Solved Paper 2014

Mathematics (Standard)

CLASS-X

Time : 3 Hours **Max. Marks : 90**

General Instructions:

(i) All questions are compulsory.

*(ii) The question paper consists of **31** questions divided into four section A, B, C and D.*

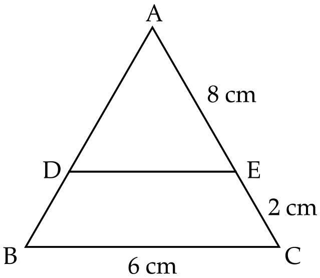
*(iii) Section A contains **4** questions of **1** mark each, Section B contains **6** questions of **2** marks each, Section C contains **10** questions of **3** marks each and Section D contains **11** questions of **4** marks each.*

(iv) Use of calculators is not permitted.

SECTION - A

1. In the given figure if $DE \parallel BC$, $AE = 8$ cm, $EC = 2$ cm and $BC = 6$ cm, then find DE. **1**

Sol. Given that,



$$AE = 8 \text{ cm}$$
$$EC = 2 \text{ cm}$$
$$BC = 6 \text{ cm}$$

As $DE \parallel BC$,

We have, $\Delta ADE \sim \Delta ABC$

So, ratio of their respective sides is equal

Then $\frac{AE}{AC} = \frac{DE}{BC}$

$\Rightarrow \frac{8}{10} = \frac{DE}{6}$

$\Rightarrow DE = 4.8$ cm

2. Evaluate: $\frac{1-\cot^2 45°}{1+\sin^2 90°}$. **1**

Sol. We have,

$\Rightarrow \frac{1-\cot^2 45°}{1+\sin^2 90°}$

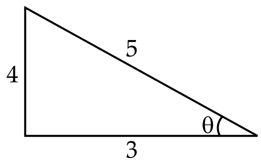
$\Rightarrow \frac{1-1}{1+1}$ $(\because \cot 45° = 1 \text{ and } \sin 90° = 1)$

$\Rightarrow \frac{0}{2} = 0$

3. If $\operatorname{cosec} \theta = \frac{5}{4}$, find the value of $\cot \theta$.

Sol. Given that,

$\Rightarrow \operatorname{cosec} \theta = \frac{5}{4}$



From the figure we have,

$\Rightarrow \cot \theta = \frac{3}{4}$

4. Following table shows sale of shoes in a store during one month:

Size of shoe	Number of pairs sold
3	4
4	18
5	25
6	12
7	5
8	1

Find the modal size of the shoes sold. **1**

Sol. From the given table,

Number of pairs sold is maximum for the size of shoe = 5

So, the modal size of the shoes sold is 5.

SECTION - B

5. Find the prime factorisation of the denominator of rational number expressed as $6.\overline{21}$ in simplest form. **2**

Sol. Let $x = 6.$

$\Rightarrow \quad x = 6.2121212121... \quad ...(i)$

Multiplying both sides by 100

$\Rightarrow \quad 100x = 621.212121... \quad ...(ii)$

Subtracting (i) from (ii),

$\Rightarrow \quad 99x = 615$

$\Rightarrow \quad x = \frac{615}{99} = \frac{205}{33}$

So, prime factorization of denominator = 3×11

6. Find a quadratic polynomial, the sum and product of whose zeroes are $\sqrt{3}$ and $\frac{1}{\sqrt{3}}$ respectively. 2

