Solved Paper 2015
Mathematics (Standard)
CLASS-X

Time : 3 Hours **Max. Marks : 90**

General Instructions:

(i) *All questions are compulsory.*

(ii) *The question paper consists of **31** questions divided into four section A, B, C and D.*

(iii) *Section A contains **4** questions of **1** mark each, Section B contains **6** questions of **2** marks each, Section C contains **10** questions of **3** marks each and Section D contains **11** questions of **4** marks each.*

(iv) *Use of calculators is not permitted.*

Delhi Set I **Code No. 30/1/1**

SECTION - A

Question number 1 to 4 carry 1 mark each.

1. If $x = -\frac{1}{2}$, is a solution of the quadratic equation $3x^2 + 2kx - 3 = 0$, find the value of k.

Sol. Given that,

One solution is $x = -\frac{1}{2}$

Now, putting $x = -\frac{1}{2}$ in given polynomial

We have,

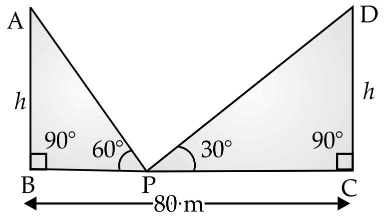
$\Rightarrow \quad \frac{3}{4} - k - 3 = 0$

$\Rightarrow \quad k = \frac{3}{4} - 3$

$\Rightarrow \quad k = -\frac{9}{4}$

2. The tops of two towers of height x and y, standing on level ground, subtend angles of 30° and 60° respectively at the centre of the line joining their feet, then find $x : y$.

Sol.



Given that, Height of tower $AB = x$

And Height of tower $CD = y$

In ΔABE, $\tan 30° = \frac{x}{BE}$

$\frac{1}{\sqrt{3}} = \frac{x}{BE}$

$BE = \sqrt{3}x$ ...(i)

In ΔCDE, $\tan 60° = \frac{y}{ED}$

$\sqrt{3} = \frac{y}{ED}$

$ED = \frac{y}{\sqrt{3}}$ ...(iii)

E is mid points

$\therefore \quad ED = BE$

So from (i) and (ii) we get

$\frac{y}{\sqrt{3}} = \sqrt{3}x$

$\Rightarrow \quad \frac{1}{3} = \frac{x}{y}$

Hence Ratio of $x : y = 1 : 3$

3. A letter of English alphabet is chosen at random. Determine the probability that the chosen letter is a consonant.

Sol. As we know there are 21 consonants in English Alphabets

So, Required probability $= \frac{21}{26}$

4. In Fig. 1, PA and PB are tangents to the circle with centre O such that ∠APB = 50°. Write the measure of ∠OAB.

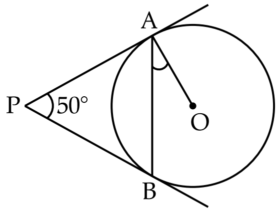


Fig. 1

Sol. We know that

ΔPAB is Isosceles

$\because \quad PB = PB$

(Tangent to circle from an external point)

$\therefore \quad \angle PAB = \angle PAB = x°$

Now, $\angle APB + \angle PAB + \angle PBA = 180°$

$50° + 2x = 180°$

$x = 65°$

So, $\angle PAB = 65°$

As, $\angle PAO = 90°$

(Tangent is perpendicular to point of contact)

$\therefore \quad \angle OAB = 90° - 65°$

$= 25°$

SECTION - B

Question numbers 5 to 10 carry 2 marks each.

5. In Fig. 2, AB is the diameter of a circle with centre O and AT is a tangent. If ∠AOQ = 58°, find ∠ATQ.

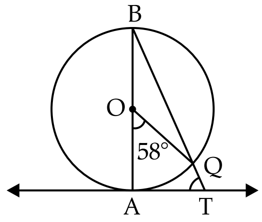


Fig. 2

Sol. $\angle ABQ = \frac{1}{2}\angle AOQ = 29°$

In ΔABT

$\angle ATQ + \angle ABQ + \angle BAT = 180°$

(Angle sum property)

$\angle ATQ = 180° - (\angle ABQ + \angle BAT)$

$= 180° - 119° = 61°$

6. Solve the following quadratic equation for *x*:

$$\mathbf{4x^2 - 4a^2x + (a^4 - b^4) = 0.}$$

Sol. The given quadratic equation can be written as

$(4x^2 - 4a^2x + a^4) - b^4 = 0$

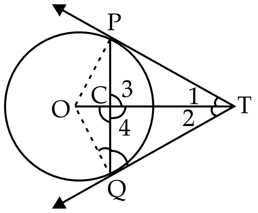
or $(2x - a^2)^2 - (b^2)^2 = 0$

$\therefore \quad (2x - a^2 + b^2)(2x - a^2 - b^2) = 0$

$\Rightarrow \quad x = \frac{a^2 - b^2}{2}, \frac{a^2 + b^2}{2}$

7. From a point T outside a circle of centre O, tangents TP and TQ are drawn to the circle. Prove that OT is the right bisector of line segment PQ.

