JEE (Main) MATHEMATICS SOLVED PAPER

2023
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Section A

Q. 1. If the maximum distance of normal ot the ellipse $\frac{x^2}{4}+\frac{y^2}{b^2}=1, b<2$, from the origin is 1, then the eccentricity of the ellipse is :

(1) $\frac{1}{2}$ **(2)** $\frac{\sqrt{3}}{4}$ **(3)** $\frac{\sqrt{3}}{2}$ **(4)** $\frac{1}{\sqrt{2}}$

Q. 2. Let a differentiable function f satisfy $f(x)+\int_3^x \frac{f(t)}{t}dt=\sqrt{x+1}, x\ge 3$. Then $12f(8)$ is equal to:

(1) 34 **(2)** 1 **(3)** 17 **(4)** 19

Q. 3. For all $z \in C$ on the curve C_1 : $|z| = 4$, let the locus of the point $z+\frac{1}{z}$ be the curve C_2. Then :

(1) the curve C_1 lies inside C_2
(2) the curve C_2 lies inside C_1
(3) the curves C_1 and C_2 intersect at 4 points
(4) the curves C_1 and C_2 intersect at 2 points

Q. 4. $y=f(x)=\sin^3\left(\frac{\pi}{3}\left(\cos\left(\frac{\pi}{3\sqrt{2}}(-4x^3+5x^2+1)^{\frac{3}{2}}\right)\right)\right)$.

Then, at $x = 1$.

(1) $\sqrt{2}y'-3\pi^2y=0$ **(2)** $y'+3\pi^2y=0$
(3) $2y'+3\pi^2y=0$ **(4)** $2y'+\sqrt{3}\pi^2y=0$

Q. 5. A wire of length 20 m is to be cut into two pieces. A piece of length l_1 is bent to make a square of area A_1 and the other piece of length l_2 is made into a circle of area A_2. If $2A_1 + 3A_2$ is minimum then $(\pi l_1): l_2$ is equal to :

(1) 1 : 6 **(2)** 6 : 1 **(3)** 3 : 1 **(4)** 4 : 1

Q. 6. Let a circle C_1 be obtained on rolling the circle $x^2 + y^2 - 4x - 6y + 11 = 0$ upwards 4 units on the tangent T to it at the point (3,2). Let C_2 be the image of C_1 in T. Let A and B be the centers of circles C_1 and C_2 respectively, and M and N be respectively the feet of perpendiculars drawn from A and B on the x-axis. Then the area of the trapezium AMNB is :

(1) $4(1+\sqrt{2})$ **(2)** $3+2\sqrt{2}$
(3) $2(1+\sqrt{2})$ **(4)** $2(2+\sqrt{2})$

Q. 7. A bag contains 6 balls. Two balls are drawn from it at random and both are found to be black. The probability that the bag contains at least 5 black balls is

(1) $\frac{3}{7}$ **(2)** $\frac{5}{7}$ **(3)** $\frac{5}{6}$ **(4)** $\frac{2}{7}$

Q. 8. Let $y = f(x)$ represent a parabola with focus $\left(-\frac{1}{2},0\right)$ and directrix $y=-\frac{1}{2}$. Then S = $\left\{x\in R : \tan^{-1}(\sqrt{f(x)})+\sin^{-1}(\sqrt{f(x)+1})=\frac{\pi}{2}\right\}$:

(1) contains exactly two elements
(2) contains exactly one element
(3) is an empty set
(4) is an infinite set

Q. 9. Let $\vec{a}=2\hat{i}+\hat{j}+\hat{k}$, and $\vec{b}$ and $\vec{c}$ be two nonzero vectors such that $|\vec{a}+\vec{b}+\vec{c}|=|\vec{a}+\vec{b}-\vec{c}|$ and $\vec{b}.\vec{c}=0$. Consider the following two statements:

(1) $|\vec{a}+\lambda\vec{c}|\ge|\vec{a}|$ for all $\lambda \in$ R.

(2) $\vec{a}$ and $\vec{c}$ are always parallel.

Then

(1) both (1) and (2) are correct
(2) only (1) is correct
(3) neither (1) nor (2) is correct
(4) only (2) is correct

Q. 10. The value of $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\frac{(2+3\sin x)}{\sin x(1+\cos x)}dx$ is equal to

(1) $\frac{10}{3}-\sqrt{3}-\log_e\sqrt{3}$ **(2)** $\frac{7}{2}-\sqrt{3}-\log_e\sqrt{3}$

(3) $-2+3\sqrt{3}+\log_e\sqrt{3}$ **(4)** $\frac{10}{3}-\sqrt{3}+\log_e\sqrt{3}$

Q. 11. Let the shortest distance between the lines $L:\frac{x-5}{-2}=\frac{y-\lambda}{0}=\frac{z+\lambda}{1}, \lambda\ge 0$ and L_1 : $x + 1 = y - 1 = 4 - z$ be $2\sqrt{6}$. If (α, β, γ) lies on L, then which of the following is NOT possible?

(1) $\alpha - 2\gamma = 19$ **(2)** $2\alpha + \gamma = 7$
(3) $2\alpha - \gamma = 9$ **(4)** $\alpha + 2\gamma = 24$

Q. 12. For the system of linear equations

$$x + y + z = 6$$
$$\alpha x + \beta y + 7z = 3$$
$$x + 2y + 3z = 14$$

which of the following is NOT true ?

(1) If $\alpha = \beta$ and $\alpha \ne 7$, then the system has a unique solution
(2) If $\alpha = \beta = 7$, then the system has no solution
(3) For every point $(\alpha, \beta) \ne (7, 7)$ on the line $x - 2y + 7 = 0$, the system has infinitely many solutions

(4) There is a unique point (α, β) on the line $x + 2y + 18 = 0$ for which the system has infinitely many solutions

Q. 13. If the domain of the function $f(x)=\frac{[x]}{1+x^2}$, where $[x]$ is greatest integer ≤ x is [2, 6), then its range is

(1) $\left(\frac{5}{26},\frac{2}{5}\right]$

(2) $\left(\frac{5}{37},\frac{2}{5}\right]-\left\{\frac{9}{29},\frac{27}{109},\frac{18}{89},\frac{9}{53}\right\}$

(3) $\left(\frac{5}{37},\frac{2}{5}\right]$

(4) $\left(\frac{5}{26},\frac{2}{5}\right]-\left\{\frac{9}{29},\frac{27}{109},\frac{18}{89},\frac{9}{53}\right\}$

Q. 14. Let R be a relation on N×N defined by (a, b) R(c, d) if and only if $ad(b-c) = bc(a-d)$. Then R is
(1) transitive but neither reflexive nor symmetric
(2) symmetric but neither reflexive nor transitive
(3) symmetric and transitive but not reflexive
(4) reflexive and symmetric but not transitive

Q. 15. (S1) $(p \Rightarrow q) \vee (p \wedge (\sim q))$ is a tautology
(S2) $((\sim p) \Rightarrow (\sim q)) \wedge ((\sim p) \vee q)$ is a contradiction.
Then
(1) both (S1) and (S2) are correct
(2) only (S1) is correct
(3) only (S2) is correct
(4) both (S1) and (S2) are wrong

