JEE (Main) MATHEMATICS SOLVED PAPER

2023
30th Jan. Shift 1

Section A

Q. 1. A straight line cuts off the intercepts OA = a and OB = b on the positive directions of x-axis and y axis respectively. If the perpendicular from origin O to this line makes an angle of $\frac{\pi}{6}$ with positive direction of y-axis and the area of ΔOAB is $\frac{98}{3}\sqrt{3}$, then $a^2 - b^2$ is equal to:

(1) $\frac{392}{3}$ **(2)** $\frac{196}{3}$ **(3)** 98 **(4)** 196

Q. 2. The minimum number of elements that must be added to the relation R = {(a, b), (b, c)} on the set {a, b, c} so that is becomes symmetric and transitive is :

(1) 3 **(2)** 4 **(3)** 5 **(4)** 7

Q. 3. If an unbiased die, marked with −2, −1, 0, 1, 2, 3 on its faces, is thrown five times, then the probability that the product of the outcomes is positive, is :

(1) $\frac{881}{2592}$ **(2)** $\frac{27}{288}$ **(3)** $\frac{440}{2592}$ **(4)** $\frac{521}{2592}$

Q. 4. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors and $\hat{n}$ is a unit vector perpendicular to $\vec{c}$ such that $\vec{a} = \alpha\vec{b} - \hat{n}$, $(\alpha \neq 0)$ and $\vec{b}.\vec{c} = 12$ then $|\vec{c} \times (\vec{a} \times \vec{b})|$ is equal to:

(1) 9 **(2)** 15 **(3)** 6 **(4)** 12

Q. 5. Among the statements :

(S1) $((p \vee q) \Rightarrow r) \Leftrightarrow (p \Rightarrow r)$

(S2) $((p \vee q) \Rightarrow r) \Leftrightarrow ((p \Rightarrow r) \vee (q \Rightarrow r))$

(1) only (S2) is a tautology

(2) only (S1) is a tautology

(3) neither (S1) nor (S2) is a tautology

(4) both (S1) and (S2) are tautologies

Q. 6. If P(h, k) be a point on the parabola $x = 4y^2$, which is nearest to the point Q (0, 33), then the distance of P from the directrix of the parabola $y^2 = 4(x + y)$ is equal to :

(1) 2 **(2)** 6 **(3)** 8 **(4)** 4

Q. 7. Let $y = x + 2, 4y = 3x + 6$ and $3y = 4x + 1$ be three tangent lines to the circle $(x - h)^2 + (y - k)^2 = r^2$. Then $h + k$ is equal to :

(1) $5(1+\sqrt{2})$ **(2)** $5\sqrt{2}$

(3) 6 **(4)** 5

Q. 8. The number of points on the curve $y = 54x^5 - 135x^4 - 70x^3 + 180x^2 + 210x$ at which the normal lines are parallel to $x + 90y + 2 = 0$ is :

(1) 4 **(2)** 2 **(3)** 0 **(4)** 3

Q. 9. If $a_n = \frac{-2}{4n^2 - 16n + 15'}$ $a_1 + a_2 + \ldots + a_{25}$ is equal to:

(1) $\frac{52}{147}$ **(2)** $\frac{49}{138}$ **(3)** $\frac{50}{141}$ **(4)** $\frac{51}{144}$

Q. 10. If $\tan 15° + \frac{1}{\tan 75°} + \frac{1}{\tan 105°} + \tan 195° = 2a$, then the value of $\left(a + \frac{1}{a}\right)$ is :

(1) 2 **(2)** $4 - 2\sqrt{3}$

(3) $5 - \frac{3}{2}\sqrt{3}$ **(4)** 4

Q. 11. If the solution of the equation $\log_{\cos x} \cot x + 4\log_{\sin x} \tan x = 1$, $x \in \left(0, \frac{\pi}{2}\right)$, is $\sin^{-1}\left(\frac{\alpha + \sqrt{\beta}}{2}\right)$, where α and β are integers, then α + β is equal to:

(1) 5 **(2)** 6 **(3)** 4 **(4)** 3

Q. 12. Let the system of linear equations

$x + y + kz = 2$

$2x + 3y - z = 1$

$3x + 4y + 2z = k$

have infinitely many solutions. Then the system

$(k + 1)x + (2k - 1)y = 7$

$(2k + 1)x + (k + 5)y = 10$

has:

(1) infinitely many solutions

(2) unique solution satisfying $x - y = 1$

(3) unique solution satisfying $x + y = 1$

(4) no solution

Q. 13. The line l_1 passes through the point (2,6,2) and is perpendicular to the plane $2x + y - 2z = 10$. Then the shortest distance between the line l_1 and the line $\frac{x+1}{2} = \frac{y+4}{-3} = \frac{z}{2}$ is:

(1) $\frac{13}{3}$ **(2)** $\frac{19}{3}$ **(3)** 7 **(4)** 9

Q. 14. Let A = $\begin{pmatrix} m & n \\ p & q \end{pmatrix}$, d = |A| ≠ 0 and |A – d(Adj A)| = 0. Then

(1) $1 + d^2 = m^2 + q^2$ **(2)** $1 + d^2 = (m + q)^2$

(3) $(1 + d)^2 = m^2 + q^2$ **(4)** $(1 + d)^2 = (m + q)^2$

Q. 15. If [t] denotes the greatest integer ≤ t, then the value of $\frac{3(e-1)}{e}\int_1^2 x^2 e^{[x]+[x^3]} dx$ is:

(1) $e^8 - 1$ **(2)** $e^7 - 1$ **(3)** $e^8 - e$ **(4)** $e^9 - e$

Q. 16. Let a unit vector $\widehat{OP}$ make angles α, β, γ with the positive directions of the co-ordinate axes OX, OY, OZ respectively, where $\beta \in \left(0, \frac{\pi}{2}\right)$. If $\widehat{OP}$ is perpendicular to the plane through points (1,2,3), (2,3,4) and (1,5,7), then which one of the following is true ?

