JEE (Main) MATHEMATICS SOLVED PAPER

2023

24th Jan. Shift 2

Section A

Q. 1. If, $f(x) = x^3 - x^2 f'(1) + xf''(2) - f'''(3)$, $x \in \mathbb{R}$ then

(1) $f(1) + f(2) + f(3) = f(0)$
(2) $2f(0) - f(1) + f(3) = f(2)$
(3) $3f(1) + f(2) = f(3)$
(4) $f(3) - f(2) = f(1)$

Q. 2. If the system of equations

$$x + 2y + 3z = 3$$
$$4x + 3y - 4z = 4$$
$$8x + 4y - \lambda z = 9 + \mu$$

has infinitely many solutions, then the ordered pair (λ, μ) is equal to:

(1) $\left(-\frac{72}{5}, \frac{21}{5}\right)$ **(2)** $\left(-\frac{72}{5}, -\frac{21}{5}\right)$

(3) $\left(\frac{72}{5}, -\frac{21}{5}\right)$ **(4)** $\left(\frac{72}{5}, \frac{21}{5}\right)$

Q. 3. If, $f(x) = \frac{2^{2x}}{2^{2x} + 2}, x \in R$, then

$f\left(\frac{1}{2023}\right) + f\left(\frac{2}{2023}\right) + \ldots + f\left(\frac{2022}{2023}\right)$ is equal to

(1) 1011 **(2)** 2010 **(3)** 1010 **(4)** 2011

Q. 4. Let $\vec{\alpha} = 4\hat{i} + 3\hat{j} + 5\hat{k}$ and $\vec{\beta} = \hat{i} + 2\hat{j} - 4\hat{k}$. Let $\vec{\beta}_1$ be parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ be perpendicular to $\vec{\alpha}$. If $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, then the value of $5\vec{\beta}_2 . (\hat{i} + \hat{j} + \hat{k})$ is

(1) 7 **(2)** 9 **(3)** 6 **(4)** 11

Q. 5. Let $y = y(x)$ be the solution of the differential equation $(x^2 - 3y^2)\,dx + 3xy\,dy = 0$, $y(1) = 1$. Then $6y^2(e)$ is equal to

(1) $2e^2$ **(2)** $3e^2$ **(3)** e^2 **(4)** $\frac{3}{2}e^2$

Q. 6. The locus of the mid points of the chords of the circle C_1: $(x - 4)^2 + (y - 5)^2 = 4$ which subtend an angle θ_1 at the centre of the circle C_1, is a circle of radius r_i. If $\theta_1 = \frac{\pi}{3}, \theta_3 = \frac{2\pi}{3}$ and $r_1^2 = r_2^2 + r_3^2$, then θ_2 is equal to

(1) $\frac{\pi}{4}$ **(2)** $\frac{\pi}{2}$ **(3)** $\frac{\pi}{6}$ **(4)** $\frac{3\pi}{4}$

Q. 7. The number of real solutions of the equation

$3\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) + 5 = 0$, is

(1) 0 **(2)** 3 **(3)** 4 **(4)** 2

Q. 8. Let A be a 3×3 matrix such that $|\text{adj}(\text{adj}(\text{adj}A))| = 12^4$ Then $|A^{-1}\text{adj}A|$ is equal to

(1) $\sqrt{6}$ **(2)** $2\sqrt{3}$ **(3)** 12 **(4)** 1

Q. 9. $\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9 - 4x^2}}\,dx$

(1) 2π **(2)** $\frac{\pi}{6}$ **(3)** $\frac{\pi}{3}$ **(4)** $\frac{\pi}{2}$

Q. 10. The number of square matrices of order 5 with entries form the set {0, 1}, such that the sum of all the elements in each row is 1 and the sum of all the elements in each column is also 1, is

(1) 125 **(2)** 225 **(3)** 150 **(4)** 120

Q. 11. If $({}^{30}C_1)^2 + 2\,({}^{30}C_2)^2 + 3\,({}^{30}C_3)^2 + \ldots + 30\,({}^{30}C_{30})^2 = \frac{\alpha 60!}{(30!)^2}$ then α is equal to:

(1) 30 **(2)** 10 **(3)** 60 **(4)** 15

Q. 12. Let the plane containing the line of intersection of the planes $P_1 : x + (\lambda + 4)y + z = 1$ and P_2: $2x + y + z = 2$ pass through the points (0, 1, 0) and (1, 0, 1). Then the distance of the point $(2\lambda, \lambda, -\lambda)$ from the plane P_2 is

(1) $4\sqrt{6}$ **(2)** $3\sqrt{6}$ **(3)** $5\sqrt{6}$ **(4)** $2\sqrt{6}$

Q. 13. Let $f(x)$ be a function such that $f(x + y) = f(x).f(y)$ for all, $x, y \in$ N. If $f(1) = 3$ and $\sum_{k=1}^{n} f(k) = 3279$, then the value of n is

(1) 9 **(2)** 6 **(3)** 8 **(4)** 7

Q. 14. Let the six numbers $a_1, a_2, a_3, a_4, a_5, a_6$, be in A.P. and $a_1 + a_3 = 10$. If the mean of these six numbers is $\frac{19}{2}$ and their variance is σ^2, then $8\sigma^2$ is equal to:

(1) 210 **(2)** 220 **(3)** 200 **(4)** 105

Q. 15. The equations of the sides AB and AC of a triangle ABC are $(\lambda + 1)x + \lambda y = 4$ and $\lambda x + (1 - \lambda)y + \lambda = 0$ respectively. Its vertex A is on the y-axis and its orthocentre is (1, 2). The length of the tangent from the point C to the part of the parabola $y^2 = 6x$ in the first quadrant is :

(1) 4 **(2)** 2 **(3)** $\sqrt{6}$ **(4)** $2\sqrt{2}$

Q. 16. Let p and q be two statements. Then $\sim (p \wedge (p \Rightarrow \sim q))$ is equivalent to

(1) $p \vee (p \wedge q)$ **(2)** $p \vee (p \wedge (\sim q))$
(3) $(\sim p) \vee q$ **(4)** $p \vee ((\sim p) \wedge q)$

Q. 17. The set of all values of a for which $\lim_{x \to a} ([x - 5] - [2x + 2]) = 0$, where $[\alpha]$ denotes the greatest integer less than or equal to α is equal to

(1) [–7.5, –6.5) **(2)** [–7.5, –6.5]
(3) (–7.5, – 6.5] **(4)** (–7.5, –6.5)

Q. 18. If the foot of the perpendicular drawn from (1, 9, 7) to the line passing through the point (3, 2, 1) and parallel to the planes $x + 2y + z = 0$ and $3y - z = 3$ is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to

(1) 3 **(2)** 1 **(3)** –1 **(4)** 5

Q. 19. The number of integers, greater than 7000 that can be formed, using the digits 3, 5, 6, 7, 8 without repetition, is

