g(x) = \sqrt{x} + 1$ & $fog(x) = x + 3 - \sqrt{x}$

$\Rightarrow f(g(x)) = x + 3 - \sqrt{x}$

$\Rightarrow f(g(x)) = (\sqrt{x})^2 + 3 - \sqrt{x}$

$f(g(x) = (g(x) - 1)^2 + 3 - (g(x) - 1)$

$f(x) = (x - 1)^2 + 3 - (x - 1)$

and $f(0) = (0 - 1)^2 + 3 - (0 - 1)$

$= 1 + 3 + 1 = 5$

17. Option (1) is correct.

Equation of plane passing through $x + 2y + az = 2$ and $x - y + z = 3$ is given by

$x + 2y + az - 2 + \lambda(x - y + z - 3) = 0$

$\Rightarrow (1 + \lambda)x + (2 - \lambda)y + (a + \lambda)z - 2 - 3\lambda = 0$

which is equivalent to $5x - 11y + bz = 6a - 1$

So $\dfrac{1+\lambda}{5} = \dfrac{2-\lambda}{-11} = \dfrac{a+\lambda}{b} = \dfrac{2+3\lambda}{6a-1}$

$\Rightarrow -11 - 11\lambda = 10 - 5\lambda$

$\Rightarrow 6\lambda = -21$ or $\lambda = \dfrac{-7}{2}$

Also, $\dfrac{1 - \frac{7}{2}}{5} = \dfrac{a - \frac{7}{2}}{b} = \dfrac{2 - 3 \times \frac{7}{2}}{6a - 1}$

$-\dfrac{1}{2} = \dfrac{a - \frac{7}{2}}{b} = \dfrac{-17}{2(6a-1)}$

$\Rightarrow a = 3, b = 1$

Now, $\dfrac{2}{\sqrt{a}} = \left|\dfrac{5a + 11c + bc - 6a + 1}{\sqrt{25 + 121 + 1}}\right|$

$\Rightarrow c = 1$

$\therefore \dfrac{a+b}{c} = \dfrac{3+1}{-1} = -4$

18. Option (3) is correct.

$\because 4^{2022} = 2^{4044} = \left(2^4\right)^{1011}$

$= (1 + 15)^{1011} = 1 + 15\lambda$

$\therefore \left\{\dfrac{4^{2022}}{15}\right\} = \left\{\dfrac{1+15\lambda}{15}\right\} = \left\{\lambda + \dfrac{1}{15}\right\} = \dfrac{1}{15}$

19. Option (1) is correct.

Given that $\dfrac{dy}{dx} = y + 7$

$\Rightarrow \displaystyle\int \frac{dy}{y+7} = \int dx \Rightarrow \log_e(y + 7) = x + c$

$y_1(0) = 0 \Rightarrow \log_e 7 = 0 + c$

$\Rightarrow c = \log_e 7 \Rightarrow \log_e(y + 7) = x + \log_e 7$

$\Rightarrow \log_e\left(\dfrac{y+7}{7}\right) = x$

or $y + 7 = 7e^x$ ...(1)

Also, $y_2(0) = 1 \Rightarrow \log_e 8 = 1 + c$

$\Rightarrow c = \log_e\left(\dfrac{8}{e}\right) \Rightarrow \log_e(y+7) = x + \log_e\left(\dfrac{8}{e}\right)$

$\Rightarrow \log_e\left(\dfrac{(y+7)e}{8}\right) = x$

$\Rightarrow (y + 7)e = 8e^x$ ...(2)

From (1) and (2)

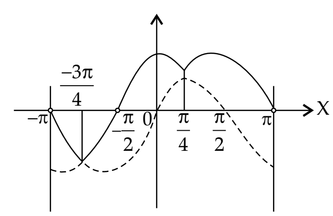
$7e^x \times e = 8e^x \Rightarrow e = \dfrac{8}{7}$ which is not true.

Hence, there is no point.

20. Option (3) is correct.

Given that $f(x) = \max\{\sin x, \cos x\}$

which can be plotted as



So, required area is $= \left|\displaystyle\int_{-\pi}^{\frac{-3\pi}{4}} \sin x\,dx\right| + \left|\int_{\frac{-3\pi}{4}}^{\frac{-\pi}{2}} \cos x\,dx\right|$

$+ \displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{4}} \cos x\,dx + + \int_{\frac{\pi}{4}}^{\pi} \sin x\,dx$

$= \left|[-\cos x]_{-\pi}^{-\frac{3\pi}{4}}\right| + \left|[\sin x]_{\frac{-3\pi}{4}}^{\frac{-\pi}{2}}\right| + [\sin x]_{-\frac{\pi}{2}}^{\frac{\pi}{4}} - [\cos x]_{\frac{\pi}{4}}^{\pi}$