Sol.



In Δs' TPC and TQC

$TP = TQ$ (Tangent)

$TC = TC$ (Common)

$\angle 1 = \angle 2$ (TP and TQ are equally inclined to OT)

$\therefore \quad \Delta TPC \cong \Delta TQC$ (SAS)

$\therefore \quad PC = QC$ and $\angle 3 = \angle 4$ (CPCT)

But $\angle 3 + \angle 4 = 180°$

$\Rightarrow \quad \angle 3 = \angle 4 = 90°$

$\therefore$ OT is the right bisector of PQ

8. Find the middle term of the A.P. 6, 13, 20, ..., 216.

Sol. The given A.P. is 6, 13, 20, ..., 216

Let n be the number of terms, $d = 7$, $a = 6$

$\therefore \quad 216 = 6 + (n - 1).7$

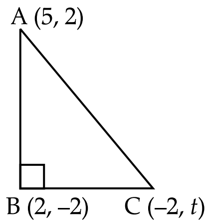
$\Rightarrow \quad n = 31$

$\therefore$ Middle term is 16^{th}

$\therefore \quad a_{16} = 6 + 15 \times 7 = 111$

9. If A(5, 2), B(2, –2) and C(–2, *t*) are the vertices of a right angled triangle with ∠B = 90°, then find the value of *t*.

Sol.



ABC is right triangle

$\therefore \quad AC^2 = BC^2 + AB^2$

$AB^2 = (2 - 5)^2 + (-2 - 2)^2 = 25$

$\Rightarrow \quad AB = 5$

$BC^2 = (-2 - 2)^2 + (t + 2)^2$

$= 16 + (t + 2)^2$

$AC^2 = (-2 - 5)^2 + (t - 2)^2$

$= 49 + (t - 2)^2$

$\therefore \quad 49 + (2 - t)^2 = 41 + (t - 2)^2$

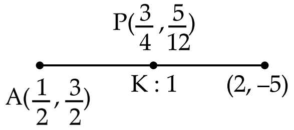
$(t - 2)^2 - (2 - t)^2 = 8$

$4 \times 2t = 8$

$\Rightarrow \quad t = 1$

10. Find the ratio in which the point $P\left(\frac{3}{4}, \frac{3}{12}\right)$ divides the line segment joining the points $A\left(\frac{1}{2}, \frac{3}{2}\right)$ and B(2, –5).

Sol.



Let P divide AB in the ratio of $k : 1$

$$\therefore \quad \frac{2K + \frac{1}{2}}{K+1} = \frac{3}{4}$$

$$\Rightarrow \quad 8K + 2 = 3K + 3$$

$$\Rightarrow \quad K = \frac{1}{5}$$

$\therefore$ Required ratio = 1 : 5

SECTION - C

Question numbers 11 to 20 carry 3 marks each.

*** 11. Find the area of the triangle ABC with A(1, –4) and mid-points of sides through A being (2, –1) and (0, –1).**

12. Find the non-zero value of k, for which the quadratic equation $kx^2 + 1 - 2(k - 1)x + x^2 = 0$ has equal roots. Hence find the roots of the equation.

Sol. The given quadratic eqn. can be written as

$$(k + 1)x^2 - 2(k - 1)x + 1 = 0$$

For equal roots $4(k - 1)^2 - 4(k + 1) = 0$

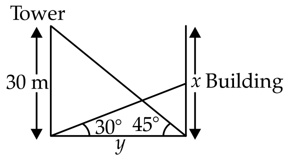
or $\quad k^2 - 3k = 0$

$\Rightarrow \quad k = 0, 3$

$\therefore$ Non-zero value of $k = 3$: Roots are $\frac{1}{2}, \frac{1}{2}$

13. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 45°. If the tower is 30 m high, find the height of the building.

Sol.



