JEE (Main) MATHEMATICS SOLVED PAPER

2023

15^{th} April Shift 1

Section A

Q. 1. Let S be the set of all values of λ, for which the shortest distance between the lines $\frac{x-\lambda}{0}=\frac{y-3}{4}=\frac{z+6}{1}$ and $\frac{x+\lambda}{3}=\frac{y}{-4}=\frac{z-6}{0}$ is 13. Then, $8\left|\sum_{\lambda\in S}\lambda\right|$ is equal to:

(1) 302 **(2)** 306 **(3)** 304 **(4)** 308

Q. 2. Let S be the set of all (λ, μ) for which the vectors $\lambda\hat{i}-\hat{j}+\hat{k}, \hat{i}+2\hat{j}+\mu\hat{k}$ and $3\hat{i}-4\hat{j}+5\hat{k}$, where $\lambda - \mu = 5$, are coplanar, then $\sum_{(\lambda,\mu)\in S} 80(\lambda^2+\mu^2)$ is equal to:

(1) 2130 **(2)** 2210 **(3)** 2290 **(4)** 2370

Q. 3. Let the foot of the perpendicular of the point P(3, –2, –9) on the plane passing through the points (–1, –2, –3), (9, 3, 4) , (9, –2, 1) be Q(α, β, γ). Then, the distance of Q from the origin is:

(1) $\sqrt{29}$ **(2)** $\sqrt{38}$ **(3)** $\sqrt{42}$ **(4)** $\sqrt{35}$

Q. 4. If the set $\left\{\text{Re}\left(\frac{z-\bar{z}+z\bar{z}}{2-3z+5\bar{z}}\right): z\in C, \text{Re}(z)=3\right\}$ is equal to the interval (α, β], then 24 (β – α) is equal to:

(1) 36 **(2)** 27 **(3)** 30 **(4)** 42

Q. 5. Let $x = x(y)$ be the solution of the differential equation $2(y+2)\log_e(y+2)\,dx + (x+4-2\log_e(y+2))\,dy = 0$, $y > -1$ with $x(e^4-2)=1$. Then, $x(e^9-2)$ is equal to:

(1) $\frac{4}{9}$ **(2)** $\frac{32}{9}$ **(3)** $\frac{10}{3}$ **(4)** 3

Q. 6. If $\int_0^1 \frac{1}{(5+2x-2x^2)(1+e^{(2-4x)})}dx = \frac{1}{\alpha}\log_e\left(\frac{\alpha+1}{\beta}\right)$, α, β > 0, then $\alpha^4-\beta^4$ is equal to

(1) 19 **(2)** –21 **(3)** 21 **(4)** 0

Q. 7. The number of common tangents, to the circles $x^2+y^2-18x-15y+131=0$ and $x^2+y^2-6x-6y-7=0$ is:

(1) 4 **(2)** 1 **(3)** 3 **(4)** 2

Q. 8. Let ABCD be a quadrilateral. If E and F are the mid points of the diagonals AC and BD, respectively and $(\overline{AB}-\overline{BC})+(\overline{AD}-\overline{DC})=k\overline{FE}$, then k is equal to:

(1) 4 **(2)** 2 **(3)** –2 **(4)** –4

Q. 9. Let $(a+bx+cx^2)^{10}=\sum_{i=0}^{20}p_i x^i$, $a, b, c, \in$ N. If $p_1 = 20$ and $p_2 = 210$, then $2(a+b+c)$ is equal to:

(1) 8 **(2)** 12 **(3)** 6 **(4)** 15

Q. 10. Let $[x]$ denote the greatest integer function and $f(x) = \max\{1+x+[x], 2+x, x+2[x]\}, 0 \le x \le 2$. Let m be the number of points in [0, 2], where f is not continuous and n be the number of points in (0, 2) where f is not differentiable. Then, $(m+n)^2$ is equal to:

(1) 6 **(2)** 3 **(3)** 2 **(4)** 11

Q. 11. A bag contains 6 white and 4 black balls. A die is rolled once and the number of ball equal to the number obtained on the die are drawn from the bag at random. The probability that all the balls drawn are white is:

(1) $\frac{1}{4}$ **(2)** $\frac{9}{50}$ **(3)** $\frac{11}{50}$ **(4)** $\frac{1}{5}$

Q. 12. If the domain of the function $f(x)=\log_e(4x^2+11x+6)+\sin^{-1}(4x+3)+\cos^{-1}\frac{10x+6}{3}$ (α, β], then 36 |α + β| is equal to:

(1) 72 **(2)** 63 **(3)** 45 **(4)** 54

Q. 13. Let the determinant of a square matrix A of order m be $m - n$, where m and n satisfy $4m + n = 22$ and $17m + 4n = 93$. If det (n adj (adj (mA))) = $3^a 5^b 6^c$, then $a + b + c$ is equal to:

(1) 101 **(2)** 84 **(3)** 109 **(4)** 96

Q. 14. The mean and standard deviation of 10 observations are 20 and 8, respectively. Later on, it was observed that one observation was recorded as 50 instead of 40. Then, the correct variance is:

(1) 14 **(2)** 11 **(3)** 12 **(4)** 13

Q. 15. If (α, β) is the orthocenter of the triangle ABC with vertices A (3, –7), B (–1,2) and C (4, 5), then 9α – 6β + 60 is equal to:

(1) 30 **(2)** 35 **(3)** 40 **(4)** 25

Q. 16. The number of real roots of the equation $x|x|-5|x+2|+6=0$, is

(1) 5 **(2)** 6 **(3)** 4 **(4)** 3

Q. 17. Let the system of linear equations

$-x+2y-9z=7$ $\quad$ $-x+3y+7z=9$

$-2x+y+5z=8$ $\quad$ $-3x+y+13z=\lambda$

have a unique solution $x=\alpha, y=\beta, z=\gamma$. Then the distance of the point (α, β, γ) from the plane $2x-2y+z=\lambda$ is:

