ISC EXAMINATION PAPER - 2024
MATHEMATICS
Class-12th
(Solved)

Maximum Marks: 80
Time allowed: Three hours

*(Candidates are allowed **additional 15 minutes** for **only** reading the paper. They must **NOT** start during this time)*

This Question Paper consists of three sections A, B and C.
*Candidates are required to attempt all questions from **Section A** and all questions **EITHER** from **Section B OR Section C**.*
Section A:** Internal choice has been provided in two questions of two marks each, **two questions of four marks each and two questions of six marks each.
Section B:** Internal choice has been provided in **one question of two marks and one question of four marks.
Section C:** Internal choice has been provided in **one question of two marks and one question of four marks.
All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.
The intended marks for questions or parts of questions are given in brackets [].
Mathematical tables and graph papers are provided

SECTION A - 65 MARKS

Question 1

In subparts (i) to (x) choose the correct options and in subparts (xi) to (xv), answer the questions as instructed.

(i) Let L be a set of all straight lines in a plane. The relation R on L defined as 'perpendicular to' is: **[1]**

(a) Symmetric and Transitive
(b) Transitive
(c) Symmetric
(d) Equivalence

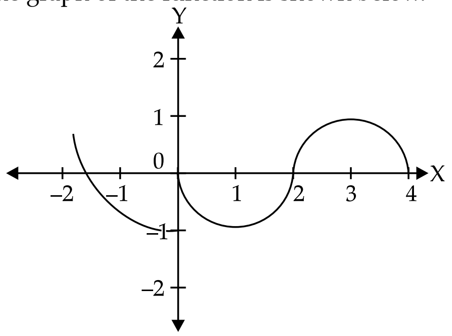
(ii) The order and the degree of differential equation $1+\left(\frac{dy}{dx}\right)^2=\frac{d^2y}{dx^2}$ are: **[1]**

(a) 2 and $\frac{3}{2}$ **(b)** 2 and 3
(c) 3 and 4 **(d)** 2 and 1

(iii) Let A be a non-empty set.
Statement 1: Identity relation on A is Reflexive.
Statement 2: Every Reflexive relation on A is an Identity relation. **[1]**

(a) Both the statements are true.
(b) Both the statements are false.
(c) Statement 1 is true and Statement 2 is false.
(d) Statement 1 is false and Statement 2 is true.

(iv) The graph of the function is shown below. **[1]**



Of the following options, at what values of x is the function / NOT differentiable?

(a) At $x = 0$ and $x = 2$
(b) At $x = 1$ and $x = 3$
(c) At $x = -1$ and $x = 1$
(d) At $x = -1.5$ and $x = 1.5$

(v) The value of $\operatorname{cosec}\left(\sin^{-1}\left(\frac{-1}{2}\right)\right)-\sec\left(\cos^{-1}\left(\frac{-1}{2}\right)\right)$ is equal to: **[1]**

(a) -4 **(b)** 0
(c) -1 **(d)** 4

(vi) The value of $\int_1^{\sqrt{3}}\frac{dx}{1+x^2}$ is: **[1]**

(a) $\frac{\pi}{2}$ **(b)** $\frac{2\pi}{3}$
(c) $\frac{\pi}{6}$ **(d)** $\frac{\pi}{12}$

(vii) Assertion: Let the matrices $A=\begin{pmatrix}-3 & 2\\ -5 & 4\end{pmatrix}$ and $B=\begin{pmatrix}4 & -2\\ 5 & -3\end{pmatrix}$ be such that $A^{100}B = BA^{100}$

Reason: $AB = BA$ implies $A^nB = BA^n$ for all positive integers n. **[1]**

(a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.
(c) Assertion is true and Reason is false.
(d) Assertion is false and Reason is true.

(viii) If $\int(\cot x - \operatorname{cosec}^2 x)e^x dx = e^x f(x) + c$ then $f(x)$ will be: **[1]**

(a) $\cot x + \operatorname{cosec} x$ (b) $\cot^2 x$
(c) $\cot x$ (d) $\operatorname{cosec} x$

(ix) In which one of the following intervals is the function $f(x) = x^3 - 12x$ increasing? **[1]**

(a) $(-2, 2)$ (b) $(-\infty, -2) \cup (2, \infty)$
(c) $(-2, \infty)$ (d) $(-\infty, 2)$

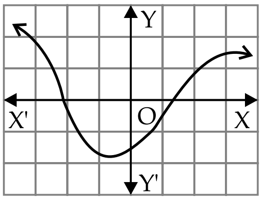
(x) If A and B are symmetric matrices of the same order, then AB – BA is: **[1]**

(a) Skew-symmetric matrix
(b) Symmetric matrix
(c) Diagonal matrix
(d) Identity matrix

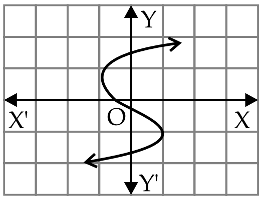
(xi) Find the derivative of $y = \log x + \frac{1}{x}$ with respect to x. **[1]**

(xii) Teena is practising for an upcoming Rifle Shooting tournament. The probability of her shooting the target in the 1st, 2nd, 3rd and 4th shots are 0.4, 0.3, 0.2 and 0.1 respectively. Find the probability of at least one shot of Teena hitting the target. **[1]**

(xiii) Which one of the following graphs is a function of x?



Graph A



Graph B

[1]

(xiv) Evaluate: $\int_0^6 |x+3|\, dx$ **[1]**

(xv) Given that $\frac{1}{y} + \frac{1}{x} = \frac{1}{12}$ and y decreases at a rate of 1 cms^{-1}, find the rate of change of x when $x = 5$ cm and $y = 1$ cm

Question 2 **[2]**

(i) Let $f : \mathrm{R} - \left\{\frac{-1}{3}\right\} \to \mathrm{R} - \{0\}$ be defined as $f(x) = \frac{5}{3x+1}$ is invertible. Find $f^{-1}(x)$

OR

(ii) If $f : R \to R$ is defined by $f(x) = \frac{2x-7}{4}$, show that $f(x)$ is one - one and onto.

Question 3 **[2]**

Find the value of the determinant given below, without expanding it at any stage.

$$\begin{vmatrix} \beta\gamma & 1 & \alpha(\beta+\gamma) \\ \gamma\alpha & 1 & \beta(\gamma+\alpha) \\ \alpha\beta & 1 & \gamma(\alpha+\beta) \end{vmatrix}$$

Question 4 **[2]**

(i) Determine the value of k for which the following function is continuous at $x = 3$.

$$f(x) = \begin{cases} \frac{(x+3)^2 - 36}{x-3} & ; \ x \neq 3 \\ k & ; \ x = 3 \end{cases}$$

OR

(ii) Find a point on the curve $y = (x-2)^2$ at which the tangent is parallel to the line joining the chord through the points (2, 0) and (4, 4).

