CBSE EXAMINATION PAPER 2024

Mathematics

Class-12th

(Solved)

(Delhi & Outside Delhi Sets)

Time : 3 Hours **Max. Marks : 80**

General Instructions:

Read the following instructions carefully and strictly follow them:

(i) This Question Paper contains ***38*** *questions.* ***All*** *questions are* ***compulsory.***
(ii) Question paper is divided into ***FIVE*** *sections Section* ***A, B, C, D*** *and* ***E.***
(iii) In ***Section A:*** *Question no.* ***1*** *to* ***18*** *are Multiple Choice Questions (MCQs) and Questions no.* ***19*** *&* ***20*** *are Assertion-Reason based questions of 1 mark each.*
(iv) In ***Section B:*** *Question no.* ***21*** *to* ***25*** *are Very Short Answer (VSA) type questions, carrying* ***2*** *marks each.*
(v) In ***Section C:*** *Question no.* ***26*** *to* ***31*** *are Short Answer (SA) type questions, carrying* ***3*** *marks each.*
(vi) In ***Section D:*** *Question no.* ***32*** *to* ***35*** *are Long Answer (LA) type questions, carrying* ***5*** *marks each.*
(vii) In ***Section E:*** *Question no.* ***36*** *to* ***38*** *are case study based questions, carrying* ***4*** *marks each.*
(viii) There is no overall choice. However, an internal choice has been provided in ***2*** *questions in Section-****B****,* ***3*** *questions in Section-****C****,* ***2*** *questions in Section-****D*** *and* ***2*** *questions in Section-****E****.*
(ix) Use of calculators is NOT allowed.

Delhi Set-1 **65/5/1**

SECTION A

This section has 20 multiple choice questions of 1 mark each.

1. A function $f : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = x^2 - 4x + 5$ is
(A) injective but not surjective
(B) surjective but not injective
(C) both injective and surjective
(D) neither injective nor surjective

2. If $A = \begin{bmatrix} a & c & -1 \\ b & 0 & 5 \\ 1 & -5 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then the value of $2a - (b + c)$ is
(A) 0 **(B)** 1
(C) –10 **(D)** 10

3. If A is a square matrix of order 3 such that the value of $|\text{adj}.A| = 8$, then the value of $|A^T|$ is
(A) $\sqrt{2}$ **(B)** $-\sqrt{2}$
(C) 8 **(D)** $2\sqrt{2}$

4. If inverse of matrix $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is the matrix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then value of λ is
(A) – 4 **(B)** 1
(C) 3 **(D)** 4

5. If $[x\ 2\ 0] \begin{bmatrix} 5 \\ -1 \\ x \end{bmatrix} = [3\ 1] \begin{bmatrix} -2 \\ x \end{bmatrix}$, then value of x is
(A) –1 **(B)** 0
(C) 1 **(D)** 2

6. Find the matrix A^2, where $A = [a_{ij}]$ is a 2×2 matrix whose elements are given by a_{ij} = maximum (i, j) – minimum (i, j)
(A) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **(B)** $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
(C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **(D)** $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

7. If $xe^y = 1$, then the value of $\frac{dy}{dx}$ at $x = 1$ is
(A) – 1 **(B)** 1
(C) $-e$ **(D)** $-\frac{1}{e}$

8. Derivative of $e^{\sin^2 x}$ with respect to $\cos x$ is
(A) $\sin x\, e^{\sin^2 x}$ **(B)** $\cos x\, e^{\sin^2 x}$
(C) $-2 \cos x\, e^{\sin^2 x}$ **(D)** $-2 \sin^2 x \cos x\, e^{\sin^2 x}$

9. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minima at x equal to
(A) 2 **(B)** 1
(C) 0 **(D)** – 2

10. Given a curve $y = 7x - x^3$ and x increases at the rate of 2 units per second. The rate at which the slope of the curve is changing, when $x = 5$ is

(A) – 60 units/s **(B)** 60 units/s

(C) – 70 units/s **(D)** –140 units/s

11. $\int \frac{1}{x(\log x)^2} dx$ is equal to

(A) $2 \log (\log x) + c$ **(B)** $-\frac{1}{\log x} + c$

(C) $\frac{(\log x)^3}{3} + c$ **(D)** $\frac{3}{(\log x)^3} + c$

12. The value of $\int_{-1}^{1} x|x| \, dx$ is

(A) $\frac{1}{6}$ **(B)** $\frac{1}{3}$

(C) $-\frac{1}{6}$ **(D)** 0

13. Area of the region bounded by curve $y^2 = 4x$ and the X-axis between $x = 0$ and $x = 1$ is

(A) $\frac{2}{3}$ **(B)** $\frac{8}{3}$

(C) 3 **(D)** $\frac{4}{3}$

14. The order of the differential equation $\frac{d^4y}{dx^4} - \sin\left(\frac{d^2y}{dx^2}\right) = 5$ is

(A) 4 **(B)** 3

(C) 2 **(D)** not defined

15. The position vectors of points P and Q are $\vec{p}$ and $\vec{q}$ respectively. The point R divides line segment PQ in the ratio 3 : 1 and S is the mid-point of line segment PR. The position vector of S is

(A) $\frac{\vec{p}+3\vec{q}}{4}$ **(B)** $\frac{\vec{p}+3\vec{q}}{8}$

(C) $\frac{5\vec{p}+3\vec{q}}{4}$ **(D)** $\frac{5\vec{p}+3\vec{q}}{8}$

16. The angle which the line $\frac{x}{1} = \frac{y}{-1} = \frac{z}{0}$ makes with the positive direction of Y-axis is

(A) $\frac{5\pi}{6}$ **(B)** $\frac{3\pi}{4}$

(C) $\frac{5\pi}{4}$ **(D)** $\frac{7\pi}{4}$

17. The Cartesian equation of the line passing through the point (1, –3, 2) and parallel to the line

$\vec{r} = (2+\lambda)\hat{i} + \lambda\hat{j} + (2\lambda - 1)\hat{k}$ is

(A) $\frac{x-1}{2} = \frac{y+3}{0} = \frac{z-2}{-1}$ **(B)** $\frac{x+1}{1} = \frac{y-3}{1} = \frac{z+2}{2}$

(C) $\frac{x+1}{2} = \frac{y-3}{0} = \frac{z+2}{-1}$ **(D)** $\frac{x-1}{1} = \frac{y+3}{1} = \frac{z-2}{2}$

18. If A and B are events such that $P(A/B) = P(B/A) \neq 0$, then

(A) $A \subset B$, but $A \neq B$ **(B)** $A = B$

(C) $A \cap B = \phi$ **(D)** $P(A) = P(B)$

Assertion - Reason Based Questions

Direction: In questions numbers **19** and **20**, two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the following options:

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. Assertion (A): Domain of $y = \cos^{-1}(x)$ is [–1, 1].

Reason (R): The range of the principal value branch of $y = \cos^{-1}(x)$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$.

20. Assertion (A): The vectors

$$\vec{a} = 6\hat{i} + 2\hat{j} - 8\hat{k}$$

$$\vec{b} = 10\hat{i} - 2\hat{j} - 6\hat{k}$$

$$\vec{c} = 4\hat{i} - 4\hat{j} + 2\hat{k}$$

represent the sides of a right angled triangle.

Reason (R): Three non-zero vectors of which none of two are collinear forms a triangle if their resultant is zero vector or sum of any two vectors is equal to the third.

SECTION B

*This section has **5** Very Short Answer questions of **2** marks each.*

21. Find value of k if $\sin^{-1}\left[k \tan\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right] = \frac{\pi}{3}$.

22. (a) Verify whether the function f defined by

$$f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$$ is continuous at $x = 0$ or not.

OR

(b) Check for differentiability of the function f defined by $f(x) = |x - 5|$, at the point $x = 5$.

23. The area of the circle is increasing at a uniform rate of 2 cm^2/s. How fast is the circumference of the circle increasing when the radius $r = 5$ cm ?

24. (a) Find: $\int \cos^3 x \, e^{\log \sin x} \, dx$

OR

(b) Find: $\int \frac{1}{5 + 4x - x^2} dx$

25. Find the vector equation of the line passing through the point (2, 3, – 5) and making equal angles with the co-ordinate axes.

SECTION C

There are 6 short answer questions in this section. Each is of 3 marks.

26. **(a)** Find $\frac{dy}{dx}$, , if $(\cos x)^y = (\cos y)^x$.

OR

(b) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x - y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

27. If x = a $\sin^3 \theta$, $y = b \cos^3 \theta$,then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

28. **(a)** Evaluate: $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx$

OR

(b) Find: $\int \frac{2x+1}{(x+1)^2(x-1)} dx$

29. **(a)** Find the particular solution of the differential equation $\frac{dy}{dx} - 2xy = 3^{x^2} e^{x^2}$; $y(0) = 5$.

OR

(b) Solve the following differential equation:

$x^2\, dy + y\,(x + y)\, dx = 0$

30. Find a vector of magnitude 4 units perpendicular to each of the vectors $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - \hat{k}$ and hence verify your answer.

31. The random variable X has the following probability distribution where *a* and *b* are some constants:

X	1	2	3	4	5
P(X)	0.2	a	a	0.2	b

If the mean $E(X) = 3$, then find values of a and b and hence determine $P(X \geq 3)$.

SECTION D

There are 4 long answer questions in this section. Each question is of 5 marks.

32. **(a)** If $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 0 & -3 \\ 1 & 2 & 0 \end{bmatrix}$, then find A^{-1} and hence solve the following system of equations:

$$x + 2y - 3z = 1$$
$$2x - 3z = 2$$
$$x + 2y = 3$$

OR

(b) Find the product of the matrices $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$ and hence solve the system of linear equations:

$$x + 2y - 3z = 4$$
$$2x + 3y + 2z = 2$$
$$3x - 3y - 4z = 11$$

33. Find the area of the region bounded by the curve $4x^2 + y^2 = 36$ using integration.

34. **(a)** Find the co-ordinates of the foot of the perpendicular drawn from the point (2, 3, – 8) to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$.

OR

(b). Find the shortest distance between the lines L_1 & L_2 given below:

L_1 : The line passing through (2, –1, 1) and parallel to $\frac{x}{1} = \frac{y}{1} = \frac{z}{3}$ L_2 : $\vec{r} = \hat{i} + (2\mu + 1)\hat{j} - (\mu + 2)\hat{k}$.

35. Solve the following L. P. P. graphically:

Maximise $Z = 60x + 40y$

Subject to
$$x + 2y \leq 12$$
$$2x + y \leq 12$$
$$4x + 5y \geq 20$$
$$x, y \geq 0$$

SECTION E

In this section there are 3 case study questions of 4 marks each.

36. **(a)** Students of a school are taken to a railway museum to learn about railways heritage and its history.

An exhibit in the museum depicted many rail lines on the track near the railway station. Let L be the set of all rail lines on the railway track and R be the relation on L defined by

$R = \{(l_1, l_2) : l_1 \text{ is parallel to } l_2\}$

On the basis of the above information, answer the following questions:

(i) Find whether the relation R is symmetric or not.

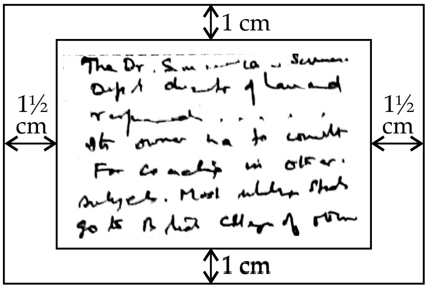
(ii) Find whether the relation R is transitive or not.

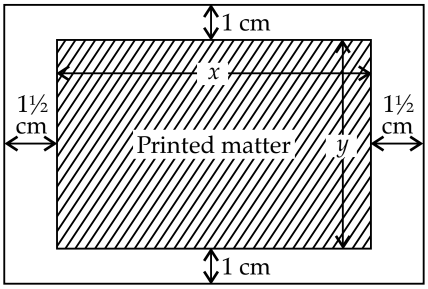
(iii) If one of the rail lines on the railway track is represented by the equation $y = 3x + 2$, then find the set of rail lines in R related to it.

OR

(b) Let S be the relation defined by $S = \{(l_1, l_2) : l_1$ is perpendicular to $l_2\}$ check whether the relation S is symmetric and transitive.

37. A rectangular visiting card is to contain 24 sq. cm. of printed matter. The margins at the top and bottom of the card are to be 1 cm and the margins on the left and right are to be $1\frac{1}{2}$ cm as shown below:





On the basis of the above information, answer the following questions:

(i) Write the expression for the area of the visiting card in terms of x.

(ii) Obtain the dimensions of the card of minimum area.

38. A departmental store sends bills to charge its customers once a month. Past experience shows that 70% of its customers pay their first month bill in time. The store also found that the customer who pays the bill in time has the probability of 0.8 of paying in time next month and the customer who doesn't pay in time has the probability of 0.4 of paying in time the next month.

Based on the above information, answer the following questions:

(i) Let E_1 and E_2 respectively, denote the event of customer paying or not paying the first month bill in time.

(ii) Let A denotes the event of customer paying second month's bill in time, then find $P(A|E_1)$ and $P(A|E_2)$.

(iii) Find the probability of customer paying second month's bill in time.

OR

(iii) Find the probability of customer paying first month's bill in time if it is found that customer has paid the second month's bill in time.

Delhi Set-2 **65/5/2**

Note: Except these, all other questions are available in Delhi Set-1

SECTION A

2. If A is a square metric of order 2 and $|A| = -2$, then value of $|5A'|$ is

(A) – 50 **(B)** – 10
(C) 10 **(D)** 50

5. The product of matrix P and Q is equal to a diagonal matrix. If the order of matrix Q is 3 × 2, then order of matrix P is

(A) 2 × 2 **(B)** 3 × 3
(C) 2 × 3 **(D)** 3 × 2

7. If $\sin(xy) = 1$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{x}{y}$ **(B)** $-\frac{x}{y}$

(C) $\frac{y}{x}$ **(D)** $-\frac{y}{x}$

11. The value of $\int_{\pi/4}^{\pi/2} \cot\theta \operatorname{cosec}^2\theta \, d\theta$ is

(A) $\frac{1}{2}$ **(B)** $-\frac{1}{2}$

(C) 0 **(D)** $-\frac{\pi}{8}$

12. The integral $\int \frac{dx}{\sqrt{9-4x^2}}$ is equal to

(A) $\frac{1}{6}\sin^{-1}\left(\frac{2x}{3}\right)+c$ **(B)** $\frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right)+c$

(C) $\sin^{-1}\left(\frac{2x}{3}\right)+c$ **(D)** $\frac{3}{2}\sin^{-1}\left(\frac{2x}{3}\right)+c$

14. The general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is:

(A) $e^x + e^{-y} = c$ **(B)** $e^{-x} + e^{-y} = c$
(C) $e^{x+y} = c$ **(D)** $2e^{x+y} = c$

SECTION B

21. If $a = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)+\cos^{-1}\left(-\frac{1}{2}\right)$ and

$b = \tan^{-1}(\sqrt{3}) - \cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

then find the value of $a + b$.

23. Sand is pouring from a pipe at the rate of 15 cm^3/minute. The falling sand forms a cone on the ground such that the height of the cone is always one-third of the radius of the base. How fast is the height of the sand cone increasing at the instant when the height is 4 cm?

SECTION C

27. Find the values of a and b so that the following function is differentiable for all values of x:

$$f(x)=\begin{cases} ax+b & x>-1 \\ bx^2-3 & x<-1 \end{cases}$$

30. Given $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$, $\vec{b}=3\hat{i}-\hat{k}$ and $\vec{c}=2\hat{i}+\hat{j}-2\hat{k}$.

Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d}=3$.

31. Bag I contains 3 red and 4 black balls, Bag II contains 5 red and 2 black balls. Two balls are transferred at random from Bag I to Bag II and then a ball is drawn at random from Bag II. Find the probability that the drawn ball is red in colour.

SECTION D

35. Solve the following Linear Programming problem graphically:

Maximise $Z = 300x + 600y$

Subject to $x + 2y \le 12$

$2x + y \le 12$

$x+\frac{5}{4}y \ge 5$

$x \ge 0, y \ge 0.$

Delhi Set-3 **65/5/3**

Note: Except these, all other questions are available in Delhi Set-1&2

SECTION A

2. For the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ \lambda & 2 & 0 \\ 1 & -2 & 3 \end{bmatrix}$ to be invertible, the value of λ is

(A) 0 **(B)** 10

(C) $\mathbb{R}-\{10\}$ **(D)** $\mathbb{R}-\{-10\}$

5. If $A = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ and $B=\begin{bmatrix} 4 & 0 \\ -1 & 1 \end{bmatrix}$, then value of x for which $A^2 = B$ is

(A) – 2 **(B)** 2

(C) 2 or –2 **(D)** 4

11. If $\int_{-2}^{3} x^2dx = k\int_{0}^{2} x^2dx + \int_{2}^{3} x^2\,dx$, then the value of k is

(A) 2 **(B)** 1

(C) 0 **(D)** $\frac{1}{2}$

12. The value of $\int_{1}^{e} \log x\,dx$ is

(A) 0 **(B)** 1

(C) e **(D)** $e \log e$

13. The area bounded by the curve $y = \sqrt{x}$, Y-axis and between the lines $y = 0$ and $y = 3$ is

(A) $2\sqrt{3}$ **(B)** 27

(C) 9 **(D)** 3

14. The order of the following differential equation

$$\frac{d^3y}{dx^3}+x\left(\frac{dy}{dx}\right)^5=4\log\left(\frac{d^4y}{dx^4}\right)$$ is

(A) not defined **(B)** 3

(C) 4 **(D)** 5

SECTION B

21. Simplify: $\cos^{-1}x + \cos^{-1}\left[\frac{x}{2}+\frac{\sqrt{3-3x^2}}{2}\right];\frac{1}{2}\le x\le 1$

27. If $y = (\tan^{-1}x)^2$, show that

$$(x^2+1)^2\frac{d^2y}{dx^2}+2x(x^2+1)\frac{dy}{dx}=2.$$

30. Find the projection of vector $(\vec{b}+\vec{c})$ on vector $\vec{a}$, where $\vec{a}=2\hat{i}+2\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+3\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}+\hat{k}$

31. An urn contains 3 red and 2 white marbles. Two marbles are drawn one by one with replacement from the urn. Find the probability distribution of the number of white balls. Also, find the mean of the number of white balls drawn.

SECTION D

35. Solve the following L.P.P. graphically:

Minimise Z = $6x + 3y$

Subject to constraints

$4x + y \ge 80;$

$x + 5y \ge 115;$

$3x + 2y \le 150$

$x, y \ge 0$

Outside Delhi Set-1 **65/4/1**

SECTION A

This section has 20 multiple choice questions of 1 mark each. 20 × 1 = 20

1. If $\begin{bmatrix} a & c & 0 \\ b & d & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is a scalar matrix, then the value of a + $2b$ +$3c$ + $4d$ is

(A) 0 **(B)** 5

(C) 10 **(D)** 25

2. Given that $A^{-1}=\frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$, matrix A is

(A) $7\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ **(B)** $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

(C) $\frac{1}{7}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ (D) $\frac{1}{49}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

3. If $A = \begin{bmatrix} 2 & -1 \\ -4 & -2 \end{bmatrix}$, then the value of $I - A + A^2 - A^3 + \ldots$ is

(A) $\begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$ (B) $\begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

4. If $A = \begin{bmatrix} -2 & 0 & 0 \\ 1 & 2 & 3 \\ 5 & 1 & -1 \end{bmatrix}$, then the value of $|A(adj.\ A)|$ is:

(A) 100 I (B) 10 I
(C) 10 (D) 1000

5. Given that $[1\ x]\begin{bmatrix} 4 & 0 \\ -2 & 0 \end{bmatrix} = 0$, the value of x is

(A) – 4 (B) – 2
(C) 2 (D) 4

6. Derivative of e^{2x} with respect to e^x, is
(A) e^x (B) $2e^x$
(C) $2e^{2x}$ (D) $2e^{3x}$

7. For what value of k, the function given below is continuous at $x = 0$?

$$f(x) = \begin{cases} \sqrt{\dfrac{4+x-2}{x}}, & x \neq 0 \\ k & x = 0 \end{cases}$$

(A) 0 (B) $\frac{1}{4}$

(C) 1 (D) 4

8. The value of $\int_0^3 \frac{dx}{\sqrt{9-x^2}}$ is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$ (D) $\frac{\pi}{18}$

9. The general solution of the differential equation $x\,dy + y\,dx = 0$ is
(A) $xy = c$ (B) $x + y = c$
(C) $x^2 + y^2 = c^2$ (D) $\log y = \log x + c$

10. The integrating factor of the differential equation $(x + 2y^2)\frac{dy}{dx} = y\ (y > 0)$ is

(A) $\frac{1}{x}$ (B) x

(C) y (D) $\frac{1}{y}$

11. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 1$, $|\vec{b}| = 2$ and $\vec{a}.\vec{b} = \sqrt{3}$, then the angle between $2\vec{a}$ and $-\vec{b}$ is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$

(C) $\frac{5\pi}{6}$ (D) $\frac{11\pi}{6}$

12. The vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 3\hat{j} - 5\hat{k}$ and $\vec{c} = -3\hat{i} + 4\hat{j} + 4\hat{k}$ represents the sides of
(A) an equilateral triangle
(B) an obtuse-angled triangle
(C) an isosceles triangle
(D) a right-angled triangle

13. Let $\vec{a}$ be any vector such that $|\vec{a}| = a$ The value of $|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2$ is

(A) a^2 (B) $2a^2$
(C) $3a^2$ (D) 0

14. The vector equation of a line passing through the point (1, – 1, 0) and parallel to Y-axis is

(A) $\vec{r} = \hat{i} - \hat{j} + \lambda(\hat{i} - \hat{j})$ (B) $\vec{r} = \hat{i} - \hat{j} + \lambda\hat{j}$

(C) $\vec{r} = \hat{i} - \hat{j} + \lambda\hat{k}$ (D) $\vec{r} = \lambda\hat{j}$

15. The lines $\frac{1-x}{2} = \frac{y-1}{3} = \frac{z}{1}$ and $\frac{2x-3}{2p} = \frac{y}{-1} = \frac{z-4}{7}$ are perpendicular to each other for p equal to

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$

(C) 2 (D) 3

16. The maximum value of $Z = 4x + y$ for a L.P.P. whose feasible region is given below is

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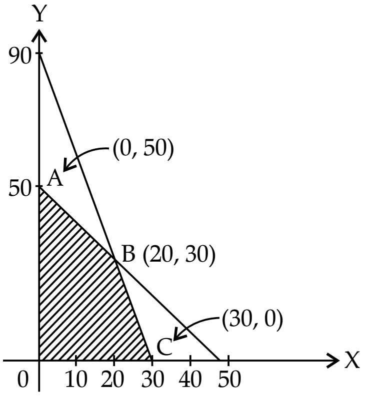


(A) 50 (B) 110
(C) 120 (D) 170

17. The probability distribution of a random variable X is

X	0	1	2	3	4
P(X)	0.1	k	$2k$	k	0.1

Where k is some unknown constant.