(1) $\alpha \in \left(0, \frac{\pi}{2}\right)$ and $\gamma \in \left(0, \frac{\pi}{2}\right)$

(2) $\alpha \in \left(0, \frac{\pi}{2}\right)$ and $\gamma \in \left(\frac{\pi}{2}, \pi\right)$

(3) $\alpha \in \left(\frac{\pi}{2}, \pi\right)$ and $\gamma \in \left(\frac{\pi}{2}, \pi\right)$

(4) $\alpha \in \left(\frac{\pi}{2}, \pi\right)$ and $\gamma \in \left(0, \frac{\pi}{2}\right)$

Q. 17. The coefficient of x^{301} in $(1 + x)^{500} + x(1 + x)^{499} + x^2(1 + x)^{498} + ... + x^{500}$ is:

(1) ${}^{500}C_{300}$ **(2)** ${}^{501}C_{200}$ **(3)** ${}^{501}C_{302}$ **(4)** ${}^{500}C_{301}$

Q. 18. Let the solution curve $y = y(x)$ of the differential equation

$$\frac{dy}{dx} - \frac{3x^5 \tan^{-1}(x^3)}{(1+x^6)^{\frac{3}{2}}} y = 2x \exp\left\{\frac{x^3 - \tan^{-1} x^3}{\sqrt{(1+x^6)}}\right\}$$

pass through the origin. Then $y(1)$ is equal to :

(1) $\exp\left(\frac{4+\pi}{4\sqrt{2}}\right)$ **(2)** $\exp\left(\frac{1-\pi}{4\sqrt{2}}\right)$

(3) $\exp\left(\frac{\pi-4}{4\sqrt{2}}\right)$ **(4)** $\exp\left(\frac{4-\pi}{4\sqrt{2}}\right)$

Q. 19. If the coefficient of x^{15} in the expansion of $\left(ax^3 + \frac{1}{bx^{1/3}}\right)^{15}$ is equal to the coefficient of x^{-15} in the expansion of $\left(ax^{1/3} - \frac{1}{bx^3}\right)^{15}$, where a and b are positive real numbers, then for each such ordered pair (a, b) :

(1) $ab = 3$ **(2)** $ab = 1$

(3) $a = b$ **(4)** $a = 3b$

Q. 20. Suppose $f : R \to (0, \infty)$ be a differentiable function such that $5f(x + y) = f(x) . f(y), \forall\ x, y \in R$. If $f(3) = 320$, then $\sum_{n=0}^{5} f(n)$ is equal to :

(1) 6875 **(2)** 6525 **(3)** 6825 **(4)** 6575

Section B

Q. 21. Let $z = 1 + i$ and $z_1 = \frac{1 + i\bar{z}}{\bar{z}(1-z) + \frac{1}{z}}$. Then $\frac{12}{\pi} \arg(z_1)$ is equal to

Q. 22. If $\lambda_1 < \lambda_2$ are two values of λ such that the angle between the planes P_1: $\vec{r}.(3\hat{i} - 5\hat{j} + \hat{k}) = 7$ and P_2: $\vec{r}.(\lambda\hat{i} + \hat{j} - 3\hat{k}) = 9$ is $\sin^{-1}\left(\frac{2\sqrt{6}}{5}\right)$. then the square of the length of perpendicular from the point $(38\lambda_1, 10\lambda_2, 2)$ to the plane P_1 is

Q. 23. Let α be the area of the larger region bounded by the curve $y^2 = 8x$ and the lines $y = x$ and $x = 2$, which lies in the first quadrant. Then the value of 3α is equal to

Q. 24. Let $\sum_{n=0}^{\infty} \frac{n^3((2n)!) + (2n-1)(n!)}{(n!)((2n)!)} = ae + \frac{b}{e} + c$, where $a, b, c \in Z$ and $e = \sum_{n=0}^{\infty} \frac{1}{n!}$ Then $a^2 - b + c$ is equal to

Q. 25. If the equation of the plane passing through the point (1,1,2) and perpendicular to the line $x - 3y + 2z - 1 = 0 = 4x - y + z$ is $Ax + By + Cz = 1$, then 140(C − B + A) is equal to

Q. 26. Number of 4-digit numbers (the repetition of digits is allowed) which are made using the digits 1, 2, 3 and 5 and are divisible by 15 , is equal to

Q. 27. Let $f'(x) = \frac{3x+2}{2x+3}, x \in R - \left\{\frac{-3}{2}\right\}$

For $n \geq 2$, define $f^n(x) = f^1$ of $f^{n-1}(x)$

If $f^5(x) = \frac{ax+b}{bx+a}, \gcd(a,b) = 1$, then $a + b$ is equal to

Q. 28. The mean and variance of 7 observations are 8 and 16 respectively. If one observation 14 is omitted and a and b are respectively mean and variance of remaining 6 observations, then $a + 3b - 5$ is equal to

Q. 29. Let S = {1,2,3,4,5,6}. Then the number of one-one functions f: S → P(S), where P(S) denote the power set of S, such that $f(n) \subset f(m)$ where $n < m$ is

Q. 30. $\lim_{x \to 0} \frac{48}{x^4} \int_0^x \frac{t^3}{t^6+1} dt$ is equal to

Answer Key

Q. No.	Answer	Topic Name	Chapter Name
1	(1)	Straight line	Coordinate geometry
2	(4)	Equivalence relation	Relations and functions
3	(4)	General rule	Probability
4	(4)	Vector triple product	Vector algebra

Q. No.	Answer	Topic Name	Chapter Name
5	(3)	Tautology	Mathematical Reasoning
6	(2)	Maxima and Minima	Application of derivative
7	(4)	Circle	Conic Section
8	(1)	Application of derivative	Coordinate Geometry
9	(3)	Special series	Sequences and series
10	(4)	Trigonometric values	Trigonometry
11	(3)	Trigonometric equation	Trigonometry
12	(3)	Ssystem of equations	Matrices and Determinants
13	(4)	Shortest distance between lines	Three dimensional geometry
14	(4)	Adjoint	Matrices and Determinants
15	(3)	Definite integral	Integral Calculus
16	(3)	Direction ratios	Three dimensional geometry
17	(2)	Coefficient of a term	Binomial theorem
18	(4)	Linear differential equation	Differential equations
19	(2)	Coefficient of a term	Binomial theorem
20	(3)	Differentiable function	Differentiability
21	[9]	Argument	Complex numbers
22	[315]	Angle between the planes	Three dimensional geometry
23	[22]	Area between the curves	Integral calculus
24	[26]	Special series	Sequences and series
25	[15]	Equation of plane	Three dimensional geometry
26	[21]	General rule	Permutation and combination
27	[3125]	Differentiable function	Differentiability
28	[37]	Mean, variance	Statistics
29	[3240]	Number of one one fuctions	Relations and functions
30	[12]	Leibnitz rule	Integral Calculus

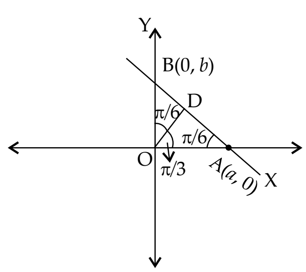
Solutions

Section A

1. Option (1) is correct.

In ΔODA, $\angle OAD = 180° - \left(\frac{\pi}{2}+\frac{\pi}{3}\right) = \frac{\pi}{6}$