Question 5 **[2]**

Evaluate: $\int_0^{2\pi} \frac{1}{1+e^{\sin x}} dx$

Question 6 **[2]**

Evaluate: $P(A \cup B)$ if $2P(A) = P(B) = \frac{5}{13}$ and $P(A|B) = \frac{2}{5}$

Question 7 **[4]**

If $y = 3\cos(\log x) + 4\sin(\log x)$, show that

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

Question 8 **[4]**

(i) Solve for x : $\sin^{-1}\left(\frac{x}{2}\right) + \cos^{-1} x = \frac{\pi}{6}$

OR

(ii) If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi$, show that $x^2 - y^2 - z^2 + 2yz\sqrt{1-x^2} = 0$

Question 9 **[4]**

(i) Evaluate: $\int x^2 \cos x \, dx$

OR

(ii) Evaluate: $\int \frac{x+7}{x^2+4x+7} dx$

Question 10 **[4]**

A jewellery seller has precious gems in white and red colour which he has put in three boxes. The distribution of these gems is shown in the table given below:

Box	Number of Gems	
	White	Red
I	1	2
II	2	3
III	3	1

He wants to gift two gems to his mother. So, he asks her to select one box at random and pick out any two gems one after the other without replacement from the selected box. The mother selects one white and one red gem.

Calculate the probability that the gems drawn are from Box II.

Question 11 **[6]**

A furniture factory uses three types of wood namely, teakwood, rosewood and satinwood for manufacturing three types of furniture, that are, table, chair and cot. The wood requirements (in tonnes) for each type of furniture are given below:

	Table	Chair	Cot
Teakwood	2	3	4
Rosewood	1	1	2
Satinwood	3	2	1

It is found that 29 tonnes of teakwood, 13 tonnes of rosewood and 16 tonnes of satinwood are available to make all three types of furniture.

Using the above information, answer the following questions:

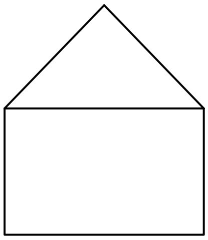
(i) Express the data given in the table above in the form of a set of simultaneous equations.

(ii) Solve the set of simultaneous equations formed in subpart (i) by matrix method.

(iii) Hence, find the number of table(s), chair(s) and cot(s) produced.

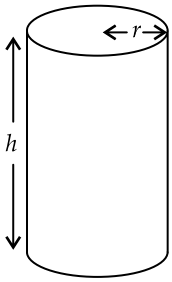
Question 12 **[6]**

(i) Mrs. Roy designs a window in her son's study room so that the room gets maximum sunlight. She designs the window in the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12 m, find the dimensions of the window that will admit maximum sunlight into the room.



OR

(ii) Sumit has bought a closed cylindrical dustbin. The radius of the dustbin is 'r' cm and height is 'h' cm. It has a volume of 20π cm^3.



(a) Express 'h' in terms of 'r', using the given volume.

(b) Prove that the total surface area of the dustbin is $2\pi r^2 + \frac{40\pi}{r}$

(c) Sumit wants to paint the dustbin. The cost of painting the base and top of the dustbin is 2 per cm^2 and the cost of painting the curved side is ₹ 25 per cm^2. Find the total cost in terms of 'r', for painting the outer surface of the dustbin including the base and top.

(d) Calculate the minimum cost for painting the dustbin.

Question 13 **[6]**

(i) Solve the following differential equation:

$$2ye^{x/y}\,dx + \left(y - 2xe^{x/y}\right)dy = 0, \text{ given } x = 0 \text{ and } y = 1$$

OR

(ii) Solve the following differential equation:

$$x(x^2 - 1)\frac{dy}{dx} = 1, y = 0, \text{ given } x = 2$$

Question 14 **[6]**

A primary school teacher wants to teach the concept of 'larger number' to the students of Class II.

To teach this concept, he conducts an activity in his class. He asks the children to select two numbers from a set of numbers given as 2, 3, 4, 5 one after the other without replacement. All the outcomes of this activity are tabulated in the form of ordered pairs given below:

	2	3	4	5
2	(2, 2)	(2, 3)	(2, 4)	
3	(3, 2)	(3, 3)		(3, 5)
4	(4, 2)		(4, 4)	(4, 5)
5		(5, 3)	(5, 4)	(5, 5)

(i) Complete the table given above.

(ii) Find the total number of ordered pairs having one larger number.

(iii) Let the random variable X denote the larger of two numbers in the ordered pair. Now, complete the probability distribution table for X given below.

X	3	4	5
$P(X = x)$			

(iv) Find the value of $P(X < 5)$

(v) Calculate the expected value of the probability distribution.

SECTION B - 15 MARKS

Question 15 **[5]**

In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(i) If $\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 4\hat{j} - 3\hat{k}$ then the value of $|\vec{a} - 2\vec{b}|$ will be:

(a) $\sqrt{85}$ **(b)** $\sqrt{86}$

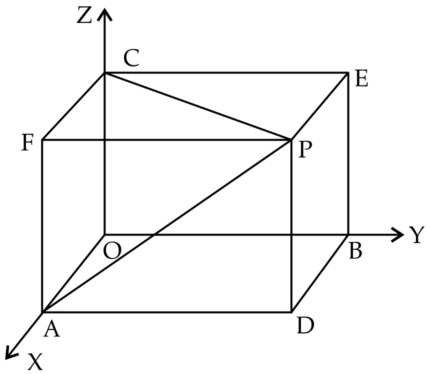
(c) $\sqrt{87}$ **(d)** $\sqrt{88}$

(ii) If a line makes an angle α, β and γ with positive direction of the coordinate axes, then the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ will be:

(a) 1 **(b)** 3

(c) – 2 **(d)** 2

(iii) In the figure. given below, if the coordinates of the point P are (a, b, c), then what are the perpendicular distances of P from XY, YZ and ZX planes respectively?



(iv) If $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 5\hat{i} - 3\hat{j} + \hat{k}$, find the projection of $\vec{b}$ on $\vec{a}$

(v) Find a vector of magnitude 20 units parallel to the vector $2\hat{i} + 5\hat{j} + 4\hat{k}$.

Question 16 **[2]**

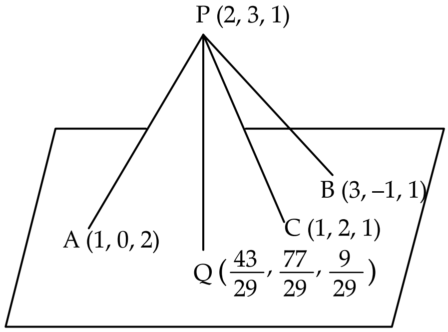
(i) If $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ where $\vec{a}, \vec{b}$ and $\vec{c}$ are non-zero vectors, then prove that either $\vec{b} = \vec{c}$ or $\vec{a}$ and $(\vec{b} - \vec{c})$ are parallel.

OR

(ii) If $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that $|\vec{a} \times \vec{b}| = \vec{a}.\vec{b}$, find the angle between $\vec{a}$ and $\vec{b}$.

Question 17 **[4]**

A mobile tower is situated at the top of a hill. Consider the surface on which the tower stands as a plane having points A(1, 0, 2), B(3, –1, 1) and C(1, 2, 1) on it. The mobile tower is tied with three cables from the points A, B and C such that it stands vertically on the ground. The top of the tower is at point P(2, 3, 1) as shown in the figure below. The foot of the perpendicular from the point P on the plane is at the point $Q\left(\frac{43}{29}, \frac{77}{29}, \frac{9}{29}\right)$



Answer the following questions.

