JEE (Main) MATHEMATICS SOLVED PAPER

2023
13th April Shift 2

Section A

Q. 1. The area of the region $\{(x, y) : x^2 \le y \le |x^2 - 4|, y \ge 1\}$ is:

(1) $\frac{3}{4}(4\sqrt{2}+1)$ **(2)** $\frac{4}{3}(4\sqrt{2}-1)$

(3) $\frac{3}{4}(4\sqrt{2}-1)$ **(4)** $\frac{4}{3}(4\sqrt{2}+1)$

Q. 2. If $\lim\limits_{x\to 0} \dfrac{e^{ax} - \cos(bx) - \dfrac{cxe^{-cx}}{2}}{1-\cos(2x)} = 17$, then $5a^2 + b^2$ is equal to:

(1) 76 **(2)** 72 **(3)** 64 **(4)** 68

Q. 3. The line that is coplanar to the line $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$ is:

(1) $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$

(2) $\frac{x+1}{1} = \frac{y-2}{2} = \frac{z-5}{5}$

(3) $\frac{x-1}{-1} = \frac{y-2}{2} = \frac{z-5}{4}$

(4) $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{4}$

Q. 4. The plane, passing through the points (0, –1, 2) and (–1, 2, 1) and parallel to the line passing through (5, 1, –7) and (1, –1, –1), also passes through the point:

(1) (0, 5, –2) **(2)** (–2, 5, 0)

(3) (2, 0, 1) **(4)** (1, –2, 1)

Q. 5. Let for a triangle ABC,

$\overline{AB} = -2\hat{i} + \hat{j} + 3\hat{k}$ $\overline{CB} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$

$\overline{CA} = 4\hat{i} + 3\hat{j} + \delta\hat{k}$

If $\delta > 0$ and the area of the triangle ABC is $5\sqrt{6}$, then $\overrightarrow{CB}.\overrightarrow{CA}$ is equal to:

(1) 108 **(2)** 60 **(3)** 54 **(4)** 120

Q. 6. Let for $A = \begin{bmatrix} 1 & 2 & 3 \\ \alpha & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix}$, $|A| = 2$. If $|2 \text{ adj } (2 \text{ adj } (2A))| = 32^n$, then $3n + \alpha$ is equal to:

(1) 10 **(2)** 9 **(3)** 12 **(4)** 11

Q. 7. The range of $f(x) = 4\sin^{-1}\left(\frac{x^2}{x^2+1}\right)$ is:

(1) $[0, \pi)$ **(2)** $[0, \pi]$ **(3)** $[0, 2\pi)$ **(4)** $[0, 2\pi]$

Q. 8. Let a_1, a_2, a_3, ... be a G. P. of increasing positive numbers. Let the sum of its 6th and 8th terms be 2 and the product of its 3rd and 5th terms be $\frac{1}{9}$. Then, $6(a_2 + a_4)(a_4 + a_6)$ is equal to:

(1) 2 **(2)** 3 **(3)** $3\sqrt{3}$ **(4)** $2\sqrt{2}$

Q. 9. If the system of equations

$2x + y - z = 5$ $2x - 5y + \lambda z = \mu$

$x + 2y - 5z = 7$

has infinitely many solutions, then $(\lambda+\mu)^2 + (\lambda-\mu)^2$ is equal to:

(1) 904 **(2)** 916 **(3)** 912 **(4)** 920

Q. 10. The statement $(p \wedge (\sim q)) \vee ((\sim p) \wedge q) \vee ((\sim p) \wedge (\sim q))$ is equivalent to____.

(1) $(\sim p) \vee (\sim q)$ **(2)** $(\sim p) \wedge (\sim q)$

(3) $p \vee (\sim q)$ **(4)** $p \vee q$

Q. 11. Let S = $z = i(z^2 + \text{Re}(z)\}$. Then, $\sum\limits_{z\in S} |z|^2$ is equal to:

(1) 4 **(2)** $\frac{7}{2}$ **(3)** 3 **(4)** $\frac{5}{2}$

Q. 12. Let α, β be the roots of the equation $x^2 - \sqrt{2}x + 2 = 0$. Then, $\alpha^{14} + \beta^{14}$ is equal to:

(1) $-128\sqrt{2}$ **(2)** $-64\sqrt{2}$

(3) -128 **(4)** -64

Q. 13. Let $|\vec{a}| = 2, |\vec{b}| = 3$ and the angle between the vectors $\vec{a}$ and $\vec{b}$ be $\frac{\pi}{4}$. Then, $\left|(\vec{a} + 2\vec{b}) \times (2\vec{a} - 3\vec{b})\right|^2$ is equal to:

(1) 482 **(2)** 841 **(3)** 882 **(4)** 441

Q. 14. The value of $\dfrac{e^{-\frac{\pi}{4}} + \int_0^{\frac{\pi}{4}} e^{-x}\tan^{50} x\,dx}{\int_0^{\frac{\pi}{4}} e^{-x}(\tan^{49} x + \tan^{51} x)\,dx}$ is:

(1) 25 **(2)** 51 **(3)** 50 **(4)** 49

Q. 15. The coefficient of x^5 in the expansion of $\left(2x^3 - \frac{1}{3x^2}\right)^5$ is:

(1) $\frac{80}{9}$ **(2)** 8 **(3)** 9 **(4)** $\frac{26}{3}$

Q. 16. The random variable X follows binomial distribution B (n, p), for which the difference of the mean and the variance is 1. If $2P(x = 2) = 3P(x = 1)$, then $n^2P(X > 1)$ is equal to:

(1) 16 **(2)** 11 **(3)** 12 **(4)** 15

Q. 17. Let the centre of a circle C be (α, β) and its radius $r < 8$. Let $3x + 4y = 24$ and $3x - 4y = 32$ be two tangents and $4x + 3y = 1$ be a normal to C. Then, $(\alpha - \beta + r)$ is equal to:

(1) 5 **(2)** 6 **(3)** 7 **(4)** 9

Q. 18. Let N be the foot of the perpendicular from the point P (1, –2, 3) on the line passing through the points (4, 5, 8) and (1, –7. 5). Then, the distance of N from the plane $2x - 2y + z + 5 = 0$ is:

(1) 6 **(2)** 7 **(3)** 9 **(4)** 8

Q. 19. All words, with or without meaning, are made using all the letters of the word MONDAY. These words are written as in a dictionary with serial numbers. The serial number of the word MONDAY is:

(1) 328 **(2)** 327 **(3)** 324 **(4)** 326

Q. 20. Let (α, β) be the centroid of the triangle formed by the lines $15x - y = 82$, $6x - 5y = -4$ and $9x + 4y = 17$. Then, $\alpha + 2\beta$ and $2\alpha - \beta$ are the roots of the equation:

(1) $x^2 - 13x + 42 = 0$ **(2)** $x^2 - 10x + 25 = 0$

(3) $x^2 - 7x + 12 = 0$ **(4)** $x^2 - 14x + 48 = 0$

Section B

Q. 21. Let A = {–4, –3, –2, 0, 1, 3, 4} and $R = \{(a, b) \in A \times A : b = |a| \text{ or } b^2 = a + 1\}$ be a relation on A. Then, the minimum number of elements that must be added to the relation R so that it becomes reflexive and symmetric, is _________.

Q. 22. Let $f_n = \int_0^{\frac{\pi}{2}} \left(\sum_{k=1}^{n} \sin^{k-1} x\right)\left(\sum_{k=1}^{n} (2k-1)\sin^{k-1} x\right) \cos x \, dx$, $n \in N$. Then, $f_{21} - f_{20}$ is equal to ________.

Q. 23. If $y = y(x)$ is the solution of the differential equation $\frac{dy}{dx} + \frac{4x}{(x^2-1)} y = \frac{x+2}{(x^2-1)^{\frac{1}{2}}}, x > 1$ such that $y(2) = \frac{2}{9}\log_e(2+\sqrt{3})$ and $y(\sqrt{2}) = \alpha \log_e (\sqrt{\alpha} + \beta) + \beta - \sqrt{\gamma}$, $\alpha, \beta, \gamma, \in N$, then $\alpha\beta\gamma$ is equal to _________.

Q. 24. Total numbers of 3-digit numbers that are divisible by 6 and can be formed by using the digits 1, 2, 3, 4, 5 with repetition, is _____.

Q. 25. The remainder, when 7^{103} is divided by 17, is ______.

Q. 26. Let $f(x) = \sum_{k=1}^{10} k x^k, x \in R$. If $2 f(2) - f'(2) = 119 (2)^n + 1$, then n is equal to _____.

Q. 27. For $x \in (-1, 1]$, the number of solutions of the equation $\sin^{-1} x = 2 \tan^{-1} x$ is equal to ____.

Q. 28. The mean and standard deviation of the marks of 10 students were found to be 50 and 12, respectively, Later, it was observed that two marks 20 and 25 were wrongly read as 45 and 50, respectively. Then, the correct variance is ______.

Q. 29. The foci of a hyperbola are $(\pm 2, 0)$ and its eccentricity is $\frac{3}{2}$. A tangent, perpendicular to the line $2x + 3y = 6$, is drawn at a point in the first quadrant on the hyperbola. If the intercepts made by the tangent on the x and y-axis are a and b, respectively, then $|6a| + |5b|$ is equal to ______.

Q. 30. Let $[\alpha]$ denote the greatest integer $\le \alpha$. Then, $[\sqrt{1}] + [\sqrt{2}] + ... + [\sqrt{120}]$ is equal to ________.

Answer Key

Q. No.	Answer	Topic name	Chapter name
1	(2)	Area between two curves	Area under curves
2	(4)	Limits using expansion	Limits
3	(1)	Equation of a plane	3D
4	(2)	Equation of a plane	3D
5	(2)	Product of two vector	Vector
6	(4)	Adjoint of a matrix	Matrix
7	(3)	Range of itf	Itf
8	(2)	Geometric progression	Sequence and series
9	(2)	Solution of system of linear equations	Matrix
10	(1)	Compound statments	Mathematical reasoning
11	(1)	Modulus of a complex number	Complex number

Q. No.	Answer	Topic name	Chapter name
12	(3)	Cube root of unity	Complex number
13	(3)	Product of two vector	Vector
14	(3)	Definite integral using indefinite	Definite integral
15	(1)	General term	Binomial theorem
16	(2)	Binomial distribution	Probability
17	(3)	Normal	Circles
18	(2)	Point . Line and plane	3D
19	(2)	Rank of a word	Permutation and combination
20	(1)	Centroid and solving equation	Straight lines
21	[7]	Reflexive and symmetric relation	Relation
22	[41]	Definite integral using properties	Definite integral
23	[6]	Linear differential equation	Differential equation
24	[16]	Divisibility problem	Basics mathematics
25	[12]	Divisibility problem	Binomial theorem
26	[10]	Function and its differentiation	Function and differentiation
27	[2]	Equation involving itf	Itf
28	[269]	Mean and variance	Statistics
29	[12]	Tangent	Hyperbola
30	[825]	Special series	Sequence and series