(1) 168 **(2)** 220 **(3)** 120 **(4)** 48

Q. 20. The value of $\left(\dfrac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$

(1) $-\frac{1}{2}(\sqrt{3}-i)$ **(2)** $-\frac{1}{2}(1-i\sqrt{3})$

(3) $\frac{1}{2}(1-i\sqrt{3})$ **(4)** $\frac{1}{2}(\sqrt{3}+i)$

Section B

Q. 21. If the shortest distance between the lines $\dfrac{x+\sqrt{6}}{2}=\dfrac{y-\sqrt{6}}{3}=\dfrac{z-\sqrt{6}}{4}$ and $\dfrac{x-\lambda}{3}=\dfrac{y-2\sqrt{6}}{4}=\dfrac{z+2\sqrt{6}}{5}$ is 6, then the square of sum of all possible values of λ is

Q. 22. Three urns A, B and C contain 4 red, 6 black; 5 red, 5 black; and λ red, 4 black balls respectively. One of the urns is selected at random and a ball is drawn. If the ball drawn is red and the probability that it is drawn from urn C is 0.4 then the square of the length of the side of the largest equilateral triangle, inscribed in the parabola $y^2 = \lambda x$ with one vertex at the vertex of the parabola, is

Q. 23. Let $S = \{\theta \in [0, 2\pi) : \tan(\pi\cos\theta) + \tan(\pi\sin\theta) = 0\}$

Then $\sum\limits_{\theta\in s}\sin^2\left(\theta+\frac{\pi}{4}\right)$ is equal to

Q. 24. If $\dfrac{1^3+2^3+3^3+\ldots \text{ up to } n \text{ terms}}{1.3+2.5+3.7+\ldots\text{up to } n \text{ terms}}=\dfrac{9}{5}$, then the value of n is

Q. 25. Let the sum of the coefficients of the first three terms in the expansion of $\left(x-\frac{3}{x^2}\right)^n, x\neq 0, n\in\mathbb{N}$, be 376. Then the coefficient of x^4 is

Q. 26. The equations of the sides AB, BC and CA of a triangle ABC are : $2x + y = 0$, $x + py = 21a$, $(a \neq 0)$ and $x - y = 3$ respectively. Let P(2, a) be the centroid of Δ ABC. Then $(BC)^2$ is equal to

Q. 27. Let $\vec{a}=\hat{i}+2\hat{j}+\lambda\hat{k}, \vec{b}=3\hat{i}-5\hat{j}-\lambda\hat{k}$,

$\vec{a}.\vec{c}=7, 2\vec{b}.\vec{c}+43=0,\ \vec{a}\times\vec{c}=\vec{b}\times\vec{c}$.

Then $|\vec{a}.\vec{b}|$ is equal to

Q. 28. The minimum number of elements that must be added to the relation R = {(a, b), (b, c), (b, d)} on the set {a, b, c, d} so that it is an equivalence relation, is

Q. 29. If the area of the region bounded by the curves $y^2 - 2y = -x$, $x + y = 0$ is A, then 8 A is equal to

Q. 30. Lef f be a differentiable function defined on $\left[0,\frac{\pi}{2}\right]$ such that $f(x) > 0$ and

$f(x)+\int_0^x f(t)\sqrt{1-(\log_e f(t))^2}\,dt = e, \forall x\in\left[0,\frac{\pi}{2}\right]$.

Then $\left(6\log_e f\left(\frac{\pi}{6}\right)\right)^2$ is equal to

Answer Key

Q. No.	Answer	Topic Name	Chapter Name
1	(2)	Higher Order Derivatives	Differential Calculus
2	(3)	System of linear equations	Matrices and Determinants
3	(1)	Algebra of Functions	Function
4	(1)	Scalar and Vector Products	Vector Algebra
5	(1)	Linear Differential Equations	Differential Equations
6	(2)	Interaction between Circle and a Line	Circle
7	(1)	Quadratic Equation and its Solution	Quadratic Equations
8	(2)	Adjoint of a Matrix	Matrices and Determinants
9	(1)	Basics of Definite Integration	Definite Integration
10	(4)	Permutations	Permutation and Combination
11	(4)	Properties of Binomial Coefficients	Binomial Theorem
12	(2)	Plane and a Point	Three Dimensional Geometry
13	(4)	Geometric Progressions	Sequences and Series

Q. No.	Answer	Topic Name	Chapter Name
14	(1)	Measures of Dispersion	Statistics
15	(4)	Tangent to a Parabola	Parabola
16	(3)	Logical Operations	Mathematical Reasoning
17	(4)	Algebra of Limits	Limits
18	(4)	Lines in 3D	Three Dimensional Geometry
19	(1)	Permutations	Permutations and Combinations
20	(1)	Representation of Complex Numbers	Complex Numbers
21	[384]	Skew Lines	Three Dimensional Geometry
22	[432]	Bayes' Theorem	Probability
23	[2]	Trigonometric Equations	Trigonometric Equations and Inequalities
24	[5]	Series of Natural Numbers and other Miscellaneous Series	Sequences and Series
25	[405]	Binomial Theorem for Positive Integral Index	Binomial Theorem
26	[122]	Interaction between Two Lines	Point and Straight Line
27	[8]	Scalar and Vector Products	Vector Algebra
28	[13]	Algebra of Relations	Set Theory and Relations
29	[36]	Area Bounded by Curves	Area under Curves
30	[27]	Variable Separable Form	Differential Equations

Solutions

Section A

1. Option (2) is correct.

Given, $f(x) = x^3 - x^2 f'(1) + xf''(2) - f'''(3), x \in \mathbb{R}$

Let $f'(1) = p, f''(2) = q$ and $f'''(3) = r$

$\Rightarrow f(x) = x^3 - px^2 + qx - r$

$\Rightarrow f'(x) = 3x^2 - 2px + q$

$\Rightarrow f''(x) = 6x - 2p \Rightarrow f'''(x) = 6$

So, $f'''(3) = 6 = r$

Now, $f'(1) = 3(1)^2 - 2p(1) + q$

$\Rightarrow p = 3 - 2p + q$

$\Rightarrow 3p = 3 + q$ ...(i)

And $f''(2) = 6(2) - 2p$

$\Rightarrow q = 12 - 2p$

$\Rightarrow 2p + q = 12$ ...(ii)

On solving equation (i) and equation (ii), we get

$p = 3, q = 6$

$\therefore f(x) = x^3 - 3x^2 + 6x - 6$

So, $f(0) = -6, f(1) = -2, f(2) = 2, f(3) = 12$

Now, $2f(0) - f(1) + f(3) = 2(-6) - (-2) + 12$

$= 2 = f(2)$

HINT:

Find $f'(x), f''(x)$ and $f'''(x)$ using $\frac{d}{dx}(x^n) = nx^{n-1}$ and solve further.

