JEE (Main) MATHEMATICS SOLVED PAPER

2023

31st Jan. Shift 2

Section A

Q. 1. The equation $e^{4x} + 8e^{3x} + 13e^{2x} - 8e^{x} + 1 = 0, x \in R$ has:

(1) four solutions two of which are negative
(2) two solutions and only one of them is negative
(3) two solutions and both are negative
(4) no solution

Q. 2. Among the relations

$S = \{(a, b): a, b \in R - \{0\}, 2 + \frac{a}{b} > 0\}$ and

$T = \{(a, b): a, b \in R, a^2 - b^2 \in Z\}$.

(1) neither S nor T is transitive
(2) S is transitive but T is not
(3) T is symmetric but S is not
(4) both S and T are symmetric

Q. 3. Let $\alpha > 0$. If $\int_0^{\alpha} \frac{x}{\sqrt{x+\alpha}-\sqrt{x}}dx = \frac{16+20\sqrt{2}}{15}$, then α is equal to:

(1) 4 **(2)** $2\sqrt{2}$ **(3)** $\sqrt{2}$ **(4)** 2

Q. 4. The complex number $z = \frac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$ is equal to:

(1) $\sqrt{2}i\left(\cos\frac{5\pi}{12}-i\sin\frac{5\pi}{12}\right)$

(2) $\sqrt{2}\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)$

(3) $\sqrt{2}\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$

(4) $\cos\frac{\pi}{12}-i\sin\frac{\pi}{12}$

Q. 5. Let $y = y(x)$ be the solution of the differential equation $(3y^2 - 5x^2)y\,dx + 2x(x^2 - y^2)\,dy = 0$ such that $y(1) = 1$. Then $|(y(2))^3 - 12y(2)|$ is equal to:

(1) $16\sqrt{2}$ **(2)** $32\sqrt{2}$ **(3)** 32 **(4)** 64

Q. 6. $\lim_{x\to\infty} \frac{(\sqrt{3x+1}+\sqrt{3x-1})^6+(\sqrt{3x+1}-\sqrt{3x-1})^6}{(x+\sqrt{x^2-1})^6+(x-\sqrt{x^2-1})^6}x^3$

(1) does not exist **(2)** is equal to 27
(3) is equal to $\frac{27}{2}$ **(4)** is equal to 9

Q. 7. The foot of perpendicular from the origin O to a plane P which meets the co-ordinate axes at the points A, B, C is $(2, a, 4)$, $a \in N$. If the volume of the tetrahedron OABC is 144 unit^3, then which of the following points is NOT on P ?

(1) (0, 6, 3) **(2)** (0, 4, 4) **(3)** (2, 2, 4) **(4)** (3, 0, 4)

Q. 8. Let $(a, b) \subset (0, 2\pi)$ be the largest interval for which $\sin^{-1}(\sin\theta) - \cos^{-1}(\sin\theta) > 0, \theta \in (0, 2\pi)$, holds. If $ax^2 + \beta x + \sin^{-1}(x^2 - 6x + 10) + \cos^{-1}(x^2 - 6x + 10) = 0$ and $\alpha - \beta = b - a$, then α is equal to :

(1) $\frac{\pi}{16}$ **(2)** $\frac{\pi}{48}$ **(3)** $\frac{\pi}{12}$ **(4)** $\frac{\pi}{8}$

Q. 9. Let the mean and standard deviation of marks of class A of 100 students be respectively 40 and α (> 0), and the mean and standard deviation of marks of class B of n students be respectively 55 and $30 - \alpha$. If the mean and variance of the marks of the combined class of $100 + n$ students are respectively 50 and 350 , then the sum of variances of classes A and B is :

(1) 650 **(2)** 450 **(3)** 900 **(4)** 500

Q. 10. The absolute minimum value of the function $f(x) = |x^2 - x + 1| + [x^2 - x + 1]$, where $[t]$ denotes the greatest integer function, in the interval $[-1, 2]$, is:

(1) $\frac{1}{4}$ **(2)** $\frac{3}{2}$ **(3)** $\frac{5}{4}$ **(4)** $\frac{3}{4}$

Q. 11. Let H be the hyperbola, whose foci are $(1 \pm \sqrt{2}, 0)$ and eccentricity is $\sqrt{2}$. Then the length of its latus rectum is

(1) $\frac{3}{2}$ **(2)** 2 **(3)** 3 **(4)** $\frac{5}{2}$

Q. 12. Let $a_1, a_2, a_3, \ldots$ be an A.P. If $a_7 = 3$, the product a_1a_4 is minimum and the sum of its first n terms is zero, then $n! - 4a_{n(n+2)}$ is equal to:

(1) 9 **(2)** $\frac{33}{4}$ **(3)** $\frac{381}{4}$ **(4)** 24

Q. 13. If a point P (α, β, γ) satisfying

$$(\alpha\ \beta\ \gamma)\begin{pmatrix}2 & 10 & 8\\ 9 & 3 & 8\\ 8 & 4 & 8\end{pmatrix} = (0\ 0\ 0)$$

lies on the plane $2x + 4y + 3z = 5$, then $6\alpha + 9\beta + 7\gamma$ is equal to :

(1) -1 **(2)** $\frac{11}{5}$ **(3)** $\frac{5}{4}$ **(4)** 11

Q. 14. Let : $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = 5\hat{i} - 3\hat{j} + 3\hat{k}$ be three vectors. If $\vec{r}$ is a vector such that, $\vec{r}\times\vec{b} = \vec{c}\times\vec{b}$ and $\vec{r}.\vec{a} = 0$ then $25|\vec{r}|^2$ is equal to

(1) 560 **(2)** 449 **(3)** 339 **(4)** 336

Q. 15. Let the plane P: $8x + a_1y + a_2z + 12 = 0$ be parallel to the line L: $\frac{x+2}{2} = \frac{y-3}{3} = \frac{z+4}{5}$. If the intercept of P on the y-axis is 1, then the distance between P and L is:

(1) $\sqrt{\frac{7}{2}}$ **(2)** $\sqrt{\frac{2}{7}}$ **(3)** $\frac{6}{\sqrt{14}}$ **(4)** $\sqrt{14}$

Q. 16. Let P be the plane, passing through the point $(1, -1, -5)$ and perpendicular to the line joining the points $(4,1, -3)$ and $(2,4,3)$. Then the distance of P from the point $(3, -2,2)$ is

(1) 5 **(2)** 4 **(3)** 7 **(4)** 6

Q. 17. The number of values of $r \in \{p, q, \sim p, \sim q\}$ for which $((p \wedge q) \Rightarrow (r \vee q)) \wedge ((p \wedge r) \Rightarrow q)$ is a tautology, is:

(1) 3 **(2)** 4 **(3)** 1 **(4)** 2

Q. 18. The set of all values of a^2 for which the line $x + y = 0$ bisects two distinct chords drawn from a point $P\left(\frac{1+a}{2}, \frac{1-a}{2}\right)$ on the circle $2x^2 + 2y^2 - (1 + a)\,x - (1 - a)\,y = 0$, is equal to:

(1) $(0, 4]$ **(2)** $(4, \infty)$ **(3)** $(2, 12]$ **(4)** $(8, \infty)$

Q. 19. If $\phi(x) = \frac{1}{\sqrt{x}}\int_{\frac{\pi}{4}}^{x}\left(4\sqrt{2}\sin t - 3\phi'(t)\right)dt$, $x > 0$, then $\phi'\left(\frac{\pi}{4}\right)$ is equal to :

(1) $\frac{8}{6+\sqrt{\pi}}$ **(2)** $\frac{4}{6+\sqrt{\pi}}$ **(3)** $\frac{8}{\sqrt{\pi}}$ **(4)** $\frac{4}{6-\sqrt{\pi}}$

Q. 20. Let $f : R - \{2, 6\} \rightarrow R$ be real valued function defined as $f(x) = \frac{x^2+2x+1}{x^2-8x+12}$. Then range of f is