$= \left|\dfrac{1}{\frac{-1}{\sqrt{2}} + 1}\right| + \left|-1 + \dfrac{1}{\sqrt{2}}\right| + \dfrac{1}{\sqrt{2}} + 1 - \left(-1 - \dfrac{1}{\sqrt{2}}\right) = 4$

Section B

21. The correct answer is (1310).

$S_{20} = 2.2^2 - 3^2 + 2.4^2 - 5^2 + 2.6^2 - 7^2 ....$

$= (2^2 - 3^2 + 4^2 - 5^2 .... \text{ upto 20 terms}) + (2^2 + 4^2 + 6^2 + ... \text{ upto 10 terms})$

$= -(5 + 9 + 13 + ... \text{ upto 10 terms}) + 2^2(1^2 + 2^2 + 3^2 + ... + \text{ upto 10 terms})$

$= -\frac{10}{2}[10 + 9 \times 4] + 4\left[\frac{10 \times 11 \times 21}{6}\right]$

$= -5 \times 46 + 2 \times 11 \times 7 \times 10$

$= -230 + 1540 = 1310$

22. The correct answer is (8).

x_i	f_i	f_ix_i	$\lvert x_i - \bar{x} \rvert$	$f_i \lvert x_i - \bar{x} \rvert$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
1	4	4	4	16	16	64
3	24	72	2	48	4	96
5	28	140	0	0	0	0
7	$\alpha =$ 16	$7\alpha =$ 112	2	32	14	64
9	8	72	4	32	16	128
	80	400		128		352

Mean = 5

$\Rightarrow \frac{\Sigma f_i x_i}{\Sigma f_i} = 5 \Rightarrow \frac{288 + 7\alpha}{64 + \alpha} = 5$

$\Rightarrow 280 + 7\alpha = 320 + 5\alpha \qquad \Rightarrow \alpha = 16$

M.D. $(\bar{x}) = \frac{\Sigma f_i \lvert x_i - \bar{x} \rvert}{\Sigma f_i} = \frac{128}{80} = \frac{8}{5} = m$

Variance $= \frac{\Sigma f_i (x_i - \bar{x})^2}{\Sigma f_i} = \frac{352}{80} = \sigma^2 = \frac{22}{5}$

Now, $\frac{3\alpha}{m + \sigma^2} = \frac{3 \times 16}{\frac{8}{5} + \frac{22}{5}} = \frac{3 \times 16 \times 5}{30} = 8$

23. The correct answer is (36).

Given that $\left(\sqrt{x} - \frac{6}{x^{\frac{3}{2}}}\right)^n, n \le 15$

$T_{r+1} = {}^nC_r (\sqrt{x})^{n-r} \left(\frac{-6}{x^{\frac{3}{2}}}\right)^r$

$= {}^nC_r (-6)^r (x)^{\frac{n-r}{2} - \frac{3r}{2}}$

For independent term $\frac{n-r}{2} - \frac{3r}{2} = 0$

$\Rightarrow n - r = 3r$ or $r = \frac{n}{4}$

Sum of coefficients of remaining terms = 649

$\Rightarrow \left(\sqrt{1} - \frac{6}{(1)^{\frac{3}{2}}}\right)^n - {}^nC_r(-6)^r = 649$

or $(-5)^n - {}^nC_{n/4}(-6)^{n/4} = 625 + 24$

$\Rightarrow (-5)^n - {}^nC_r(-6)^r = 649$

$(-5)^n - {}^nC_{n/4}(-6)^{n/4} = (-5)^4 - {}^4C_{4/4}(-6)^{4/4}$

By comparing, $n = 4$ and $r = 1$

Now, for coefficient of $x^{n-n}, \frac{n-r}{2} - \frac{3r}{2} = -n = -4$

$\Rightarrow n - r - 3r = -8$ or $4 - 4r = -8 \Rightarrow r = 3$

So, coefficient of $x^{-n} = {}^4C_3(-6)^3$

$\lambda\alpha = {}^4C_1(-216)$

But $\lambda = {}^4C_1(-6) \Rightarrow \alpha = 36$

24. The correct answer is (24).

Given that

$\omega = z\bar{z} + k_1 z + k_2 iz + \lambda(1 + i), k_1, k_2 \in R$

Let $z = x + iy$, then

$\omega = x^2 + y^2 + k_1 x + ik_1 y + i k_2 x - k_2 y + \lambda + i\lambda$

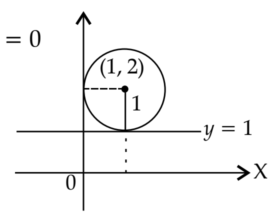
$\Rightarrow \text{Re}(\omega) = 0 \Rightarrow = x^2 + y^2 + k_1 x - k_2 y + \lambda = 0$