2. Option (3) is correct.

Given: System of equations

$x + 2y + 3z = 3$

$4x + 3y - 4z = 4$

$8x + 4y - \lambda z = 9 + \mu$

As we know for infinite many solutions, $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

Now, $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 3 & -4 \\ 8 & 4 & -\lambda \end{vmatrix} = 0$

$\Rightarrow \lambda = \frac{72}{5}$

Now, $\Delta_3 = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 3 & 4 \\ 8 & 4 & 9+\mu \end{vmatrix} = 0$

$\Rightarrow \mu = -\frac{21}{5}$

3. Option (1) is correct.

Given: $f(x) = \frac{2^{2x}}{2^{2x}+2}, x \in R$

$\Rightarrow f(x) = \frac{4^x}{4^x+2}$

Now, $f(1-x) = \frac{4^{(1-x)}}{4^{(1-x)}+2}$

$\Rightarrow f(1-x) = \frac{4}{4+2.4^x}$

$\Rightarrow f(1-x) = \frac{2}{2+4^x}$

So, $f(x) + f(1-x) = \frac{4^x}{4^x+2} + \frac{2}{4^x+2} = 1$

Let A $= f\left(\frac{1}{2023}\right) + f\left(\frac{2}{2023}\right) + ... + f\left(\frac{2022}{2023}\right)$

$\Rightarrow A = f\left(\frac{1}{2023}\right) + f\left(\frac{2022}{2023}\right) + f\left(\frac{2}{2023}\right) + f\left(\frac{2021}{2023}\right) + ...$
$+ f\left(\frac{1011}{2023}\right) + f\left(\frac{1012}{2023}\right)$

$\Rightarrow$ A = 1 + 1 + 1 + ... up to 1011 terms

$\{\because f(x) + f(1 - x) = 1\}$

$\Rightarrow$ A = 1011

HINT:

Use $f(x) + f(1 - x) = 1$ and solve further.

4. Option (1) is correct.

Given: $\vec{\alpha} = 4\hat{i} + 3\hat{j} + 5\hat{k}$

$\vec{\beta} = \hat{i} + 2\hat{j} - 4\hat{k}$

$\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$

$\because$ $\vec{\beta}_1$ is parallel to $\vec{\alpha}$

$\Rightarrow \beta_1 = \mu(4\hat{i} + 3\hat{j} + 5\hat{k}); \mu \in R$

Also given that $\vec{\beta}_2$ is perpendicular to α

$\Rightarrow \vec{\beta}_2\, \alpha = 0$

Since, $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$

$\Rightarrow \vec{\beta} = \mu\vec{\alpha} + \vec{\beta}_2$

$\Rightarrow \vec{\beta}.\vec{\alpha} = \mu|\vec{\alpha}^2| + \vec{\beta}_2.\vec{\alpha}$

$\Rightarrow (4\hat{i} + 3\hat{j} + 5\hat{k}).(\hat{i} + 2\hat{j} - 4\hat{k}) = \mu(\sqrt{16 + 9 + 25})^2 + 0$

$\Rightarrow 4 + 6 - 20 = \mu(50)$

$\Rightarrow \mu = -\frac{1}{5}$

Now, $\vec{\beta} = -\frac{1}{5}\vec{\alpha} + \vec{\beta}_2$

$\Rightarrow 5\vec{\beta}_2 = 5\vec{\beta} + \vec{\alpha}$

$\Rightarrow 5\vec{\beta}_2 = 5(\hat{i} + 2\hat{j} - 4\hat{k}) + (4\hat{i} + 3\hat{j} + 5\hat{k})$

$\Rightarrow 5\vec{\beta}_2 = 9\hat{i} + 13\hat{j} - 15\hat{k}$

Now, $5\vec{\beta}_2.(\hat{i} + \hat{j} + \hat{k}) = 9 + 13 - 15 = 7$

HINT:

(1) Use if $\vec{a}$ is parallel to $\vec{b}$, then $\vec{b} = k\vec{a}; k \in R$

(2) Use if $\vec{u}$ is perpendicular to $\vec{v}$, then $\vec{u}.\vec{v} = 0$

(3) If $\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$ and $\vec{b} = b_1\vec{i} + b_2\vec{j} + b_3\vec{k}$, then $\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

5. Option (1) is correct.

Given, differential equation $(x^2 - 3y^2)\,dx + 3xy\,dy = 0$, $y(1) = 1$

$\Rightarrow 3xy\frac{dy}{dx} - 3y^2 = -x^2$

$\Rightarrow y\frac{dy}{dx} - \frac{y^2}{x} = -\frac{x}{3}$

$\Rightarrow 2y\frac{dy}{dx} - \frac{2y^2}{x} = -\frac{2x}{3}$

Let $y^2 = v$

$\Rightarrow 2y\frac{dy}{dx} = \frac{dv}{dx}$

So, $\frac{dv}{dx} - \frac{2v}{x} = \frac{-2x}{3}$ which is linear differential equation

Now, I.F. $= e^{\int -\frac{2}{x}dx}$

$\Rightarrow$ I.F. $= e^{-2\ln x}$

$\Rightarrow$ I.F. $= e^{\ln x^{-2}} \Rightarrow$ I.F. $= \frac{1}{x^2}$

Now, solution of linear differential equation is

$v\,(\text{I.F.}) = \int \frac{-2x}{3}(\text{I.F.})\,dx + c$

$\Rightarrow v\frac{(1)}{(x^2)} = \int \frac{-2x}{3} \times \frac{1}{x^2}dx + c$

$\Rightarrow \frac{v}{x^2} = -\frac{2}{3}\int \frac{1}{x}dx + c$

$\Rightarrow \frac{v}{x^2} = -\frac{2}{3}\ln x + c$

$\Rightarrow \frac{y^2}{x^2} = -\frac{2}{3}\ln x + c$ $\qquad \because y(1) = 1$

$\Rightarrow c = 1$

So, $\frac{y^2}{x^2} = -\frac{2}{3}\ln x + 1$

$\Rightarrow y^2 = -\frac{2}{3}x^2\ln x + x^2$

$\Rightarrow y^2(e) = -\frac{2}{3}e^2\ln e + e^2$

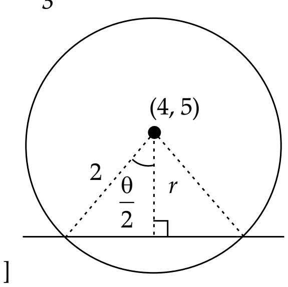
$\Rightarrow y^2(e) = \frac{e^2}{3} \Rightarrow 6y^2(e) = 2e^2$

6. Option (2) is correct.

Given: Circle c_1: $(x - 4)^2 + (y - 5)^2 = 4$

$\Rightarrow$ Centre = (4, 5) and radius = 2

Also given that $\theta_1 = \frac{\pi}{3}, \theta_3 = \frac{2\pi}{3}$ and $r_1^2 = r_2^2 + r_3^2$



So, $\cos\left(\frac{\theta_i}{2}\right) = \frac{r_i}{2}$

$\Rightarrow r_i = 2\cos\left(\frac{\theta_i}{2}\right)$

$[\because r_1^2 = r_2^2 + r_3^2]$

$\Rightarrow \cos^2\frac{\theta_1}{2} = \cos^2\frac{\theta_2}{2} + \cos^2\frac{\theta_3}{2}$

$\Rightarrow \cos^2\left(\frac{\pi}{6}\right) = \cos^2\left(\frac{\theta_2}{3}\right) + \cos^2\left(\frac{\pi}{3}\right)$