(1) 7 **(2)** 9 **(3)** 13 **(4)** 11

Q. 18. Let A_1 and A_2 be two arithmetic means and G_1, G_2, G_3 be three geometric means of two distinct positive numbers. Then, $G_1^4+G_2^4+G_3^4+G_3^4+G_1^2G_3^2$ is equal to:

(1) $2(A_1+A_2)G_1G_3$ **(2)** $(A_1+A_2)^2G_1G_3$

(3) $2(A_1+A_2)G_1^2G_3^2$ **(4)** $(A_1+A_2)G_1^2G_3^2$

Q. 19. Negation of $p \wedge (q \wedge \sim (p \wedge q))$ is:
(1) $(\sim p \wedge q)) \wedge q$ **(2)** $\sim (p \vee q)$
(3) $p \vee q$ **(4)** $\sim (p \wedge q) \vee p$

Q. 20. The total number of three-digit numbers, divisible by 3, which can be formed using the digits 1, 3, 5, 8, if repetition of digits is allowed, is:
(1) 21 **(2)** 18 **(3)** 20 **(4)** 22

Section B

Q. 21. Let A = {1, 2, 3, 4} and R be a relation on the set A×A defined by R={((a, b, (c, d) : 2a + 3b = 4c + 5d}. Then, the number of elements in R is ________.

Q. 22. The number of elements in the set $\{n \in \mathbb{N} : 10 \le n \le 100$ and $3^n - 3$ is a multiple of 7} is ________.

Q. 23. Let an ellipse with centre (1,0) and latus rectum of length $\frac{1}{2}$ have its major axis along the x-axis. If its minor axis subtends an angle 60° at the foci, then the square of the sum of the lengths of its minor and major axes is equal to ________.

Q. 24. If the area bounded by the curve $2y^2 = 3x$, lines $x + y = 3$, $y = 0$ and outside the circle $(x - 3)^2 + y^2 = 2$ is A , then $4(\pi + 4A)$ is equal to ________.

Q. 25. Consider the triangles with vertices A(2, 1), B(0, 0) and C(t, 4), $t \in [0, 4]$. If the maximum and the minimum perimeters of such triangles are obtained at $t = \alpha$ and $t = \beta$, respectively, then $6\alpha + 21\beta$ is equal to ________.

Q. 26. Let the plane P contain the line $2x + y - z - 3 = 0 = 5x - 3y + 4z + 9$ and be parallel to the line $\frac{x+2}{2} = \frac{3-y}{-4} = \frac{z-7}{5}$. Then, the distance of the point A(8, –1, –19) from the plane P measured parallel to the line $\frac{x}{-3} = \frac{y-5}{4} = \frac{2-z}{-12}$ is equal to _____.

Q. 27. If the sum of the series $\left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{2^2} - \frac{1}{2.3} + \frac{1}{3^2}\right) + \left(\frac{1}{2^3} - \frac{1}{2^2.3} + \frac{1}{2.3^2} - \frac{1}{3^3}\right) + \left(\frac{1}{2^4} - \frac{1}{2^2.3} + \frac{1}{2^2.3^2} - \frac{1}{2.3^2} + \frac{1}{3^4}\right) + \ldots$ is $\frac{\alpha}{\beta}$, where α and β are co-prime, then $\alpha + 3\beta$ is equal to ________.

Q. 28. A person forgets his 4-digit ATM pin code. But he remembers that in the code all the digits are different, the greatest digit is 7 and the sum of the first two digits is equal to the sum of the last two digits. Then, the maximum number of trials necessary to obtain the correct code is______

Q. 29. If the line $x = y = z$ intersects the line $x \sin A + y \sin B + z \sin C - 18 = 0 = x \sin 2A + y \sin 2B + z \sin 2C - 9$, where A, B, C are the angles of a triangle ABC, then $80\left(\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right)$ is equal to ________.

Q. 30. Let $f(x) = \int \frac{dx}{(3+4x^2)\sqrt{4-3x^2}}$, $|x| < \frac{2}{\sqrt{3}}$. If $f(0) = 0$ and $f(1) = \frac{1}{\alpha\beta}\tan^{-1}\left(\frac{\alpha}{\beta}\right)$, $\alpha, \beta > 0$, then $\alpha^2 + \beta^2$ is equal to ________.

Answer Key

Q. No.	Answer	Topic name	Chapter name
1	(2)	Straight Line	3D
2	(3)	Coplanar	Vectors
3	(3)	Plane	3D
4	(3)	Modulus	Complex number
5	(2)	Linear differential equation	Differential equation
6	(3)	Definite Integral	Definite Integral
7	(3)	Circles	Conic Sections
8	(4)	Mid-points	Vectors
9	(2)	Coefficient of terms	Binomial theorem
10	(2)	Continuity and differentiable	Differentiation
11	(4)	Basic probability	Probability
12	(3)	Domain	Functions
13	(4)	Adjoint of Matrices	Matrices
14	(4)	Mean and variance	Statistics
15	(4)	Orthocentre	Straight Line

Q. No.	Answer	Topic name	Chapter name
16	(4)	Quadratic equation	Complex Number
17	(1)	System of equations	Determinants
18	(2)	A.M. and G.M.	Sequence and series
19	(4)	Negation	Mathematical Reasoning
20	(4)	Permutation	Permutation and combination
21	[6]	Relation	Relation and Function
22	[15]	A.P.	Sequence and series
23	[9]	Ellipse	Conic sections
24	[42]	Area bounded relation	Application of Integration
25	[48]	Perimeter of triangle	Straight line
26	[26]	Plane	3D
27	[7]	Special series	Sequence and Series
28	[72]	Permutation	Permutation and combination
29	[5]	Identity	Trigonometry
30	[28]	Substitution Method	Indefinite Integration

