JEE (Main) MATHEMATICS SOLVED PAPER

2023
24th Jan. Shift 1

Section A

Q. 1. Let $\vec{u} = \hat{i} - \hat{j} - 2\hat{k}, \vec{v} = 2\hat{i} + \hat{j} - \hat{k}, \vec{v}.\vec{w} = 2$ and $\vec{v} \times \vec{w} = \vec{u} + \lambda\vec{v}$. Then $\vec{u}.\vec{w}$ is equal to

(1) 2 **(2)** $\frac{3}{2}$ **(3)** 1 **(4)** $-\frac{2}{3}$

Q. 2. $\lim_{t\to 0}\left(1^{\frac{1}{\sin^2 t}} + 2^{\frac{1}{\sin^2 t}} + \ldots + n^{\frac{1}{\sin^2 t}}\right)^{\sin^2 t}$ is equal to

(1) n^2 **(2)** $\frac{n(n+1)}{2}$

(3) n **(4)** $n^2 + n$

Q. 3. Let α be a root of the equation $(a - c)x^2 + (b - a)x (c - b) = 0$ where a, b, c are distinct real numbers such that the matrix $\begin{bmatrix} \alpha^2 & \alpha & 1 \\ 1 & 1 & 1 \\ a & b & c \end{bmatrix}$ is singular.

Then, the value of $\frac{(a-c)^2}{(b-a)(c-b)} + \frac{(b-a)^2}{(a-c)(c-b)} + \frac{(c-b)^2}{(a-c)(b-a)}$ is

(1) 12 **(2)** 9 **(3)** 3 **(4)** 6

Q. 4. The area enclosed by the curves $y^2 + 4x = 4$ and $y - 2x = 2$ is:

(1) 9 **(2)** $\frac{22}{3}$ **(3)** $\frac{23}{3}$ **(4)** $\frac{25}{3}$

Q. 5. Let $p, q \in \mathbb{R}$ and $(1 - \sqrt{3}i)^{200} = 2^{199}(p + iq)$, $i = \sqrt{-1}$. Then $p + q + q^2$ and $p - q + q^2$ are roots of the equation.

(1) $x^2 - 4x - 1 = 0$ **(2)** $x^2 - 4x + 1 = 0$
(3) $x^2 + 4x - 1 = 0$ **(4)** $x^2 + 4x + 1 = 0$

Q. 6. Let N denote the number that turns up when a fair die is rolled. If the probability that the system of equations

$x + y + z = 1$ $2x + Ny + 2z = 2$
$3x + 3y + Nz = 3$

has unique solution is $\frac{k}{6}$, then the sum of value of k and all possible values of N is

(1) 21 **(2)** 18 **(3)** 20 **(4)** 19

Q. 7. For three positive integers p, q, r, $x^{pq^2} = y^{qr} = z^{p^2 r}$ and $r = pq + 1$ such that 3, $3\log_y x$, $3\log_z y$, $7\log_x z$ are in A.P. with common difference $\frac{1}{2}$. Then $r - p - q$ is equal to

(1) –6 **(2)** 12 **(3)** 6 **(4)** 2

Q. 8. The relation R = $\{(a, b): gcd(a, b) = 1, 2a \neq b, a, b \in Z\}$ is:

(1) reflexive but not symmetric
(2) transitive but not reflexive
(3) symmetric but not transitive
(4) neither symmetric nor transitive

Q. 9. Let PQR be a triangle. The points A, B and C are on the sides QR, RP and PQ respectively such that $\frac{QA}{AR} = \frac{RB}{BP} = \frac{PC}{CQ} = \frac{1}{2}$. Then $\frac{\text{Area}(\Delta PQR)}{\text{Area}(\Delta ABC)}$ is equal to

(1) 4 **(2)** 3 **(3)** 1 **(4)** 2

Q. 10. Let $y = y(x)$ be the solution of the differential equation $x^3 dy + (xy - 1)\, dx = 0$, $x > 0$, $y\left(\frac{1}{2}\right) = 3 - e$. Then $y(1)$ is equal to

(1) 1 **(2)** e **(3)** 3 **(4)** $2 - e$

Q. 11. If A and B are two non-zero $n \times n$ matrices such that $A^2 + B = A^2 B$, then

(1) $A^2 = I$ or $B = I$ **(2)** $A^2B = I$
(3) $AB = I$ **(4)** $A^2B = BA^2$

Q. 12. The equation $x^2 - 4x + [x] + 3 = x[x]$, where $[x]$ denotes the greatest integer function, has :

(1) a unique solution in $(-\infty, 1)$
(2) no solution
(3) exactly two solutions in $(-\infty, \infty)$
(4) a unique solution in $(-\infty, \infty)$

Q. 13. Let a tangent to the curve $y^2 = 24x$ meet the curve $xy = 2$ at the points A and B. Then the mid points of such line segment AB lie on a parabola with the

(1) length of latus rectum $\frac{3}{2}$
(2) directrix $4x = -3$
(3) length of latus rectum 2
(4) directrix $4x = 3$

Q. 14. Let Ω be the sample space and $A \subseteq \Omega$ be an event. Given below are two statements :

(S_1) : If P(A) = 0, then A = ϕ
(S_2): If P(A) = 1, then A = Ω, Then

(1) both (S_1) and (S_2) are true
(2) only (S_1) is true
(3) only (S_2) is true
(4) both (S_1) and (S_2) are false

Q. 15. The value of $\sum_{r=0}^{22} {}^{22}C_r \, {}^{23}C_r$ is

(1) ${}^{44}C_{23}$ **(2)** ${}^{45}C_{23}$ **(3)** ${}^{44}C_{22}$ **(4)** ${}^{45}C_{24}$

Q. 16. The distance of the point (–1, 9, –16) from the plane $2x + 3y - z = 5$ measured parallel to the line $\frac{x+4}{3} = \frac{2-y}{4} = \frac{z-3}{12}$ is

(1) 31 **(2)** $13\sqrt{2}$ **(3)** $20\sqrt{2}$ **(4)** 26

Q. 17. $\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right) + \sec^{-1}\left(\sqrt{\frac{8+4\sqrt{3}}{6+3\sqrt{3}}}\right)$ is equal to:

(1) $\frac{\pi}{3}$ **(2)** $\frac{\pi}{4}$ **(3)** $\frac{\pi}{6}$ **(4)** $\frac{\pi}{2}$

Q. 18. Let $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & , x \neq 0 \\ 0 & , x = 0 \end{cases}$

Then at $x = 0$

(1) f is continuous but not differentiable
(2) f and f' both are continuous
(3) f' is continuous but not differentiable
(4) f is continuous but f' is not continuous

