Solved Paper 2013

Mathematics (Standard)

CLASS-X

Time : 3 Hours **Max. Marks : 90**

General Instructions:

(i) All questions are compulsory.

*(ii) The question paper consists of **34** questions divided into four section A, B, C and D.*

*(iii) Section A contains **8** questions of one mark each, which are multiple choice type questions, Section B contains **6** questions of two marks each, Section C contains **10** questions of three marks each, and Section D contains **10** questions of four marks each.*

(iv) Use of calculators is not permitted.

SECTION - A

Question Number 1 to 8 carry one mark each. In each of these questions, four alternative choices have been provided of which only one is correct. Select the correct choice.

1. The common difference of the AP $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}$, ... is:

(a) p **(b)** $-p$

(c) -1 **(d)** 1

Ans. Option (c) is correct.

Explanation: First term = $\frac{1}{p}$ and

second term is $\frac{1-p}{p}$

$$\text{Common difference} = \text{second term} - \text{first term}$$

$$= \frac{1-p}{p} - \frac{1}{p}$$

$$= \frac{1-p-1}{p} = -1$$

2. If Fig. 1, PA and PB are two tangents drawn from an external point P to a circle with centre C and radius 4 cm. If PA ⊥ PB, then the length of each tangent is:

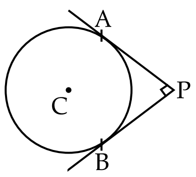


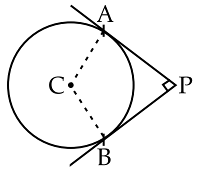
Fig. 1

(a) 3 cm **(b)** 4 cm

(c) 5 cm **(d)** 6 cm

Ans. Option (b) is correct.

Explanation:



CA is perpendicular to AP and CB is perpendicular to BP

$PA \perp PB$ (Given)

$\therefore$ All angle are of 90°

Radius of the circle $= AC = BC = 4$ cm

Also, $AP = BP$

(tangents drawn from point P)

So, BPAC is a square.

$\Rightarrow$ $AP = BP = BC = CA = 4$ cm

So, length of tangents are 4 cm each.

3. In Fig. 2, a circle with centre O is inscribed in a quadrilateral ABCD such that, it touches the sides BC, AB, AD and CD at points P, Q, R and S respectively. If $AB = 29$ cm, $AD = 23$, $\angle B = 90°$, and $DS = 5$ cm, then the radius of the circle (in cm) is:

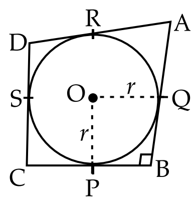


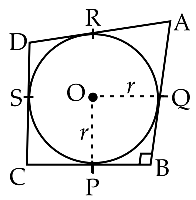
Fig. 2

(a) 11 **(b)** 18

(c) 6 **(d)** 15

Ans. Option (a) is correct.

Explanation:



As, Tangents segments to a circle from the same external point are equal in length.

$\therefore \quad AQ = AR \quad$ ...(i)

$DS = DR \quad$ ...(ii)

$CP = CS \quad$ ...(iii)

$PB = BQ \quad$ ...(iv)

Also, $\quad DS = 5$ cm $\quad$ (Given)

$\therefore \quad DR = 5$ cm

Now, $\quad AD = 23$ cm

$AR + DR = 23$ cm

$AR = 23 - 5$

$= 18$ cm

Again $\quad AB = 29$ cm

$AQ + QB = 29$ cm

$QB = 29 - 18 = 11$ cm

Now, In quadrilateral OPBQ $\angle B = 90°$

Also, $\quad \angle OPB = \angle OQB = 90°$

(Tangent is perpendicular to radius at point of contact)

So, $\quad \angle POQ = 90°$

Further, $\quad PB = BQ \quad$ (From in)

$\therefore$ OPBQ is a square

Hence Rad $\quad O = BQ = 11$ cm

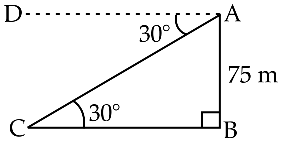
4. The angles of depression of a car, standing on the ground, from the top of a 75 m high tower, is 30°. The distance of the car from the base of the tower (in m) is:

(a) $25\sqrt{3}$ **(b)** $50\sqrt{3}$

(c) $75\sqrt{3}$ **(d)** 150

Ans. Option (c) is correct.