The probability that the random variable X takes the value 2 is

(A) $\frac{1}{5}$ (B) $\frac{2}{5}$

(C) $\frac{4}{5}$ (D) 1

18. The function $f(x) = kx - \sin x$ is strictly increasing for

(A) $k > 1$ (B) $k < 1$

(C) $k > -1$ (D) $k < -1$

Assertion - Reason Based Questions

Questions No. 19 & 20, are Assertion (A) and Reason (R) based questions carrying 1 mark each. Two statements are given. one labelled Assertion (A) and the other labelled Reason (R).

Select the correct answer from the codes (A), (B), (C) and (D) as given below:

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).

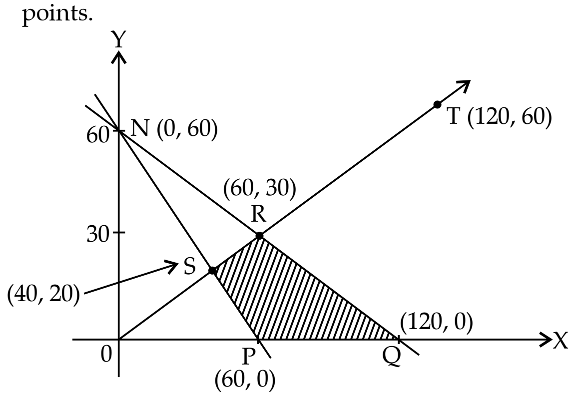
(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. Assertion (A): The relation $R = \{(x, y) : (x + y)$ is a prime number and $x, y \in N\}$

Reason (R): The number '$2n$' is composite for all natural numbers n.

20. Assertion (A): The corner points of the bounded feasible region of a L. P. P. are shown below. The maximum value of $Z = x + 2y$ occurs at infinite points.



Reason (R): The optimal solution of a LPP having bounded feasible region must occur at corner points.

SECTION - B

In this section there are 5 very short answer type questions of 2 marks each.

21. (a) Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$, Where $\frac{-\pi}{2} < x < \frac{\pi}{2}$ in the simplest form.

(b) Find the principal value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$.

22. (a) If $y = \cos^3(\sec^2 2t)$, find $\frac{dy}{dt}$.

OR

(b) If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.

23. Find the interval in which the function $f(x) = x^4 - 4x^3 + 10$ is strictly decreasing.

24. The volume of a cube is increasing at the rate of 6 cm^3/s. How fast is the surface area of cube increasing, when the length of an edge is 8 cm ?

25. Find: $\int \frac{1}{x(x^2-1)}\, dx$.

SECTION C

In this section there are 6 short answer type questions of 3 marks each.

26. Given that $y = (\sin x)^x \cdot x^{\sin x} + a^x$, find $\frac{dy}{dx}$.

27. (a) Evaluate : $\int_0^{\frac{\pi}{4}} \frac{x\,dx}{1+\cos 2x+\sin 2x}$

OR

(b) Find: $\int e^x \left[\frac{1}{(1+x^2)^{\frac{3}{2}}} + \frac{x}{\sqrt{1+x^2}}\right] dx$

28. Find: $\int \frac{3x+5}{\sqrt{x^2+2x+4}}\, dx$

29. (a) Find the particular solution of the differential equation $\frac{dy}{dx} = y \cot 2x$, given that $y\left(\frac{\pi}{4}\right) = 2$.

OR

(b) Find the particular solution of the differential equation $\left(xe^{\frac{y}{x}} + y\right) dx = x\, dy$, given that $y = 1$ when $x = 1$.

30. Solve the following linear programming problem graphically:

Maximise $Z = 2x + 3y$

subject to the constraints:

$$x + y \le 6$$
$$x \ge 2$$
$$y \le 3$$
$$x, y \ge 0.$$

31. (a) A card from a well shuffled deck of 52 playing cards is lost. From the remaining cards of the pack, a card is drawn at random and is found to be a king. Find the probability of the lost card being a king.

OR

(b) A biased die is twice as likely to show an even number as an odd number. If such a die is thrown twice, find the probability distribution of the number of sixes. Also, find the mean of the distribution.

SECTION D

In the section there are 4 long answer type questions of 5 marks each.

32. (a) Sketch the graph of $y = x\,|x|$ and hence find the area bounded by this curve, X-axis and the ordinates $x = -2$ and $x = 2$, using integration.

OR

(b) Using integration, find the area bounded by the ellipse $9x^2 + 25y^2 = 225$, the lines $x = -2$, $x = 2$, and the X-axis.

33. (a) Let $A = R - \{5\}$ and $B = R - \{1\}$. Consider the function $f : A \to B$, defined by $f(x) = \frac{x-3}{x-5}$ Show that f is one-one and onto.

(b) Check whether the relation S in the set of real numbers R defined by $S = \{(a, b) :$ where $a - b + \sqrt{2}$ is an irrational number$\}$ is reflexive, symmetric or transitive.

34. If $A = \begin{bmatrix} 2 & 1 & -3 \\ 3 & 2 & 1 \\ 1 & 2 & -1 \end{bmatrix}$, find A^{-1} and hence solve the following system of equations:

$$2x + y - 3z = 13$$
$$3x + 2y + z = 4$$
$$x + 2y - z = 8$$

35. (a) Find the distance between the line $\frac{x}{2} = \frac{2y-6}{4} = \frac{1-z}{-1}$ and another line parallel to it passing through the point $(4, 0, -5)$.

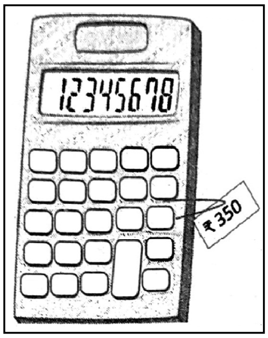
OR

(b) If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-7}$ are perpendicular to each other, find the value of k and hence write the vector equation of a line perpendicular to these two lines and passing through the point $(3, -4, 7)$.

SECTION E

In this section, there are 3 case study based questions of 4 marks each.

36. A store has been selling calculators at ₹ 350 each. A market survey indicates that a reduction in price (p) of calculator increases the number of units (x) sold. The relation between the price and quantity sold is given by the demand function $p = 450 - \frac{1}{2}x$.

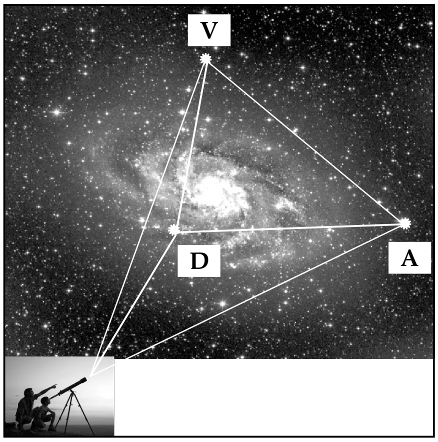


Based on the above information, answer the following questions:

(i) Determine the number of units (x) that should be sold to maximise the revenue $R(x) = xp(x)$. Also, verify the result.

(ii) What rebate in price of calculator should the store give to maximise the revenue ?

37. An instructor at the astronomical centre shows three among the brightest stars in a particular constellation. Assume that the telescope is located at O (0, 0, 0) and the three stars have their locations at the points D, A and V having position vectors $2\hat{i}+3\hat{j}+4\hat{k}$, $7\hat{i}+5\hat{j}+8\hat{k}$, and $-3\hat{i}+7\hat{j}+11\hat{k}$ respectively.



Based on the above information, answer the following questions:

(i) How far is the star V from star A ?

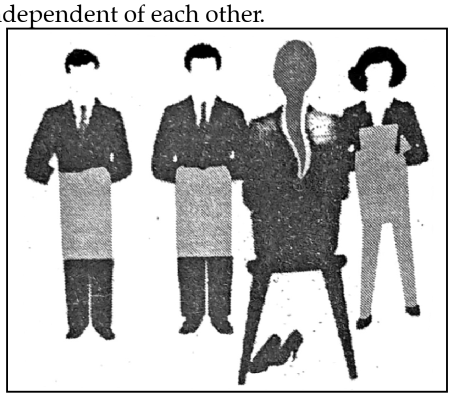
(ii) Find a unit vector in the direction of $\overrightarrow{DA}$.

(iii) Find the measure of $\angle$VDA.

OR

(iii) What is the projection of vector $\overrightarrow{DV}$ on vector $\overrightarrow{DA}$?

38. Rohit, Jaspreet and Alia appeared for an interview for three vacancies in the same post. The probability of Rohit's Selection is $\frac{1}{5}$, Jaspreet's selection is $\frac{1}{3}$ and Alia's selection is $\frac{1}{4}$. The event of selection is independent of each other.



Based on the above information, answer the following questions:

(i) What is the probability that at least one of them is selected ?

(ii) Find P (G | $\overline{H}$) where G is the event of Jaspreet's selection and $\overline{H}$ denotes the event that Rohit is not selected.

(iii) Find the probability that exactly one of them is selected.

OR

(iii) Find the probability that exactly two of them are selected.

Outside Delhi Set-2 — 65/4/2

Note: Except these, all other questions are available in Outside Delhi Set-1

SECTION A

4. If $A = [a_{ij}] = \begin{bmatrix} 2 & -1 & 5 \\ 1 & 3 & 2 \\ 5 & 0 & 4 \end{bmatrix}$ and C_{ij} is the cofactor of element a_{ij} then the value of $a_{21} \cdot c_{11} + a_{22} \cdot c_{12} + a_{23} \cdot c_{13}$ is

(A) – 57 **(B)** 0
(C) 9 **(D)** 57

5. If $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$ and $A^2 - kA - 5I = 0$, then the value of k is

(A) 3 **(B)** 5
(C) 7 **(D)** 9

6. If $e^{x^2 y} = c$, then $\frac{dy}{dx}$ is

(A) $\frac{xe^{x^2 y}}{2y}$ **(B)** $\frac{-2y}{x}$

(C) $\frac{2y}{x}$ **(D)** $\frac{x}{2y}$

7. The value of the constant c that makes the function f defined by

$$f(x) = \begin{cases} x^2 - c^2, & \text{if } x < 4 \\ cx + 20, & \text{if } x \geq 4 \end{cases}$$

continuous for all real numbers is

(A) – 2 **(B)** – 1
(C) 0 **(D)** 2

8. The value of $\int_{-1}^{1} |x|\, dx$ is

(A) – 2 **(B)** – 1
(C) 1 **(D)** 2

9. The number of arbitrary constants in the particular solution of the differential equation $\log\left(\frac{dy}{dx}\right) = 3x + 4y;\ y(0) = 0$ is/are

(A) 2 **(B)** 1
(C) 0 **(D)** 3

14. A vector perpendicular to the line $\vec{r} = \hat{i} + \hat{j} - \hat{k} + \lambda(3\hat{i} - \hat{j})$ is

(A) $5\hat{i} + \hat{j} + 6k$ **(B)** $\hat{i} + 3\hat{j} + 5\hat{k}$

(C) $2\hat{i} - 2\hat{j}$ **(D)** $9\hat{i} - 3\hat{j}$

SECTION B

23. Show that $f(x) = \frac{4\sin x}{2 + \cos x} - x$ is an increasing function of x in $\left[0, \frac{\pi}{2}\right]$.

25. Evaluate: $\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos x . \log\left(\frac{1+x}{1-x}\right) dx$

SECTION C

26. Given that $x^y + y^x = a^b$, where a and b are positive constants, find $\frac{dy}{dx}$.

28. Find: $\int \frac{2x+3}{x^2(x+3)} dx$

30. Solve the following L.P.P. graphically:
Maximise $Z = x + 3y$
subject to the constraints:

$$x + 2y \leq 200$$
$$x + y \leq 150$$
$$y \leq 75$$
$$x, y = 0$$

SECTION D

34. Use the product of matrices $\begin{pmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{pmatrix}$ to solve the following system of equations:

$$x + 2y - 3z = 6$$
$$3x + 2y - 2z = 3$$
$$2x - y + z = 2$$

Outside Delhi Set-3 — 65/4/3

Note: Except these, all other questions are available in Outside Delhi Set-1&2

SECTION A

4. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $A^2 + 7I = kA$, then the value of k is

(A) 1 **(B)** 2
(C) 5 **(D)** 7

5. Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$ and $B = \frac{1}{3}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & \lambda \end{bmatrix}$. If $AB = I$, then the value of λ is

(A) $\frac{-9}{4}$ **(B)** -2

(C) $\frac{-3}{2}$ **(D)** 0

6. Derivative of x^2 with respect to x^3, is

(A) $\frac{2}{3x}$ **(B)** $\frac{3x}{2}$

(C) $\frac{2x}{3}$ **(D)** $6x^5$

7. The function $f(x) = |x|+|x-2|$ is

(A) continuous, but not differentiable at $x = 0$ and $x = 2$.

(B) differentiable but not continuous at $x = 0$ and $x = 2$.

(C) continuous but not differentiable at $x = 0$ only.

(D) neither continuous nor differentiable at $x = 0$ and $x = 2$.

8. The value of $\int_0^{\pi} \tan^2\left(\frac{\theta}{3}\right) d\theta$ is

(A) $\pi + \sqrt{3}$ **(B)** $3\sqrt{3} - \pi$

(C) $\sqrt{3} - \pi$ **(D)** $\pi - \sqrt{3}$

9. The integrating factor of the differential equation $\frac{dy}{dx} + \frac{2}{x}y = 0, x \neq 0$ is

(A) $\frac{2}{x}$ **(B)** x^2

(C) $e^{\frac{2}{x}}$ **(D)** $e^{\log(2x)}$

14. The cartesian equation of a line passing through the point with position vector $\vec{a} = \hat{i} - \hat{j}$ and parallel to the line $\vec{r} = \hat{i} + \hat{k} + \mu(2\hat{i} - \hat{j})$, is

(A) $\frac{x-2}{1} = \frac{y+1}{0} = \frac{z}{1}$ **(B)** $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{0}$

(C) $\frac{x+1}{2} = \frac{y+1}{-1} = \frac{z}{0}$ **(D)** $\frac{x-1}{2} = \frac{y}{-1} = \frac{z-1}{0}$

SECTION B

23. Show that the function f given by $f(x) = \sin x + \cos x$, is strictly decreasing in the interval $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$.

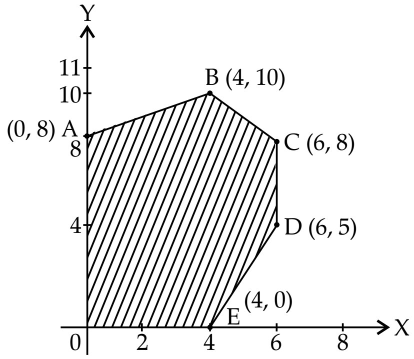
25. Find: $\int \frac{2x}{(x^2+1)(x^2-4)} dx$.

SECTION C

26. Find $\frac{dy}{dx}$, if $y = (\cos x)^x + \cos^{-1}\sqrt{x}$ is given.

28. Find: $\int \sec^3 \theta \, d\theta$

30. The corner points of the feasible region determined by the system of linear constraints are as shown in the following figure:



(i) If $Z = 3x - 4y$ be the objective function, then find the maximum value of Z.

(ii) If $Z = px + qy$ where $p, q, > 0$ be the objective function. Find the condition on p and q so that maximum value of Z occurs at B(4, 10) and C(6, 8).

SECTION D

34. Find A^{-1}, if $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 1 & 0 & 1 \end{bmatrix}$. Hence, solve the following system of equations:

$$x + 2y + z = 5$$
$$2x + 3y = 1$$
$$x - y + z = 8$$

ANSWERS

Delhi Set-1 **65/5/1**

SECTION A

1. **Option (D) is correct.**

Explanation: Given, $f: R \to R$

$$f(x) = x^2 - 4x + 5$$

One-One (Injective)

$$f(x_1) = f(x_2)$$

$$\Rightarrow \quad x_1^2 - 4x_1 + 5 = x_1^2 - 4x_2 + 5$$

$$\Rightarrow \quad x_1^2 - x_2^2 = 4(x_1 - x_2)$$

$$\Rightarrow \quad (x_1 - x_2)(x_1 + x_2) = 4(x_1 - x_2)$$

$$\Rightarrow \quad x_1 + x_2 = 4$$

$$\Rightarrow \quad x_1 = 4 - x_2$$

Thus, $f(x)$ is not an injective mapping.

onto surjective

let $\quad y = x^2 - 4x + 5$

$$\Rightarrow \quad y = (x-2)^2 + 1$$

$$\Rightarrow \quad y - 1 = (x-2)^2$$

$$\Rightarrow \quad x = \sqrt{(y-1)} + 2$$

Thus, for any value of $y < 1$, $x \notin$ R. So, we don't have a pre-image for all $y \in$ R in $x \in$ R.

Thus, $\quad f(x) = x^2 - 4x + 5$ is not surjective.

2. **Option (A) is correct.**

Explanation: We know that, if matrix A is skew symmetric, then

$$A' = -A$$

$$\therefore \quad \begin{bmatrix} a & b & 1 \\ c & 0 & -5 \\ -1 & 5 & 0 \end{bmatrix} = \begin{bmatrix} -a & -c & 1 \\ -b & 0 & -5 \\ -1 & 5 & 0 \end{bmatrix}$$

On comparing matrices, we get

$$a = -a \Rightarrow 2a = 0 \Rightarrow a = 0$$

$$b = -c \text{ and } c = -b$$

Now, $\quad 2a + (b + c) = 2 \times 0 + (b - b) = 0$

3. **Option (D) is correct.**

Explanation: We know that

$$|\text{adj } A| = |A|^{n-1}$$

where n is the order of square matrix A

Given $\quad |\text{adj } A| = 8$

$$\therefore \quad |A|^{n-1} = 8$$

$$\Rightarrow \quad |A|^{3-1} = 8$$

$$\Rightarrow \quad |A|^2 = 8$$

$$\Rightarrow \quad |A| = 2\sqrt{2}$$

Also, determinant of a matrix and its transpose has same values.