(1) $\left(-\infty, -\frac{21}{4}\right) \cup (0, \infty)$

(2) $\left(-\infty, -\frac{21}{4}\right] \cup [1, \infty)$

(3) $\left(-\infty, -\frac{21}{4}\right] \cup \left[\frac{21}{4}, \infty\right)$

(4) $\left(-\infty, -\frac{21}{4}\right] \cup [0, \infty)$

Section B

Q. 21. Let $A = [a_{ij}]$, $a_{ij} \in Z \cap [0, 4]$, $1 \le i, j \le 2$. The number of matrices A such that the sum of all entries is a prime number $p \in (2, 13)$ is

Q. 22. Let A be a $n \times n$ matrix such that $|A| = 2$. If the determinant of the matrix Adj (2 . Adj (2 A^{-1})).is 2^{84} then n is equal to

Q. 23. If the constant term in the binomial expansion of $\left(\frac{x^{\frac{5}{2}}}{2} - \frac{4}{x^l}\right)^9$ is –84 and the coefficient of x^{-3l} is $2^{\alpha}\beta$, where $\beta < 0$ is an odd number, then $|\alpha l - \beta|$ is equal to

Q. 24. Let S be the set of all $a \in N$ such that the area of the triangle formed by the tangent at the point $P(b,c)$, $b, c \in N$, on the parabola $y^2 = 2ax$ and the lines $x = b$, $y = 0$ is 16 unit^2, then $\sum_{a \in S} a$ is equal to

Q. 25. Let the area of the region $\{(x, y): |2x - 1| \le y \le |x^2 - x|, 0 \le x \le 1\}$ be A. Then $(6A + 11)^2$ is equal to

Q. 26. The coefficient of x^{-6}, in the expansion of $\left(\frac{4x}{5} + \frac{5}{2x^2}\right)^9$, is

Q. 27. Let A be the event that the absolute difference between two randomly choosen real numbers in the sample space $[0, 60]$ is less than or equal to a. If $P(A) = \frac{11}{36}$, then a is equal to

Q. 28. If ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 11 : 21$, then $n^2 + n + 15$ is equal to :

Q. 29. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $|\vec{a}| = \sqrt{31}, 4|\vec{b}| = |\vec{c}| = 2$ and $2(\vec{a} \times \vec{b}) = 3|\vec{c} \times \vec{a}|$. If the angle between $\vec{b}$ and $\vec{c}$ is $\frac{2\pi}{3}$, then $\left(\frac{\vec{a} \times \vec{c}}{\vec{a}.\vec{b}}\right)^2$ is equal to

Q. 30. The sum $1^2 - 2.\,3^2 + 3.5^2 - 4\,.\,7^2 + 5.9^2 - ... + 15.29^2$ is

Answer Key

Q. No.	Answer	Topic's Name	Chapter's Name
1	(3)	Quadratic equation	Complex number and Quadratic equation
2	(3)	Types of Relations	Relations and Functions
3	(4)	Definite Integral	Integral calculus
4	(3)	De-Morgan's Law	Complex number
5	(2)	Homogeneous differential equation	Differential equations
6	(2)	Limits	Limits, Continuity and Differentiability
7	(3)	Tetrahedron	3D

Q. No.	Answer	Topic's Name	Chapter's Name
8	(3)	Inverse Trigonometrical Identities	Inverse Trigonometry
9	(4)	Standard Deviation	Statistics
10	(4)	Maxima and Minima	Differentiation
11	(2)	Hyperbola	Conic Section
12	(4)	A.P.	Sequence and series
13	(4)	Solution of simultaneous equations	Matrix
14	(3)	Vector	Vector Calculus
15	(4)	Parallel Plane	3D
16	(1)	Perpendicular Plane	3D
17	(4)	Prepositional Logics	Mathematical Reasoning
18	(4)	Circle	2D
19	(1)	Definite Integral	Integral Calculus
20	(4)	Range of a Function	Function or Mapping
21	[204]	Matrix	Matrix
22	[5]	Adjoint	Matrix
23	[98]	Binomial Expansion	Binomial Theorem
24	[146]	Area of Triangle	Conic Section
25	[125]	Area Bounded	Definite Integral
26	[5040]	Binomial expansion	Binomial Theorem
27	[10]	Probability	Probability
28	[45]	Permutation	Permutations and combinations
29	[3]	Dot and cross product	Vector Algebra
30	[6952]	Special Series	Sequence and Series

Solutions

Section A

1. Option (3) is correct.

Given equation

$e^{4x} + 8e^{3x} + 13e^{2x} - 8e^{x} + 1 = 0 \qquad x \in R$

Let $e^x = t$

$t^4 + 8t^3 + 13t^2 - 8t + 1 = 0 \qquad ...(1)$

Dividing by t^2

$$t^2 + 8t + 13 - \frac{8}{t} + \frac{1}{t^2} = 0$$

$$\left(t^2 + \frac{1}{t^2}\right) + 8\left(t - \frac{1}{t}\right) + 13 = 0$$

Let $t - \frac{1}{t} = p$

$$\therefore \left(t - \frac{1}{t}\right)^2 = p^2$$

$$\therefore t^2 + \frac{1}{t^2} = p^2 + 2$$

$$\Rightarrow p^2 + 8p + 15 = 0$$

$p^2 + 5p + 3p + 15 = 0$

$(p + 5)(p + 3) = 0$

$t - \frac{1}{t} + 5 = 0 \Rightarrow t^2 - 1 + 5t = 0$

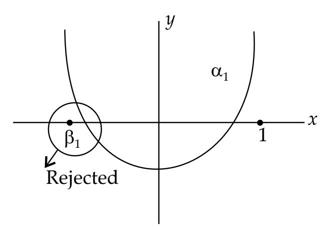
$\Rightarrow t^2 + 5t - 1 = 0$

$t - \frac{1}{t} + 3 = 0$

$\Rightarrow t^2 + 3t - 1 = 0$

$$\Rightarrow t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

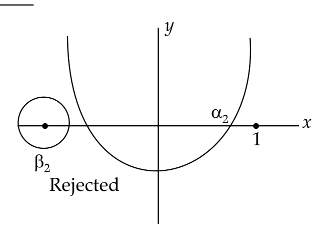
$$= \frac{-5 \pm \sqrt{25 - 4 \times 1 \times -1}}{2} = \frac{-5 \pm \sqrt{29}}{2}$$



and $t = \frac{-3 \pm \sqrt{9 - 4 \times 1 \times -1}}{2}$

$t = \frac{-3 \pm \sqrt{13}}{2}$

$0 < \alpha_1 < 1 \quad t = e^x$



$\Rightarrow x = \log \alpha_1 < 0$

$x = \log \alpha_2 < 0$

Two solution and both are –ve.

2. Option (3) is correct.

S $= \{(a, b) : a, b \in R - \{0\}, 2 + \frac{a}{b} > 0\}$

and T $= \{(a, b) : a, b \in R, a^2 - b^2 \in Z\}$

be two relations
Symmetric relation
aRb and bRa $\forall\ a, b \in$ A

$S:\ 2+\dfrac{a}{b}>0 \quad \therefore \dfrac{a}{b}>-2$

Let $(-1, 2) \in S \left(\because \dfrac{-1}{2}>-2\right)$

and $(2, -1) \notin S\left(\because \dfrac{2}{-1}\text{ not greater than }-2\right)$

So S is not symmetric.

For T, if $(a, b) \in T \Rightarrow a^2 - b^2 \in z$

$\Rightarrow -(b^2-a^2) \in Z$

$b^2-a^2 \in z \Rightarrow (b, a) \in T$

So T is symmetric relation.