Solutions

Section A

1. Option (2) is correct.

Given that SD = 13

$$\Rightarrow \frac{\begin{vmatrix} 2\lambda & 3 & -12 \\ 0 & 4 & 1 \\ 3 & -4 & 0 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 4 & 1 \\ 3 & -4 & 0 \end{vmatrix}} = 13$$

$$\Rightarrow \frac{|2\lambda(4)-3(-3)-12(-12)|}{|4\hat{i}+3\hat{j}-12\hat{k}|} = 13$$

$$\lambda = \frac{16}{8}, \frac{-322}{8}$$

$$\text{So } 8\left|\sum_{\lambda\in S}\lambda\right| = 8\left|\frac{16}{8}-\frac{322}{8}\right| = \frac{8(306)}{8} = 306$$

2. Option (3) is correct.

Given vectors are coplanar, if

$$\begin{vmatrix} \lambda & -1 & 1 \\ 1 & 2 & \mu \\ 3 & -4 & 5 \end{vmatrix} = 0$$

$\Rightarrow \lambda\,(10 + 4\mu) + 1\,(5 - 3\,\mu) + (-10) = 0$

$\Rightarrow \mu = \frac{-15}{4}$ or $\mu = -3$ $\Rightarrow \lambda = \frac{5}{4}$ or $\lambda = 2$

$$\text{So } \sum_{(\lambda,\mu)\in S} 80(\lambda^2+\mu^2) = 80\left[\frac{225}{16}+\frac{25}{16}+9+4\right]$$

$$= 80\left[\frac{250+13\times 16}{16}\right]$$

$= 5\,[250 + 208] = 5 \times 458 = 2290$

3. Option (3) is correct.

Eqn. of the plane passing through the given points (–1, –2, –3), (9, 3, 4), (9, –2, 1) is

$$\begin{vmatrix} x+1 & y+2 & z+3 \\ 10 & 5 & 7 \\ 10 & 0 & 4 \end{vmatrix} = 0$$

$\Rightarrow 2x + 3y - 5z - 7 = 0$

Also, Q (α, β, γ) is the foot of the perpendicular from P (3, –2, –9) on plane $2x + 3y - 5z - 7 = 0$

$$\frac{\alpha-3}{2} = \frac{\beta+2}{3} = \frac{\gamma+9}{-5} = \frac{-(6-6+45-7)}{4+9+25} = \frac{-38}{38}$$

$\Rightarrow \alpha = 1, \beta = -5, \gamma = -4$

Hence, OQ $= \sqrt{1+25+16} = \sqrt{42}$

4. Option (3) is correct.

Let $z = 3 + iy$

$$\therefore \frac{z-\bar{z}+z\bar{z}}{2-3z+5\bar{z}} = \frac{3+iy-3+iy+9+y^2}{2-9-3iy+15-5iy}$$

$$= \frac{(9+y^2)+2iy}{8(1-iy)} \times \frac{1+iy}{1+iy}$$

$$\text{Re}\left(\frac{z-\bar{z}+z\bar{z}}{2-3z+5\bar{z}}\right) = \frac{9+y^2-2y^2}{8(1+y^2)} = \frac{9-y^2}{8(1+y^2)}$$

$$= \frac{10-(1+y^2)}{8(1+y^2)} = \frac{1}{8}\left[\frac{10}{1+y^2}-1\right]$$

But $1 \le 1 + y^2 < \infty$

$$0 < \frac{10}{1+y^2} \le 10$$

$$-1 < \frac{1}{1+y^2} - 1 \le 9$$

$$-\frac{1}{8} < \frac{1}{8}\left[\frac{1}{1+y^2} - 1\right] \le \frac{9}{8}$$

Hence, $\text{Re}\left(\frac{z - \bar{z} + z\bar{z}}{2 - 3z + 5\bar{z}}\right) \in \left(-\frac{1}{8}, \frac{9}{8}\right] \equiv (\alpha, \beta]$

$$\Rightarrow \alpha = -\frac{1}{8}, \beta = \frac{9}{8}$$

and $24(\beta - \alpha) = 24\left(\frac{9}{8} + \frac{1}{8}\right) = \frac{24 \times 10}{8} = 30$

5. **Option (2) is correct.**

Given that

$2(y + 2)\log_e(y + 2)\,dx + (x + 4 - 2\log_e(y + 2))dy = 0$

$$\Rightarrow \left(x + 4 - 2\log_e(y+2)\right)\frac{dy}{dx} = -2(y+2)\log(y+2)$$

Let $t = \log_e(y + 2)$

$$\frac{dt}{dx} = \frac{1}{y+2} \times \frac{dy}{dx}$$

$$\Rightarrow (x + 4 - 2t)(y+2)\frac{dt}{dx} = -2(y+2) \times t$$

$$\Rightarrow x + 4 - 2t = -2t\frac{dx}{dt}$$

$$\Rightarrow \frac{dx}{dt} - 1 + \frac{x+4}{2t} = 0$$

$$\frac{dx}{dt} + \left(\frac{1}{2t}\right)x + \left(\frac{2}{t} - 1\right) = 0$$

$$\frac{dx}{dt} + \left(\frac{1}{2t}\right)x = 1 - \frac{2}{t} \text{ which is L.D.E.}$$

$$\text{I.F.} = e^{\int \frac{1}{2x}dt} = e^{\frac{1}{2}\log t} = (t)^{\frac{1}{2}}$$

$$x \times (t)^{\frac{1}{2}} = \int \left(1 - \frac{2}{t}\right)t^{\frac{1}{2}}\, dt + C$$

$$x\sqrt{t} = \frac{2}{3}t^{\frac{3}{2}} - 2 \times 2\, t^{\frac{1}{2}} + C$$

$$x = \frac{2}{3}t - 4 + C\, t^{-\frac{1}{2}}$$

$$x = \frac{2}{3}\log(y+2) - 4 + C\left(\log(y+2)\right)^{-\frac{1}{2}}$$

putting $x = 1$ & $y = e^4 - 2$

$$1 = \frac{2}{3} \times 4 - 4 + C \times \frac{1}{2} \Rightarrow C = \frac{14}{3}$$

$$\Rightarrow x = \frac{2}{3} \times 9 - 4 + \frac{14}{3} \times \frac{1}{3} = 6 - 4 + \frac{14}{9} = \frac{32}{9}$$