4. **Option (D) is correct.**

Explanation: Given,

$$A = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

and $\quad A^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{bmatrix}$

Since, $\quad AA^{-1} = I$

$$\therefore \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 12-3\lambda & 0 \\ 0 & -3+\lambda & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

On comparing matrices, we get

$12 - 3\lambda = 0$ and $-3 + \lambda = 1$

$\Rightarrow \lambda = 4$ and $\lambda = 4$

5. **Option (A) is correct.**

Explanation: We have,

$$[x \quad 2 \quad 0]\begin{bmatrix} 5 \\ -1 \\ x \end{bmatrix} = [3 \quad 1]\begin{bmatrix} -2 \\ x \end{bmatrix}$$

$$5x - 2 = -6 + x$$

$$4x = -4$$

$$x = -1$$

6. **Option (C) is correct.**

Explanation: Given for matrix $A = [a_{ij}]_{2\times 2}$

we have $\quad a_{ij} = \max(i, j) - \min(i, j)$

$$\therefore \quad a_{11} = 1 - 1 = 0$$

$$a_{12} = 2 - 1 = 1$$

$$a_{21} = 2 - 1 = 1$$

$$a_{22} = 2 - 2 = 0$$

$$\therefore \quad A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Now, $\quad A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

7. **Option (A) is correct.**

Explanation: We have,

$$xe^y = 1$$

Differentiating w.r.t. 'x' both sides

$$1.e^y + xe^y \frac{dy}{dx} = 0$$

$$xe^y \frac{dy}{dx} = -e^y$$

$$\frac{dy}{dx} = -\frac{1}{x}$$

$$\therefore \quad \left.\frac{dy}{dx}\right|_{x=1} = -1$$

8. Option (C) is correct.

Explanation: let $u = e^{\sin^2 x}$ and $v = \cos x$

$$\therefore \quad \frac{du}{dx} = e^{\sin^2 x}(2\sin x)\cos x$$

and $$\frac{dv}{dx} = -\sin x$$

Thus, $$\frac{du}{dv} = \frac{du/dx}{dv/dx}$$

$$= \frac{e^{\sin^2 x}.2\sin x.\cos x}{-\sin x}$$

$$= -2\cos x\, e^{\sin^2 x}$$

9. Option (A) is correct.

Explanation: We have,

$$f(x) = \frac{x}{2} + \frac{2}{x}$$

$$f'(x) = \frac{1}{2} - \frac{2}{x^2}$$

Put, $$f'(x) = 0 \Rightarrow \frac{1}{2} - \frac{2}{x^2} = 0$$

$$\Rightarrow \quad \frac{1}{2} = \frac{2}{x^2}$$

$$\Rightarrow \quad x^2 = 4$$

$$\Rightarrow \quad x = \pm 2$$

Now, $$f''(x) = \frac{6}{x^3}$$

At $x = 2$, $$f''(2) = \frac{6}{8} > 0$$

At $x = -2$, $$f''(-2) = -\frac{6}{8} < 0$$

Thus, $f(x)$ has local minima at $x = 2$.

10. Option (A) is correct.

Explanation: Given,

$$y = 7x - x^3$$

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = 7 - 3x^2$$

Thus, slope, $s = 7 - 3x^2$

Now, differentiating w.r.t. 't', we get

$$\frac{ds}{dt} = -6x\frac{dx}{dt}$$

$$\therefore \quad \left.\frac{ds}{dt}\right|_{\text{at } x=5} = -6 \times 5 \times 2$$

$$\left[\because \frac{dx}{dt} = 2 \text{ units/s (given)}\right]$$

$$= -60 \text{ units/s}$$

11. Option (B) is correct.

Explanation: Let $I = \int \frac{1}{x(\log x)^2} = dx$

$$\log x = t \Rightarrow \frac{1}{x}dx = dt$$

$$\therefore \quad I = \int \frac{dt}{t^2} = -\frac{1}{t} + C$$

Thus, $$I = -\frac{1}{\log x} + C$$

12. Option (D) is correct.

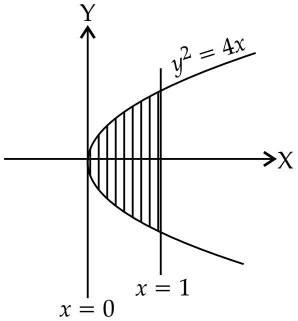
Explanation: We have

$$I = \int_{-1}^{1} x|x|\,dx = 0$$

[as $x\,|x|$ is an odd function]

13. Option (B) is correct.

Explanation: Given



$$\text{Required Area} = 2\int_0^1 y\,dx$$

$$= 2\int_0^1 2\sqrt{x}\,dx$$

$$= 4\int_0^1 x^{1/2}\,dx$$

$$= 4\left[\frac{x^{3/2}}{3/2}\right]_0^1$$

$$= \frac{8}{3} \text{ square units}$$

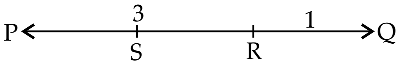
14. Option (A) is correct.

Explanation: Highest order derivative present in the given differential equation is 4.

Thus, order of differential equation = 4

15. Option (D) is correct.

Explanation:



Position vector of R,

$$\vec{r} = \frac{3\vec{q} + \vec{p}}{3+1}$$

or $$\vec{r} = \frac{3\vec{q} + \vec{p}}{4}$$

Given S is the mid-point of PR

$\therefore$ Position vector of S,

$$\vec{S} = \frac{\vec{p} + \vec{r}}{2}$$

$$= \frac{\vec{p}\left(\frac{3\vec{q} + \vec{p}}{4}\right)}{2}$$

$$= \frac{5\vec{p}+3\vec{q}}{8}$$

16. Option (B) is correct.

Explanation: Given line is

$$\frac{x}{1} = \frac{y}{-1} = \frac{z}{0}$$

$\therefore$ Direction ratios of line are: 1, –1, 0

We know that direction ratio of y-axis are 0, 1, 0

$$\cos\theta = \frac{\vec{b}_1 . \vec{b}_2}{|\vec{b}_1|.|\vec{b}_2|}$$

$$= \frac{(\hat{i}-\hat{j}).(\hat{j})}{\sqrt{(1)^2+(-1)^2} \cdot \sqrt{(1)^2}}$$

$$[\because \vec{b}_1 = \hat{i}-\hat{j} \text{ and } \vec{b}_2 = \hat{j}]$$

$$= \frac{-1}{\sqrt{2}}$$

$$\therefore \quad \cos\theta = \cos\left(\frac{3\pi}{4}\right)$$

$$\Rightarrow \quad \theta = \frac{3\pi}{4}$$

17. Option (D) is correct.

Explanation: Given line is

$$\vec{r} = (2+\lambda)\hat{i}+\lambda\hat{j}+(2\lambda-1)\hat{k}$$

$$\vec{r} = (2\hat{i}-\hat{k})+\lambda(\hat{i}+\hat{j}+2\hat{k})$$

which is of the form

$$\vec{r} = a+\lambda\vec{b}$$

$\therefore$ Required line is

$$\frac{x-1}{1} = \frac{y+3}{1} = \frac{z-2}{2}$$

[As lines passes through (1, –3, 2) and having Direction cosines (1, 1, 2)]

18. Option (D) is correct.

Explanation: Given

$$P\left(\frac{A}{B}\right) = P\left(\frac{B}{A}\right)$$

$$\Rightarrow \quad \frac{P(A\cap B)}{P(B)} = \frac{P(B\cap A)}{P(A)}$$

$$\Rightarrow \quad \frac{P(A\cap B)}{P(B)} = \frac{P(A\cap B)}{P(A)}$$

$$\Rightarrow \quad P(A) = P(B)$$

19. Option (C) is correct.

Explanation: Range of $\cos^{-1} x$ is $[0, \pi]$

20. Option (B) is correct.

Explanation: **Assertion:**

$$|\vec{a}| = \sqrt{(6)^2+(2)^2+(-8)^2} = \sqrt{104}$$

$$|\vec{b}| = \sqrt{(10)^2+(-2)^2+(-6)^2} = \sqrt{140}$$

$$|\vec{c}| = \sqrt{(4)^2+(-4)^2+(2)^2} = \sqrt{36}$$

$$\left(\sqrt{140}\right)^2 = \left(\sqrt{104}\right)^2+\left(\sqrt{36}\right)^2$$

Hence, it forms a right angle triangle.

Reason: Reason is correct but not the explanation of Assertion.

SECTION B

21. We have,

$$\sin^{-1}\left[k\tan\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right] = \frac{\pi}{3}$$

$$k\left[\tan\left(2\cos^{-1}\left(\cos\frac{\pi}{6}\right)\right)\right] = \sin\frac{\pi}{3}$$

$$k\tan\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$k\sqrt{3} = \frac{\sqrt{3}}{2}$$

$$k = \frac{1}{2}$$

22. (a) Given, $$f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right), & x\neq 0 \\ 0, & x=0 \end{cases}$$

for a continuous function,

$$\text{LHL} = \text{RHL} = f(a)$$

Now, $$\text{LHL} = \lim_{x\to 0^-} x\sin\frac{1}{x}$$

$$= \lim_{h\to 0}(0-h)\sin\left(\frac{1}{0-h}\right)$$

$$= \lim_{h\to 0} -h\sin\left(-\frac{1}{h}\right)$$

$$= \lim_{h\to 0} h\sin\left(\frac{1}{h}\right)$$

$$[\sin(-\infty) = -\sin\theta]$$

$$= 0\times\sin(\infty)$$

$$= 0$$

and $$\text{RHL} = \lim_{x\to 0^+} x\sin\frac{1}{x}$$

$$= \lim_{h\to 0}(0+h)\sin\left(\frac{1}{0+h}\right)$$

$$= \lim_{h\to 0} h\sin\left(\frac{1}{h}\right)$$

$$= 0.\sin(\infty) = 0$$

So, $\text{LHL} = \text{RHL} = f(0)$

Hence, $f(x)$ is continuous at $x = 0$

OR

(b) Given $$f(x) = |x-5|$$

$$\therefore \quad f(x) = \begin{cases} (x-5) & x\geq 5 \\ -(x-5) & x<5 \end{cases}$$

Here, LHD $= \lim_{h\to 0} \frac{f(x-h)-f(x)}{-h}$

$= \lim_{h\to 0} \frac{f(5-h)-f(5)}{-h}$

$= \lim_{h\to 0} \frac{-(5-h-5)-0}{-h}$

$= \lim_{h\to 0} \frac{h}{-h} = -1$

and RHD $= \lim_{h\to 0} \frac{f(5+h)-f(5)}{h}$

$= \lim_{h\to 0} \frac{(5+h-5)-0}{h}$

$= \lim_{h\to 0} \frac{h}{h}$

$= 1$

$\because$ LHD $\neq$ RHD Hence $f(x)$ is not differentiable.

23. Let radius of the circle be r cm.

Given $\frac{dA}{dt} = 2\text{ cm}^2/\text{s}$

since $A = \pi r^2$

$\therefore \quad \frac{dA}{dt} = 2\pi r\frac{dr}{dt}$ ...(i)

Also, circumference, $C = 2\pi r$

$\therefore \quad \frac{dC}{dt} = 2\pi\frac{dr}{dt}$ ...(ii)

from (i), $2 = 2\pi r\frac{dr}{dt}$

$\Rightarrow \quad \frac{dr}{dt} = \frac{1}{\pi r}$

Now, substituting the value of $\frac{dr}{dt}$ in eq. (ii) we get

$\frac{dc}{dt} = 2\pi\frac{1}{\pi r} = \frac{2}{r}$

Now, $\left.\frac{dC}{dt}\right|_{\text{at } r=5} = \frac{2}{5}$

$= 0.4$ cm/s

Thus, circumference of circle increases at the rate of 0.4 cm/s.

24. **(a)** Let $I = \int \cos^3 x\, e^{\log \sin x} dx$

or $I = \int \cos^3 x \sin x\, dx$

Put $\cos x = t \Rightarrow -\sin x\, dx = dt$

$\therefore \quad I = -\int t^3 dt$

or, $I = -\frac{t^4}{4}+C$

or, $I = -\frac{\cos^4 x}{4}+C$

OR

(b) Let $I = \int\frac{dx}{5+4x-x^2}$

$= \int\frac{dx}{5+4-4+4x-x^2}$

$= \int\frac{dx}{(3)^2-(x-2)^2}$

$= \frac{1}{2\times 3}\log\left|\frac{3+(x-2)}{3-(x-2)}\right|+c$

[using $\int\frac{1}{a^2-x^2}dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right|+C$]

$= \frac{1}{6}\log\left|\frac{x+1}{5-x}\right|+C$

25. Given $\alpha = \beta = \gamma$

Now,

$\cos^2\alpha + \cos^2\beta + \cos^2\beta = 1$

$\therefore \quad 3\cos^2\alpha = 1$ $[\because \alpha = \beta = \gamma]$

or, $\cos^2\alpha = \frac{1}{3}$

or, $\cos\alpha = \pm\frac{1}{\sqrt{3}}$

Thus, $\cos\alpha = \cos\beta = \cos\gamma = \pm\frac{1}{\sqrt{3}} = l = m = n$

Let required equation of line is $\vec{r} = \vec{a}+k\vec{b}$

Here, $\vec{a} = 2\hat{i}+3\hat{j}-5\hat{k}$

$\vec{b} = \pm\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$

Thus, required equation is

$\vec{r} = (2\hat{i}+3\hat{j}-5\hat{k})\pm\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$

SECTION C

26. **(a)** Given, $(\cos x)^y = (\cos y)^x$

Taking log on both sides, we get

$y\log(\cos x) = x\log(\cos y)$

Differentiating both sides w.r.t. x, we get

$\frac{dy}{dx}.\log(\cos x)+y.\left(\frac{-\sin x}{\cos x}\right)$

$= 1.\log(\cos y)+x.\left(\frac{-\sin y}{\cos y}\right)\frac{dy}{dx}$

$\frac{dy}{dx}.\log(\cos x)-\frac{y\sin x}{\cos x} = \log(\cos y)-\frac{x\sin y}{\cos y}\frac{dy}{dx}$

$\frac{dy}{dx}\left[\log(\cos x)+\frac{y\sin y}{\cos y}\right] = \log(\cos y)+\frac{y\sin x}{\cos x}$

$\frac{dy}{dx}(\log\cos x + x\tan y) = \log(\cos y) + y\tan x$

$\frac{dy}{dx} = \frac{\log(\cos y)+y\tan x}{\log(\cos x)+\tan y}$

OR

(b) Given, $\sqrt{1-x^2}+\sqrt{1-y^2} = a(x-y)$

Put $x = \sin\theta$ and $y = \sin\phi$

$\therefore$ $\theta = \sin^{-1}x$ and $\phi = \sin^{-1}y$

Now, $\sqrt{1-\sin^2\theta}+\sqrt{1-\sin^2\phi} = a(\sin\theta - \sin\phi)$

$$\cos\theta + \cos\phi = a(\sin\theta - \sin\phi)$$

$$2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right) = 2a\cos\left(\frac{\theta+\phi}{2}\right).\sin\left(\frac{\theta-\phi}{2}\right)$$

$$\frac{\cos\left(\frac{\theta-\phi}{2}\right)}{\sin\left(\frac{\theta-\phi}{2}\right)} = a$$

$$\cot\left(\frac{\theta-\phi}{2}\right) = a$$

$$\frac{\theta-\phi}{2} = \cot^{-1}a$$

$$\theta-\phi = 2\cot^{-1}a$$

$\therefore$ $\sin^{-1}x - \sin^{-1}y = 2\cot^{-1}a$

Differentiating w.r.t. x, we get

$$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$$

Hence Proved

27. Given, $x = a\sin^3\theta$ and $y = a\cos^3\theta$

$$\frac{dx}{d\theta} = 3a\sin^2\theta\cos\theta$$

and $$\frac{dy}{d\theta} = -3a\cos^2\theta\sin\theta$$

$\therefore$ $$\frac{dy}{dx} = \frac{-3a\cos^2\theta\sin\theta}{3a\sin^2\theta\cos\theta}$$

$$= -\cot\theta$$

and $$\frac{d^2y}{dx^2} = \frac{d}{d\theta}(-\cot\theta)\frac{d\theta}{dx}$$

$$= \operatorname{cosec}^2\theta\frac{1}{3a\sin^2\theta\cos\theta}$$

$$= \frac{1}{3a\sin^4\theta\cos\theta}$$

Now, $$\left.\frac{d^2y}{dx^2}\right|_{\text{at } x=\frac{\pi}{4}} = \frac{1}{3.a\left(\frac{1}{\sqrt{2}}\right)^4.\frac{1}{\sqrt{2}}}$$

$$= \frac{1}{3.a\frac{1}{4}.\frac{1}{\sqrt{2}}} = \frac{4\sqrt{2}}{3a}$$

28. (a) Let $$I = \int_0^\pi \frac{e^{\cos x}}{e^{\cos x}+e^{-\cos x}}dx \quad ...(i)$$

$\therefore$ $$I = \int_0^\pi \frac{e^{\cos(\pi-x)}}{e^{\cos(\pi-x)}+e^{-\cos(\pi-x)}}dx$$

$$\left[\text{Using} \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

or, $$I = \int_0^\pi \frac{e^{-\cos x}}{e^{-\cos x}+e^{\cos x}}dx \quad ...(ii)$$

On adding eqs. (i) & (ii), we get

$$2I = \int_0^\pi \frac{e^{\cos x}+e^{-\cos x}}{e^{\cos x}+e^{-\cos x}}dx$$

$$= \int_0^\pi dx$$

$$= [x]_0^\pi = \pi$$

$\therefore$ $$I = \frac{\pi}{2}$$

OR

(b) Let $$I = \int\frac{2x+1}{(x+1)^2(x-1)}dx$$

Let $$\frac{2x+1}{(x+1)^2(x-1)} = \frac{A}{(x+1)}+\frac{B}{(x+1)^2}+\frac{C}{(x-1)}$$

$$2x+1 = A(x+1)(x-1)+B(x-1)+C(x+1)^2$$

$$2x+1 = A(x^2-1)+Bx-B+C(x^2+2x+1)$$

On comparing, we get

$A + C = 0$; $B + 2C = 2$ and $-A-B+C = 1$

On solving, we get

$A = \frac{-3}{4}$, $B = \frac{1}{2}$ and $C = \frac{3}{4}$

$\therefore$ $$\frac{2x+1}{(x+1)^2(x-1)} = \frac{-3}{4(x+1)}+\frac{1}{2(x+1)^2}+\frac{3}{4(x-1)}$$

Thus, $$\int\frac{2x+1}{(x+1)^2(x-1)}dx$$

$$= -\frac{3}{4}\int\frac{1}{(x+1)}dx+\frac{1}{2}\int\frac{1}{(x+1)^2}dx+\frac{3}{4}\int\frac{1}{(x-1)}dx$$

$$= -\frac{3}{4}\log|x+1|+\frac{1}{2}I_1+\frac{3}{4}\log|x-1|+C_1$$

How, $$I_1 = \int\frac{1}{(x+1)^2}dx$$

let $x+1 = u \Rightarrow dx = du$

$\therefore$ $$I_1 = \int\frac{1}{u^2}du = \frac{u^{-2+1}}{-2+1}+C_2$$

$$= -\frac{1}{u}+C_2 = -\frac{1}{(x+1)}+C_2$$

Therefore, $\int \frac{2x+1}{(x+1)^2(x-1)}dx$

$$= -\frac{3}{4}\log|x+1| - \frac{1}{2(x+1)} + \frac{3}{4}\log|x-1| + C$$

Where $C = C_1 + C_2$

29. **(a)** Given differential equation is

$$\frac{dy}{dx} - 2xy = 3x^2e^{x^2}$$

On comparing the above equation with

$$\frac{dy}{dx} + Py = Q,$$

We get $P = -2x$, $Q = 3x^2e^{x^2}$

$$\therefore \quad I = e^{\int Pdx} = e^{-\int 2xdx}$$

$$= e^{-2\left(\frac{x^2}{2}\right)} = e^{-x^2}$$

$$y.e^{-x^2} = \int 3x^2e^{x^2}.(e^{-x^2})dx + C$$

or, $$y.e^{-x^2} = 3\int x^2dx + C$$

or, $$\frac{y}{e^{x^2}} = 3\left[\frac{x^3}{3}\right] + C$$

or, $$\frac{y}{e^{x^2}} = x^3 + C$$

or, $$y = e^{x^2}x^3 + Ce^{x^2}$$

Given $y(0) = 5$

$\therefore$ $5 = 0 + C e^0 \Rightarrow C = 5$

Thus, required solution is

or $$y = e^{x^2}(x^3 + 5)$$

OR

(b) We have, $x^2dy + y(x + y)dx = 0$

$$\frac{x^2}{y}\frac{dy}{dx} + (x+y) = 0$$

$$\frac{x^2}{y^2}\frac{dy}{dx} + \frac{x}{y} + 1 = 0 \quad ...(i)$$

Put $y = vx$

$$\Rightarrow \quad \frac{dy}{dx} = V + x\frac{dV}{dx}$$

From eq (i), we get

$$\frac{1}{V^2}\left(V + x\frac{dV}{dx}\right) + \frac{1}{V} + 1 = 0$$

$$\Rightarrow \quad x\frac{dV}{dx} = -(2V + V^2)$$

$$\Rightarrow \quad \frac{1}{2}\left[\frac{1}{V} - \frac{1}{V+2}\right]dV = -\frac{dx}{x}$$

