CBSE EXAMINATION PAPER- 2024

Applied Mathematics

Class-12th

(Solved)

(Delhi & Outside Delhi sets)

Time : 3 Hours **Max. Marks : 80**

General Instructions:

Read the following instructions carefully and follow them:

*(i) This question paper contains **38** questions. **All** questions are **compulsory**.*

*(ii) This question paper is divided into **FIVE** sections - Section **A, B, C, D** and **E.***

*(iii) In **Section A,** Question no. **1** to **18** are multiple choice questions (MCQs) and question number **19** and **20** are Assertion-Reason based questions of 1 mark each.*

*(iv) In **Section B,** Question no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

*(v) In **Section C,** Question no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

*(vi) In **Section D,** Question no. **32** to **35** are log answer (LA) type questions carrying **5** marks each.*

*(vii) In **Section E,** Question no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and 3 questions in Section E.

*(ix) Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions of 1 mark each.

1. In a 1 km race, player P beats player Q by 18 meters or 9 seconds. What is P's time to complete the race.

(A) 512 seconds **(B)** 502 seconds

(C) 491 seconds **(D)** 481 seconds

2. If $x > y$ and $z < 0$, then

(A) $xz > yz$ **(B)** $xz \geq yz$

(C) $\frac{x}{z} > \frac{y}{z}$ **(D)** $\frac{x}{z} < \frac{y}{z}$

3. If $AB = A$ and $BA = B$, then $(B^2 + B)$ equals

(A) 2A **(B)** O

(C) 2I **(D)** 2B

4. The value of $\Delta = \begin{vmatrix} 42 & 2 & 5 \\ 79 & 7 & 9 \\ 29 & 5 & 3 \end{vmatrix}$ is

(A) 0 **(B)** 1

(C) –3 **(D)** –15

5. If $y = e^{-2x}$, then $\frac{d^3y}{dx^3}$ is equal to

(A) $2e^{-2x}$ **(B)** e^{-4x}

(C) $4e^{-4x}$ **(D)** $-8e^{-2x}$

6. The function $f(x) = x^2 - x + 1$ is

(A) increasing in (0, 1)

(B) decreasing in (0, 1)

(C) increasing in $\left(0, \frac{1}{2}\right)$ and decreasing in $\left(\frac{1}{2}, 1\right)$

(D) increasing in $\left(\frac{1}{2}, 1\right)$ and decreasing in $\left(0, \frac{1}{2}\right)$

7. The order and the degree of the differential equation

$$ydx + x\log\left(\frac{y}{x}\right)dy - 2xdy = 0$$

are respectively:

(A) 1, 1 **(B)** 1, 2

(C) 2, 1 **(D)** 1, not defined

8. A fair coin is tossed twice and outcomes are noted. If the random variable X represents the number of heads that appeared in the experiment, then the mathematical expectation of X is

(A) 1 **(B)** $\frac{1}{2}$

(C) $\frac{1}{4}$ **(D)** $1\frac{1}{2}$

9. What time will it be after 1275 hours, if the present time is 9:00 p.m.?

(A) 11 p.m. **(B)** 12 p.m.

(C) 9 p.m. **(D)** 9 a.m.

10. If for a Poisson variate X, $P(X = k) = P(X = k + 1)$, then the variance of X is

(A) $k - 1$ **(B)** k

(C) $k + 1$ **(D)** $k + 2$

11. If the calculated value of $|t| < t_v(\alpha)$ (critical value of t), then the null hypothesis

(A) is rejected. **(B)** is accepted.

(C) is neither accepted nor rejected. **(D)** cannot be determined.

12. For testing the significance of difference between the means of two independent samples, the degree of freedom (v) is taken as

(A) $n_1 - n_2 + 2$ **(B)** $n_1 - n_2 - 2$

(C) $n_1 + n_2 - 2$ **(D)** $n_1 + n_2 + 2$

13. For the given values 23, 32, 40, 47, 58, 33, 42; the 5-yearly moving averages are

(A) 38, 40, 42 **(B)** 40, 42, 44

(C) 40, 42, 46 **(D)** 42, 44, 46

14. Using flat rate method, the EMI to repay a loan of ₹ 20,000 in $2\frac{1}{2}$ years at an interest rate of 8% p.a. is

(A) ₹ 700 **(B)** ₹ 800

(C) ₹ 900 **(D)** ₹ 100

15. A mobile phone costs ₹ 12,000 and its scrap value after a useful life of 3 years is ₹ 3,000. Then, the book value of the mobile phone at the end of 2 years is:

(A) ₹ 3,000 **(B)** ₹ 6,000

(C) ₹ 5,000 **(D)** ₹ 7,000

16. What sum of money should be deposited at the end of every 6 months to accumulate ₹ 50,000 in 8 years, if money is worth 6% p.a. compounded semi-annually? [Given: $(1.03)^{16} = 1.6047$]

(A) ₹ 3,432.53 **(B)** ₹ 2,783.08

(C) ₹ 2,480.57 **(D)** ₹ 2,149.93

17. The graph of the inequation $2x + 3y > 6$ is the

(A) entire XOY-plane

(B) half-plane that contains the origin

(C) half-plane that neither contains the origin nor the points on the line $2x + 3y = 6$

(D) whole XOY-plane excluding the points on the line $2x + 3y = 6$

18. In an LPP, if the objective function $Z = ax + by$ has same maximum value on two corner points of the feasible region, then the number of points at which maximum value of Z occurs is

(A) 0 **(B)** 2

(C) finite **(D)** infinite

Question number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below:

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A):** The function $f(x) = x^2 - x + 1$ is strictly increasing on (–1, 1).