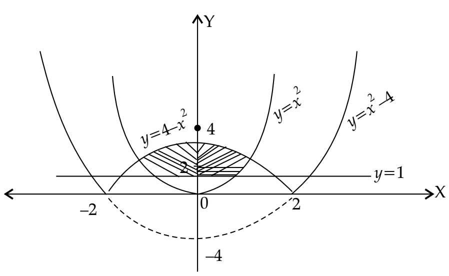
Solutions

Section A

1. Option (2) is correct.

Given that $x^2 \le y \le |x^2-4|$, $y \ge 1$

Here, $x^2 = y$ and $y = |x^2-4|$ both represent parabola, which can be plotted as



Solving $x^2 = y$ & $y = |x^2-4|$

We get $y = \pm(y-4)$

$y = -y + 4 \Rightarrow y = 2$

so, the required area

$$= 2\left[\int_1^2 \sqrt{y}\,dy + \int_2^4 2\sqrt{4-y}\,dy\right]$$

$$= 2\times\left\{\left[\frac{2}{3}y^{\frac{3}{2}}\right]_1^2 - \left[\frac{2}{3}(4-y)^{\frac{3}{2}}\right]_2^4\right\}$$

$$= 2\left\{\frac{2}{3}\times 2\sqrt{2} - \frac{2}{3} + \frac{2}{3}\times 2\sqrt{2}\right\}$$

$$= 2\left\{\frac{8}{3}\sqrt{2} - \frac{2}{3}\right\} = \frac{4}{3}\left\{4\sqrt{2}-1\right\}$$

2. Option (4) is correct.

Given that

$$\lim_{x\to 0} \frac{e^{ax} - \cos bx - \frac{cx}{2}\times e^{-cx}}{1-\cos(2x)} = 17$$

$$\lim_{x\to 0} \frac{\left\{1+ax+\frac{a^2x^2}{2}....\right\} - \left\{1-\frac{b^2x^2}{2}+\frac{b^4x^4}{24}-....\right\} - \frac{cx}{2}\left\{1-cx+\frac{c^2x^2}{2}.....\right\}}{1-\left\{1-\frac{4x^2}{2}+\frac{16x^4}{24}....\right\}}$$

$$\lim_{x\to 0} \frac{\left(a-\frac{c}{2}\right)x + \left(\frac{a^2}{2}+\frac{b^2}{2}+\frac{c^2}{2}\right)x^2 + ...}{2x^2 - \frac{x^4}{3}} = 17$$

This limit will exist if $a - \frac{c}{2} = 0$

$\Rightarrow c = 2a$ ...(1)

and $\dfrac{\frac{a^2}{2}+\frac{b^2}{2}+\frac{c^2}{2}}{2} = 17$

$\Rightarrow a^2 + b^2 + c^2 = 34 \times 2$

or $a^2 + b^2 + 4a^2 = 34 \times 2$ (from eqn (1)
$\Rightarrow 5a^2 + b^2 = 68$

3. Option (1) is correct.

Given line passes through the point (–3, 1, 5) whose d.r's are (–3, 1, 5)

Option (1): Here $\begin{vmatrix} -2 & -1 & 0 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix}$

$= -2(-5) + 1(-10) + 0 = 10 - 10 = 0$

So, no need to check other options.

4. Option (2) is correct.

Given that the plane passes through the points (0, –1, 2) & (–1, 2, 1)
$\Rightarrow$ d.r's of this line (1, –3, 1)
Also, the plane is parallel to line passing through (5, 1, –7) and (1, –1, –1)
So dr's of this line (4, 2, –6)
Now, if (a, b, c) one the d.r's of the normal to the plane then

$$a\hat{i} + b\hat{j} + c\hat{k} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 4 & 2 & -6 \end{vmatrix} = 16\hat{i} + 10\hat{j} + 14\hat{k}$$

$\Rightarrow (a, b, c) \equiv (16, +10, +14)$ or $(8, +5, +7)$
So the eqn. of the plane is
$ax + by + cz + d = 0$
or $8x + 5y + 7z + d = 0$
which passes through (0, –1, 2)
$\Rightarrow 0 - 5 + 14 + d = 0 \Rightarrow d = -9$
so $8x + 5y + 7z - 9 = 0$'

Clearly the point (–2, 5, 0) lies on the above plane.

5. Option (2) is correct.

Given that $\overrightarrow{AB} = -2\hat{i} + \hat{j} + 3\hat{k}$

$\overrightarrow{CB} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$

$\overrightarrow{CA} = 4\hat{i} + 3\hat{j} + \delta\hat{k}$

Since $\overrightarrow{CA} + \overrightarrow{AB} = \overrightarrow{CB}$

$\Rightarrow 2\hat{i} + 4\hat{j} + (3+\delta)\hat{k} = \overrightarrow{CB}$

Now

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & 3 \\ -4 & -3 & -\delta \end{vmatrix} = (9-\delta)\hat{i} - (2\delta + 12)\hat{j} + 10\hat{k}$$

$\Rightarrow |\overrightarrow{AB} \times \overrightarrow{AC}|^2 = (9-\delta)^2 + (2\delta+12)^2 + 100$