Sol. Given that,

$$\text{Sum of zeroes} = \sqrt{3}$$

$$\text{Product of zeroes} = \frac{1}{\sqrt{3}}$$

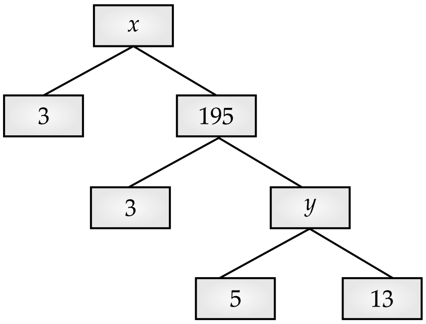
Required Quadratic Polynomial is

$\Rightarrow x^2 - (\text{Sum of zeroes}).x + \text{Product of zeroes}$

$\Rightarrow x^2 - \sqrt{3}x + \frac{1}{\sqrt{3}} = 0$

$\Rightarrow \sqrt{3}x^2 - 3x + 1 = 0$

7. Complete the following factor tree and find the composite number x. 2



Sol. From the factor tree we have,

$\Rightarrow \quad x = 195 \times 3 = 585$

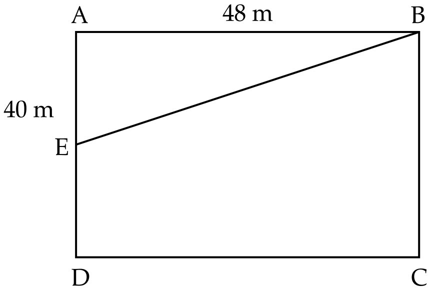
$\Rightarrow \quad y = 13 \times 5 = 65$

8. In a rectangle ABCD, E is middle point of AD. If $AD = 40$ m and $AB = 48$ m, then find EB. 2

Sol. Given that,

E is the midpoint of AD

$AD = 40$ m and $AB = 48$ m



In right angled triangle ABE

$(BE)^2 = (AE)^2 + (AB)^2$

$\Rightarrow \quad (BE)^2 = (20)^2 + (48)^2$

$\Rightarrow \quad (BE)^2 = 400 + 2304$

$\Rightarrow \quad (BE)^2 = 2704$

$\Rightarrow \quad BE = 52$ m

9. If $x = p \sec\theta + q \tan\theta$ and $y = p \tan\theta + q \sec\theta$, then prove that $x^2 - y^2 = p^2 - q^2$. 2

Sol. Given that,

$\Rightarrow \quad x = p \sec\theta + q \tan\theta$

$\Rightarrow \quad y = p \tan\theta + q \sec\theta$

Taking LHS,

$\Rightarrow x^2 - y^2 = (p \sec\theta + q \tan\theta)^2 - (p \tan\theta + q \sec\theta)^2$

$\Rightarrow x^2 - y^2 = p^2(\sec^2\theta - \tan^2\theta) + q^2(\tan^2\theta - \sec^2\theta) + 2pq \sec\theta \tan\theta - 2pq \sec\theta \tan\theta$

$\Rightarrow x^2 - y^2 = p^2(1) + q^2(-1) \quad [\sec^2 x - \tan^2 x = 1]$

$\Rightarrow x^2 - y^2 = p^2 - q^2$ **Hence proved.**

10. Given below is the distribution of weekly pocket money received by students of a class. Calculate the pocket money that is received by most of the students. 2

Pocket Many (in ₹)	No. of Students
0 – 20	**2**
20 – 40	**2**
40 – 60	**3**
60 – 80	**12**
80 – 100	**18**
100 – 120	**5**
120 – 140	**2**

Sol. We have,

Pocket Many (in ₹)	No. of Students
0 – 20	2
20 – 40	2
40 – 60	3
60 – 80	12
80 – 100	18
100 – 120	5
120 – 140	2

Modal class = 80 – 100

So, $l = 80$

$\Rightarrow f_1 = 18, f_0 = 12, f_2 = 5, h = 20$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

On putting values,

Mode = 80 + 6.3157

Mode = 86.32

So, the pocket money received by most of the students is ₹ 86.32

SECTION - C

11. Prove that $3+2\sqrt{3}$ is an irrational number. **3**

Sol. To prove: $3+2\sqrt{3}$ is an irrational number

If possible, let $3+2\sqrt{3}$ be rational.

Then, $3+2\sqrt{3}$ is rational and 3 is rational.

$[(3+2\sqrt{3})-3]$ is rational.

[Difference of two rational number is rational]

$2\sqrt{3}$ is rational.

$\sqrt{3}$ is rational.

Let the simplest form of $\sqrt{3}$ be $\frac{a}{b}$

Then, a and b are integers having no common factor other than 1

Now, $\sqrt{3} = \frac{a}{b}$

$3b^2 = a^2$

3 divides a^2. [3 divides $3b^2$]

3 divides a

Let $a = 3c$ for some integer c.

Therefore,

$3b^2 = 9c^2$

$b^2 = 3c^2$

3 divides b^2 [3 divides $3c^2$]

3 divides b

Thus, 3 is a common factor of a and b.