Q. 16. If the sum and product of four positive consecutive terms of a G.P., are 126 and 1296 , respectively, then the sum of common ratios of all such GPs is

(1) 7 **(2)** 3 **(3)** $\frac{9}{2}$ **(4)** 14

Q. 17. Let $A=\begin{pmatrix}1 & 0 & 0\\ 0 & 4 & -1\\ 0 & 12 & -3\end{pmatrix}$. Then the sum of the diagonal elements of the matrix $(A + I)^{11}$ is equal to
(1) 6144 **(2)** 2050 **(3)** 4097 **(4)** 4094

Q. 18. The number of real roots of the equation $\sqrt{x^2-4x+3}+\sqrt{x^2-9}=\sqrt{4x^2-14x+6}$ is:
(1) 3 **(2)** 1 **(3)** 2 **(4)** 0

Q. 19. If $\sin^{-1}\frac{\alpha}{17}+\cos^{-1}\frac{4}{5}-\tan^{-1}\frac{77}{36}=0$, $0 < \alpha < 13$, then $\sin^{-1}(\sin \alpha) + \cos^{-1}(\cos \alpha)$ is equal to
(1) 16 **(2)** 0 **(3)** π **(4)** 16 – 5π

Q. 20. Let $a \in (0, 1)$ and $\beta = \log_e(1-\alpha)$. Let $P_n(x) = x+\frac{x^2}{2}+\frac{x^3}{3}+...+\frac{x^n}{n}, x\in(0,1)$. Then the integral $\int_0^{\alpha}\frac{t^{50}}{1-t}dt$ is equal to

(1) $\beta + P_{50}(\alpha)$ **(2)** $P_{50}(\alpha) - \beta$
(3) $\beta - P_{50}(\alpha)$ **(4)** $-(\beta + P_{50}(\alpha))$

Section B

Q. 21. Let α > 0, be the smallest number such that the expansion of $\left(x^{\frac{2}{3}}+\frac{2}{x^3}\right)^{30}$ has a term $\beta x^{-\alpha}$, β ∈ N. Then α is equal to

Q. 22. Let for $x \in$ R.

$$f(x)=\frac{x+|x|}{2} \text{ and } g(x)=\begin{cases}x, & x<0\\ x^2, & x\geq 0\end{cases}$$

Then area bounded by the curve $y = (f \cdot g)(x)$ and the lines $y = 0$, $2y - x = 15$ is equal to

Q. 23. Number of 4-digit numbers that are less than or equal to 2800 and either divisible by 3 or by 11 , is equal to

Q. 24. If the variance of the frequency distribution

x_i	2	3	4	5	6	7	8
Frequency f_i	3	6	16	α	9	5	6

is 3, then α is equal to

Q. 25. Let θ be the angle between the planes P_1 : $\vec{r}.(\hat{i}+\hat{j}+2\hat{k}) = 9$ and P_2: $\vec{r}.(2\hat{i}-\hat{j}+\hat{k}) = 15$. Let L be the lines that meets P_2 at the point (4, –2, 5) and makes an angle θ with the normal of P_2. If α is the angle between L and P_2, then $(\tan^2\theta)(\cot^2\alpha)$ is equal to

Q. 26. Let 5 digit numbers be constructed using the digits 0, 2, 3, 4, 7, 9 with repetition allowed, and are arranged in ascending order with serial numbers. Then the serial number of the number 42923 is

Q. 27. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}|=\sqrt{14}, |\vec{b}|=\sqrt{6}$ and $|\vec{a}\times\vec{b}|=\sqrt{48}$. Then $(\vec{a}.\vec{b})^2$ is equal to

Q. 28. Let the line L: $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z-3}{1}$ intersect the plane $2x + y + 3z = 16$ at the point P. Let the point Q be the foot of perpendicular from the point R(1, −1, −3) on the line L. If α is the area of triangle PQR, then α^2 is equal to

Q. 29. Let $a_1, a_2, ..., a_n$, be in A.P. If $a_5 = 2a_7$ and $a_{11} = 18$ then

$$12\left(\frac{1}{\sqrt{a_{10}}+\sqrt{a_{11}}}+\frac{1}{\sqrt{a_{11}}+\sqrt{a_{12}}}+...+\frac{1}{\sqrt{a_{17}}+\sqrt{a_{18}}}\right)$$

Q. 30. The remainder on dividing 5^{99} by 11 is :

Answer Key

Q. No.	Answer	Topic Name	Chapter Name
1	(3)	Eccentricity of ellipse	Conic Section
2	(3)	Linear differential equation of first order	Differential Equations
3	(3)	Function of complex variable	Complex Number
4	(3)	Differentiation	Differentiation
5	(2)	Maxima and Minima	Application of Derivative
6	(1)	Circle	Conic Section
7	(2)	Probability	Probability
8	(1)	Parabola	Conic Section
9	(2)	Vectors	Vector Algebra
10	(4)	Definite Integral	Integral Calculus
11	(4)	Shortest distance between lines	Three Dimensional Geometry
12	(3)	Solution of simul-taneous equations	Matrices and Determinants
13	(3)	Domain and Range	Relations and Function
14	(2)	Types of relations	Relation
15	(2)	Tautology and Contradiction	Mathematical Reasoning
16	(1)	A.P. and G.P.	Sequence and Series
17	(3)	Matrix	Matrices and Determinants
18	(2)	Nature of Roots	Quadratic equations
19	(3)	Inverse Trigonometry	Trigonometry
20	(4)	Definite Integral	Integral Calculus
21	[2]	Binomial expansion	Binomial Theorem
22	[72]	Area bounded by curve	Integral Calculus
23	[710]	Number systems	Sequence and Series
24	[5]	Variance	Statistics
25	[9]	Angle between planes	3D
26	[2997]	General Rule	Permutations and Combinations
27	[36]	Dot and Cross Product	Vector Algebra
28	[180]	Intersection of line and plane	3D
29	[8]	A.P. and G.P.	Sequence and Series
30	[9]	Remainder Theorem	Binomial Theorem

Solutions

Section A

1. Option (3) is correct.

Given equation of ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, b<2$$

$\Rightarrow a = 2, b < 2$

Normal to the ellipse at point ($a\cos\theta$, $b\sin\theta$) is

$$\frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2 \quad ...(1)$$

Then its distance from origin is

$$P=\frac{|a^2-b^2|}{\sqrt{\left(\frac{a}{\cos\theta}\right)^2+\left(\frac{-b}{\sin\theta}\right)^2}}=\frac{|a^2-b^2|}{\sqrt{a^2\sec^2\theta+b^2\operatorname{cosec}^2\theta}}$$

$$P=\frac{|(a-b)(a+b)|}{\sqrt{a^2+b^2+2ab+(a\tan\theta-b\cot\theta)^2}}$$

$$P_{max}=\frac{|(a-b)(a+b)|}{a+b}=|a-b|$$

$\because P_{max}=1 \therefore |a-b|=1$

$\Rightarrow |2-b|=1 \Rightarrow b=1 \ (\because b<2)$

$$\text{Eccentricity } (e)=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{1}{4}}=\frac{\sqrt{3}}{2}$$