(i) $\frac{30}{y} = \tan 45° = 1$

$\Rightarrow \quad y = 30$

(ii) $\frac{x}{y} = \tan 30° = \frac{1}{\sqrt{3}}$

$$\Rightarrow \quad x = \frac{y}{\sqrt{3}} = \frac{30}{\sqrt{3}}$$

$$= 10\sqrt{3}$$

$\therefore$ Height of building $10\sqrt{3}$ m

* *Out of Syllabus*

14. Two different dice are rolled together. Find the probability of getting:

(i) the sum of numbers on two dice to be 5.

(ii) even numbers on both dice.

Sol. Total possible out comes = 36

(i) The possible outcomes are (2, 3), (3, 2), (1, 4), (4, 1) : Number : 4

$\therefore$ Required Probability $= \frac{4}{36} = \frac{1}{9}$

(ii) The possible outcomes are

(2, 2), (2, 4), (2, 6), (4, 2), (4, 4), (4, 6), (6, 2), (6, 4), (6, 6)

Number of favorable outcomes = 9

$\therefore$ Required Probability $= \frac{9}{36} = \frac{1}{4}$

15. If S_n, denotes the sum of first n terms of an A.P., prove that $S_{12} = 3(S_8 - S_4)$.

Sol. Let a be the first term and d the common difference

$$S_{12} = 6[2a + 11d] = 12a + 66d$$
$$S_8 = 4[2a + 7d] = 8a + 28d$$
$$S_4 = 2[2a + 3d] = 4a + 6d$$
$$3(S_8 - S_4) = 3(4a + 22d)$$
$$= 12a + 66d = S_{12}$$

Hence Proved

16. In Fig. 3, APB and AQO are semicircles, and $AO = OB$. If the perimeter of the figure is 40 cm, find the area of the shaded region. $\left[\text{Use } \pi = \frac{22}{7}\right]$

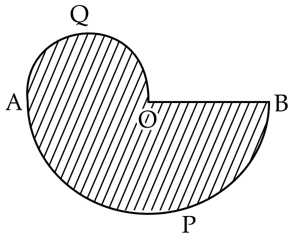


Fig. 3

Sol. Let $\quad OA = OB = r$

$$\therefore \quad 40 = \frac{22}{7} \times \frac{r}{2} + \frac{22}{7} \times r + r$$

$$\Rightarrow \quad 280 = 40r$$

$$r = 7 \text{ cm}$$

$$\therefore \text{ Shaded area} = \left(\frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} + \frac{1}{2} \times \frac{22}{7} \times 7 \times 7\right) \text{cm}^2$$

$$= \left(77 \times \frac{5}{4}\right) \text{cm}^2$$

or $\quad \frac{385}{4} \text{ cm}^2 = 96\frac{1}{4} \text{ cm}^2$

*** 17. In Fig. 4, from the top of a solid cone of height 12 cm and base radius 6 cm, a cone of height 4 cm is removed by a plane parallel to the base. Find the total surface area of the remaining solid.**

$\left(\text{Use } \pi = \frac{22}{7} \text{ and } \sqrt{5} = 2.236\right)$

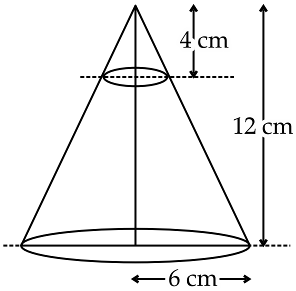
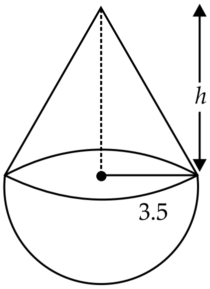


Fig. 4

18. A solid wooden toy in the form of a hemisphere surmounted by a cone of same radius. The radius of hemisphere is 3.5 cm and the total wood used in the making of toy is $166\frac{5}{6}$ cm^3. Find the height of the toy. Also, find the cost of painting the hemispherical part of the toy at the rate of ₹ 10 per cm^2. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Sol.