This contradicts the fact that a and b have no common factor other than 1.

So, 3 is irrational.

Hence, $3+2\sqrt{3}$ is irrational.

12. Solve by elimination:

$$\mathbf{3x = y + 5}$$

$$\mathbf{5x - y = 11}$$ **3**

Sol. To solve given equations by elimination method

$\Rightarrow \quad 3x - y = 5$

$\Rightarrow \quad 5x - y = 11$

$\quad - \quad + \quad -$

$-2x = -6$

$x = 3$

$y = 3(3) - 5 = 9 - 5 = 4$

Hence, $x = 3$ and $y = 4$

13. A man earns ₹ 600 per month more than his wife. One-tenth of the man's salary and one-sixth of the wife's salary amount to ₹ 1,500, which is saved every month. Find their incomes. **3**

Sol. Given that,

Man earns ₹ 600 per month more than his wife

Let the income of man be x

Income of wife = $(x - 600)$

According to given condition,

$\Rightarrow \quad \frac{1}{10}x + \frac{1}{6}(x-600) = 1500$

$\Rightarrow \quad \frac{x}{10} + \frac{x}{6} - 100 = 1500$

$\Rightarrow \quad \frac{8x}{30} = 1600$

$\Rightarrow \quad x = 6000$

So,

Income of man = ₹ 6000

Income of wife = ₹ 6000 – ₹ 600

= ₹ 5400

*** 14. Check whether polynomial $x - 1$ is a factor of the polynomial $x^3 - 8x^2 + 19x - 12$. Verify by division algorithm.** **3**

15. If the perimeters of two similar triangles ABC and DEF are 50 cm and 70 cm respectively and one side of ΔABC = 20 cm, then find the corresponding side of ΔDEF. **3**

Sol. Given that,

Perimeters of two similar triangles ABC and DEF are 50 cm and 70 cm

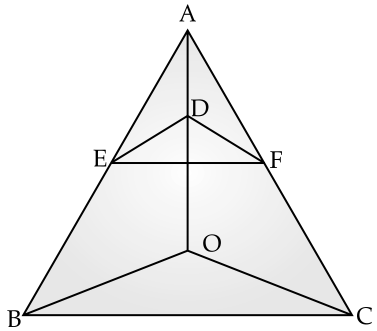
One side of triangle ABC = 20 cm

As both triangles are similar so the ratio of their corresponding sides is equal to the ratio of their perimeters

$\Rightarrow \quad \frac{50}{70} = \frac{20}{\text{other side}}$

Corresponding other side of triangle = 28 cm

16. In the figure if DE || OB and EF || BC, then prove that DF || OC. **3**



Sol. In ΔAOB, we have

$DE \parallel OB$ [Given]

Therefore, by basic proportionality theorem, we have

$$\frac{AE}{EB} = \frac{AD}{DO} \quad \ldots(i)$$

In ΔABC, we have

$EF \parallel BC$ [Given]

Therefore, by basic proportionality theorem, we have

$$\frac{AE}{EB} = \frac{AF}{FC} \quad \ldots(ii)$$

From (i) and (ii), we have

$$\frac{AF}{FC} = \frac{AD}{DO}$$

$\Rightarrow \quad DF \parallel OC$

[By the converse of Basic Proportionality Theorem]

* *Out of Syllabus*

17. Prove the identify:
(sec A – cos A).(cot A + tan A) = tan A.sec A. **3**

Sol. To prove:

(sec A – cos A).(cot A + tan A) = tan A. sec A

We have,

LHS.

(Sec A – cos A).(cot A + tan A)

$$\Rightarrow \left(\frac{1}{\cos A} - \cos A\right).\left(\frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}\right)$$

$$\Rightarrow \left(\frac{1-\cos^2 A}{\cos A}\right).\left(\frac{\cos^2 A + \sin^2 A}{\sin A \cos A}\right)$$

$$\Rightarrow \left(\frac{\sin^2 A}{\cos A}\right).\left(\frac{1}{\sin A \cos A}\right)$$