***Explanation*:**



Suppose AB is the tower and C is the position of the car from the base of the tower.

It is given that, $\quad AB = 75$ m

Now, $\quad \angle ACB = \angle CAD = 30° \quad$ (Alt $\angle$S)

In rt. ΔABC,

$$\tan 30° = \frac{AB}{BC}$$

$$\left[\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}\right]$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{75}{BC}$$

$$\Rightarrow \quad BC = 75\sqrt{3} \text{ m}$$

5. The probability of getting an even number, when a die is thrown once, is:

(a) $\frac{1}{2}$ **(b)** $\frac{1}{3}$

(c) $\frac{1}{6}$ **(d)** $\frac{5}{6}$

Ans. Option (a) is correct.

***Explanation*:**

Sample space $= \{1, 2, 3, 4, 5, 6\}$

Even numbers $= \{2, 4, 6\}$

$$P(\text{Even numbers}) = \frac{3}{6} = \frac{1}{2}$$

6. A box contains 90 discs, numbered from 1 to 90. If one disc is drawn at random from the box, the probability that it bears a prime-number less than 23, is:

(a) $\frac{7}{90}$ **(b)** $\frac{10}{90}$

(c) $\frac{4}{45}$ **(d)** $\frac{9}{89}$

Ans. Option (c) is correct.

***Explanation*:** Prime numbers less than 23

2, 3, 5, 7, 11, 13, 17, 19

Total no. of possible outcomes = 90

No. of favourable outcomes = 8

$$P(\text{prime number less than 23}) = \frac{8}{90} = \frac{4}{45}$$

*** 7. In Fig. 3, the area of triangle ABC (in sq. units) is:**

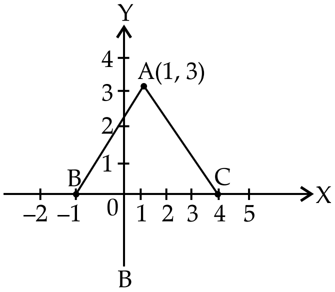


Fig. 3

(a) 15 **(b)** 10

(c) 7.5 **(d)** 2.5

8. If the distance between the circumference and the radius of a circle is 37 cm, then using $\pi = \frac{22}{7}$, the circumference (in cm) of the circle is:

(a) 154 **(b)** 44

(c) 14 **(d)** 7

Ans. Option (b) is correct.

***Explanation*:** Let C be the circumference and r be the radius of the circle.

According to the Question,

Circumference – Radius = 37

$$2\pi r - r = 37$$

$$\Rightarrow \quad r(2\pi - 1) = 37$$

$$\Rightarrow \quad r\left(2\times\frac{22}{7} - 1\right) = 37$$

* *Out of Syllabus*

$$\Rightarrow \quad r\left(\frac{44}{7}-1\right) = 37$$

$$\Rightarrow \quad r\left(\frac{37}{7}\right) = 37$$

$$\Rightarrow \quad r = 7 \text{ cm}$$

Therefore, circumference $= 2\pi r$

$$= 2\times\frac{22}{7}\times 7 = 44 \text{ cm}$$

SECTION - B

Question Number 9 to 14 carry two marks each.

9. Solve the following quadratic equation for x:

$$\mathbf{4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0}$$

Sol. The quadratic equation is:

$$4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$$

$$\Rightarrow \quad 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$$

$$\Rightarrow \quad 4x(\sqrt{3}x+2) - \sqrt{3}(\sqrt{3}x+2) = 0$$

$$\Rightarrow \quad (4x-\sqrt{3})(\sqrt{3}x+2) = 0$$

$$\Rightarrow \quad 4x - \sqrt{3} = 0$$

$$\text{or} \quad \sqrt{3}x + 2 = 0$$

$$\Rightarrow \quad x = \frac{\sqrt{3}}{4}$$

$$\text{or} \quad x = \frac{-2}{\sqrt{3}}$$

10. How many three digit natural numbers are divisible by 7?

Sol. The A.P. formed by the numbers divisible by 7 are,

105, 112, 119, …………. , 994

$a = 105$, $d = 112 - 105 = 7$, $a_n = 994$

$$a_n = a + (n-1)d$$

$$994 = 105 + (n-1)7$$

$$994 - 105 = (n-1)7$$

$$889 = (n-1)7$$

$$n - 1 = 127$$

$$n = 128$$

Hence, there are 128 three digit numbers divisible by 7.