3. **Option (4) is correct.**

$\alpha > 0$

$$\int_0^\alpha \frac{xdx}{\sqrt{x+\alpha}-\sqrt{x}}=\frac{16+20\sqrt{2}}{15}$$

$$\int_0^\alpha \frac{x\times(\sqrt{x+a}+\sqrt{x})}{(x+\alpha-x)}dx=\frac{16+20\sqrt{2}}{15}$$

$$\frac{1}{\alpha}\int_0^\alpha\left[x\sqrt{x+\alpha}+x^{\frac{3}{2}}\right]dx=\frac{16+20\sqrt{2}}{15}$$

Let $I_1 = \int_0^\alpha x\sqrt{x+\alpha}\,dx$

$I_2 = \int_0^\alpha x^{\frac{3}{2}}\,dx$

We have $I_1 = \int_0^\alpha (x+\alpha-\alpha)\sqrt{x+\alpha}\,dx$

$$=\int_0^\alpha (x+\alpha)^{\frac{3}{2}}dx-\alpha\int_0^\alpha\sqrt{x+\alpha}\,dx$$

$$=\frac{2}{5}\left((x+\alpha)^{\frac{5}{2}}\right)_0^\alpha-\left(\alpha\frac{2}{3}(x+\alpha)^{\frac{3}{2}}\right)_0^\alpha$$

$$I_1=\frac{2}{5}\left[(2\alpha)^{\frac{5}{2}}-\alpha^{\frac{5}{2}}\right]-\frac{2}{3}\alpha\left[(2\alpha)^{\frac{3}{2}}-\alpha^{\frac{3}{2}}\right]$$

$$=\frac{2}{5}\times(2\alpha)^{\frac{5}{2}}-\frac{2}{5}(\alpha)^{\frac{5}{2}}-\frac{2\alpha}{3}(2\alpha)^{\frac{3}{2}}+\frac{2}{3}\alpha^{\frac{5}{2}}$$

$$=\frac{2}{5}(2\alpha)^{\frac{5}{2}}-\frac{2}{5}\alpha^{\frac{5}{2}}-\frac{1}{3}(2\alpha)^{\frac{5}{2}}+\frac{2}{3}\alpha^{\frac{5}{2}}$$

$$=(2\alpha)^{\frac{5}{2}}\left[\frac{2}{5}-\frac{1}{3}\right]-\alpha^{\frac{5}{2}}\left[\frac{2}{5}-\frac{2}{3}\right]$$

$$=(2\alpha)^{\frac{5}{2}}\left[\frac{1}{15}\right]-2\alpha^{\frac{5}{2}}\left[\frac{-2}{15}\right]$$

$$=(2\alpha)^{\frac{5}{2}}\left(\frac{1}{15}\right)+\frac{4\alpha^{\frac{5}{2}}}{15}=\frac{4\alpha^{\frac{5}{2}}}{15}[\sqrt{2}+1]$$

$$I_2=\int_0^\alpha x^{3/2}dx=\left[\frac{2}{5}x^{\frac{5}{2}}\right]_0^\alpha=\frac{2}{5}\left[\alpha^{\frac{5}{2}}\right]$$

$$I=\frac{1}{\alpha}[I_1+I_2]=\frac{16+20\sqrt{2}}{15}$$

$$\text{So, }\frac{1}{\alpha}\left[\frac{4\alpha^{\frac{5}{2}}}{15}(\sqrt{2}+1)+\frac{2}{5}\alpha^{\frac{5}{2}}\right]=\frac{16+20\sqrt{2}}{15}$$

$$\Rightarrow\frac{2}{15}\alpha^{\frac{3}{2}}[2\sqrt{2}+5]=\frac{16+20\sqrt{2}}{15}$$

$\Rightarrow \alpha = 2$

4. **Option (3) is correct.**

The complex number

$$z=\frac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$$

$$e^{i\theta}=\cos\theta+i\sin\theta$$

$$z=\frac{(i-1)}{e^{i\frac{\pi}{3}}}$$

Also, $i-1=\sqrt{2}\left(\dfrac{i}{\sqrt{2}}-\dfrac{1}{\sqrt{2}}\right)$

$$=\sqrt{2}\left(\frac{-1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)$$

$$=\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)=\sqrt{2}e^{i\frac{3\pi}{4}}$$

$$z=\frac{\sqrt{2}e^{i\frac{3\pi}{4}}}{e^{i\frac{\pi}{3}}}=\sqrt{2}e^{i\frac{3\pi}{4}}.e^{-i\frac{\pi}{3}}$$

$$=z\sqrt{2}e^{i\left(3\frac{\pi}{4}-\frac{\pi}{3}\right)}=\sqrt{2}e^{i\frac{5\pi}{12}}=\sqrt{2}\left[\cos\left(\frac{5\pi}{12}\right)+i\sin\left(\frac{5\pi}{12}\right)\right]$$

5. **Option (2) is correct.**

Given $y(3y^2-5x^2)\,dx + 2x(x^2-y^2)\,dy = 0$

$Mdx + Ndy = 0$

$M = y(3y^2-5x^2),\ N = 2x(x^2-y^2)$

We check the condition of exactness

$$\frac{\partial M}{\partial y}=9y^2-5x^2,\ \frac{\partial N}{\partial x}=6x^2-2y^2$$

$\dfrac{\partial M}{\partial y}\neq\dfrac{\partial N}{\partial x}$ which is not exact differential equation

$y(3y^2-5x^2)\,dx = -2x(x^2-y^2)\,dy$

$\therefore \dfrac{dy}{dx}=\dfrac{-y(3y^2-5x^2)}{2x(x^2-y^2)}$ which is homogeneous differential equation.

Put $y = v.x$

$$\frac{dy}{dx}=v+\frac{xdx}{dv}$$

$$v+\frac{xdv}{dx}=\frac{-vx(3v^2x^2-5x^2)}{2x(x^2-v^2x^2)}$$

$$=\frac{-v}{2}\frac{x^2(3v^2-5)}{x^2(1-v^2)}=\frac{-v(3v^2-5)}{2(1-v^2)}$$

$$\frac{xdv}{dx}=\frac{-3v^3+5v}{2(1-v^2)}-v$$

$$= \frac{-3v^3 + 5v - 2v(1-v^2)}{2(1-v^2)} = \frac{-3v^3 + 5v - 2v + 2v^3}{2(1-v^2)}$$

$$\frac{xdv}{dx} = \frac{-v^3 + 3v}{2(1-v^2)} = \frac{v(3-v^2)}{2(1-v^2)}$$ by separation of variables

$$\int \frac{2(1-v^2)}{v(3-v^2)} dv = \int \frac{dx}{x} \Rightarrow \int \frac{1-v^2}{v(3-v^2)} dv = \int \frac{dx}{2x}$$

$$\Rightarrow \frac{1}{3}\int \frac{3(1-v^2)}{(3v-v^3)} dv = \int \frac{dx}{2x}$$

Let $(3v - v^3) = t$

$\therefore (3 - 3v^2)\, dv = dt$

$3\,(1 - v^2)\, dv = dt$

$$\frac{1}{3}\int \frac{dt}{t} = \frac{1}{2}\int \frac{1}{x} dx$$

$$\therefore \frac{1}{3}\log t = \frac{1}{2}\log x + \log c$$

$$\log(t)^{\frac{1}{3}} = \log\sqrt{x} + \log c$$

$2\log t = 3\log x + 6\log c$

$t^2 = x^3 c'$ $\qquad c' = c^6$

$(3v - v^3)^2 = x^3 c'$

$$\left[\frac{3y}{x} - \left(\frac{y}{x}\right)^3\right]^2 = x^3 c' \qquad \text{Put } v = \frac{y}{x}$$