Q. 19. The compound statement $(\sim(P \wedge Q)) \vee ((\sim P) \wedge Q) \Rightarrow ((\sim P) \wedge (\sim Q))$ is equivalent to

(1) $(\sim Q) \vee P$
(2) $((\sim P) \vee Q) \wedge (\sim Q)$
(3) $(\sim P) \vee Q$
(4) $((\sim P) \vee Q) \wedge ((\sim Q) \vee P)$

Q. 20. The distance of the point (7, −3, −4) from the plane passing through the points (2, −3, 1), (−1, 1, −2) and (3, −4, 2) is :

(1) 5 **(2)** 4 **(3)** $5\sqrt{2}$ **(4)** $4\sqrt{2}$

Section B

Q. 21. Let $\lambda \in \mathbb{R}$ and let the equation E be $|x|^2 - 2|x| + |\lambda - 3| = 0$. Then the largest element in the set S $= \{x + \lambda : x$ is an integer solution of E$\}$ is

Q. 22. Let a tangent to the curve $9x^2 + 16y^2 = 144$ intersect the coordinate axes at the points A and B. Then, the minimum length of the line segment AB is

Q. 23. The shortest distance between the lines $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-6}{2}$ and $\frac{x-6}{3} = \frac{1-y}{2} = \frac{z+8}{0}$ is equal to

Q. 24. Suppose $\sum_{r=0}^{2023} r^2 \, {}^{2023}C_r = 2023 \times \alpha \times 2^{2022}$. Then the value of α is

Q. 25. The value of $\frac{8}{\pi}\int_0^{\frac{\pi}{2}} \frac{(\cos x)^{2023}}{(\sin x)^{2023} + (\cos x)^{2023}} dx$ is

Q. 26. The number of 9 digit numbers, that can be formed using all the digits of the number 123412341 so that the even digits occupy only even places, is

Q. 27. A boy needs to select five courses from 12 available courses, out of which 5 courses are language courses. If he can choose at most two language courses, then the number of ways he can choose five courses is

Q. 28. The 4th term of a GP is 500 and its common ratio is $\frac{1}{m}, m \in \mathbb{N}$. Let S_n denote the sum of the first n terms of this GP. If $S_6 > S_5 + 1$ and $S_7 < S_6 + \frac{1}{2}$, then the number of possible values of m is

Q. 29. Let C be the largest circle centred at (2,0) and inscribed in the ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$. If $(1, \alpha)$ lies on C, then $10\,\alpha^2$ is equal to

Q. 30. The value of $12\int_0^3 |x^2 - 3x + 2|\, dx$ is

Answer Key

Q. No.	Answer	Topic Name	Chapter Name
1	(3)	Triple Products	Vector Algebra
2	(3)	Methods of Evaluation of Limits	Limits
3	(3)	Quadratic Equation and its Solution	Quadratic Equations
4	(1)	Area Bounded by Curves	Area under Curves
5	(2)	Euler's law	Complex Numbers
6	(3)	System of linear equations	Matrices and Determinants
7	(4)	Arithmetic Progressions	Sequences and Series
8	(4)	Equivalence Relations	Set Theory and Relations
9	(2)	Scalar and Vector Products	Vector Algebra
10	(1)	Solution of Linear Differential Equations	Differential Equations
11	(4)	Operations on Matrices	Matrices and Determinants
12	(4)	Quadratic Equations and its solution	Quadratic Equations

Q. No.	Answer	Topic Name	Chapter Name
13	(4)	Interaction Between Two Conics	Hyperbola
14	(1)	Basics of Probability	Probability
15	(2)	Properties of Binomial Coefficients	Binomial Theorem
16	(4)	Intersection of a Line and a Plane	Three Dimensional Geometry
17	(1)	Basics of Inverse Trigonometric Functions	Inverse Trigonometric Functions
18	(4)	Differentiability of a Function	Continuity and Differentiability
19	(3)	Logical Operations	Mathematical Reasoning
20	(3)	Plane and a Point	Three Dimensional Geometry
21	[5]	Algebra of Functions	Function
22	[7]	Properties of Ellipse	Ellipse
23	[14]	Skew Lines	Three Dimensional Geometry
24	[1012]	Properties of Binomial Coefficients	Binomial Theorem
25	[2]	Properties of Definite Integrals	Definite Integration
26	[60]	Permutations	Permutations and Combinations
27	[546]	Combinations	Permutations and Combinations
28	[12]	Geometric Progressions	Sequences and Series
29	[118]	Interaction between Two Conics	Ellipse
30	[22]	Properties of Definite Integrals	Definite Integration

Solutions

Section A

1. Option (3) is correct.

Given, $\vec{v} \times \vec{w} = \vec{u} + \lambda\vec{v}$

Taking dot product with $\vec{v}$

$\Rightarrow (\vec{v} \times \vec{w}).\vec{v} = (\vec{u} + \lambda\vec{v}).\vec{v}$

$\Rightarrow 0 = \vec{u}.\vec{v} + \lambda\vec{v}.\vec{v}$

$\Rightarrow 0 = (\hat{i} - \hat{j} - 2\hat{k}).(2\hat{i} + \hat{j} - \hat{k}) + \lambda\,|\vec{v}|^2$

$\Rightarrow 0 = 2 - 1 + 2 + \lambda\,(4 + 1 + 1)$

$\Rightarrow \lambda = \dfrac{-3}{6} = \dfrac{-1}{2}$

So, $\vec{v} \times \vec{w} = \vec{u} + \lambda\vec{v} = \vec{u} - \dfrac{1}{2}\vec{v}$

Taking dot product with $\vec{w}$

$\Rightarrow (\vec{v} \times \vec{w}).\vec{w} = \vec{u}.\vec{w} - \dfrac{1}{2}\vec{v}.\vec{w}$

$\Rightarrow 0 = \vec{u}.\vec{w} - \dfrac{1}{2}(2)$

$\Rightarrow \vec{u}.\vec{w} = 1$

2. Option (3) is correct.

$$\text{Let } l = \lim_{t\to 0}\left(1^{\frac{1}{\sin^2 t}} + 2^{\frac{1}{\sin^2 t}} + \ldots + n^{\frac{1}{\sin^2 t}}\right)^{\sin^2 t}$$

$$= \lim_{t\to 0} n\left[\left(\frac{1}{n}\right)^{\text{cosec}^2 t} + \left(\frac{2}{n}\right)^{\text{cosec}^2 t} + \ldots + \left(\frac{n}{n}\right)^{\text{cosec}^2 t}\right]^{\sin^2 t}$$

$$= n\lim_{t\to 0}\left[\sum_{r=1}^{n-1}\left(\frac{r}{n}\right)^{\text{cosec}^2 t} + 1\right]^{\sin^2 t} = n\lim_{t\to 0}\left[\sum_{r=1}^{n-1} 0 + 1\right]^{\sin^2 t}$$

$$\left[\text{as}\left(\frac{r}{n}\right)^{\text{cosec}^2 t} \to 0 \because 0 < \frac{r}{n} < 1\ \&\ \text{cosec}^2 t \to \infty\right]$$