$\therefore \tan\frac{\pi}{6} = \frac{OB}{OA}$



$\Rightarrow \frac{1}{\sqrt{3}} = \frac{b}{a}$

$\Rightarrow a = \sqrt{3}b$ ...(i)

Area of $\Delta AOB = \frac{1}{2}ab$

$\Rightarrow \frac{98}{3}\sqrt{3} = \frac{1}{2}ab$

$\Rightarrow 2\times\frac{98}{3}\sqrt{3} = \sqrt{3}b^2$ [from (i)]

$\Rightarrow b^2 = \frac{196}{3}$ ...(ii)

From (i) $\Rightarrow a^2 = 3b^2$

$a^2 = 3\times\frac{196}{3} = 196$

Now, $a^2 - b^2 = 196 - \frac{196}{3} = \frac{2\times 196}{3} = \frac{392}{3}$

2. Option (4) is correct.

For symmetric

Given that $(a, b) \in R$ then $(b, a) \in R$ and $(b, c) \in R$ then $(c, b) \in R$

So, for symmetric, we added (b, a) and (c, b)

For transitive

Since, $(a, b) \in R$ and $(b, a) \in R \Rightarrow (a, a) \in R$

$(b, a) \in R$ and $(a, b) \in R \Rightarrow (b, b) \in R$

$(c, b) \in R$ and $(b, c) \in R \Rightarrow (c, c) \in R$

$(c, b) \in R$ and $(b, a) \in R \Rightarrow (c, a) \in R$

$\because (c, a) \in R$ then $(a, c) \in R$

So, minimum number of elements = 7.

3. Option (4) is correct.

Total possible outcomes = 6^5

Product of otucomes is positive if

Case 1: All numbers are positive

P (Product of the outcomes is positive)

$= {}^5C_5 \left(\frac{3}{6}\right)^5$

Case 2: 3 times positive and 2 times negative
P (product of the outcomes is positive)

$= {}^5C_3 \left(\frac{3}{6}\right)^3 \left(\frac{2}{6}\right)^2$

Case 3: 1 time positive and 4 times negative

$= {}^5C_1 \left(\frac{3}{6}\right)\left(\frac{2}{6}\right)^4$

$\therefore$ Required probability $= {}^5C_5\left(\frac{3}{6}\right)^5 + {}^5C_3\left(\frac{3}{6}\right)^3\left(\frac{2}{6}\right)^2 + {}^5C_1\left(\frac{3}{6}\right)\left(\frac{2}{6}\right)^4$

$= \left(\frac{3}{6}\right)^5 + 10\times\frac{27\times 4}{6^5} + 5.\frac{3\times 16}{6^5}$

$= \frac{1563}{6^5} = \frac{521}{2592}$

4. Option (4) is correct.

Given that $|\hat{n}| = 1, \vec{a} = \alpha\vec{b} - \hat{n}, \vec{b}.\vec{c} = 12$ and $\vec{c}.\vec{n} = 0$

Now, $\vec{c}\times(\vec{a}\times\vec{b}) = (\vec{c}.\vec{b})\vec{a} - (\vec{c}.\vec{a})\vec{b}$

$= 12\vec{a} - [\vec{c}.(\alpha\vec{b} - \hat{n})]\vec{b}$

$= 12\vec{a} - [\alpha(\vec{c}.\vec{b}) - (\vec{c}.\hat{n})]\vec{b}$

$= 12\vec{a} - [12\alpha - 0]\vec{b}$

$= 12\vec{a} - 12\alpha\vec{b} = 12[\vec{a} - \alpha\vec{b}]$ ...(i)

$\because \vec{a} = \alpha\vec{b} - \hat{n} \Rightarrow \vec{a} - \alpha\vec{b} = -\hat{n}$

$\Rightarrow |\vec{a} - \alpha\vec{b}| = 1$ ...(ii)

So, $|\vec{c}\times(\vec{a}\times\vec{b})| = 12|\vec{a} - \alpha\vec{b}|$ [From (i)]
$= 12$ [From (ii)]

5. Option (3) is correct.

For, S_1: $((p \vee q \Rightarrow r) \Leftrightarrow (p \Rightarrow r)$

p	q	r	$p \vee q$	$(p \vee q) \Rightarrow r$	$p \Rightarrow r$	S_1
T	T	T	T	T	T	T
T	T	F	T	F	F	T
T	F	T	T	T	T	T
T	F	F	T	F	F	T
F	T	T	T	T	T	T
F	T	F	T	F	T	F
F	F	T	F	T	T	T
F	F	F	F	T	T	T

$\therefore S_1$ is not a tautology.

For S_2: $((p \vee q) \Rightarrow r) \Leftrightarrow ((p \Rightarrow r) \vee (q \Rightarrow r))$

p	q	r	$p \vee q$	$p \vee q \Rightarrow r$	A: $p \Rightarrow r$	B: $q \Rightarrow r$	$A \vee B$	S_2
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	T	T	T	T	T	T
T	F	F	T	F	F	T	T	F
F	T	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T	F
F	F	T	F	T	T	T	T	T
F	F	F	F	T	T	T	T	T

$\therefore S_2$ is not a tautology.

6. Option (2) is correct.

Since P (h, k) lies on $x = 4y^2$

$\therefore h = 4k^2$ ...(i)

$PQ = \sqrt{(h-0)^2 + (k-33)^2} = \sqrt{h^2 + k^2 - 66k + (33)^2}$

$PQ^2 = 16k^4 + k^2 - 66k + 1089$

$\Rightarrow \frac{d(PQ^2)}{dk} = 64k^3 + 2k - 66 = 0$

$\Rightarrow (k-1)(64k^2 + 64k + 66) = 0$

$\Rightarrow k - 1 = 0 \Rightarrow k = 1$

$\frac{d^2(PQ^2)}{dk^2} = 192k^2 + 2$

So, $\left(\frac{d^2PQ^2}{dk^2}\right)_{k=1} = 192 + 2 = 194 > 0$

$\therefore$ Distance is minimum when $k = 1$.