Reason (R): If $f(x)$ is continuous on $[a, b]$ and derivable on (a, b), then $f(x)$ is strictly increasing on $[a, b]$ if $f'(x) > 0$ for all $x \in (a, b)$

20. In a binomial distribution, $n = 200$ and $p = 0.04$. Taking Poisson distribution as an approximation to the binomial distribution:

Assertion (A): Mean of Poisson distribution = 8

Reason (R): $P(X = 4) = \dfrac{512}{3e^8}$.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. **(a)** If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$, find the value of k such that $A^2 - 8A + kI = 0$.

OR

(b) If $\begin{bmatrix} x-y & 2x+z \\ 2x-y & 3z+w \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$, find the values of x, y, z and w.

22. Using Cramer's rule, solve the following system of equations

$2x_1 + 3x_2 = 5$

$11x_1 - 5x_2 = 6$

23. Find the solution to the following linear programming problem (if it exists) graphically:

Maximize $Z = x + y$

$x - y \leq -1$

$-x + y \leq 0$

$x, y \geq 0.$

24. At 6% p.a., compounded quarterly, find the present value of a perpetuity of ₹ 600 payable at the end of each quarter.

25. **(a)** Assume an investment's starting value is ₹ 20,000 and it grows to ₹ 50,000 in 3 years. Calculate CAGR (Compounded Annual Growth Rate) [Use: $(2.5)^{1/3} = 1.355$]

OR

(b) A man bought an item for ₹ 12,000. At the end of the year, he decided to sell it for ₹ 15,000. If the inflation rate was 6%, find the nominal and real rate of return.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. A container has 50 litres of juice in it. 5 litres of juice is taken out and is replaced by 5 litres of water. This process is repeated 4 more times. Determine the quantity of juice in the container after final replacement.

[Use $(0.9)^5 = 0.59049$]

27. **(a)** Evaluate: $\int_0^2 x^2 dx$ and hence show the region on the graph whose area it represents.

OR

(b) Evaluate: $\int_0^1 \frac{e^{-x}}{1+e^x} dx$

28. Find the differential equation of all circles in the first quadrant which touches both the coordinate axes.

29. Given that the scores of a set of candidates on an IQ test are normally distributed. If the IQ test has a mean of 100 and a standard deviation of 10, determine the probability that a candidate who takes the test will score between 90 and 110.

[Given $P(Z < 1) = 0.8413$ and $P(Z < -1) = 0.1587$]

30. The mean weekly sales of a 4-wheeler was 50 units per agency in 20 agencies. After an advertising campaign, the mean weekly sales increased to 55 units per agency with standard deviation of 10 units. Test whether the advertising campaign was successful.
[Given $\sqrt{5} = 2.24$, $t_{19}(0.05 = 1.729$]

31. **(a)** A recent accounting graduate opened a new business and installed a computer system that costs ₹ 45,200. The computer system will be depreciated linearly over 3 years and will have a scrap value of ₹ 0.

(i) What is the rate of depreciation?

(ii) Give a linear equation that describes the computer system's book value at the end of t^{th} year, where $0 \leq t \leq 3$.

(iii) What will be the computer system's book value at the end of the first year and a half?

OR

(b) Find the effective rate which is equivalent to normal rate of 10% p.a. compounded:

(i) semi-annually.

(ii) quarterly.

[Given $(1.05)^2 = 1.1025$, $(1.025)^4 = 1.1038$]

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. A cistern has three pipes A, B and C. Pipes A and B are inlet pipes whereas C is an outlet pipe. Pipes A and B can fill the cistern separately in 3 hours and 4 hours respectively; while pipe C can empty the completely filled cistern in 1 hour. If the pipes A, B and C are opened in order at 5, 6 and 7 a.m. respectively, at what time will the cistern be empty?

33. **(a)** Find all the points of local maxima and local minima of the function:

$$f(x) = -\frac{3}{4}x^4 - 8x^3 - \frac{45}{2}x^2 + 105$$

OR

(b) Find the intervals in which the following function f is strictly increasing or strictly decreasing:
$f(x) = 20 - 9x + 6x^2 - x^3$

34. **(a)** Let X denote the number of hours a Class 12 student studies during a randomly selected school day. The probability that X can take the values x_i, for an unknown constant 'k':

$$P(X = k) = \begin{cases} 0.1 & \text{if} \quad x_i = 0 \\ kx_i & \text{if} \quad x_i = 1 \text{ or } 2 \\ k(5 - x_i) & \text{if} \quad x_i = 3 \text{ or } 4 \end{cases}$$

(i) Find the value of k.
(ii) Determine the probability that the student studied for at least 2 hours.
(iii) Determine the probability that the student studied for at most 2 hours.

OR

(b) A river near a small town floods and overflows twice in every 10 years on an average. Assuming that the Poisson distribution is appropriate, what is the mean expectation? Also, calculate the probability of 3 or less overflows and floods in a 10-year interval.

[Given $e^{-2} = 0.13534$]

35. Amrita buys a car for which she makes a down payment of ₹ 2,50,000 and the balance is to be paid in 2 years by monthly instalments of ₹ 25,448 each. If the financer charges interest at the rate of 20% p.a, find the actual price of the car.