Centre $\left(\frac{-k_1}{2}, \frac{k_2}{2}\right) \equiv (1, 2)$

$\Rightarrow k_1 = -2$ & $k_1 = 4$

and radius = 1

$\Rightarrow \sqrt{1 + 4 - \lambda} = 1 \quad \Rightarrow \lambda = 4$



Im $(\omega) = 0 \Rightarrow k_1 y + k_2 x + \lambda = 0$

$\Rightarrow -2y + 4x + 4 = 0$

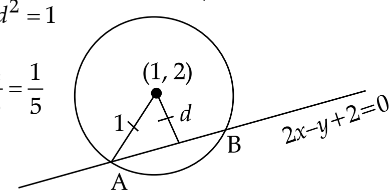
or $2x - y + 2 = 0$

so $d = \frac{2}{\sqrt{4+1}} = \frac{2}{\sqrt{5}}$

Now, $\left(\frac{AB}{2}\right)^2 + d^2 = 1$

$\Rightarrow \frac{(AB)^2}{2} = 1 - \frac{4}{5} = \frac{1}{5}$

or $(AB)^2 = \frac{4}{5}$



and $30\,(AB)^2 = 30 \times \frac{4}{5} = 24$

25. The correct answer is (66).

Given that $\vec{a} = 3\hat{i} + \hat{j} - \hat{k}, \vec{c} = 2\hat{i} - 3\hat{j} + 3\hat{k}$

$\Rightarrow \lvert \vec{a} \rvert = \sqrt{11}, \lvert \vec{c} \rvert = \sqrt{22}, \lvert \vec{b} \rvert^2 = 50$

$\vec{a} = \vec{b} \times \vec{c} \quad \Rightarrow \lvert \vec{a} \rvert = \lvert \vec{b} \times \vec{c} \rvert$

$\Rightarrow \sqrt{11} = \lvert \vec{b} \rvert \lvert \vec{c} \rvert \sin\theta$

$\Rightarrow \sqrt{11} = \sqrt{50} \times \sqrt{22} \sin\theta$

$\Rightarrow \sin\theta = \frac{1}{10} \Rightarrow \cos\theta = \frac{\sqrt{99}}{10}$

Now, $\lvert \vec{b} + \vec{c} \rvert^2 = \lvert \vec{b} \rvert^2 + \lvert \vec{c} \rvert^2 + 2\,\vec{b}.\vec{c}$

$= 50 + 22 + 2 \lvert \vec{b} \rvert \lvert \vec{c} \rvert \cos\theta$

$= 72 + 2\sqrt{50} \times \sqrt{22} \times \frac{\sqrt{99}}{10}$

$= 72 + \frac{2 \times 5\sqrt{2} \times 3 \times \sqrt{2} \times 11}{10}$

$\lvert \vec{b} + \vec{c} \rvert^2 = 72 + 66$

So, $72 - \lvert \vec{b} + \vec{c} \rvert^2 = -66$

$\lvert 72 - \lvert \vec{b} + \vec{c} \rvert^2 \rvert = 66$

26. The correct answer is (8).

We know that eqn. of tangent to hyperbola $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$ is given by $y = mx \pm \sqrt{a^2-b^2m^2}$ which pass through P(4, 1)

So, $1 = 4m \pm \sqrt{25-16m^2}$

$\Rightarrow (1-4m)^2 = \left(\pm\sqrt{25-16m^2}\right)^2$

$\Rightarrow 1 + 16m^2 - 8m = 25 - 16m^2$

$\Rightarrow 32m^2 - 8m - 24 = 0$

$\Rightarrow 4m^2 - m - 3 = 0$

$\Rightarrow (4m + 3)(m - 1) = 0$

$\Rightarrow m = \frac{-3}{4}$ or $m = 1 \Rightarrow |m_1| = \frac{3}{4}$ & $|m_2| = 1$

So, equation of tangent with slopes $|m_1|$ & $|m_2|$ are

$\Rightarrow 4y = 3x - 16$ and $y = x - 3$

Putting $y = 0$ in both eqns.

$\alpha = \frac{16}{3}$ and $\beta = 3$

Also, on solving the above tangents

We get Q (–4, –7)

$\Rightarrow PQ = \sqrt{(4+4)^2+(1+7)^2} = \sqrt{64+64} = 8\sqrt{2}$

and $\frac{(PQ)^2}{\alpha\beta}$

$\frac{(PQ)^2}{\alpha\beta} = \frac{64\times 2}{\frac{16}{3}\times 3} = 8$

27. The correct answer is (15).

If P(α, β, γ) be the image of point $\left(\frac{5}{3},\frac{5}{3},\frac{8}{3}\right)$ in plane $x - 2y + z - 2 = 0$, then