$[(3yx^2 - y^3)]^2 = x^9 c'$ $\qquad$ Given $y(1) = 1$

$(3-1)^2 = c'$ $\quad \therefore c' = 4$ $\qquad$ at $x = 1, y = 1$

$[3yx^2 - y^3]^2 = 4x^9$

Let $x = 2$

$[3y(2) \times 4 - (y(2)^3]^2 = 4 \times (2)^9$

$$3y(2) \times 4 - (y(2))^3 = \sqrt{4 \times 2^9}$$

$$12y(2) - [y(2)]^3 = 2 \times 2^{9/2} = 2 \times 2^4 \times \sqrt{2} = 32\sqrt{2}$$

$$|(y(2)^3 - 12y(2)| = 32\sqrt{2}$$

6. Option (2) is correct.

Given

$$\lim_{x\to\infty} \frac{(\sqrt{3x+1} + \sqrt{3x-1})^6 + (\sqrt{3x+1} - \sqrt{3x-1})^6}{(x+\sqrt{x^2-1})^6 + (x-\sqrt{x^2-1})^6} \times x^3$$

$$\lim_{x\to\infty} \frac{x^3\left[\left(\sqrt{3+\frac{1}{x}} + \sqrt{3-\frac{1}{x}}\right)^6 + \left(\sqrt{3+\frac{1}{x}} - \sqrt{3-\frac{1}{x}}\right)^6\right]}{x^6\left[\left(1+\sqrt{1-\frac{1}{x^2}}\right)^6 + \left(1-\sqrt{1-\frac{1}{x^2}}\right)^6\right]} \times x^3$$

$$= \left[\frac{(\sqrt{3}+\sqrt{3})^6 + (\sqrt{3}-\sqrt{3})^6}{(1+1)^6 + (1-1)^6}\right]$$

$$= \frac{(2\sqrt{3})^6}{(2)^6} = \frac{(2)^6}{2^6}(\sqrt{3})^6 = (3)^{\frac{6}{2}} = 3^3 = 27$$

7. Option (3) is correct.

Given volume of tetrahedron OABC is 144 unit^3

$$\vec{n} = (2, a, 4)$$

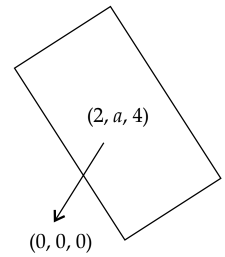
Equation of plane is $2x + ay + 4z$

$= 4 + a^2 + 16 = 20 + a^2$

$$A\left(\frac{20+a^2}{2}, 0, 0\right) B\left(0, \frac{20+a^2}{a}, 0\right), C\left(0, 0, \frac{20+a^2}{4}\right)$$

We have value of tetrahedron

$$v = 144$$



$$\frac{1}{6}\frac{(20+a^2)^3}{8a} = 144 = 2^4 \times 3^2$$

$(20 + a^2)^3 = 2^8 \times 3^3 \,.\, a$

$$20 + a^2 = (4a)^{\frac{1}{3}}.12$$

$a = 2$ satisfies above equation

So, $2x + 2y + 4z = 24$, $x + y + 2z = 12$

$P = (2, 2, 4)$

8. Option (3) is correct.

$x^2 - 6x + 10 = x^2 - 6x + 9 + 1 = (x-3)^2 + 1 \geq 1$

So $x = 3$ is the only element in the domain

So, $\alpha x^2 + \beta x + \sin^{-1}(x^2 - 6x + 10)$

$\qquad + \cos^{-1}(x^2 - 6x + 10) = 0$

$$\Rightarrow 9\alpha + 3\beta + \frac{\pi}{2} = 0$$

$\sin^{-1}(\sin\theta) - \cos^{-1}(\cos\theta) \geq 0$

$$\sin^{-1}(\sin\theta) - \left(\frac{\pi}{2} - \sin^{-1}(\sin\theta)\right) > 0$$

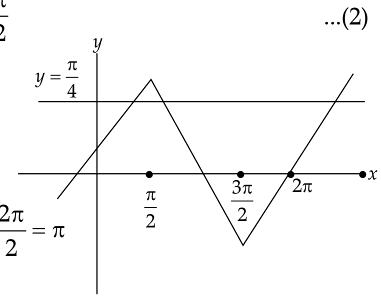
$$2\sin^{-1}(\sin\theta) > \frac{\pi}{2} \Rightarrow \sin^{-1}(\sin\theta) > \frac{\pi}{4}$$

$$\text{So } \theta = \left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$$

$$\alpha - \beta = \frac{3\pi}{4} - \frac{\pi}{4} = \frac{\pi}{2} \qquad \text{...(1)}$$

and $9\alpha + 3\beta = -\dfrac{\pi}{2}$ $\qquad$ ...(2)

From (1) and (2)



$$3\alpha - 3\beta = \frac{3\pi}{2}$$

$$9\alpha + 3\beta = -\frac{\pi}{2}$$

$$12\alpha = \frac{3\pi}{2} - \frac{\pi}{2} = \frac{2\pi}{2} = \pi$$

$$\Rightarrow \alpha = \frac{\pi}{12}$$

9. Option (4) is correct.

$M_A = 40$, $Sd_A = \alpha > 0$, $n_A = 100$

$M_B = 55$, $Sd_B = 30 - \alpha$, $n_B = n$

$M_{A\cup B} = 50$ variance$_{A\cup B} = 350$, $n_{A\cup B} = 100 + n$

$A = \{x_1 \ldots x_{100}\}$, $B = \{y_1 \ldots y_n\}$

$\Sigma x_i = 4000$, $\Sigma y_i = 55n$

$\Sigma(x_i + y_i) = 50\,(100 + n)$

$4000 + 55n = 5000 + 50n$

$5n = 1000 \Rightarrow n = 200$

Using formula of S.D.

$$\alpha^2 = \frac{\Sigma x_i^2}{100} - (40)^2$$

$$(30-\alpha)^2 = \frac{\Sigma y_i^2}{200} - (55)^2$$

$\Sigma x_i^2 = 100\,(1600 + \alpha^2)$

$\Sigma y_i^2 = 200\,(55)^2 + (30-\alpha)^2)$

$$350 = \frac{\Sigma(x_i^2 + y_i^2)}{300} - (50)^2$$

$\Sigma x_i^2 + \Sigma y_i^2 = ((50)^2 + 350) \times 300$

$\Rightarrow 160000 + 100\alpha^2 + 200\,(55)^2 + 200\,(30-\alpha)^2$

$= (50)^2 \times 300 + 350 \times 300$

$\Rightarrow 1600 + \alpha^2 + 6050 + 2\,(30-\alpha)^2 = 7500 + 1050$

$\Rightarrow \alpha^2 + 1800 - 120\,\alpha + 2\alpha^2 - 900 = 0$

$\Rightarrow 3\alpha^2 - 120\alpha + 900 = 0$	If $\alpha = 10$
$\Rightarrow \alpha^2 - 40\alpha + 300 = 0$	$\text{var}_A = 100$ and $\text{var}_B = 400$
$\Rightarrow (\alpha-10)\,(\alpha-30) = 0$	$\text{var}_A + \text{var}_B = 500$
$\Rightarrow \alpha = 10, \alpha = 30$	

10. Option (4) is correct.

Given $f(x) = |x^2 - x + 1| + [x^2 - x + 1]\; x \in [-1, 2]$

$= x^2 - x + 1 > 0\ \forall\, x \in R \quad \because f'(x) = 2x - 1 = 0 \quad \therefore x = \frac{1}{2}$

Minimum value of $x^2 - x + 1$ occurs at $x = \frac{1}{2} \in [-1, 2]$

So mini $f(x) = f\left(\frac{1}{2}\right) = \frac{3}{4} + \left[\frac{3}{4}\right] = \frac{3}{4}$

11. Option (2) is correct.

Given focus of hyperbola $(1 \pm \sqrt{2}, 0) \quad e = \sqrt{2}$

We have foci of hyperbola $(\pm\, ae, 0)$

$2ae = 1 + \sqrt{2} - (1 - \sqrt{2}) = 1 + \sqrt{2} - 1 + \sqrt{2} = 2\sqrt{2}$

$2a\sqrt{2} = 2\sqrt{2} \quad \therefore a = 1$ ...(1)

where $a^2e^2 = a^2 + b^2 \quad \therefore 1\,(2) = 1 + b^2 \quad \therefore b^2 = 2 - 1$

$b^2 = 1 \therefore b = \pm 1 \Rightarrow b = 1$ ...(2)