(i) Find the equation of the plane containing the points A, B and C.

(ii) Find the equation of the line PQ.

(iii) Calculate the height of the tower.

Question 18 **[4]**

(i) Using integration, find the area bounded by the curve $y^2 = 4ax$ and the line $x = a$

OR

(ii) Using integration, find the area of the region bounded by the curve $y^2 = 4x$ and $x^2 = 4y$

SECTION C - 15 MARKS

Question 19 **[5]**

In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(i) A company sells hand towels at ₹ 100 per unit. The fixed cost for the company to manufacture hand towels is ₹ 35,000 and variable cost is estimated to be 30% of total revenue. What will be the total cost function for manufacturing hand towels?

(a) $35000 + 3x$ **(b)** $35000 + 30x$

(c) $35000 + 100x$ **(d)** $35000+10x$

(ii) If the correlation coefficient of two sets of variables (X, Y) is $\frac{-3}{4}$, which one of the following statements is true for the same set of variables?

(a) Only one of the two regression lines has a negative coefficient.

(b) Both regression coefficients are positive.

(c) Both regression coefficients are negative.

(d) One of the lines of regression is parallel to the x-axis.

(iii) If the total cost function is given by $C = x + 2x^3 - \frac{7}{2}x^2$, find the Marginal Average Cost function (MAC).

(iv) The equations of two lines of regression are $4x + 3y + 7 = 0$ and $3x + 4y + 8 = 0$. Find the mean value of x and y.

(v) The manufacturer of a pen fixes its selling price at ₹ 45, and the cost function is $C(x) = 30x + 240$. The manufacturer will begin to earn profit if he sells more than 16 pens. Why? Give one reason.

Question 20 **[2]**

(i) The Average Cost function associated with producing and marketing x units of an item is given by $AC = x + 5 + \frac{36}{x}$

(a) Find the Total Cost function.

(b) Find the range of values of x for which Average Cost is increasing.

OR

(ii) A monopolist's demand function is $x = 60 - \frac{p}{5}$. At what level of output will marginal revenue be zero?

Question 21 **[4]**

(i) XYZ company plans to advertise some vacancies. The Manager is asked to suggest the monthly salary for these vacancies based on the years of experience. To do so, the Manager studies the years of service and the monthly salary drawn by the existing employees in the company.

Following is the data that the Manager refers to:

Years of service (X)	11	7	9	5	8	6	10
Monthly salary (in ₹ 1000) (Y)	10	8	6	5	9	7	11

(a) Find the regression equation of monthly salary on the years of service.

(b) If a person with 13 years of experience applies for a job in this company, what monthly salary will be suggested by the Manager?

OR

The line of regression of marks in Statistics (X) and marks in Accountancy (Y) for a class of 50 students is $3y - 5x + 180 = 0$. The average score in Accountancy is 44 and the variance of marks in Statistics is $\left(\frac{9}{16}\right)^{th}$ of variance of marks in Accountancy.

(a) Find the average score in Statistics.

(b) Find the coefficient of correlation between marks in Statistics and marks in Accountancy.

Question 22 **[4]**

Aman has ₹ 1500 to purchase rice and wheat for his grocery shop. Each sack of rice and wheat costs ₹ 180 and ₹ 120 respectively. He can store a maximum number of 10 bags in his shop. He will earn a profit of ₹ 11 per bag of rice and ₹ 9 per bag of wheat.

(i) Formulate a Linear Programming Problem to maximise Aman's profit.

(ii) Calculate the maximum profit.

SOLUTIONS

SECTION A

1. (i) Option (c) is correct.

Explanation: $L_1 \perp L_2$, $L_2 \perp L_3$ then $L_1 \parallel L_3$

$\therefore$ relation is not transitive

$L_1 \perp L_2$ then $L_2 \perp L_1$

$\therefore$ relation is symmetric.

(ii) Option (d) is correct.

Explanation: Highest derivative of D.E. is $\left(\dfrac{d^2y}{dx^2}\right)^1$

Hence, order and degree are 2 and 1 respectively.

(iii) Option (c) is correct.

Explanation: Every identity relation on a non empty set A is a reflexive relation but not conversely.

(iv) Option (a) is correct.

Explanation: Function is not differentiable.

at $x = 0$, $x = 2$ and $x = 4$

(v) Option (b) is correct.

Explanation: $\text{cosec}\left(\sin^{-1}\left(-\frac{1}{2}\right) - \sec\left(\cos^{-1}\left(-\frac{1}{2}\right)\right)\right)$

$$= \text{cosec}\left(\sin^{-1}\left(-\sin\frac{\pi}{6}\right)\right) - \sec\left(\cos^{-1}\left(\cos\frac{2\pi}{3}\right)\right)$$

$$= \text{cosec}\left(-\frac{\pi}{6}\right) - \sec\left(\frac{2\pi}{3}\right)$$

$$= -\text{cosec}\frac{\pi}{6} + \sec\frac{\pi}{3}$$

$$= -2 + 2 = 0$$

(vi) Option (d) is correct.

Explanation: $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$

$$= \left[\tan^{-1}x\right]_1^{\sqrt{3}}$$

$$= \tan^{-1}\sqrt{3} - \tan^{-1}1$$

$$= \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$$

(vii) Option (a) is correct.

Explanation: $AB = \begin{bmatrix} -3 & 2 \\ -5 & 4 \end{bmatrix}\begin{bmatrix} 4 & -2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$

$$BA = \begin{bmatrix} 4 & -2 \\ 5 & -3 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ -5 & 4 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$$

Reason, $AB = BA$

$A.A.B = A.B.A$

{$\therefore$ Both side multiply by A}

$A^2B = B.A.A$ {$AB = BA$}

$A^2B = BA^2$

So, $A^nB = BA^n$

Hence, Reason is true and correct explanation of Assertion

(viii) Option (c) is correct.

Explanation: $\int(\cot x - \text{cosec}^2 x)e^x dx$

$f(x) = \cot x$ then $f'(x) = -\text{cosec}^2 x$

$\int e^x f(x) + f'(x) dx = e^x f(x) + c$

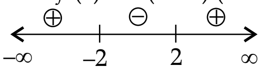
$\therefore \quad f(x) = \cot x$

(ix) Option (b) is correct.

Explanation: $f(x) = x^3 - 12x$

$f'(x) = 3x^2 - 12$

$f'(x) = 3(x-2)(x+2)$



Increasing function $(-\infty, -2) \cup (2, \infty)$

(x) Option (a) is correct.

Explanation: Let A and B be symmetric matrices then $A = A^t$ and $B = B^t$

$$(AB - BA)^t = (AB)^t - (BA)^t$$
$$= (B^tA^t) - (A^tB^t)$$
$$= BA - AB$$
$$= -(AB - BA)$$

Hence it is skew-symmetric.