On integrating both sides, we get

$$\Rightarrow \quad \frac{1}{2}(\log V - \log V + 2) = -\log x + C_1$$

$$\Rightarrow \quad \frac{1}{2}\log\left(\frac{V}{V+2}\right) = -\log x + C_1$$

$$\Rightarrow \quad \log\left(\frac{V}{V+2}\right) = -2\log x + \log k$$

where $k = e^{2C_1}$

$$\Rightarrow \quad \log\left(\frac{V}{V+2}\right) = \log\left(\frac{k}{x^2}\right)$$

$$\Rightarrow \quad \frac{V}{V+2} = \frac{k}{x^2}$$

Substituting $V = \frac{y}{x}$, we get

$$\frac{y}{2x+y} = \frac{k}{x^2}$$

$\Rightarrow x^2y = C^2(2x + y)$ where $k = C^2$

30. Given vectors are:

$$\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\vec{b} = \hat{i} + \hat{j} - \hat{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= \hat{i}(1-1) - \hat{j}(-2-1) + \hat{k}(2+1)$$

$$= 3\hat{j} + 3\hat{k}$$

Now, $$|\vec{a} \times \vec{b}| = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9}$$

$$= \sqrt{18} = 3\sqrt{2}$$

Therefore required vector is, $4\left(\frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}\right)$

$$= \frac{4}{3\sqrt{2}}(3\hat{j} + 3\hat{k})$$

$$= \frac{2\sqrt{2}}{3} \times 3(\hat{j} + \hat{k})$$

$$= 2\sqrt{2}(\hat{j} + \hat{k})$$

31. We have,

X	1	2	3	4	5
$P(X)$	0.2	a	a	0.2	b

Given $E(X) = 3$

Since, $E(X) = \Sigma XP(X)$

$\therefore$ $3 = 0.2 + 2a + 3a + 0.8 + 5b$

or, $3 = 5a + 5b + 1$

or, $5a + 5b = 2$ ...(i)

Also, $\Sigma P(X) = 1$

$\therefore \quad 0.2 + a + a + 0.2 + b = 1$

or, $\quad 2a + b = 1 - 0.4$

or, $\quad 2a + b = 0.6 \quad \text{...(ii)}$

eq (i) × 5 – eq (i), we get

$$\begin{aligned} 10a + 5b &= 3 \\ 5a + 5b &= 2 \\ - \quad - \quad . \; - & \\ \hline 5a &= 1 \end{aligned}$$

$$a = \frac{1}{5} = 0.2$$

Substituting value of a in eq (i), we get

$$5\left(\frac{1}{5}\right) + 5b = 2$$

$$5b = 1$$

$$b = \frac{1}{5} = 0.2$$

Thus, $a = 0.2$ and $b = 0.2$

Now, $\quad P(X \geq 3) = P(X = 3) + P(X = 4) + P(X = 5)$

$$= a + 0.2 + b$$

$$= 0.2 + 0.2 + 0.2$$

$$= 0.6$$

SECTION D

32. (a) Given, $\quad A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 0 & -3 \\ 1 & 2 & 0 \end{bmatrix}$

$$|A| = 1(0 + 6) - 2(0 + 3) - 3(4 - 0)$$

$$= 6 - 6 - 12 = -12 \neq 0$$

⇒ inverse exists co-factors are

$$C_{11} = (-1)^2 \begin{vmatrix} 0 & -3 \\ 2 & 0 \end{vmatrix} = 6$$

$$C_{12} = (-1)^3 \begin{vmatrix} 2 & -3 \\ 1 & 0 \end{vmatrix} = -3$$

$$C_{13} = (-1)^4 \begin{vmatrix} 2 & 0 \\ 1 & 2 \end{vmatrix} = 4$$

$$C_{21} = (-1)^3 \begin{vmatrix} 2 & -3 \\ 2 & 0 \end{vmatrix} = -6$$

$$C_{22} = (-1)^4 \begin{vmatrix} 1 & -3 \\ 1 & 0 \end{vmatrix} = 3$$

$$C_{23} = (-1)^5 \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = 0$$

$$C_{31} = (-1)^4 \begin{vmatrix} 2 & -3 \\ 0 & -3 \end{vmatrix} = -6$$

$$C_{32} = (-1)^5 \begin{vmatrix} 1 & -3 \\ 2 & -3 \end{vmatrix} = -3$$

$$C_{33} = (-1)^6 \begin{vmatrix} 1 & 2 \\ 2 & 0 \end{vmatrix} = -4$$

The adjoint of the matrix A is given by

$$\text{adj}(A) = \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -6 & -6 \\ -3 & 3 & -3 \\ 4 & 0 & -4 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{(\text{adj } A)}{|A|} = -\frac{1}{12} \begin{bmatrix} 6 & -6 & -6 \\ -3 & 3 & -3 \\ 4 & 0 & -4 \end{bmatrix}$$

$$= \frac{1}{12} \begin{bmatrix} -6 & 6 & 6 \\ 3 & -3 & 3 \\ -4 & 0 & 4 \end{bmatrix}$$

Given system of linear equations are

$$x + 2y - 3z = 1$$

$$2x - 3z = 2$$

$$x + 2y = 3$$

Represent it in matrix form

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 0 & -3 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Which is in the form $AX = B$

$$X = A^{-1}B$$

$$= \frac{1}{12} \begin{bmatrix} -6 & 6 & 6 \\ 3 & -3 & 3 \\ -4 & 0 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= \frac{1}{12} \begin{bmatrix} 24 \\ 6 \\ 8 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ \frac{1}{2} \\ \frac{2}{3} \end{bmatrix}$$

$$\therefore \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ \frac{1}{2} \\ \frac{2}{3} \end{bmatrix}$$

or, $x = 2, y = \frac{1}{2}$ and $z = \frac{2}{3}$

OR

(b) We have,

$$AB = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -6+28+45 & 17+10-27 & 13-16+3 \\ -12+42-30 & 34+15+18 & 26-24-2 \\ -18-42+60 & 51-15-36 & 39+24+4 \end{bmatrix}$$

$$= \begin{bmatrix} 67 & 0 & 0 \\ 0 & 67 & 0 \\ 0 & 0 & 67 \end{bmatrix}$$

$$= 67\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Thus, $AB = 67\,I$

$\Rightarrow \quad A\left(\frac{1}{67}B\right) = I$

$\Rightarrow \quad A^{-1} = \frac{1}{67}(B)$

Given system of linear equation is

$$x + 2y - 3z = -4$$
$$2x + 3y + 2z = 2$$
$$3x - 3y - 4z = 11$$

Represent it in matrix form as

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$

which is of the form $AX = D$

$\therefore \quad X = A^{-1}D$

$$= \frac{1}{67}(B).D$$

$$= \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}\begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$

$$= \frac{1}{67}\begin{bmatrix} 201 \\ -134 \\ 67 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

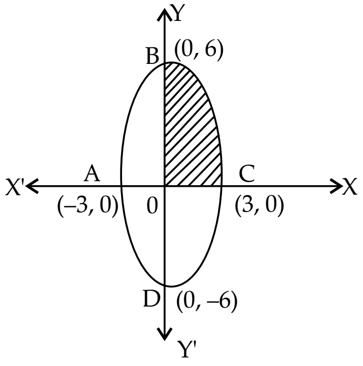
$\therefore x = 3, y = -2$ and $z = 1$

33. Given curve is $4x^2 + y^2 = 36$

or, $\quad \frac{x^2}{9} + \frac{y^2}{36} = 1$

or, $\quad \frac{x^2}{3^2} + \frac{y^2}{6^2} = 1$

since, ellipse is symmetrical along



x-axis and y-axis

Area of ellipse = Area of ABCD

$= 4 \times$ Area of OBC

$$= 4 \times \int_0^3 y\,dx$$

$$= 4 \times \int_0^3 (2\sqrt{9-x^2})\,dx$$

$$= 8\int_0^3 \sqrt{9-x^2}\,dx$$

[since OBC is above x-axis]

$$= 8\int_0^3 \sqrt{3^2-x^2}\,dx$$

$$= 8\left[\frac{x}{2}\sqrt{3^2-x^2} + \frac{3^2}{2}\sin^{-1}\frac{x}{3}\right]_0^3$$

$$= 8\left[\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\sin^{-1}\frac{x}{3}\right]_0^3$$

$$= 8\left[0 + \frac{9}{2}\sin^{-1}(1) - (0+0)\right]$$

$$= 8 \times \frac{9}{2} \times \frac{\pi}{2}$$

$= 2 \times 9\pi$

$= 18\pi$ sq. units

34. (a) Given line is $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$

or, $\quad \frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3} = \lambda$

Now, the co-ordinates of any points Q on the line are $(-2\lambda + 4, 6\lambda, -3\lambda + 1)$.

Also, the given point is $p(2, 3, -8)$

The direction ratios of PQ are $-2\lambda + 4 - 2, 6\lambda - 3, -3\lambda + 1 + 8$ i.e., $-2\lambda + 2, 6\lambda - 3, -3\lambda + 9$

Also, the direction cosines of the given line are $-2, 6, -3$.

If PQ $\perp$ line, then

$$-2(-2\lambda + 2) + 6(6\lambda - 3) - 3(-3\lambda + 9) = 0$$
$$4\lambda - 4 + 36\lambda - 18 + 9\lambda - 27 = 0$$
$$49\lambda - 49 = 0$$
$$\lambda = 1$$

Now, the foot of the perpendicular is

$[-2(1) + 4, 6(1), -3(1) + 1]$ i.e., $(2, 6, -2)$

Hence, the distance PQ is

$$= \sqrt{(2-2)^2 + (3-6)^2 + (-8+2)^2}$$

$$= \sqrt{0+9+36} = \sqrt{45} = 3\sqrt{5} \text{ units}$$

OR

(b) Given, L_1: The line passing through (2, –1, 1) and parallel to $\frac{x}{1} = \frac{y}{1} = \frac{z}{3}$ ...(i)

Direction ratios of line (i) are (1, 1, 3)

$\therefore$ Vector equation of line passing through (2, –1, 1) having direction ratios is

$$\vec{r} = (2\hat{i} - \hat{j} + \hat{k}) + \lambda(\hat{i} + \hat{j} + 3\hat{k}) \quad ...(ii)$$

Given, L_2: $\vec{r} = \hat{i} + (3\mu + 1)\hat{j} - (\mu + 2)\hat{k}$

or, $\vec{r} = (\hat{i} + \hat{j} - 2\hat{k}) + \mu(3\hat{j} - \hat{k})$ ...(iii)

Now, shortest distance between lines (ii) & (iii) is given by

$$d = \frac{(\vec{b}_1 \times \vec{b}_2).(\vec{a}_2 \times \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|}$$

Here, $\vec{a}_1 = (2\hat{i} - \hat{j} + \hat{k})\,\vec{a}_2 = (\hat{i} + \hat{j} - 2\hat{k})$

$\vec{b}_1 = (\hat{i} + \hat{j} + 3\hat{k}), \vec{b}_2 = (3\hat{j} - \hat{k})$

Now, $\vec{a}_2 - \vec{a}_1 = (1-2)\hat{i} + (1+1)\hat{j} + (-2-1)\hat{k}$

$$= -\hat{i} + 2\hat{j} - 3\hat{k}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 3 \\ 0 & 3 & -1 \end{vmatrix}$$

$$= \hat{i}(-1-9) - \hat{j}(-1-0) + \hat{k}(3-0)$$

$$= -10\hat{i} + \hat{j} + 3\hat{k}$$

and $\left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{(-10)^2 + (1)^2 + (3)^2}$

$$= \sqrt{100+1+9}$$

$$= \sqrt{110}$$

$\therefore$ $d = \left|\frac{(-\hat{i} + 2\hat{j} - 3\hat{k})(-10\hat{i} + \hat{j} + 3\hat{k})}{\sqrt{110}}\right|$

$$= \left|\frac{10+2-3}{\sqrt{110}}\right| = \frac{9}{\sqrt{110}} \text{ unit}$$

35. Given LPP is

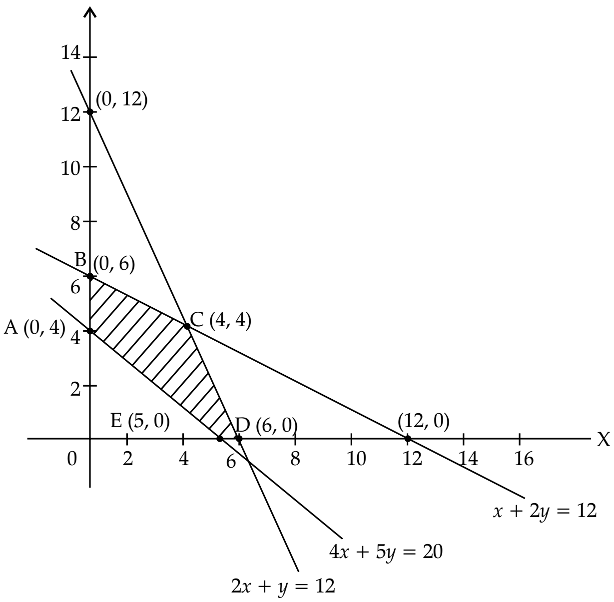
Max $Z = 60x + 40y$

Subject to $x + 2y \le 12$ ...(i)

$2x + y \le 12$ ...(ii)

$4x + 5y \ge 20$ ...(iii)

$x, y \ge 0$



Corner points	Z = 60x + 40y
A(0, 4)	1600
B(0, 6)	2400
C(4, 4)	4000 (max)
D(6, 0)	3600
E (5, 0)	3000

Thus, maximum value of Z is 4000 which occurs at the point c (4, 4).

SECTION E

36. (a) (i) Given Relation, $R = \{(l_1, l_2): d_1$ is parallel to l_2

$\because l_1 \parallel l_2 \Rightarrow l_2 \parallel l_1$

Hence, given relation is symmetric.

(ii) Let $l_1 \parallel l_2$ and $l_2 \parallel l_3 \Rightarrow l_1 \parallel l_3$

[As parallel lines are parallel to each other]

(iii) Given equation of line : $y = 3x + 2$

Comparing with $y = mx + C$

m = slope = 3

Slope of parallel, lines is same

$\therefore$ Equation of set of rall line is given by

$y = 3x + \lambda \qquad \lambda \in R$

OR

(b) Given relation, $S = \{(l_1, l_2): l_1$ is perpendicular to $l_2\}$

Symmetric $l_1 \perp l_2 \Rightarrow l_2 \perp l_1$

(Two lines are perpendicular to each other)

Hence, S is symmetric.

Transitive:

Let $L_1 \perp L_2$ and $l_2 \perp l_3 \Rightarrow l_1 \parallel l_3$

Thus, relation S is not transitive.

37. (i) Given, x = length of printed area

y = breadth of printed area

Area of printed part = xy

or, $24 = xy$

or, $y = \frac{24}{x}$ ...(i)

Now, area of visiting card = $(x + 3)(y + 2)$

or, $A = (x + 3)(y + 2)$

or, $A = (x+3)\left(\frac{24}{x} + 2\right)$

or, $A = \frac{(x+3)(24+2x)}{x}$

or, $A = \frac{2x^2+30x+72}{x}$

or $A = 2x+30+\frac{72}{x}$

(ii) We have, Area, $A = 2x+30+\frac{72}{x}$

$\therefore \quad \frac{dA}{dx} = 2+0-\frac{72}{x^2}$

and $\frac{d^2A}{dx^2} = \frac{144}{x^3}$

for minimum area, put $\frac{dA}{dx} = 0$

i.e., $2-\frac{72}{x^2} = 0$

or, $x^2 = 36 \Rightarrow x = \pm 6$

At $x = 6. \frac{d^2A}{dx^2} > 0$ Hence, minimum

Therefore, length of vising card

$= x + 3 = 6 + 3 = 9$ cm

and breadth of visiting card $= y + 2 = \frac{24}{x}+2$

$= \frac{24}{6}+2 = 6$ cm

38. (i) $P(E_1)$ = Probability that customer pyas the bill in first time

$P(E_2)$ = Probability that customer does not pay the bill in first time.

$\therefore \quad P(E_1) = \frac{70}{100} = \frac{7}{10}$

and $P(E_2) = \frac{30}{100} = \frac{3}{10}$

(ii) A = Event that customer pay the bill in 2nd month on time

$P\left(\frac{A}{E_1}\right) = 0.8 = \frac{8}{10}$

and $P\left(\frac{A}{E_2}\right) = 0.4 = \frac{4}{10}$

(iii) $P(A) = P(E_1).P\left(\frac{A}{E_1}\right)+P(E_2).P\left(\frac{A}{E_2}\right)$

$= \frac{7}{10}\times\frac{8}{10}+\frac{3}{10}\times\frac{4}{10}$

$= \frac{56}{100}+\frac{12}{100}$

$= \frac{68}{100} = \frac{17}{25}$

OR

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1).P\left(\frac{A}{E_1}\right)}{P(E_1).P\left(\frac{A}{E_1}\right)+P(E_2).P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{7}{10}\times\frac{8}{10}}{\frac{7}{10}\times\frac{8}{10}+\frac{3}{10}\times\frac{4}{10}}$$

$$= \frac{\frac{56}{100}}{\frac{68}{100}} = \frac{56}{68} = \frac{14}{17}$$

Delhi Set-2 **65/5/2**

SECTION A

**2. Option (A) is **

7. Option (D) is correct.

Explanation: $\sin(xy) = 1$

$x.y = \sin^{-1} 1$

$\frac{dy}{dx}(x.y) = \frac{dy}{dx}(\sin_1^{-1})$

$x.\frac{dy}{dx}+y = 0$

$\frac{dy}{dx} = -\frac{y}{x}$

11. Option (A) is correct.

Explanation: Since, $\int_{\pi/4}^{\pi/2} \cot\theta.\text{cosec}^2\,\theta\, d\theta$

Let $\cot\theta = t$

$-\text{cosec}^2\,\theta\, d\theta = dt$

$\theta = \frac{\pi}{4}, t = 1$

$\theta = \frac{\pi}{2}, t = 0$

So, $I = \int_1^0 t(-dt)$

$I = \int_0^1 t dt$

$= \left[\frac{t^2}{2}\right]_0^1$

$= \frac{1}{2}$

12. Option (B) is correct.

Explanation: $\int \frac{1}{\sqrt{9-4x^2}} dx$

$\frac{1}{\sqrt{4}} \int \frac{1}{\sqrt{\left(\frac{3}{2}\right)^2 - x^2}}$

using formula $\frac{1}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}+c$

$$\frac{1}{2}\sin^{-1}\frac{x}{3/2}+C$$

$$\frac{1}{2}\sin^{-1}\frac{2x}{3}+C$$

14. Option (A) is correct.

***Explanation*:**

$$\frac{dy}{dx} = e^{x+y}$$

$$dy = e^x.e^y dx$$

$$\frac{dy}{e^y} = e^x dx$$

$$\int e^{-y}dy = \int e^x dx$$

$$\frac{e^{-y}}{-1} = e^x + c_1$$

$$-e^{-y} = e^x + c_1 \qquad [c_1 = -c]$$

$$c = e^x + e^y$$

21. Since

$$a = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)+\cos^{-1}\left(\frac{-1}{2}\right)$$

$$= \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)+\cos^{-1}\left(\frac{-1}{2}\right)$$

$$= \frac{\pi}{4}+\frac{2\pi}{3}$$

$$= \frac{11\pi}{12}$$

$$b = \tan^{-1}(\sqrt{3})-\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$$

$$= \frac{\pi}{3}-\frac{2\pi}{3}$$

$$= \frac{-\pi}{3}$$

Now, $a + b$

$$= \frac{11\pi}{12}-\frac{\pi}{3}$$

$$= \frac{11\pi-4\pi}{12}$$

$$= \frac{7\pi}{12}$$

23. Given

$$\frac{dv}{dt} = 15 \text{ cm}^3/\text{min}$$

$$h = \frac{1}{3}r, h = 4$$

Since,

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi(3h)^2\times h$$

$$v = \frac{9}{3}h^3\pi = 3h^3\pi$$

$$\frac{dv}{dt} = 9h^2\pi\frac{dh}{dt}$$

$$15 = 9h^2\pi\frac{dh}{dt}$$

$$\frac{15}{9\times(4)^2\pi} = \frac{dh}{dt} \Rightarrow \frac{5}{48\pi}$$

27.

$$f(x) = \begin{Bmatrix} ax+b & x>-1 \\ bx^2-3 & x\le -1 \end{Bmatrix}$$

$$f'(x) = \begin{Bmatrix} ax & x>-1 \\ 2bx & x\le -1 \end{Bmatrix}$$

$$Lf'(-1) = a(-1) = -a$$
$$Rf'(-1) = 2b(-1) = -2b$$
$$Lf'(-1) = Rf'(-1)$$
$$-a = -2b$$
$$a = 2b$$

The function is differentiable so, the function continuity

$$\text{L.H.S.} = \text{R.H.S.}$$

$$\lim_{x\to -1^-} bx^2-3 = \lim_{x\to -1^+} ax+b$$

$$b-3 = -a+b, \quad a = 3$$

and $b = 6$

30.

$$\vec{a} = 2\hat{i}-\hat{j}+\hat{k}$$

$$\vec{b} = 3\hat{i}-\hat{k}$$

$$\vec{c} = 2\hat{i}+\hat{j}-2\hat{k}$$

$$\vec{d} = (x\hat{i}+y\hat{j}+z\hat{k})$$

since, $\vec{d}$ is perpendicular to $\vec{a}$ and $\vec{b}$

$$\vec{d}.\vec{a} = (x\hat{i}+y\hat{j}+2\hat{k})(2\hat{i}-\hat{j}+\hat{k})$$

$$= 2x-y+2$$

$$\vec{d}.\vec{b} = (x\hat{i}+y\hat{j}+z\hat{k})(3\hat{i}-\hat{k})$$

$$= 3x-z$$

$$\vec{c}.\vec{d} = (2\hat{i}+\hat{j}-2\hat{k})(x\hat{i}+y\hat{j}+z\hat{k})$$

$$= 2x+y-2z = 3$$

On solving $x = 3, y = 15, z = 9$

$$\text{vector } d = x\hat{i}+y\hat{j}+z\hat{k}$$

$$= 3\hat{i}+15\hat{j}+9\hat{k}$$

31. Let, E_1, E_2, E_3 and A be event

E_1 = Both transferred ball from Bag I to bag II are red.