$$\Rightarrow \left(\frac{\sin A}{\cos A}\right).\left(\frac{1}{\cos A}\right)$$

$\Rightarrow \tan A . \sec A$

LHS = RHS **Hence proved.**

18. Given 2 cos 3θ = $\sqrt{3}$, find the value of θ. **3**

Sol. Given that,

$$2 \cos 3\theta = \sqrt{3}$$

$$\cos 3\theta = \frac{\sqrt{3}}{2}$$

$$\cos 3\theta = \cos 30°$$

On comparing,

$$3\theta = 30°$$
$$\theta = 10°$$

19. For helping poor girls of their class, students saved pocket money as shown in the following table:

Money saved (in ₹)	Number of Students
5 – 7	6
7 – 9	3
9 – 11	9
11 – 13	5
13 – 15	7

Find mean and median for this data. **3**

Sol. We have,

Class Interval	x_i	f_i	$f_i x_i$	c.f.
5 – 7	6	6	36	6
7 – 9	8	3	24	9
9 – 11	10	9	90	18
11 – 13	12	5	60	23
13 – 15	14	7	98	30
Total		$\Sigma f_i = 30$	$\Sigma f_i x_i = 308$	

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{308}{30} = 10.26$$

$$\text{Median} = l + \left[\frac{\frac{N}{2-CF}}{2-CF}\right] \times h$$

where $l = 9$, $\frac{N}{2} = 15$, $CF = 9$, $h = 2$, $f = 9$

On putting values in the formula,

$$\Rightarrow \quad \text{Median} = 9 + \left[\frac{15-9}{9}\right] \times 2$$

$$= 9 + 1.33 = 10.33$$

*** 20. Monthly pocket money of students of a class is given in the following frequency distribution:** **3**

Pocket money (in ₹)	Number of Students
100 – 125	14
125 – 150	8
150 – 175	12
175 – 200	5
200 – 225	11

Find mean pocket money using step deviation method.

SECTION - D

21. If two positive integers x and y are expressible in terms of primes as $x = p^2q^3$ and $y = p^3q$, what can you say about their LCM and HCF. Is LCM a multiple of HCF? Explain. **4**

Sol. Given that,

$$\Rightarrow \quad x = p^2q^3 \text{ and } y = p^3q$$
$$\text{LCM} = p^3q^3$$
$$\text{HCF} = p^2q$$

As there are common factors between HCF and LCM

Hence, HCF is a multiple of the LCM.

22. Sita Devi wants to make a rectangular pond on the road side for the purpose of providing drinking water for street animals. The area of the pond will be decreased by 3 square feet if its length is decreased by 2 ft. and breadth is increased by 1 ft. Its area will be increased by 4 square feet if the length is increased by 1 ft. and breadth remains same. Find the dimensions of the pond. What motivated Sita Devi to provide water point for street animals? **4**

Sol. Let length of the rectangular pond = x ft.

Breadth of rectangular pond = y ft.

Area of rectangular pond = xy

According to the question,

$$(x - 2)(y + 1) = (xy - 3)$$

* *Out of Syllabus*

$$xy + x - 2y - 2 = xy - 3$$
$$x - 2y = -1 \quad \ldots(i)$$
$$(x + 1).y = (xy + 4)$$
$$xy + y = xy + 4$$
$$y = 4 \quad \ldots(ii)$$

Putting the value of y in eq. (i), we get

$$x - 2(4) = -1$$
$$x - 8 = -1$$
$$\Rightarrow \quad x = -1 + 8 = 7$$

So, we get

Length of rectangular pond = 7 ft.

Breadth of rectangular pond = 4 ft.

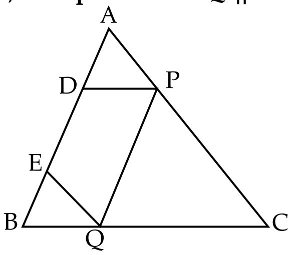
Values:

1. Water is essential for living beings.
2. Animals are also living beings and they also need basic amenities.

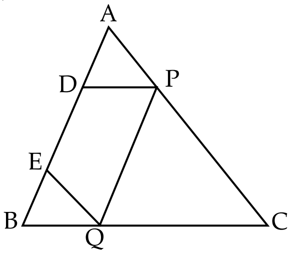
*** 23. If a polynomial $x^4 + 5x^3 + 4x^2 - 10x - 12$ has two zeroes as –2 and –3, then find the other zeroes.**