[Given $\left(\frac{61}{60}\right)^{-24} = 0.67253$]

SECTION E

This section comprises 3 case study-based questions of 4 marks each.

Case Study – 1

36. On her birthday, Prema decides to donate some money to children of an orphanage home.

If there are 8 children less, everyone gets ₹ 10 more. However, if there are 16 children more, everyone gets ₹ 10 less.

Let the number of children in the orphanage home be x and the amount to be donated to each child be ₹ y.

Based on the above information, answer the following questions:

(i) Write the system of linear equations in x and y formed of the given situation.

(ii) Write the system of linear equations, obtained in (i) above, in matrix form $AX = B$.

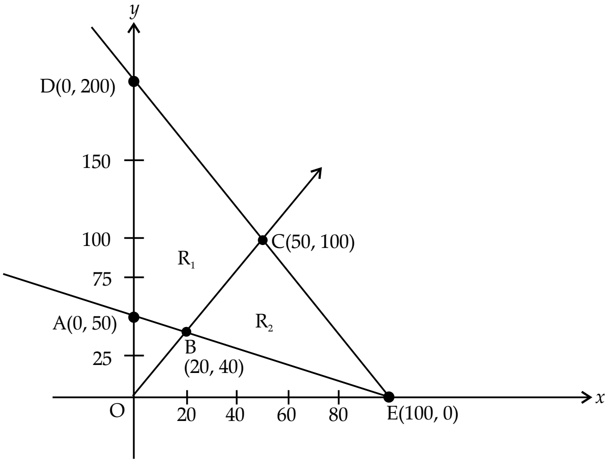
(iii)(a) Find the inverse of matrix A.

OR

(b) Determine the values of x and y.

Case Study – 2

37. In number theory, it is often important to find factors of an integer N. The number N has two trivial factors, namely 1 and N. Any other factor, if exists, is called non-trivial factor of N. Naresh has plotted a graph of some constraints (linear inequations) with points A(0, 50), B(20, 40), C(50, 100), D(0, 200) and E(100, 0). This graph is constructed using three non-trivial constraints and two trivial constraints. One of the non-trivial constraints is $x + 2y \geq 100$.



Based on the above information, answer the following questions:

(i) What are the two trivial constraints?

(ii) (a) If R_1 is the feasible region, then what are the other two non-trivial constraints?

OR

(b) If R_2 is the feasible region, then what are the other two non-trivial constraints?

(iii) If R_1 is the feasible region, then find the maximum value of the objective function $z = 5x + 2y$.

Case Study – 3

38. When observed over a long period of time, a time series data can predict trends that can forecast increase or decrease or stagnation of a variable under consideration. Such analytical studies can benefit a business for forecasting or predication of future estimated sales or production.

The table below shows the sale of an item in a district during 1996-2001:

Year:	1996	1997	1998	1999	2000	2001
Sales (in lakh ₹)	6.5	5.3	4.3	6.1	5.6	7.8

Based on the above information, answer the following questions:

(i) Determine the equation of the straight-line trend.

(ii) (a) Tabulate the trend values of the years and also compute expected sales trend for the year 2002.

OR

(b) Fit's straight-line trend by the method of least squares for the following data:

Year:	2004	2005	2006	2007	2008	2009	2010
Profit (₹ '000)	114	130	126	144	138	156	164

■■

ANSWERS

SECTION A

1. Option (C) is correct.

Explanation: P beats Q by 18 m in 9 s

$\therefore$ Q covers 18 m in 9 s

Q covers 1 km in $\frac{9}{18} \times 1000 = 500$ s

$\therefore$ P's time to complete the race

$500 - 9 = 491$ s

2. Option (D) is correct.

Explanation:

$$x > y \quad \text{(if } z < 0\text{)}$$

$$xz < yz \quad \therefore z^2 > 0$$

$$\frac{xz}{z^2} < \frac{yz}{z^2}$$

$$\frac{x}{z} < \frac{y}{z}$$

3. Option (D) is correct.

Explanation:

$$\begin{aligned} B^2 &= B.B \\ &= BA.B \\ &= B(A.B) \\ &= B.A \\ &= B \\ \therefore \quad B^2 + B &= B + B \\ &= 2B \end{aligned}$$

4. Option (A) is correct.

Explanation:

$$\Delta = \begin{vmatrix} 42 & 2 & 5 \\ 79 & 7 & 9 \\ 29 & 5 & 3 \end{vmatrix}$$

$$C_1 \rightarrow C_1 - 8C_3$$

$$\Delta = \begin{vmatrix} 42-40 & 2 & 5 \\ 79-72 & 7 & 9 \\ 29-24 & 5 & 3 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 2 & 2 & 5 \\ 7 & 7 & 9 \\ 5 & 5 & 3 \end{vmatrix}$$

$$\Delta = 0 \quad (C_1 = C_2)$$

5. Option (D) is correct.

Explanation:

$$y = e^{-2x}$$

$$\frac{dy}{dx} = \frac{d}{dx}(e^{-2x}) = -2e^{-2x}$$

$$\frac{d^2y}{dx^2} = -2(-2)e^{-2x} = 4e^{-2x}$$

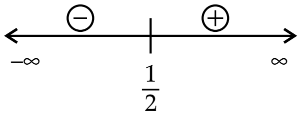
$$\frac{d^3y}{dx^3} = -8e^{-2x}$$

6. Option (D) is correct.

Explanation:

$$f(x) = x^2 - x + 1$$

$$f'(x) = 2x - 1$$



Increasing $\left(\frac{1}{2}, \infty\right) \Rightarrow \left(\frac{1}{2}, 1\right)$

decreasing $\left(-\infty, \frac{1}{2}\right) \Rightarrow \left(0, \frac{1}{2}\right)$

7. Option (A) is correct.

Explanation: $ydx + x\log\left(\frac{y}{x}\right)dy - 2xdy = 0$

$$ydx = \left\{2x - x\log\left(\frac{y}{x}\right)\right\}dy$$

$$\frac{dy}{dx} = \frac{y}{2x - x\log\left(\frac{y}{x}\right)}$$

$\therefore$ Order and degree is 1, 1 respectively.