(xi) $y = \log x + \frac{1}{x}$

$$\frac{dy}{dx} = \frac{d}{dx}\log x + \frac{d}{dx}\frac{1}{x}$$

$$\frac{dy}{dx} = \frac{1}{x} - \frac{1}{x^2} = \frac{x-1}{x^2}$$

(xii) Probability of at least one hitting the target

$= 0.4 \times 0.3 \times 0.2 \times 0.1$

$= 0.0024$

Probability of not hitting the target is

$= (1 - 0.4)(1 - 0.3)(1 - 0.2)(1 - 0.1)$

$= 0.6 \times 0.7 \times 0.8 \times 0.9$

$= 0.336 \times 0.9$

$= 0.3024$

Probability of at least one shot hitting the target is

$1 - 0.3024 = 0.6976$

(xiii) A relation is a function if each input x is associated with exactly one output. This means that for each value of x, there must be only one corresponding value of y. so Graph A is a function but Graph B is not a function because at $x = 0$ then we get three value of y.

(xiv) $\int_0^6 |x+3|$

$f(x) = |x + 3|$

$f(x) = x + 3, \quad x > -3$

$$\therefore \quad \int_0^6 |x+3|\,dx = \int_0^6 (x+3)dx = \left[\frac{x^2}{2} + 3x\right]_0^6$$

$= 18 + 18 - 0 - 0$

$= 36$

(xv)

$$\frac{1}{y}+\frac{1}{x} = \frac{1}{12}$$

$$\frac{d}{dt}\frac{1}{y}+\frac{d}{dt}\frac{1}{x} = \frac{d}{dt}\frac{1}{12}$$

$$-\frac{1}{y^2}\frac{dy}{dt}-\frac{1}{x^2}\frac{dx}{dt} = 0$$

$$\frac{1}{x^2}\frac{dx}{dt} = -\frac{1}{y^2}\frac{dy}{dt}$$

$$\frac{dx}{dt} = -\frac{x^2}{y^2}\frac{dy}{dt}$$

$$\frac{dx}{dt} = -\frac{25}{1}(-1) = 25 \text{ cm/s}$$

2. (i)

$$f(x) = \frac{5}{3x+1}$$

$$y = \frac{5}{3x+1}$$

$$3xy + y = 5$$

$$x = \frac{5-y}{3y}$$

$$f^{-1}(x) = \frac{5-x}{3x}$$

OR

(ii)

$$f(x_1) = f(x_2)$$

$$\frac{2x_1-7}{4} = \frac{2x_2-7}{4}$$

$$2x_1 - 7 = 2x_2 - 7$$

$$x_1 = x_2$$

$\therefore$ $f(x)$ is one-one

onto $\quad f(x) = y$

$$\frac{2x-7}{4} = y$$

$$2x - 7 = 4y$$

$$x = 2y - \frac{7}{2}$$

For any value of y co-domain, we can find the domain x.

3.

$$A = \begin{vmatrix} \beta\gamma & 1 & \alpha(\beta+\gamma) \\ \gamma\alpha & 1 & \beta(\gamma+\alpha) \\ \alpha\beta & 1 & \gamma(\alpha+\beta) \end{vmatrix}$$

$C_1 : C_1 + C_3$

$$A = \begin{vmatrix} \beta\gamma+\alpha\beta+\alpha\gamma & 1 & \alpha(\beta+\gamma) \\ \gamma\alpha+\beta\gamma+\beta\alpha & 1 & \beta(\gamma+\alpha) \\ \alpha\beta+\gamma\alpha+\gamma\beta & 1 & \gamma(\alpha+\beta) \end{vmatrix}$$

$$A = (\alpha\beta+\beta\gamma+\gamma\alpha)\begin{vmatrix} 1 & 1 & \alpha(\beta+\gamma) \\ 1 & 1 & \beta(\gamma+\alpha) \\ 1 & 1 & \gamma(\alpha+\beta) \end{vmatrix}$$

$$A = (\alpha\beta + \beta\gamma + \gamma\alpha) \times 0 = 0 \quad (C_1 = C_2)$$

4. (i)

$$f(x) = \frac{(x+3)^2-36}{x-3}$$

$$\lim_{x\to 3} f(x) = \lim_{x\to 3}\frac{(x+3)^2-36}{x-3}$$

$$= \lim_{x\to 3}\frac{(x+3+6)(x+3-6)}{(x-3)}$$

$$= \lim_{x\to 3}(x+9) \qquad (x \neq 3)$$

$$= 12$$

$f(x)$ is continuous at $x = 3$

$\therefore \quad k = 12$

OR

(ii)

$$y = (x-2)^2$$

$$\frac{dy}{dx} = 2(x-2)$$

$$\left(\frac{dy}{dx}\right)_{(x_1, y_1)} = 2(x_1 - 2)$$

$$\text{Slope of chord} = \frac{y_2-y_1}{x_2-x_1} = \frac{4-0}{4-2} = 2$$

Slope of tangent = slope of chord

$$2(x_1 - 2) = 2$$

$\therefore \quad x_1 = 3$

Required point (3, 1)

5.

$$I = \int_0^{2\pi}\frac{1}{1+e^{\sin x}}dx \qquad \text{...(i)}$$

$$I = \int_0^{2\pi}\frac{1}{1+e^{\sin(2\pi-x)}}dx$$

$$I = \int_0^{2\pi}\frac{1}{1+e^{-\sin x}}dx$$

$$I = \int_0^{2\pi}\frac{e^{\sin x}}{1+e^{\sin x}}dx \qquad \text{...(ii)}$$

From (i) & (ii)

$$2I = \int_0^{2\pi}\frac{1+e^{\sin x}}{1+e^{\sin x}}dx$$

$$= \int_0^{2n} dx = [x]_0^{2\pi}$$

$$2I = 2\pi - 0$$

$$I = \pi$$

6.

$$P(A|B) = \frac{P(A\cap B)}{P(B)}$$

$$\frac{2}{5} = \frac{P(A\cap B)}{\frac{5}{13}}$$

$$\frac{2}{5}\times\frac{5}{13} = \frac{2}{13} = P(A\cap B)$$

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$= \frac{5}{26}+\frac{5}{13}-\frac{2}{13}$$

$$= \frac{5}{26}+\frac{3}{13}$$

$$= \frac{5+6}{26} = \frac{11}{26}$$

7.
$$y = 3\cos(\log x) + 4\sin(\log x)$$
$$\frac{dy}{dx} = -3\sin(\log x)\frac{d}{dx}\log x + 4\cos(\log x)\frac{d}{dx}\log x$$
$$\frac{dy}{dx} = \frac{-3\sin(\log x)}{x} + \frac{4\cos(\log x)}{x}$$
$$\frac{xdy}{dx} = -3\sin(\log x) + 4\cos(\log x)$$
$$\frac{xd^2y}{dx^2} + \frac{dy}{dx} = -\frac{\cos(\log x)}{x} - \frac{4\sin(\log x)}{x}$$
$$\frac{x^2d^2y}{dx^2} + \frac{xdy}{dx} = -(3\cos(\log x) + 4\sin(\log x))$$
$$\frac{x^2d^2y}{dx^2} + \frac{xdy}{dx} + y = 0$$
Hence Proved