11. In Fig. 4, a circle inscribed in triangle ABC touches its sides AB, BC and AC at points D, E and F respectively. If AB = 12 cm, BC = 8 cm and AC = 10 cm, then find the length of AD, BE and CF.

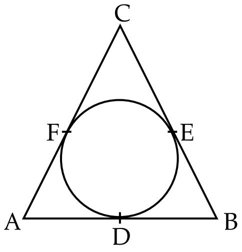
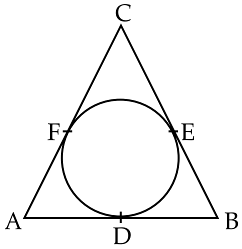


Fig. 4

Sol.



Let $AD = x$, $BE = y$ and $CF = z$

Also, the length of tangents drawn from same external point are equal in lengths,

$$\therefore \quad AD = AF = x$$

$$BD = BE = y$$

$$CF = CE = z$$

According to the question,

$$x + y = 12 \quad \ldots\text{(i)}$$

$$x + z = 10 \quad \ldots\text{(ii)}$$

$$z + y = 8 \quad \ldots\text{(iii)}$$

Add (i), (ii) and (iii) we get

$$2x + 2y + 2z = 30$$

Or $\quad x + y + z = 15 \quad \ldots$(iv)

From (i) and (iv), we get

$$12 + z = 15$$

Or $\quad z = 3$

From (ii) and (iv), we get

$$y + 10 = 15$$

Or $\quad y = 5$

From (iii) and (iv), we get

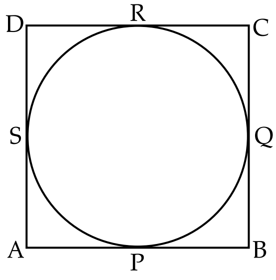
$$x + 8 = 15$$

$$x = 7$$

Hence, $AD = 7$ cm, $BE = 5$ cm and $CF = 3$ cm

12. Prove that the parallelogram circumscribing a circle is a rhombus.

Sol.



Given: ABCD is a parallelogram circumscribing a circle with centre O.

To prove: ABCD is a rhombus

Proof: As the length of tangents drawn from same external point are equal in length

Therefore,

$$AP = AS \quad \ldots(i)$$
$$BP = BQ \quad \ldots(ii)$$
$$CR = CQ \quad \ldots(iii)$$
$$DR = DS \quad \ldots(iv)$$

Add (i), (ii), (iii) and (iv)

$$AP + BP + CR + DR = AS + DS + CQ + BQ$$
$$AB + CD = AD + BC$$
$$2AB = 2AD$$
$$(\because AB = CD \text{ and } AD = BC \text{ Opposite side of } ||\text{gm})$$
$$AB = AD$$

Similarly, $AB = BC = CD = DA$

$\Rightarrow$ ABCD is a rhombus.

13. A card is drawn at random from a well shuffled pack of 52 playing cards. Find the probability that the drawn card is neither a king nor a queen.

Sol. Total cards = 52

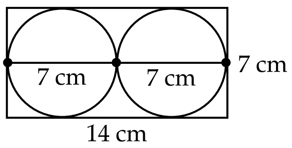
Number of cards being king or queen = 4 + 4 = 8

Number of cards neither a king or a queen = 52 – 8 = 44

$$P(\text{Neither a king nor a queen}) = \frac{44}{52} = \frac{11}{13}$$

14. Two circular pieces of equal radii and maximum area, touching each other are cut out from a rectangular card board of dimensions 14 cm × 7 cm. Find the area of the remaining card board. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Sol.