8. Option (A) is correct.

Explanation:

X	0	1	2
P(*X*)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

$$\begin{aligned} E(X) &= 0 \times \frac{1}{4} + 1 \times \frac{1}{2} + 2 \times \frac{1}{4} \\ &= 0 + \frac{1}{2} + \frac{1}{2} \\ &= 1 \end{aligned}$$

9. Option (B) is correct.

Explanation: Present time 9:00 p.m.

after $\quad 1275 \text{ h} = 24 \times 53 + 3 \text{ h}$

$\therefore$ 12 p.m. or 12 mid-night

10. Option (C) is correct.

Explanation: $P(X = k) = P(X = k + 1)$

$$\frac{e^{-\lambda}\lambda^k}{k!} = \frac{e^{-\lambda}\lambda^{k+1}}{(k+1)!}$$

$$\frac{\lambda^k}{k!} = \frac{\lambda^k.\lambda}{(k+1)k!}$$

$$1 = \frac{\lambda}{k+1}$$

$$\lambda = k + 1$$

$$\text{Variance} = \text{mean} = \lambda = k + 1$$

11. Option (A) is correct.

Explanation: If the absolute value of the t-value is less than the critical value than reject the null hypothesis.

12. Option (C) is correct.

Explanation: Significance of difference between the means of two independent samples the degree of freedom is $(n_1 + n_2 - 2)$

13. Option (B) is correct.

Explanation:

23	–	5 years
32	–	moving average
40	200	$\frac{200}{5} = 40$
47	210	$\frac{210}{5} = 42$
58	220	$\frac{220}{5} = 44$
33	–	–
42	–	–

14. Option (B) is correct.

Explanation: $P = ₹\, 20{,}000$

$r = 8\%$ p.a.

$t = 2\frac{1}{2}$ year

$$\text{S.I.} = \frac{P \times r \times t}{100}$$

$$= \frac{20000 \times 8 \times \frac{5}{2}}{100}$$

$$= ₹\, 4{,}000$$

$$\text{E.M.I.} = \frac{P + \text{S.I}}{x}$$

$$= \frac{20000 + 4000}{30}$$

$$= ₹\, 800$$

15. Option (B) is correct.

Explanation:

Book value after	1 Year	2 Year	3 Year
	12000 – 3000	9000 – 3000	6000 – 3000
	= ₹ 9000	= ₹ 6000	= ₹ 3000

16. Option (C) is correct.

Explanation: Amount $A = ₹\, 50{,}000$, time = 8 years

rate, $r = 6\%$ p.a.

No. of payment $i = 8 \times 2 = 16$

$$A = P\left[\frac{(1+i)^n - 1}{r}\right]$$

$$50000 = P\left[\frac{(1+0.3)^{16} - 1}{0.03}\right]$$

$$50000 = P\left[\frac{1.6047 - 1}{0.03}\right]$$

$$50000 = P\left(\frac{0.6047}{0.03}\right)$$

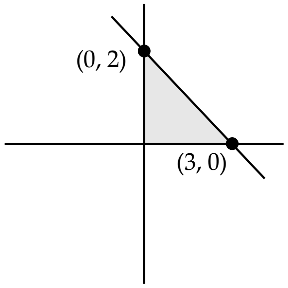
$$\frac{50000 \times 0.03}{0.6047} = P$$

$$P = 2480.568$$

$$\therefore \quad P = ₹\, 2480.57$$

17. Option (C) is correct.

Explanation: $2x + 3y > 6$ represents



half plane that neither contains origin nor the points on the line $2x + 3y = 6$

18. Option (D) is correct.

Explanation: If the objective function has same maximum value on two corner points of the feasible region, then the number of points at which maximum value is joining these two corners by straight line and all points lie on the straight line have the maximum value.

19. Option (D) is correct.

***Explanation*:** $f(x) = x^2 - x - 1$

$$f'(x) = 2x - 1 - 0$$

for increasing function

$$f'(x) > 0$$

$$2x - 1 > 0$$

$$x > \frac{1}{2}$$

$f(x)$ is decreasing $\left(-1, \frac{1}{2}\right)$

$f(x)$ is increasing $\left(\frac{1}{2}, 1\right)$

$\therefore$ Assertion is false.

20. Option (A) is correct.

***Explanation*:** mean, $\lambda = np = 200 \times 0.04 = 8$

$\therefore$ Assertion is true

$$P(x = 4) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$= \frac{8^4 e^{-8}}{4!}$$

$$= \frac{8 \times 8^3 x^{-8}}{24}$$

$$= \frac{512}{3e^8}$$

$\therefore$ Reason is true.