$\frac{\alpha-\frac{5}{3}}{1} = \frac{\beta-\frac{5}{3}}{-2} = \frac{\gamma-\frac{8}{3}}{1} = \frac{-2\left(\frac{5}{3}-\frac{10}{3}+\frac{8}{3}-2\right)}{1+4+1}$

$= \frac{-2}{6}\left(\frac{5-10+8-6}{3}\right) = \frac{-1}{3}\left(\frac{-3}{3}\right) = \frac{1}{3}$

$\Rightarrow \alpha = \frac{1}{3}+\frac{5}{3}, \beta = \frac{5}{3}-\frac{2}{3}, \gamma = \frac{8}{3}+\frac{1}{3}$

$\Rightarrow \alpha = 2, \beta = 1, \gamma = 3$

or P (2, 1, 3), Q (6, –2, α)

$PQ = 13$ or $PQ^2 = 169$

$\Rightarrow (4)^2 + (-3)^2 + (\alpha - 3)^2 = 169$

$\Rightarrow 16 + 9 + (\alpha - 3)^2 = 169$

$\Rightarrow (\alpha - 3)^2 = 144$

$\Rightarrow \alpha - 3 = \pm 12 \Rightarrow \alpha = 15\ (\alpha > 0)$

28. The correct answer is (18).

Given that

$S_k(x) = C_k x + k\int_0^x S_{k-1}(t)\,dt$

Putting $k = 2$ and $x = 3$, we get,

$S_3(3) = C_2(3) + 2\int_0^3 S_1(t)\,dt$ ...(1)

Putting $k = 1$

$S_1(x) = C_1 x + \int_0^x S_0(t)\,dt = C_1 x + \frac{x^2}{2}$

Putting in eqn. (1)

$S_2(3) = 3C_2 + 2\int_0^3 \left(C_1 t + \frac{t^2}{2}\right)dt$

$= 3C_2 + 2\left[\frac{C_1 t^2}{2}+\frac{t^3}{6}\right]_0^3$

$S_2(3) = 3C_2 + 9C_1 + 9$

But $C_1 = 1 - \int_0^1 S_0(x)\,dx = \frac{1}{2}$

$C_2 = 1 - \int_0^1 S_1(x)\,dx = 0$

$C_3 = 1 - \int_0^1 S_2(x)\,dx = 1 - \int_0^1\left(C_2 x + C_1 x^2 + \frac{x^3}{3}\right)dx$

$= 1 - \left[\frac{C_2 x^2}{2}+\frac{C_1 x^3}{3}+\frac{x^4}{12}\right]_0^1$

$= 1 - \left[0 + \frac{1}{2}\times\frac{1}{3}+\frac{1}{12}\right] = 1 - \frac{1}{4} = \frac{3}{4}$

then value is ${}^3C_2 + {}^9C_1 + 9 + {}^6C_3$

$= 0 + 9/2 + 9 + 6(3/4) = 18$

29. The correct answer is (0).

Given that

$\sin^{-1}\left(\frac{x+1}{\sqrt{(x+1)^2+1}}\right) - \sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right) = \frac{\pi}{4}$

$\sin^{-1}\left(\frac{x+1}{\sqrt{(x+1)^2+1}}\right) = \frac{\pi}{4} + \sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right)$

$\Rightarrow \frac{x+1}{\sqrt{(x+1)^2+1}} = \sin\left(\frac{\pi}{4}+\sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right)\right)$

$= \frac{1}{\sqrt{2}}\cos\left(\sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right)\right) + \frac{1}{\sqrt{2}}\times\frac{x}{\sqrt{x^2+1}}$

$= \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{x^2+1}}+\frac{x}{\sqrt{x^2+1}}\right]$

$\Rightarrow \frac{x+1}{\sqrt{(x+1)^2+1}} = \frac{1+x}{\sqrt{2}(\sqrt{x^2+1})}$

$\Rightarrow x + 1 = 0 \Rightarrow x = -1$

and $(x + 1)^2 + 1 = 2(x^2 + 1)$

$\Rightarrow x^2 - 2x = 0 \Rightarrow x = 0, 2$

2, –1 will not satisfy the given condition hence S = (0)

Hence, required value is $\sin\frac{5\pi}{2-\cos 5\pi} = 0$

30. The correct answer is (413).

Given that digits are 1, 2, 3, 4

And $x_1, x_2, x_3, \ldots\ldots x_7$ are the numbers, where

$x_1 + x_2 + \ldots + x_7 = 12, x_i \in \{1, 2, 3, 4\}$

Hence, no. of required solutions are

$= {}^{12-1}C_{7-1} - \frac{7!}{6!} - \frac{7!}{5!}$

$= {}^{11}C_6 - 7 - 7\times 6$

$= \frac{11\times10\times9\times8\times7}{5\times4\times3\times2} - 7 - 42 = 462 - 49 = 413$

❑❑