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 1}{1} = 2$

12. Option (4) is correct.

$a_7 = 3 \Rightarrow a_1 + 6d = 3$ ...(1)

$a.(a + 3d) \to$ is minimum

$S_n = 0 \Rightarrow \frac{n}{2}[2a_1 + (n-1)d] = 0$

$2a_1 + (n-1)\,d = 0$ ...(2)

$a(a + 3d) \to$ is minimum

Let $f(d) = (3 - 6d)\,(3 - 6d + 3d)$

$f(d) = (3 - 6d)\,.\,(3 - 6d + 3d) = (3 - 6d)\,(3 - 3d)$

$f(d) = 9 - 18d - 9d + 18d^2 = 18d^2 - 27d + 9$

$f(d) = 36d - 27 = 0 \qquad \therefore d = \frac{27}{36} = \frac{3}{4}$ putting in (1)

$a_1 + 6 \times \frac{3}{4} = 3 \quad \therefore a_1 = 3 - \frac{9}{2} = \frac{-3}{2}$

$a_1 = \frac{-3}{2}, d = \frac{3}{4}$

So, $2\left(\frac{-3}{2}\right) + (n-1)\left(\frac{3}{4}\right) = 0$

$\frac{3}{4}(n-1) = 3 \quad \therefore n - 1 = \frac{12}{3} = 4 \qquad \therefore n = 5$ and $5! = 120$

So, $a_{n(n+2)}$

$a_{5\times7} = a_{35} = \frac{-3}{2} + (34) \times \frac{3}{4} = -\frac{3}{2} + \frac{51}{2} = \frac{48}{2} = 24$

So, $5! - 4 \times 24 = 120 - 96 = 24$

13. Option (4) is correct.

Given, $(\alpha\beta\gamma)\begin{bmatrix} 2 & 10 & 8 \\ 9 & 3 & 8 \\ 8 & 4 & 8 \end{bmatrix} = (0, 0, 0)$

$2\alpha + 9\beta + 8\gamma = 0$ ...(1)

$10\alpha + 3\beta + 4\gamma = 0$ ...(2)

$8\alpha + 8\beta + 8\gamma = 0$ ...(3)

Given equation of plane

$2x + 4y + 3z = 5$ ...(4)

We use cross multiplication method (1) and (2)

$2\alpha + 9\beta + 8\gamma = 0$

$10\alpha + 3\beta + 4\gamma = 0$

$$\frac{\alpha}{36 - 24} = \frac{-\beta}{8 - 80} = \frac{+\gamma}{6 - 90}$$

$$\frac{\alpha}{12} = \frac{-\beta}{-72} = \frac{\gamma}{-84} \qquad \therefore \frac{\alpha}{1} = \frac{\beta}{6} = \frac{\gamma}{-7} = k, (\text{ say})$$

$\alpha = k \quad \beta = 6k \quad \gamma = -7k$ which lies an the plane

$2k + 4 \times 6k - 7 \times 3k = 5$

$2k + 24k - 21k = 5 \quad \therefore 5k = 5 \quad \therefore k = 1$

Hence $\alpha = 1, \beta = 6, \gamma = -7$

Then the value of $6\alpha + 9\beta + 7\gamma = 6 + 9 \times 6 - 7 \times 7$

$= 6 + 54 - 49 = 60 - 49 = 11$

14. Option (3) is correct.

Given $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = 5\hat{i} - 3\hat{j} + 3\hat{k}$

$\vec{r}$ is a vector such that

$\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r}.\vec{a} = 0$

$\vec{r} \times \vec{b} - \vec{c} \times \vec{b} = 0 \Rightarrow (\vec{r} - \vec{c}) \times \vec{b} = \vec{0}$

$\vec{r} - \vec{c} = \lambda\vec{b} \qquad \therefore \vec{r} = \lambda\vec{b} + \vec{c} = (\lambda + 5)\hat{i} - (\lambda + 3)\hat{j} + (2\lambda + 3)\hat{k}$

and $\vec{r}.\vec{a} = 0$

$1(\lambda + 5) - 2\,(\lambda + 3) + 3\,(2\,\lambda + 3) = 0$

$5\lambda + 8 = 0 \Rightarrow \lambda = \frac{-8}{5}$

$\vec{r} = \frac{17}{5}\hat{i} - \frac{7}{5}\hat{j} + \frac{1}{5}\hat{k}$

$$\therefore 25\,|\vec{r}\,|^2 = 25\left(\sqrt{\left(\frac{17}{5}\right)^2 + \left(\frac{-7}{5}\right)^2 + \left(\frac{1}{5}\right)^2}\right)^2$$

$$= 25 \times \frac{1}{25}((17)^2 + (-7)^2 + 1) = 339$$

15. Option (4) is correct.

Given equation of plane

$P = 8x + \alpha_1 y + \alpha_2 z + 12 = 0$

Which is parallel to the line

L : $\frac{x+2}{2} = \frac{y-3}{3} = \frac{z+4}{5}$

Given y-intercept $= \frac{-12}{\alpha_1} = 1 \quad \therefore \alpha_1 = -12$

and $\vec{n} = (8, \alpha_1, \alpha_2)$

$\bar{l} = (2,3,5)\ (dr.s)$

$\bar{n}.\bar{l} = 0$ ($\because$ Plane P and line L are parallel)

$16 + 3\alpha_1 + 5\alpha_2 = 0$ ...(1)

Put $\alpha_1 = -12$

$16 - 12 \times 3 + 5\alpha_2 = 0 \Rightarrow 5\alpha_2 = 20$

$\Rightarrow \alpha_2 = 4$

Equation of plane P:

$8x - 12y + 4z + 12 = 0$ ...(2)

$\Rightarrow 2x - 3y + z + 3 = 0$

(–2, 3, –4) is a point lie on the line.

$\perp$ Distance between the point (–2, 3, –4) and the plane P is

$$d = \frac{|-4-9-4+3|}{\sqrt{2^2+3^2+1^2}} = \frac{|-14|}{\sqrt{14}} = \frac{14}{\sqrt{14}} = \sqrt{14}$$

16. Option (1) is correct.

Let A (4, 1, –3) and B (2, 4, 3)

$\vec{n} = \overline{AB} = (-2,3,6)$

Plane P is

$-2(x-1) + 3(y+1) + 6(z+5) = 0$

$\Rightarrow -2x + 2 + 3y + 3 + 6z + 30 = 0$

$\Rightarrow -2x + 3y + 6z + 35 = 0$

$\Rightarrow 2x - 3y - 6z = 35$ ...(1)

Distance of plane from the point (3, –2, 2) is

$$= \frac{|6+6-12-35|}{\sqrt{2^2+3^2+6^2}} = \frac{35}{7} = 5$$

17. Option (4) is correct.

The number of values of $r \in \{p, q, \sim p, \sim q\}$ for which $((p \wedge q) \Rightarrow (r \vee q)) \wedge \Rightarrow ((p \wedge r) \Rightarrow (q))$ is a tautology is

$[(p \wedge q) \Rightarrow (r \vee q)] \wedge [(p \wedge r) \Rightarrow q]$

We have

$p \rightarrow q = \sim p \vee q$

De Morgan's law

by $\sim (p \wedge q) = \sim p \vee \sim q$

$(p \wedge q) \Rightarrow (r \vee q)$

$\Rightarrow \sim (p \wedge q) \vee (r \vee q) \Rightarrow \sim p \vee \sim q \vee r \vee q \rightarrow$ tautology ...(1)

$(p \wedge r) \Rightarrow q$

$\sim (p \wedge r) \vee q \Rightarrow \sim p \vee \sim r \vee q$ ...(2)

If $r = p$ $\quad \sim p \vee \sim p \vee q \rightarrow$ Not tautology

If $r = \sim p$ $\quad \sim p \vee p \vee q \rightarrow$ tautology

If $r = q$ $\quad \sim p \vee q \vee q \rightarrow$ tautology

If $r = \sim q$ $\quad \sim p \vee z\, q \vee q \rightarrow$ Not a tautology

Two values.