2. Option (3) is correct.

Given

$$f(x)+\int_3^x \frac{f(t)}{t}dt=\sqrt{x+1}, x\geq 3$$

Differentiate with respect to x both sides

$$f'(x)+\frac{f(x)}{x}=\frac{1}{2}(x+1)^{1/2-1}, x\geq 3$$

$$\Rightarrow f'(x)+\frac{1}{x}f(x)=\frac{1}{2\sqrt{x+1}} \quad ...(1)$$

$\Rightarrow \frac{df}{dx}+\frac{1}{x}f=\frac{1}{2\sqrt{x+1}}$ which is linear differential equation of first order.

$$P=\frac{1}{x},\ Q=\frac{1}{2\sqrt{x+1}}$$

$$\therefore \text{ I.F.}=e^{\int Pdx}$$

$$\Rightarrow \text{I.F.}=e^{\int \frac{1}{x}dx}=e^{\log x}=x$$

So, the solution is

$$f(x)\,.\,(\text{I.F.}) = \int (\text{I.F.})Q\,dx+C$$

$$\Rightarrow f(x).x=\int\frac{x}{2\sqrt{x+1}}dx+C$$

$$\Rightarrow f(x).x=\frac{1}{2}\int\frac{x+1-x}{\sqrt{x+1}}dx+C$$

$$\Rightarrow xf(x)=\frac{1}{2}\left[\int\sqrt{x+1}-\int\frac{1}{\sqrt{x+1}}dx\right]+C$$

$$\Rightarrow xf(x)=\frac{1}{2}\left[\frac{2}{3}(x+1)^{3/2}-\frac{2}{1}(x+1)^{1/2}\right]+C$$

$$\Rightarrow xf(x)=\left[\frac{1}{3}(x+1)^{3/2}-\sqrt{(x+1)}\right]+C \quad (\because f(3)=2)$$

$$\Rightarrow 3\times 2=\left[\frac{1}{3}\times 8-2\right]+C \Rightarrow 6=\left(\frac{8}{3}-2\right)+C$$

$$\Rightarrow 6=\frac{2}{3}+C \Rightarrow C=6-\frac{2}{3}=\frac{18-2}{3}=\frac{16}{3}$$

So, $x\,f(x)=\frac{1}{3}(x+1)^{3/2}-\sqrt{x+1}+\frac{16}{3}$

Put $x = 8$

$$f(8).8=\frac{1}{3}(27)-3+\frac{16}{3}$$

$$\Rightarrow 8f(8)=6+\frac{16}{3}=\frac{18+16}{3}$$

$$\Rightarrow 8f(8)=\frac{34}{3}$$

$$\Rightarrow 12f(8) = 17$$

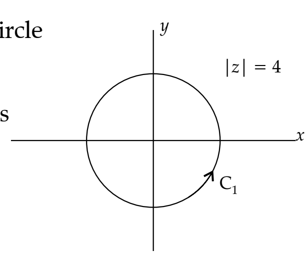
3. Option (3) is correct.

Given $C_1 : |z| = 4$ is a circle

$\Rightarrow z\bar{z}=16$

$\Rightarrow x^2+y^2=16^2$ of radius

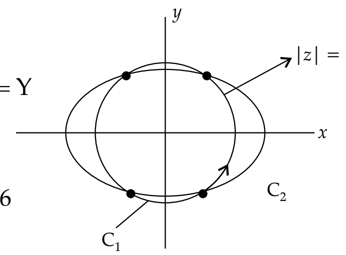
We have $|z|^2=z\bar{z}$



$$z=\frac{16}{\bar{z}}$$

$$z+\frac{1}{z}=z+\frac{1}{\frac{16}{\bar{z}}}\Rightarrow z+\frac{\bar{z}}{16}=(x+iy)+\frac{1}{16}(x-iy)$$

$$=\frac{17}{16}x+\frac{15}{16}yi$$



$\frac{17}{16}x=X$ and $\frac{15}{16}y=Y$

$\because x^2+y^2=16$

$$\frac{X^2}{\left(\frac{17}{16}\right)^2}+\frac{Y^2}{\left(\frac{15}{16}\right)^2}=16$$

$$\therefore \frac{X^2}{\left(\frac{17}{4}\right)^2}+\frac{Y^2}{\left(\frac{15}{4}\right)^2}=1\rightarrow C_2 \text{ is an ellipse.}$$

Hence curve C_1 and C_2 are intersect at 4 points.

4. Option (3) is correct.

Given

$$y=f(x)=\sin^3\left(\frac{\pi}{3}\left(\cos\frac{\pi}{3\sqrt{2}}(-4x^3+5x^2+1)^{3/2}\right)\right)$$

Put $x = 1$

$$y=f(x)=\sin^3\left(\frac{\pi}{3}\left(\cos\frac{\pi}{3\sqrt{2}}(-4+5+1)^{3/2}\right)\right)$$

$$=\sin^3\left(\frac{\pi}{3}\left(\cos\frac{\pi}{3\sqrt{2}}\times 2\sqrt{2}\right)=\sin^3\left(\frac{\pi}{3}\cos\frac{2\pi}{3}\right)\right)$$

Let $g(x)=\frac{\pi}{3\sqrt{2}}(-4x^3+5x^2+1)^{3/2}$

$$\Rightarrow g(1)=\frac{\pi}{3\sqrt{2}}\times 2\sqrt{2}=\frac{2\pi}{3}$$

$$y=f(x)=\sin^3\left(\frac{\pi}{3}(\cos g(x))\right)$$

Differentiate with respect to x both sides

$$\frac{dy}{dx}=f'(x)=3\sin^2\left(\frac{\pi}{3}\cos g(x)\right)\times\cos\left(\frac{\pi}{3}\cos g(x)\right)$$

$$\times\frac{\pi}{3}(-\sin g(x)).g'(x)$$

$$g(x)=\frac{\pi}{3\sqrt{2}}(-4x^3+5x^2+1)^{3/2}$$

$$g'(x)=\frac{\pi}{3\sqrt{2}}\times\frac{3}{2}(-4x^3+5x^2+1)^{1/2}(-12x^2+10x)$$

$$=\frac{\pi}{2\sqrt{2}}(-4x^3+5x^2+1)^{1/2}\times 2(5x-6x^2)$$

$$g'(1)=\frac{\pi}{\sqrt{2}}(-4\times(1)+5+1)^{1/2}\times(5\times 1-6\times 1)$$

$$g'(1)=\frac{\pi}{\sqrt{2}}(2)^{1/2}\times(-1)=\frac{-\sqrt{2}\pi}{\sqrt{2}}=-\pi$$