Diameter of each of the circular piece is $\frac{14}{2}$ cm

Radius of each circular pieces = $\frac{7}{2}$ cm

Therefore, Sum of area of two circular pieces = $2\pi r^2$

$$= 2\times\frac{22}{7}\times\left(\frac{7}{2}\right)^2$$
$$= 2\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}$$
$$= 77 \text{ cm}^2$$

Area of card board = Length × breadth

$$= 14 \times 7 = 98 \text{ cm}^2$$

Area of remaining cardboard

= Area of card board – Area of two circular pieces

= 98 – 77

= 21 cm^2

SECTION - C

Question Number 15 to 24 carry three marks each.

15. For what value of k, are the roots of the quadratic equation $kx(x - 2) + 6 = 0$ equal?

Sol. The roots of the quadratic equation $kx(x-2) + 6 = 0$ are equal,

Therefore, Discriminant = 0

$$kx^2 - 2kx + 6 = 0$$
$$a = k, b = -2k \text{ and } c = 6$$
$$D = b^2 - 4ac$$
$$\Rightarrow (-2k)^2 - 4(k)(6) = 0$$
$$\Rightarrow 4k^2 - 24k = 0$$
$$\Rightarrow 4k(k-6) = 0$$
$$\Rightarrow k = 0 \text{ or } 6$$

16. Find the number of terms of the AP 18, $15\frac{1}{2}$, 13, ..., $-49\frac{1}{2}$, and find the sum of all its terms.

Sol. 18, $15\frac{1}{2}$, 13, ..., $-49\frac{1}{2}$

$$d = \text{second term} - \text{first term}$$
$$= \frac{31}{2} - 18 = \frac{-5}{2}$$

As $a_n = a + (n-1)d$

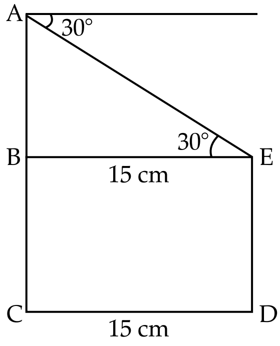
$$\Rightarrow \frac{-99}{2} = 18 + (n-1)\frac{-5}{2}$$
$$\Rightarrow \frac{-99}{2} - 18 = (n-1)\frac{-5}{2}$$
$$\Rightarrow \frac{-135}{2} = (n-1)\frac{-5}{2}$$
$$\Rightarrow n - 1 = \frac{-135}{2}\times\frac{2}{-5}$$
$$\Rightarrow n - 1 = 27$$
$$\Rightarrow n = 28$$
$$S_n = \frac{n}{2}[a+1]$$
$$= \frac{28}{2}\left[18 - \frac{99}{2}\right]$$
$$= 14\left[\frac{36-99}{2}\right]$$
$$= 14\times\frac{-63}{2}$$
$$= -441$$

*** 17. Construct a triangle with sides 5 cm, 4 cm and 6 cm. Then construct another triangle whose sides are $\frac{2}{3}$ times the corresponding sides of first triangle.**

* *Out of Syllabus*

18. The horizontal distance between two poles is 15 m. The angle of depression of the top of first pole as seen from the top of second pole is 30°. If the height of the second pole is 24 m, find the height of the first pole. [Use $\sqrt{3}$ = 1.732]

Sol.



Let AC and ED are the two poles such that

$$AC = 24 \text{ m and } CD = 15 \text{ m}$$

Let $\quad BC = ED = h$ m

Also, $\quad AB = 24 - h$

In ΔABE, $\quad \tan 30° = \dfrac{AB}{BE}$

$$\left[\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}\right]$$

$$\tan 30° = \frac{24-h}{15}$$

$$\frac{1}{\sqrt{3}} = \frac{24-h}{15}$$

$$\frac{15}{1.73} = 24 - h$$

$$h = 24 - 8.67$$

$$h = 15.32 \text{ m}$$

19. Prove that the points (7, 10), (–2, 5) and (3, –4) are the vertices of an isosceles right triangle.

Sol. Let A(7, 10) B(–2, 5) and C(3, –4) are the vertices of an isosceles right triangle

$$\text{Distance formula} = \sqrt{(x_2-x_1)^2-(y_2-y_1)^2}$$

$$AB = \sqrt{(-2-7)^2+(5-10)^2}$$

$$= \sqrt{81+25} = \sqrt{106}$$

$$BC = \sqrt{(3+2)^2+(-4-5)^2}$$

$$= \sqrt{25+81} = \sqrt{106}$$

$$AC = \sqrt{(3-7)^2+(-4-10)^2}$$

$$= \sqrt{16+196} = \sqrt{212}$$

Therefore, $\quad AB = BC$

$\Rightarrow \Delta$ABC is an isosceles triangle

Also, $\quad AB^2 + BC^2 = 106 + 106$

$$= 212$$

$$= AC^2$$

It Verifies Pythagoras theorem,

So, ΔABC is a isosceles right triangle.