*** 24. Find all the zeroes of the polynomial $8x^4 + 8x^3 - 18x^2 - 20x - 5$, if it is given that two of its zeroes are $\sqrt{\frac{5}{2}}$ and $-\sqrt{\frac{5}{2}}$.** **4**

25. In the figure, there are two points D and E on side AB of ΔABC such that $AD = BE$. If DP || BC and EQ || AC, then prove that PQ || AB. **4**



Sol. We have,



In ΔABC, we have

DP || BC and EQ || AC

$$\therefore \quad \frac{AD}{DB} = \frac{AP}{PC}$$

$$\Rightarrow \quad \frac{BE}{EA} = \frac{BQ}{QC} \quad \ldots(i)$$

Also, we have

$$\Rightarrow \quad \frac{AD}{DB} = \frac{AP}{PC}$$

$$\Rightarrow \quad \frac{AD}{DB} = \frac{BQ}{QC} \quad \ldots(ii)$$

From equations (i) and (ii), we get

$$\Rightarrow \quad \frac{AP}{PC} = \frac{BQ}{QC}$$

So, in ΔABC, P and Q divide sides CA and CB respectively in the same ratio.

Hence, PQ || AB

[By the converse of Basic Proportionality Theorem]

26. In ΔABC, altitudes AD and CE intersect each other at the point P. Prove that

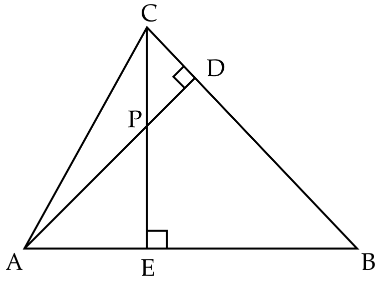
(i) ΔAPE ~ ΔCPD

(ii) $AP \times PD = CP \times PE$

(iii) ΔADB ~ ΔCEB

(iv) $AB \times CE = BC \times$ AD **4**

Sol. We have,



In ΔAEP and ΔCDP,

∠APE = ∠CPD

(Vertically opposite angle)

∠AEP = ∠CDP = 90°

∴ By AA criterion of similarity, ΔAEP ~ ΔCDP

As ΔAEP ~ ΔCDP

So, the ratio of their corresponding sides is equal

$$\text{Hence,} \quad \frac{AP}{CP} = \frac{PE}{PD}$$

$$\text{Also,} \quad AP \times PD = CP \times PE$$

In ΔABD and ΔCBE

∠ADB = ∠CEB = 90°

∠B is common

∴ By AA criterion of similarity, ΔABD ~ ΔCBE

So, the ratio of their corresponding sides are equal

$$\text{Hence,} \quad \frac{AB}{BC} = \frac{AD}{CE}$$

$$\text{Also,} \quad AB \times CE = BC \times AD$$

27. Prove that:

(cot A + sec B)^2 – (tan B – cosec A)^2 = 2 (cot A . sec B + tan B . cosec A). **4**

Sol. $(\cot A + \sec B)^2 - (\tan B - \text{cosec } A)^2 = 2(\cot A . \sec B + \tan B . \text{cosec } A)$

Proof:

Taking LHS,

$(\cot^2 A + \sec^2 B + 2 \cot A \sec B) - (\tan^2 B + \text{cosec}^2 A - 2 \tan B \text{ cosec } A)$

$= \cot^2 A - \text{cosec}^2 A + \sec^2 B - \tan^2 B + 2 \cot A \sec B + 2 \tan B \text{ cosec } A$

$= (-1) + 1 + 2 \cot A \sec B + 2 \tan B \text{ cosec } A$

$= 2 \cot A \sec B + 2 \tan B \text{ cosec } A$

$= 2(\cot A \sec B + \tan B \text{ cosec } A)$

LHS = RHS **Hence, proved.**

* *Out of Syllabus*

28. Prove that: $(\sin\theta + \cos\theta + 1).(\sin\theta - 1 + \cos\theta).\sec\theta.\text{cosec}\,\theta = 2$. **4**

Sol. To prove:

$(\sin\theta + \cos\theta + 1).(\sin\theta - 1 + \cos\theta).(\sec\theta\,\text{cosec}\,\theta) = 2$