Volume of solid wooden toy

$$166\frac{5}{6} = \frac{2}{3}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times\frac{7}{2}+\frac{1}{3}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times h$$

or $$\frac{1001}{6} = \frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}[7+h]$$

$$\Rightarrow \quad 7 + h = \frac{1001\times7}{22\times7} = 13$$

$$\Rightarrow \quad h = 6 \text{ cm}$$

Area of hemispherical part of toy

$$= \left(2\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\right) \text{cm}^2$$

$$= 77 \text{ cm}^2$$

$\therefore$ Cost of painting = ₹ (77 × 10) = ₹ 770

19. In Fig. 5, from a cuboidal solid metallic block, of dimensions 15 cm × 10 cm × 5 cm, a cylindrical hole of diameter 7 cm is drilled out. Find the surface area of the remaining block. $\left[\text{Use } \pi = \frac{22}{7}\right]$

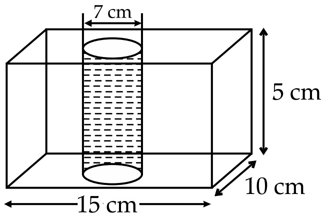


Fig. 5

Sol. Total surface area of solid cuboidal block

$$= 2(15 \times 10 + 10 \times 5 + 15 \times 5) \text{ cm}^2$$
$$= 550 \text{ cm}^2$$

Area of two circular bases

$$= 2\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2} = 77 \text{ cm}^2$$

Area of curved surface of cylinder

$$= 2\pi rh = 2\times\frac{22}{7}\times\frac{7}{2}\times5$$
$$= 110 \text{ cm}^2$$

Required area = $(550 + 110 - 77)$ cm^2

$$= 583 \text{ cm}^2$$

20. In Fig. 6, find the area of the shaded region [Use π = 3.14]

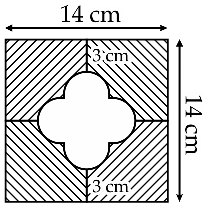
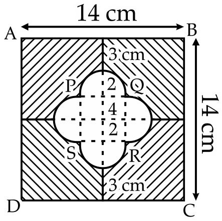


Fig. 6

Sol.



Area of Sq. ABCD = 14^2 or 196 cm^2

Area of Small Sq. = 4^2 or 16 cm^2

Area of 4 semi circles = $\left[4.\frac{1}{2}3.14(2)^2\right]$ cm^2

$$= 25.12 \text{ cm}^2$$

$\therefore$ Required area = $(196 - 16 - 25.12)$ cm^2

$$= 154.88 \text{ cm}^2$$

* *Out of Syllabus*

SECTION - D

Question numbers 21 to 31 carry 4 marks each.

21. The numerator of a fraction is 3 less than its denominator. If 2 is added to both the numerator and the denominator, then the sum of the new fraction and original fraction is $\frac{29}{20}$. Find the original fraction.

Sol. Let the fraction be $\frac{x-3}{x}$

By the given condition, new fraction

$$\frac{x-3+2}{x+2} = \frac{x-1}{x+2}$$

$$\therefore \quad \frac{x-3}{x}+\frac{x-1}{x+2} = \frac{29}{20}$$

$\Rightarrow$ $20[(x-3)(x+2)+x(x-1)] = 29(x^2+2x)$

$$20(x^2-x-6+x^2-x) = 29x^2+58x$$

or $\quad 11x^2-98x-120 = 0$

or $\quad 11x^2-110x-12x-120 = 0$

$$(11x+12)(x-10) = 0$$

$\Rightarrow \quad x = 10$

$\therefore$ The Fraction is $\frac{7}{10}$

22. Ramkali required ₹ 2500 after 12 weeks to send her daughter to school. She saved ₹ 100 in the first week and increased her weekly saving by ₹ 20 every week. Find whether she will be able to send her daughter to school after 12 weeks.

What value is generated in the above situation?

Sol. Money required for Ramkali for admission of daughter = ₹ 2500

A.P. formed by saving

100, 120, 140, ... upto 12 terms

$$\text{Sum of AP} = \frac{12}{2}[2\times 100+11\times 20]$$

$$= 6[420]$$

$$= ₹\ 2520$$

$\therefore$ She can get her daughter admitted

Value : Small saving can fulfill your big desires or any else

23. Solve for x:

$$\frac{2}{x+1}+\frac{3}{2(x-2)} = \frac{23}{5x},\ x\neq 0, -1, 2$$

Sol.
$$\frac{2}{x+1}+\frac{3}{2(x-2)} = \frac{23}{5x}$$

or $5x[4(x-2)+3x+3] = 46(x+1)(x-2)$

$$5x(7x-5) = 46(x^2-x-2)$$

$\Rightarrow \quad 11x^2-21x-92 = 0$

$\Rightarrow \quad x = \frac{21\pm\sqrt{441+4048}}{22}$

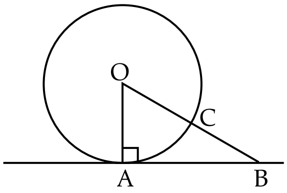
$$= \frac{21\pm 67}{22}$$

$$= 4, \frac{-23}{11}$$

24. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

Sol. Correctly stated

Given:



$OA = OC$ (Radii of circle)

Now $\quad OB = OC + BC$

$\therefore \quad OB > OC$ (OC being radius and B any point on tangent)

$\Rightarrow \quad OA < OB$

B is an arbitrary point on the tangent.