SECTION B

21. (a)

$$A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0+0 \\ -1-7 & 0+49 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix}$$

$$A^2 - 8A + kI = 0$$

$$\begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} - 8\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} + k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1-8+k & 0+0+0 \\ -8+8+0 & 49-56+k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} k-7 & 0 \\ 0 & k-7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$k - 7 = 0$$

$$\Rightarrow \quad k = 7$$

OR

(b) $$\begin{bmatrix} x-y & 2x+z \\ 2x-y & 3z+w \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

By comparing

$x - y = -1$ $\qquad$ $2x + z = 5$

$2x - y = 0$ $\qquad$ $3z + w = 13$

so, $-x = -1$ $\qquad$ and $\qquad$ $2x + z = 5$

$x = 1$ $\qquad$ $2 + z = 5$

and $y = 2$ $\qquad$ $z = 3$

$$3z + w = 13$$

$$9 + w = 13$$

$$w = 4$$

$x = 1, y = 2, z = 3$ and $w = 4$

22.

$$2x_1 + 3x_2 = 5$$

$$11x_1 - 5x_2 = 6$$

$$|A| = \begin{vmatrix} 2 & 3 \\ 11 & -5 \end{vmatrix}$$

$$= -10 - 33$$

$$= -43$$

$$|A_{x_1}| = \begin{vmatrix} 5 & 3 \\ 6 & -5 \end{vmatrix}$$

$$= -25 - 18$$

$$= -43$$

$$|A_{x_2}| = \begin{vmatrix} 2 & 5 \\ 11 & 6 \end{vmatrix}$$

$$= 12 - 55$$

$$= -43$$

$$x_1 = \frac{|A_{x_1}|}{|A|}$$

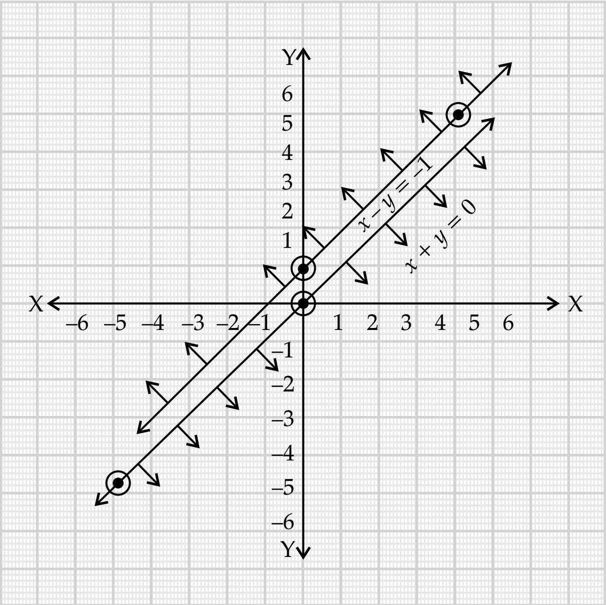
$$= \frac{-43}{-43} = 1$$

$$x_2 = \frac{|A_{x_2}|}{|A|}$$

$$= \frac{-43}{-43} = 1$$

$x_1 = 1$ and $x_2 = 1$

23.



24.

$$P.V. = \frac{C}{r}, C = ₹\,600$$

$$r = \frac{6}{400}$$

$$= 0.015$$

$$P.V. = \frac{600}{0.015}$$

$$= \frac{600000}{15}$$

Present value $= ₹\,40000$

25. (a)

$$CAGR = \left(\frac{A}{P}\right)^{\frac{1}{t}} - 1$$

A = Final Amount

P = Beginning Amount

t = time in years

$$\therefore \quad CAGR = \left(\frac{50000}{20000}\right)^{\frac{1}{3}} - 1$$

$$= (2.5)^{\frac{1}{3}} - 1$$

$$= 1.355 - 1$$

$= 0.355$ or 35.5%

OR

(b) $C.P. = ₹\,12000, S.P. = ₹\,15000$

inflation rate $= 6\%$

$$\text{Nominal rate of return} = \frac{15000 - 12000}{12000} \times 100$$

$$= \frac{3}{12} \times 100$$

$$= 25\%$$

Real rate of return $= (25 - 6)\%$

$$= 19\%$$

SECTION C

26. Quantity of juice in the container $= x\left[1 - \left(\frac{y}{x}\right)\right]^n$

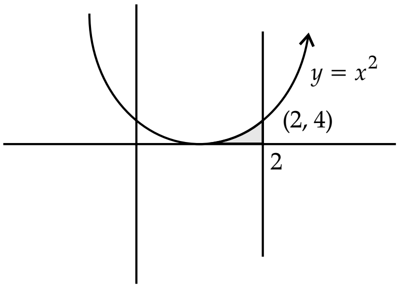
$$= 50\left[1 - \left(\frac{5}{50}\right)\right]^5$$

$$= 50[1 - 0.1]^5 = 50 \times (0.9)^5$$

$$= 50 \times 0.59049$$

$= 29.52450$ litre

27. (a)



$$\text{Area} = \int_0^2 x^2 dx$$

$$= \left[\frac{x^3}{3}\right]_0^2 = \frac{8}{3} - 0$$

Required Area $= \frac{8}{3}$ sq. units

Hence, Shaded portion is the required Area.