$= n$

HINT:

(1) As $n \to \infty, \dfrac{r}{n} \to 0,$ for $0 < r < n$

(2) As $x \to 0$, $\text{cosec}^2 x \to \infty$

3. Option (3)is correct.

$$\text{Let P} = \begin{bmatrix} \alpha^2 & \alpha & 1 \\ 1 & 1 & 1 \\ a & b & c \end{bmatrix}$$

Given that P is singular

$\Rightarrow |P| = 0$

$\Rightarrow \alpha^2 (c - b) - \alpha (c - a) + 1(b - a) = 0$

$\Rightarrow \alpha^2 (c - b) + \alpha (a - c) + (b - a) = 0$

Put $\alpha = 1$, $c - b + a - c + b - a = 0$

So, $\alpha = 1$ is a root

$$\text{Now, } \sum \frac{(a-c)^2}{(b-a)(c-b)} = \text{M (say)}$$

$$= \sum \frac{(a-c)^2}{(b-a)(c-b)}.\left(\frac{a-c}{a-c}\right) = \sum\frac{(a-c)^3}{(b-a)(c-b)(a-c)}$$

$$= \frac{(a-c)^3+(c-b)^3+(b-a)^3}{(b-a)(c-b)(a-c)}$$

Here, $(a-c)+(c-b)+(b-a)=0$

$\Rightarrow (a-c)^3+(c-b)^3+(b-a)^3 = 3(a-c)(c-b)(b-a)$

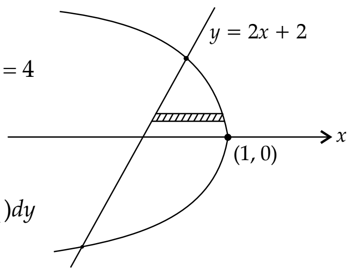
$$\therefore \text{M} = \frac{3(a-c)(c-b)(b-a)}{(b-a)(c-b)(a-c)}$$

$\Rightarrow \text{M} = 3$

4. Option (1) is correct.

Let $C_1 : y^2 + 4x = 4$ & $C_2 : y - 2x = 2$

Solving C_1 & C_2



$$\Rightarrow y^2 + 4\left(\frac{y}{2}-1\right) = 4$$

$\Rightarrow y = -4, 2$

$$\text{Area} = \int (x_2 - x_1)dy$$

$$\Rightarrow \text{Area} = \int_{-4}^{2}\left\{\left(\frac{4-y^2}{4}\right)-\left(\frac{y-2}{2}\right)\right\}dy$$

$$\Rightarrow \text{Area} = \int_{-4}^{2}\left(2-\frac{y}{2}-\frac{y^2}{4}\right)dy$$

$$\Rightarrow \text{Area} = \left[2y-\frac{y^2}{4}-\frac{y^3}{12}\right]_{-4}^{2}$$

$$\Rightarrow \text{Area} = 2(2-(-4))-\frac{1}{4}(4-16)-\frac{1}{12}(8-(-64))$$

$\Rightarrow \text{Area} = 12 + 3 - 6$

$\Rightarrow \text{Area} = 9$ sq. units

HINT:

(1) Solve both the curves to get the point of intersection.

(2) Plot both the curves & think of a strip either horizontal or vertical and then integrate.

5. Option (2) is correct.

Given $2^{199}(p+iq) = (1-\sqrt{3}i)^{200}$

$$\Rightarrow \frac{2^{199}}{2^{200}}(p+iq) = \left(\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)^{200}$$

$$\Rightarrow p + iq = 2\left(\text{C is}\left(-\frac{\pi}{3}\right)\right)^{200}$$

$$\Rightarrow p + iq = 2\left(e^{-i\frac{\pi}{3}}\right)^{200}$$

$$\Rightarrow p + iq = 2e^{-i\frac{200\pi}{3}}$$

$$\Rightarrow p + iq = 2\left(\cos\left(\frac{200\pi}{3}\right)-i\sin\left(\frac{200\pi}{3}\right)\right)$$

$$\Rightarrow p + iq = 2\left[\cos\left(66\pi+\frac{2\pi}{3}\right)-i\sin\left(66\pi+\frac{2\pi}{3}\right)\right]$$

$$\Rightarrow p + iq = 2\left[\cos\left(\frac{2\pi}{3}\right)-i\sin\left(\frac{2\pi}{3}\right)\right]$$

$$\Rightarrow p + iq = 2\left(-\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)$$

$\Rightarrow p = -1, q = -\sqrt{3}$

Now, $p + q + q^2 = -1-\sqrt{3}+(-\sqrt{3})^2 = 2-\sqrt{3}$

$p - q + q^2 = -1-(-\sqrt{3})+(-\sqrt{3})^2 = 2+\sqrt{3}$

Equation whose roots are $p + q + q^2$ & $p - q + q^2$ having

Sum of roots (S) $= (2-\sqrt{3})+(2+\sqrt{3}) = 4$

and product of roots (P) $= (2-\sqrt{3})(2+\sqrt{3}) = 4-3 = 1$

Required quadratic equation is $x^2 - (\text{S})\,x + \text{P} = 0$

$\Rightarrow x^2 - 4x + 1 = 0$

HINT:

(1) Use $\cos\theta + i\sin\theta = e^{i\theta}$

(2) Quadratic equation whose roots are α and β is
$x^2 - (\alpha+\beta)\,x + \alpha\beta = 0$

6. Option (3) is correct.

System of equations is,

$x + y + z = 1$
$2x + \text{N}y + 2z = 2$
$3x + 3y + \text{N}z = 3$

For unique solution, $\Delta \neq 0$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 2 & \text{N} & 2 \\ 3 & 3 & \text{N} \end{vmatrix} \neq 0$$

$\Rightarrow 1(\text{N}^2-6)-(2\text{N}-6)+(6-3\text{N}) \neq 0$

$\Rightarrow \text{N}^2 - 5\text{N} + 6 \neq 0$

$\Rightarrow (\text{N}-2)(\text{N}-3) \neq 0$

Therefore $\text{N} \neq 2, \text{N} \neq 3$

So, favourable cases are {1, 4, 5, 6}

Total cases ≡ {1, 2, 3, 4, 5, 6}

$$\text{Hence, probability} = \frac{\text{Number of favourable cases}}{\text{Total cases}}$$

$$\Rightarrow \frac{k}{6} = \frac{4}{6} \Rightarrow k = 4$$

So, sum of all possible values of k & N
$= 4 + (1 + 4 + 5 + 6) = 20$

7. Option (4) is correct.

Given than, $x^{pq^2} = y^{qr} = z^{p^2r}$ ...(1)