18. Option (4) is correct.

Given equation of circle.

$2x^2 + 2y^2 - (1+a)x - (1-a)y = 0$ ...(1)

$$\Rightarrow x^2 + y^2 - \frac{(1+a)}{2}x - \frac{(1-a)y}{2} = 0$$

We have $x(x - x_1) + y(y - y_1) = 0$

$$\Rightarrow x\left(x - \frac{1+a}{2}\right) + y\left(y - \frac{(1-a)}{2}\right) = 0$$

$y - y_1 = m(x - x_1)$ ...(2)

$$(x_1, y_1) = \left(\frac{1+a}{2}, \frac{1-a}{2}\right)$$

and $x + y = 0$

$-x - y_1 = mx - mx_1$

$$x = \frac{mx_1 - y_1}{1+m} \text{ and } y = \frac{y_1 - mx_1}{1+m}$$

$$\frac{\frac{y_1}{2} - y}{\frac{x_1}{2} - x} = \frac{-1}{m} \Rightarrow \frac{\frac{y_1}{2} - \left(\frac{y_1 - mx_1}{1+m}\right)}{\frac{x_1}{2} - \left(\frac{mx_1 - y_1}{1+m}\right)} = -\frac{1}{m}$$

$$\Rightarrow \frac{(1+m)y_1 - 2y_1 + 2mx_1}{(1+m)x_1 - 2mx_1 + 2y_1} = \frac{-1}{m}$$

$$\Rightarrow \frac{m(y_1 + 2x_1) - y_1}{-mx_1 + x_1 + 2y_1} = \frac{-1}{m}$$

$\Rightarrow m^2(y_1 + 2x_1) - my_1 = mx_1 - x_1 - 2y_1$

$\Rightarrow m^2(y_1 + 2x_1) - (y_1 + x_1)m + x_1 + 2y_1 = 0$

$D > 0$

$(y_1 + x_1)^2 - 4(y_1 + 2x_1)(x_1 + 2y_1) > 0$ ($\because b^2 - 4ac > 0$)

$$x_1 = \frac{1+a}{2},\ y_1 = \frac{1-a}{2} \quad \therefore x_1 + y_1 = 1$$

$$y_1 + 2x_1 = \frac{1-a}{2} + 1 + a = \frac{3}{2} + \frac{a}{2} = \frac{3+a}{2}$$

$$x_1 + 2y_1 = \frac{1+a}{2} + 1 - a = \frac{3}{2} - \frac{a}{2} = \frac{3-a}{2}$$

$$1 - 4\left(\frac{3-a}{2}\right)\left(\frac{3+a}{2}\right) > 0$$

$1 - (9 - a^2) > 0 \ \therefore a^2 - 8 > 0$

$\therefore a^2 > 8 \rightarrow (8, \infty)$

19. Option (1) is correct.

Given $\phi(x) = \frac{1}{\sqrt{x}}\int_{\frac{\pi}{4}}^{x}(4\sqrt{2}\sin t - 3\phi'(t))dt$

$$\sqrt{x}\,\phi(x) = \int_{\pi/4}^{x}\left(4\sqrt{2}\sin t - 3\phi'(t)\right)dt$$

Diff. w.r.t. x

$$\phi'(x)\sqrt{x} + \phi(x)\frac{1}{2}x^{\frac{1}{2}-1} = 4\sqrt{2}\sin x - 3\phi'(x)$$

$$\Rightarrow (\sqrt{x}+3)\phi'(x) + \frac{1}{2\sqrt{x}}\phi(x) = 4\sqrt{2}\sin x$$

$$\Rightarrow \phi'(x) + \frac{1}{2\sqrt{x}(\sqrt{x}+3)}\phi(x) = \frac{4\sqrt{2}\sin x}{\sqrt{x}+3}$$

Put $x = \frac{\pi}{4}$ $\quad \phi'\left(\frac{\pi}{4}\right) + 0 = \dfrac{4\sqrt{2} \times \frac{1}{\sqrt{2}}}{\sqrt{\frac{\pi}{4}} + 3}$ $\quad \left(\because \phi\left(\frac{\pi}{4}\right) = 0\right)$

$$\phi'\left(\frac{\pi}{4}\right) = \frac{4 \times 2}{\sqrt{\pi}+6} = \frac{8}{\sqrt{\pi}+6}$$

20. Option (4) is correct.

$f: R \rightarrow (2, 6) \rightarrow R$

be real valued function

$$y = f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12}$$

$y(x^2 - 8x + 12) = x^2 + 2x + 1$

$\Rightarrow x^2y - 8xy + 12y = x^2 + 2x + 1$ $\quad \therefore (b^2 - 4ac \geq 0)$

$\Rightarrow x^2(y-1) - (8y+2)x + 12y - 1 = 0$ $\quad D \geq 0$

$\Rightarrow (8y + 2)^2 - 4 \times (y - 1) \times (12y - 1) \geq 0$

$\Rightarrow 16y^2 + 8y + 1 - (12y^2 - 13y + 1) \geq 0$

$\Rightarrow 4y^2 + 21y \geq 0$

$$\Rightarrow 4y\left[y + \frac{21}{4}\right] \geq 0$$

$$\Rightarrow y \in \left(-\infty, \frac{-21}{4}\right] \cup [0, \infty)$$

Section B

21. Correct answer is [204].

Let A = $[a_{ij}]$, $a_i \in z \cap [0,4]$ $1 \leq i, j \leq 2$

$a_{ij} = \{0, 1, 2, 3, 4\}$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$a_{11} + a_{12} + a_{21} + a_{22} = P$

where P is a prime number

$(1 + x + x^2 + x^3 + x^4)^4$

$= (1 - x^5)^4 (1 - x)^{-4}$

$(1 - 4x^5 + 6x^{10} - 4x^{15} + x^{20})\ (1 + {}^4C_1 x + {}^5C_2 x^2 + {}^6C_3 x^3 + {}^7C_4 x^7 + \ldots.\}$