OR

(b)

$$I = \int_0^1 \frac{e^{-x}}{1 + e^x} dx$$

$$I = \int_0^1 \frac{dx}{e^x(1 + e^x)}$$

Let $e^x = t$

$$e^x\, dx = dt$$

$$dx = \frac{dt}{t}$$

$x = 0$ then $t = 1$

$x = 1$ then $t = e$

$$\therefore \quad I = \int_1^e \frac{dt}{t^2(1 + t)}$$

$$\frac{1}{t^2(t+1)} = \frac{A}{t}+\frac{B}{t^2}+\frac{C}{t+1}$$

$1 = At(t+1) + B(t+1) + Ct^2$

$t = 0 \quad 1 = B$

$t = -1 \quad 1 = C$

$t = 1$

$1 = 2A + 2B + C$

$1 = 2A + 2 + 1$

$A = -1$

$$\int_1^e \frac{1}{t^2(t+1)}dt = \int_1^e\left[-\frac{1}{t}+\frac{1}{t^2}+\frac{1}{t+1}\right]dt$$

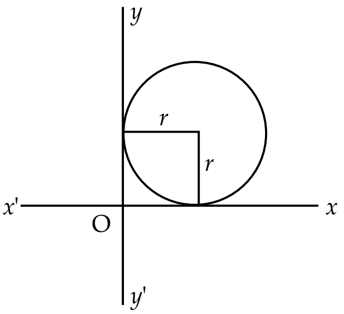
$$= \left[-\log|t|-\frac{1}{t}+\log(t+1)\right]_1^e$$

$$= \left[\log\left|\frac{t+1}{t}\right|-\frac{1}{t}\right]_1^e$$

$$= \log\frac{e+1}{e}-\frac{1}{e}-\log 2+1$$

$$= \log\frac{e+1}{2e}+1-\frac{1}{e}$$

28. Equation of circle touches the both axis in first quadrant



$(x-r)^2 + (y-r)^2 = r^2$ ...(i)

Differentiation w.r.t. x

$$2(x-r)+2(y-r)\frac{dy}{dx} = 0$$

$$(x-r) = -(y-r)\frac{dy}{dx} \quad ...(ii)$$

$$x-r+\frac{ydy}{dx}-\frac{rdy}{dx} = 0$$

$$x+\frac{ydy}{dx} = \left(1+\frac{dy}{dx}\right)_r$$

$$r = \frac{x+\frac{ydy}{dx}}{1+\frac{dy}{dx}} \quad ...(iii)$$

From (i) & (ii)

$$\left[-(y-r)\frac{dy}{dx}\right]^2+(y-r)^2 = r^2$$

$$(y-r)^2\left(\frac{dy}{dx}\right)^2+(y-r)^2 = r^2$$

$$\left[1+\left(\frac{dy}{dx}\right)^2\right](y-r)^2 = r^2$$

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]\left[y-\frac{x+y\frac{dy}{dx}}{1+\frac{dy}{dx}}\right]^2 = \left(\frac{x+y\frac{dy}{dx}}{1+\frac{dy}{dx}}\right)^2$$

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^2\frac{\left[y+y\frac{dy}{dx}-x-y\frac{dy}{dx}\right]^2}{\left(1+\frac{dy}{dx}\right)^2} = \left(\frac{x+y\frac{dy}{dx}}{1+\frac{dy}{dx}}\right)^2$$

$$\left(1+\frac{dy}{dx}\right)^2(y-x)^2 = \left(x+y\frac{dy}{dx}\right)^2$$

29. $90 = 100 - 10 = \text{S.D.}$

$110 = 100 + 10 = \text{S.D.}$

∴ Score between 90 and 110 is equal is ±S.D.

Hence, 68% of people would be expected between 90 and 110

$$z = \frac{x-\text{Mean}}{\text{S.D.}}$$

$$z_{90} = \frac{90-100}{10} = -1$$

$$z_{110} = \frac{110-100}{10} = 1$$

Score between 90 and 110 is

$P(z < 1) - P(z < -1) = 0.8413 - 0.1587$

$= 0.6826 = 68.26\%$

30. Sample size $(n) = 20$

Sample S.D. $(\sigma) = 10$ units

Sample mean $(\bar{x}) = 55$ units

Population mean $\mu = 50$ units

Null Hypothesis

The advertising campaign can not be successful

∴ $H_0 : \mu = 50$

Alternate Hypothesis

$H_1 : \mu > 50$

The advertising campaign was successful

$$z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n-1}}}$$

$$= \frac{\frac{55-50}{10}}{\sqrt{20-1}}$$

$$= \frac{5\times\sqrt{19}}{10}$$

$$= \frac{5\times 4.350}{10}$$

$$= \frac{21.794}{10}$$

$$= 2.179$$

$$z > t_{0.05} = 1.729$$

Thus the advertising campaign was successful.

31. **(a)** Computer system cost = ₹ 45200

(i) Rate of depreciation $= \dfrac{\text{Intial value} - \text{Scrap value}}{\text{Number of Years}}$

$$= \frac{45200-0}{3}$$

$$= ₹\ 15066.66 = ₹\ 15066\frac{2}{3}$$

(ii) Linear equation for book value

$$A = 45200 - \frac{45200}{3}t$$

$$A = 45200\left(1-\frac{t}{3}\right)$$

$$A = \frac{45200}{3}(3-t)$$

(iii) When $t = \dfrac{1}{2}$ year $= \dfrac{3}{2}$ year

$$A = \frac{45200}{3}\left(3-\frac{3}{2}\right)$$

$$= \frac{45200}{3}\left(\frac{3}{2}\right)$$

$$= ₹\ 22600$$

$\therefore$ Computer system's book value at the end of $1\frac{1}{2}$ year will be ₹ 22600.