$\therefore h = 4k^2 = 4$

So, P(4, 1)

Given parabola is $y^2 = 4(x + y)$

$y^2 - 4y = 4x \Rightarrow (y-2)^4 = 4(x + 1)$

Directrix is $x + 1 = -1 \Rightarrow x = -2$

Distance of P(4, 1) from the direction $x = -2$

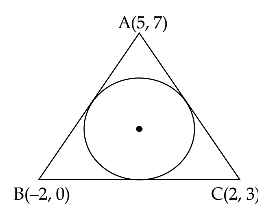
$= \frac{4+2}{\sqrt{1^2}} = 6$

7. Option (4) is correct.

Let AB : $y = x + 2$ ...(i)

BC : $4y = 3x + 6$ ...(ii)

AC : $3y = 4x + 1$ ...(iii)



On solving equations (i) and (ii)

we get $x = -2, y = 0$ i.e., B(–2, 0)

On solving equations (ii) and (iii)

we get $x = 2$ and $y = 3$ i.e., C(2, 3)

On solving equations (i) and (iii)

We get $x = 5$ and $y = 7$ i.e., A(5, 7)

$AB = \sqrt{(5+2)^2 + (7-0)^2} = \sqrt{49+49} = 7\sqrt{2}$

$BC = \sqrt{(2+2)^2 + (3-0)^2} = \sqrt{16+9} = 5$

$AC = \sqrt{(5-2)^2 + (7-3)^2} = \sqrt{9+16} = 5$

Incentre of triangle is (h, k)

$= \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$

$= \left(\frac{5(-2) + 2\times 7\sqrt{2} + 5\times 5}{5+5+7\sqrt{2}}, \frac{5\times 0 + 3\times 7\sqrt{2} + 5\times 7}{5+5+7\sqrt{2}}\right)$

$= \left(\frac{14\sqrt{2}+15}{10+7\sqrt{2}}, \frac{21\sqrt{2}+35}{10+7\sqrt{2}}\right)$

Now, $h + k = \frac{14\sqrt{2} + 15 + 21\sqrt{2} + 35}{10 + 7\sqrt{2}} = \frac{35\sqrt{2}+50}{10+7\sqrt{2}} = 5$

8. Option (1) is correct.

$\because y = 54x^5 - 135x^4 - 70x^3 + 180x^2 + 210x$

$\Rightarrow \frac{dy}{dx} = 270x^4 - 540x^3 - 210x^2 + 360x + 210$

Slope of normal = slope of $x + 90y + 2 = 0$

$\therefore 270x^4 - 540x^3 - 210x^2 + 360x + 210 = \frac{-1}{-\frac{1}{90}}$

$\Rightarrow 270x^4 - 540x^3 - 210x^2 + 360x + 210 = 90$

$\Rightarrow 270x^4 - 540x^3 - 210x^2 + 360x + 120 = 0$

$\Rightarrow 9x^4 - 18x^3 - 7x^2 + 12x + 4 = 0$

$\Rightarrow (x-1)(x-2)(3x+1)(3x+2) = 0$

$\Rightarrow x = 1, 2, -\frac{1}{3}, -\frac{2}{3}$

Numbers of points are 4.

9. Option (3) is correct.

Given that $a_n = \frac{-2}{4n^2 - 16n + 15} = \frac{-2}{(2n-3)(2n-5)}$

$= \frac{(2n-5)-(2n-3)}{(2n-3)(2n-5)} = \frac{1}{2n-3} - \frac{1}{2n-5}$

Now, $a_1 + a_2 + ... + a_{25}$

$= \frac{-1}{1} + \frac{1}{3} + \frac{1}{1} + \frac{1}{1} + \frac{1}{3} - \frac{1}{1} + \frac{1}{5} - \frac{1}{3} + \frac{1}{7} - \frac{1}{5} + ... + \frac{1}{47} - \frac{1}{45}$

$= \frac{1}{3} + \frac{1}{47} = \frac{47+3}{141} = \frac{50}{141}$

10. Option (4) is correct.

Given that $\tan 15° + \frac{1}{\tan 75°} + \frac{1}{\tan 105°} + \tan 195° = 2a$

$\Rightarrow \tan 15° + \cot 75° + \cot 105° + \tan 195° = 2a$

$\Rightarrow \tan 15° + \tan(90° - 75°) + \cot(90 + 15°) + \tan(180° + 15°) = 2a$

$\Rightarrow \tan 15° + \tan 15° - \tan 15° + \tan 15° = 2a$

$\Rightarrow 2\tan 15° = 2a \Rightarrow a = \tan 15°$

Now, $a + \frac{1}{a} = \tan 15° + \frac{1}{\tan 15°}$

$= \tan 15° + \cot 15° = 2 - \sqrt{3} + 2 + \sqrt{3}$

$= 4$

11. Option (3) is correct.

Given that $\log_{\cos x} \cot x + 4 \log_{\sin x} \tan x = 1$

$\Rightarrow \log_{\cos x}\left(\frac{\cos x}{\sin x}\right) + 4\log_{\sin x}\left(\frac{\sin x}{\cos x}\right) = 1$

$\Rightarrow \log_{\cos x} \cos x - \log_{\cos x} \sin x + 4 [\log_{\sin x} \sin x - \log_{\sin x} \cos x] = 1$

$\Rightarrow 1 - \log_{\cos x} \sin x + 4 - \frac{4}{\log_{\cos x} \sin x} = 1$

Let $\log_{\cos x} \sin x = t$

$\Rightarrow t + \frac{4}{t} = 4 \Rightarrow t^2 - 4t + 4 = 0$

$\Rightarrow (t-2)^2 = 0 \Rightarrow t = 2$

$\Rightarrow \log_{\cos x} \sin x = 2$

$\Rightarrow \sin x = \cos^2 x \Rightarrow \sin x = 1 - \sin^2 x$

$\Rightarrow \sin^2 x + \sin x - 1 = 0$

$\Rightarrow \sin x = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$

$\therefore \sin x = \frac{-1+\sqrt{5}}{2} \quad \left[\because x \in \left(0, \frac{\pi}{2}\right)\right]$

$x = \sin^{-1}\left(\frac{-1+\sqrt{5}}{2}\right)$

$\therefore \alpha = -1$ and $\beta = 5$

Now, $\alpha + \beta = -1 + 5 = 4$

12. Option (3) is correct.

Given system of linear equations have infinitely many solutions.

$\therefore \begin{vmatrix} 1 & 1 & k \\ 2 & 3 & -1 \\ 3 & 4 & 2 \end{vmatrix} = 0$

$\Rightarrow 1(6+4) - 1(4+3) + k(8-9) = 0$

$\Rightarrow 10 - 7 - k = 0 \Rightarrow k = 3$

$\therefore$ Another given system of equations are

$4x + 5y = 7$ ...(i)

$7x + 8y = 10$ ...(ii)

$\because \frac{4}{7} \neq \frac{5}{8}$ i.e., $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

So, system of equations have unique solution.