$\Rightarrow \frac{3}{4} = \frac{1}{4} + \cos^2\frac{\theta_2}{2}$

$\Rightarrow \cos^2\frac{\theta_2}{2} = \frac{1}{2}$

$\Rightarrow \frac{\theta_2}{2} = \frac{\pi}{4}$

$\Rightarrow \theta_2 = \frac{\pi}{2}$

7. Option (1) is correct.

Given: $3\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) + 5 = 0$

$\Rightarrow 3\left[\left(x + \frac{1}{x}\right)^2 - 2\right] - 2\left(x + \frac{1}{x}\right) + 5 = 0$

Let $x + \frac{1}{x} = v$

$\Rightarrow 3\,[v^2 - 2] - 2v + 5 = 0$

$\Rightarrow 3v^2 - 2v - 1 = 0$

$\Rightarrow (3v + 1)\,(v - 1) = 0$

$\Rightarrow v = 1, -\frac{1}{3}$

As we know $x + \frac{1}{x} \geq 2$ or $x + \frac{1}{x} \leq -2$

But $x + \frac{1}{x} = v = 1, -\frac{1}{3}$

So, no real solution of the given equation is possible.

HINT:

(1) Convert given equation into quadratic equation by substituting $x + \frac{1}{x} = v$ and solve further.

(2) Use $x + \frac{1}{x} \in (-\infty, -2] \cup [2, \infty)$

8. Option (2) is correct.

Given $|\text{adj (adj (adj A))}| = 12^4$

$\Rightarrow |A|^{(n-1)^3} = 12^4$

$\Rightarrow |A|^{(2)^3} = 12^4$

$\Rightarrow |A|^8 = 12^4 \Rightarrow |A| = \sqrt{12}$

Now, $|A^{-1}\,\text{adj}A| = |A^{-1}|\,|\text{adj }A|$

$= \frac{1}{|A|}|A|^2$ $\qquad \{\because |\text{adj }A\} = |A|^{n-1};$ where n = order of matrix A$\}$

$= |A| = \sqrt{12} = 2\sqrt{3}$

9. Option (1) is correct.

Let $I = \int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9 - 4x^2}}dx$

As we know $\int \frac{1}{\sqrt{a^2 - (bx)^2}}dx = \frac{1}{b}\sin^{-1}\left(\frac{bx}{a}\right)$

$\Rightarrow I = \frac{48}{2}\left[\sin^{-1}\left(\frac{2x}{3}\right)\right]_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}}$

$\Rightarrow I = 24\left[\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right]$

$\Rightarrow I = 24\left[\frac{\pi}{3} - \frac{\pi}{4}\right] \Rightarrow I = 24\left[\frac{\pi}{12}\right] \Rightarrow I = 2\pi$

10. Option (4) is correct.

$$\begin{bmatrix} - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \end{bmatrix}$$

$\because$ Sum of all the elemetns in each row and in each column is 1

$\therefore$ In every row and every column there would be exactly one 1 and four zeroes.

So, number of required matrices

$= {}^5C_1 \times {}^4C_1 \times {}^3C_1 \times {}^2C_1 \times {}^1C_1$

$= 5 \times 4 \times 3 \times 2 \times 1 = 120$

HINT:

(1) In every row and every column there would be exactly one 1 and four zeroes.

(2) Recall multiplication principle of counting.

11. Option (4) is correct.

Given, $({}^{30}C_1)^2 + 2({}^{30}C_2)^2 + 3\,({}^{30}C_3)^2 + \ldots. + 30\,({}^{30}C_{30})^2$

$= \frac{\alpha 60!}{(30!)^2}$

Let $P = 0\,({}^{30}C_0)^2 + 1\,({}^{30}C_1)^2 + 2\,({}^{30}C_2)^2 + \ldots. + 30({}^{30}C_{30})^2$ ...(i)

$P = 30\,({}^{30}C_{30})^2 + 29\,({}^{30}C_{29})^2 + 28\,({}^{30}C_{28})^2 + \ldots. + 0({}^{30}C_0)^2$ ...(ii)

Adding equation (i) and equation (ii), we get

$2P = 30\left[\left({}^{30}C_0^2\right) + \left({}^{30}C_1^2\right) + \left({}^{30}C_2^2\right) + \ldots. + \left({}^{30}C_{30}^2\right)\right]$

As we know $\sum_{r=0}^{n}\left({}^nC_r\right)^2 = {}^{2n}C_n$

So, $P = 15\,{}^{60}C_{30}$

$\Rightarrow P = 15\,\frac{60!}{(30!)^2} \Rightarrow \alpha = 15$

12. Option (2) is correct.

Given planes $P_1 : x + (\lambda + 4)\,y + z = 1$

$P_2 : 2x + y + z = 2$

Equation of plane containing the line of intersection of the plane P_1 and P_2 is given by

$P : [x + (\lambda + 4)\,y + z - 1] + k\,[2x + y + z - 2] = 0$

$\because$ Plane P passes through (0, 1, 0)

$\Rightarrow \lambda + 4 - 1 + k\,(1 - 2) = 0$

$\Rightarrow \lambda - k + 3 = 0$ ...(i)

Plane P also passes through (1, 0, 1)

$\Rightarrow 1 + k\,(2 + 1 - 2) = 0$

$\Rightarrow k = -1$

Put the value of $k = -1$ in equation (i), we get

$\lambda = -4$

So, point $(2\lambda, \lambda, -\lambda) = (-8, -4, 4)$

Now, distance of (–8, –4, 4) from plane P_2 is

$d = \left|\frac{2(-8) - 4 + 4 - 2}{\sqrt{2^2 + 1^2 + 1^2}}\right|$

$\Rightarrow d = \left|\frac{-18}{\sqrt{6}}\right|$

$\Rightarrow d = 3\sqrt{6}$

HINT:

(1) Equation of plane containing the line of intersection of the plane P_1 and P_2 is given by $P_1 + \lambda P_2 = 0$.

(2) Perpendicular distance of point (x_1, y_1, z_1) from plane $ax + by + cz + d = 0$ is given by

$$D = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$$

13. Option (4) is correct.

Given: $f(x + y) = f(x).\, f(y)$

$\Rightarrow f(x) = p^x$ $\quad [\because f(1) = 3]$

$\Rightarrow p = 3$

So, $f(x) = 3^x$

Also given that $\sum_{k=1}^{n} f(k) = 3279$

$\Rightarrow f(1) + f(2) + ... + f(n) = 3279$

$\Rightarrow 3 + 3^2 + ... + 3^n = 3279$

$\Rightarrow \dfrac{3(3^n - 1)}{3 - 1} = 3279$

$\Rightarrow 3^n = 2187$

$\Rightarrow 3^n = 3^7$

$\Rightarrow n = 7$

14. Option (1) is correct.

Given $a_1, a_2, a_3, a_4, a_5, a_6$ are in A.P. and $a_1 + a_3 = 10$

And mean of $a_1, a_2, a_3, a_4, a_5, a_6 = \dfrac{19}{2}$

So, $\dfrac{a_1 + a_2 + a_3 + a_4 + a_5 + a_6}{6} = \dfrac{19}{2}$

$\Rightarrow a_1 + a_2 + a_3 + a_4 + a_5 + a_6 = 57$

Let common difference of AP be d.