for 3, ${}^6C_3 = \dfrac{6 \times 5 \times 4}{6} = 20$

for 5, $(-4) + {}^8C_5 = -4 + 56 = 52$

for 7, $(-4)\ {}^5C_2 + {}^{10}C_7 = -40 + 120 = 80$

for 11, ${}^{14}C_{11} + (-4)\ {}^9C_6 + 6\ ({}^4C_1) = 364 - 336 + 24 = 52$

Total number of matrix = 204

$a_{11} + a_{12} + a_{22} + a_{12} = 3$

$0\quad 0\quad 0\quad 3 \;=\; \dfrac{4!}{3!} = 4$

$0\quad 0\quad 2\quad 1 \;=\; \dfrac{4!}{2!} = 12$

$0\quad 1\quad 1\quad 1 \;=\; \dfrac{4!}{3!} = 4$

Total – 20

$a_{11} + a_{12} + a_{13} + a_{22} = 5$

$0\quad 0\quad 1\quad 4 \;=\; \dfrac{4!}{2!} = 12$

$0\quad 0\quad 2\quad 3 \;=\; \dfrac{4!}{2!} = 12$

$0\quad 1\quad 1\quad 3 \;=\; \dfrac{4!}{2!} = 12$

$0\quad 1\quad 2\quad 2 \;=\; \dfrac{4!}{2!} = 12$

$1\quad 1\quad 1\quad 2 \;=\; \dfrac{4!}{3!} = 4$

Total – 52

$a_{11} + a_{12} + a_{21} + a_{22} = 7$

$0\quad 0\quad 3\quad 4 \;=\; \dfrac{4!}{2!} = 12$

$0\quad 1\quad 3\quad 3 \;=\; \dfrac{4!}{2!} = 12$

$0\quad 1\quad 2\quad 4 \;=\; 4! = 24$

$1\quad 1\quad 1\quad 4 \;=\; \dfrac{4!}{3!} = 4$

$0\quad 2\quad 2\quad 3 \;=\; \dfrac{4!}{2!} = 12$

$1\quad 1\quad 2\quad 3 \;=\; \dfrac{4!}{2!} = 12$

$1\quad 2\quad 2\quad 2 \;=\; \dfrac{4!}{3!} = 4$

Total – 80

$a_{11} + a_{12} + a_{21} + a_{22} = 11$

$0\quad 3\quad 4\quad 4 \;=\; \dfrac{4!}{2!} = 12$

$1\quad 2\quad 4\quad 4 \;=\; \dfrac{4!}{2!} = 12$

$1\quad 3\quad 3\quad 4 \;=\; \dfrac{4!}{2!} = 12$

$2\quad 2\quad 3\quad 4 \;=\; \dfrac{4!}{3!} = 12$

$2\quad 3\quad 3\quad 3 \;=\; \dfrac{4!}{3!} = 4$

Total – 52

Total matrix = 20 + 52 + 80 + 52 = 204

22. Correct answer is [5].

Given $|A| = 2$

$|\text{Adj}\ (2\ \text{Adj}\ 2\ A^{-1})| = 2^{84}$

We have

$|2\ \text{Adj}\ (2A^{-1})|^{n-1} = 2^{84}$

$2^n\ |\text{Adj}\ (2A^{-1})|^{n-1} = 2^{84}$

$2^n \times (2^{n-1})^n\ |\text{Adj}\ (A^{-1})|^{n-1} = 2^{84}$

$$\left(2^n \times 2^{n(n-1)} \times \left(\frac{1}{2}\right)^{n-1}\right)^{n-1} = 2^{84}$$

$$\left(2^{n+n^2-n+(1-n)}\right)^{n-1} = 2^{84}$$

$$\left(2^{n^2-n+1}\right)^{n-1} = 2^{84}$$

$(n^2 - n + 1)\ (n - 1) = 84 \Rightarrow n = 5$

$n = 5$

23. Correct answer is [98].

$$\left(\frac{x^{\frac{5}{2}}}{2} - \frac{4}{xl}\right)^9$$

$$T_{r+1} = {}^9C_r \left(\frac{x^{\frac{5}{2}}}{2}\right)^{9-r} \left(\frac{-4}{x^l}\right)^r$$

$$= {}^9C_r \left(\frac{1}{2}\right)^{9-r} (-4)^r\, x^{\frac{45-5r}{2} - lr}$$

for constant

$${}^9C_r \left(\frac{1}{2}\right)^{9-r} (-1)^r\, 2^{2r} = -84$$

$= {}^9C_r\ 2^{r-9} . 2^{2r} (-1)^r = -84$

$= {}^9C_r 2^{3r-9}(-1)^r = -84 \Rightarrow r = 3$

So, $\frac{45-5r}{2} - lr = 0 \Rightarrow \frac{45-15}{2} - 3l = 0$

$15 - 3l = 0 \Rightarrow l = 5$

For coefficient of x^{-15} is

$\frac{45-5r}{2} - 5r = -15$

$\Rightarrow 45 - 5r - 10r = -30 \Rightarrow 75 = 15r \Rightarrow r = 5$

For coefficient of x^{-15} is ${}^9C_5\left(\frac{1}{2}\right)^4(-4)^5$

$= \frac{9\times8\times7\times6}{4\times3\times2\times1}\times\frac{1}{2^4}\times2^{10}\times(-1)$

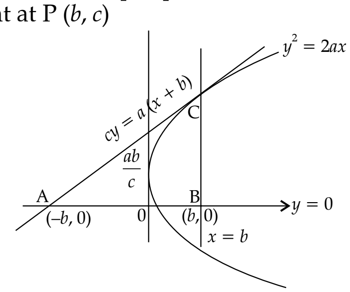
$= 9 \times 2 \times 7 \times 2^6 \times (-1) = 2^7(-63) = 2^\alpha\beta$

$\alpha = 7, \beta = -63$

$\Rightarrow |\alpha l - \beta| = |7 \times 5 + 63| = |35 + 63| = 98$

24. Correct answer is [146].

Tangent at P (b, c)



Area $= \left|\frac{1}{2}\times2b\times\frac{2ba}{c}\right| = 16$

$\frac{2b^2a}{c} = 16 \quad \therefore \frac{b^2a}{c} = 8$

$\because$ P(b, c) lies on $y^2 = 2ax$ $\qquad \therefore c^2 = 2ab$

$\Rightarrow \frac{b^4a^2}{c^2} = 64 \Rightarrow \frac{b^4a^2}{2ab} = 64$

$b^3a = 128 \Rightarrow a = \frac{128}{b^3}$

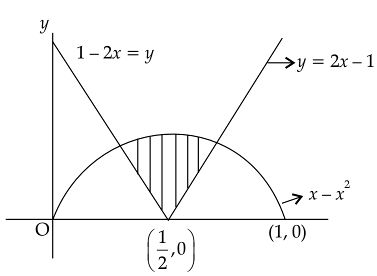
a can be 128, 16, 2 Then S = {2, 16, 128}

$\Sigma a \in S\, a = 146$

25. Correct answer is [125].

Given bounded region

$|2x - 1| \le y \le |x^2 - x|, 0 \le x \le 1$



$x - x^2 = 1 - 2x$

$\therefore x^2 - 3x + 1 = 0$

$x = \frac{3\pm\sqrt{9-4}}{2} = \frac{3\pm\sqrt{5}}{2}$

$x = \frac{3-\sqrt{5}}{2}$ and $0 < x < \frac{1}{2}$

Area $= 2\int_{\frac{3-\sqrt{5}}{2}}^{\frac{1}{2}}[(x - x^2) - (1 - 2x)]dx$

$= 2\int_{\frac{3-\sqrt{5}}{2}}^{\frac{1}{2}}[3x - x^2 - 1]dx$

$= 2\left[\frac{3x^2}{2} - \frac{x^3}{3} - x\right]_{\frac{3-\sqrt{5}}{2}}^{\frac{1}{2}}$

for $x = \frac{3-\sqrt{5}}{2}$ $\qquad x^2 = 3x - 1$

$x^3 = 3x^2 - x = 3(3x - 1) - x = 8x - 3$

$\frac{3}{2}x^2 - \frac{1}{3}x^3 - x = \frac{3}{2}(3x-1) - \frac{1}{3}(8x-3) - x$

$= \frac{9x-3}{2} - \frac{(8x-3)}{3} - x$

$= \frac{27x - 9 - (16x - 6)}{6} - x$

$= \frac{11x-3}{6} - x = \frac{5x-3}{6}$

for $x = \frac{1}{2}$

$\frac{3}{2}x^2 - \frac{1}{3}x^3 - x = \frac{3}{2}\left(\frac{1}{4}\right) - \frac{1}{3}\left(\frac{1}{8}\right) - \frac{1}{2}$

$= \frac{9-1-12}{24} = \frac{-4}{24} = \frac{-1}{6}$

Area $= 2\left[-\frac{1}{6} - \left(\frac{5\left(\frac{3-\sqrt{5}}{2}\right) - 3}{6}\right)\right]$

$= 2\left[-\frac{1}{6} - \left(\frac{9-5\sqrt{5}}{12}\right)\right]$

$= 2\left[\frac{-2-9+5\sqrt{5}}{12}\right] = \frac{5\sqrt{5}-11}{6}$

$\Rightarrow (6A + 11)^2 = (5\sqrt{5})^2 = 125$

26. Correct answer is [5040].

The coefficient of x^{-6} in the expansion of

$\left(\frac{4x}{5} + \frac{5}{2x^2}\right)^9$

$T_{r+1} = {}^nC_r \cdot P^{n-r} q^r$

$T_{r+1} = {}^9C_r (P)^{n-r} q^r = {}^9C_r\left(\frac{4}{5}x\right)^{9-r}\left(\frac{5}{2x^2}\right)^r$

$= {}^9C_r\left(\frac{4}{5}\right)^{9-r}(x)^{9-r}\left(\frac{5}{2}\right)^r x^{-2r}$