6. **Option (3) is correct.**

Let $I = \int_0^1 \frac{1}{(5 + 2x - 2x^2)(1 + e^{2-4x})}dx$ ...(i)

Replacing x by $1 - x$

$$\Rightarrow I = \int_0^1 \frac{1}{(5 + 2(1-x) - 2(1-x)^2)(1 + e^{2-4(1-x)})}dx$$

$$= \int_0^1 \frac{1}{(5 + 2 - 2x - 2 - 2x^2 + 4x)(1 + e^{-2+4x})}dx$$

$$= \int_0^1 \frac{1}{(5 + 2x - 2x^2)\left(1 + \frac{1}{e^{2-4x}}\right)}dx$$

$$I = \int_0^1 \frac{e^{2-4x}}{(5 + 2x - 2x^2)(1 + e^{2-4x})}dx \quad ...(ii)$$

Adding (1) and (ii) we get,

$$2I = \int_0^1 \frac{1}{5 + 2x - 2x^2}dx = \int_0^1 \frac{dx}{2\left(\frac{5}{2} + x - x^2 + \frac{1}{4} - \frac{1}{4}\right)}$$

$$= \frac{1}{2}\int_0^1 \frac{dx}{\left(\frac{\sqrt{11}}{2}\right)^2 - \left(x - \frac{1}{2}\right)^2} = \frac{1 \times 2}{2 \times 2 \times \sqrt{11}}$$

$$\log\left|\frac{\frac{\sqrt{11}}{2} + x - \frac{1}{2}}{\frac{\sqrt{11}}{2} - x + \frac{1}{2}}\right|_0^1$$

$$= \frac{1}{2\sqrt{11}}\left\{\log\left|\frac{\frac{\sqrt{11}}{2} + 1 - \frac{1}{2}}{\frac{\sqrt{11}}{2} - 1 + \frac{1}{2}}\right| - \log\left|\frac{\frac{\sqrt{11}}{2} - \frac{1}{2}}{\frac{\sqrt{11}}{2} + \frac{1}{2}}\right|\right\}$$

$$= \frac{1}{2\sqrt{11}}\left\{\log\left|\frac{\sqrt{11}+1}{\sqrt{11}-1}\right| - \log\left|\frac{\sqrt{11}-1}{\sqrt{11}+1}\right|\right\}$$

$$= \frac{1}{2\sqrt{11}}\log\left(\frac{\sqrt{11}+1}{\sqrt{11}-1}\right)^2$$

$$\Rightarrow I = \frac{1}{\sqrt{11}}\log\left(\frac{\sqrt{11}+1}{\sqrt{10}}\right) = \frac{1}{\alpha}\log\left(\frac{\alpha+1}{\beta}\right)$$

$$\Rightarrow \alpha = \sqrt{11}, \beta = \sqrt{10}$$

Hence, $\alpha^4 - \beta^4 = 121 - 100 = 21$

7. **Option (3) is correct.**

For the first circle

$C_1\left(9, \frac{15}{2}\right)$ and $r_1 = \sqrt{81 + \frac{225}{4} - 131} = \frac{5}{2}$

For the second circle, $C_2(3, 3)$, $r_2 = \sqrt{9 + 9 + 7} = 5$

$$C_1C_2 = \sqrt{(9-3)^2 + \left(\frac{15}{2} - 3\right)^2} = \sqrt{36 + \frac{81}{4}} = \frac{15}{2}$$

and $r_1 + r_2 = \frac{5}{2} + 5 = \frac{15}{2}$

Here, $C_1C_2 = r_1 + r_2$, so both circle touch externally.

Hence, the number of common tangents are 3.

8. **Option (4) is correct.**

Let position vectors of

A, B, C, D, E, F are $\vec{a}, \vec{b}, \vec{c}, \vec{d}, \vec{e}, \vec{f}$, respectively.

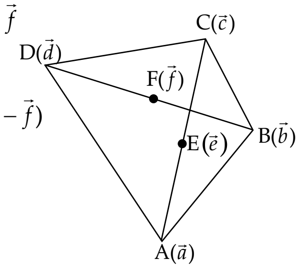
So, $(\overrightarrow{AB} - \overrightarrow{BC}) + (\overrightarrow{AD} - \overrightarrow{DC}) = k\overrightarrow{FE}$

$$\Rightarrow \vec{b} - \vec{a} - \vec{c} + \vec{b} + \vec{d} - \vec{a} - \vec{c} + \vec{d} = k(\vec{e} - \vec{f})$$

$$\Rightarrow 2\vec{b} - 2\vec{a} - 2\vec{c} + 2\vec{d} = k(\vec{e} - \vec{f}) \quad ...(1)$$

But E and F are the mid points of AC and BD

so $\frac{\vec{a}+\vec{c}}{2}=\vec{e}$ and $\frac{\vec{b}+\vec{d}}{2}=\vec{f}$



From eqn. (1)

$2(\vec{b}+\vec{d})-2(\vec{a}+\vec{c})=k(\vec{e}-\vec{f})$

$4\vec{f}-4\vec{e}=k(\vec{e}-\vec{f})$

$\Rightarrow -4(\vec{e}-\vec{f})=k(\vec{e}-\vec{f})$

$\Rightarrow k = -4$

9. Option (2) is correct.

Given that $(a+bx+cx^2)^{10}=\sum_{i=0}^{20} p_i x^i$

$\Rightarrow (a + bx + cx^2)^{10} = p_0 + p_1 x + p_2 x^2 + \ldots + p_{20} x^{20}$

Comparing coefficients of x and x^2, we get

$p_1 = 20 = \frac{10!}{9!1!}\times a^9 b$ $\Rightarrow a^9 b = 2 = 1^9 \times 2$ (as $a, b, c \in$ N