8.
$$\cos^{-1}x + \sin^{-1}\frac{x}{2} = \frac{\pi}{6}$$
$$\frac{\pi}{2} - \sin^{-1}x + \sin^{-1}\frac{x}{2} = \frac{\pi}{6}$$
$$-\sin^{-1}x + \sin^{-1}\frac{x}{2} = \frac{\pi}{6} - \frac{\pi}{2}$$
$$\sin^{-1}\frac{x}{2} = -\frac{\pi}{3} + \sin^{-1}x$$
$$\frac{x}{2} = \sin\left(\sin^{-1}x - \frac{\pi}{3}\right)$$
$$\frac{x}{2} = \sin\sin^{-1}x\cos\frac{\pi}{3} - \cos\sin^{-1}x\sin\frac{\pi}{3}$$
$$\frac{x}{2} = x\times\frac{1}{2} - \sqrt{1-x^2}\frac{\sqrt{3}}{2}$$
$$x = x - \sqrt{3}\sqrt{1-x^2}$$
$$\sqrt{3}\sqrt{1-x^2} = 0$$
$$3(1-x^2) = 0$$
$$x^2 = 1$$
$$\Rightarrow \quad x = \pm 1$$

$x = -1$ is not satisfied the equation

$\therefore \quad x = 1$

OR

$$\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$$
$$\sin^{-1}x + \sin^{-1}y = \pi - \sin^{-1}z$$
$$\sin^{-1}(x\sqrt{1-y^2} + y(\sqrt{1-x^2}) = \pi - \sin^{-1}z$$
$$x\sqrt{1-y^2} + y\sqrt{1-x^2} = \sin(\pi - \sin^{-1}z)$$
$$x\sqrt{1-y^2} + y\sqrt{1-x^2} = \sin\pi\cos\sin^{-1}z - \cos\pi\sin\sin^{-1}z$$
$$x\sqrt{1-y^2} + y\sqrt{1-x^2} = z$$
$$x\sqrt{1-y^2} = z - y\sqrt{1-x^2}$$

Squaring both sides

$$x^2(1-y^2) = z^2 + y^2(1-x^2) - 2yz\sqrt{1-x^2}$$
$$x^2 - x^2y^2 = z^2 + y^2 - x^2y^2 - 2yz\sqrt{1-x^2}$$
$$x^2 - y^2 - z^2 + 2yz\sqrt{1-x^2} = 0$$
Hence Proved

9. $\int x^2\cos x\,dx$

$$= x^2\int\cos x\,dx - \int\left[\frac{d}{dx}x^2\int\cos x\,dx\right]dx$$
$$= x^2\sin x - \int 2x\sin x\,dx$$
$$= x^2\sin x - 2\left[-x\cos x - \int 1(-\cos x)dx\right]$$
$$= x^2\sin x + 2x\cos x - \sin x + c$$

OR

$$\int\frac{x+7}{x^2+4x+7}dx$$
$$= \int\frac{\frac{1}{2}(2x+4)+5}{x^2+4x+7}dx$$
$$= \frac{1}{2}\int\frac{2x+4}{x^2+4x+7}dx + \int\frac{5}{x^2+4x+7}dx$$
$$= \frac{1}{2}\log|x^2+4x+7| + \int\frac{5}{(x+2)^2+3}$$
$$\left[\int\frac{f'(x)}{f(x)}dx = \log|f(x)|\right]$$
$$= \frac{1}{2}\log|x^2+4x+7| + \frac{5}{\sqrt{3}}\tan^{-1}\left|\frac{x+2}{\sqrt{3}}\right| + C$$

10. Let E_1, E_2, E_3 and A the events of selecting the box I, II, III and 1 white and 1 red gem.

$$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$
$$P\left(\frac{A}{E_1}\right) = \frac{{}^1C_1\times{}^2C_1}{{}^3C_2} = \frac{1\times 2}{3} = \frac{2}{3}$$
$$P\left(\frac{A}{E_2}\right) = \frac{{}^2C_1\times{}^3C_1}{{}^5C_2} = \frac{2\times 3}{\frac{5\times 4}{2}} = \frac{3}{5}$$
$$P\left(\frac{A}{E_3}\right) = \frac{{}^3C_1\times{}^1C_1}{{}^4C_2} = \frac{3\times 1}{\frac{4\times 3}{2}} = \frac{1}{2}$$
$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)\times P\left(\frac{A}{E_2}\right)}{P(E_1)\times P\left(\frac{A}{E_1}\right) + P(E_2)\times P\left(\frac{A}{E_2}\right) + P(E_3)\times P\left(\frac{A}{E_3}\right)}$$
$$= \frac{\frac{1}{3}\times\frac{3}{5}}{\frac{1}{3}\times\frac{2}{3} + \frac{1}{3}\times\frac{3}{5} + \frac{1}{3}\times\frac{1}{2}}$$

$$= \frac{\frac{3}{5}}{\frac{2}{3}+\frac{3}{5}+\frac{1}{2}}$$

$$= \frac{\frac{3}{5}}{\frac{20+18+15}{30}} = \frac{3}{5}\times\frac{30}{53}$$

Required probability $= \frac{18}{53}$

11. **(i)** Let the number of table, chair and cot be x, y and z respectively

$$2x + 3y + 4z = 29$$
$$x + y + 2z = 13$$
$$3x + 2y + z = 16$$

(ii) Equations can be rearranged in matrix form

$$\begin{bmatrix} 2 & 3 & 4 \\ 1 & 1 & 2 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 29 \\ 13 \\ 16 \end{bmatrix}$$

$$AX = B \qquad \text{Where } A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 1 & 2 \\ 3 & 2 & 1 \end{bmatrix}$$

$$A^{-1}AX = A^{-1}B \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 29 \\ 13 \\ 16 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 3 & 4 \\ 1 & 1 & 2 \\ 3 & 2 & 1 \end{vmatrix}$$

$$= 2\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} - 3\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} + 4\begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix}$$

$$= -6 + 15 - 4 = 5$$

$C_{11} = (1-4) = -3$ | $C_{21} = -(3-8) = 5$
$C_{12} = -(1-6) = 5$ | $C_{22} = (2-12) = -10$
$C_{13} = (2-3) = -1$ | $C_{23} = -(4-9) = 5$
$C_{31} = (6-4) = 2$, $C_{32} = -(4-4) = 0$, $C_{33} = 2-3 = -1$

$$\text{Adj } A = \begin{bmatrix} -3 & 5 & -1 \\ 5 & -10 & 5 \\ 2 & 0 & -1 \end{bmatrix}^t$$

$$= \begin{bmatrix} -3 & 5 & 2 \\ 5 & -10 & 0 \\ -1 & 5 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj } A}{|A|} = \frac{1}{5}\begin{bmatrix} -3 & 5 & 2 \\ 5 & -10 & 0 \\ -1 & 5 & -1 \end{bmatrix}$$

$$X = A^{-1}B$$

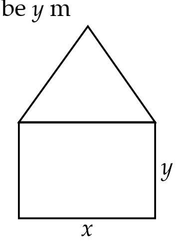
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -3 & 5 & 2 \\ 5 & -10 & 0 \\ -1 & 5 & -1 \end{bmatrix}\begin{bmatrix} 29 \\ 13 \\ 16 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 10 \\ 15 \\ 20 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