Thus, OA is shorter than any other line segment joining O to any point on tangent.

Shortest distance of a point from a given line is the perpendicular distance from that line.

Hence, the tangent at any point of circle is perpendicular to the radius.

25. In Fig. 7, tangents PQ and PR are drawn from an external point P to a circle with centre O, such that ∠RPQ = 30°. A chord RS is drawn parallel to the tangent PQ.

Find ∠RQS.

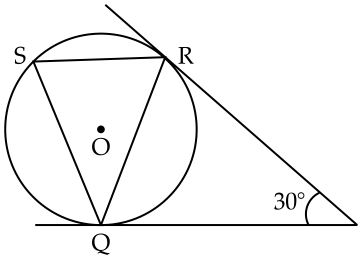
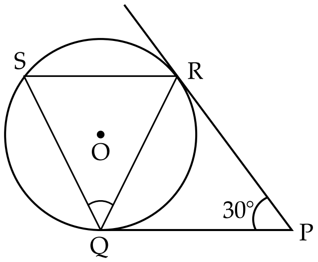


Fig. 7

Sol.



Join OR and OQ

$$PR = PQ$$

$\Rightarrow \quad \angle PRQ = \angle PQR$

$$= \frac{(180-30)^\circ}{2} = 75^\circ$$

$SR \parallel QP$ and QR is a transversal

$\Rightarrow \quad \angle SRQ = 75^\circ$

$\therefore \quad \angle ORQ = \angle RQO$

$= 90^\circ - 75^\circ = 15^\circ$

$\therefore \quad \angle QOR = (180 - 2 \times 15)^\circ = 150^\circ$

$\Rightarrow \quad \angle QSR = 75^\circ$

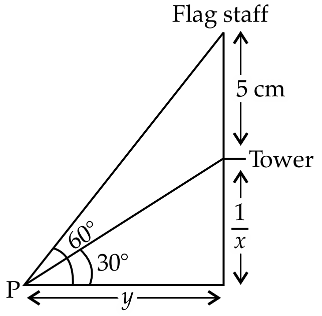
$\angle RQS = 180^\circ - (\angle SRQ + \angle QSR)$

$= 30^\circ$

*** 26. Construct a triangle ABC with BC = 7 cm, ∠B = 60° and AB = 6 cm. Construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of △ABC.**

27. From a point P on the ground the angle of elevation of the top of a tower is 30° and that of the top of a flag staff fixed on the top of the tower, is 60°. If the length of the flag staff is 5 m, find the height of the tower.

Sol.



Writing the trigonometric equations

(i) $$\frac{x}{y} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$\Rightarrow \quad y = \sqrt{3}x$

(ii) $$\frac{x+5}{y} = \tan 60^\circ = \sqrt{3}$$

Subsitute value of y

or $$\frac{x+5}{\sqrt{3}x} = \sqrt{3}$$

$\Rightarrow \quad 3x = x + 5$

or $\quad x = 2.5$

$\therefore$ Height of Tower = 2.5 m

28. A box contains 20 cards numbered from 1 to 20. A card is drawn at random from the box. Find the probability that the number on the drawn card is

(i) divisible by 2 or 3

(ii) a prime number

Sol. (i) Numbers divisible by 2 or 3 from 1 to 20 are 2, 4, 6, 8, 10, 12, 14, 16, 18, 3, 9, 15, 20 Their number is 13

$\therefore$ Required Probability $= \frac{13}{20}$

(ii) Prime numbers from 1 to 20 are 2, 3, 5, 7, 11, 13, 17, 19 : 8 in number

$\therefore$ Required Probability $= \frac{8}{20}$ or $\frac{2}{5}$

*** 29. If A(–4, 8), B(–3, –4), C(0, –5) and D(5, 6) are the vertices of a quadrilateral ABCD, find its area.**

30. A well of diameter 4 m is dug 14 m deep. The earth taken out is spread evenly all around the well to form a 40 cm high embankment. Find the with of the embankment.