$\Rightarrow a = 1, b = 2$

$p_2 = 210 = \frac{10!}{9!1!}\times a^9 c + \frac{10!}{8!2!}\times 2a^8 b^2$

$\Rightarrow 210 = 10\times 1\times C + \frac{10\times 9}{2}\times 1\times 4$

$\Rightarrow 210 = 10C + 180$

$\Rightarrow C = 3$

Hence, $2(a + b + c) = 2(1 + 2 + 3) = 12$

10. Option (2) is correct.

Given that $f(x) = \max\{1 + x + [x], 2 + x, x + 2[x]\}$

Case I: If $x \in [0, 1)$, then $f(x) = \max\{1 + x + 0, 2 + x, x + 2 \times 0\}$

$= \max\{1 + x, 2 + x, x\} = 2 + x$

Case II: If $x \in [1, 2)$, then $f(x) = \max\{1 + x + 1, 2 + x, x + 2 \times 1\}$

$= \max\{2 + x, 2 + x, 2 + x\} = 2 + x$

Case III: if $x = 2$, then $f(x) = \max\{3 + x, 2 + x, x + 4\}$

$= \max\{5, 4, 6\} = 6$

So, $f(x)=\begin{cases} 2+x, & 0\le x<2 \\ 6, & x=2 \end{cases}$

$\Rightarrow f(x)$ is discontinuous at $x = 2$ and differentiable in $(0, 2)$

$\Rightarrow m = 1$ and $n = 0$

Hence, $(m + n)^2 + 2 = (1 + 0)^2 + 2 = 3$

11. Option (4) is correct.

Given that there are 6 white and 4 black balls.

Hence, the required probability

$=\frac{1}{6}\times\frac{^6C_1}{^{10}C_1}+\frac{1}{6}\times\frac{^6C_2}{^{10}C_2}+\ldots+\frac{1}{6}\times\frac{^6C_6}{^{10}C_6}$

$=\frac{1}{6}\left(\frac{126+70+35+15+5+1}{210}\right)=\frac{1}{\cancel{6}}\times\frac{\overset{42}{\cancel{252}}}{210}=\frac{1}{5}$

12. Option (3) is correct.

Given that

$f(x) = \log_e(4x^2 + 11x + 6) + \sin^{-1}(4x + 3) + \cos^{-1}\left(\frac{10x+6}{3}\right)$

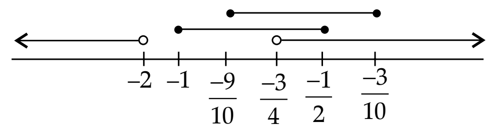
Now, to find domain-

$4x^2 + 11x + 6 > 0, -1 \le 4x + 3 \le 1,\ -1\le\frac{10x+6}{3}\le 1$

$\Rightarrow (4x + 3)(x + 2) > 0$ and $-4 \le 4x \le -2$ and $-3 \le 10x + 6 \le 3$

$\Rightarrow \left(x<-2 \text{ or } x>\frac{-3}{4}\right)$ and $-1\le x\le -\frac{1}{2}$

and $\frac{-9}{10}\le x\le -\frac{3}{10}$



$\Rightarrow x\in\left(-\frac{3}{4},\frac{-1}{2}\right]\equiv(\alpha,\beta]$

$\Rightarrow \alpha=\frac{-3}{9}$ & $\beta=\frac{-1}{2}$

Hence, $36|\alpha+\beta| = 36\left|\frac{-3}{4}-\frac{1}{2}\right| = 36\times\frac{5}{4} = 45$

13. Option (4) is correct.

Given that $|A| = m - n, 4m + n = 22$ ...(1)

and $17m + 4n = 93$ ...(2)

Solving (1) and (2) $\Rightarrow m = 5, n = 2$

$\Rightarrow |A| = m - n = 3$

Now, $(n \text{ adj}(\text{adj}(mA))) = |2 \text{ adj}(\text{adj}(5A))|$

$= 2^5 |\text{adj}(\text{adj}(5A))|$

$= 2^5 |5A|^{16}$

$= 2^5 \times 5^{16\times 5} |A|^{16}$

$= 3^{11} \times 5^{80} \times 6^5 = 3^a \times 5^b \times 6^c$

$\Rightarrow a = 11, b = 80, c = 5$

and $a + b + c = 96$

14. Option (4) is correct.

Given that $\bar{x}=20, \sigma=8, n=10$

$\Rightarrow \frac{\Sigma x_i}{10}=20, \frac{\Sigma x_i^2}{10}-(20)^2=8^2$

$\Rightarrow \Sigma x_i = 200, \Sigma x_i^2 = 4640$

Now, correct mean $=\frac{200-50+40}{10}=\frac{190}{10}=19$

and correct variance $=\frac{4640-(50)^2+(40)^2}{10}-(19)^2$

$=\frac{4640-900}{10}-361 = 374 - 361 = 13$

15. Option (4) is correct.

Given that O (α, β) is the orthocentre if triangle ABC

$\Rightarrow$ slope of OA $\times$ slope of BC $= -1$

$\Rightarrow \frac{\beta+7}{\alpha-3}\times\frac{5-2}{4+1}=-1$

$\Rightarrow 5\alpha + 3\beta + 6 = 0$ ...(1)

Slope of OB $\times$ slope of AC $= -1$

$$\Rightarrow \frac{\beta-2}{\alpha+1}\times\frac{5+7}{4-3}=-1$$

$\Rightarrow 12\beta - 24 = \alpha - 1$

$\Rightarrow \alpha + 12\beta - 23$ ...(2)