$x = 2, y = 3, z = 4$

(iii) No. of chairs = 2
No of tables = 3
No, of Cots = 4

12. Let the length of window be x m and breadth of rectangle part be y m



$$\text{Perimeter} = 12 \text{ m}$$
$$3x + 2y = 12$$
$$y = 6 - \frac{3x}{2} \qquad \text{...(i)}$$

Area of window

$$A = xy + \frac{\sqrt{3}}{4}x^2$$

$$A = x\left(6 - \frac{3x}{2}\right) + \frac{\sqrt{3}}{4}x^2$$

$$A = 6x - \frac{3x^2}{2} + \frac{\sqrt{3}}{4}x^2$$

$$\frac{dA}{dx} = 6 + \left(\frac{-3}{2} + \frac{\sqrt{3}}{4}\right)2x$$

for maxima/min. $\frac{dA}{dx} = 0$

$$6 + \left(\frac{\sqrt{3}}{4} - \frac{3}{2}\right)2x = 0$$

$$\left(\frac{\sqrt{3}-6}{4}\right)x = -3$$

$$x = \frac{12}{6-\sqrt{3}} = \frac{12(6+\sqrt{3})}{36-3}$$

$$= \frac{4}{11}(6+\sqrt{3})$$

$$\frac{d^2A}{dx^2} = 0 + 2\left(\frac{\sqrt{3}}{4} - \frac{3}{2}\right) < 0$$

Hence Area is maximum at $x = \frac{4}{11}(6+\sqrt{3})$

$$y = 6 - \frac{3x}{2}$$

$$y = 6 - \frac{12}{22}(6+\sqrt{3})$$

$$y = \frac{66-36-6\sqrt{3}}{11} = \frac{30-6\sqrt{3}}{11}$$

$$y = \frac{6(5-\sqrt{3})}{11}$$

Hence dimension of window be $\frac{4}{11}(6+\sqrt{3})$ m and $\frac{6}{11}(5-\sqrt{3})$ m

OR

(ii) (a) Volume of cylinder = 20π

$$\pi r^2 h = 20\pi$$

$$h = \frac{20}{r^2}$$

(b) $\text{T.S.A} = 2\pi r^2 + 2\pi rh$

$$= 2\pi r^2 + \frac{40\pi}{r}$$

(c) Cost of Painting $= 2 \times 2\pi r^2 + 25 \times \frac{40\pi}{r}$

$$= 4\pi r^2 + \frac{1000\pi}{r}$$

$$C(r) = 4\pi r^2 + \frac{1000\pi}{r}$$

$$\frac{dC}{dr} = 8\pi r - \frac{1000\pi}{r^2}$$

For max/min $\frac{dC}{dr} = 0$

$$8\pi r - \frac{1000\pi}{r} = 0$$

$$r^3 = \frac{1000}{8}$$

$$r = \frac{10}{2} = 5 \text{ cm}$$

$$\frac{d^2C}{dr^2} = 8\pi + \frac{2000\pi}{r^3}$$

$$\left(\frac{d^2C}{dr^2}\right)_{r=5} = 8\pi + \frac{2000\pi}{125} > 0$$

Hence cost is minimum at r = 5cm

(d) Minimum cost $C(r) = 4\pi(5)^2 + \frac{1000\pi}{5}$

$$= ₹\, 300\pi$$

$$= ₹\, \frac{300 \times 22}{7}$$

$$= ₹\, 942.85 \quad \text{(Approx)}$$

13. $2ye^{x/y}\,dx + (y - 2xe^{x/y})\,dy = 0$

$$\frac{dx}{dy} = -\left(\frac{y-2xe^{x/y}}{2ye^{x/y}}\right)$$

Let, $x = Vy$

$$\frac{dx}{dy} = V + y\frac{dV}{dy}$$

$$V + y\frac{dV}{dy} = -\left(\frac{y-2Vye^V}{2ye^V}\right)$$

$$y\frac{dV}{dy} = -\left(\frac{1-2Ve^V}{2e^V}\right) - V$$

$$y\frac{dV}{dy} = \frac{-1+2Ve^V-2Ve^V}{2e^V}$$

$$y\frac{dV}{dy} = -\frac{1}{2e^V}$$

$$e^V dV = -\frac{1}{2}\frac{dy}{y}$$

$$\int e^V dV = -\frac{1}{2}\int\frac{dy}{y}$$

$$e^V = -\frac{1}{2}\log y + C$$

$$e^{x/y} = -\frac{1}{2}\log y + C$$

$$x = 0, y = 1$$

$$e^{\circ} = -\frac{1}{2}\log 1 + C$$

$$C = 1$$

$$\therefore \quad e^{x/y} = -\frac{1}{2}\log y + 1$$

$$2e^{x/y} + \log y = 2$$

$$\log y = 2\,(1 - e^{x/y})$$

OR

$$x\,(x^2-1)\,\frac{dy}{dx} = 1$$

$$dy = \frac{dx}{x(x^2-1)}$$

$$dy = \frac{dx}{x(x-1)(x+1)}$$

$$\frac{1}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$$

$$1 = A(x^2-1) + Bx(x+1) + Cx(x-1)$$

$x = 0$, $\quad -1 = A$, $\quad x = 1 \quad 1 = 2B \Rightarrow B = \frac{1}{2}$

At $x = -1$,

$$1 = 2C \Rightarrow C = \frac{1}{2}$$

$$\int dy = \int\frac{dx}{x(x^2-1)}$$

$$y = \int\left(\frac{-1}{x} + \frac{1}{2(x+1)} + \frac{1}{2(x-1)}\right)dx$$

$$y = -\log x + \frac{1}{2}\log(x+1) + \frac{1}{2}\log(x-1) + \log C$$

$$y = \log\frac{\sqrt{(x^2-1)}C}{x}$$

$$0 = \log\frac{\sqrt{3}}{2}C$$

$$\log 1 = \log\frac{\sqrt{3}}{2}C$$

$$\frac{\sqrt{3}}{2}C = 1 \Rightarrow C = \frac{2}{\sqrt{3}}$$

$$\therefore \quad y = \log\frac{\frac{2}{\sqrt{3}}\sqrt{x^2-1}}{x}$$

$$e^y = \frac{2}{\sqrt{3}}\frac{\sqrt{x^2-1}}{x}$$

14. (i)

	2	3	4	5
2	(2, 2)	(2, 3)	(2, 4)	(2, 5)
3	(3, 2)	(3, 3)	(3, 4)	(3, 5)
4	(4, 2)	(4, 3)	(4, 4)	(4, 5)
5	(5, 2)	(5, 3)	(5, 4)	(5, 5)

(ii) Total no. of order of pairs having one larger number are

$$16 - 4 = 12$$

(iii)

X	3	4	5
P ($X = x$)	2/12	4/12	6/12

(iv) $$P(x < 5) = \frac{2}{12} + \frac{4}{12} = \frac{6}{12} = \frac{1}{2}$$

(v) Expected value $(\bar{X}) = 3\times\frac{1}{6} + 4\times\frac{1}{3} + 5\times\frac{1}{2}$

$$= \frac{1}{2} + \frac{4}{3} + \frac{5}{2}$$

$$= \frac{3+8+15}{6}$$

$$= \frac{26}{6} = \frac{13}{3}$$

$\therefore$ Required answer $= \frac{13}{3}$

SECTION B

15. (i) Option (b) is correct.