Also, 3, $3\log_y x$, $3\log_z y$, $7\log_x z$ are in A.P.

$$\Rightarrow 3+\frac{1}{2} = 3\log_y x$$

$$\Rightarrow \log_y x = \frac{7}{6}$$

$\Rightarrow x = (y)^{7/6} \Rightarrow x^6 = y^7$

$$3\log_z y = 3+2\left(\frac{1}{2}\right)=4 \quad \text{...(2)}$$

$$\Rightarrow \log_z y = \frac{4}{3} \Rightarrow y = (z)^{4/3}$$

$$\Rightarrow y^3 = z^4 \quad \text{...(3)}$$

$$7\log_x(z) = 3+3\left(\frac{1}{2}\right)=\frac{9}{2}$$

$$\Rightarrow \log_x(z) = \frac{9}{14} \Rightarrow z = (x)^{9/14}$$

$$\Rightarrow z^{14} = x^9 \quad \text{...(4)}$$

Now from (1), we have

$$x^{pq^2} = \left(x^{\frac{6}{7}}\right)^{qr} = \left(x^{\frac{9}{14}}\right)^{p^2 r}$$

$$\Rightarrow pq^2 = \frac{6}{7}qr = \frac{9}{14}p^2 r$$

$$\Rightarrow pq = \frac{6}{7}r, q^2 = \frac{9}{14}pr$$

Also, $r = pq + 1$

$$\Rightarrow r = \frac{6}{7}r+1 \Rightarrow \frac{r}{7}=1 \Rightarrow r = 7$$

Now, $q^2 = \frac{9}{14}pr$

$$\Rightarrow q(q^2) = \left(\frac{9}{14}r\right)(pq)$$

$$\Rightarrow q^3 = \left(\frac{9}{14}\right)r\left(\frac{6}{7}\right)r$$

$$\Rightarrow q^3 = \frac{9\times 6}{14\times 7}\times(7)^2$$

$$\Rightarrow q^3 = 27 \Rightarrow q = 3$$

And, $pq = \frac{6}{7}r$

$$\Rightarrow p = \frac{6}{7}\times\frac{7}{3} \Rightarrow p = 2$$

So, $r-p-q = 7-2-3 = 2$

8. **Option (4) is correct.**

R = $\{(a, b) : gcd(a, b) = 1, 2a \neq b, a, b \in Z\}$

Reflexive:

Check for (a, a)

$gcd(a, a) = a$

So, R is not reflexive

Symmetric:

$(a, b) \in R \Rightarrow gcd\ (a, b) = 1$

Now check for (b, a)

$gcd\ (b, a) = gcd\ (a, b) = 1$

But gcd (1, 2) = gcd (2, 1) = 1

But $b \neq 2a$, So R is not symmetric.

Transitive:

Consider, $gcd(2, 3) = 1$

$\Rightarrow (2, 3) \in R$

Now, gcd (3, 4) = 1

$\Rightarrow (3, 4) \in R$

Again, gcd (2, 4) = $2 \neq 1$

$\Rightarrow (2, 4) \notin R$

$\Rightarrow$ R is not Transitive.

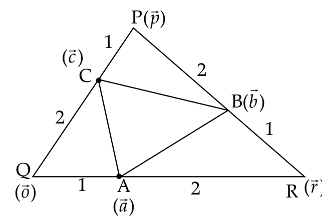
HINT:

(1) R is reflexive if $(a, a) \in R\ \forall\ a \in A$

(2) If $a, b \in A, (a, b) \in R \Rightarrow (b, a) \in R$, then R is symmetric.

(3) If $a, b, c \in A, (a, b) \in R, (b, c) \in R \Rightarrow (a, c) \in R$, then R is transitive.

9. **Option (2) is correct.**



Let position vector of Q, P, and R be $\vec{o}, \vec{p}$ and $\vec{r}$ respectively.

Again, let position vector of points A, B, C be $\vec{a}, \vec{b}$ & $\vec{c}$ respectively.

Using section formula

$$\vec{a} = \frac{\vec{r}}{3}, \vec{b} = \frac{\vec{p}+2\vec{r}}{3}, \vec{c} = \frac{2\vec{p}}{3}$$

$$\text{Area of } \Delta PQR = \left|\frac{1}{2}|\overrightarrow{QP}\times\overrightarrow{QR}|\right| = \frac{1}{2}|\vec{r}\times\vec{p}|$$

$$\text{Area of } \Delta ABC = \frac{1}{2}|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|$$

$$= \frac{1}{2}\left|\left(\frac{\vec{r}}{3}\times\left(\frac{\vec{p}+2\vec{r}}{3}\right)\right)+\left(\frac{\vec{p}+2\vec{r}}{3}\right)\times\left(\frac{2\vec{p}}{3}\right)+\left(\frac{2\vec{p}}{3}\times\frac{\vec{r}}{3}\right)\right|$$

$$= \frac{1}{2}\left|\frac{\vec{r}\times\vec{p}}{9}+4\left(\frac{\vec{r}\times\vec{p}}{9}\right)+\frac{2}{9}(\vec{p}\times\vec{r})\right|, \text{ as } \vec{r}\times\vec{r}=0$$

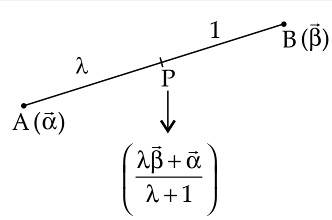
$$= \frac{1}{18}|\vec{r}\times\vec{p}+4(\vec{r}\times\vec{p})-2(\vec{r}\times\vec{p})| \text{ as } \vec{p}\times\vec{r} = -(\vec{r}\times\vec{p})$$

$$= \frac{|\vec{r}\times\vec{p}|}{6}$$

$$\text{So, } \frac{\text{area of } \Delta PQR}{\text{area of } \Delta ABC} = 3$$

HINT:

(1)



(2) Think of calculating the area of triangle using cross product.