So, $\dfrac{6}{2}(2a_1 + 5d) = 57$

$\{\because$ Sum of n terms of AP is given by $\dfrac{n}{2}[2a + (n-1)d]$

where a = first term and d = common difference$\}$

$\Rightarrow 2a_1 + 5d = 19$ ...(i)

$[\because a_1 + a_3 = 10]$

$\Rightarrow a_1 + a_1 + 2d = 10$

$\Rightarrow 2a_1 + 2d = 10$

$\Rightarrow a_1 + d = 5$ ...(ii)

On solving equation (i) and equation (ii), we get

$a_1 = 2$ and $d = 3$

Now, variance $= \sigma^2 = \dfrac{\Sigma x_i^2}{n} - (\bar{x})^2$

$\Rightarrow \sigma^2 = \dfrac{2^2 + 5^2 + 8^2 + 11^2 + 14^2 + 17^2}{6} - \left(\dfrac{19}{2}\right)^2$

$\Rightarrow \sigma^2 = \dfrac{699}{6} - \dfrac{361}{4}$

$\Rightarrow \sigma^2 = \dfrac{105}{4} \Rightarrow 8\sigma^2 = 210$

15. Option (4) is correct.

Given: Equation of AB : $(\lambda + 1)\, x + \lambda y = 4$

AC : $\lambda x + (1 - \lambda)\, y + \lambda = 0$

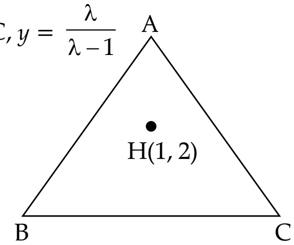
$\because$ Vertex A lies on y-axis

$\therefore$ x-coordinate of point A = 0

So, $x = 0$ will satisfy the equation of AB and AC

So, from equation of AB, $y = \dfrac{4}{\lambda}$

And from equation of AC, $y = \dfrac{\lambda}{\lambda - 1}$



So, $\dfrac{4}{\lambda} = \dfrac{\lambda}{\lambda - 1}$

$\Rightarrow 4\lambda - 4 = \lambda^2$

$\Rightarrow (\lambda - 2)^2 = 0$

$\Rightarrow \lambda = 2$

So, A = (0, 2)

Now, AB : $3x + 2y = 4$ and AC : $2x - y = -2$

Slope of AB, $m_{AB} = -\dfrac{3}{2}$

$\because$ $\mu(1, 2)$ is orthocentre of ΔABC

$\therefore m_{CH} \,.\, m_{AB} = -1$

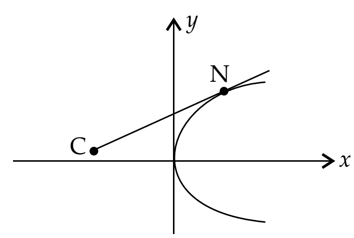
$\Rightarrow m_{CH} = \dfrac{2}{3}$

Let the coordinates of point C be (P, 2P + 2)

$\Rightarrow \dfrac{2P + 2 - 2}{P - 1} = \dfrac{2}{3}$

$\Rightarrow P = -\dfrac{1}{2}$

$\therefore C = \left(-\dfrac{1}{2}, 1\right)$



Given equation of parabola is $y^2 = 6x$

Now, equation of tangent to the parabola $y^2 = 6x$ in parametric form is given by $ty = x + \dfrac{3}{2}t^2$.

$\because$ Tangent is passing through $C\left(-\dfrac{1}{2}, 1\right)$

$\therefore t = -\dfrac{1}{2} + \dfrac{3}{2}t^2$

$\Rightarrow 3t^2 - 2t - 1 = 0$

$\Rightarrow (3t + 1)(t - 1) = 0 \Rightarrow t = 1$

So, coordinates of point of contact N = $(at^2, 2at)$

$= \left(\dfrac{3}{2}, 3\right)$

Now, NC $= \sqrt{\left(\dfrac{3}{2} + \dfrac{1}{2}\right)^2 + (3 - 1)^2}$

$\Rightarrow$ NC $= \sqrt{4 + 4} = 2\sqrt{2}$

16. Option (3) is correct.

As we know $A \Rightarrow B = \sim A \vee B$

So, $p \Rightarrow \sim q = \sim p \vee \sim q$

Now, $p \wedge (p \Rightarrow \sim q) = p \wedge (\sim p \vee \sim q)$

$= (p \wedge \sim p) \vee (p \wedge \sim q) = F \vee (p \wedge \sim q)$

Now, $\sim [p \wedge (p \Rightarrow \sim q)] = \sim [F \vee (p \wedge \sim q)]$

$= \sim F \wedge \sim (p \wedge \sim q) = T \wedge (\sim p \vee q) = \sim p \vee q$

HINT:

(1) Use $A \Rightarrow B = \sim A \vee B$

(2) Use $A \wedge (B \vee C) = (A \wedge B) \vee (A \wedge C)$

17. Option (4) is correct.

Given, $\lim_{x \to a}([x - 5] - [2x + 2]) = 0$

$\Rightarrow [a-5]-[2a+2]=0$

$\Rightarrow [a]-5-[2a]-2=0$

$\Rightarrow [a]-[2a]=7$ ...(i)

If $a \in z$, we have $a = -7$

For $a \in (-7.5, -7)$, $[a]-[2a] = -8+15 = 7$

So, $a \in (-7.5, -7)$ satisfy the given equation.

For $a \in (-7, -6.5)$, $[a]-[2a] = -7+14 = 7$

So, $a \in (-7, -6.5)$ satisfy the given equation

At $a = -7.5$

$[a]-[2a] = -8+15 = 7$

So, $a = -7.5$ satisfy the equation (i)

Now, at $a = -6.5$

$[a]-[2a] = -7+13 = 6$

So, $a = -6.5$ doesn't satisfy the equation (i)

$\because x \to a$

$\therefore a \neq -6.5$ or -7.5

So, $a \in (-7.5, -6.5)$

HINT:
Solve given limit using the definition of greatest integer function.