Explanation: $\vec{a} - 2\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k} - 2(2\hat{i} - 4\hat{j} - 3\hat{k})$

$$= 3\hat{i} - 2\hat{j} + \hat{k} - 4\hat{i} + 8\hat{j} + 6\hat{k}$$

$$= -\hat{i} + 6\hat{j} + 7\hat{k}$$

$$|\vec{a} - 2\vec{b}| = |\sqrt{1^2+6^2+7^2}| = \sqrt{86} \text{ unit}$$

(ii) Option (d) is correct.

Explanation: $\sin^2\alpha + \sin^2\beta + \sin^2\alpha$

$1 - \cos^2\alpha + 1 - \cos^2\beta + 1 - \cos^2\gamma$

$3 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$

$3 - 1 = 2$

(iii) Distance from XY plane c unit

YX plane a unit

ZX plane b unit

(iv) Projection b on $a = \frac{\vec{a}.\vec{b}}{|\vec{a}|}$

$$= \frac{(2\hat{i}+\hat{j}+2\hat{k}).(5\hat{i}-3\hat{j}+\hat{k})}{\sqrt{2^2+1^2+2^2}}$$

$$= \frac{10-3+2}{3}$$

$= 3$ units

(v) $$\vec{a} = 2\hat{i}+5\hat{j}+4\hat{k}$$

$$\vec{a} = \frac{2\hat{i}+5\hat{j}+4\hat{k}}{\sqrt{2^2+5^2+4^2}}$$

$$\vec{a} = \frac{2\hat{i}+5\hat{j}+4\hat{k}}{3\sqrt{5}}$$

20 units magnitude vector parallel to $\vec{a}$ is

$$\frac{20\times(2\hat{i}+5\hat{j}+4\hat{k})}{3\sqrt{5}}$$

$$\frac{4\sqrt{5}}{3}(2\hat{i}+5\hat{j}+4\hat{k})$$

$$\frac{8\sqrt{5}}{3}\hat{i} + \frac{20\sqrt{5}}{3}\hat{j} + \frac{16\sqrt{5}}{3}\hat{k}$$

16. (i) $$\vec{a}\times\vec{b} = \vec{a}\times\vec{c}$$

$$\vec{a}\times\vec{b} - \vec{a}\times\vec{c} = 0$$

$$\vec{a}\times(\vec{b}-\vec{c}) = 0$$

Either $\vec{a}$ and $\vec{b}-\vec{c}$ is parallel vectors

or $$\vec{b}-\vec{c} = 0$$

$$\vec{b} = \vec{c}$$

OR

(ii) $$|\vec{a}\times\vec{b}| = \vec{a}.\vec{b}$$

$$|\vec{a}||\vec{b}|\sin\theta = |\vec{a}||\vec{b}|\cos\theta$$

$$\tan\theta = 1$$

$$\theta = \frac{\pi}{4} \text{ or } 45°$$

17. (i) $A(1, 0, 2)$ $B(3, -1, 1)$ $C(1, 2, 1)$

Equation of the plane passing through the three points is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y-0 & z-2 \\ 2 & -1 & -1 \\ 0 & 2 & -1 \end{vmatrix} = 0$$

$$(x-1)\begin{vmatrix} -1 & -1 \\ 2 & -1 \end{vmatrix} + y\begin{vmatrix} -1 & 2 \\ -1 & 0 \end{vmatrix} + (z-2)\begin{vmatrix} 2 & -1 \\ 0 & 2 \end{vmatrix} = 0$$

$$3(x-1) + 2y + (z-2)(4) = 0$$

$$3x + 2y + 4z - 11 = 0$$

(ii) Equation of line PQ

$P(2, 3, 1)$ $Q\left(\frac{43}{29}, \frac{77}{29}, \frac{9}{29}\right)$

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

$$\frac{x-2}{\frac{43}{29}-2} = \frac{y-3}{\frac{77}{29}-3} = \frac{z-3}{\frac{9}{29}-1}$$

$$\frac{x-2}{-\frac{15}{29}} = \frac{y-3}{-\frac{10}{29}} = \frac{z-3}{-\frac{20}{29}}$$

$$\frac{x-2}{15} = \frac{y-3}{10} = \frac{z-1}{20}$$

$$\frac{x-2}{3} = \frac{y-3}{2} = \frac{z-1}{4}$$

(iii) Height of tower (PQ)

$$\left|\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}\right|$$

$$\left|\sqrt{\left(\frac{-15}{29}\right)^2+\left(\frac{-10}{29}\right)^2+\left(\frac{-20}{29}\right)^2}\right|$$

$$\frac{1}{29}\left|\sqrt{225+100+400}\right|$$

$$\frac{5\sqrt{29}}{29} \text{ units}$$

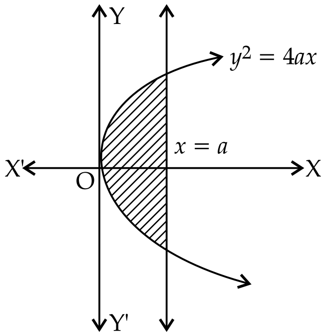
18. (i) Required area $= 2\int_0^a y\,dx$

$$= 2\int_0^a 2\sqrt{a}\sqrt{x}\,dx$$

$$= 2.2.\sqrt{a}\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^a$$

$$= \frac{8\sqrt{a}}{3}(a^{3/2}-0)$$

$$= \frac{8a^2}{3} \text{ units}^2$$



OR

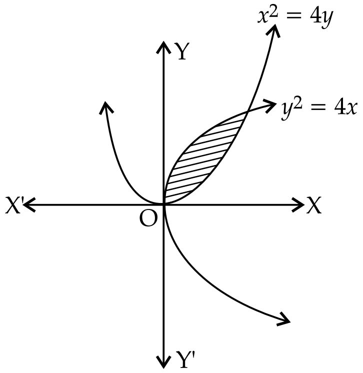
$$y^2 = 4x$$
$$x^2 = 4y$$

the curve (0, 0) and (4, 4)

Required area $= \int_0^4 (y_2 - y_1)dx$

$$= \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right)dx$$

$$= \left(\frac{2x^{3/2}}{\frac{3}{2}} - \frac{x^3}{12}\right)_0^4$$



Required Area $= \left[\frac{4}{3}x^{3/2} - \frac{x^3}{12}\right]_0^4$

$$= \left[\frac{4}{3}\times 8 - \frac{64}{12} - 0\right]$$

$$= \frac{32}{3} - \frac{16}{3}$$

$$= \frac{16}{3} \text{ units}^2$$

SECTION - C

19. (i) Option (b) is correct

Explanation: Let no of travels sold be x

$\therefore$ Revenue $= ₹\,100x$

Variable cost $= \frac{30}{100}\times 100x = 30x$

$\therefore$ Total cost $= 35000 + 30x$

(ii) Option (c) is correct.