10. Option (1) is correct.

$x^3dy + (xy - 1)\,dx = 0$

Also, $y\left(\frac{1}{2}\right) = 3 - e \;\&\; x > 0$

Now, $x^3\frac{dy}{dx} + xy - 1 = 0$

$\Rightarrow x^3\frac{dy}{dx} = 1 - xy \Rightarrow x^3\frac{dy}{dx} + xy = 1$

$\Rightarrow \frac{dy}{dx} + \left(\frac{1}{x^2}\right)y = \frac{1}{x^3}$

This is a linear differential equation

I.F. $= e^{\int\frac{1}{x^2}dx} = e^{\frac{-1}{x}}$

So, differential equation becomes,

$ye^{\frac{-1}{x}} = \int e^{\frac{-1}{x}}.\frac{1}{x^3}dx$

Put $\frac{-1}{x} = t \Rightarrow \frac{1}{x^2}dx = dt$

R.H.S. $= \int -\underset{\text{I}}{t}\,\underset{\text{II}}{e^t}dt$

Integrating by parts

R.H.S. $= -\,[te^t - \int e^t dt] = -te^t + e^t + c$

So, solution of differential equation is

$ye^{\frac{-1}{x}} = -e^{\frac{-1}{x}}\left(\frac{-1}{x} - 1\right) + c$

$\Rightarrow y = \left(\frac{1}{x} + 1\right) + ce^{\frac{1}{x}}$

At $x = \frac{1}{2}, y = 3 - e$

$\Rightarrow 3 - e = \frac{1}{\left(\frac{1}{2}\right)} + 1 + ce^{\frac{1}{\left(\frac{1}{2}\right)}}$

$\Rightarrow 3 - e = (2 + 1) + ce^2$

$\Rightarrow ce^2 = -e$

$\Rightarrow c = \frac{-1}{e}$

So, $y = \left(\frac{1}{x} + 1\right) + \left(-\frac{1}{e}\right)e^{\frac{1}{x}}$

Now, at $x = 1$, $y = \left(\frac{1}{1} + 1\right) + \left(\frac{-1}{e}\right)e^{\frac{1}{1}}$

$\Rightarrow y = 2 - 1 = 1$

11. Option (4) is correct.

Given, $A^2 + B = A^2B$ ...(1)

$\Rightarrow A^2 - A^2B + B = 0$

$\Rightarrow A^2(I - B) - (-B + I - I) = 0$

$\Rightarrow A^2(I - B) - (I - B) = -I$

$\Rightarrow (A^2 - I)(I - B) = -I$

$\Rightarrow (I - A^2)(I - B) = I$

$\Rightarrow (I - B)(I - A^2) = I$

$\Rightarrow I - B - A^2 + BA^2 = I$

$\Rightarrow BA^2 - B - A^2 = 0$ ...(2)

$(1) + (2) \Rightarrow A^2B = BA^2$

HINT:	
(1) $-(-A) = A$	**(2)** $A(BC^2) = ABC^2$
(3) $(A^2B)C = A^2BC$	

12. Option (4) is correct.

$x^2 - 4x + [x] + 3 = x\,[x]$, where [.] = GIF

$\Rightarrow x^2 - 4x + 3 = x\,[x] - [x]$

$\Rightarrow (x - 1)(x - 3) = (x - 1)\,[x]$

$\Rightarrow (x - 1)(x - 3 - [x]) = 0$

$\Rightarrow x = 1, x - 3 = [x]$

$\Rightarrow x = 1, x - [x] = 3$

$\Rightarrow x = 1, \{x\} = 3$, where {.} = Fractional part function

But $\{x\} \in [0, 1)$

So, $\{x\} \neq 3$

$\Rightarrow x = 1$

HINT:

(1) Take terms containing GIF on R.H.S. & factorize

(2) For $y = \{x\}$, where {.} = FPF, $y \in [0, 1)$

13. Option (4) is correct.

$P \equiv (at^2, 2at)$, here $a = 6$

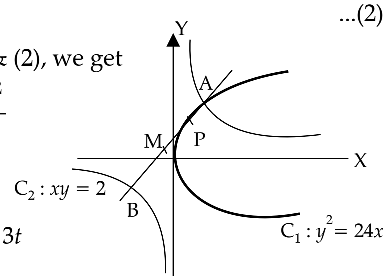
$\Rightarrow P \equiv (6t^2, 12t)$

Tangent to parabola at P(t)

$\Rightarrow ty = x + 6t^2$ ...(1)

Let M (h, k) be the mid-point of chord AB to hyperbola $xy = 2$

$AB \equiv \frac{x}{h} + \frac{y}{k} = 2$ ...(2)



Comparing (1) & (2), we get

$\frac{-1}{\left(\frac{1}{h}\right)} = \frac{t}{\left(\frac{1}{k}\right)} = \frac{6t^2}{2}$

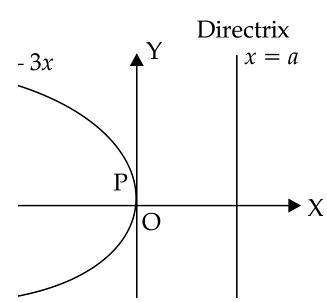
$\Rightarrow -h = tk = 3t^2$

$\Rightarrow h = -3t^2, k = 3t$

So, $h = -3\left(\frac{k}{3}\right)^2$

$\Rightarrow k^2 = -3h$

$\Rightarrow y^2 = -3x$



But $4a = 3$

$\Rightarrow a = \frac{3}{4}$

$\therefore$ Directrix is $x = \frac{3}{4}$

HINT:

(1) Write equation of tangent to parabola in parametric form.

(2) Write equation of chord to rectangular hyperbola $xy = c^2$ whose middle point is given as (h, k).

14. Option (1) is correct.

Let $\Omega \equiv \{1, 2, 3, 4, 5, 6\}$ i.e., they are outcome of throwing a dice.

Let A ≡ Getting a number 7

Now, $P(A) = \frac{\text{Favourable cases}}{\text{Total cases}} = \frac{0}{6} = 0$

But $A = \phi$

$\Rightarrow S_2$ is true.

B ≡ Getting a number < 7

$P(B) = \frac{\text{Favourable cases}}{\text{Total cases}}$

$\Rightarrow P(B) = \frac{6}{6} = 1$

Since, $B = \Omega$

$\Rightarrow S_2$ is true.

HINT:

(1) Think of a case that can never happen.
(2) Think of a case that will always happen.