20. Find the ratio in which the *y*-axis divides the line segment joining the points (–4, –6) and (10, 12). Also find the coordinates of the point of division.

Sol. Let Point (0, y) divide the line segment joining points (–4, –6) and (10, 12) in the ratio k : 1

According to the section formula,

$$(x, y) = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

$$(0, y) = \left(\frac{k(10)+1(-4)}{k+1}, \frac{k(12)+1(-6)}{k+1}\right)$$

$$0 = \frac{10k-4}{k+1}$$

$$0 = 10k - 4$$

$$10k = 4$$

$$\Rightarrow \quad k = \frac{4}{10} \Rightarrow \frac{2}{5}$$

Therefore ratio is 2 : 5

Coordinates of point of division

$$y = \frac{12k-6}{k+1}$$

$$y = \frac{12\times\frac{2}{5}-6}{\frac{2}{5}+1}$$

$$= \frac{\frac{24-30}{5}}{\frac{2+5}{5}}$$

Therefore, Required point is $\left(0, -\dfrac{6}{7}\right)$.

21. In Fig. 5, AB and CD are two diameters of a circle with centre O, which are perpendicular to each other. OB is the diameter of the smaller circle. If *OA* = 7 cm, find the area of the shaded region.
$\left[\textbf{Use } \pi = \dfrac{\textbf{22}}{\textbf{7}}\right]$

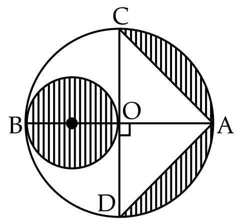
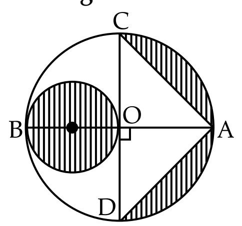


Fig. 5

Sol.



Radius of bigger circle $= OA = R = 7$ cm

Radius of smaller circle $= r = \frac{7}{2}$ cm

Area of bigger circle $= \pi r^2 = \frac{22}{7} \times 7 \times 7$

$= 154 \text{ cm}^2$

Area of semicircle $= \frac{154}{2} = 77 \text{ cm}^2$

Area of smaller circle $= \pi r^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$= \frac{77}{2} \text{ cm}^2$

Area of unshaded triangle ΔADC

$= \frac{1}{2} \times DC \times AO$

$= \frac{1}{2} \times 14 \times 7 = 49 \text{ cm}^2$

Area of shaded portion = Area of smaller circle + Area of semicircle – Area of ΔABC

$= \frac{77}{2} + 77 - 49$

$= 66.5 \text{ cm}^2$

22. A vessel is in the form of a hemispherical bowl surmounted by a hollow cylinder of same diameter. The diameter of the hemispherical bowl is 14 cm and the total height of the vessel is 13 cm. Find the total surface area of the vessel. $\left[\textbf{Use } \pi = \frac{22}{7}\right]$

Sol. Let the height of the cylinder be h cm and radius be r cm

Radius of the hemispherical bowl = 7 cm

Total height of the vessel = 13 cm

Therefore, Height of the cylinder = 13 – 7 = 6 cm

As the vessel is hollow,

So total surface area of the vessel

= 2(Curved surface area of the cylinder + Curved surface area of the hemisphere)

$= 2[2\pi rh + 2\pi r^2]$

$= 4\pi r(h + r)$

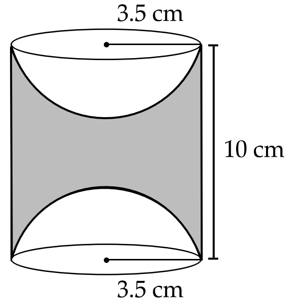
$= 4 \times \frac{22}{7} \times 7(6 + 7)$

$= 88 \times 13$

$= 1144 \text{ cm}^2$

23. A wooden toy was made by scooping out a hemisphere of same radius from each end of a solid cylinder. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the volume of wood in the toy. $\left[\textbf{Use } \pi = \frac{22}{7}\right]$

Sol.