E_2 = Both transferred ball from bag I to bag II are black

E_3 = out of two transferred ball one is red and other is black.

Now,

$$P(E_1) = \frac{{}^3C_1}{{}^7C_2} = \frac{1}{7}$$

$$P(E_2) = \frac{{}^4C_2}{{}^7C_2} = \frac{2}{7}$$

$$P(E_3) = \frac{{}^3C_1\times{}^4C_1}{{}^7C_2} = \frac{4}{7}$$

Required probability

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)\times P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)\times P\left(\frac{A}{E_2}\right)+P(E_3)\times P\left(\frac{A}{E_3}\right)}$$

$$P\left(\frac{A}{E_1}\right) = \frac{7}{9}, P\left(\frac{A}{E_2}\right)=\frac{2}{9}, P\left(\frac{A}{E_3}\right)=\frac{6}{9}$$

Now,
$$= \frac{\frac{2}{7}\times\frac{2}{9}}{\frac{1}{7}\times\frac{7}{9}+\frac{2}{7}\times\frac{2}{9}+\frac{4}{7}\times\frac{6}{9}}$$

$$= \frac{4}{35}$$

35. Since, $Z = 300x + 600y$

$x + 2y \le 12$

$2x + y \le 12$

$x+\frac{5}{4}y \ge 5$

$x \ge 0\ , y \ge 0.$

Now, $x + 2y = 12,$

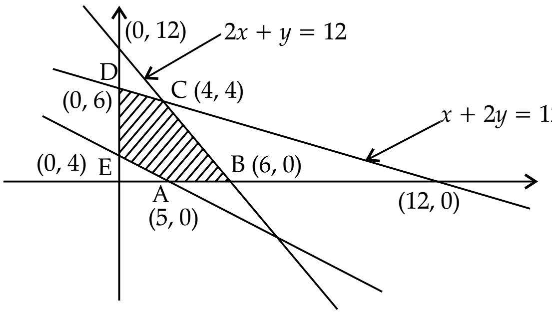
x	0	12	4
y	6	0	4

$2x + y = 12$

x	0	6	4
y	12	0	4

$x+\frac{5}{4}y = 5$

x	0	5
y	4	0



Corner point	$z = 300x + 600y$
A(5,0)	$z = 1500$
B(6,0)	$z = 1800$
C(4,4)	$z = 3600$ Maximum
D(0,6)	$z = 3600$ Maximum
E(0,4)	$z = 2400$

The maximum of objective function at two points at (4, 4) and (0, 6)

Delhi Set-3 **65/5/3**

SECTION A

2. Option (D) is correct.

Explanation: $A = \begin{vmatrix} 2 & -1 & 1 \\ \lambda & 2 & 0 \\ 1 & -2 & 3 \end{vmatrix}$

For invertible, the Determinant of A should not be equal to zero.

$|A| = 2(6-0)-(-1)(3\lambda-0) + 1(-2\lambda-2)$
$|A| = 12 + 3\lambda - 2\lambda - 2$
$|A| = 10 + \lambda \neq 0$

Therefore, $\lambda \neq -10$

5. Option (A) is correct.

Explanation: $A = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 0 \\ -1 & 1 \end{bmatrix}$

Given that, $A^2 = B$

$$\begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}.\begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x^2 & 0 \\ x+1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ -1 & 1 \end{bmatrix}$$

$x + 1 = -1$
$x = -1-1$
$x = -2$

11. Option (A) is correct.

Explanation:

$$\int_{-2}^{3} x^2 dx = k\int_0^2 x^2 dx + \int_2^3 x^2 dx$$

$$\Rightarrow \left[\frac{x^3}{3}\right]_{-2}^{3} = k\left[\frac{x^3}{3}\right]_0^2 + \left[\frac{x^3}{3}\right]_2^3$$

$$\Rightarrow \left[9+\frac{8}{3}\right] = k\left[\frac{8}{3}\right]+\left[9-\frac{8}{3}\right]$$

$\Rightarrow k = 2$

12. Option (B) is correct.

Explanation: $I = \int_1^e \log x\, dx$

$= [x \log x - x]_1^e$
$= [e \log e - e] - [\log 1 - 1]$
$= e \log e - e - \log 1 + 1$
{Since $\text{Log}\, e = 1, \text{Log}\, 1 = 0$}
$= e - e - 0 + 1$
$I = 1$

13. Option (C) is correct.

Explanation: Area $= \int_0^3 x.dy$

$= \int_0^3 y^2 dy$

Since $x = y^2$

$$= \left[\frac{y^3}{3}\right]_0^3 = \left[\frac{3^3}{3} - 0\right] = 9$$

Area $= 9$

14. Option (C) is correct.

Explanation: Order = 4

21. $\cos^{-1} x + \cos^{-1} x\left[\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right], \; \frac{1}{2} \le x \le 1$

Let $\cos^{-1} x = \alpha, x = \cos\alpha$

Now,
$$= \alpha + \cos^{-1}\left[\frac{\cos\alpha}{2} + \frac{\sqrt{3-3\cos^2\alpha}}{2}\right]$$

$$= \alpha + \cos^{-1}\left[\frac{\cos\alpha}{2} + \frac{\sqrt{3}.\sqrt{1-\cos^2\alpha}}{2}\right]$$

$$= \alpha + \cos^{-1}\left[\frac{\cos\alpha}{2} + \frac{\sqrt{3}}{2}\sin\alpha\right]$$

$$= \alpha + \cos^{-1}\left[\cos\frac{\pi}{3}\cos\alpha + \sin\frac{\pi}{3}\sin\alpha\right]$$

$$= \alpha + \cos^{-1}\left[\cos\left(\frac{\pi}{3} - \alpha\right)\right]$$

$$= \frac{\pi}{3}$$

27. Given that $y = [\tan^{-1}x]^2$

$$\frac{dy}{dx} = \frac{d[\tan^{-1}]^2}{dx}$$

$$\frac{dy}{dx} = 2\tan^{-1}x \times \frac{1}{1+x^2}$$

$$\frac{dy}{dx} = \frac{2\tan^{-1}x}{1+x^2} \quad \text{...(i)}$$

Again Differentiating,

$$\frac{d^2y}{dx^2} = \frac{(1+x^2)\frac{d}{dx}(2\tan^{-1}x) - (2\tan^{-1}x)\frac{d}{dx}(1+x^2)}{(1+x^2)^2}$$

$$\frac{d^2y}{dx^2} = \frac{(1+x^2)\times\frac{2}{(1+x^2)} - 2\tan^{-1}x \times 2x}{(1+x^2)^2}$$

$$\frac{d^2y}{dx^2} = \frac{2 - 4x\tan^{-1}x}{(1+x^2)^2}$$

$$(1+x^2)^2\frac{d^2y}{dx^2} = 2 - 4x\tan^{-1}x \quad \text{...(ii)}$$

$$\text{L.H.S} = (1+x^2)^2\frac{d^2y}{dx^2} + 2x(1+x^2)\frac{dy}{dx}$$

$$= 2 - 4x\tan^{-1}x + 2x(1+x^2)\frac{2\tan^{-1}x}{(1+x^2)}$$

$$= 2 - 4x\tan^{-1}x + 4x\tan^{-1}x$$
$$= 2 = \text{R.H.S}$$

30. $\vec{b} + \vec{c} = \hat{i} + 3\hat{j} + \hat{k} + \hat{i} + \hat{k} = 2\hat{i} + 3\hat{j} + 2\hat{k}$

Projection of $(\vec{b} + \vec{c})$ on $\vec{a} = \dfrac{(\vec{b} + \vec{c}).\vec{a}}{|\vec{a}|}$

$$= \frac{(2\hat{i} + 3\hat{j} + 2\hat{k}).(2\hat{i} + 2\hat{j} + \hat{k})}{\sqrt{4+4+1}}$$

$$= \frac{4+6+2}{3} = \frac{12}{3} = 4$$

Therefore, Projection of $(\vec{b} + \vec{c})$ on $\vec{a} = 4$

31. Sample Space $= \{R_1, R_2, R_3, W_1, W_2\}$

Prob. of getting a white ball $= \dfrac{2}{5}$

Prob. of not getting a white ball $= \dfrac{3}{5}$

Let x be a random variable of getting a white ball.

Therefore, $x = \{0, 1, 2\}$

$P(x = 0)$ = Not getting a white ball × Not getting white ball

$$= \frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$$

$P(x = 1)$ = Not getting a white ball × getting a white ball + Getting a white ball × Not getting a white ball

$$= \frac{3}{5} \times \frac{2}{5} + \frac{2}{5} \times \frac{3}{5}$$

$$= \frac{6}{25} + \frac{6}{25} = \frac{12}{25}$$

$P(x = 2)$ = getting a white ball × getting white ball

$$= \frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$$

Probability Distribution Table is as:

x	0	1	2
P(x)	$\frac{9}{25}$	$\frac{12}{25}$	$\frac{4}{25}$

$$\text{Mean} = \Sigma xP(x) = 0 \times \frac{9}{25} + 1 \times \frac{12}{25} + 2 \times \frac{4}{25}$$

$$= \frac{12}{25} + \frac{8}{25} = \frac{20}{25} = 0.8$$

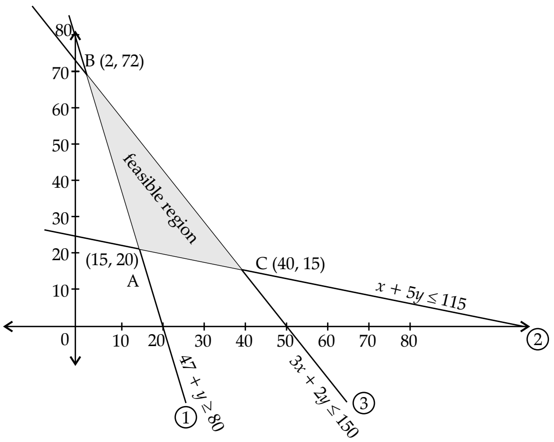
35. Min $Z = 6x + 3y$

S.T.C. $4x + y \ge 80$

$x + 5y \ge 115$

$3x + 2y \le 150$

The feasible region determined by the constraints is given below:



Corner Points	coordinates	$Z = 6x + 3y$
A	(15,20)	150 Minimum
B	(2,72)	228
C	(40,15)	285

Hence, the minimum value of Z is 150 attains at (15, 20).

Outside Delhi Set-1 **65/4/1**

SECTION A

1. Option (D) is correct.

Explanation: In scalar matrix all diagonal elements are equal

so, $a = d = 5$

$c = b = 0$

value of $a + 2b + 3c + 4d$

$= 5 + 20$

$= 25$

2. Option (B) is correct.

Explanation: We know that

$$(A^{-1})^{-1} = A$$

$$A^{-1} = \begin{bmatrix} \frac{2}{7} & \frac{1}{7} \\ \frac{-3}{7} & \frac{2}{7} \end{bmatrix}$$

$$\Rightarrow \quad A = \frac{adj(A^{-1})}{|A^{-1}|}$$

$$\Rightarrow \quad A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$$

3. Option (A) is correct.

Explanation: $A = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix} I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The value of $I - A + A^2 - A^3 \ldots\ldots = \begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$

4. Option (B) is correct.

Explanation: $A = \begin{bmatrix} -2 & 0 & 0 \\ 1 & 2 & 3 \\ 5 & 1 & -1 \end{bmatrix}$

$$\text{adj}(A) = \begin{bmatrix} -5 & 0 & 0 \\ 16 & 0 & 6 \\ -9 & 2 & -4 \end{bmatrix}$$

$$A.\text{adj}(A) = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$$|A.\text{adj}(A)| = 10\, I$$

5. Option (C) is correct.

Explanation:

$$[1 \quad x]\begin{bmatrix} 4 & 0 \\ -2 & 0 \end{bmatrix} = 0$$

$$4 - 2x = 0$$

$$x = 2$$

6. Option (C) is correct.

Explanation: Let $u = e^{2x}, v = e^x$

$$\frac{du}{dx} = e^{2x} \times 2, \frac{dv}{dx} = e^x$$

$$\frac{du}{dx} / \frac{dv}{dx} = 2e^x$$

7. Option (B) is correct.

Explanation: $f(x) = \begin{cases} \dfrac{\sqrt{4+x}-2,}{x} & x \neq 0 \\ k, & x = 0 \end{cases}$

For continuity at $x = 0$

$$\lim_{x\to 0^+}[f(x)] = \lim_{x\to 0^-}[f(x)] = f(0)$$

$$f(x) = \frac{\sqrt{4+x}-2}{x} \times \frac{\sqrt{4+x}+2}{\sqrt{4+x}+2}$$

$$f(x) = \frac{4+x-4}{x(\sqrt{4+x}+2)}$$

$$f(x) = \frac{1}{(\sqrt{4+x}+2)}$$

$$= \frac{1}{\sqrt{4+x}+2}$$

Let, $x = 0 + h$

$$= \lim_{h\to 0}\frac{1}{\sqrt{4+0+h+2}} = \frac{1}{4}$$

Given $f(0) = k$

So $\left[k = \frac{1}{4}\right]$

8. Option (C) is correct.

Explanation: $\int_0^3 \frac{dx}{\sqrt{9-x^2}}$ $\left[\therefore \frac{1}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}\right]$

$$I = \int_0^3 \frac{dx}{\sqrt{3^2-x^2}}$$

$$I = \left[\sin^{-1}\frac{x}{3}\right]_0^3$$

$$= \left[\sin^{-1}\frac{3}{3} - \sin^{-1}\frac{0}{3}\right]$$

$$= \frac{\pi}{2}$$

9. Option (A) is correct.

Explanation: Given,

Differential Eqn. is $xdy + ydx = 0$

$$xdy = -ydx$$

$$\Rightarrow \int\frac{dy}{y} = -\int\frac{dx}{x}$$

$$\Rightarrow \log y = -\log x + \log c$$

$$\Rightarrow \log(yx) = \log c$$

$$\Rightarrow xy = c$$

10. Option (D) is correct.

Explanation: Differential Eqn. $(x+2y^2)\frac{dy}{dx} = y$

find, I.F.

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x+2y^2}$$

$$\Rightarrow \frac{dx}{dy} = \frac{x+2y^2}{y}$$

$$\Rightarrow \frac{dx}{dy} = \frac{x}{y}+2y$$

As we know that

$$\Rightarrow \frac{dx}{dy}-\frac{x}{y} = 2y$$

$$\frac{dx}{dy}+Px = Q, P = \frac{-1}{y}, Q = 2y$$

$$\text{I.F.} = e^{\int Pdy}$$

$$= e^{\int -\frac{1}{y}dy} = e^{-\log y} = \frac{1}{y}$$

$$\text{I.F.} = \frac{1}{y}$$

11. Option (D) is correct.

Explanation: $|\vec{a}| = 1$ $\vec{a}.\vec{b} = \sqrt{3}$

$$|\vec{b}| = 2$$

$$\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$\sqrt{3} = 2\cos\theta$$

$$\cos\theta = \frac{\sqrt{3}}{2} \quad \theta = \frac{\pi}{6}$$

$$(2\vec{a}).(-\vec{b}) = 2|\vec{a}||\vec{b}|\cos\theta$$

$$\theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$$

12. Option (D) is correct.

Explanation: Given vectors

$$\vec{a} = 2\hat{i}-\hat{j}+\hat{k},$$

$$\vec{b} = \hat{i}-3\hat{j}-5\hat{k},$$

$$\vec{c} = -3\hat{i}+4\hat{j}+4\hat{k},$$

when two vectors are perpendicular to each other

$$\vec{A}.\vec{B} = 0$$

So, $\vec{a}.\vec{b} = (2\hat{i}-\hat{j}+\hat{k}).(\hat{i}-3\hat{j}-5\hat{k})$

$$= [2+3-5]$$

$$\vec{a}.\vec{b} = 0$$

So, a Right angled Triangle are formed.

13. Option (B) is correct.

Explanation: Let $\vec{a} = a_1\hat{i}+a_2\hat{j}+a_3\hat{k}.$

Given $|\vec{a}| = a$

Then $\vec{a}\times\hat{i} = -a_2\hat{k}+a_3\hat{j}$

$$[\therefore \hat{i}\times\hat{i} = \hat{j}\times\hat{i} = \hat{k}\times\hat{k} = 0]$$

$$\vec{a}\times\hat{j} = a_1\hat{k}-a_3\hat{j}$$

$$\vec{a}\times\hat{k} = -a_1\hat{j}+a_2\hat{i}$$

and $\therefore |\vec{a}\times\hat{i}|^2+|\vec{a}\times\hat{j}|^2+|\vec{a}\times\hat{k}|^2$

$$[\therefore |\vec{a}| = \sqrt{a_1^2+a_2^2+a_3^2}]$$

$$= a_2^2+a_3^2+a_1^2+a_3^2+a_1^2+a_2^2$$

$$= 2[a_1^2+a_2^2+a_3^2]$$

$$= 2[|\vec{a}|]^2$$

$$= 2a^2$$

14. Option (D) is correct.

Explanation: We know that vector eqn. of a line is

$$\vec{r} = \vec{a}+\lambda\vec{b}$$

Given $\vec{a} = (1,-1,0)$

passing through the point line is parallel to y-axis.

$$\vec{b} = (0,1,0)$$

$$\vec{r} = \hat{i}-\hat{j}+\lambda\hat{j}$$

15. Option (C) is correct.

Explanation:

$$L_1: \quad \frac{1-x}{2} = \frac{y-1}{3} = \frac{z}{1}$$

$$\frac{x-1}{-2} = \frac{y-1}{3} = \frac{z}{1} \qquad \text{...(i)}$$

$$L_2: \quad \frac{2x-3}{2p} = \frac{y}{-1} = \frac{z-4}{7}$$

$$\frac{x-\frac{3}{2}}{P} = \frac{y}{-1} = \frac{z-4}{7} \quad \text{...(ii)}$$

On Comparing with

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

Direction ratio of line (i) are

$$a_1 = -2, b_1 = 3, c_1 = 1$$

Direction ratio of line (ii) are

$$a_2 = p, b_2 = -1, c_2 = 7$$

when $L_1 \perp L_2$ then $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$-2\times p + 3\times(-1) + 1\times 7 = 0$$
$$-2p - 3 + 7 = 0$$
$$-2p + 4 = 0$$
$$p = 2$$

16. Option (C) is correct.

Explanation: Given L.P.P. is Maximise $z = 4x + y$

Corner points	value of z ($z = 4x + y$)
A (0, 50)	$z = 4 \times 0 + 50 = 50$
B (20, 30)	$z = 4 \times 20 + 30 = 110$
C (30, 0)	$z = 4 \times 30 + 0 = 120$
D (0, 0)	$z = 0 + 0 = 0$

Maximum value is 120.

17. Option (B) is correct.

Explanation: We have

$$P(0) + P(1) + P(2) + P(3) + P(4) = 1$$
$$0.1 + k + 2k + k + 0.1 = 1$$
$$0.2 + 4k = 1$$
$$4k = 0.8$$
$$k = 0.2 = \frac{1}{5}$$

Given $\quad P(2) = 2k$

$$= 2\times\frac{1}{5} = \frac{2}{5}$$

18. Option (?) is correct.

Explanation: $\quad f(x) = kx - \sin x$ is strictly increasing for all $x \in R$

$$f'(x) > 0 \ \forall \ x \in R$$

$\therefore \quad \lambda(0) > 0$

$\Rightarrow \quad k - \cos x > 0$

$\Rightarrow \quad k - \cos 0 > 0$

$\Rightarrow \quad k - 1 > 0 \Rightarrow k > 1$

19. Option (D) is correct.

Explanation: Assertion:

$R = \{(x, y) : (x + y)$ is a Prime Number and $x, y \in N\}$

Reflexive: For (1 + 1) is Prime Number $(1, 1) \in R$

For (2 + 2) is not prime number $(2, 2) \notin R$

So, R is not reflexive.

Reason: A composite number is a natural number or positive integer that has more than two factors.

So; $(2n)$ is more than two factor and it is not prime.

20. Option (D) is correct.

Explanation: Assertion:

Corner points	value of z ($z = x + 2y$)
P (60, 0)	60
Q (120, 0)	120
R (60, 30)	120
S (40, 20)	80

No, there is only two points. Q and R are shown maximum value.