On solving equations (i) and (ii)

We get $x = -2$ and $y = 3$

which satisfying $x + y = 1$.

13. Option (4) is correct.

Since line l_1, passes through the point (2, 6, 2) and is perpendicular to the plane

$2x + y - 2z = 10$

$\therefore \vec{a}_1 = 2\hat{i} + 6\hat{j} + 2\hat{k}$

$\vec{b}_1 = 2\hat{i} + \hat{j} - 2\hat{k}$

$\because$ Equation of another line is

$\frac{x+1}{2} = \frac{y+4}{-3} = \frac{z}{2}$

$\vec{a}_2 = -\hat{i} - 4\hat{j}; \vec{b}_2 = 2\hat{i} - 3\hat{j} + 2\hat{k}$

$\vec{a}_2 - \vec{a}_1 = -3\hat{i} - 10\hat{j} - 2\hat{k}$

$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 2 & -3 & 2 \end{vmatrix} = -4\hat{i} - 8\hat{j} - 8\hat{k}$

Shortest distance $= \frac{|(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$

$= \frac{|(-3\hat{i} - 10\hat{j} - 2\hat{k}).(-4\hat{i} - 8\hat{j} - 8\hat{k})|}{\sqrt{16+64+64}}$

$= \frac{|12 + 80 + 16|}{\sqrt{144}} = \frac{108}{12} = 9$

14. Option (4) is correct.

$\because$ A = $\begin{bmatrix} m & n \\ p & q \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} m & n \\ p & q \end{vmatrix} = mq - np$

$\therefore d = mq - np$ ...(i)

Now, $A - d(\text{Adj } A) = \begin{bmatrix} m & n \\ p & q \end{bmatrix} - d\begin{bmatrix} q & -n \\ -p & m \end{bmatrix}$

$\Rightarrow \begin{bmatrix} m-dq & n+dn \\ p+pd & q-dm \end{bmatrix}$

$\Rightarrow |A - d\,(\text{Adj } A)| = \begin{vmatrix} m-dq & n(1+d) \\ p(1+d) & q-dm \end{vmatrix} = 0$

$\Rightarrow (m - dq)(q - dm) - np(1 + d)^2 = 0$

$\Rightarrow mq - dm^2 - dq^2 + d^2qm = np(1 + d)^2$

$\Rightarrow mq - dm^2 - dq^2 + d^2qm = (mq - d)(1 + d)^2$

$\Rightarrow mq - dm^2 - dq^2 + d^2qm = mq + 2mqd + mqd^2 - d(1 + d)^2$

$\Rightarrow d(1 + d)^2 = dm^2 + dq^2 + 2mqd$

$\Rightarrow d(1 + d)^2 = d(m^2 + q^2 + 2mq)$

$\Rightarrow (1 + d)^2 = (m + q)^2$

15. Option (3) is correct.

Let $I = \frac{3(e-1)}{e}\int_1^2 x^2 e^{[x]+[x^3]}dx$

$= \frac{3(e-1)}{e}\int_1^2 x^2 e^{1+[x^3]}dx$

$= \frac{3(e-1)}{e} \times e\int_1^2 x^2 e^{[x^3]}dx$

Let $x^3 = t \Rightarrow 3x^2\,dx = dt$

when $x = 1$ then $t = 1$

and when $x = 2$ then $t = 8$

$\therefore I = \frac{3(e-1)}{3}\int_1^8 e^{[t]}dt$

$= (e-1)\left[\int_1^2 e\,dt + \int_2^3 e^2 dt + \int_3^4 e^3 dt + \int_4^5 e^4 dt + \int_5^6 e^5 dt + \int_6^7 e^6 dt + \int_7^8 e^7 dt\right]$

$= (e - 1)[e(2 - 1) + e^2(3 - 2) + e^3(4 - 3) + e^4(5 - 4) + e^5(6 - 5) + e^6(7 - 6) + e^7(8 - 7)]$

$= (e - 1)[e + e^2 + e^3 + e^4 + e^5 + e^6 + e^7]$

$= (e-1)\left[\frac{e(e^7-1)}{e-1}\right] = e^8 - e$

16. Option (3) is correct.

Direction cosines of $\widehat{OP}$ are $\cos\alpha$, $\cos\beta$, $\cos\gamma$ and $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Normal vector of plane = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ (2-1) & (3-2) & (4-3) \\ (1-1) & (5-2) & (7-3) \end{vmatrix}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 0 & 3 & 4 \end{vmatrix} = \hat{i} - 4\hat{j} + 3\hat{k}$

$\therefore$ Direction cosines of normal plane are

$\left[\pm\frac{1}{\sqrt{26}}, \mp\frac{4}{\sqrt{26}}, \pm\frac{3}{\sqrt{26}}\right]$

$\therefore$ Direction cosines of $\widehat{OP}$

$= \left[-\frac{1}{\sqrt{26}}, \frac{4}{\sqrt{26}}, \frac{-3}{\sqrt{26}}\right]$ $\left[\because \beta \in \left(0, \frac{\pi}{2}\right)\right]$

Hence, $\alpha \in \left(\frac{\pi}{2}, \pi\right)$ and $\gamma \in \left(\frac{\pi}{2}, \pi\right)$

17. Option (2) is correct.

$(1 + x)^{500} + x(1 + x)^{499} + x^2(1 + x)^{498} + \dots + x^{500}$

$= \frac{(1+x)^{500}\left[\left(\frac{x}{1+x}\right)^{501} - 1\right]}{\frac{x}{1+x} - 1}$

$= \frac{(1+x)^{500}\frac{[x^{501} - (1+x)^{501}]}{(1+x)^{501}}}{\frac{-1}{1+x}}$

$= (1 + x)^{501} - x^{501}$

$\therefore$ Coefficient of x^{301} in given expression is ${}^{501}C_{301}$ or ${}^{501}C_{200}$

18. Option (4) is correct.

Given differential equation is

$\frac{dy}{dx} - \frac{3x^5\tan^{-1}(x^3)}{(1+x^6)^{\frac{3}{2}}}y = 2xe^{\frac{x^3 - \tan^{-1}x^3}{\sqrt{1+x^6}}}$

I.F. $= e^{-\int\frac{3x^5\tan^{-1}x^3}{(1+x^6)^{\frac{3}{2}}}dx}$

Let $\tan^{-1}x^3 = t \Rightarrow \frac{3x^2}{1+x^6}dx = dt$

$\therefore$ I.F. $= e^{-\int\frac{x^3 t}{\sqrt{1+x^6}}dt}$

$= e^{-\int\frac{t\tan t}{\sqrt{1+\tan^2 t}}dt} = e^{-\int\frac{t\tan t}{\sec t}dt}$