OR

(b) Effective interest rate $(r) = \left(1+\dfrac{i}{n}\right)^n - 1$

where i = interest rate

n = number of compounding period per year

$$r = \left(1+\frac{i}{n}\right)^n - 1$$

(i) $r = \left(1+\dfrac{10}{200}\right)^2 - 1$

$= (1.05)^2 - 1$

$= 1.1025 - 1$

$= 0.1025$

$\therefore \quad r = 10.25\%$

(ii) $r = \left(1+\dfrac{10}{400}\right)^4 - 1$

$= (1.025)^4 - 1$

$= 1.1038 - 1 = 0.1038$

$\therefore \quad r = 10.38\%$

SECTION D

32. Let the pipe C running x h.

Then pipe B running $x + 1$ hr and pipe A running $x + 2$ h.

$$\frac{x+2}{3} + \frac{x+1}{4} - \frac{x}{1} = 1$$

$$\frac{4x+8+3x+3-12x}{12} = 1$$

$$\frac{-5x+11}{12} = 1$$

$$-5x = 12 - 11 = +1$$

$$x = -\frac{1}{5}$$

$$x = \frac{1}{5}\text{ h}$$

Tank will empty at 7:12 a.m.

[(–) sign shows that tank is not fill]

33. **(a)** $f(x) = -\dfrac{3}{4}x^4 - 8x^3 - \dfrac{45}{2}x^2 + 105$

$f'(x) = -3x^3 - 24x^2 - 45x + 0$

$f'(x) = -3x(x^2 + 8x + 15)$

$f'(x) = -3x(x + 5)(x + 3)$

For max/min $f'(x) = 0$

$-3x(x + 5)(x + 3) = 0$

$\therefore \quad x = 0, -5, -3$

$f'(x) = -3x^3 - 24x^2 - 45x$

$f''(x) = -9x^2 - 48x - 45$

$= -3(3x^2 + 16x + 15)$

$f''(-3) = -3\{27 - 48 + 15\}$

$= 18 > 0$

$\therefore f(x)$ is minimum at $x = -3$

$f''(-5) = -3(75 - 80 + 15)$

$= -30 < 0$

$\therefore$ $f(x)$ is maximum at $x = -5$

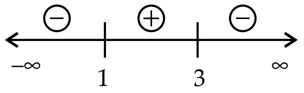
$$f''(3) = -3(0 + 0 + 15)$$
$$= -45 < 0$$

$\therefore$ $f(x)$ is maximum at $x = 0$

$f(x)$ is maximum at $x = 0$ and -5 and minimum at $x = -3$

OR

(b)
$$f(x) = 20 - 9x + 6x^2 - x^3$$
$$f'(x) = 0 - 9 + 12x - 3x^2$$
$$f'(x) = -3(x^2 - 4x + 3)$$
$$f'(x) = -3(x - 3)(x - 1)$$



For increasing function

$$f'(x) < 0$$
$$-3(x - 3)(x - 1) < 0$$

$\therefore$ $x < 1$ and $x > 3$

$f(x)$ is increasing $(1, 3)$

$f(x)$ is decreasing $(-\infty, 1) \cup (3, \infty)$

34. (a)
$$P(X = k) = \begin{cases} 0.1 & \text{if} \quad x_i = 0 \\ kx_i & \text{if} \quad x_i = 1 \text{ or } 2 \\ k(5 - x_i) & \text{if} \quad x_i = 3 \text{ or } 4 \end{cases}$$

(i) $P(0) + P(1) + P(2) + P(3) + P(4) = 1$

$$0.1 + k + 2k + 2k + k = 1$$
$$6k = 0.9$$
$$k = \frac{9}{60}$$
$$= \frac{3}{20}$$

(ii) $P(x = k \geq 2)$

$$2k + 2k + k = 5k$$
$$= 5 \times \frac{3}{20}$$
$$= \frac{3}{4}$$

(iii) $P(x = k \leq 2)$

$$0.1 + k + 2k$$
$$0.1 + 3k = \frac{1}{10} + \frac{3 \times 3}{20}$$
$$= \frac{11}{20}$$

(b) Average of flood overflow in every 10 years is 2

$\therefore$ Mean $(\lambda) = 2$

Probability of 3 or less overflow, in every 10 years

$\therefore$ $P(X \leq 3)$

$\therefore$ $P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$

$$\frac{2^0 e^{-2}}{01} + \frac{2^1 e^{-2}}{11} + \frac{2^2 e^{-2}}{21} + \frac{2^3 e^{-2}}{31}$$

$$e^{-2}\left[1 + 2 + \frac{4}{2} + \frac{8}{6}\right]$$

$$e^{-2}\left(\frac{19}{3}\right) \qquad \left[P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}\right]$$

$$= 0.13534 \times \frac{19}{3}$$

$= 0.85715$ Approx

Required probability is 0.85715

35. Case-down payment = ₹ 250000, EMI = ₹ 25448, time = 2 years, rate = 20% p.a.

$$\text{E.M.I.} = \frac{P \times i(1+i)^n}{(1+i)^n - 1}$$

$$P = \frac{\text{EMI}[(1+i)^n - 1]}{i(1+i)^n}$$

$$P = \frac{\text{EMI}}{i}\left[1 - \frac{1}{(1+i)^n}\right]$$

$$P = \frac{\text{EMI}}{i}[1 - (1+i)^{-n}]$$

$$P = \frac{25448}{\frac{20}{1200}}\left[1 - \left(1 + \frac{20}{1200}\right)^{-24}\right]$$