Reason: Theorem 2: Let R be a feasible region For L.P.P. and Let $z = ax + by$ be the objective function if R is bounded, then the objective function Z has both maximum or minimum value on R and each of these occurs at a corner point of R. If the feasible region is unbounded, then a maximum or a minimum may not exist. However, it it exist. It must occur at a corner point of R.

SECTION B

21. (a) $\tan^{-1}\left(\dfrac{\cos x}{1-\sin x}\right) \quad \left[\because \cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2}\right]$

$$= \tan^{-1}\left[\frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2}\right] \quad [a^2 - b^2 = (a+b)(a-b)]$$

$$= \tan^{-1}\left[\frac{\cos\frac{x}{2} + \sin\frac{x}{2}}{\cos\frac{x}{2} - \sin\frac{x}{2}}\right]$$

$$= \tan^{-1}\left[\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right]$$

$$= \tan^{-1}\left[\frac{\tan\frac{\pi}{4} + \tan\frac{x}{2}}{1-\tan\frac{\pi}{4}\tan\frac{x}{2}}\right]$$

$$\left[\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A.\tan B}\right]$$

$$= \tan^{-1}\left(\tan\frac{\pi}{4} + \frac{x}{2}\right)$$

$$= \frac{\pi}{4} + \frac{x}{2}$$

OR

(b) $\tan(1)^{-1} + \cos^{-1}\left(-\dfrac{1}{2}\right) + \sin^{-1}\left(-\dfrac{1}{\sqrt{2}}\right)$

$$= \tan^{-1}\left(\tan\frac{\pi}{4}\right) + \cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\left(-\frac{\pi}{4}\right)\right)$$

$$= \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{4}$$

$$= \frac{2\pi}{3}$$

22. **(a)** $y = \cos^3(\sec^2 2t)$

Applying chair Rule.

$$\frac{dy}{dt} = 3\cos^2(\sec^2 2t) \times [(-\sin(\sec^2 2t)] \times 2\sec 2t$$
$$\times \sec 2t\ x \tan 2t \times 2$$
$$= -12\cos^2(\sec^2 2t) \times (\sec^2 2t) \times \sin(\sec^2 2t) \times \tan 2t$$

OR

(b) $x^y = e^{x-y}$

taking log both sides

$$y \log x = x - y \ \log e \qquad [\therefore \log e = 1]$$
$$y \log x + y = x$$
$$y = \frac{x}{\log x + 1}$$

Now we apply quotient rule

$$\frac{dy}{dx} = \frac{(\log x + 1)\frac{d}{dx}(x) - x\frac{d}{dx}(\log x + 1)}{(1+\log x)^2}$$
$$= \frac{(\log x + 1) - x \times \frac{1}{x}}{(1+\log x)^2}$$
$$= \frac{\log x + 1 - 1}{(1+\log x)^2}$$
$$= \frac{\log x}{(1+\log x)^2}$$

Hence proved

$$\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$$

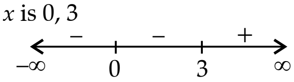
23. We have $f(x) = x^4 - 4x^3 + 10$

$\Rightarrow \quad f'(x) = 4x^3 - 12x^2$

$\Rightarrow \quad f'(x) = 4x^2(x-3)$

For $f(x)$ is strictly decreasing, we must have $f'(x) < 0$

Value of x is 0, 3



Strictly Decreasing $(-\infty, 0) \cup (0, 3)$

24. Given $\frac{dv}{dt} = 6\text{ cm}^3/\text{s} \quad l = 8\text{ cm}$

find, $\frac{dS}{dt}$

V = volume of cube = l^3

$$\Rightarrow \quad \frac{dv}{dt} = 3l^2\frac{dl}{dt}$$
$$\Rightarrow \quad 6 = 3l^2\frac{dl}{dt}$$
$$\Rightarrow \quad \frac{2}{l^2} = \frac{dl}{dt} \qquad \text{...(i)}$$

Surface Area of cube

$$S = 6l^2$$
$$\Rightarrow \quad \frac{dS}{dt} = 12l\frac{dl}{dt} \qquad \text{...(ii)}$$

Now put eqn (i) in enq (ii)

$$\Rightarrow \quad \frac{dS}{dt} = 12l \times \frac{2}{l^2}$$
$$\Rightarrow \quad \frac{dS}{dt} = \frac{24}{8}$$
$$\Rightarrow \quad \frac{dS}{dt} = 3\text{ cm}^2/\text{s}$$

25.
$$\int \frac{1}{x(x^2-1)}dx = \int \frac{x}{x^2(x^2-1)}dx$$

$x^2 = t$

$2xdx = dt$

$$= \frac{1}{2}\int \frac{dt}{t(t-1)}$$
$$= \frac{1}{2}\left[\int \frac{dt}{t-1} - \int \frac{dt}{t}\right]$$
$$= \frac{1}{2}[\log(t-1) - \log(t)] + c$$
$$= \frac{1}{2}\log\left(\frac{x^2-1}{x^2}\right) + C$$

26. $y = (\sin x)^x \cdot x^{\sin x} + a^x$

Let $u = (\sin x)^x$; $v = x^{\sin x}$

$$y = u.v + a^x \qquad \text{...(i)}$$
$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx} + a^x \log a$$
$$u = (\sin x)^x$$

taking log both sides

$$= x \log \sin x$$

Differentiate $\log u = \frac{1}{u}\frac{du}{dx} = \frac{x}{\sin x} \times \cos x$

$$+ \log \sin x$$
$$= \frac{du}{dx} = (\sin)^x[x\cot x + \log\sin x] \qquad \text{...(ii)}$$
$$v = x^{\sin x}$$
$$\log v = \sin x \log x$$
$$\frac{1}{v}\frac{dv}{dx} = \frac{\sin x}{x} + \log x \cos x$$
$$\frac{dv}{dx} = x^{\sin x}\left[\frac{\sin x}{x} + \log\cos x\right] \qquad \text{...(iii)}$$

Put eq (ii) and (iii) in eq (i)

$$\frac{dy}{dx} = (\sin x)^x . x^{\sin x}\left[\frac{\sin x}{x} + \log\cos x\right]$$
$$+ x^{\sin x}.(\sin x)^x[x\cot x + \log\sin x] + a^x \log a$$

27. **(a)** $I = \int_0^{\frac{\pi}{4}} \frac{x\,dx}{1+\cos 2x + \sin 2x} \qquad \text{...(i)}$

by using property '4' we get

$$I = \int_0^{\frac{\pi}{4}} \frac{\left(\frac{\pi}{4} - x\right)dx}{1 + \cos\left(\frac{\pi}{2} - 2x\right) + \sin\left(\frac{\pi}{2} - 2x\right)}$$
$$I = \int_0^{\frac{\pi}{4}} \frac{\left(\frac{\pi}{4} - x\right)dx}{1 + \sin 2x + \cos 2x} \qquad \text{...(ii)}$$

Now add eq (i) and (ii)

$$2I = \int_0^{\frac{\pi}{4}} \frac{\frac{\pi}{4}}{1+\sin 2x+\cos 2x}\,dx$$

$$2I = \frac{\pi}{4}\int_0^{\frac{\pi}{4}} \frac{dx}{2\cos^2 x+2\sin x\cos 2x}$$

$[\therefore\ 1 + \cos 2\theta = 2\cos^2\theta,\ \sin 2\theta = 2\sin\theta\cos\theta]$

$$\Rightarrow \quad 2I = \frac{\pi}{8}\int_0^{\frac{\pi}{4}} \frac{\frac{1}{\cos^2 x}}{1+\tan x}$$

$$\Rightarrow \quad I = \frac{\pi}{16}\int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{1+\tan x}dx$$

Let $1 + \tan x = t$

$\sec^2 x dx = dt$

New limit

$1 + \tan 0 = t \Rightarrow t = 1$

$1+\tan\frac{\pi}{4}t \Rightarrow t = 2$

$$\Rightarrow \quad I = \frac{\pi}{16}\int_1^2 \frac{dt}{t}$$

$$\Rightarrow \quad I = \frac{\pi}{16}|\log t|_1^2$$

$$I = \frac{\pi}{16}\log(2)-\log(1)$$

$$I = \frac{\pi}{16}\log 2$$

OR

(b) Let $\quad I = \int e^x\left[\frac{1}{(1+x^2)^{3/2}}+\frac{x}{\sqrt{1+x^2}}\right]$

we know that

$\therefore\ \int e^x[f(x)+f'(x)]$ the $[e^x f(x) + C]$ is a solution

$$f(x) = \frac{x}{\sqrt{1+x^2}}$$

$$f'(x) = \frac{\sqrt{1+x^2}\times(1)-x\times\frac{1}{2\sqrt{1+x^2}}\times 2x}{(\sqrt{1+x^2})^2}$$

$$f'(x) = \frac{\sqrt{1+x^2}-\frac{x^2}{\sqrt{1+x^2}}}{(1+x^2)}$$

$$f'(x) = \frac{1+x^2-x^2}{(1+x^2)^{1/2+1}} = \frac{1}{(1+x^2)^{3/2}}$$

$$I = \frac{xe^x}{\sqrt{1+x^2}}+C$$

28. $\int\frac{3x+5}{\sqrt{x^2+2x+4}}$

Let $\quad I = \int\frac{3x+5}{\sqrt{x^2+2x+1+3}}$

$$= \int\frac{3(x+1)}{x^2+2x+4}+\frac{2}{\sqrt{(x+1)^2+(\sqrt{3})^2}}$$

$$I_1 = \int\frac{3x+5}{\sqrt{(x+1)^2+(\sqrt{3})^2}}$$

$I = I_1 + I_2$

$$I = 3\int\frac{x+1}{\sqrt{x^2+2x+4}}$$

$$I_1 = \frac{3}{2}\int\frac{dt}{\sqrt{t}}$$

Let $x^2 + 2x + 4 = t$

$(2x + 2)\,dx = dt$

$(x + 1)\,dx = \frac{dt}{2}$

$$I_1 = \frac{3}{2}\int t^{-\frac{1}{2}}dt$$

$$I_1 = \frac{3}{2}\frac{t^{\frac{1}{2}}}{\frac{1}{2}}$$

$$I_1 = 3\sqrt{x^2+2x+4}$$

$$I_2 = 2\int\frac{1}{\sqrt{(x+1)^2+(\sqrt{3})^2}}$$

$$I_2 = 2\log|x+1+\sqrt{x^2+2x+4}|$$

$$I = 3\sqrt{x^2+2x+4}+2\log$$

$$|x+1+\sqrt{x^2+2x+4}|+C$$

29. (a) $\quad \frac{dy}{dx} = y\cot 2x$

Given $y = 2,\ x = \frac{\pi}{4}$

$$\Rightarrow \quad \frac{dy}{y} = \cot 2x\,dx$$

$$\Rightarrow \quad \int\frac{dy}{y} = \int\cot 2x\,dx$$

$$\Rightarrow \quad \log y = \frac{1}{2}\log|\sin 2x|+\log C$$

$$\Rightarrow \quad \log\frac{4}{\sqrt{\sin 2x}} = \log C$$

$$\Rightarrow \quad \frac{y}{\sqrt{\sin^2 x}} = C \qquad \text{...(i)}$$

Put $x = \frac{\pi}{4}$, $y = 2$ in eq (i)

$\Rightarrow \quad C = 2$

Hence $[y = 2\sqrt{\sin 2x}\,]$

OR

(b) $\quad (xe^{\frac{y}{x}}+y)dx = xdy \qquad$ given $y = 1,\ x = 1$

The given differential eqn is homogeneous function of degree zero.

To solve it we make substitution

$$y = vx, \qquad \frac{dy}{dx} = v+x\frac{dv}{dx} \qquad \text{...(i)}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{xe^{\frac{y}{x}} + y}{x}$$

$$\Rightarrow \quad \frac{dy}{dx} = e^{\frac{y}{x}} + \frac{y}{x}$$

from (i)

$$v + x\frac{dv}{dx} = e^v + v$$

$$\Rightarrow \quad \int\frac{dv}{e^v} = \int\frac{dx}{x}$$

$$-e^{-v} = \log x + C$$

or $\quad -e^{-\frac{y}{x}} = \log|x| + C$

It is given that $x = 1, y = 1$

So $\quad -e^{-1} = \log(1) + C$

or $\quad C = -e^{-1}$

Hence required solution

$$e^{-1} - e^{-\frac{y}{x}} = \log|x|$$

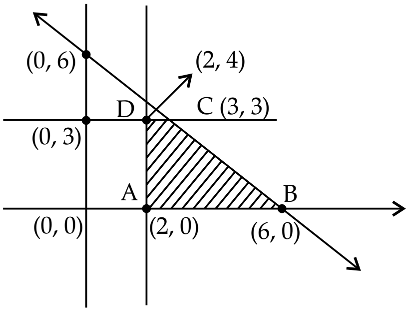
30. $\quad Z = 2x + 3y$

constraints:

$$x + y \le 6$$
$$x \ge 2$$
$$y \le 3, \quad x, y \ge 0$$

Let $\quad x + y = 6$

x	0	6	1
y	6	0	5



Corner points	$z = 2x + 3y$
A (2, 0)	$2 \times 2 + 3 \times 0 = 4$
B (6, 0)	$2 \times 6 + 3 \times 0 = 12$
C (3, 3)	$2 \times 3 + 3 \times 3 = 15$
D (2, 4)	$2 \times 2 + 3 \times 4 = 16$

Maximum value of Z is 16 at D (2, 4)

31. (a) E_1: Lost card is king

E_2: Lost card is not king

Let A: a card drawn from remaining pack is king

$$P(E_1) = \frac{4}{52}, \quad P(E_2) = \frac{48}{52}$$

$$P\left(\frac{A}{E_1}\right) = \frac{3}{51} \quad P\left(\frac{A}{E_2}\right) = \frac{4}{51}$$

$$P\left(\frac{E_1}{A}\right) = \frac{\frac{4}{52}\times\frac{3}{51}}{\frac{4}{52}\times\frac{3}{51}+\frac{48}{52}\times\frac{4}{51}}$$

$$= \frac{4\times 3}{4\times 3 + 48\times 4}$$

$$P\left(\frac{E_1}{A}\right) = \frac{12}{204}$$

OR

(b) Let the probability for odd Number $= p$

and probability for even number $= 2p$

$\Rightarrow \quad P + 2P = 1 \quad [\therefore P(E) + P(\bar{E}) = 1]$

$\Rightarrow \quad 3P = 1$

$$P = \frac{1}{3}$$

Probability for odd No $= \frac{1}{3}$

Probability for even No $= \frac{2}{3}$

Probability Distribution is X

$$P(X = 0) = {}^2C_0\left(\frac{1}{3}\right)^{2-0}\left(\frac{2}{3}\right)^0 = 1\times\frac{1}{9} = \frac{1}{9}$$

$$P(X = 1) = {}^2C_1\left(\frac{1}{3}\right)^{2-1}\left(\frac{2}{3}\right)^1$$

$$= 2\times\frac{1}{3}\times\frac{2}{3} = \frac{4}{9}$$

$$P(X = 2) = {}^2C_2\left(\frac{1}{3}\right)^{2-2}\left(\frac{2}{3}\right)^2$$

$$= 1\times 1\times\frac{4}{9} = \frac{4}{9}$$

X	0	1	2
$P(X)$	$\frac{1}{9}$	$\frac{4}{9}$	$\frac{4}{9}$
$XP(x)$	0	$\frac{4}{9}$	$\frac{8}{9}$

Mean $= \Sigma_{P_i x_i} = \frac{12}{9}$

32. (a) $\quad y = x|x|$

$y = x^2$ if $x > 0$

and $\quad y = -x^2$ if $x < 0$

$$\text{Required Area} = \int_{-2}^{2} y\,dx$$

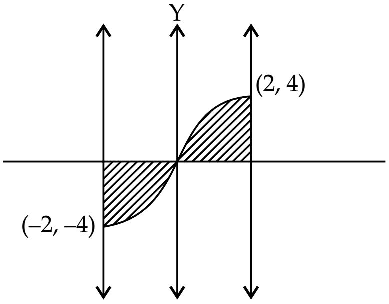
$$= \int_{-2}^{2} x|x|\,dx$$

$$= \int_{-2}^{0} +x^2 dx + \int_{0}^{2} x^2 dx$$

$$= +\left[\frac{x^3}{3}\right]_{-2}^{0} + \left[\frac{x^3}{3}\right]_{0}^{2}$$

$$= +\left[0-\left[\frac{(-2)^3}{3}\right]\right]+\left[\frac{2^3}{3}-0\right]$$

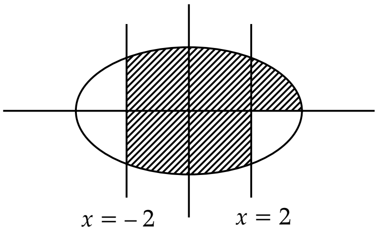
$$= +\frac{8}{3}+\frac{8}{3} = \frac{16}{3} \text{ unit}$$



OR

(b) $$9x^2 + 25y^2 = 225$$

$$\frac{x^2}{25}+\frac{y^2}{9} = 1$$



$$A = \int_{-2}^{2} \frac{3}{9}\sqrt{25-x^2}$$

$$A = \frac{6}{9}\int_{-2}^{2} \sqrt{25-x^2}$$

$$A = \frac{6}{9}\left[\frac{x}{2}\sqrt{25-x^2}+\frac{25}{2}\sin^{-1}\frac{x}{5}\right]_{-2}^{2}$$

$$A = \frac{6}{9}\left[\frac{2}{2}\sqrt{25-2^2}+\frac{25}{2}\sin^{-1}\frac{2}{5}\right]$$

$$-\left[\frac{-2}{2}\sqrt{25-(-2)^2}+\frac{25}{2}\sin^{-1}\frac{-2}{5}\right]$$

$$A = \frac{6}{9}\left[1\times\sqrt{21}+\frac{25}{2}\sin^{-1}\frac{2}{5}+\sqrt{21}+\frac{25}{2}\sin^{-1}\frac{2}{5}\right]$$

$$A = \frac{6}{9}\left[2\sqrt{21}+25\sin^{-1}\frac{2}{5}\right]$$

$$A = \frac{12}{9}\left[\sqrt{21}+25\sin^{-1}\frac{2}{5}\right]$$

33. (a) Given, $A = R - \{5\}, B = R - \{1\}$

$$f : A \to B \qquad f(x) = \frac{x-3}{x-5}$$

for f is one-one

Let $x_1, x_2 \in R$

$$f(x_1) = f(x_2)$$

$$\frac{x_1-3}{x_1-5} = \frac{x_2-3}{x_2-5}$$

$$(x_1 - 3)(x_2 - 5) = (x_2 - 3)(x_1 - 5)$$

$$x_1x_2 - 5x_1 - 3x_2 + 15 = x_1x_2 - 5x_2 - 3x_1 + 15$$

$$-5x_1 + 3x_1 = -5x_2 + 3x_2$$

$$-2x_1 = -2x_2$$

$$\therefore \quad x_1 = x_2$$

f is one-one.

For if is onto

Let y be any element of R

$$y = f(x)$$

$$\Rightarrow \quad y = \frac{x-3}{x-5}$$

$$\Rightarrow \quad y(x-5) = x-3$$

$$\Rightarrow \quad yx-5y = x-3$$

$$\Rightarrow \quad yx-x = 5y-3$$

$$\Rightarrow \quad x = \frac{5y-3}{y-1}$$

$$f(x) = \frac{x-3}{x-5}$$

$$f\left(\frac{5y-3}{y-1}\right) = \frac{\frac{5y-3}{y-1}-3}{\frac{5y-3}{y-1}-5}$$

$$\Rightarrow \quad = \frac{\frac{5y-3-3y+3}{y-1}}{\frac{5y-3-5y+5}{y-1}}$$

$$f\left(\frac{5y-3}{y-1}\right) = \frac{2y}{2} = y$$

f is onto

OR

(b) For a Relation to be Reflexive aRa

For real a

$aRa \Rightarrow a-a+\sqrt{2} = \sqrt{2}$

$\sqrt{2}$ is an irrational number

aRa is Reflexive

For a Relation to be symmetric

$$aRb \Rightarrow bRa$$

For real number a and b

$$aRb \Rightarrow a-b+\sqrt{2} \qquad \Rightarrow aRb \neq bRa$$

$$bRa \Rightarrow b-a+\sqrt{2} \qquad \text{It is not symmetric}$$

For Transitive

$$aRb = bRc = aRc$$

For real number a, b and c

Let $\quad a = -\sqrt{2}$

$$b = 3\sqrt{2}$$

$$c = 2$$

$aRb \Rightarrow \quad a-b+\sqrt{2} = -\sqrt{2}-3\sqrt{2}+\sqrt{2}$

$= -3\sqrt{2}$ is an irrational

$bRc \Rightarrow 3\sqrt{2}-2+\sqrt{2}$

$= 4\sqrt{2}-2$ is an irrational

$aRc \Rightarrow -\sqrt{2}-2+\sqrt{2}$

$= -2$ is not an irrational

aRb, bRc then a is not related to b.

the Relation is not transitive.