$= e^{-\int t\sin t\,dt} = e^{t\cos t - \sin t}$

$= e^{\frac{\tan^{-1}x^3}{\sqrt{1+x^6}} - \frac{x^3}{\sqrt{1+x^6}}}$

Solution is

$y\left(e^{\frac{\tan^{-1}x^3}{\sqrt{1+x^6}} - \frac{x^3}{\sqrt{1+x^6}}}\right)$

$= \int 2xe^{\frac{x^3 - \tan^{-1}x^3}{\sqrt{1+x^6}}} \cdot e^{\frac{\tan^{-1}x^3 - x^3}{\sqrt{1+x^6}}}dx$

$= \int 2x\,dx = x^2 + C$

Since it passes through (0, 0)

$\therefore 0 = 0 + C \Rightarrow C = 0$

Now put $x = 1$

$y\left(e^{\frac{\tan^{-1}1 - 1}{\sqrt{2}}}\right) = 1 \Rightarrow y\left(e^{\frac{\frac{\pi}{4} - 1}{\sqrt{2}}}\right) = 1$

$\Rightarrow y = e^{\frac{4-\pi}{4\sqrt{2}}} = e \times p\left(\frac{4-\pi}{4\sqrt{2}}\right)$

19. Option (2) is correct.

General term of expansion $\left(ax^3 + \frac{1}{bx^{\frac{1}{3}}}\right)^{15}$

$T_{r+1} = {}^{15}C_r (ax^3)^{15-r}\left(\frac{1}{bx^{\frac{1}{3}}}\right)^r$

$= {}^{15}C_r a^{15-r} x^{45-3r} b^{-r} x^{-\frac{r}{3}}$

$= {}^{15}C_r\, a^{15-r} b^{-r} x^{45-3r-\frac{r}{3}}$

$x^{45-3r-\frac{r}{3}} = x^{15}$

$\Rightarrow 45 - 3r - \frac{r}{3} = 15 \Rightarrow 30 = 3r + \frac{r}{3}$

$\Rightarrow 90 = 10r \Rightarrow r = 9$

$\therefore$ Coefficient of $x^{15} = {}^{15}C_9 a^6 b^{-9}$

General term of expansion $\left(ax^{\frac{1}{3}} - \frac{1}{bx^3}\right)^{15}$

$T_{r+1} = {}^{15}C_r (ax^{1/3})^{15-r} (-1)^r (b)^{-r}, (x^3)^{-r}$

$= {}^{15}C_r a^{15-r} (b)^{-r} (-1)^{-r} x^{\frac{15-r}{3}-3r}$

$x^{\frac{15-r}{3}-3r} = x^{-15} \Rightarrow \frac{15-r}{3} - 3r = -15$

$\Rightarrow 15 - r - 9r = -45 \Rightarrow 10r = 60$

$\Rightarrow r = \frac{60}{10} = 6$

$\therefore$ Coefficient of $x^{-15} = {}^{15}C_6 a^9 b^{-6}$

According to question

${}^{15}C_9 a^6 b^{-9} = {}^{15}C_6 a^9 b^{-6}$ $\quad \left[\because {}^nC_r = {}^nC_{n-r}\right]$

$\Rightarrow \frac{a^9 b^{-6}}{a^6 b^{-9}} = 1 \Rightarrow a^3 b^3 = 1 \Rightarrow ab = 1$

20. Option (3) is correct.

Given that $5f(x + y) = f(x). f(y)$ and $f(3) = 320$

Put $x = 3$ and $y = 0$, we get

$5f(3) = f(3) f(0) \Rightarrow f(0) = 5$

Put $x = 1$ and $y = 1$, we get

$5f(2) = f(1) f(1) = f^2(1)$ ...(i)

Put $x = 2$ and $y = 1$, we get

$5 f(3) = f(2) f(1)$

$\Rightarrow 5 \times 320 = \frac{f^3(1)}{5}$ [from (i)]

$\Rightarrow f^3(1) = 25 \times 320 = 125 \times 64$

$\Rightarrow f(1) = 5 \times 4 = 20$

$\Rightarrow f(2) = 80$ [From (i)]

Put $x = 2$ and $y = 2$, we get

$\Rightarrow 5f(4) = f(2) f(2) \Rightarrow 5f(4) = 80 \times 80$

$f(4) = 1280$

Put $x = 3$ and $y = 2$, we get

$\Rightarrow 5f(5) = f(3) f(2)$

$f(5) = \frac{320 \times 80}{5} = 5120$

$\sum_{n=0}^{5} f(x) = f(0) + f(1) + f(2) + f(3) + f(4) + f(5)$

$= 5 + 20 + 80 + 320 + 1280 + 5120$

$= 5 (1 + 2^2 + 2^4 + 2^6 + 2^8 + 2^{10})$

$= \frac{5((2^2)^6 - 1)}{2^2 - 1} = \frac{5(2^{12} - 1)}{3} = 6825$

21. The correct answer is [9].

$z_1 = \frac{1 + i\bar{z}}{\bar{z}(1 - z) + \frac{1}{z}} = \frac{1 + i(1 - i)}{(1 - i)(1 - 1 - i) + \frac{1}{1 + i} \times \frac{1 - i}{1 - i}}$

$= \frac{1 + i - i^2}{(1 - i)(-i) + \frac{1 - i}{1 - i^2}} = \frac{1 + i + 1}{-i + i^2 + \frac{1 - i}{2}}$

$= \frac{2 + i}{-i - 1 + \frac{1 - i}{2}} = \frac{2 + i}{\frac{-2i - 2 + 1 - i}{2}}$

$= \frac{2(2 + i)}{-1 - 3i} = \frac{-2(2 + i)}{1 + 3i} \times \frac{1 - 3i}{1 - 3i}$

$= \frac{-2(2 - 6i + i - 3i^2)}{1 - 9i^2} = \frac{-2(5 - 5i)}{10} = -1 + i$

$\arg z_1 = \pi - \tan^{-1}\left|\frac{-1}{1}\right| = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

Now, $\frac{12}{\pi}\arg(z_1) = \frac{12}{\pi} \times \frac{3\pi}{4} = 9$

22. The correct answer is [315].

$\because \vec{b}_1 = 3\hat{i} - 5\hat{j} + \hat{k}; \vec{b}_2 = \lambda\hat{i} + \hat{j} - 3\hat{k}$