But area $(\Delta ABC) = 5\sqrt{6}$

$\Rightarrow \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = 5\sqrt{6}$

$\Rightarrow (9-\delta)^2 + (2\delta + 12)^2 + 100 = 4 \times 25 \times 6$
$\Rightarrow 81 + \delta^2 - 18\delta + 4\delta^2 + 144 + 48\delta + 100 = 600$
$\Rightarrow 5\delta^2 + 30\delta - 275 = 0$
$\Rightarrow \delta^2 + 6\delta - 55 = 0$
$\Rightarrow (\delta + 11)(\delta - 5) = 0$
$\Rightarrow \delta = 5$
so $\overrightarrow{CB} = 2\hat{i} + 4\hat{j} + 8\hat{k}$
$\overrightarrow{CA} = 4\hat{i} + 3\hat{j} + 5\hat{k}$
and $\overrightarrow{CB} \cdot \overrightarrow{CA} = 8 + 12 + 40 = 60$

6. Option (4) is correct.

Given that $A = \begin{bmatrix} 1 & 2 & 3 \\ \alpha & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix}, |A| = 2$

Now, adj (2A) = 2^2 adj (A)
$\Rightarrow$ 2 adj (2A) = 8 adj (A)
$\Rightarrow$ adj (2 adj (2A)) = adj (8 adj (A))
$= 8^2$ adj (adj (A))
$\Rightarrow$ |2 adj (2adj (2A))| = |2 × 64 adj (adj (A))|
$= (128)^3$ |adj (adj (A))|
$= (2^7)^3 |A|^{(3-1)2}$
$= 2^{21} \times |A|^4 = 2^{21} \times 2^4 = 2^{25}$
So |2 adj (2adj (2A))| $= 32^n$
$\Rightarrow 2^{25} = 2^{5n}$
$\Rightarrow 5n = 25 \Rightarrow n = 5$
Also, $|A| = 2$

$$\Rightarrow \begin{vmatrix} 1 & 2 & 3 \\ \alpha & 3 & 1 \\ 1 & 1 & 2 \end{vmatrix} = 2$$

$\Rightarrow 5 - 2(2\alpha - 1) + 3(\alpha - 3) = 2$
$\Rightarrow 5 - 4\alpha + 2 + 3\alpha - 9 = 2$
$\Rightarrow \alpha = -4$
Hence, $3n + \alpha = 3 \times 5 - 4 = 11$

7. Option (3) is correct.

Given that $f(x) = 4\sin^{-1}\left(\frac{x^2}{x^2+1}\right)$

Since, $0 \le \frac{x^2}{x^2+1} < 1 \quad \forall\, x \in R$

Therefore, $\sin^{-1}(0) \le \sin^{-1}\left(\frac{x^2}{x^2+1}\right) < \sin^{-1} 1$

$4 \times 0 \le f(x) < 4 \times \frac{\pi}{2}$

$\Rightarrow f(x) \in [0, 2\pi)$

8. Option (2) is correct.

Given that $a_1, a_2, a_3, \ldots$ are in G.P.

Also, $a_6 + a_8 = 2, a_3 \times a_5 = \frac{1}{9}$

$\Rightarrow ar^5 + ar^7 = 2$ and $ar^2 \times ar^4 = \frac{1}{9}$

$\Rightarrow ar^5(1 + r^2) = 2$ and $ar^3 = \frac{1}{3}$

$\Rightarrow ar^3 \times r^2(1 + r^2) = 2 \Rightarrow \frac{1}{3}[r^2 + r^4] = 2$

$\Rightarrow r^4 + r^2 - 6 = 0$
$\Rightarrow (r^2 + 3)(r^5 - 2) = 0$
$\Rightarrow r^2 = 2$

and $ar^3 = \frac{1}{3} \Rightarrow a \times 2^{3/2} = \frac{1}{3}$

$\Rightarrow a = \frac{1}{3 \times 2^{\frac{3}{2}}}$

So $6(a_2 + a_4)(a_4 + a_6)$

$= 6(ar + ar^3)(ar^3 + ar^5)$

$= 6\,ar \times ar^3 (1 + r^2)(1 + r^2)$

$= 6a^2r^4 (1 + r^2)^2$

$= 6 \times \frac{1}{9 \times 8} \times 4(1+2)^2 = \frac{6 \times 4 \times 9}{9 \times 8} = 3$

9. Option (2) is correct.

Since given system of equations have infinitely many solutions, so

$\rho\,[A : B] < \rho\,[A] = 2$

Now

$$[A:B] = \begin{bmatrix} 2 & 1 & -1 & : & 5 \\ 2 & -5 & \lambda & : & \mu \\ 1 & 2 & -5 & : & 7 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$ and then $R_2 \leftrightarrow R_1$

$$\sim \begin{bmatrix} 1 & 2 & -5 & : & 7 \\ 2 & 1 & -1 & : & 5 \\ 2 & -5 & \lambda & : & \mu \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & -5 & : & 7 \\ 0 & -3 & 9 & : & -9 \\ 0 & -9 & \lambda+10 & : & \mu-14 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & -5 & : & 7 \\ 0 & -3 & 9 & : & -9 \\ 0 & 0 & \lambda-17 & : & \mu+13 \end{bmatrix}$$

For inifinite solutions

$\lambda = 17$ and $\mu = -13$

Hence $(\lambda + \mu)^2 + (\lambda - \mu)^2 = (17 - 13)^2 + (17 + 13)^2$

$= (30)^2 + (4)^2 = 900 + 16 = 916$

10. Option (1) is correct.

Given that

$(p \wedge (\sim q)) \vee ((\sim p) \wedge q) \vee ((\sim p) \wedge (\sim q))$

$\equiv (p \wedge (\sim q)) \vee (((\sim p) \wedge q) \vee ((\sim p) \wedge (\sim q)))$