15. Option (2) is correct.

Let $S = \sum_{r=0}^{22} ({}^{22}C_r)({}^{23}C_r)$

Consider, $(1 + x)^{22} = {}^{22}C_0 x^0 + {}^{22}C_1 x^1 + ... + {}^{22}C_{22} x^{22}$ ...(1)

Again, $(x + 1)^{23} = {}^{23}C_0 x^{23} + {}^{23}C_1 x^{22} + ... + {}^{23}C_{23} x^0$ ...(2)

(1) × (2) gives

$(1 + x)^{45} = ({}^{22}C_0 x^0 + {}^{22}C_1 x^1 + ... + {}^{22}C_{22} x^{22}) \times ({}^{23}C_0 x^{23} + {}^{23}C_1 x^{22} + ... + {}^{23}C_{23} x^0)$ ...(3)

Again, $S = \sum_{r=0}^{22} ({}^{22}C_{22-r})({}^{23}C_r)$,

Using ${}^nC_r = {}^nC_{n-r}$

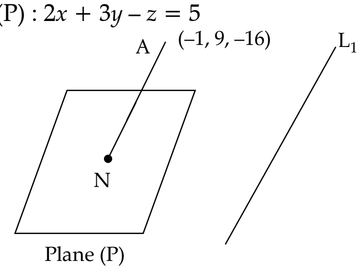
$\Rightarrow S = ({}^{22}C_{22})({}^{23}C_0) + ({}^{22}C_{21})({}^{23}C_1) + ... + ({}^{22}C_0)({}^{23}C_{22})$

From (3), comparing coefficient of x^{23} on both sides

or $S = {}^{45}C_{23}$

16. Option (4) is correct.

Plane (P) : $2x + 3y - z = 5$



where $A \equiv (-1, 9, -16)$ & $L_1 \equiv \frac{x+4}{3} = \frac{y-2}{-4} = \frac{z-3}{12}$

Equation of line AN is,

$\frac{x-(-1)}{3} = \frac{y-9}{-4} = \frac{z-(-16)}{12} = \lambda$ (say)

$\Rightarrow x = 3\lambda - 1, y = -4\lambda + 9, z = 12\lambda - 16$

This point lies on plane (P)

$\Rightarrow 2(3\lambda - 1) + 3(-4\lambda + 9) - (12\lambda - 16) = 5$

$\Rightarrow -18\lambda + 41 = 5 \Rightarrow \lambda = 2$

So, $N \equiv (3(2) - 1, -4(2) + 9, 12(2) - 16)$

$\Rightarrow N \equiv (5, 1, 8)$

$AN = \sqrt{(5-(-1))^2 + (1-9)^2 + (8-(-16))^2}$

$= \sqrt{36 + 64 + 576} = \sqrt{676} = 26$

HINT:

(1) Write equation of line passing through (–1, 9, –16) & parallel to $\frac{x+4}{3} = \frac{2-y}{4} = \frac{z-3}{12}$

(2) Find point where it intersects the plane and then the required distance.

17. Option (1) is correct.

$\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right) + \sec^{-1}\left(\sqrt{\frac{8+4\sqrt{3}}{6+3\sqrt{3}}}\right)$

$= \tan^{-1}\left(\frac{\sqrt{3}+1}{\sqrt{3}(\sqrt{3}+1)}\right) + \sec^{-1}\left(\sqrt{\frac{4(2+\sqrt{3})}{3(2+\sqrt{3})}}\right)$

$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) + \sec^{-1}\left(\sqrt{\frac{4}{3}}\right)$

$= \frac{\pi}{6} + \sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3}$

HINT:

Take common & simplify the given expressions.

18. Option (4) is correct.

Given: $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & , x \neq 0 \\ 0 & , x = 0 \end{cases}$

As we know function is said to be continuous at a point if limiting value of the function at that point is equal to the functional value of the function at that point.

Now, $\lim_{x\to 0} f(x) = \lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) = 0 = f(0)$

$\therefore f(x)$ is continuous at $x = 0$.

As we know function is said to be differentiable at a point if LHD = RHD at that point.

Now, L.H.D. at $x = 0, f'(0^-) = \lim_{h\to 0} \frac{f(0-h) - f(0)}{0-h}$

$= \lim_{h\to 0} \frac{-h^2 \sin\left(\frac{1}{h}\right) - 0}{-h} = \lim_{h\to 0} h \sin\left(\frac{1}{h}\right) = 0$

Now, R.H.D. at $x = 0, f'(0^+) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h}$

$= \lim_{h\to 0} h \sin\left(\frac{1}{h}\right) = 0$

$\because$ L.H.D. = R.H.D. at $x = 0$

$\therefore f(x)$ is differentiable at $x = 0$

Now, $f'(x) = \begin{cases} 2x\sin\left(\frac{1}{x}\right) + x^2\cos\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right) & ,x \neq 0 \\ 0 & ,x = 0 \end{cases}$

$f'(x) = \begin{cases} 2x\sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right) & ,x \neq 0 \\ 0 & ,x = 0 \end{cases}$

$\because$ limit of $f'(x) = 0$ oscillates

$\therefore f'(x)$ is not continuous at $x = 0$

19. Option (3) is correct.

As we know $A \Rightarrow B \equiv \sim A \vee B$

So, $\sim(\sim P \wedge Q) \Rightarrow (\sim (P \vee Q)$

$= \sim[\sim(\sim P \wedge Q)] \vee (\sim P \vee Q)$

$= (\sim P \wedge Q) \vee (\sim P \vee Q)$

$= [\sim P \vee (\sim P \vee Q)] \wedge [Q \vee (\sim P \vee Q)]$

$= [\sim P \vee Q] \wedge [\sim P \vee Q] \equiv \sim P \vee Q$

HINT:

(1) Use $A \Rightarrow B = \sim A \vee B$

(2) Use $\sim (A \vee B) = \sim A \wedge \sim B$

20. Option (3) is correct.

As we know equation of plane passing through the points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_1-x_2 & y_1-y_2 & z_1-z_2 \\ x_1-x_3 & y_1-y_3 & z_1-z_3 \end{vmatrix} = 0$$

So, equation of plane passing through (2, –3, 1), (–1, 1, –2) and (3, –4, 2) is given by

$$\begin{vmatrix} x-2 & y+3 & z-1 \\ 3 & -4 & 3 \\ -1 & 1 & -1 \end{vmatrix} = 0$$

$\Rightarrow x - z - 1 = 0$

Now, distance of the point (7, –3, –4) from plane $x - z - 1 = 0$ is

$d = \left|\frac{7-(-4)-1}{\sqrt{2}}\right| \Rightarrow d = 5\sqrt{2}$

Section B

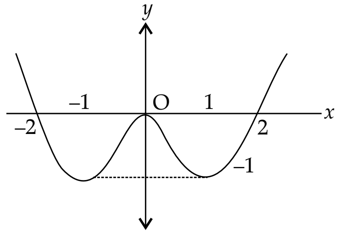
21. Correct answer is (5).

Given, E : $|x|^2 - 2|x| + |\lambda - 3| = 0$

S = $\{x + \lambda : x$ is an integer solution of E$\}$.