Note: Angle between planes is θ

$\therefore \cos\theta = \frac{\vec{b}_1.\vec{b}_2}{|\vec{b}_1||\vec{b}_2|} = \frac{(3\hat{i} - 5\hat{j} + \hat{k}).(\lambda\hat{i} + \hat{j} - 3\hat{k})}{\sqrt{9 + 25 + 1}\sqrt{\lambda^2 + 1 + 9}}$

$\Rightarrow \cos\theta = \frac{3\lambda - 5 - 3}{\sqrt{35}\sqrt{\lambda^2 + 10}} = \frac{3\lambda - 8}{\sqrt{35}\sqrt{\lambda^2 + 10}}$

$\Rightarrow \sqrt{1 - \sin^2\theta} = \frac{3\lambda - 8}{\sqrt{35}\sqrt{\lambda^2 + 10}}$

$1 - \left(\frac{2\sqrt{6}}{5}\right)^2 = \frac{(3\lambda - 8)^2}{35(\lambda^2 + 10)}$

$\Rightarrow \frac{1}{25} = \frac{9\lambda^2 - 48\lambda + 64}{35(\lambda^2 + 10)}$

$\Rightarrow 7\lambda^2 + 70 = 5(9\lambda^2 - 48\lambda + 64)$

$\Rightarrow 19\lambda^2 - 120\lambda + 125 = 0$

$\Rightarrow \lambda = 5, \frac{25}{19}$

$\therefore \lambda_1 = \frac{25}{19}, \lambda_2 = 5 \quad (\because \lambda_1 < \lambda_2)$

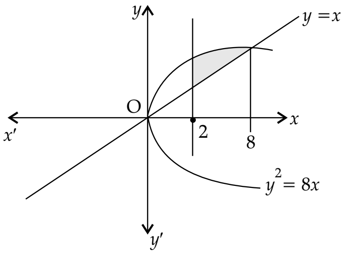
$\therefore$ Point $(38\lambda_1, 10\lambda_2, 2) = (50, 50, 2)$

$\therefore$ Distance of point (50, 50, 2) from plane $P_1 : 3x - 5y + z - 7 = 0$

$d = \left|\frac{150 - 250 + 2 - 7}{\sqrt{9 + 25 + 1}}\right| = \frac{105}{\sqrt{35}} = 3\sqrt{35}$

$\Rightarrow d^2 = (3\sqrt{35})^2 = 315$

23. The correct answer is [22].



Area $= \int_2^8 2\sqrt{2}\sqrt{x}dx - \int_2^8 x\,dx$

$$\Rightarrow \alpha = 2\sqrt{2} \times \frac{2}{3}\left[x^{\frac{3}{2}}\right]_2^8 - \left[\frac{x^2}{2}\right]_2^8$$

$$= \frac{4\sqrt{2}}{3}[8 \times 2\sqrt{2} - 2\sqrt{2}] - [32 - 2]$$

$$= \frac{4\sqrt{2}}{3} \times 14\sqrt{2} - 30 = \frac{112}{3} - 30 = \frac{22}{3}$$

Now, $3\alpha = 3.\frac{22}{3} = 22$

24. The correct answer is [26].

$$\sum_{n=0}^{\infty} \frac{n^3((2n)!) + (2n-1)(n!)}{(n!)((2n)!)}$$

$$= \sum_{n=0}^{\infty} \frac{n^3((2n)!)}{(n!)((2n)!)} + \frac{(2n-1)(n!)}{(n!)((2n)!)}$$

$$= \sum_{n=0}^{\infty} \frac{n^3}{n!} + \frac{2n-1}{(2n)!} = \sum_{n=0}^{\infty} \frac{n^2}{(n-1)!} + \sum_{n=0}^{\infty} \frac{2n-1}{(2n)!}$$

$$= \sum_{n=0}^{\infty} \frac{n^2-1+1}{(n-1)!} + \sum_{n=0}^{\infty} \frac{2n}{(2n)!} - \sum_{n=0}^{\infty} \frac{1}{(2n)!}$$

$$= \sum_{n=2}^{\infty} \frac{n+1}{(n-2)!} + \sum_{n=1}^{\infty} \frac{1}{(n-1)!} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)!} - \sum_{n=0}^{\infty} \frac{1}{(2n)!}$$

$$= \sum_{n=2}^{\infty} \frac{(n-2)+3}{(n-2)!} + \sum_{n=1}^{\infty} \frac{1}{(n-1)!} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)!} - \sum_{n=0}^{\infty} \frac{1}{(2n)!}$$

$$= \sum_{n=3}^{\infty} \frac{1}{(n-3)!} + 3\sum_{n=2}^{\infty} \frac{1}{(n-2)!} + \sum_{n=1}^{\infty} \frac{1}{(n-1)!} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)!} - \sum_{n=0}^{\infty} \frac{1}{(2n)!}$$

$$= e + 3e + e + \left(\frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} \ldots\right) - \left(1 + \frac{1}{2!} + \frac{1}{4!} \ldots\right) \quad \left[\because e = \sum_{n=0}^{\infty} \frac{1}{n!}\right]$$

$$= 5e - \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \ldots\ldots\right)$$

$$= 5e - e^{-1} = 5e - \frac{1}{e} = ae + \frac{b}{e} + c$$

$\therefore$ $a = 5$, $b = -1$ and $c = 0$

Now, $a^2 - b + c = 25 + 1 + 0 = 26$

25. The correct answer is [15].

Direction vector of line

$$\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 4 & -1 & 1 \end{vmatrix} = -\hat{i} + 7\hat{j} + 11\hat{k}$$

$\therefore$ Normal vector of plane $= -\hat{i} + 7\hat{j} + 11\hat{k}$

Equation of plane is

$$[\vec{r} - (\hat{i} + \hat{j} + 2\hat{k})].(-\hat{i} + 7\hat{j} + 11\hat{k}) = 0$$

$$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k})(-\hat{i} + 7\hat{j} + 11\hat{k}) - (-1 + 7 + 22) = 0$$

$$\Rightarrow -x + 7y + 11z = 28$$

$$\Rightarrow -\frac{x}{28} + \frac{7y}{28} + \frac{11z}{28} = 1$$

Now, 140 (C – B + A) $= 140\left(\frac{11}{28} - \frac{7}{28} - \frac{1}{28}\right)$

$$= 140 \times \frac{3}{28} = 15$$

26. The correct answer is [21].

Since number is divisible by 15. So it is divisible by 3 and 5.

Since number divisible by 5, then unit place must have 5.

For divisible by 3, the sum of digits is divisible by 3.