$= {}^9C_r\left(\frac{4}{5}\right)^{9-r}\left(\frac{5}{2}\right)^r (x)^{9-3r}$

For coefficient of term x^{-6} let $9 - 3r = -6$

$3r = 15, r = 5$

$$= {}^9C_5 \left(\frac{4}{5}\right)^{9-5}\left(\frac{5}{2}\right)^5 = \frac{9!}{5!4!}\left(\frac{4}{5}\right)^4\left(\frac{5}{2}\right)^5$$

$$= \frac{9\times8\times7\times6\times5!}{5!\times4\times3\times2}\times\frac{(4)^4}{5^4}\times\frac{5^5}{2^5}$$

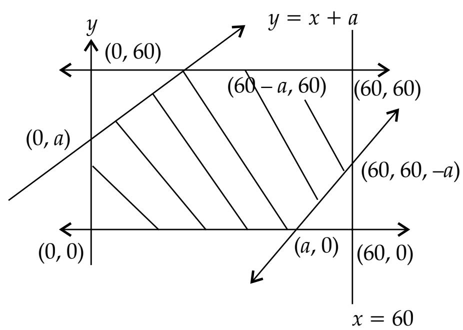
$$\frac{9\times8\times7\times6}{4\times6}\times\frac{4^4\times5}{2\times2\times2\times2\times2} = \frac{9\times8\times7\times4\times4\times4\times5}{2\times2\times2\times2\times2}$$

$= 9 \times 8 \times 7 \times 10 = 72 \times 70 = 5040$

27. Correct answer is [10].

Given $|x - y| < a \rightarrow -a < x - y$ and $x - y < a$

$x, y \in [0, 60]$



$$P(A) = \frac{\text{Shaded Area}}{\text{Total Area}}$$

$$= \frac{(60)^2 - \left[\frac{1}{2}(60-a)^2 + \frac{1}{2}\times(60-a)^2\right]}{(60)^2}$$

$$= \frac{(60)^2 - (60-a)^2}{(60)^2}$$

$$P(A) = \frac{(60)^2 - (60-a)^2}{(60)^2} \Rightarrow \frac{11}{36} = \frac{120a - a^2}{3600}$$

$\Rightarrow 1100 = 120a - a^2$

$\Rightarrow a^2 - 120a + 1100 = 0$

$\Rightarrow a^2 - 110a - 10a + 1100 = 0$

$\Rightarrow (a - 10)(a - 110) = 0$

$\Rightarrow a = 10$ $\quad (\because$ for $a = 110$, P(A) = 1)

28. Correct answer is [45].

$$^{2n+1}P_{n-1} : {}^{2n-1}P_n = 11 : 21$$

$$\Rightarrow \frac{\dfrac{\lfloor 2n+1}{\lfloor 2n+1-n+1}}{\dfrac{\lfloor 2n-1}{\lfloor 2n-1-n}} = \frac{11}{21}$$

$$\Rightarrow \frac{\lfloor 2n+1}{\lfloor n+2}\times\frac{\lfloor n-1}{\lfloor 2n-1} = \frac{11}{21}$$

$$\Rightarrow \frac{(2n+1).2n\lfloor 2n-1\times\lfloor n-1}{(n+2)(n+1)n\lfloor n-1\times\lfloor 2n-1} = \frac{11}{21}$$

$\Rightarrow 2(2n + 1) \times 21 = 11(n + 1)(n + 2)$

$\Rightarrow 42(2n + 1) = 11(n^2 + 3n + 2)$

$\Rightarrow 84n + 42 = 11n^2 + 33n + 22$

$\Rightarrow 11n^2 - 51n - 20 = 0 \Rightarrow n = 5$

Then $n^2 + n + 15 = 25 + 5 + 15 = 45$

29. Correct answer is [3].

Given $|\vec{a}| = \sqrt{31}$

$|\vec{b}| = \frac{1}{2}, |\vec{c}| = 2$

$2(\vec{a}\times\vec{b}) = 3(\vec{c}\times\vec{a}) = -3(\vec{a}\times\vec{c})$

$\Rightarrow \vec{a}\times 2\vec{b} + \vec{a}\times 3\vec{c} = 0$

$\Rightarrow \vec{a}\times(2\vec{b}+3\vec{c}) = 0 \Rightarrow \vec{a} = \lambda(2\vec{b}+3\vec{c})$ ...(1)

$|\vec{a}|^2 = \lambda^2|(2\vec{b}+3\vec{c})|^2$ $\quad \left(\because \vec{b}.\vec{c} = \frac{1}{2}\times2\times\left(\frac{-1}{2}\right) = \frac{-1}{2}\right)$

$31 = \lambda^2\left[4|\vec{b}|^2 + 9|\vec{c}| + 12\vec{b}.\vec{c}\right]$

$= \lambda^2\left[4\times\frac{1}{4} + 9\times4 + 12\times\frac{-1}{2}\right]$

$\Rightarrow 31 = \lambda^2[1 + 36 - 6] \Rightarrow 31 = 31\lambda^2$

$\Rightarrow \lambda^2 = 1 \Rightarrow \lambda = \pm1$

$\therefore \vec{a} = \pm(2\vec{b}+3\vec{c})$ ...(2)

Now, $2|\vec{a}\times\vec{b}| = 3|\vec{a}\times\vec{c}| \Rightarrow \frac{2}{3}|\vec{a}\times\vec{b}| = |\vec{a}\times\vec{c}|$

$\vec{a} = \pm(2\vec{b}+3\vec{c})$

$\vec{a}\times\vec{b} = \pm3(\vec{c}\times\vec{b})$ $\qquad \vec{b}\times\vec{b} = 0$

$\vec{a}\times\vec{c} = \pm2(\vec{b}\times\vec{c})$ $\qquad \vec{c}\times\vec{c} = 0$

$|\vec{a}\times\vec{b}| = 3|\vec{c}\times\vec{b}| = 3|\vec{c}||\vec{b}|\sin\frac{2\pi}{3}$

$= 3\times2\times\frac{1}{2}\times\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$

$|\vec{a}\times\vec{c}| = \frac{2}{3}|\vec{a}\times\vec{b}| = \frac{2}{3}\times\frac{3\sqrt{3}}{2} = \sqrt{3}$

$\Rightarrow |\vec{a}\times\vec{c}| = \sqrt{3}$

We have $(\vec{a}.\vec{b})^2 + |\vec{a}\times\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 = 31\times\frac{1}{4}$

$\Rightarrow (\vec{a}.\vec{b})^2 + \frac{27}{4} = \frac{31}{4}$ $\quad \therefore (\vec{a}.\vec{b})^2 = 1$

$\Rightarrow \left(\frac{\vec{a}\times\vec{c}}{a.b}\right)^2 = \frac{3}{1} = 3$

30. Correct answer is [6952].

$1^2 - 2.3^2 + 3.5^2 - 4.7^2 + 5.9^2 - .... + 15.29^2$

$S = 15.29^2 - 14.27^2 + ... + 3.5^2 - 2.3^2 + 1^2$

$(n + 1)(2n + 1)^2 - n(2n - 1)^2$

$= n(4n^2 + 4n + 1) + 4n^2 + 4n + 1 - n(4n^2 - 4n + 1)$

$= 12n^2 + 4n + 1$

$S = \Sigma(12n^2 + 4n + 1)$ for $n = 2, 4, 6, 8, 10, 12, 14] + 1$

$$S_1 = \sum_{k=1}^{7} 12(2k)^2 + 4(2k) + 1$$

$$= \sum_{k=1}^{7}[48k^2 + 8k + 1] = 48\sum_{k=1}^{7}k^2 + 8\sum_{k=1}^{7}k + \sum_{k=1}^{7}1$$

$$= \frac{48\times7\times8\times15}{6} + \frac{8(7)(8)}{2} + 7 = 6951$$

$S = 6951 + 1 = 6952$

❑❑