Solving (1) and (2), we get

$$\alpha=\frac{-47}{19}, \beta=\frac{121}{57}$$

Hence, $9\alpha - 6\beta + 60 = \frac{-9\times 47}{19}-\frac{6\times 121}{57}+60$

$$=\frac{-423}{19}-\frac{242}{19}+\frac{60\times 19}{19}$$

$$=\frac{-423-242+1140}{19}=\frac{475}{14}=25$$

16. Option (4) is correct.

Given that $x\,|x| - 5\,|x+2| + 6 = 0$

Case I: If $x < -2$

then $-x\,(x) + 5\,(x+2) + 6 = 0$

$\Rightarrow x^2 - 5x - 16 = 0$

$$\Rightarrow x=\frac{5\pm\sqrt{25+64}}{2}=\frac{5\pm\sqrt{89}}{2}$$

So, $x=\frac{5-\sqrt{89}}{2}$ as $x < -2$

Case II: If $-2 \le x < 0$

then $-x\,(x) - 5\,(x+2) + 6 = 0$

$\Rightarrow x^2 + 5x + 4 = 0$

$\Rightarrow (x+1)(x+4) = 0$

$\Rightarrow x = -1$ or $x = -4$ (Rejected) as $-2 \le x < 0$

Case III: $x \ge 0$

then $x(x) - 5\,(x+2) + 6 = 0$

$\Rightarrow x^2 - 5x - 4 = 0$

$$\Rightarrow x=\frac{5\pm\sqrt{41}}{2}$$

So, $x=\frac{5+\sqrt{41}}{2}$ as $x \ge 0$

Hence, there are total 3 solutions possible.

17. Option (1) is correct.

The augmented matrix of the first 3 equations can be written as

$$[A:B]=\begin{bmatrix}-1 & 2 & -9 & : & 7\\ -1 & 3 & 7 & : & 9\\ -2 & 1 & 5 & : & 8\end{bmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - 2R_1$

$$\sim\begin{bmatrix}-1 & 2 & -9 & : & 7\\ 0 & 1 & 16 & : & 2\\ 0 & -3 & 23 & : & -6\end{bmatrix}$$

Applying $R_3 \to R_3 + 3R_2$

$$[A:B]\sim\begin{bmatrix}-1 & 2 & -9 & : & 7\\ 0 & 1 & 16 & : & 2\\ 0 & 0 & 71 & : & 0\end{bmatrix}$$

which can be written as

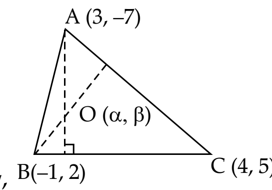
$-x + 2y - 9z = 7$

$y + 16z = 2$

$71z = 0$

$\Rightarrow z = 0, y = 2, x = -3$

$\Rightarrow (\alpha, \beta, \gamma) \equiv (-3, 2, 0)$



Putting the values of (x, y, z) in $-3x + y + 13z = \lambda$

We get, $9 + 2 + 0 = \lambda = 11$

Hence, the distance of $(-3, 2, 0)$ from the plane

$2x - 2y + z - 11 = 0$ is $\frac{|-6-4-11|}{\sqrt{4+4+1}}=\frac{21}{3}=7$

18. Option (2) is correct.

Let the two numbers be a and b

Now, a, A_1, A_2, b are in AP

$$\Rightarrow d=\frac{b-a}{3}, A_1=a+\frac{b-a}{3}=\frac{2a+b}{3}$$

and $A_2=a+2\left(\frac{b-a}{3}\right)=\frac{a+2b}{3}$

a, G_1, G_2, G_3, b are in G.P.

$$\Rightarrow r=\left(\frac{b}{a}\right)^{\frac{1}{4}},\ G_1 = ar, G_2 = ar^2, G_3 = ar^3$$

Now, $G_1^4 + G_2^4 + G_3^4 + G_1^2 G_3^2$

$= a^4r^4 + a^4r^8 + a^4r^{12} + a^2r^2 \times a^2r^6$

$$=a^4\times\frac{b}{a}+a^4\times\frac{b^2}{a^2}+a^4\times\frac{b^3}{a^3}+a^4\times\frac{b^2}{a^2}$$

$= a^3b + 2a^2b^2 + ab^3$

$= ab\,(a^2 + 2ab + b^2)$

$= ab\,(a+b)^2$

$$=a^2\frac{b}{a}\left(\frac{2a+b}{3}+\frac{a+2b}{3}\right)^2$$

$= G_1G_3\,(A_1 + A_2)^2$

19. Option (4) is correct.

negation of $p \wedge (q \wedge \sim (p \wedge q))$

$= \sim p \vee \sim (q \wedge \sim (p \wedge q))$

$= \sim p \vee (\sim q \vee (p \wedge q))$

$= \sim p \vee ((\sim q \vee p) \wedge (\sim q \vee q))$

$= \sim p \vee ((\sim q \vee p) \wedge t)$

$= \sim p \vee (\sim q \vee p)$

$= (\sim p \vee \sim q) \vee p$

$= \sim (p \wedge q) \vee p$

20. Option (4) is correct.

Given digits are 1, 3, 5, 8

Now, if 3-digit number is divisible 3, then possible triplets are (1, 1, 1), (3, 3, 3), (5, 5, 5), (8, 8, 8), (5, 5, 8), (8, 8, 5), (1, 3, 5), (1, 3, 8)

Hence, total numbers are

$$= 1 + 1 + 1 + 1 + \frac{3!}{2!}+\frac{3!}{2!}+3!+3!$$

$= 4 + 3 + 3 + 6 + 6 = 22$

Section B

21. Correct answer is [6].

Given that A = {1, 2, 3, 4}

and R $\{((a, b), (c, d)) : 2a + 3b = 4c + 5d\}$

$\Rightarrow 2a = \{2, 4, 6, 8\}, 3b = \{3, 6, 9, 12\}, 4c = \{4, 8, 12, 16\}$

and $5d = \{5, 10, 15, 20\}$

$$\Rightarrow 2a+3b=\begin{cases}5,8,11,14\\7,10,13,16\\9,12,15,18\\11,14,17,20\end{cases} \text{ and } 3b+4c=\begin{cases}9,14,19,24\\13,18,23,28\\17,22,27,32\\21,26,31,36\end{cases}$$

Hence, possible values for which $2a + 3b = 4c + 5d$ are 9, 13, 14, 14, 17, 18.