Proof:

Taking LHS,

$(\sin\theta + \cos\theta + 1).(\sin\theta - 1 + \cos\theta).(\sec\theta\,\text{cosec}\,\theta)$

$= (\sin^2\theta - \sin\theta + \sin\theta\cos\theta + \cos\theta\sin\theta - \cos\theta + \cos^2\theta + \sin\theta - 1 + \cos\theta)(\sec\theta\,\text{cosec}\,\theta)$

$= (1 - 1 + 2\sin\theta\cos\theta)\left(\frac{1}{\cos\theta}\right)\left(\frac{1}{\sin\theta}\right)$

$= (2\sin\theta\cos\theta)\left(\frac{1}{\cos\theta}\right)\left(\frac{1}{\sin\theta}\right)$

$= 2$

LHS = RHS **Hence proved.**

29. If $\tan(20° - 3\alpha) = \cot(5\alpha - 20°)$, then find the value of a and hence evaluate:
$\sin\alpha.\sec\alpha.\tan\alpha - \text{cosec}\,\alpha.\cos\alpha.\cot\alpha$. **4**

Sol. Given that,

$\Rightarrow \tan(20° - 3\alpha) = \cot(5\alpha - 20°)$

$\Rightarrow \tan(20° - 3\alpha) = \tan(90° - 5\alpha + 20°)$

$\Rightarrow \tan(20° - 3\alpha) = \tan(110° - 5\alpha)$

On comparing,

$\Rightarrow 20° - 3\alpha = 110° - 5\alpha$

$\Rightarrow 2\alpha = 90°$

$\Rightarrow \alpha = 45°$

Now,

$\sin\alpha.\sec\alpha\tan\alpha - \text{cosec}\,\alpha\cos\alpha\cot\alpha$

On putting $\alpha = 45°$

We have,

$\sin 45°\sec 45°\tan 45° - \text{cosec}\,45°\cos 45°\cot 45°$

$= \frac{1}{\sqrt{2}}\times\frac{\sqrt{2}}{1}\times 1 - \frac{\sqrt{2}}{1}\times\frac{1}{\sqrt{2}}\times 1$

$= 1 - 1$

$= 0$

*** 30. The frequency distribution of weekly pocket money received by a group of students is given below:**

Pocket money (in ₹)	Number of Students
More than or equal to 20	90
More than or equal to 40	76
More than or equal to 60	60
More than or equal to 80	55
More than or equal to 100	51
More than or equal to 120	49
More than or equal to 140	33
More than or equal to 160	12
More than or equal to 180	8
More than or equal to 200	4

Draw a 'more than type' ogive and from it, find median. Verify median by actual calculations. **4**

31. Cost of living Index for some period is given in the following frequency distribution:

Index	Number of weeks
1500 – 1600	3
1600 – 1700	11
1700 – 1800	12
1800 – 1900	7
1900 – 2000	9
2000 – 2100	8
2100 – 2200	2

Find the mode and median for above data. **4**

Sol. We have,

Index	Number of weeks (f)	Cumulative frequency (Cf)
1500-1600	3	3
1600-1700	11	14
1700-1800	12	26
1800-1900	7	33
1900-2000	9	42
2000-2100	8	50
2100-2200	2	52
Total	**$N = 52$**	

We have,

Modal class is 1700-1800

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)\times h$$

So, $l = 1700$

$\Rightarrow f_1 = 12, f_0 = 11, f_2 = 7, h = 100$

On putting values,

$$\text{Mode} = 1700 + \frac{1}{6}\times 100$$

$= 1700 + 16.66$

$= 1716.66$

Now, $\frac{N}{2} = 26$

Median class is 1700-1800

$$\text{Median} = l + \left[\frac{\left(\frac{N}{2} - CF\right)}{f}\right]\times h$$

where $l = 1700$, $\frac{N}{2} = 26$, $CF = 14$, $h = 100$, $f = 12$

On putting values,

$$\text{Median} = 1700 + \left[\frac{26 - 14}{12}\right]\times 100$$

$= 1700 + (1)100$

$= 1700 + 100 = 1800$

Hence, median $= 1800$

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* *Out of Syllabus*