18. Option (4) is correct.

Let the normals of the plane $x + 2y + z = 0$ and $3y - z = 3$ be $\vec{n}_1$ & $\vec{n}_2$

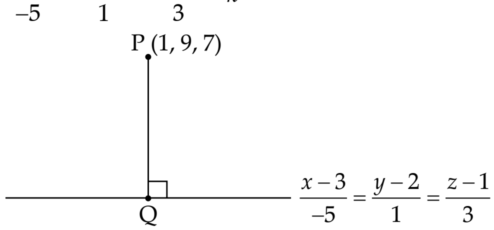
$\Rightarrow \vec{n}_1 = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{n}_2 = 3\hat{j} - \hat{k}$

And the direction ratio of the line = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{vmatrix}$

$= \hat{i}(-2-3) - \hat{j}(-1+0) + \hat{k}(3-0) = -5\hat{i} + \hat{j} + 3\hat{k}$

So the equation of the line passing through (3, 2, 1) is

$$\frac{x-3}{-5} = \frac{y-2}{1} = \frac{z-1}{3} = k$$



Let the coordinates of point Q be $(-5k+3, k+2, 3k+1)$

Now, direction ratios of PQ $= -5k+3-1, k+2-9, 3k+1-7$

$= -5k+2, \; k-7, 3k-6$

$\because$ PQ $\perp$ Line

So, $(-5k+2)(-5) + (k-7)(1) + (3k-6)3 = 0$

$\Rightarrow 35k = 35$

$\Rightarrow k = 1$

$\therefore$ Foot of perpendicular Q $= (-5+3, 1+2, 3+1)$
$= (-2, 3, 4)$

So, $\alpha + \beta + \gamma = -2 + 3 + 4 = 5$

19. Option (1) is correct.

Given digits : 3, 5, 6, 7, 8

All five digits number is greater than 7000

So, number of five digits number = 5! = 120

For 4 digits number greater than 7000

For 1000th place we can take only 7 or 8 from given digits and for remaining places we can take any digit from given digits.

So, number of 4 digits number greater than 7000

$= 2 \times 4 \times 3 \times 2 = 48$

$\therefore$ Number of integer, greater than 7000

$= 120 + 48 = 168$

HINT:
First find number of 5 digits numbers and then find 4 digit numbers of taking 7 or 8 on 1000th place using the fundamental principle of counting.

20. Option (1) is correct.

Let A = $\left(\dfrac{1+\sin\left(\frac{2\pi}{9}\right)+i\cos\left(\frac{2\pi}{9}\right)}{1+\sin\left(\frac{2\pi}{9}\right)-i\cos\left(\frac{2\pi}{9}\right)}\right)^3$

$\Rightarrow A = \left(\dfrac{1+\cos\left(\frac{\pi}{2}-\frac{2\pi}{9}\right)+i\sin\left(\frac{\pi}{2}-\frac{2\pi}{9}\right)}{1+\cos\left(\frac{\pi}{2}-\frac{2\pi}{9}\right)-i\sin\left(\frac{\pi}{2}-\frac{2\pi}{9}\right)}\right)^3$

$\Rightarrow A = \left(\dfrac{1+\cos\left(\frac{5\pi}{18}\right)+i\sin\left(\frac{5\pi}{18}\right)}{1+\cos\left(\frac{5\pi}{18}\right)-i\sin\left(\frac{5\pi}{18}\right)}\right)^3$

$\Rightarrow A = \left(\dfrac{2\cos^2\frac{5\pi}{36}+2i\sin\left(\frac{5\pi}{36}\right)\cos\left(\frac{5\pi}{36}\right)}{2\cos^2\frac{5\pi}{36}-2i\sin\left(\frac{5\pi}{36}\right)\cos\left(\frac{5\pi}{36}\right)}\right)^3$

$\Rightarrow A = \left(\dfrac{\cos\frac{5\pi}{36}+i\sin\frac{5\pi}{36}}{\cos\frac{5\pi}{36}-i\sin\frac{5\pi}{36}}\right)^3$

$\Rightarrow A = \left(\dfrac{e^{i\frac{5\pi}{36}}}{e^{-i\frac{5\pi}{36}}}\right)^3$

$\Rightarrow A = \left(e^{i\frac{5\pi}{18}}\right)^3 \Rightarrow A = e^{i\frac{5\pi}{6}}$

$\Rightarrow A = \cos\frac{5\pi}{6} + i\sin\left(\frac{5\pi}{6}\right)$

$\Rightarrow A = -\frac{\sqrt{3}}{2} + i\frac{1}{2}$

Section B

21. The correct answer is [384].

Given lines L_1: $\dfrac{x+\sqrt{6}}{2} = \dfrac{y-\sqrt{6}}{3} = \dfrac{z-\sqrt{6}}{4}$

$\Rightarrow L_1$: $\vec{r} = (-\sqrt{6}\hat{i} + \sqrt{6}\hat{j} + \sqrt{6}\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$

And L_2: $\dfrac{x-\lambda}{3} = \dfrac{y-2\sqrt{6}}{4} = \dfrac{z+2\sqrt{6}}{5}$

$\Rightarrow L_2$: $\vec{r} = (\lambda\hat{i} + 2\sqrt{6}\hat{j} - 2\sqrt{6}\hat{k}) + \mu(3\hat{i} + 4\hat{j} + 5\hat{k})$

As we know shortest distance between two lines

$\vec{r} = \vec{a} + \lambda\vec{p}$ and $\vec{r} = \vec{b} + \mu\vec{q}$ is given by

$$d = \left|\frac{(\vec{b}-\vec{a}).(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right|$$

So, $\vec{a} = -\sqrt{6}\hat{i} + \sqrt{6}\hat{j} + \sqrt{6}\hat{k}$
$\vec{b} = \lambda\hat{i} + 2\sqrt{6}\hat{j} - 2\sqrt{6}\hat{k}$

$\vec{p} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{q} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

Now, $\vec{b} - \vec{a} = (\lambda + \sqrt{6})\hat{i} + \sqrt{6}\hat{j} - 3\sqrt{6}\hat{k}$

$$\vec{p}\times\vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

$\Rightarrow \vec{p}\times\vec{q} = -\hat{i} + 2\hat{j} - \hat{k}$

$\Rightarrow |\vec{p}\times\vec{q}| = \sqrt{1+4+1} = \sqrt{6}$

Now, $(\vec{b}-\vec{a}).(\vec{p}\times\vec{q}) = -\lambda - \sqrt{6} + 2\sqrt{6} + 3\sqrt{6} = -\lambda + 4\sqrt{6}$

So, shortest distance $= \left|\frac{-\lambda + 4\sqrt{6}}{\sqrt{6}}\right| = 6$

$\Rightarrow |-\lambda + 4\sqrt{6}| = 6\sqrt{6}$

$\Rightarrow -\lambda + 4\sqrt{6} = \pm 6\sqrt{6}$

$\Rightarrow \lambda = 4\sqrt{6} \mp 6\sqrt{6}$

$\Rightarrow \lambda = -2\sqrt{6}, 10\sqrt{6}$

Sum of all possible values of $\lambda = -2\sqrt{6} + 10\sqrt{6} = 8\sqrt{6}$

$\therefore (8\sqrt{6})^2 = 384$

22. The correct answer is [432].

Given, Urn A contains 4 Red, 6 Black

Urn B contains 5 Red, 5 Black

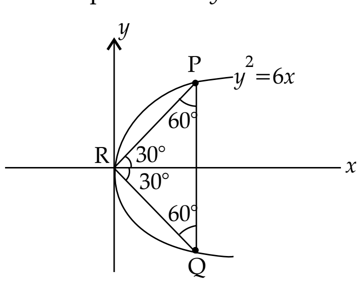
Urn C contains λ Red, 4 Black

Also P(Red ball from urn C) = 0.4

$$\Rightarrow \frac{\frac{1}{3}\times\frac{\lambda}{\lambda+4}}{\frac{1}{3}\times\frac{4}{10} + \frac{1}{3}\times\frac{5}{10} + \frac{1}{3}\times\frac{\lambda}{\lambda+4}} = \frac{4}{10}$$

$$\Rightarrow \frac{\frac{\lambda}{\lambda+4}}{\frac{2}{10} + \frac{\lambda}{\lambda+4}} = \frac{4}{10} \Rightarrow 24\lambda = 144 \Rightarrow \lambda = 6$$