At $x = 1$

$$y'(1)=-g'(1)\sin g(1)\sin^2\left(\frac{\pi}{3}\cos g(1)\right)-\pi\cos\left(\frac{\pi}{3}\cos g(1)\right)$$

$$=\pi^2\sin\frac{2\pi}{3}.\sin^2\left(\frac{\pi}{3}\cos\frac{2\pi}{3}\right)\cos\left(-\frac{\pi}{6}\right)=\frac{3\pi^2}{16}$$

$$y(1)=\sin^3\left(\frac{\pi}{3}\cos\frac{2\pi}{3}\right)=\frac{-1}{8}$$

$$2y'(1) + 3\pi^2 y(1) = 2 \times \frac{3\pi^2}{16} + 3\pi^2 \times \frac{-1}{8}$$

$$= \frac{3\pi^2}{8} - \frac{3\pi^2}{8} = 0$$

$\Rightarrow 2y'(1) + 3\pi^2 y(1) = 0$

5. Option (2) is correct.

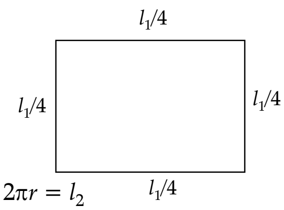
Given length of wire = 20 m

$l_1 + l_2 = 20$ m ...(1)

Length l_2 is bent to make a circle of area A_2

Length l_1 is bent to make a square of area A_1

Area of square $A_1 = \left(\frac{l_1}{4}\right)^2$



Area of circle $A_2 = \pi r^2$

Circumference of circle = $2\pi r = l_2$

$$\Rightarrow r = \frac{l_2}{2\pi}$$

$$\therefore\ A_2 = \pi\left(\frac{l_2}{2\pi}\right)^2 = \frac{\pi l_2^2}{4\pi^2} = \frac{l_2^2}{4\pi}$$

Let S= $2A_1 + 3A_2$

$$\Rightarrow S = 2 \times \frac{l_1^2}{16} + 3 \times \frac{l_2^2}{4\pi} = \frac{l_1^2}{8} + \frac{3l_2^2}{4\pi} \quad ...(1)$$

S is minimum $S = \frac{l_1^2}{8} + \frac{3l_2^2}{4\pi}$

$$\Rightarrow \frac{dS_1}{dl_1} = \frac{2l_1}{8} + \frac{6l_2}{4\pi} \times \frac{dl_2}{dl_1} = 0$$

$$\therefore \frac{l_1}{4} + \frac{3l_2}{2\pi} \times (-1) = 0 \qquad \left[\begin{array}{l} \because l_1 + l_2 = 20 \\ 1 + \frac{dl_2}{dl_1} = 0 \\ \therefore \frac{dl_2}{dl_1} = -1 \end{array}\right]$$

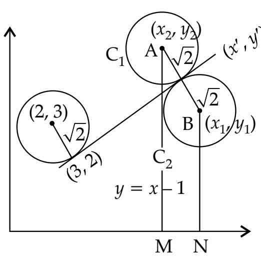
$$\frac{l_1}{4} = \frac{3l_2}{2\pi}$$

$$\Rightarrow \frac{\pi l_1}{6l_2} = \frac{1}{1} \Rightarrow \frac{\pi l_1}{l_2} = \frac{6}{1} = 6:1$$

$\Rightarrow \pi l_1 : l_2 = 6 : 1$

6. Option (1) is correct.

Let (x', y') point lies an line $y = x - 1$ having distance 4 unit from (3, 2)



$$x' = \frac{4}{\sqrt{2}} + 3 = 2\sqrt{2} + 3$$

$$y' = \frac{4}{\sqrt{2}} + 2 = 2\sqrt{2} + 2$$

Slope of line AB is –1 for point A and B

i.e., $\tan\theta = -1$ then $\sin\theta = \frac{1}{\sqrt{2}}, \cos\theta = \frac{-1}{\sqrt{2}}$

$$x = \pm\sqrt{2}\left(\frac{-1}{\sqrt{2}}\right) + (2\sqrt{2} + 3)$$

$$y = \pm\sqrt{2}\left(\frac{1}{\sqrt{2}}\right) + (2\sqrt{2} + 2)$$

for point A we take – ve sign

$(x_2, y_2) = (2\sqrt{2} + 2, 2\sqrt{2} + 3)$

for point B we take +ve sign

$(x_1, y_1) = (2\sqrt{2} + 4, 2\sqrt{2} + 1)$

MN = $|x_2 - x_1|$ = 2

AM + BN = $2\sqrt{2} + 3 + 2\sqrt{2} + 1 = 4 + 4\sqrt{2}$

Area of trapezium= $\frac{1}{2} \times 2 \times (4 + 4\sqrt{2})$

$= 4 + 4\sqrt{2} = 4(1 + \sqrt{2})$

7. Option (2) is correct.

Total balls in a bag = 6

Probability of at least 5 black balls in a bag

$$= \frac{\text{Favorable cases}}{\text{Total outcomes}} = \frac{{}^5C_2 + {}^6C_2}{{}^2C_2 + {}^3C_2 + {}^4C_2 + {}^5C_2 + {}^6C_2}$$

$$= \frac{10 + 15}{1 + 3 + 6 + 10 + 15} = \frac{25}{35} = \frac{5}{7}$$

8. Option (1) is correct.

Given focus of parabola $\left(\frac{-1}{2}, 0\right) = (a, k + p)$

and directrix $y = \frac{-1}{2} = k - p$

$\therefore$ Equation of parabola

$(x - a)^2 = 4p(y - k)$

$$k + p = 0$$
$$k - p = -\frac{1}{2}$$
$$2k = -\frac{1}{2}$$

$$\Rightarrow k = -\frac{1}{4}$$

$$p = \frac{1}{4}$$

$$\left(x + \frac{1}{2}\right)^2 = \cancel{4} \times \frac{1}{\cancel{4}}\left(y + \frac{1}{4}\right)$$

$$x^2 + \frac{1}{\cancel{4}} + x = y + \frac{1}{\cancel{4}}$$

$\therefore y = f(x) = x^2 + x$ ...(1)

$$S = \left\{x \in R : \tan^{-1}(\sqrt{f(x)}) + \sin^{-1}(\sqrt{f(x) + 1}) = \frac{\pi}{2}\right\}$$

$$\tan^{-1}\left(\sqrt{f(x)}\right) + \sin^{-1}(\sqrt{f(x) + 1}) = \frac{\pi}{2}$$

$f(x) \geq 0$ and $\sqrt{f(x) + 1}$ cannot greater then 1.

So $f(x)$ must be 0

i.e., $f(x) = 0$

$\Rightarrow x^2 + x = 0 \Rightarrow x(x + 1) = 0$

$\Rightarrow x = 0, -1$

$\therefore$ S contain 2 elements.