34. Given
$$A = \begin{bmatrix} 2 & 1 & -3 \\ 3 & 2 & 1 \\ 1 & 2 & -1 \end{bmatrix}$$
$$|A| = 2(-2-2) - 1(-3-1) - 3(6-2)$$
$$= 2(-4) - 1(-4) - 3(4)$$
$$= -8 + 4 - 12$$
$$|A| = -16$$

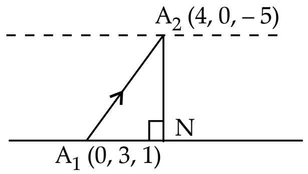
Minor of Matrices A	Cofactor
$M_{11} = (-2-2) = -4$	$A_{11} = -4$
$M_{12} = (-3-1) = -4$	$A_{12} = 4$
$M_{13} = (6-2) = 4$	$A_{13} = 4$
$M_{21} = (-1+6) = 5$	$A_{21} = -5$
$M_{22} = (-2+3) = 1$	$A_{22} = 1$
$M_{23} = (4-1) = 3$	$A_{23} = -3$
$M_{31} = (1+6) = 7$	$A_{31} = 7$
$M_{32} = (2+9) = 11$	$A_{32} = -11$
$M_{33} = (4-3) = 1$	$A_{33} = 1$

$$\text{adj } A = \begin{bmatrix} -4 & 4 & 4 \\ -5 & 1 & -3 \\ 7 & -11 & 1 \end{bmatrix}^T$$
$$\text{adj } A = \begin{bmatrix} -4 & -5 & 7 \\ 4 & 1 & -11 \\ 4 & -3 & 1 \end{bmatrix}$$
$$\therefore \quad A^{-1} = \frac{(\text{adj } A)}{|A|}$$
$$A^{-1} = \frac{1}{-16}\begin{bmatrix} -4 & -5 & 7 \\ 4 & 1 & -11 \\ 4 & -3 & 1 \end{bmatrix}$$

Given eqn can be written in matrix form as
$$AX = B$$
$$X = A^{-1}B \qquad \text{...(i)}$$
Where, $A = \begin{bmatrix} 2 & 1 & -3 \\ 3 & 2 & 1 \\ 1 & 2 & -1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $B = \begin{bmatrix} 13 \\ 4 \\ 8 \end{bmatrix}$

from (i)
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-16}\begin{bmatrix} -4 & -5 & 7 \\ 4 & 1 & -11 \\ 4 & -3 & 1 \end{bmatrix}\begin{bmatrix} 13 \\ 4 \\ 8 \end{bmatrix}$$
$$= \frac{1}{-16}\begin{bmatrix} -52-20+56 \\ 52+4-88 \\ 52-12+8 \end{bmatrix}$$
$$= \frac{1}{-16}\begin{bmatrix} -16 \\ -32 \\ 48 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$$
$\therefore x = 1, y = 2, z = -3$

35. (a) The given line cartesian form
$$\frac{x}{2} = \frac{2y-6}{4} = \frac{1-z}{-1} \qquad \text{...(i)}$$



It passing through the point

A_1 with P.V. $\vec{a_1} = 3\hat{j} + \hat{k}$

and it is parallel to the vector
$$\vec{b} = a\hat{i} + 2\hat{j} + \hat{k}$$
The equation of the line passing through the point $A_2(4, 0, -5)$ with P.V.

$\vec{a_2} = 4\hat{i} - 5\hat{k}$ and parallel to the line (i) is
$$\vec{r} = 4\hat{i} - 5\hat{k} + \mu(2\hat{j} + \hat{k}) \qquad \text{...(ii)}$$
Now $\quad \vec{a_2} - \vec{a_1} = 4\hat{i} + 0\hat{j} - 5\hat{k} - 0\hat{i} - 3\hat{j} - \hat{k}$
$$= 4\hat{i} - 3\hat{j} - 6\hat{k}$$
$$|\vec{b}| = \sqrt{(0)^2 + (2)^2 + (1)^2}$$
$$= \sqrt{5}$$
and $\quad \vec{b} \times (\vec{a_2} - \vec{a_1}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & 1 \\ 4 & -3 & -6 \end{vmatrix}$
$$= \hat{i}(-12-3) - \hat{j}(0-4) + \hat{k}(0-8)$$
$$= -15\hat{i} + 4\hat{j} - 8\hat{k}$$
$$|\vec{b} \times (\vec{a_2} - \vec{a_1})| = \sqrt{(-15)^2 + (4)^2 + (-8)^2}$$
$$= \sqrt{225 + 16 + 64}$$
$$= \sqrt{305}$$
$\therefore$ The distance the parallel lines

(i) and (ii) $\quad = \dfrac{\vec{b} \times (\vec{a_2} - \vec{a_1})}{|\vec{b}|} = \dfrac{\sqrt{305}}{\sqrt{5}}$

OR

(b) The equation of the given lines are
$$L_1: \quad \frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$$
$$L_2: \quad \frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-7}$$
Their Direction Ratio are $(-3, 2k, 2)$ and $(3k, 1, -7)$ when $L_1 \perp L_2$
$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$
$$(-3)(+3k) + (2k)(1) + (2)(-7) = 0$$
$$-9k + 2k - 14 = 0$$

$$-7k - 14 = 0$$
$$k = -2$$

Now

L_1: $$\frac{x-1}{-3} = \frac{y-2}{-4} = \frac{z-3}{2}$$

L_2: $$\frac{x-1}{-6} = \frac{y-1}{1} = \frac{z-6}{-7}$$

We know that equation of any line through a given point (x_1, y_1, z_1) are

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

$\therefore$ Equation of any line through $(3, -4, -7)$

$$\frac{x-3}{a} = \frac{y+4}{b} = \frac{z+7}{c}$$

If it is perpendicular to the lines

then $-3a - 4b + 2c = 0$

$-6a + b - 7c = 0$

$$\frac{a}{28-2} = \frac{b}{-12-21} = \frac{c}{-3-24}$$

$$\frac{a}{+26} = \frac{b}{-33} = \frac{c}{-27}$$

Equation of line

$$\frac{x-3}{26} = \frac{y+4}{-33} = \frac{z+7}{-27}$$

36. (i) The Demand function

$$P = 450 - \frac{1}{2}x$$

Revenue function $R(x) = Px$

$$= \left(450 - \frac{1}{2}x\right) \times x$$

$$R = 450x - \frac{1}{2}x^2$$

$$\frac{dR}{dx} = 450 - x$$

$$\frac{dR}{dx} = 0$$

$$450 - x = 0$$
$$x = 450$$

$$\frac{d^2R}{dx^2} = -1$$

$$\frac{d^2R}{dx^2} < 0$$

when $x = 450$

To find number of unit x for R is maximum,

We should find 'x' where $\frac{dR}{dx} = 0$, $\frac{d^2R}{dx^2} < 0$

$$P = 450 - \frac{1}{2} \times 450$$

$$P = 450 - 225$$
$$P = ₹\,125$$

$\therefore$ The revenue is maximum when $P = 125$, the price per unit is ₹ 125 and 450 unit are demanded.

(ii) Rebate in price of calculator

$$= 350 - 125 = ₹\,125$$
$$P = ₹\,125$$

37. (i) Distance between VA

$$\vec{VA} = \text{P.V. of } A - \text{P.V. of } V$$

$$= (7\hat{i} + 5\hat{j} + 8\hat{k}) - (-3\hat{i} + 7\hat{j} + 11\hat{k})$$

$$\vec{VA} = 10\hat{i} - 2\hat{j} - 3\hat{k}$$

$$|\vec{VA}| = \sqrt{100+4+9} = \sqrt{113}$$

(ii) Unit vector in direction $\vec{DA}$

$$= \frac{\vec{DA}}{|\vec{DA}|}$$

$$\vec{DA} = \text{P.V. of } A - \text{P.V. of } D$$

$$= (7\hat{i} + 5\hat{j} + 8\hat{k}) - (2\hat{i} + 3\hat{j} + 4\hat{k})$$

$$\vec{DA} = 5\hat{i} + 2\hat{j} + 4\hat{k}$$

$$\text{unit vector} = \frac{5\hat{i} + 2\hat{j} + 4\hat{k}}{\sqrt{25+4+16}}$$

$$= \frac{5\hat{i} + 2\hat{j} + 4\hat{k}}{\sqrt{45}}$$

(iii) Angle of $\angle$VDA

Let $\vec{DA} = \vec{a}$

$\vec{DV} = \vec{b}$

So $\vec{a} = 5\hat{i} + 2\hat{j} + 4\hat{k}$

$\vec{b} = -5\hat{i} + 4\hat{j} + 7\hat{k}$

Angle $$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

$$= \frac{-25+8+28}{\sqrt{25+4+16}\sqrt{25+16+49}}$$

$$\cos\theta = \frac{11}{\sqrt{45}\sqrt{90}} = \frac{11}{45\sqrt{2}}$$

$$\theta = \cos^{-1}\left(\frac{11}{45\sqrt{2}}\right)$$

OR

(iii) Projection of vector $\vec{DV}$ on vector $\vec{DA}$

$$= \frac{\vec{DV}\cdot\vec{DA}}{|\vec{DA}|}$$

$$= \frac{11}{\sqrt{45}}$$

38. Let
$$P(R) = \frac{1}{5}$$
$$P(J) = \frac{1}{3}$$
$$P(A) = \frac{1}{4}$$

(i) At least one of them is selected
$$= 1 - P(\bar{R})\, P(\bar{J})\, P(\bar{A})$$
$$= 1 - \frac{4}{5} \times \frac{2}{3} \times \frac{3}{4}$$
$$= 1 - \frac{2}{5}$$
$$= \frac{3}{5}$$

(ii) Given, G is the event of Jaspreet's selection
$$P(G) = \frac{1}{3}$$

$\bar{H}$: Rohit is not selected
$$P(\bar{H}) = \frac{4}{5};\ P(H) = \frac{1}{5}$$
$$P\left(\frac{G}{\bar{H}}\right) = \frac{P(G \cap \bar{H})}{P(\bar{H})}$$
$$P(G \cap \bar{H}) = P(G) - P(G \cap H)$$
$$= \frac{1}{3} - \frac{1}{3} \times \frac{1}{5} = \frac{4}{15}$$
$$P\left(\frac{G}{\bar{H}}\right) = \frac{\frac{4}{15}}{\frac{4}{5}}$$
$$P\left(\frac{G}{\bar{H}}\right) = \frac{1}{3}$$

(iii) Exactly one of them is selected
$$= P(R)\, P(\bar{J})\, P(\bar{A}) + P(\bar{R})\, P(J)\, P(\bar{A}) + P(\bar{R})\, P(\bar{J})\, R(A)$$
$$= \frac{1}{5} \times \frac{2}{3} \times \frac{3}{4} + \frac{4}{5} \times \frac{1}{3} \times \frac{3}{4} + \frac{4}{5} \times \frac{2}{3} \times \frac{1}{4}$$
$$= \frac{6}{60} + \frac{12}{60} + \frac{8}{60}$$
$$= \frac{26}{60}$$

OR

Exactly two of them is selected
$$= P(R)\, P(J)\, P(\bar{A}) + P(R)\, P(\bar{J})\, P(A) + P(\bar{R})\, P(J)\, R(A)$$
$$= \frac{1}{5} \times \frac{1}{3} \times \frac{3}{4} + \frac{1}{5} \times \frac{2}{3} \times \frac{1}{4} + \frac{4}{5} \times \frac{1}{3} \times \frac{1}{4}$$
$$= \frac{3}{60} + \frac{2}{60} + \frac{4}{60}$$
$$= \frac{9}{60}$$

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SECTION A

4. Option (B) is correct.

Explanation: Given,
$$A = [a_{ij}] = \begin{bmatrix} 2 & -1 & 5 \\ 1 & 3 & 2 \\ 5 & 0 & 4 \end{bmatrix}$$

$a_{21} = 1,\ a_{22} = 3,\ a_{23} = 2$
$$C_{11} = (-1)^{1+1} \begin{vmatrix} 3 & 2 \\ 0 & 4 \end{vmatrix}$$
$$= (1)\,(12 - 0)$$
$$= 12$$
$$C_{12} = (-1)^{1+2} \begin{vmatrix} 1 & 2 \\ 5 & 4 \end{vmatrix}$$
$$= (-1)\,(4 - 10)$$
$$= (-1)\,(-6)$$
$$= 6$$
$$C_{13} = (-1)^{1+3} \begin{vmatrix} 1 & 3 \\ 5 & 0 \end{vmatrix}$$
$$= (1)\,(0 - 15)$$
$$= -15$$

Now, $a_{21}c_{11} + a_{22}c_{12} + a_{23}c_{13} = (1)(12) + (3)\,(6) + (2)\,(-15)$
$$= 12 + 18 - 30$$
$$= 30 - 30$$
$$= 0$$

5. Option (B) is correct.

Explanation: Given,
$$A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$$
$$A^2 = A.\,A$$
$$= \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$$
$$= \begin{bmatrix} (1)(1)+(3)(3) & (1)(3)+(3)(4) \\ (3)(1)+(4)(3) & (3)(3)+(4)(4) \end{bmatrix}$$
$$= \begin{bmatrix} 1+9 & 3+12 \\ 3+12 & 9+16 \end{bmatrix}$$
$$= \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix}$$

Now, $A^2 - KA - 5I = 0$ [Given]
$$\begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix} - k\begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$
$$\begin{bmatrix} 10-k-5 & 15-3k-0 \\ 15-3k-0 & 25-4k-5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$
$$\begin{bmatrix} 5-k & 15-3k \\ 15-3k & 20-4k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

On comparing

$$5 - k = 0$$
$$k = 5$$

OR $\quad 15 - 3k = 0$

$$k = 5$$

OR, $\quad 20 - 4k = 0$

Hence, $\quad k = 5$

6. **Option (B) is correct.**

Explanation: Given,

$$e^{x^2 y} = C$$

d.w.r to x

$$\frac{d}{dx}(e^{x^2 y}) = \frac{d}{dx}C$$

$$e^{x^2 y}\left[x^2\frac{dy}{dx} + y(2x)\right] = 0$$

$$x^2 e^{x^2 y}\frac{dy}{dx} + 2xy\, e^{x^2 y} = 0$$

$$x^2 e^{x^2 y}\frac{dy}{dx} = -2xy\, e^{x^2 y}$$

$$\frac{dy}{dx} = \frac{-2xye^{x^2 y}}{x^2 e^{x^2 y}}$$

$$= \frac{-2y}{x}$$

7. **Option (A) is correct.**

Explanation:

$$\text{L.H.L.} = \lim_{x\to 4^-} f(x)$$
$$= \lim_{h\to 0} f(4-h)$$
$$= \lim_{h\to 0}(4-h)^2 - c^2$$
$$= (4-0)^2 - c^2$$
$$= 16 - c^2$$
$$\text{R.H.L.} = \lim_{x\to 4^+} f(x)$$
$$= \lim_{h\to 0} f(4+h)$$
$$= \lim_{h\to 0} c(4+h) + 20$$
$$= c(4+0) + 20$$
$$= 4c + 20$$

Given function is continuous, Then

$$\text{L.H.L.} = \text{R.H.L.}$$
$$16 - c^2 = 4c + 20$$
$$c^2 + 4c + 4 = 0$$
$$(c+2)^2 = 0$$
$$c + 2 = 0$$
$$c = -2$$

8. **Option (C) is correct.**

Explanation: G

$$\int_{-1}^{1}|x|\,dx = \int_{-1}^{0} -x\,dx + \int_0^1 x\,dx$$

$$= -\left[\frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_0^1$$

$$= -\left[\frac{(0)^2}{2} - \frac{(-1)^2}{2}\right] + \left[\frac{(1)^2}{2} - \frac{(0)^2}{2}\right]$$

$$= -\left[0 - \frac{1}{2}\right] + \left[\frac{1}{2} - 0\right]$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

9. **Option (C) is correct.**

Explanation: We know that there is no arbitrary constants exist in the particular solution.

14. **Option (B) is correct.**

Explanation: $\vec{r} = \hat{i} + \hat{j} + \hat{k} + x(3\hat{i} - \hat{j} + 0\hat{k})$

$a_1 = 3, b_1 = -1, c_1 = 0$

On checking options one by one taking option (B) as

$a_2 = 1, b_2 = 3, c_2 = 5$

We know that, vector perpendicular to line, only when

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$
$$= (3)(1) + (-1)(3) + (0)(5)$$
$$= 3 - 3 + 0$$
$$= 0$$

SECTION B

23. $$f(x) = \frac{4\sin x}{2+\cos x} - x$$

d.w.r to x

$$f'(x) = \frac{(2+\cos x)\frac{d}{dx}(4\sin x) - (4\sin x)\frac{d}{dx}(2+\cos x)}{(2+\cos x)^2} - \frac{dx}{dx}$$

$$= \frac{(2+\cos x)(4\cos x) - (4\sin x)(0-\sin x)}{(2+\cos)^2} - 1$$

$$= \frac{8\cos x + 4\cos^2 x + 4\sin^2 x}{(2+\operatorname{co} x)^2} - 1$$

$$= \frac{8\cos x + 4(\cos^2 x + \sin^2 x)}{(2+\cos x)^2} - 1$$

$$= \frac{8\cos x + 4(1)}{(2+\cos x)^2} - 1$$

$$= \frac{8\cos x + 4 - 4\cos^2 x - 4\cos x}{(2+\cos x)^2}$$

$$= \frac{4\cos x - \cos^2 x}{(2+\cos x)^2}$$

$$= \frac{\cos x\,(4-\cos x)}{(2+\cos x)^2}$$

Here, $(2+\cos x)^2 > 0$

$\cos x \geq 0, x \in \left[0, \frac{\pi}{2}\right]$

Hence, $\quad f(x) = \frac{4\sin x}{2+\cos x} - x$ is an increasing function of x in $\left[0, \frac{\pi}{2}\right]$

25. Let, $I = \int_{-\frac{1}{2}}^{\frac{1}{2}} \cos x.\log\left(\frac{1+x}{1-x}\right)dx \quad$ ...(i)

$$\left[\text{using property } \int_{-a}^{b} f(x) = \int_{-a}^{b} f(a+b-x)dx\right]$$

$$I = \int_{-\frac{1}{2}}^{\frac{1}{2}} \cos\left(-\frac{1}{2}+\frac{1}{2}-x\right).\log\left[\frac{1+\left(-\frac{1}{2}+\frac{1}{2}-x\right)}{1-\left(-\frac{1}{2}+\frac{1}{2}-x\right)}\right]dx$$

$$= \int_{-\frac{1}{2}}^{\frac{1}{2}} \cos(-x).\log\left(\frac{1-x}{1+x}\right)dx$$

$$I = \int_{-\frac{1}{2}}^{\frac{1}{2}} \cos x \log\left(\frac{1-x}{1+x}\right)dx \quad ...(ii)$$

On adding eq. (i) and eq (ii)

$$I + I = \int_{-\frac{1}{2}}^{\frac{1}{2}} \cos x \log\left(\frac{1-x}{1+x}\right)dx + \cos x \log\left(\frac{1-x}{1+x}\right)dx$$

$$2I = \int_{-\frac{1}{2}}^{\frac{1}{2}} \cos x\left[\log\left(\frac{1+x}{1-x}\right)+\log\left(\frac{1-x}{1+x}\right)\right]dx$$

[using $\log m + \log n = \log mn$]

$$2I = \int_{-\frac{1}{2}}^{\frac{1}{2}} \cos x \log\left(\frac{1+x}{1-x}\right)\left(\frac{1-x}{1+x}\right)dx$$

$$2I = \int_{-\frac{1}{2}}^{\frac{1}{2}} \cos x \log 1 dx$$

$$2I = \int_{-\frac{1}{2}}^{\frac{1}{2}} \cos x\,(0)\,dx \quad [\log 1 = 0]$$

$2I = 0$

$I = 0$

26. Given, $x^y + y^x = a^b$

$e^{\log x^y} + e^{\log y^x} = a^b$ [we know that: $e^{\log m} = m$]