So, $|x|^2 - 2|x| = -|\lambda - 3|$

Lets draw the graph of $f(x) = |x|^2 - 2|x|$



It is clear from the figure, $-1 \leq |x|^2 - 2|x| < \infty$ and $-|\lambda - 3| \leq 0$

So, given equation holds only when $|\lambda - 3| \leq 1$ and $x \in [-2, 2]$

$\Rightarrow -1 \leq \lambda - 3 \leq 1 \Rightarrow 2 \leq \lambda \leq 4$

For $x = 0$, $\lambda = 3$

For $x = \{-1, 1\}$, $\lambda = 4$ or 2

For $x = \{-2, 2\}$, $\lambda = 3$

So, largest element in the set S is 5

HINT:

(1) Write given equation as $|x|^2 - 2|x| = -|\lambda - 3|$ and draw the graph of $|x|^2 - 2|x|$ and analyse further using the concept of modulus function.

(2) Quadratic function $f(x) = x^2 + bx + c$; $a > 0$ has minimum value at $x = \frac{-b}{2a}$.

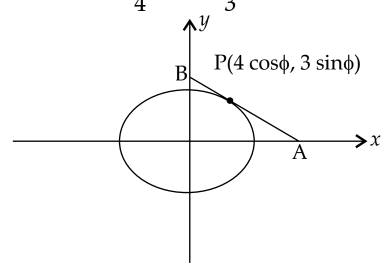
22. Correct answer is (7).

Given equation of curve is $9x^2 + 16y^2 = 144$

$\Rightarrow \frac{x^2}{16} + \frac{y^2}{9} = 1$

As we know equation of tangent to ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at any point $(a\cos\phi, b\sin\phi)$ is $\frac{x}{a}\cos\phi + \frac{y}{b}\sin\phi = 1$

So, equation of tangent to given ellipse at $(4\cos\phi, 3\sin\phi)$ is $\frac{x}{4}\cos\phi + \frac{y}{3}\sin\phi = 1$



So, coordinates of A = $(4\sec\phi, 0)$

Coordinates of B = $(0, 3\operatorname{cosec}\phi)$

Now, AB = $\sqrt{16\sec^2\phi + 9\operatorname{cosec}^2\phi}$

$\Rightarrow (AB)_{min} = \sqrt{25 + 24} = 7$

HINT:

(1) Equation of tangent to ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at any point $(a\cos\phi, b\sin\phi)$ is $\frac{x}{a}\cos\phi + \frac{y}{b}\sin\phi = 1$

(2) Use $1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \operatorname{cosec}^2\theta$.

23. Correct answer is (14).

Given lines $L_1 : \frac{x-2}{3} = \frac{y+1}{2} = \frac{z-6}{2}$

$L_2 : \frac{x-6}{3} = \frac{1-y}{2} = \frac{z+8}{0}$

L_1 can be written as in vector from

$\vec{r} = (2\hat{i} - \hat{j} + 6\hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 2\hat{k})$

L_2 can be written as in vector from

$\vec{r} = (6\hat{i} + \hat{j} - 8\hat{k}) + \mu(3\hat{i} - 2\hat{j})$

As we know shortest distance between two lines $\vec{r} = \vec{a} + \lambda\vec{p}$ and $\vec{r} = \vec{b} + \mu\vec{q}$ is given by

$$d = \left|\frac{(\vec{b} - \vec{a}).(\vec{p} \times \vec{q})}{|\vec{p} \times \vec{q}|}\right|$$

Here, $\vec{a} = 2\hat{i} - \hat{j} + 6\hat{k}$, $\vec{b} = 6\hat{i} + \hat{j} - 8\hat{k}$,

$\vec{p} = 3\hat{i} + 2\hat{j} + 2\hat{k}$, $\vec{q} = 3\hat{i} - 2\hat{j}$

$$\text{Now, } \vec{p} \times \vec{q} = \begin{vmatrix} i & j & k \\ 3 & 2 & 2 \\ 3 & -2 & 0 \end{vmatrix}$$

$\Rightarrow \vec{p} \times \vec{q} = 4\hat{i} + 6\hat{j} - 12\hat{k} = 2(2\hat{i} + 3\hat{j} - 6\hat{k})$

Now, $\vec{b} - \vec{a} = 4\hat{i} + 2\hat{j} - 14\hat{k} = 2(2\hat{i} + \hat{j} - 7\hat{k})$

So, $(\vec{b} - \vec{a}).(\vec{p} \times \vec{q}) = 4[4 + 3 + 42] = 196$

And $|\vec{p} \times \vec{q}| = 2\sqrt{4 + 9 + 36} = 14$

$\therefore$ Shortest distance between given lines is

$$d = \left|\frac{196}{14}\right| = 14$$

24. Correct answer is (1012).

Given: $\sum_{r=0}^{2023} r^2 \, {}^{2023}C_r = 2023 \times \alpha \times 2^{2022}$

Let A = $\sum_{r=0}^{2023} r^2 \, {}^{2023}C_r$

$\Rightarrow$ A = $\sum_{r=0}^{2023} r^2 \frac{2023}{r} \, {}^{2022}C_{r-1}$ $\quad \left\{\because {}^nC_r = \frac{n}{r} \, {}^{n-1}C_{r-1}\right\}$

$\Rightarrow$ A = $\sum_{r=1}^{2023} r(2023) \, {}^{2022}C_{r-1}$

$\Rightarrow$ A = $2023\left\{\sum_{r=1}^{2023} (r-1) \, {}^{2022}C_{r-1} + \sum_{r=1}^{2023} {}^{2022}C_{r-1}\right\}$

$\Rightarrow$ A = 2023 $\{2022 \, 2^{2021} + 2^{2022}\}$

$\{\because {}^nC_1 + 2\,{}^nC_2 + 3\,{}^nC_3 + ... + n\,{}^nC_n = n\,2^{n-1}$ and

${}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n = 2^n\}$

$\Rightarrow$ A = $2023 \times 2022 \times 2^{2021} + 2^{2022} \times 2023$

$\Rightarrow$ A = $2023 \times 2^{2022}(1011 + 1)$

$\Rightarrow$ A = $1012 \times 2023 \times 2^{2022}$

$\Rightarrow 2023 \times \alpha \times 2^{2022} = 1012 \times 2023 \times 2^{2022}$

$\Rightarrow \alpha = 1012$

25. Correct answer is (2).

Let I = $\int_0^{\frac{\pi}{2}} \frac{(\cos x)^{2023}}{(\sin x)^{2023} + (\cos x)^{2023}} dx$ ...(1)