So, total number of pairs are 6.

22. Correct answer is [15].

Given that

$\{n \in \mathrm{N} : 10 \le n \le 100 \text{ and } 3^n - 3 \text{ is a multiple of } 7\}$

$\Rightarrow 3^n - 3 = 7k, k \in \mathrm{I}, n \in \mathrm{N}, n \in [10, 100]$

$\Rightarrow n = 1, 7, 13, 20, ..., 97$ which is an A.P.

Now, $97 = 1 + (p - 1)\, 6$

$\Rightarrow \frac{96}{6} = p - 1 \Rightarrow p = 16 + 1 \Rightarrow p = 17$

but $\{1, 7\} \notin [10, 100]$

Hence, number of possible values of n are 15.

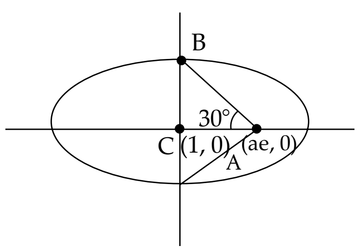
23. Correct answer is [9].

Given that the centre of ellipse is at (1, 0)

So, eqn. of ellipse is $\frac{(x-1)^2}{a^2} + \frac{y^2}{b^2} = 1$

$\text{L.R.} = \frac{1}{2} \Rightarrow \frac{2b^2}{a} = \frac{1}{2}$

$\Rightarrow a = 4b^2$



In Δ ABC

$\tan 30° = \frac{BC}{AC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{b}{ae}$

$\Rightarrow 3b^2 = a^2e^2 = a^2 - b^2$

$\Rightarrow a^2 = 4b^2 = a$

$\Rightarrow a = 1$ and $b^2 = \frac{1}{4} \Rightarrow (2b)^2 = 1$

Hence, sum of the square of the length of major & minor axes

$= ((2a) + (2b))^2 = (2 + 1)^2 = 9$

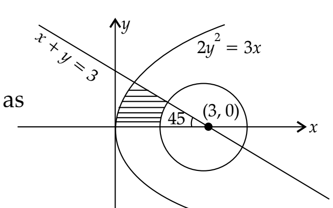
24. Correct answer is [42].

Given curves are $2y^2 = 3x$

and $x + y = 3, y = 0$

$(x - 3)^2 + y^2 = 2,$

which can be plotted as



Solving $2y^2 = 3x$

and $x + y = 3$

We get $y = \frac{3}{2}$

Hence, the required area

$$A = \int_0^{3/2} (x_{\text{line}} - x_{\text{parabola}})dy - \frac{\text{Area of circle}}{8}$$

$$\int_0^{\frac{3}{2}} \left((3-y) - \frac{2y^2}{3}\right)dy - \frac{\pi\times(\sqrt{2})^2}{8}$$

$$A = \left[3y - \frac{y^2}{2} - \frac{2\times y^3}{9}\right]_0^{\frac{3}{2}} - \frac{\pi}{4}$$

$$\Rightarrow 4A + \pi = 4\left[\frac{9}{2} - \frac{9}{8} - \frac{27}{36}\right]$$

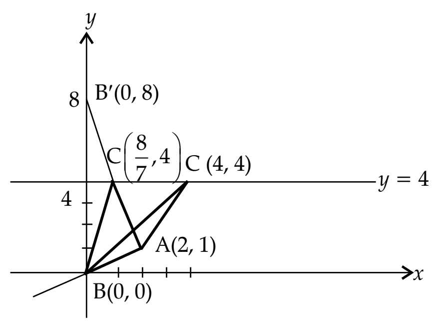
$$\Rightarrow 4A + \pi = 4\left[\frac{36-9-6}{8}\right] = \frac{21}{2}$$

$\Rightarrow 4\,(4A + \pi) = 42$

25. Correct answer is [48].

Image of B w.r.t. $y = 4$ is (0, 8) B'

and slope of line AB′ is $\frac{8-1}{0-2} = \frac{-7}{2}$



Equation of line AB′

$$y - 8 = \frac{-7}{2}(x - 0)$$

Putting $y = 4 \Rightarrow -4 = \frac{-7}{2}(x)$

$$\Rightarrow x = \frac{8}{7}$$

$\therefore C\left(\frac{8}{7}, 4\right)$ when ΔABC is of minimum perimeter

$$\Rightarrow \beta = \frac{8}{7}$$

Also, for maximum perimeter, t can be maximum 4

$\Rightarrow \alpha = 4$

Hence, $6\alpha + 21\beta = 24 + 24 = 48$

26. Correct answer is [26].

Let $P_1 + \lambda P_2 = 0$

$\Rightarrow (2x + y - z - 3) + \lambda\,(5x - 3y + 4z + 9) = 0$

$\Rightarrow (5\lambda + 2)\,x + (1 - 3\lambda)\,y + (4\lambda - 1)\,z + 9\lambda - 3 = 0$

This plane is || to line whose d.r's are (2, +4, 5)

so $2\,(5\lambda + 2) + 4\,(1 - 3\lambda) + 5\,(4\lambda - 1) = 0$

$\Rightarrow 10\lambda + 4 + 4 - 12\lambda + 20\lambda - 5 = 0$

$\Rightarrow 18\lambda = -3 \Rightarrow \lambda = \frac{-1}{6}$

$\therefore$ Plane $7x + 9y - 10z - 27 = 0$

Eqn. of line through A (8, –1, –19) whose d.r's are (–3, 4, –12) is

$$\frac{x-8}{-3}=\frac{y+1}{4}=\frac{z+19}{12}=\lambda(\text{say})$$

$\Rightarrow (x, y, z) \equiv (-3\lambda + 8, 4\lambda - 1, 12\lambda - 19)$ lies on plane