9. Option (2) is correct.

Given $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$

and $\vec{b}, \vec{c}$ be two non zero vectors

such that $|\vec{a} + \vec{b} + \vec{c}| = |\vec{a} + \vec{b} - \vec{c}|$

and $\vec{b}.\vec{c} = 0$ $\quad \{\because \vec{a}.\vec{b} = ab\cos\theta\}$

$|\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a} + \vec{b} - c|^2$

$|\bar{a}|^2 + |\bar{b}|^2 + |\bar{c}|^2 + 2\bar{a}\bar{b} + 2\bar{b}\bar{c} + 2\bar{a}\bar{c}$

$= |\bar{a}|^2 + |\bar{b}|^2 + |\bar{c}|^2 + 2\bar{a}\bar{b} - 2\bar{b}\bar{c} - 2\bar{a}\bar{c}$

$\Rightarrow 4\vec{b}.\vec{c} + 4\vec{a}.\vec{c} = 0$

$\Rightarrow \vec{b}.\vec{c} + \vec{a}.\vec{c} = 0 \quad (\because \vec{b}.\vec{c} = 0)$

$\Rightarrow \vec{a}.\vec{c} = 0$ (2 is incorrect)

$|\vec{a} + \lambda\vec{c}|^2 \geq |\vec{a}|^2$

$\Rightarrow |\vec{a}|^2 + \lambda^2|\vec{c}|^2 + 2\lambda\vec{a}\vec{c} \geq |\vec{a}|^2$

$\Rightarrow \lambda^2|\vec{c}|^2 + 2\lambda\vec{a}\vec{c} \geq 0$

$\Rightarrow \lambda^2 c^2 \geq 0, \quad \forall \lambda \in R$

True (1 is correct)

10. Option (4) is correct.

Given $\int_{\pi/3}^{\pi/2} \frac{(2+3\sin x)}{\sin x(1+\cos x)}dx$

$$\int_{\pi/3}^{\pi/2} \frac{2}{\sin x(1+\cos x)}dx + 3\int_{\pi/3}^{\pi/2} \frac{\sin x}{\sin x(1+\cos x)}dx$$

$$= 2\int_{\pi/3}^{\pi/2} \frac{1}{\sin x(1+\cos x)}dx + 3\int_{\pi/3}^{\pi/2} \frac{1}{1+\cos x}dx$$

$$\left(\because \sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)$$

$$= 2\int_{\pi/3}^{\pi/2} \frac{1+\tan^2\frac{x}{2}}{2\tan\frac{x}{2}\left(1+\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}dx$$

$$+ 3\int_{\pi/3}^{\pi/2} \frac{1}{(1+\cos x)} \times \frac{(1-\cos x)}{(1-\cos x)}dx$$

$$\left(\because \cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)$$

$$= 2\int_{\pi/3}^{\pi/2} \frac{\left(1+\tan^2\frac{x}{2}\right)}{2\tan\frac{x}{2}\left(\frac{2}{1+\tan^2\frac{x}{2}}\right)}dx + 3\int_{\pi/3}^{\pi/2} \frac{1-\cos x}{1-\cos^2 x}dx$$

$$= \frac{1}{2}\int_{\pi/3}^{\pi/2} \frac{\left(1+\tan^2\frac{x}{2}\right)\left(1+\tan^2\frac{x}{2}\right)}{\tan\frac{x}{2}}dx + 3\int_{\pi/3}^{\pi/2} \frac{1-\cos x}{\sin^2 x}dx$$

$$= \frac{1}{2}\int_{\pi/3}^{\pi/2} \frac{\sec^2\frac{x}{2}\left(1+\tan^2\frac{x}{2}\right)}{\tan\frac{x}{2}}dx + 3\int_{\pi/3}^{\pi/2} (\text{cosec}^2 x - \cot x\ \text{cosec} x)dx$$

Let $\tan\frac{x}{2} = t \Rightarrow \frac{1}{2}\sec^2\frac{x}{2}dx = dt$

$$= \int_{1/\sqrt{3}}^{1} \left(\frac{1+t^2}{t}\right)dt + 3\int_{\pi/3}^{\pi/2} (\text{cosec}^2 x - \text{cosec} x \cot x)dx$$

$$= \int_{1/\sqrt{3}}^{1} \left(\frac{1}{t} + t\right)dt + 3[-\cot x + \text{cosec} x]_{\pi/3}^{\pi/2}$$

$$= \left[\log t + \frac{t^2}{2}\right]_{1/\sqrt{3}}^{1} + 3[\text{cosec} x - \cot x]_{\pi/3}^{\pi/2}$$

$$= \frac{1}{3} + 3 + \log\sqrt{3} - \sqrt{3} = \frac{10}{3} + \log\sqrt{3} - \sqrt{3}$$

11. Option (4) is correct.

Given shortest distances between lines L and L_1 is $2\sqrt{6}$

$L: \frac{x-5}{-2} = \frac{y-\lambda}{0} = \frac{z+\lambda}{1}, \lambda \geq 0$

$L_1: \frac{x+1}{1} = \frac{y-1}{1} = \frac{z-4}{-1},$

Let $\vec{a}_1 = <-2, 0, 1>$ $\vec{b}_1 = <5, \lambda, -\lambda>$

$\vec{a}_2 = <1, 1, -1>$ $\vec{b}_2 = <-1, 1, 4>$

Normal vector of both lines is

$$\vec{a}_1 \times \vec{a}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$= \hat{i}(0-1) - \hat{j}(2-1) + \hat{k}(-2-0)$

$= -\hat{i} - \hat{j} - 2\hat{k} = <-1, -1, -2>$

$\vec{b}_1 - \vec{b}_2 = <6, \lambda-1, -\lambda-4>$

Shortest distance $d = \left|\frac{(\vec{b}_1 - \vec{b}_2).(\vec{a}_1 \times \vec{a}_2)}{|\vec{a}_1 \times \vec{a}_2|}\right|$

$$2\sqrt{6} = \left|\frac{<6, \lambda-1, -\lambda-4> \times <-1, -1, -2>}{\sqrt{1+1+4}}\right|$$

$\Rightarrow 12 = |-6 - \lambda + 1 + 2\lambda + 8|$

$\Rightarrow |\lambda + 3| = 12$

$\Rightarrow \lambda = 9, -15$

$\Rightarrow \lambda = 9, (\lambda \geq 0)$

$\because (\alpha, \beta, \gamma)$ lies on line L then

$\frac{\alpha-5}{-2} = \frac{\beta-9}{0} = \frac{\gamma+9}{1} = k$

$\Rightarrow \alpha = 5 - 2k, \beta = 9, \gamma = -9 + k$

$\alpha + 2\gamma = 5 - 2k - 18 + 2k = -13 \neq 24$

Therefore $\alpha + 2\gamma = 24$ is not possible

12. Option (3) is correct.

System of equations

$x + y + z = 6$ ...(1)

$\alpha x + \beta y + 7z = 3$ ...(2)

$x + 2y + 3z = 14$ ...(3)

Augmented matrix [A : B]

$$C = \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ \alpha & \beta & 7 & : & 3 \\ 1 & 2 & 3 & : & 14 \end{bmatrix}$$

$R_2 \to R_2 - \alpha R_1$ and $R_3 \to R_3 - R_1$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & \beta-\alpha & 7-\alpha & : & 3-6\alpha \\ 0 & 1 & 2 & : & 8 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & 8 \\ 0 & \beta-\alpha & 7-\alpha & : & 3-6\alpha \end{bmatrix}$$

$R_3 \to R_3 - (\beta - \alpha)R_2$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & 8 \\ 0 & 0 & \alpha-2\beta+7 & : & 2\alpha-8\beta+3 \end{bmatrix}$$