Radius of the cylinder = Radius of hemisphere

$= r = 3.5$ cm

Height of the cylinder = 10 cm

Volume of the toy = Volume of the cylinder – Volume of both hemispheres

$= \pi r^2 h - 2 \times \frac{2}{3}\pi r^3$

$= \pi r^2\left[h - \frac{4}{3}r\right]$

$= \frac{22}{7} \times 3.5 \times 3.5\left[10 - \frac{4}{3} \times 3.5\right]$

$= \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10}\left[10 - \frac{14}{3}\right]$

$= 205.33 \text{ cm}^3$ (approx..)

24. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre.
Find: (i) the length of the arc (ii) area of the sector formed by the arc. $\left[\textbf{Use } \pi = \frac{22}{7}\right]$

Sol. As, $\theta = 60°$

And $r = 21$ cm

(i) Length of arc $= \frac{\theta}{360} 2\pi r$

$= \frac{60}{360} \times 2 \times \frac{22}{7} \times 21$

$= 22$ cm

(ii) Area of sector $= \frac{\theta}{360}\pi r^2$

$= \frac{60}{360} \times \frac{22}{7} \times 21 \times 21$

$= 231 \text{ cm}^2$

SECTION - D

Question Number 25 to 34 carry four marks each.

25. Solve the following for x:

$$\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$$

Sol.
$$\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$$

Or,
$$\frac{1}{2a+b+2x} - \frac{1}{2x} = \frac{1}{2a} + \frac{1}{b}$$

Or, $\dfrac{2x-2a-b-2x}{4ax+2bx+4x^2} = \dfrac{b+2a}{2ab}$

Or, $(-2a-b)(2ab) = (b+2a)(4ax+2bx+4x^2)$

Or, $\dfrac{-(b+2a)(2ab)}{b+2a} = 4ax+2bx+4x^2$

Or, $4x^2+2bx+4ax+2ab = 0$

Or, $2x(2x+b)+2a(2a+b) = 0$

Or, $(2x+2a)(2x+b) = 0$

Or, $x = -a$ or $x = -b$

26. Sum of the areas of two squares is 400 cm^2. If the difference of their perimeters is 16 cm, find the sides of the two squares.

Sol. Let the side of the squares be x and y respectively.

Area of the first square $= x^2$ and

area of the second square $= y^2$

According to the question,

Sum of both areas $x^2 + y^2 = 400$ …(i)

And difference between the perimeters

$= 4x - 4y = 16$

Or $x - y = 4$ …(ii)

Taking (ii),

$x - y = 4$

Or $x = 4 + y$ …(iii)

Put this value of x in (i),

$(4+y)^2 + y^2 = 400$

$\Rightarrow$ $16 + y^2 + 8y + y^2 = 400$

$\Rightarrow$ $2y^2 + 8y - 384 = 0$

$\Rightarrow$ $y^2 + 4y - 192 = 0$

$\Rightarrow$ $y^2 + 16y - 12y - 192 = 0$

$\Rightarrow y(y+16) - 12(y+16) = 0$

$\Rightarrow$ $(y-12)(y+16) = 0$

$\Rightarrow$ $y = 12$ and -16

Sides cannot be negative

$\therefore$ $y = 12$ cm

Equating y in (iii) equation

$x = 4 + 12$

$= 16$ cm

Hence sides of two squares are 16 cm and 12 cm

27. If the sum of first 7 terms of an AP is 49 and that of first 17 terms is 289, find the sum of its first *n* terms.

Sol. Let a be the first term and d be the common difference,

Also, $S_n = \dfrac{n}{2}[2a + (n-1)d]$

Therefore,

$$S_7 = \frac{7}{2}[2a + (7-1)d] = 49$$

Or $2a + 6d = 14$

Or $a + 3d = 7$ …(i)

Also,

$$S_{17} = \frac{17}{2}[2a + (17-1)d] = 289$$

Or $2a + 16d = 34$

Or $a + 8d = 17$ …(ii)

Solving (i) and (ii),

We get, $a = 1$ and $d = 2$

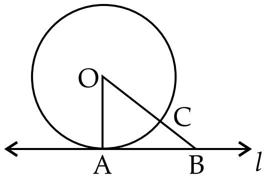
Thus, $S_n = \dfrac{n}{2}[2(1) + (n-1)2]$

$= \dfrac{n}{2}[2 + 2n - 2]$

$= \dfrac{n}{2}[2n] = n^2$

28. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

Sol.