$\equiv (p \wedge (\sim q)) \vee ((\sim p) \wedge (q \vee (\sim q)))$

$\equiv (p \wedge (\sim q)) \vee (\sim p \wedge t)$

$\equiv (p \wedge (\sim q)) \vee (\sim p)$

$\equiv (p \vee (\sim p)) \wedge ((\sim p) \vee (\sim q))$

$\equiv t \wedge ((\sim p) \vee (\sim q))$

$\equiv (\sim p) \vee (\sim q)$

11. Option (1) is correct.

Given that $S = \{z \in C : \bar{z} = i(z^2 + \text{Re}(\bar{z}))\}$

Let $z = x + iy$

then $\bar{z} = i(z^2 + \text{Re}(\bar{z}))$

$\Rightarrow x - iy = i(x^2 - y^2 + 2ixy + x)$

$= -2xy + i(x^2 - y^2 + x)$

Comparing real & imaginary parts -

$x = -2xy$ and $-y = x^2 - y^2 + x$

$\Rightarrow x(1 + 2y) = 0$ and $(x + y)(x - y) + (x + y) = 0$

$\Rightarrow x = 0$ or $y = \frac{-1}{2}$ and $(x + y)(x - y + 1) = 0$

Now, if $x = 0$, then $y = 0, 1$

If $y = \frac{-1}{2}$ then $\left(x - \frac{1}{2}\right)\left(x + \frac{1}{2} + 1\right) = 0$

$\Rightarrow x = \frac{1}{2}, \frac{-3}{2}$

$\therefore z = 0 + 0i,\ = 0 + i,\ \frac{1}{2} - i\left(\frac{1}{2}\right), \frac{-8}{2} - i\left(\frac{1}{2}\right)$

$\sum_{z \in S} |z|^2 = 0 + 0 + 0 + 1 + \frac{1}{4} + \frac{1}{4} + \frac{9}{4} + \frac{1}{4} = 4$

12. Option (3) is correct.

Given that $x^2 - \sqrt{2}x + 2 = 0$

$\Rightarrow x = \frac{\sqrt{2} \pm \sqrt{2 - 4 \times 2}}{2 \times 1} = \frac{\sqrt{2} \pm i\sqrt{6}}{2}$

$x = \sqrt{2}\left(\frac{1 \pm i\sqrt{3}}{2}\right) = -\sqrt{2}\omega, -\sqrt{2}\omega^2$

$\Rightarrow \therefore \alpha = -\sqrt{2}\omega$ and $\beta = -\sqrt{2}\omega^2$

and $\alpha^{14} + \beta^{14} = 2^7\omega^{14} + 2^7\omega^{28}$

$= 2^7(\omega^3)^4\omega^2 + 2^7(\omega^3)^9\omega$

$= 2^7[1.\omega^2 + 1.\omega] = 2^7(-1) = -128$

13. Option (3) is correct.

Given that $|\vec{a}| = 2, |\vec{b}| = 3, \theta = \frac{\pi}{4}$

$|(\vec{a} + 2\vec{b}) \times (2\vec{a} - 3\vec{b})|^2$

$= |2\vec{a} \times \vec{a} - 3\vec{a} \times \vec{b} + 4\vec{b} \times \vec{a} - 6\vec{b} \times \vec{b}|^2$

$= |0 - 3\vec{a} \times \vec{b} - 4\vec{a} \times \vec{b} - 0|^2$

$= |-7\vec{a} \times \vec{b}|^2$

$= 49|\vec{a}|^2|\vec{b}|^2\left(\sin\frac{\pi}{4}\right)^2$

$= 49 \times 4 \times 9 \times \frac{1}{2} = 882$

14. Option (3) is correct.

We have

$\int_0^{\frac{\pi}{4}} e^{-x}(\tan^{49}x + \tan^{51}x\,dx)$

$= \int_0^{\frac{\pi}{4}} e^{-x}\tan^{49}x(1 + \tan^2 x)dx$

$= \int_0^{\frac{\pi}{4}} \underset{\text{I}}{e^{-x}}\, \underbrace{\tan^{44}x\sec^2}_{\text{II}} x\,dx$

$$= \left[e^{-x}\frac{\tan^{50}x}{50}\right]_0^{\frac{\pi}{4}} + \int_0^{\frac{\pi}{4}} e^{-x}\frac{\tan^{50}x}{50}dx$$

$$\int_0^{\frac{\pi}{4}} e^{-x}\left(\tan^{44}x + \tan^{51}x\right)dx = \frac{e^{-\frac{\pi}{4}}}{50} + \frac{1}{50}\int_0^{\frac{\pi}{4}} e^{-x}\tan^{50}x\,dx$$

$$\Rightarrow \frac{e^{-\frac{\pi}{4}} + \int_0^{\frac{\pi}{4}} e^{-x}\tan^{50}x\,dx}{\int_0^{\frac{\pi}{4}} e^{-x}(\tan^{49}x + \tan^{51}x)dx} = 50$$

15. Option (1) is correct.

Given that $\left(2x^3 - \frac{1}{3x^2}\right)^5$

$T_{r+1} = {}^5C_r(2x^3)^{5-r}\left(-\frac{1}{3x^2}\right)^r$

$= {}^5C_r(-1)^r(2)^{5-r} \times \left(\frac{1}{3}\right)^r x^{15-5r}$

For coefficient of x^5, $15 - 5r = 5$

$\Rightarrow r = 2$

So, coefficient of x^5 is ${}^5C_2(-1)^2(2)^3 \times \left(\frac{1}{3}\right)^2$

$= 10 \times 8 \times \frac{1}{9} = \frac{80}{9}$

16. Option (2) is correct.

Given that 2P (X = 2) = 3P (X = 1)