Explanation: The correlation coefficient ranges from –1 to 1, where–1 indicates a perfect negative correlation, 0 indicates no correlation, and 1 indicates a perfect positive correlation. A coefficient greater than 0 indicates a positive relationship, meaning as one variable increases, the other tends to increase as well. Conversely, a coefficient less than 0 indicates a negative relationship, meaning as one variable increases, the other tends to decrease. Therefore, a correlation coefficient of –3/4 indicates a negative relationship between the variables X and Y.

(iii)
$$C = x + 2x^3 - \frac{7}{2}x^2$$
$$\text{Arg cost} = \frac{C}{x} = 1 + 2x^2 - \frac{7}{2}x$$
$$\text{M.A.C} = \frac{d}{dx}\left(1 + 2x^2 - \frac{7}{2}x\right) = 4x - \frac{7}{2}$$

(iv)
$$4x + 3y + 7 = 0 \quad \text{...(i)}$$
$$3x + 4y + 8 = 0 \quad \text{...(ii)}$$
from (i) & (ii) $7x + 7y + 15 = 0$

by adding $x + y + \frac{15}{7} = 0$ ...(iii)

by subtracting $x - y - 1 = 0$ ....(iv)

from (iii) & (iv) $2x + \frac{8}{7} = 0$
$$x = -\frac{4}{7}$$
from (iii) $y = -\frac{11}{7}$

mean value of x and y is $-\frac{4}{7}$ and $-\frac{11}{7}$

(v)
$$R(x) = p \times x = 45x$$
$$C(x) = 30x + 240$$
$$P(x) = R(x) - C(x)$$
$$= 45x - 30x - 240$$
$$= 15x - 240$$
To get profit $p(x) > 0$
$$15x - 240 > 0$$
$$x > \frac{240}{15}$$
$$x > 16$$

20.
$$\text{A.C} = x + 5 + \frac{36}{x}$$

(a) Total cost function = $x \times$ A.C = $x^2 + 5x + 36$

(b)
$$AC = x + 5 + \frac{36}{x}$$
$$\frac{d(AC)}{dx} = 1 + 0 - \frac{36}{x^2}$$
for increasing $\frac{d(AC)}{dx} > 0$
$$1 - \frac{36}{x^2} > 0$$
$$x^2 - 36 > 0$$
$$(x - 6)(x + 6) > 0$$

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←——|——|——→
–6 6

Range of increasing average cost is $x > 6$

OR
$$x = 60 - \frac{p}{5}$$
$$\Rightarrow \quad 300 - 5x = p$$
$$R(x) = p \times x = (300 - 5x)$$
$$\text{M.R.} = \frac{d}{dx}(300x - 5x^2) = 300 - 10x$$
$$\text{M.R} = 0$$
$$300 - 10x = 0$$
$$\therefore \quad x = 30$$

21.

Year of Series (X)	Monthly salary in (₹ 1000) (Y)	x^2	y^2	xy
11	10	121	100	110
7	8	49	64	56
9	6	81	36	54
5	5	25	25	25
8	9	64	81	72
6	7	36	49	42
10	11	100	121	110
56	**56**	**476**	**476**	**469**

$n = 7$
$$b_{yx} = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$$
$$b_{yx} = \frac{7 \times 469 - 56 \times 56}{7 \times 476 - (56)^2}$$
$$= \frac{3283 - 3136}{3332 - 3136}$$
$$b_{yx} = \frac{147}{196} = 0.75$$
$$\bar{x} = \frac{\sum x}{7} = \frac{56}{7} = 8, \; \bar{y} = \frac{\sum y}{n} = \frac{56}{7} = 8$$

Regression line y on x
$$y - \bar{y} = b_{yx}(x - \bar{x})$$
$$y - 8 = 0.75(x - 8)$$
$$y - 8 = \frac{3}{4}(x - 8)$$
$$y = \frac{3x}{4} - 6 + 8$$
$$y = \frac{3}{4}x + 2$$

(b) If 13 years of service i.e., $x = 13$
$$y = \frac{(3/4) \times 13 + 2}{4}$$

$y = 9.75 + 2 = 11.75$

$y = 11.75 \times 1000 = ₹\, 11750$

OR

(ii) $3y - 5x + 180 = 0$

$\bar{y} = 44, n = 50$

Variance of X $= \frac{9}{16}$ Variance of Y

Regression equation X on Y is

$5x = 3y + 180$

$x = \frac{3}{5}y + \frac{180}{5} = \frac{3}{5}y + 36$

$\therefore \quad b_{xy} = \frac{3}{5} = 0.6$

(a) $a = \bar{x} - b_{xy}\,\bar{y}$

$\bar{x} = a + b_{xy}\,\bar{y} = 36 + 0.6 \times 44$

$\bar{x} = 36 + 26.4 = 62.4$

(b) $\sigma_x^2 = \frac{9}{16}\sigma_y^2$

$\therefore \quad \frac{\sigma_x}{\sigma_y} = \frac{3}{4}$

$b_{xy} = r \times \frac{\sigma_x}{\sigma_y}$

$\frac{3}{5} = r \times \frac{3}{4}$

$\therefore \quad r = \frac{4}{5} = 0.8$

22. $Z = 11x + 9y$

$x + y \le 10$

$180x + 120y \le 1500$

$18x + 12y \le 150$

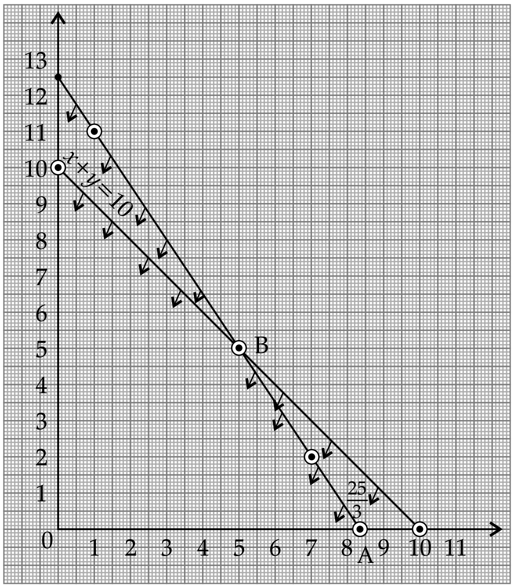
$3x + 2y \le 25$

$x + y = 10$

x	0	10	5
y	10	0	5

$3x + 2y = 25$

x	1	5	7
y	11	5	2



Feasible region is OABCO

$Z = 11x + 9y$

At (0, 0) $\quad Z = 11 \times 0 + 9 \times 0 = 0$

At $A(25/3, 0)$ $\quad Z = 11 \times \frac{25}{3} + 9 \times 0 = \frac{275}{3} = 91.66$

At $B(5, 5)$ $\quad Z = 11 \times 5 + 9 \times 5 = 55 + 45 = 100$

At $C(0, 10)$ $\quad Z = 11 \times 0 + 9 \times 10 = 0 + 90 = 90$

Maximum profit at (5, 5) is ₹ 100