So, equation of parabola is $y^2 = 6x$



Let parametric coordinates of point P be $\left(\frac{3}{2}t^2, 3t\right)$

Now, slope of PR = tan30°

$$\Rightarrow \frac{3t}{\frac{3}{2}t^2} = \frac{1}{\sqrt{3}} \Rightarrow t = 2\sqrt{3}$$

$\therefore$ Coordinates of P $= (18, 6\sqrt{3})$

Now, PR $= \sqrt{(18)^2 + (6\sqrt{3})^2}$

$\Rightarrow$ PR $= \sqrt{432} \Rightarrow (\text{PR})^2 = 432$

23. The correct answer is [2].

Given: S = {θ ∈[0, 2 λ); tan (π cos θ) + tan (π sin θ) = 0}

So, tan (π cos θ) = – tan (π sin θ)

⇒ tan (π cos θ) = tan (–π sin θ)

As we know if tan θ = tan α, then θ = $n\pi$ + α; n ∈ I

∴ π cos θ = $n\pi$ – π sin θ; n ∈ I

⇒ π cos θ+ π sin θ = $n\pi$

⇒ cos θ + sin θ = n

Since, $-\sqrt{2} \le \cos\theta + \sin\theta \le \sqrt{2}$

∴ n = –1, 0, 1

Case 1: If n = –1

cos θ + sin θ = –1

$$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$$

$$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \cos\left(\frac{3\pi}{4}\right)$$

$$\Rightarrow \theta - \frac{\pi}{4} = 2k\pi \pm \frac{3\pi}{4} \Rightarrow \theta = \pi, \frac{3\pi}{2}$$

Case-2: If n = 0

cos θ + sin θ = 0

$$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = 0$$

$$\Rightarrow \theta - \frac{\pi}{4} = 2k\pi \pm \frac{\pi}{2} \Rightarrow \theta = \frac{3\pi}{4}, \frac{7\pi}{4}$$

Case-3: If n = 1

$$\cos\theta + \sin\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right)$$

$$\Rightarrow \theta - \frac{\pi}{4} = 2k\pi \pm \frac{\pi}{4} \Rightarrow \theta = \frac{\pi}{2}, 0$$

$$\therefore \theta = \left\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \frac{3\pi}{4}, \frac{7\pi}{4}\right\}$$

$$\text{So, } \sin\left(\theta + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$$

$$\text{Now, } \sum_{\theta\in S} \sin^2\left(\theta + \frac{\pi}{4}\right) = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 2$$

HINT:

(1) General solution of tanθ = tanα is θ= $n\pi$ + α; $\alpha \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ n ∈ I

(2) Use $-\sqrt{a^2+b^2} \le a\sin x + b\cos x \le \sqrt{a^2+b^2}$

24. The correct answer is [5].

Given: $\frac{1^3+2^3+3^3+\text{.... upto } n \text{ terms}}{1.3+2.5+3.7+\text{....upto } n \text{ terms}}=\frac{9}{5}$

As we know, sum of cubes of n natural numbers

$$=\left\{\frac{n(n+1)}{2}\right\}^2$$

$$\Rightarrow \frac{\left\{\frac{n(n+1)}{2}\right\}^2}{\sum_{x=1}^{n} x(2x+1)}=\frac{9}{5}$$

$$\Rightarrow \frac{\frac{n^2(n+1)^2}{4}}{\sum_{x=1}^{n}(2x^2+x)}=\frac{9}{5}$$

$$\Rightarrow \frac{\frac{n^2(n+1)^2}{4}}{2\sum_{x=1}^{n}x^2+\sum_{x=1}^{n}x}=\frac{9}{5}$$

$$\Rightarrow \frac{\frac{n^2(n+1)^2}{4}}{2\left\{\frac{n(n+1)(2n+1)}{6}\right\}+\left\{\frac{n(n+1)}{2}\right\}}=\frac{9}{5}$$

{∵Sum of squares of n natural numbers $=\frac{n(n+1)(2n+1)}{6}$ and sum of n natural numbers $=\frac{n(n+1)}{2}$}

$$\Rightarrow \frac{\frac{n^2(n+1)^2}{4}}{\frac{n(n+1)(2n+1)}{3}+\frac{n(n+1)}{2}}=\frac{9}{5}$$

$$\Rightarrow \frac{\frac{n^2(n+1)^2}{4}}{n(n+1)\left\{\frac{2n+1}{3}+\frac{1}{2}\right\}}=\frac{9}{5}$$

$$\Rightarrow \frac{\frac{n(n+1)}{4}}{\frac{(4n+2+3)}{6}}=\frac{9}{5}\Rightarrow\frac{3n(n+1)}{2(4n+5)}=\left(\frac{9}{5}\right)$$

$\Rightarrow 5n^2+5n=24n+30$

$\Rightarrow n=5$

25. The correct answer is [405].

Given: Sum of coefficients of first 3 terms of $\left(x-\frac{3}{x^2}\right)^n=376$

General term of given expansion is

$$T_{r+1}={}^nC_r\,x^{n-r}\left(\frac{-3}{x^2}\right)^r.$$

So, coefficients of first three terms are ${}^nC_0$, $-3\,{}^nC_1$, $9\,{}^nC_2$

$\therefore {}^nC_0-3\,{}^nC_1+9\,{}^nC_2=376$

$$\Rightarrow 1-3n+\frac{9n(n-1)}{2}=376$$

$\Rightarrow n=10,\ \frac{-25}{3}$ (not possible)

For coefficient of x^4, $n-3r=4$

$\Rightarrow 10-3r=4$

$\Rightarrow r=2$

$\therefore$ Coefficient of $x^4={}^{10}C_2(-3)^2=\frac{10\times9}{2\times1}\times9=405$

> **HINT:**
> General term of binomial expansion $(a+b)^n$ is given by $T_{r+1}={}^nC_r\,a^{n-r}b^r$

26. The correct answer is [122].

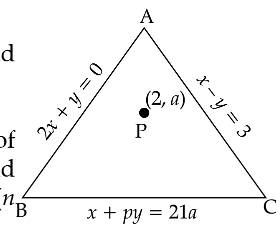
Given equation of sides are

AB : $2x+y=0$ ...(i)

BC : $x+py=21a$ ...(ii)

CA : $x-y=3$ ...(iii)

Centroid of ΔABC = P $(2, a)$



On solving equation (i) and equation (iii), we get

A = (1, –2)

Let the coordinates of point B be $(m, -2m)$ and coordinates of point C be $(n+3, n)$

∵Centroid of Δ ABC = $(2, a)$

$$\Rightarrow \frac{m+n+3+1}{3}=2 \text{ and } \frac{n-2m-2}{3}=a$$

$\Rightarrow m+n=2$ ...(iv)