If $\alpha - 2\beta + 7 = 0$

and $2\alpha - 8\beta + 3 = 0$

Then $\rho(A) = \rho(C) = 2 < 3$ then system is consistent and have infinite many solutions

$\alpha - 2\beta = -7$ ...(1) × 2

$2\alpha - 8\beta = -3$ ...(2)

$$\begin{array}{r} 2\alpha - 4\beta = -14 \\ 2\alpha - 8\beta = -3 \\ - \quad + \quad + \\ \hline 4\beta = -11 \end{array} \Rightarrow \beta = \frac{11}{4}$$

$\alpha - 2\beta = -7$

$$\Rightarrow \alpha - \cancel{2} \times \left(\frac{-11}{\cancel{4}_{2}}\right) = -7 \Rightarrow \alpha = \frac{-25}{2}, \beta = \frac{-11}{4}$$

Hence $(\alpha, \beta) \neq (7, 7)$ on the line $x - y + 7 = 0$

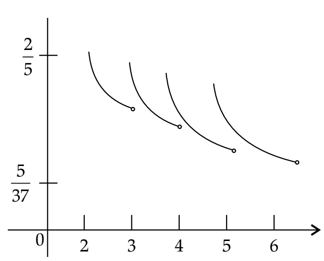
The system has infinite many solutions

13. Option (3) is correct.

Given $f(x) = \dfrac{|x|}{1+x^2}, x \in [2, 6)]$

Then

$$f(x) = \begin{cases} \dfrac{2}{1+x^2}, x \in [2,3) \\ \dfrac{3}{1+x^2}, x \in [3,4) \\ \dfrac{4}{1+x^2}, x \in [4,5) \\ \dfrac{5}{1+x^2}, x \in [5,6) \end{cases}$$



$\because f(x)$ is decreasing sign in $x \in [2, 6)$

Range is $\left(\dfrac{5}{37}, \dfrac{2}{5}\right]$

14. Option (2) is correct.

N × N defined a relation

(a, b) R (c, d) if and only if

$ad(b - c) = bc(a - d)$

(i) Reflexive : aRa $\forall a \in$ A

Let (a, b) R (a, b)

$\Rightarrow ab(b - a) = ba(a - b)$

But $ab(b - a) \neq ba(a - b)$

$\therefore$ R is not reflexive.

(ii) Symmetric: a R b and b R c $\forall a, b, c \in$ A

(a, b) R $(c, d) \Rightarrow ad(b - c) = bc(a - d)$

Then (c, d) R $(a, b) \Rightarrow cb(d - a) = ba(c - b)$

$\Rightarrow cb(a - d) = ba(b - c)$

R is symmetric

(iii) Transitive:

(a, b) R $(c, d) \Rightarrow ad(b - c) = bc(a - d)$

(2, 3) R (3, 2) and (3, 2) R (5, 30)

But (2, 3) is not related to (5, 30)

R is not transitive

15. Option (2) is correct.

S_1 $(p \Rightarrow q) \vee (p \wedge (\sim q))$ is a tautology.

Truth Table

			A	B	
p	q	$\sim q$	$p \Rightarrow q$	$p \wedge \sim q$	A ∨ B
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	T	F	T
F	F	T	T	F	T

Hence S_1 statement is a tautology.

S_2 : $((\sim p) \Rightarrow (\sim q)) \wedge ((\sim p) \vee q)$ is a contradiction.

Truth Table

				A	B	
p	q	$\sim p$	$\sim q$	$\sim p \Rightarrow \sim q$	$\sim p \vee q$	A ∧ B
T	T	F	F	T	T	T
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	T	T	T

Hence S_2 statement is not a contradiction only S_1 is correct.

16. Option (1) is correct.

Let four term of G.P.

$\dfrac{a}{r^3}, \dfrac{a}{r}, ar, ar^3$

So, $\dfrac{a}{r^3} + \dfrac{a}{r} + ar + ar^3 = 126$ ...(1)

And $\dfrac{a}{r^3} \times \dfrac{a}{r} \times ar \times ar^3 = 1296$ ...(2)

$\Rightarrow a^4 = 1296$

$\Rightarrow a^4 = 6^4 \Rightarrow a = 6$

From (1)

$\dfrac{6}{r^3} + \dfrac{6}{r} + 6r + 6r^3 = 126$

$$\Rightarrow \frac{1}{r^3}+\frac{1}{r}+r+r^3=21$$

$$\Rightarrow \left(r^3+\frac{1}{r^3}\right)+\left(r+\frac{1}{r}\right)=21$$

$$\Rightarrow \left(r+\frac{1}{r}\right)+\left(r+\frac{1}{r}\right)^3-3\left(r+\frac{1}{r}\right)=21$$

$$\Rightarrow \left(r+\frac{1}{r}\right)^3-2\left(r+\frac{1}{r}\right)=21$$

Let $r+\frac{1}{r}=A$

$\Rightarrow A^3-2A-21=0 \Rightarrow A^3-2A=21$

A = 3 is a root

So, $r+\frac{1}{r}=3 \Rightarrow r^2+1=3r$

$\Rightarrow r^2-3r+1=0$

$$\Rightarrow r=\frac{3\pm\sqrt{9-4}}{2}=\frac{3\pm\sqrt{5}}{2}$$

Sum of common ratio

$$=\frac{9}{4}+\frac{5}{4}+\frac{3\sqrt{5}}{2}+\frac{9}{4}+\frac{5}{4}-\frac{3\sqrt{5}}{2}=7$$

17. Option (3) is correct.

Given $A=\begin{bmatrix}1&0&0\\0&4&-1\\0&12&-3\end{bmatrix}$

$$A^2=A\times A=\begin{bmatrix}1&0&0\\0&4&-1\\0&12&-3\end{bmatrix}\times\begin{bmatrix}1&0&0\\0&4&-1\\0&12&-3\end{bmatrix}=\begin{bmatrix}1&0&0\\0&4&-1\\0&12&-3\end{bmatrix}=A$$

$A^3=A^4=A^5=\ldots\ldots=A$

We have by binomial expansion $(x+a)^n={}^nC_0x^na^0+{}^nC_1x^{n-1}a+\ldots\ldots+{}^nC_na^n$