$\Rightarrow 2 \times {}^nC_2\, p^2(1-p)^{n-2} = 3 \times {}^nC_1\, p\,(1-p)^{n-1}$

$\Rightarrow \frac{2(n)(n-1)}{2}p^2(1-p)^{n-2} = 3 \times n \times p\,(1-p)^{n-1}$

$\Rightarrow (n-1)\,p = 3\,(1-p)$

$\Rightarrow np - p = 3 - 3p$

$\Rightarrow np + 2p = 3$ ...(1)

Also,

mean – variance = 1

$\Rightarrow np - npq = 1$

$\Rightarrow 3 - 2p - (3 - 2p)(1 - p) = 1$

$\Rightarrow 3 - 2p - 3 + 3p + 2p - 2p^2 = 1$

$\Rightarrow 2p^2 - 3p + 1 = 0$

$\Rightarrow (2p - 1)(p - 1) = 0$

$\Rightarrow p = \frac{1}{2}$ ($p = 1$) rejected also $q = \frac{1}{2}$

and $\frac{n}{2} + 1 = 3 \Rightarrow n = 4$

Now, n^2P(X > 1) = (1 – P (X = 0) – P(X = 1)n^2

$= \left(1 - {}^4C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^4 - {}^4C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^3\right)4^2$

$= \left(1 - \frac{1}{16} - \frac{4}{16}\right)16$

$= \left(\frac{16-5}{16}\right)16 = 11$

17. Option (3) is correct.

Since, $3x + 4y = 24$ and $3x - 4y = 32$ be two tangents & $4x + 3y = 1$ be a normal and (α, β) be the centre, then the centre lies on the normal

$\Rightarrow 4\alpha + 3\beta = 1$ ...(1)

Also, $\left|\frac{3\alpha + 4\beta - 24}{5}\right| = \left|\frac{3\alpha - 4\beta - 32}{5}\right|$

$\Rightarrow 3\alpha + 4\beta - 24 = \pm(3\alpha - 4\beta - 32)$

Taking (+) sign

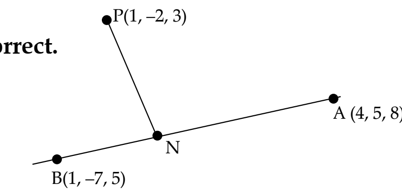
$3\alpha + 4\beta - 24 = 3\alpha - 4\beta - 32$

$\Rightarrow \beta = -1$ and $\alpha = 1$

and $r = \frac{|3 \times 1 + 4 \times (-1) - 24|}{5} = \frac{25}{5} = 5 < 8$

Here, $\alpha - \beta + \gamma = 1 + 1 + 5 = 7$

18. Option (2) is correct.



Eqn. of line AB

$\frac{x-1}{3} = \frac{y+7}{12} = \frac{z-5}{3} = \lambda$

$\Rightarrow$ N $(x, y, z) \equiv (3\lambda + 1, 12\lambda - 7, 3\lambda + 5)$

Also, PN $\perp$ AB

$\Rightarrow 3(3\lambda + 1 - 1) + 12(12\lambda - 7 + 2) + 3(3\lambda + 5 - 3) = 0$

$\Rightarrow 9\lambda + 144\lambda - 60 + 9\lambda + 6 = 0$

$\Rightarrow 162\lambda = 53 \Rightarrow \lambda = \frac{1}{3}$

So, N is (2, –3, 6)

and distance of $2x - 2y + z + 5 = 0$ from N (2, –3, 6) is

$= \left|\frac{2 \times 2 + 3 \times 2 + 6 \times 1 + 5}{\sqrt{4+4+1}}\right| = \frac{21}{3} = 7$

19. Option (2) is correct.

Given word is MONDAY which can be written as ADMNOY

No. of words starting with A are 5!

No. of words starting with D are 5!

No. of words starting with MA are 4!

No. of words starting with MN are 4!

No. of words starting with MON are 4!

No. of words starting with MOD are 3!

No. of words starting with MONA are 2!

No. of words starting with MONDAY are 1!

Hence, required rank is = 5! × 2 + 4! × 3 + 3! × 2 + 2 + 1

= 240 + 72 + 12 + 3 = 327

20. Option (1) is correct.

Given eqn of sides are $15x - y = 82$ ...(1)

$6x - 5y = -4$ ...(2)

and $9x + 4y = 17$ ...(2)

Solving eqns (1), (2), (3) simultaneously, we get the vertices of Δ as (6, 8), (1, 2) and (5, –7)

If (α, β) is the centroid of Δ, then

$\alpha = \frac{6+1+5}{3} = \frac{12}{3} = 4$

$\beta = \frac{8+2-7}{3} = \frac{3}{3} = 1$

So, $\alpha + 2\beta = = 4 + 2 = 6$

and $2\alpha - \beta = 8 - 1 = 7$

Hence, required quadratic eqn is

$x^2 - (6 + 7)x + 6 \times 7 = 0$

$\Rightarrow x^2 - 13x + 42 = 0$

Section B

21. Correct answer is [7].

Given that A = {–4, –3, –2, 0, 1, 3, 4}

$R = \{(a, b) \in A \times A : b = |a|, \text{ or } b^2 = a + 1|$

$\Rightarrow$ R = {(–4, 4), (4, 4), (–3, 3), (3, 3), (0, 0), (0, 1), (1, 1), (3, –2)}