& $n-2m=3a+2$ ...(v)

Put $n=2-m$ from equation (iv) to (v), we get

$m=-a$

Point B satisfy the equation of BC

So, $m-2mp=21a$

$\Rightarrow m(1-2p)=21a$

$\Rightarrow 2p-1=21$

$\Rightarrow p=11$

Point C also satisfy the equation of BC

So, $n+3+p(n)=21a$

$\Rightarrow 12n+3=-21m$

$\Rightarrow 12n+21m+3=0$ ...(vi)

On solving equation (iv) and equation (vi), we get

$m=-3, n=5$

$\therefore$ B = (–3, 6) and C = (8, 5)

Now, BC $=\sqrt{(11)^2+1^2}$

$\Rightarrow$ BC $=\sqrt{122}$

$\Rightarrow BC^2=122$

27. The correct answer is [8].

Given: $\vec{a}=\hat{i}+2\hat{j}+\lambda\hat{k}$

$\vec{b}=3\hat{i}-5\hat{j}-\lambda\hat{k}$

$\vec{a}.\vec{c} = 7$

$2\vec{b}.\vec{c} + 43 = 0$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{c}$

$\because \vec{a} \times \vec{c} = \vec{b} \times \vec{c}$

$\Rightarrow (\vec{a} - \vec{b}) \times \vec{c} = 0$

$\Rightarrow (\vec{a} - \vec{b}) \,||\, \vec{c}$

$\Rightarrow \vec{c} = k(\vec{a} - \vec{b}) \Rightarrow \vec{c} = k(-2\hat{i} + 7\hat{j} + 2\lambda\hat{k})$

$\because \vec{a}.\vec{c} = 7$

$\Rightarrow k(-2 + 14 + 2\lambda^2) = 7$

$\Rightarrow k(2\lambda^2 + 12) = 7$ ...(i)

Also, $2\vec{b}.\vec{c} = -43$

$\Rightarrow 2k(-6 - 35 - 2\lambda^2) = -43$

$\Rightarrow 2k(-41 - 2\lambda^2) = -43$ ...(ii)

From equation (i) and equation (ii), we get

$$\frac{2\lambda^2 + 12}{2(41 + 2\lambda^2)} = \frac{7}{43}$$

$\Rightarrow 43(\lambda^2 + 6) = 7(2\lambda^2 + 41)$

$\Rightarrow 29\lambda^2 = 29$

$\Rightarrow \lambda^2 = 1$

Now, $\vec{a}.\vec{b} = 3 - 10 - \lambda^2 = -8$

$\Rightarrow |\vec{a}.\vec{b}| = 8$

28. The correct answer is [13].

Given: Relation R = {(*a*, *b*), (*b*, *c*), (*b*, *d*)} on set {*a*, *b*, *c*, *d*} for a relation to be equivalence relation, it must be reflexive, symmetric and transitive.

For reflexive relation, (*a*, *a*), (*b*, *b*), (*c*, *c*), (*d*, *d*) must be added in relation R.

So, R = {(*a*, *a*), (*b*, *b*), (*c*, *c*), (*d*, *d*), (*a*, *b*), (*b*, *c*), (*b*, *d*)}

For symmetric relation, if $(x, y) \in R \Rightarrow (y, x) \in R$

Now, as $(a, b) \in R \Rightarrow (b, a) \in R$

and $(b, c) \in R \Rightarrow (c, b) \in R$

and $(b, d) \in R \Rightarrow (d, b) \in R$

So, R = {(*a*, *a*), (*b*, *b*), (*c*, *c*), (*d*, *d*), (*a*, *b*), (*b*, *c*), (*b*, *d*), (*b*, *a*), (*c*, *b*), (*d*, *b*)}

For transitive relation, if $(x, y) \in R$ and $(y, z) \in R$ $\Rightarrow (x, z) \in R$

So, R = {(*a*, *a*), (*b*, *b*), (*c*, *c*), (*d*, *d*), (*a*, *b*), (*b*, *c*), (*b*, *d*), (*b*, *a*), (*c*, *b*), (*d*, *b*), (*a*, *c*), (*a*, *d*), (*c*, *a*), (*c*, *d*), (*d*, *c*), (*d*, *a*)}

So, total number of elements added = 13

29. The correct answer is [36].

Given curves $y^2 - 2y = -x$ and $x + y = 0$

Now, $y^2 - 2y + 1 = -x + 1$

$\Rightarrow (y - 1)^2 = -(x - 1)$ ...(i)

Let's find intersecting points of both curves

$y^2 - 2y - y = 0$

$\Rightarrow y^2 - 3y = 0$

$\Rightarrow y = 0, 3$

$\Rightarrow x = 0, -3$

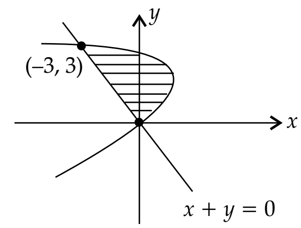
$\therefore$ Intersecting points are (0, 0) and (–3, 3)

So, required area $= \int_0^3 \{(2y - y^2) - (-y)\}dy$

$\Rightarrow A = \int_0^3 (3y - y^2)dy$

$\Rightarrow A = \left[\frac{3y^2}{2} - \frac{y^3}{3}\right]_0^3$

$\Rightarrow 8A = 36$



HINT:

Draw the figure of both curves and identify the bounded region and use the concept of vertical strip and solve further.

30. The correct answer is [27].

Given: $f(x) + \int_0^x f(t)\sqrt{1 - (\log_e f(t))^2}\,dt = e, \forall x \in \left[0, \frac{\pi}{2}\right]$ ...(i)

Differentiate the above equation, we get

$f'(x) + f(x)\sqrt{1 - [\log_e f(x)]^2} = 0$

$\Rightarrow \frac{dy}{dx} + y\sqrt{1 - (\log_e y)^2} = 0$

$\Rightarrow \frac{dy}{dx} = -y\sqrt{1 - \log_e^2 y}$

$\Rightarrow \int \frac{dy}{y\sqrt{1 - \log_e^2 y}} = \int -1\,dx$

Let $\log_e y = u$

$\Rightarrow \frac{1}{y}dy = du \Rightarrow \int \frac{du}{\sqrt{1 - u^2}} = -x + c$

$\Rightarrow \sin^{-1} u = -x + c$

$\Rightarrow \sin^{-1} \log_e y = -x + c$

Put $x = 0$ in equation (i), we get

$f(0) = e$ i.e., $y(0) = e$

So, at $x = 0$, $\sin^{-1}(1) = c$

$\Rightarrow c = \frac{\pi}{2}$

$\therefore \sin^{-1} \log_e y = -x + \frac{\pi}{2}$

$\Rightarrow \log_e y = \sin\left(\frac{\pi}{2} - x\right) \Rightarrow \log_e y = \cos x$

At $x = \frac{\pi}{6}, \log_e f\left(\frac{\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

So, $\left[6\log_e f\left(\frac{\pi}{6}\right)\right]^2 = \left[6 \times \frac{\sqrt{3}}{2}\right]^2 = 27$

❑❑