Number of ways of digits (1, 1, 2, 5) $= \frac{3!}{2!} = 3$

Number of ways of digits (1, 3, 3, 5) $= \frac{3!}{2!} = 3$

Number of ways of digits (1, 1, 5, 5) $= \frac{3!}{2!} = 3$

Number of ways of digits (2, 2, 3, 5) $= \frac{3!}{2!} = 3$

Number of ways of digits (2, 3, 5, 5) = 3! = 6

Number of ways of digits (3, 5, 5, 5) $= \frac{3!}{2!} = 3$

Total numbers = 3 + 3 + 3 + 3 + 6 + 3 = 21

27. The correct answer is [3125].

Given that $f'(x) = \frac{3x+2}{2x+3}, x \in R - \left\{\frac{-3}{2}\right\}$

and $f^n(x) = f'(f^{n-1}(x))$

$\therefore$ $f^2(x) = f'(f'(x)) = f'\left(\frac{3x+2}{2x+3}\right)$

$$= \frac{3\left(\frac{3x+2}{2x+3}\right) + 2}{2\left(\frac{3x+2}{2x+3}\right) + 3} = \frac{13x+12}{12x+13}$$

$$f^3(x) = f'(f^2(x)) = f'\left(\frac{13x+12}{12x+13}\right)$$

$$= \frac{3\left(\frac{13x+12}{12x+13}\right) + 2}{2\left(\frac{13x+12}{12x+13}\right) + 3} = \frac{63x+62}{62x+63}$$

$$f^4(x) = f'(f^3(x)) = f'\left(\frac{63x+62}{62x+63}\right)$$

$$= \frac{3\left(\frac{63x+62}{62x+63}\right)+2}{2\left(\frac{63x+62}{62x+63}\right)+3} = \frac{313x+312}{312x+313}$$

$$f^5(x) = f^1(f^4(x)) = f^1\left(\frac{313x+312}{312x+313}\right)$$

$$= \frac{3\left(\frac{313x+312}{312x+313}\right)+2}{2\left(\frac{313x+312}{312x+313}\right)+3}$$

$$= \frac{1563x+1562}{1562x+1563} = \frac{ax+b}{bx+a}$$

$\Rightarrow a = 1563$ and $b = 1562$

So, $a + b = 1563 + 1562 = 3125$.

28. The correct answer is [37].

Since, mean of 7 observations is 8.

$\therefore$ Sum of observations $= \Sigma x_i = 7 \times 8 = 56$

New sum of observations $= 56 - 14 = 42$

New number of observations $= 7 - 1 = 6$

New mean $(a) = \frac{42}{6} = 7$

Variance $= \frac{\Sigma x_i^2}{N} - (\text{Mean})^2$

$$\Rightarrow 16 = \frac{\Sigma x_i^2}{7} - (8)^2$$

$$\Rightarrow \frac{\Sigma x_i^2}{7} = 16 + 64 = 80$$

$\Rightarrow \Sigma x_i^2 = 560$

New, $\Sigma x_i^2 = 560 - (14)^2 = 560 - 196 = 364$

New variance $= \frac{\text{New } \Sigma x_i^2}{6} - (\text{New mean})^2$

$$\Rightarrow b = \frac{364}{6} - (7)^2 = \frac{182}{3} - 49 = \frac{35}{3}$$

Now, $a + 3b - 5 = 7 + 35 - 5 = 37$

29. The correct answer is [3240].

$n(S) = 6$

P(S) = {ϕ, {1}, {2} .... {6}, {1, 2}.... {5, 6}, ...
{1, 2, 3, 4, 5, 6} = 64 elements.

Case 1:

$f(6)$ = S i.e., 1 option
$f(5)$ = any 5 elements subset A of S i.e., 6 options
$f(4)$ = any 4 elements subset B of A i.e., 5 options
$f(3)$ = any 3 elements subset C of B i.e., 4 options
$f(2)$ = any 2 elements subset D of C i.e., 3 options
$f(1)$ = any 1 element subset E of D or empty subsets i.e., 3 options
Total functions = 1080

Case 2:

$f(6)$ = any 5 elements subset A of S i.e. 6 options
$f(5)$ = any 4 elements subset B of A i.e. 5 options
$f(4)$ = any 3 elements subset C of B i.e. 4 options
$f(3)$ = any 2 elements subset D of C i.e. 3 options
$f(2)$ = any 1 element subset E of D i.e. 2 options
$f(1)$ = empty subset i.e., 1 option
Total functions = 720

Case 3:

$f(6)$ = S i.e., 1 option
$f(5)$ = any 4 elements subset A of S i.e., 15 options
$f(4)$ = any 3 elements subset B of A i.e., 4 options
$f(3)$ = any 2 elements subset C of B i.e., 3 options
$f(2)$ = any 1 element subset D of C i.e., 2 options
$f(1)$ = empty subset i.e., 1 option
Total functions = 360

Case 4:

$f(6)$ = S i.e., 1 option
$f(5)$ = any 5 elements subset A of S i.e., 6 options
$f(4)$ = any 3 elements subset B of A i.e., 10 options
$f(3)$ = any 2 elements subset C of B i.e., 3 options
$f(2)$ = any 1 element subset D of C i.e., 2 options
$f(1)$ = empty subset i.e., 1 option
Total functions = 360

Case 5:

$f(6)$ = S i.e., 1 option
$f(5)$ = any 5 elements subset A of S i.e., 6 options
$f(4)$ = any 4 elements subset B of A i.e., 5 options
$f(3)$ = any 2 elements subset C of B i.e., 6 options
$f(2)$ = any 1 element subset D of C i.e., 2 options
$f(1)$ = empty subset i.e., 1 option
Total functions = 360

Case 6:

$f(6)$ = S i.e., 1 option
$f(5)$ = any 5 elements subset A of S i.e., 6 options
$f(4)$ = any 4 elements subset B of A i.e., 5 options
$f(3)$ = any 3 elements subset C of B i.e., 4 options
$f(2)$ = any 1 element subset D of C i.e., 3 options
$f(1)$ = empty subset i.e., 1 option
Total functions = 360

Hence, number of one-one functions = 3240

30. The correct answer is [12].

$$\lim_{x\to 0} \frac{48}{x^4}\int_0^x \frac{t^3}{t^6+1}dt \left(\frac{0}{0} \text{ form}\right)$$

Using L Hospital Rule

$$= \lim_{x\to 0} \frac{48\frac{x^3}{x^6+1}}{4x^3} = \lim_{x\to 0} \frac{12}{x^6+1} = 12.$$

❑❑