$\Rightarrow 7(8-3\lambda) + 9(4\lambda - 1) - 10(12\lambda - 19) = 27$

$\Rightarrow \lambda = 2$

$\therefore$ B (2, 7, 5)

and AB = $\sqrt{(6)^2+(8)^2+(24)^2}=26$

27. Correct answer is [7].

Let $a=\frac{1}{2}$ and $b=\frac{1}{3}$ and S be the sum of the series, then

$S = (a-b) + (a^2 - ab + b^2) + (a^2 - a^2b + ab^2 - b^3) + \ldots$

$S(a+b) = (a^2 - b^2) + (a^3 + b^3) + (a^4 - b^4) + \ldots.$

$S(a+b) = \{a^2 + a^3 + a^4 + \ldots\} - \{b^2 - b^3 + b^4 \ldots\}$

$S(a+b) = \frac{a^2}{1-a}-\frac{b^2}{1+b}$

$$\Rightarrow S\left(\frac{1}{2}+\frac{1}{3}\right)=\frac{\frac{1}{4}}{1-\frac{1}{2}}-\frac{\frac{1}{9}}{1+\frac{1}{3}} \quad \Rightarrow S=\frac{1}{2}=\frac{\alpha}{\beta}$$

Hence, $\alpha = 1, \beta = 2$

and $\alpha + 3\beta = 1 + 6 = 7$

28. Correct answer is [72].

Given that the maximum digit is 7 and the sum of first two digits = sum of last two digits = α (say)

Case I: If $\alpha = 7$, then

Total ways are $(1 \times 2) \times (6 \times 2) = 24$

Case II: If $\alpha = 8$, then

Total ways are $(1 \times 2) \times (4 \times 2) = 16$

Case III: If $\alpha = 9$, then

Total ways are $(1 \times 2) \times (4 \times 2) = 16$

Case IV: If $\alpha = 10$, then

Total ways are $(1 \times 2) \times (2 \times 2) = 8$

Case V: If $\alpha = 11$, then

Total ways are $(1 \times 2) \times (2 \times 2) = 8$

Hence, total number of ways are $24 + 2 \times 16 + 2 \times 8 = 72$

29. Correct answer is [5].

Since, we know that, in ΔABC

sin2A + sin2B + sin2C = 4 sinA sinB sin C

and sinA + sin B + sin C = $4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

Also putting $x = y = z$ in the given line

$x \sin A + y \sin B + z \sin C - 18 = 0 = x \sin 2A + y \sin 2B + z \sin 2C - 9$

$\Rightarrow \sin A + \sin B + \sin C = \frac{18}{x}.\frac{1}{2}$

$\sin 2A + \sin 2B + \sin 2C = \frac{9}{x}=\frac{18}{x}.\frac{1}{2}$

so, sinA + sinB + sinC = 2 (sin2A + sin2B + sin2C)

$\Rightarrow 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2} = 2 \times 4\sin A \sin B \sin C$

$= 8 \times 2\sin\frac{A}{2}\cos\frac{B}{2}\times 2\sin\frac{B}{2}\cos\frac{B}{2}\times 2\sin\frac{C}{2}\cos\frac{C}{2}$

$\Rightarrow \sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}=\frac{1}{16}$

$\Rightarrow 80\left(\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right)=5$

30. Correct answer is [28].

Given that $f(x)=\int\frac{dx}{(3+4x^2)\sqrt{4-3x^2}}$

putting $x=\frac{1}{t}, dx=\frac{-1}{t^2}dt$

$$\Rightarrow f\left(\frac{1}{t}\right)=\int\frac{-\frac{1}{t^2}dt}{\left(3+\frac{4}{t^2}\right)\sqrt{4-\frac{3}{t^2}}}=-\int\frac{t\,dt}{(3t^2+4)\sqrt{4t^2-3}}$$

putting $4t^2 - 3 = z^2$, $8t\,dt = 2z\,dz$

$$\Rightarrow f\left(\sqrt{\frac{z^2+3}{4}}\right)=-\frac{1}{4}\int\frac{z\,dz}{\left(3\left(\frac{z^2+3}{4}\right)+4\right)\times z}$$

$$=-\frac{1}{3}\int\frac{dz}{z^2+\left(\frac{5}{\sqrt{3}}\right)^2}$$

$$=\frac{-1}{3}\times\frac{\sqrt{3}}{5}\tan^{-1}\left(\frac{\sqrt{3}z}{5}\right)+C$$

$$\Rightarrow f\left(\frac{1}{t}\right)=-\frac{1}{5\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}}{5}\sqrt{4t^2-3}\right)+C$$

$$\Rightarrow f(x)=-\frac{1}{5\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}}{5}\frac{\sqrt{4-3x^2}}{x^2}\right)+C$$

But $f(0) = 0 \Rightarrow 0 = -\frac{1}{5\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}}{5}\times\infty\right)=\frac{-\pi}{10\sqrt{3}}+C$

$\Rightarrow C=\frac{\pi}{10\sqrt{3}}$

$$\therefore f(x)=\frac{\pi}{10\sqrt{3}}-\frac{1}{5\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}}{5x}\sqrt{4-3x^2}\right)$$

$$\Rightarrow f(1)=\frac{\pi}{10\sqrt{3}}-\frac{1}{5\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}}{5}\right)$$

$$=\frac{1}{5\sqrt{3}}\left(\frac{\pi}{2}-\tan^{-1}\left(\frac{\sqrt{3}}{5}\right)\right)=\frac{1}{5\sqrt{3}}\cot^{-1}\left(\frac{\sqrt{3}}{5}\right)$$

$$\frac{1}{\alpha\beta}\tan^{-1}\left(\frac{\alpha}{\beta}\right)=\frac{1}{5\sqrt{3}}\tan^{-1}\left(\frac{5}{\sqrt{3}}\right)$$

$\Rightarrow \alpha = 5, \beta = \sqrt{3}$

and $\alpha^2 + \beta^2 = 25 + 3 = 28$

❑❑