Using $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$\Rightarrow$ I = $\int_0^{\frac{\pi}{2}} \frac{(\sin x)^{2023}}{(\cos x)^{2023} + (\sin x)^{2023}} dx$ ...(2)

(1) + (2), 2I = $\int_0^{\frac{\pi}{2}} dx$

$\Rightarrow$ I = $\frac{1}{2}\left(\frac{\pi}{2} - 0\right) \Rightarrow$ I = $\frac{\pi}{4}$

So, $\frac{8}{\pi}(I) = \frac{8}{\pi}.\frac{\pi}{4} = 2$

HINT:

Use property: $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

26. Correct answer is (60).

$1 \rightarrow 3$ times

$2 \rightarrow 2$ times

$3 \rightarrow 2$ times

$4 \rightarrow 2$ times

_ X _ X _ X _ X _

O E O E O E O E O

O - odd, E - even

Number of ways for even digits to occupy even places

$= \frac{4!}{2!2!} = \frac{24}{2 \times 2} = 6$

Total number of 9 digit numbers = $(6)\left(\frac{5!}{3!2!}\right)$

$= (6)\left(\frac{120}{6 \times 2}\right) = 60$

27. Correct answer is (546).

Number of ways

Total courses = 12

5 Language Courses (LA)

7 Other Course (OT)

= {O(LA), 5 (OT)} + {1(LA), 4 (OT)} + {2 (LA), 3 (OT)}

$\Rightarrow ({}^5C_0)({}^7C_5) + ({}^5C_1)({}^7C_4) + ({}^5C_2)({}^7C_3)$

$\Rightarrow (1)({}^7C_2) + (5)({}^7C_3) + \left(\frac{5 \times 4}{2}\right)({}^7C_3)$

$= \left(\frac{7 \times 6}{2}\right) + (5)\left(\frac{7 \times 6 \times 5}{6}\right) + (10)\left(\frac{7 \times 6 \times 5}{6}\right)$

= 21 + 175 + 350 = 546

28. Correct answer is (12).

Let first term of G.P. be 'a'.

Given, $T_4 = a\left(\frac{1}{m}\right)^3 = 500$

$\Rightarrow \frac{a}{m^3} = 500$

$\Rightarrow a = 500\, m^3$

Consider, $S_n - S_{n-1}$

$= a\left(\frac{1-r^n}{1-r}\right) - a\left(\frac{1-r^{n-1}}{1-r}\right)$, where $r = \frac{1}{m}$

$= \frac{a}{(1-r)}[1-r^n-1+r^{n-1}] = \frac{ar^{n-1}}{(1-r)}(1-r) = ar^{n-1}$

$\Rightarrow S_n - S_{n-1} = \frac{a}{m^{n-1}} \Rightarrow S_n - S_{n-1} = \frac{500m^3}{m^{n-1}}$

$\Rightarrow S_n - S_{n-1} = 500\, m^{4-n}$

Given, $S_6 - S_5 > 1$

$\Rightarrow 500\, m^{4-6} > 1$

$\Rightarrow \frac{500}{m^2} > 1$ ...(1)

Again, $S_7 - S_6 < \frac{1}{2} \Rightarrow 500\, m^{4-7} < \frac{1}{2}$

$\Rightarrow \frac{500}{m^3} < \frac{1}{2}$ ...(2)

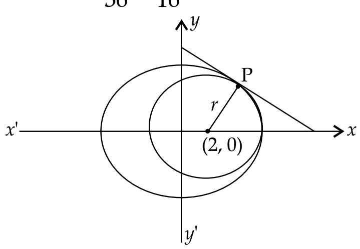
(1) $\Rightarrow m^2 < 500$ (2) $\Rightarrow m^3 > 1000$

So, $m \in \{11, 12, 13, ..., 22\}$

$\therefore$ Number of possible values of m is 12.

29. Correct answer is (118).

Let ellipse be E: $\frac{x^2}{36} + \frac{y^2}{16} = 1$



Circle (C) $\equiv (x-2)^2 + (y-0)^2 = r^2$

For C to be largest possible circle, its radius has to be maximum.

Point P on ellipse $\equiv (6\cos\theta, 4\sin\theta)$

T = 0

Normal at P on ellipse should also be normal to circle as ellipse and circle are touching each other at P.

Normal $\equiv 6x\sec\theta - 4y\,\text{cosec}\theta = 36 - 16$

It also passes through centre of circle i.e., (2, 0)

$\Rightarrow 12\sec\theta = 20$

$\Rightarrow \cos\theta = \frac{3}{5}$

So, $\sin\theta = \frac{4}{5}$

$P \equiv \left(6\times\frac{3}{5}, 4\times\frac{4}{5}\right) \equiv \left(\frac{18}{5}, \frac{16}{5}\right)$

Now, $r = \sqrt{\left(2-\frac{18}{5}\right)^2 + \left(0-\frac{16}{5}\right)^2}$

$\Rightarrow r = \sqrt{\frac{64}{25} + \frac{256}{25}}$

$\Rightarrow r = \frac{\sqrt{320}}{5} = \frac{8\sqrt{5}}{5} \Rightarrow r = \frac{8}{\sqrt{5}}$

Now $(1, \alpha)$ lies on c.

$\therefore \sqrt{(2-1)^2 + (0-\alpha)^2} = \frac{8}{\sqrt{5}}$

$1 + \alpha^2 = \frac{64}{5} \Rightarrow \alpha^2 = \frac{64}{5} - 1 = \frac{59}{5}$

$10\alpha^2 = \frac{59}{5} \times 10 = 118$

HINT:

(1) Think of common normal and remember that normal of circle passes through its centre.

(2) Normal to ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at P $(a\cos\theta, b\sin\theta)$ is $(a\sec\theta)\,x - (b\,\text{cosec}\theta)\,y = a^2 - b^2$

(3) Finally find radius of circle and make it equal to the distance between $(1, \alpha)$ and $(2, 0)$.

30. Correct answer is (22).

Let $I = \int_0^3 |x^2 - 3x + 2|\,dx$

$|x^2 - 3x + 2| = \begin{cases} x^2 - 3x + 2 & x \in (-\infty, 1] \cup [2, \infty) \\ -(x^2 - 3x + 2), & x \in (1, 2) \end{cases}$

So, $I = \int_0^1 (x^2 - 3x + 2)dx - \int_1^2 (x^2 - 3x + 2)dx + \int_2^3 (x^2 - 3x + 2)dx$

$\Rightarrow I = \left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right]_0^1 - \left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right]_1^2 + \left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right]_2^3$

$\Rightarrow I = \frac{5}{6} + \frac{1}{6} + \frac{5}{6} = \frac{11}{6}$

So, $12I = 12\left(\frac{11}{6}\right) = 22$

❑❑