$$P = 25448 \times 60\left[1 - \left(\frac{61}{60}\right)^{-24}\right]$$

$$= 1526880(1 - 0.67253)$$
$$= 1526880\ (0.32747)$$
$$= ₹\ 500007.39$$

Actual Price = ₹ 500007.39 + 250000

= ₹ 750007.39

SECTION E

36. Let the number of children be x, amount for children be ₹ y

(i) $\therefore$ Total amount ₹ xy

$$(x - 8)(y + 10) = xy$$
$$xy + 10x - 8y - 80 = xy$$

$$10x - 8y = 80$$
$$5x - 4y = 40 \quad ...(i)$$
$$(x + 16)(y - 10) = xy$$
$$xy - 10x + 16y - 160 = xy$$
$$-10x + 16y = 160$$
$$-5x + 8y = 80 \quad ...(ii)$$

(ii) $\begin{bmatrix} 5 & -4 \\ -5 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ 80 \end{bmatrix}$

(iii) $A = \begin{bmatrix} 5 & -4 \\ -5 & 8 \end{bmatrix}$

(a) $|A| = \begin{bmatrix} 5 & -4 \\ -5 & 8 \end{bmatrix}$

$$= 40 - 20 = 20$$

$$\text{Adj } A = \begin{bmatrix} 8 & 4 \\ 5 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{20}\begin{bmatrix} 8 & 4 \\ 5 & 5 \end{bmatrix}$$

OR

(b) $AX = B$

$$A^{-1}AX = A^{-1}B$$
$$IX = A^{-1}B$$
$$X = A^{-1}B$$
$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{20}\begin{bmatrix} 8 & 4 \\ 5 & 5 \end{bmatrix}\begin{bmatrix} 40 \\ 80 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{20}\begin{bmatrix} 320+320 \\ 200+400 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{20}\begin{bmatrix} 640 \\ 600 \end{bmatrix}$$
$$= \begin{bmatrix} 32 \\ 30 \end{bmatrix}$$

$\therefore$ $x = 32$ and $y = 30$

37. Case Study – 2

Given, constraint $x + 2y \geq 100$

(i) $y \geq 2x$ and $2x + y \leq 200$

(ii) (a) for R_1 $y \geq 2x$ and $2x + y \leq 200$

OR

(b) for R_2 $y \leq 2x$ and $2x + y \leq 200$

(iii) $z = 5x + 2y$

R_1 is feasible region then corner points are D(0, 200) C (50, 100) B(20, 40) and A(0, 50)

$$z = 5x + 2y$$

At A(0, 50) $z = 100$

B(20, 40) $z = 100 + 80 = 180$

C(50, 100) $z = 250 + 200 = 450$

D(0, 200) $z = 0 + 400 = 400$

Max value at C(50, 100) is 450

38. Case Study – 3

(i)

Year	Sales (y) (in lakh ₹)	x	xy	x^2	Trend Value
1996	6.5	–5	–32.5	25	2.93
1997	5.3	–3	–15.9	9	4.13
1998	4.3	–1	–4.3	1	5.33
1999	6.1	1	6.1	1	6.53
2000	5.6	3	16.8	9	7.83
2001	7.8	5	39.0	25	8.93
$N = 6$	35.6	0	4.2	70	

$$a = \frac{\Sigma y - b\Sigma x}{N}$$
$$= \frac{35.6 - 0}{6}$$
$$= 5.933$$
$$b = \frac{N\Sigma xy - \Sigma x\Sigma y}{N\Sigma x^2 - (\Sigma x)^2}$$
$$b = \frac{6(4.2) - 0(35.6)}{6(70) - (0)^2}$$
$$= \frac{4.2}{70} = 0.6$$

$\therefore$ $y = a + bx$

$$= \frac{35.6}{6} + 0.6x$$
$$y = \frac{89}{15} + 0.6x$$

(ii) (a) Trend value for the year 2002

$$y = \frac{89}{15} + 0.6x$$

for 2002 $x = 7$

$$y = \frac{89}{15} + 0.6 \times 7$$
$$y = \frac{89}{15} + 4.2$$
$$= \frac{89}{15} + \frac{21}{5}$$

$$y = \frac{89+63}{15}$$

$$= \frac{152}{15}$$

y = ₹ 10.133 lakh

OR

(b)

Year	Profit (y) (₹ 000)	x	xy	x^2
2004	114	–3	–342	9
2005	130	–2	–260	4
2006	126	–1	–126	1
2007	144	0	0	0
2008	138	1	138	1
2009	156	2	312	4
2010	164	3	492	9
$N = 7$	972	0	214	28

$$a = \frac{\Sigma y - b\Sigma x}{N}$$

$$= \frac{972-0}{7}$$

$$a = \frac{972}{7}$$

$= 138.857$ (Approx)

$$b = \frac{N\Sigma xy - \Sigma x\Sigma y}{N\Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{7 \times 214 - 0}{7 \times 28 - 0}$$

$$= \frac{214}{28}$$

$$b = \frac{107}{14}$$

$= 7.642$ (Approx)

$y = a + bx$

$y = 138.857 + 7.642\,x$

■■