Now, for reflexive R must contain (–2, –2), (–3, –3), (–4, –4)

and for symmetric R must contain (4, –4), (3, –3), (–2, 3), (1, 0)

22. Correct answer is [41].

Given that

$$f_n = \int_0^{\frac{\pi}{2}} \left(\sum_{k=1}^{n} \sin^{k-1} x\right)\left(\sum_{k=1}^{n} (2k-1)\sin^{k-1} x\right)\cos x\, dx$$

Putting $t = \sin x \Rightarrow dt = \cos x\, dx$

$x = 0, t = 0$ and $x = \frac{\pi}{2}, t = 1$

$$\Rightarrow f_n = \int_0^1 \left(\sum_{k=1}^{n} t^{k-1}\right)\left(\sum_{k=1}^{n} (2k-1)t^{k-1}\right)dt$$

$$\Rightarrow f_n = \int_0^1 \underbrace{\{1 + t + t^2 + ... + t^{n-1}\}}_{a}\ \underbrace{\{1 + 3t + 5t^2 + ... + (2n-1)t^{n-1}\}}_{b}dt$$

$$\Rightarrow f_n = \int_0^1 a\,b\,dt \qquad ...(1)$$

and $$f_{n+1} = \int_0^1 (a + t^n)(b + (2n+1)t^n)dt \qquad ...(2)$$

from (1) and (2)

$$f_{n+1} - f_n = \int_0^1 \{(a + t^n)(b + (2n+1)t^n) - ab\}dt$$

$$= \int_0^1 [ab + t^n\{(2n+1)a + b\} + (2n+1)t^n - ab]dt$$

$$= \int_0^1 t^n\{(2n+1)t^n + (2n+2) + (2n+4)t + (2n+6)t^2 + ... + 4nt^{n-1}\}dt$$

$$= \int_0^1 \{(2n+2)t^n + (2n+4)t^{n+1} + (2n+6)t^{n+1} + ... + 4nt^{2n-1} + (2n+1)t^{2n}\}dt$$

Putting $n = 20$

$$f_{21} - f_{20} = \int_0^1 [42t^{20} + 44t^{21} + 46t^{22} + ... + 80t^{39} + 41t^{40}]dt$$

$$= \frac{42}{21} + \frac{44}{22} + \frac{45}{23} + ... + \frac{80}{40} + \frac{41}{41}$$

$$= \underbrace{2 + 2 + 2 + ... + 2}_{20 \text{ times}} + 1$$

$= 40 + 1 = 41$

23. Correct answer is [6].

Given that

$$\frac{dy}{dx} + \frac{4x}{x^2 - 1}y = \frac{x+2}{(x^2-1)^{\frac{5}{2}}}$$

which is LDE, where $P = \frac{4x}{x^2 - 1}$ and $Q = \frac{x+2}{(x^2-1)^{5/2}}$

So I.F. $= e^{\int P dx} = e^{\int \frac{4x}{x^2-1}dx} = e^{2\log(x^2-1)} = (x^2-1)^2$

So, solution of DE is given by

$$y \times (x^2-1)^2 = \int \frac{x+2}{(x^2-1)^{\frac{5}{2}}} \times (x^2-1)^2 dx + C$$

$$= \int \frac{x+2}{(x^2-1)^{\frac{1}{2}}} dx + C$$

$$= \int \frac{x}{\sqrt{x^2-1}} + 2\int \frac{1}{\sqrt{x^2-1}} dx + C$$

$$y(x^2-1)^2 = \sqrt{x^2-1} + 2\log\left|x + \sqrt{x^2-1}\right| + C$$

Putting $y(2) = \frac{2}{9}\log(2 + \sqrt{3})$

$$\frac{2}{9}\log(2+\sqrt{3}) \times 9 = \sqrt{3} + 2\log|2 + \sqrt{3}| + C$$

$\Rightarrow C = -\sqrt{3}$

putting $x = \sqrt{2}$

$y(1)^2 = \sqrt{1} + 2\log|\sqrt{2} + 1| - \sqrt{3}$

$\Rightarrow y = 1 - \sqrt{3} + 2\log|1 + \sqrt{2}| = \alpha\log(\sqrt{\alpha} + \beta) + \beta - \sqrt{\gamma}$

On comparing $\Rightarrow \alpha = 1, \beta = 1, \gamma = 3$

and $\alpha \,.\, \beta \,.\, \gamma = 2 \times 1 \times 3 = 6$

24. Correct answer is [16].

Given numbers are 1, 2, 3, 4, 5

Now, a number is divisible by 6, if it is divisible by both 3 and 2.

Case I: If 2 is at unit place, then possible pairs of first two places are (1, 3), (3, 1), (2, 2), (2, 5), (5, 2), (3, 4), (4, 3), (5, 5)

Case II: If 4 is at unit place, then possible pairs of first two places are (1, 1), (1, 4), (4, 1), (2, 3), (3, 2), (4, 4), (3, 5), (5, 8)

Hence, total numbers are 8 + 8 = 16

25. Correct answer is [12].

$7^{103} \equiv 7 \times 7^{102}$

$= 7\,(7^2)^{51}$

$= 7\,(51 - 2)^{51}$

$= 7\,(51k - 2^{51})$ where $k \in z^+$

$= 7 \times 51k - 7 \times 2^3 \times (2^4)^{12}$

$= 7 \times 51k - 7 \times 8 \times (17 - 1)^{12}$

$= 7 \times 51k - 56 \times (17q + 1)$

$= 7 \times 51k - 56 \times 17q - 56$

$= 17\,\{7 \times 3k - 56q - 4\} + 12$

$= 17p + 12$

Hence, remainder is 12.