$(A+I)^{11}={}^{11}C_0A^{11}I^0+{}^{11}C_1A^{10}I+\ldots.+{}^{11}C_{11}I^{11}$

$=A^{11}+11A^{10}I+\ldots.+I^{11}$

$=A+11AI+\ldots.+I^{11}$

$=A(1+11+\ldots.+)+I$

$=A(2^{11}-1)+I$

$=2047A+I$

Sum of diagonal elements $=2047(1+4-3)+3$

$=4097$

18. Option (2) is correct.

Given

$$\sqrt{x^2-4x+3}+\sqrt{x^2-9}=\sqrt{4x^2-14x+6}$$

$$\sqrt{x^2-3x-x+3}+\sqrt{(x-3)(x+3)}=\sqrt{4x(x-3)-2(x-3)}$$

$$\Rightarrow \sqrt{(x-3)(x-1)}+\sqrt{(x-3)(x+3)}=\sqrt{(x-3)(4x-2)}$$

$\sqrt{x-3}=0 \therefore x=3$ is a root

$$\Rightarrow \sqrt{x-1}+\sqrt{x+3}=\sqrt{(4x-2)}$$

On squaring both sides

$$x-1+x+3+2\sqrt{(x-1)(x+3)}=4x-2$$

$$\Rightarrow 2x+2+2\sqrt{(x-1)(x+3)}=4x-2$$

$$\Rightarrow 2\sqrt{(x-1)(x+3)}=4x-2-2x-2$$

$$\Rightarrow \sqrt{(x-1)(x+3)}=x-2$$

$\Rightarrow 6x=7 \Rightarrow x=\frac{7}{6}$ (Not possible)

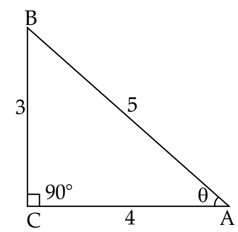
$\therefore$ Number of real root is only one, $x=3$

19. Option (3) is correct.

$$\sin^{-1}\left(\frac{\alpha}{17}\right)+\cos^{-1}\left(\frac{4}{5}\right)-\tan^{-1}\left(\frac{77}{36}\right)=0, 0<\alpha<13$$

$$\Rightarrow \sin^{-1}\left(\frac{\alpha}{17}\right)=\tan^{-1}\left(\frac{77}{36}\right)-\cos^{-1}\left(\frac{4}{5}\right)$$

$$=\tan^{-1}\left(\frac{77}{36}\right)-\tan^{-1}\left(\frac{3}{4}\right)$$



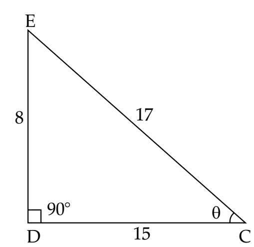
$$\sin^{-1}\left(\frac{\alpha}{17}\right)=\tan^{-1}\left(\frac{\frac{77}{36}-\frac{3}{4}}{1+\frac{77}{36}\times\frac{3}{4}}\right)$$

We have $\tan^{-1}x=\tan^{-1}\left\{\frac{\frac{77-27}{36}}{\frac{48+77}{48}}\right\}$

$$\tan^{-1}y=\tan^{-1}\left(\frac{x-y}{1+xy}\right)$$

$$=\tan^{-1}\left\{\frac{50}{36}\times\frac{48}{125}\right\}=\tan^{-1}\left\{\frac{8}{15}\right\}=\sin^{-1}\left(\frac{8}{17}\right)$$

$$\Rightarrow \sin^{-1}\left(\frac{\alpha}{17}\right)=\sin^{-1}\left(\frac{8}{17}\right)$$



$\Rightarrow \alpha=8$

$\therefore \sin^{-1}(\sin\alpha)+\cos^{-1}(\cos\alpha)$

$=\sin^{-1}(\sin 8)+\cos^{-1}(\cos 8)$

$=-8+3\pi+8-2\pi=\pi$

20. Option (4) is correct.

$\alpha\in(0,1), \beta=\log_e(1-\alpha)$

Let $P_n(x)= x+\frac{x^2}{2}+\frac{x^3}{3}+\ldots.+\frac{x^n}{n}, x\in(0,1)$

$$\int_0^\alpha \frac{t^{50}}{1-t}dt=\int_0^\alpha \frac{t^{50}-1+1}{1-t}dt$$

$$=-\int_0^\alpha \frac{1-t^{50}}{1-t}dt+\int_0^\alpha \frac{1}{1-t}dt$$

$$=-\int_0^\alpha (1+t+t^2+\ldots.+t^{49})dt+\left[-\log(1-t)\right]_0^\alpha$$

$$=-\left[t+\frac{t^2}{2}+\frac{t^3}{3}+\ldots.+\frac{t^{50}}{50}\right]_0^\alpha-\log[(1-\alpha)]$$

$$= -\left[\alpha + \frac{\alpha^2}{2} + \frac{\alpha^3}{3} + ... + \frac{\alpha^{50}}{50}\right] - \log(1-\alpha)$$

$- P_{50}(\alpha) - \log(1 - \alpha) = -(\beta + P_{50}(\alpha))$

Section B

21. Correct answer is [2].

We have by binomial expansion

$(x + a)^n = {}^nC_0 x^n a^0 + {}^nC_1 x^{n-1} a^1 + ... {}^nC_r x^n a^{n-r} + ...$

$$\left(x^{\frac{2}{3}} + \frac{2}{x^3}\right)^{30}$$

$$T_{r+1} = {}^{30}C_r (x^{2/3})^{30-r} \left(\frac{2}{x^3}\right)^r$$

$$= {}^{30}C_r 2^r (x)^{\frac{2(30-r)}{3} - 3r}$$

$$= {}^{30}C_r .2^r . (x)^{\frac{60-2r-9r}{3}} = {}^{30}C_r .2^r . (x)^{\frac{60-11r}{3}}$$

$$\frac{60-11r}{3} < 0 \quad \therefore 11r > 60$$

$$\Rightarrow r > \frac{60}{11}$$

$r = 6$

$T_7 = {}^{30}C_6 .2^6 . x^{-2}$ Then

$\beta = {}^{30}C_6 \times 2^6 \in N \therefore \alpha = 2$

22. Correct answer is [72].

$$f(x) = \frac{x+|x|}{2} \text{ and } g(x) = \begin{cases} x & x < 0 \\ x^2 & x \geq 0 \end{cases}$$

$$fog(x) = f[g(x)] = \begin{cases} g(x) & g(x) \geq 0 \\ 0 & g(x) < 0 \end{cases}$$

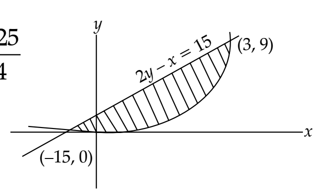
$$fog(x) = \begin{cases} x^2 & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Given lines are $2y - x = 15$ and $y = 0$

$$\text{Area} = \int_0^3 f(x)dx = \int_0^3 \left(\frac{x+15}{2} - x^2\right)dx + \frac{1}{2} \times \frac{15}{2} \times 15$$

$$= \left[\frac{x^2}{4} + \frac{15x}{2} - \frac{x^3}{3}\right]_0^3 + \frac{225}{4}$$

$$= \frac{9}{4} + \frac{45}{2} - 9 + \frac{225}{4} = 72$$



Area = 72

23. Correct answer is [710].

4 digit number which are less than 2800 are 1000 – 2799

Numbers which are divisible by 3

$T_n = a + (n - 1)d$

$2799 = 1002 + (n - 1) 3$

$\therefore n = 600$

Numbers which are divisible by 11 in 1000 – 2799

= (Number which are divisible by 11 in 1 – 2799)

– (Number which are divisible by 11 in 1 – 999)

$$= \frac{2799}{11} - \frac{999}{11} = 164$$

Number which are divisible by 33 in 1000 – 2799

= (Number which are divisible by 33 in 1 – 2799)