$e^{y\log x} + e^{x\log y} = a^b$ [$\log m^n = n \log m$]

d.w.r to x

$$e^{y\log x}\left[y\times\frac{1}{x}+\log x\frac{dy}{dx}\right]+e^{x\log y}\left[x\times\frac{1}{y}\frac{dy}{dx}+\log y\times 1\right] = \frac{d}{dx}a^b$$

$$e\log x^y\left[\frac{y}{x}+\log x\frac{dy}{dx}\right]+e^{\log y^x}\left[\frac{x}{y}\frac{dy}{dx}+\log y\right] = 0$$

$$x^y\left[\frac{y}{x}+\log x\frac{dy}{dx}\right]+y^x\left[\frac{x}{y}\frac{dy}{dx}+\log y\right] = 0$$

$$yx^{y-1}+x^y\log x\frac{dy}{dx}+xy^{x-1}\frac{dy}{dx}+y^x\log y = 0$$

$$(x^y\log x+xy^{x-1})\frac{dy}{dx} = -(y^x\log y+yx^{y-1})$$

$$\frac{dy}{dx} = \frac{-(y^x\log y+yx^{y-1})}{(x^y\log x+xy^{x-1})}$$

28. $$I = \int\frac{2x+3}{x^2(x+3)}dx \quad ...(i)$$

$$\frac{2x+3}{x^2(x+3)} = \frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+3} \quad ...(ii)$$

$$\frac{2x+3}{x^2(x+3)} = \frac{A(x)(x+3)+B(x+3)+C(x^2)}{x^2(x+3)}$$

$2x + 3 = Ax(x + 3) + B(x + 3) + Cx^2$

Put $x = 0$

$2(0) + 3 = A(0)(0 + 3) + B(0 + 3) + C(0)^2$

$3 = 0 + 3B + 0$

$B = 1$

Put $x = -3$

$2(-3) + 3 = A(-3)(-3 + 3) + B(-3 + 3) + C(-3)^2$

$-3 = 0 + 0 + 9C$

$C = -\frac{1}{3}$

Put $x = 1$

$2(1) + 3 = A(1)(1 + 3) + B(1 + 3) + C(1)^2$

$5 = 4A + 4B + C$

$5 = 4A + 4(1) + \left(-\frac{1}{3}\right)$

$4A = 5-4+\frac{1}{3}$

$4A = 1+\frac{1}{3}$

$4A = \frac{4}{3}$

$A = \frac{1}{3}$

value of A, B and C put in eq (ii)

$$\frac{2x+3}{x^2(x+3)} = \frac{1}{3x}+\frac{1}{x^2}-\frac{1}{3(x+3)} \quad ...(iii)$$

from eq (i) and eq (iii)

$$I = \int\left(\frac{1}{3x}+\frac{1}{x^2}-\frac{1}{3(x+3)}\right)dx$$

$$I = \frac{1}{3}\int\frac{1}{x}dx+\int\frac{1}{x^2}dx-\frac{1}{3}\int\frac{1}{x+3}dx$$

$$I = \frac{1}{3}\log x+\frac{x^{-1}}{-1}-\frac{1}{3}\log(x+3)+C$$

$$= \frac{1}{3}\log x-\frac{1}{x}-\frac{1}{3}\log(x+3)+C$$

30. Maximise $Z = x + 3y$

subject to the constraints:

$x + 2y \leq 200$

$x + y \leq 150$

$y \leq 75$

$x, y \geq 0$

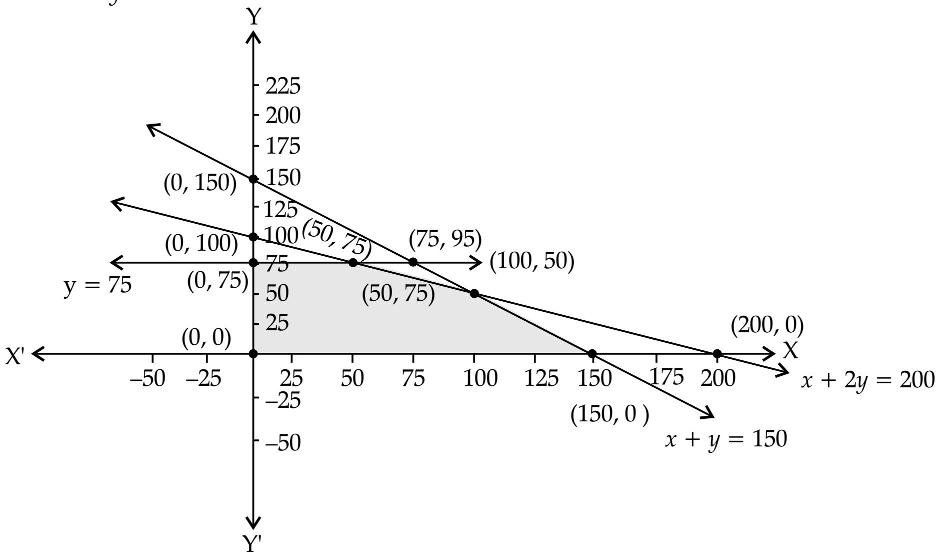
Now, $x + 2y = 200$

x	0	200	100	50
y	100	0	50	75

$x + y = 150$

x	0	150	75	100
y	150	0	75	50

$y = 75$



Corner point	$z = x + 3y$
(0, 0)	$0 + 3(0) = 0$
(0, 75)	$0 + 3 \times 75 = 225$
(50, 75)	$50 + 3 \times 75 = 275 \Rightarrow$ (Maximise)
(100, 50)	$100 + 3 \times 50 = 250$
(150, 0)	$150 + 3 \times 0 = 150$

Maximum value of $z = 275$ at $x = 50, y = 75$

34. Let $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$

and $B = \begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 0-14+21 & 1+14-15 & 2-14+12 \\ 0-14+14 & 3+14-10 & 6-14+8 \\ 0+7-7 & 2-7+5 & 4+7-4 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

$$= 7\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$AB = 7I$$

$$\frac{1}{7}(AB) = I$$

$$\frac{1}{7}B = A^{-1}$$

$$A^{-1} = \frac{1}{7}\begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}$$

Now, The given system of equation is

$$x + 2y - 3z = 6$$
$$3x + 2y - 2z = 3$$
$$2x - y + z = 2$$

$$\begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$$

$$AX = C$$
$$X = A^{-1}C$$

$$X = \frac{1}{7}\begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}\begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$$

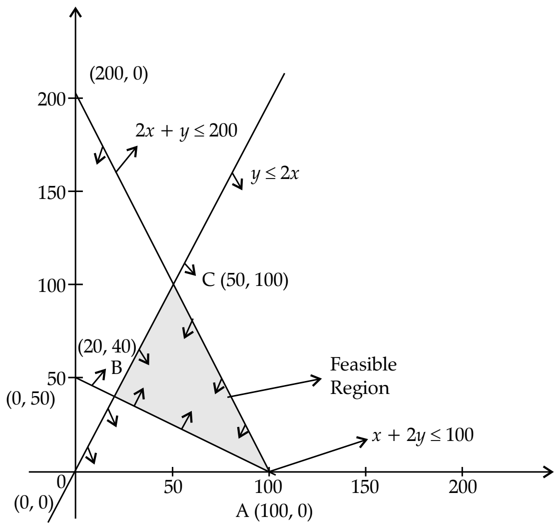
[from eq 1]

$$X = \frac{1}{7}\begin{bmatrix} 0+3+4 \\ -42+21-14 \\ -42+15-8 \end{bmatrix}$$

$$X = \frac{1}{7}\begin{bmatrix} 7 \\ -35 \\ -35 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ -5 \end{bmatrix}$$

$\Rightarrow x = 1, y = -5, z = -5,$



Outside Delhi Set-3 **65/4/3**

SECTION A

4. Option (C) is correct.

Explanation: Given

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

Let $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3\times3+1\times-1 & 3\times1+1\times2 \\ -1\times3+2\times-1 & -1\times1+2\times2 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

Now $A^2 + 7C = kA$

$$\Rightarrow \quad = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = k\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\Rightarrow \quad = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} = k\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\Rightarrow \quad = 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = k\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\Rightarrow \quad k = 5$$

5. Option (B) is correct.

Explanation: Given

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$$

$$B = \frac{1}{3}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & \lambda \end{bmatrix}$$

Here, $AB = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \times \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & \lambda \end{bmatrix} \times \frac{1}{3}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2-9+12 & 0-2+2 & 1+3+2\lambda \\ 18-18 & 0+4-3 & 0-6-3\lambda \\ -6-18+24 & 0-4+4 & 3+6+4\lambda \end{bmatrix} \times \frac{1}{3}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1+3+2\lambda \\ 0 & 1 & -6-3\lambda \\ 0 & 0 & 3+6+4\lambda \end{bmatrix} \times \frac{1}{3}$$

After equating $\quad 0 = \dfrac{1+3+2\lambda}{3}$

$$\lambda = -2$$

6. Option (A) is correct.

Explanation:

Let $\quad y = x^2$

& $\quad z = x^3$

then $\quad \dfrac{dy}{dx} = 2x \quad$ ...(i)

& $\quad \dfrac{dz}{dx} = 3x^2 \quad$ ...(ii)

(i) ÷ (ii)

$$\frac{dy}{dz} = \frac{2x}{3x^2} = \frac{2}{3x}$$

$$\Rightarrow \quad = \frac{d(x^2)}{d(x^3)} = \frac{\left(\dfrac{dx^2}{dx}\right)}{\left(\dfrac{dx^3}{dx}\right)} = \frac{2}{3x}$$

7. Option (A) is correct.

Explanation: $f(x) = |x| + |x-2|$

To break modulus

Put $\quad x = 0$

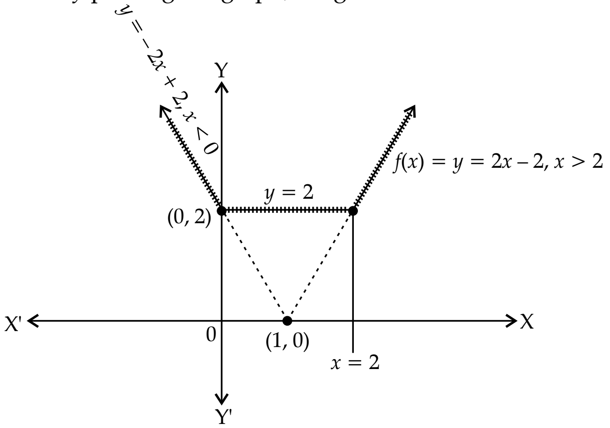
& $\quad x-2 = 0$

$x = 0, 2$

$$\xleftrightarrow[\quad 0 \qquad 2 \quad]{}$$

Now $f(x) = \begin{cases} -x-x+2=-2x+2 & ; \quad x<0 \\ x-x+2=2 & ; \quad 0 \le x \le 2 \\ x+x-2=2x-2 & ; \quad x>2 \end{cases}$

by plotting the graph, we get



so $f(x)$ is continuous everywhere but not differentiable at $x = 0$ and $x = 2$

8. Option (B) is correct.

***Explanation*:**

$$\int_0^{\pi} \tan^2\left(\frac{\theta}{3}\right) d\theta = \int_0^{\pi}\left[\sec^2\left(\frac{\theta}{3}\right)-1\right].d\theta$$

$$= \int_0^{\pi} \sec^2\left(\frac{\theta}{3}\right) d\theta - \int_0^{\pi} d\theta$$

$$= \int_0^{\pi} \sec^2\left(\frac{\theta}{3}\right).d\theta - \int_0^{\pi} d\theta$$

$$= \left[\frac{\tan\frac{\theta}{3}}{\frac{1}{3}}\right]_0^{\pi} - [\theta]_0^{\pi}$$

$$= \left(3\tan\frac{\pi}{3} - 3\tan\frac{0}{3}\right) - (\pi - 0)$$

$$= \left(3\times\sqrt{3} - \pi\right)$$

9. Option (B) is correct.

***Explanation*:** Compare this with $\frac{dy}{dx}+\frac{2y}{x}=0$

$$\frac{dy}{dx}+Py = Q$$

we get $P = \frac{2}{x}$

and $Q = 0$

Integrating factor I.F.

$$= e^{\int P.dx}$$

$$= e^{\int \frac{2}{x}.dx}$$

$$= e^{2lnx}$$

$$= e^{lnx^2}$$

$$I.F. = x^2$$

14. Option (B) is correct.

***Explanation*:** Here,

$$\vec{a} = \hat{i} - \hat{j}$$

and $\vec{r} = \hat{i} + \hat{k} + \mu(2\hat{i} - \hat{j})$

Let $\vec{m}$ be a line passing through $\vec{a}$ and parallel to $\vec{r}$

$\Rightarrow \vec{m}$ be a line passing through $\vec{a}$ and parallel to $(2\hat{i} - \hat{j}) = \vec{b}$ (say)

So we know that a line through a point with position vector $\vec{a}$ and parallel to $\vec{b}$ is given by the equation.

$$\vec{m} = \vec{a} + \lambda\vec{b}$$

$$\Rightarrow \left(x\hat{i} + y\hat{j} + z\hat{k}\right) = (\hat{i} - \hat{j}) + \lambda(2\hat{i} - \hat{j})$$

$$= \hat{i}(1+2\lambda) + \hat{j}(-1-\lambda)$$

So its Cartesian equation is

$$\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-0}{0} \quad \text{(After equating)}$$

23. Given $f(x) = \sin x + \cos x$

$f'(x) = \cos x - \sin x$

putting $f'(x) = 0$

$\cos x = \sin x$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$ (for $x \in [0, 2\pi]$)

plotting points

$$\xleftrightarrow[\quad 0 \qquad \frac{\pi}{4} \qquad \frac{5\pi}{4} \qquad 2\pi \quad]{}$$

Here, when

$x \in \frac{\pi}{4}, \frac{5\pi}{4}$

putting $f'(x) = \cos x - \sin x$

at $x = \frac{\pi}{2} \in \left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

$$f'\left(\frac{\pi}{2}\right) = \cos\frac{\pi}{2} - \sin\frac{\pi}{2} = -1 < 0$$

thus $f'(x) < 0$ for $x \in \left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

$\Rightarrow f$ is strictly decreasing

in $x \in \left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ **Hence proved**

25. Let $I = \int \frac{2x.dx}{(x^2+1)(x^2-4)}$

Put $x^2 = y \Rightarrow 2x.dx = dy$

$$I = \int \frac{dy}{(y+1)(y-4)}$$

$$= \int\left(\frac{A}{y+1} + \frac{B}{y-4}\right).dy$$

Let $\frac{A}{(y+1)}+\frac{B}{(y-4)} = \frac{1}{(y+1)(y-4)}$

$\Rightarrow A(y-4) + B(y+1) = 1$

$\Rightarrow (A+B)y - 4A + B = 1 + 0\,y$

After equating, we get

$A + B = 0$ & $B - 4A = 1$

Solving we get

$$A = \frac{-1}{5} \;\&\; B = \frac{1}{5}$$

put value of A & B in I, we get

$$I = \frac{-1}{5}\int\frac{dy}{(y+1)}+\frac{1}{5}\int\frac{dy}{(y-4)}$$

$$= \frac{-1}{5}\log(y+1)+\frac{1}{5}\log(y-4)+C$$

$$= \frac{-1}{5}\log(x^2+1)+\frac{1}{5}\log(x^2-4)+C$$

26. Given $\quad y = \underset{p}{(\cos x)^x} + \underset{q}{\cos^{-1}\sqrt{x}}$ ...(i)

Let $\quad p = (\cos x)^x \;\&\; q = \cos^{-1}\sqrt{x}$

for $\quad p = (\cos x)^x$

Taking log both side

$$\log p = x\log|(\cos x)|$$

Differentiating both side w.r.t. x

$$\frac{1}{p}\frac{dp}{dx} = \frac{x}{\cos x}(-\sin x)+\log|(\cos x)|$$

$$\frac{dp}{dx} = p[\log|(\cos x)| - x\tan x]$$

$$= (\cos x)^x[\log|\cos x| - x\tan x] \quad \text{...(ii)}$$

Also $\quad q = \cos^{-1}\sqrt{x}$ ...(iii)

$$\Rightarrow \frac{dq}{dx} = \frac{-1}{\sqrt{1-(\sqrt{x})^2}}\times\frac{1}{2\sqrt{x}}$$

$$= \frac{-1}{2\sqrt{x}-\sqrt{1-x}}$$

Now $\quad y = (\cos x)^x + \cos^{-1}\sqrt{x}$ (from (i))

$$y = p + q$$

$$\Rightarrow \frac{dy}{dx} = \frac{dp}{dx}+\frac{dq}{dx} \quad \text{(from (ii) \& (iii))}$$

$$\frac{dy}{dx} = (\cos x)^x[\log|\cos x| - x\tan x] - \frac{1}{2\sqrt{x}-\sqrt{1-x}}$$

28. Let $\quad I = \int\sec^3\theta.d\theta$

Integrating by parts, we have

$u = \sec\theta,\; v = \sec^2\theta$

So, $\quad I = \int u.v.d\theta$

$$I = u\int v.d\theta - \int\left(\frac{du}{d\theta}.\int v.d\theta\right).d\theta$$

$$I = \sec\theta\int\sec^2\theta.d\theta - \int\left\{\frac{d(\sec\theta)}{d\theta}.\int\sec^2\theta.d\theta\right\}d\theta$$

$$I = \sec\theta.\tan\theta - \int\sec\theta.\tan\theta.\tan\theta.\,d\theta$$

$$I = \sec\theta.\tan\theta - \int\sec\theta.\tan^2\theta.\,d\theta$$

$$I = \sec\theta.\tan\theta - \int\sec\theta.(\sec^2\theta-1).\,d\theta$$

$$I = \sec\theta.\tan\theta - \int\sec^3\theta + \int\sec\theta.\,d\theta$$

$$I = \sec\theta.\tan\theta - I + \ln|\sec\theta+\tan\theta| + C$$

$$2I = \sec\theta.\tan\theta + \ln|\sec\theta+\tan\theta| + C$$

$$I = \frac{1}{2}[\sec\theta.\tan\theta + \ln|\sec\theta+\tan\theta| + C]$$

30. Given

$$Z = 3x - 4y$$

(i) $Z(A) = Z(0, 8) = 3\times 0 - 8\times 4 = -32$

$Z(B) = Z(4, 10) = 12 - 40 = -28$

$Z(C) = Z(6, 8) = 18 - 32 = -14$

$Z(D) = Z(6, 5) = 18 - 20 = -2$

$Z(E) = Z(4, 0) = 12 - 0 = -12$

So, maximum value of Z = 12

(ii) Given $Z = px + qy$, where $p, q > 0$

Let Z be the maximum value of Z

then it is given, maximum value of z occurs at B (4, 10) and C (6, 8)

$\Rightarrow \quad Z_0 = p.4 + q.10$

$\quad = p.6 + q.8$

$\Rightarrow \quad 4p + 10q = 6p + 8q$

$\Rightarrow \quad 2p = 2q$

$\Rightarrow \quad p = q$

Hence, this is the required condition.

34. Given $\quad A = \begin{bmatrix}1 & 2 & 1\\ 2 & 3 & -1\\ 1 & 0 & 1\end{bmatrix}$

Here, $\quad |A| = 1(3-0) - 2(2+1) + 1(0-3)$

$= 3 - 6 - 3$

$= -6 \neq 0$

So, A^{-1} exists.

Now, $\quad M_{11} = \begin{vmatrix}3 & -1\\ 0 & 1\end{vmatrix} = 3$

$\quad M_{12} = \begin{vmatrix}2 & -1\\ 1 & 1\end{vmatrix} = 3$

$$M_{13} = \begin{vmatrix} 2 & 3 \\ 1 & 0 \end{vmatrix} = -3$$

$$M_{21} = \begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} = 2$$

$$M_{22} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$$

$$M_{23} = \begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} = -2$$

$$M_{31} = \begin{vmatrix} 2 & 1 \\ 3 & -1 \end{vmatrix} = -5$$

$$M_{32} = \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = -3$$

$$M_{33} = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = -1$$

Thus the minor matrix of A

$$= \begin{bmatrix} 3 & 3 & -3 \\ 2 & 0 & -2 \\ -5 & -3 & -1 \end{bmatrix}$$

co-factor matrix of $A = \begin{bmatrix} 3 & -3 & -3 \\ -2 & 0 & 2 \\ -5 & 3 & -1 \end{bmatrix}$

Also, $\quad adj\, A = \begin{bmatrix} 3 & -2 & -5 \\ -3 & 0 & 3 \\ -3 & 2 & -1 \end{bmatrix}$

Therefore, $\quad A^{-1} = \dfrac{adj\, A}{|A|}$

$$= \frac{1}{-6}\begin{bmatrix} 3 & -2 & -5 \\ -3 & 0 & 3 \\ -3 & 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{2} & \frac{1}{3} & \frac{5}{6} \\ \frac{1}{2} & 0 & \frac{-1}{2} \\ \frac{1}{2} & -\frac{1}{3} & \frac{1}{6} \end{bmatrix}$$

Now, Given

$$x + 2y + z = 5$$
$$2x + 3y = 1$$
$$x - y + z = 8$$

writing equation as

$$AX = B$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 0 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 8 \end{bmatrix}$$

i.e., $\quad A'X = B$

$\Rightarrow \quad X = (A')^{-1}B$

$\quad = (A^{-1})'B$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{3} & \frac{5}{6} \\ \frac{1}{2} & 0 & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{3} & \frac{1}{6} \end{bmatrix}\begin{bmatrix} 5 \\ 1 \\ 8 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & 0 & -\frac{1}{3} \\ \frac{5}{6} & \frac{-1}{2} & \frac{1}{6} \end{bmatrix}\begin{bmatrix} 5 \\ 1 \\ 8 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{5}{2} + \frac{1}{2} + \frac{8}{2} \\ \frac{5}{3} + 0 - \frac{8}{3} \\ \frac{25}{6} - \frac{1}{2} + \frac{8}{6} \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}$$

$\therefore \quad x = 2, y = -1, z = 5$