26. Correct answer is [10].

Given that

$$f(x) = \sum_{k=1}^{10} kx^k, k \in \mathrm{R}$$

$\Rightarrow f(x) = x + 2x^2 + 3x^3 + ... + 10x^{10}$

$xf(x) = \quad x^2 + 2x^3 + \quad + 9x^{10} + 10x^{11}$

$- \quad - \quad - \quad - \quad -$

$(1 - x)\,f(x) = x + x^2 + x^3 + ... + x^{10} - 10x^{11}$

$$(1-x)f(x) = \frac{x(1-x^{10})}{1-x} - 10x^{11}$$

of $(1 - x)^2 f(x) = x(1 - x^{10}) - 10\,x^{11}(1 - x)$

Putting $x = 2$

$f(2) = 2\,(1 - 2^{10}) + 10 \times 2^{11}$

Also, $(1 - x)^2 f'(x) - 2(1 - x)\,f(x) = 1 - x^{10} - x\,(10x^9)$
$+ 10x^{11} - 110x^{10}\,(1 - x)$

Putting $x = 2$

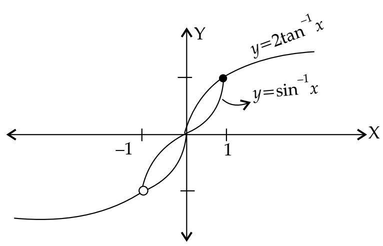
$f'(2) + 2f(2) = 1 - 2^{10} - 10 \times 2^{10} + 10 \times 2^{11} + 110 \times 2^{10}$

$= 1 + 119 \times 2^{10}$

Hence, by comparing we get $n = 10$

27. Correct answer is [2].

Let $y = \sin^{-1} x$ and $y = 2\tan^{-1} x$



Since, two graphs intersect at 3 points but $x \in (-1, 1]$, therefore, there are two solutions.

28. Correct answer is [269].

Mean = 50,

$$\Rightarrow \frac{\Sigma x_i}{10} = 50$$

$\Rightarrow \Sigma x_i = 500$

Variance = 12^2

$$\frac{\Sigma x_i^2}{10} - (\bar{x})^2 = 12^2$$

$\Rightarrow \Sigma x_i^2 = (144 + 2500) \times 10 = 26440$

Now, correct mean $= \dfrac{500 - 45 - 50 + 20 + 25}{10} = \dfrac{450}{10}$
$= 45$

Also, correct $\Sigma x_i^2 = 26440 - (45)^2 - (50)^2 + (20)^2 + (25)^2$

$= 26440 - 70 \times 20 - 70 \times 30 = 22940$

$\therefore$ Correct variance $= \dfrac{22940}{10} -$ (Correct mean)^2

$= 2294 - (45)^2 = 2294 - 2025 = 269$

29. Correct answer is [12].

Given that $2ae = 4$

$$\Rightarrow 2a\left(\frac{3}{2}\right) = 4 \Rightarrow a = \frac{4}{3}$$

Also, $e = \sqrt{1 + \dfrac{b^2}{a^2}} \Rightarrow \dfrac{3}{4} = \sqrt{1 + b^2\left(\dfrac{9}{16}\right)}$

$$\Rightarrow b^2 = \frac{20}{9}$$

Slope of tangent $m = \dfrac{3}{2}$

So eqn. of tangent is given by

$$y = mx \pm \sqrt{a^2m^2 - b^2}$$

$$\Rightarrow y = \frac{3}{2}x \pm \sqrt{\frac{16}{9} \times \frac{9}{4} - \frac{20}{9}} = \frac{3x}{2} \pm \frac{4}{3}$$

Putting $y = 0 \Rightarrow a = \pm\dfrac{8}{9}$

$x = 0 \Rightarrow b = \pm\dfrac{4}{3}$

and $|6a| + |5b| = 12$

30. Correct answer is [825].

Since

$1 \le \sqrt{1}, \sqrt{2}, \sqrt{3} < 2$

$2 \le \sqrt{4}, \sqrt{5}, ...\sqrt{8} < 3$

$3 \le \sqrt{9}, \sqrt{10}, ...\sqrt{15} < 4$

$4 \le \sqrt{16}, \sqrt{17}, ...\sqrt{24} < 4$

:

:

$10 \le \sqrt{100}, \sqrt{101}, ..., \sqrt{120} < 11$

Hence, $\left[\sqrt{1}\right] + \left[\sqrt{2}\right] + \left[\sqrt{3}\right] + ... + \left[\sqrt{120}\right]$

$= 1 \times 3 + 2 \times 5 + 3 \times 7 + 4 \times 9 + ... + 10 \times 21$

$= 1\,(2 \times 1 + 1) + 2\,(2 \times 2 + 1) + 3\,(2 \times 3 + 1) +$
$.... 10\,(2 \times 10 + 1)$

$= 2\{1^2 + 2^2 + 3^2 + ... + 10^2\} + \{1 + 2 + 3 + ... + 10\}$

$$= 2 \times \frac{10 \times 11 \times 21}{6} + \frac{10 \times 11}{2}$$

$= 770 + 55 = 825$

❑❑