Given: A circle with centre O and radius r. Also, a tangent l at point A.

To prove: $OA \perp l$

Construction: Take a point B, on the tangent l. Join OB intersecting the circle at C.

Proof: $OA = OC$ (Radius of the circle)

$OB = OC + BC$

$\Rightarrow$ $OB > OC$

$\Rightarrow$ $OB > OA$

$\Rightarrow$ $OA < OB$

B is an arbitrary point on the tangent l. Thus OA is shorter than any other line segment joining O to any point on l.

And, the shortest distance between a point and a line is the perpendicular distance.

Hence $OA \perp l$

Therefore, Tangent at any point of a circle is perpendicular to the radius through the point of contact.

29. In Fig. 6, *l* and *m* are two parallel tangents to a circle with centre O, touching the circle at A and B respectively. Another tangent at C intersects the line *l* at D and *m* at E. Prove that $\angle$DOE = 90°.

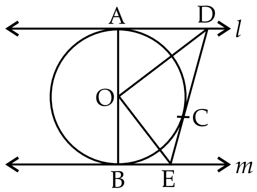
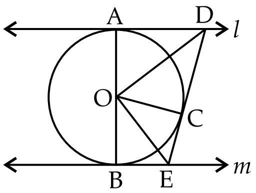


Fig. 6

Sol.



Firstly Join OC,

In ΔODA and ΔODC

$$OA = OC$$

(Radii of the same circle)

$$AD = DC$$

(Length of tangent drawn from an external point to a circle are equal)

$$DO = OD \quad \text{(Common side)}$$

Therefore, $\Delta ODA \cong \Delta ODC$ (SSS Criteria)

So, $\angle DOA = \angle COD$ [BY CPCT]

Similarly, $\Delta OEB \cong \Delta OEC$

So, $\angle EOB = \angle COE$ [BY CPCT]

AOB is a diameter of the circle.

Hence, it is a straight line.

$$\angle DOA + \angle COD + \angle COE + \angle EOB = 180°$$

$$\Rightarrow \quad 2\angle COD + 2\angle COE = 180°$$

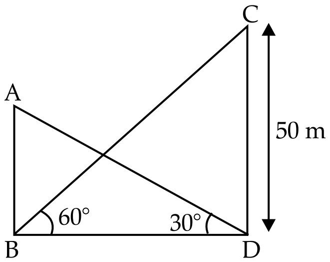
$$\Rightarrow \quad \angle COD + \angle COE = 90°$$

$$\Rightarrow \quad \angle DOE = 90°$$

Hence proved.

30. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

Sol.



Height of the tower $= CD = 50$ m

From the fig,

In ΔCDB

$$\tan 60° = \frac{CD}{BD}$$

$$\left[\text{Using } \tan\theta = \frac{\text{Perpendicular}}{\text{Base}}\right]$$

$$\frac{50}{BD} = \sqrt{3}$$

$$BD = \frac{50}{\sqrt{3}} \quad \ldots\text{(i)}$$

In ΔABD,

$$\tan 30° = \frac{AB}{BD}$$

$$\frac{AB}{BD} = \frac{1}{\sqrt{3}}$$

$$AB = \frac{1}{\sqrt{3}} \times \frac{50}{\sqrt{3}} \quad \text{[From (i)]}$$

$$AB = \frac{50}{3} \text{ m}$$

Therefore, height of the building is $\frac{50}{3}$ m.

31. A group consists of 12 persons, of which 3 are extremely patient, other 6 are extremely honest and rest are extremely kind. A person from the group is selected at random. Assuming that each person is equally likely to be selected, find the probability of selecting a person who is (i) extremely patient (ii) extremely kind or honest. Which of the above values you prefer more.