– (Number which are divisible by 33 in 1 – 999)

$$= \frac{2799}{33} - \frac{999}{33} = 84 - 30 = 54$$

Total numbers = $n(3) + n(11) - n(33)$

= 600 + 164 – 54 = 710

24. Correct answer is [5].

Given variance $\sigma^2 = 3$

x	f	$d = x - 5$	fd	d^2	$f.d^2$
2	3	–3	–9	9	27
3	6	–2	–12	4	24
4	16	–1	–16	1	16
5	α	0	0	0	0
6	9	1	9	1	9
7	5	2	10	4	20
8	6	3	18	9	54
	$\Sigma f = 45+\alpha$		$\Sigma f.d = 0$		$\Sigma fd^2 = 150$

We have variance

$$\sigma^2 = \frac{\Sigma fd^2}{\Sigma f} - \left(\frac{\Sigma fd}{\Sigma f}\right)^2$$

$$\Rightarrow \frac{150}{45+\alpha} - 0 = 3$$

$\Rightarrow 150 = 3 \times (45 + \alpha)$

$\Rightarrow 50 = 45 + \alpha \qquad \Rightarrow \alpha = 5$

25. Correct answer is [9].

Given equation of planes

$P_1 : \vec{r}.(\hat{i} + \hat{j} + 2\hat{k}) = 9$

$\Rightarrow P_1: x + y + 2z = 9$

$P_2 : \vec{r}.(2\hat{i} - \hat{j} + \hat{k}) = 15$

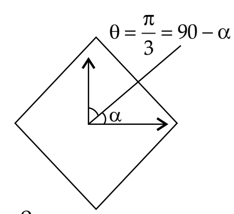
$P_2 : 2x - y + z = 15$

$$\cos\theta = \frac{l_1l_2 + m_1m_2 + n_1n_2}{\sqrt{l_1^2 + m_1^2 + n_1^2}\sqrt{l_2^2 + m_2^2 + n_2^2}}$$

$$\Rightarrow \cos\theta = \frac{2\times1 + 1\times-1 + 2\times1}{\sqrt{6}\times\sqrt{6}} = \frac{2-1+2}{6} = \frac{3}{6} = \frac{1}{2}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

$$\Rightarrow 90 - \alpha = \frac{\pi}{3} \Rightarrow \alpha = 90^o - 60^o = 30^o$$



$$\Rightarrow \alpha = \frac{\pi}{6}$$

Then, $(\tan^2\theta).(\cot^2\alpha)$

$$= \tan^2\frac{\pi}{3}.\cot^2\frac{\pi}{6} = (\sqrt{3})^2.(\sqrt{3})^2 = 9$$

26. Correct answer is [2997].

5 digit numbers be constructed using the digits 0, 2, 3, 4, 7, 9

2 .... $\to 6^4 = 1296$

3 .... $\to$ 1296

4 0 ..... $\to 6^3 = 216$

4 2 0 .... $\to 6^2 = 36$

4 2 2 .... $\to$ 36

4 2 3 .... $\to$ 36

4 2 4 .... $\to$ 36

4 2 7 .... $\to$ 36

4 2 9 0 .... $\to 6^1 = 6$

4 2 9 2 0 .... $\to$ 1

4 2 9 2 2 .... $\to$ 1

4 2 9 2 3 .... $\to$ 1

$6^4 \times 2 + 6^3 + 6^2 \times 5 + 6 + 3 = 2997$ So the rank of the number 42923 is 2997

27. Correct answer is [36].

Given $|\vec{a}| = \sqrt{14}, |\vec{b}| = \sqrt{6}$

and $|\vec{a} \times \vec{b}| = \sqrt{48}$

We have

$$|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 . |\vec{b}|^2 - (\vec{a}.\vec{b})^2$$

$$48 = 14 \times 6 - (\vec{a}.\vec{b})^2$$

$$\therefore (\vec{a}.\vec{b}) = 84 - 48 = 36$$

28. Correct answer is [180].

Given equation of line L: $\dfrac{x-1}{2} = \dfrac{y+1}{-1} = \dfrac{z-3}{1}$

Let the point on line L is $(2\lambda + 1, -\lambda - 1, \lambda + 3)$

equation of plane $2x + y + 3z = 16$

$\Rightarrow 2(2\lambda + 1) - \lambda - 1 + 3(\lambda + 3) = 16$

$\Rightarrow 4\lambda + 2 - \lambda - 1 + 3\lambda + 9 = 16$

$6\lambda + 10 = 16$ $\qquad \therefore 6\lambda = 16 - 10 = 6$

Point P = (3, –2, 4) $\qquad \lambda = 1$

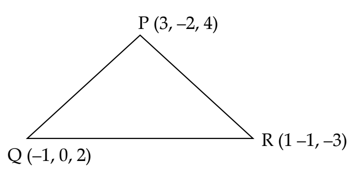
Dr's of QR = $<2\lambda, -\lambda, \lambda + 6>$

$4\lambda + \lambda + \lambda + 6 = 0$

$\therefore \lambda = -1$

$\overline{QR} = 2\hat{i} - \hat{j} - 5\hat{k}$

$\overline{QP} = 4\hat{i} - 2\hat{j} + 2\hat{k}$



$$\overline{QR} \times \overline{QP} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & -5 \\ 4 & -2 & 2 \end{vmatrix} = -12\hat{i} - 24\hat{j}$$

$$\alpha = \frac{1}{2} \times \sqrt{144 + 576}$$

$$\therefore \alpha^2 = \frac{720}{4} = 180 \quad \therefore \alpha^2 = 180$$

29. Correct answer is [8].

Given $a_5 = 2a_7$ and $a_{11} = 18$

$a_5 = 2a_7$

$a_1 + 4d = 2(a_1 + 6d)$

$a_1 + 8d = 0$

Also, $a_1 + 10d = 18$

On solving, we get

$\Rightarrow a_1 = -72, d = 9$

$\Rightarrow a_{18} = a_1 + 17d = -72 + 153 = 81$

$\Rightarrow a_{10} = a_1 + 9d = 9$

$$12\left(\frac{\sqrt{a_{11}} - \sqrt{a_{10}}}{d} + \frac{\sqrt{a_{12}} - \sqrt{a_{11}}}{d} + \ldots + \frac{\sqrt{a_{18}} - \sqrt{a_{17}}}{d}\right)$$

$$= 12\left(\frac{\sqrt{a_{18}} - \sqrt{a_{10}}}{d}\right) = \frac{12 \times (9-3)}{9} = 8$$

30. Correct answer is [9].

Find remainder 5^{99} divided by 11

$(5) . (25)^{49}$

$\downarrow$

$(5) . (3)^{49}$

$(15) . (3)^{48}$

$\downarrow$

$(15) . (9)^{24}$

$\downarrow$

$(4)\ (-2)^{24}$

$(4) . (2)^{24}$

$(4) . (2)^4 . (32)^4$

$\downarrow$

$(4) . (5)\ (-1)^4$

20

$\downarrow$

9 Remainder

❑❑