Sol. Total persons in the group = 12

(i) Number of persons who are extremely patient = 3

$$P(\text{Selecting person who is extremely patient}) = \frac{3}{12} = \frac{1}{4}$$

(ii) Number of persons who are extremely honest = 6

Number of persons who are extremely kind

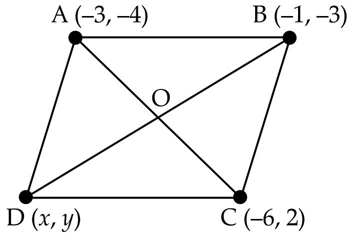
$$= 12 - 3 - 6 = 3$$

Therefore, P(selecting a person who is extremely kind or extremely honest) $= \frac{3+6}{12} = \frac{9}{12} = \frac{3}{4}$

From the given three values, we prefer honesty. Honesty can get rid of rampant corruption which is a burning issue of the present society.

32. The three vertices of a parallelogram ABCD are A(3, –4), B(–1, –3) and C(–6, 2). Find the coordinates of vertex D and find the area of ABCD.

Sol.



ABCD is a parallelogram with the vertices A(3, – 4), B(– 1, – 3) and C(– 6, 2).

Let the fourth vertex D be (a, b)

According to the property of a parallelogram that diagonals of a parallelogram bisect each other at the same point.

Therefore, Mid-point of the diagonal AC

= Mid-point of the diagonal BD

$$\Rightarrow \quad \left(\frac{3-6}{2}, \frac{-4+2}{2}\right) = \left(\frac{-1+x}{2}, \frac{-3+y}{2}\right)$$

$$\Rightarrow \quad \left(\frac{-3}{2}, \frac{-2}{2}\right) = \left(\frac{-1+x}{2}, \frac{-3+y}{2}\right)$$

$$\Rightarrow \quad \frac{-3}{2} = \frac{-1+x}{2}$$

$$\text{or} \quad -1 = \frac{-3+y}{2}$$

$$\Rightarrow \quad -3 = -1 + x$$

$$\text{or} \quad -2 = -3 + y$$

$$\Rightarrow \quad -2 = x$$

$$\text{or} \quad 1 = y$$

$$\Rightarrow \quad x = -2 \text{ or } y = 1$$

So, the coordinates of the vertex D is(– 2, 1)

33. Water is flowing through a cylindrical pipe, of internal diameter 2 cm, into a cylindrical tank of base radius 40 cm, at the rate of 0.4 m/s. Determine the rise in level of water in the tank in half an hour.

Sol. Diameter of circular end of pipe = 2 cm

Therefore, Radius (R) of circular end of pipe

$$= \frac{2}{200} \text{ m} = 0.01 \text{ m}$$

Area of cross-section $= \pi R^2$

$$= \pi(0.01)^2$$

$$= 0.0001\ \pi \text{ m}^2$$

Speed of water = 0.4 m/sec

Volume of water that flows in 1 minute from pipe

$$= 24 \times 0.0001\pi \text{ m}^3$$

Volume of water that flows in 30 minute from pipe

$$= 0.072\pi \text{ m}^3$$

Radius of base of cylindrical tank = 40 cm or 0.4 m

Let the cylindrical tank be filled up to h m in 30 minutes

Volume of water filled in tank in 30 minutes is equal to volume of water flowed in 30 minutes from the pipe.

$$\therefore \quad \pi R^2 h = 0.072\pi$$

$$\Rightarrow \quad (0.4)^2 \times h = 0.072$$

$$\Rightarrow \quad 0.16 \times h = 0.072$$

$$\Rightarrow \quad h = \frac{0.072}{0.16}$$

$$\Rightarrow \quad h = 0.45 \text{ m} = 45 \text{ cm}$$

*** 34. A bucket open at the top, and made up of a metal sheet is in the form of a frustum of a cone. The depth of the bucket is 24 cm and the diameter of its upper and lower circular ends are 30 cm and 10 cm respectively. Find the cost of metal sheet used in it at the rate of ₹ 10 per 100 cm^2. [Use π = 3.14]**

❑❑

* *Out of Syllabus*