Sol. Volume of earth taken out after digging the well

$$= \left(\frac{22}{7} \times 2 \times 2 \times 14\right) \text{ cu. m}$$

$= 176$ cu. m ...(i)

Let x be the width of embankment formed by using (i)

Volume of embankment = Volume of earth taken out

$$\frac{22}{7}[(2+x)^2 - (2)^2] \times \frac{40}{100} = 176$$

$\Rightarrow \quad x^2 + 4x - 140 = 0$

$\Rightarrow \quad (x + 14)(x - 10) = 0$

$\Rightarrow \quad x = 10$

$\therefore$ Width of embankment = 10 m

31. Water is flowing at the rate of 2.52 km/h through a cylindrical pipe into a cylindrical tank, the radius of whose base is 40 cm, If the increase in the level of water in the tank, in half an hour is 3.15 m, find the internal diameter of the pipe.

Sol. Let x m be the internal radius of the pipe

Radius of base of tank = 40 cm = $\frac{2}{5}$ m

Level of water raised in the tank = 3.15 or $\frac{315}{100}$

2.52 km/hour ⇒ 1.26 km in half hour = 1260 m

$\therefore$ Getting the equation

Volume of pipe = Volume of tank

$$\pi x^2.1260 = \pi.\frac{2}{5}.\frac{2}{5} \times \frac{315}{100}$$

$$\Rightarrow \quad x^2 = \frac{4}{25}.\frac{315}{100} \times \frac{1}{1260}$$

$$= \frac{1}{2500}$$

$$\Rightarrow \quad x = \frac{1}{50} \text{ m} = 2 \text{ cm}$$

$\therefore$ Internal diameter of pipe = 4 cm

* *Out of Syllabus*

Delhi Set II — Code No. 30/1/2

SECTION - B

10. Find the middle term of the A.P. 213, 205, 197, ..., 37.

Sol. Here $a = 213, d = -8, a_n = 37$, where n is the number of terms

$$\therefore \quad 37 = 213 + (n-1)(-8)$$

$$\frac{-176}{-8} = n - 1$$

$$\Rightarrow \quad n = 23$$

$$\therefore \quad \text{Middle term} = a_{12} = 213 + 11(-8) = 125$$

SECTION - C

*** 14. In Fig. 4, from the top of a solid cone of height 12 cm and base radius 6 cm, a cone of height 4 cm is removed by a plane parallel to the base. Find the total surface area of the remaining solid.**

$$\left(\textbf{Use } \pi = \frac{22}{7} \textbf{ and } \sqrt{5} = 2.236\right)$$

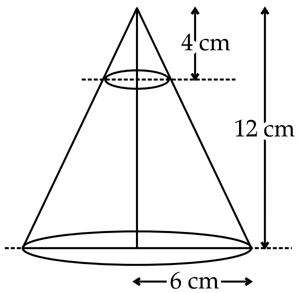


Fig. 4

18. If the sum of the first n terms of an A.P. is $\frac{1}{2}(3n^2 + 7n)$, then find its n^{th} term.

Sol.

$$S_n = \frac{1}{2}(3n^2 + 7n)$$

$$\Rightarrow \quad S_1 = a_1 = \frac{1}{2}(10) = 5$$

$$S_2 = \frac{1}{2}(26) = 13$$

$$a_2 = S_2 - S_1$$

$$\Rightarrow \quad a_2 = 8$$

$\therefore$ It is an A.P. with $a = 5$ and $d = 3$

$$\therefore \quad a_n = 5 + (n-1)3 = 3n + 2$$

19. Three distinct coins are tossed together. Find the probability of getting

(i) at least 2 heads

(ii) at most 2 heads

Sol. The total number of possible outcomes = 8

(i) $P(\text{at least two heads}) = \frac{4}{8} = \frac{1}{2}$

(ii) $P(\text{at most two heads}) = \frac{7}{8}$

20. Find the value of p for which the quadratic equation $(p + 1)x^2 - 6(p + 1)x + 3(p + 9) = 0$, $p \neq -1$ has equal roots. Hence find the roots of the equation.

Sol. For the given quadratic equation to have equal roots

$$D = 0$$

$$\Rightarrow \quad b^2 - 4ac = 0$$

Here $b = 6(p + 1)$, $a = (p + 1)$ and $C = 3(p + 9)$

$$[6(p+1)]^2 - 4(p+1).3(p+9) = 0$$

$$\text{or} \quad 36(p+1)^2 - 12(p+1)(p+9) = 0$$

$$12(p+1)[3p + 3 - p - 9] = 0$$

As $p \neq -1$, $2p = 6$ or $p = 3$

Roots are 3, 3

SECTION - D

28. To fill a swimming pool two pipes are to be used. If the pipe of larger diameter is used for 4 hours and the pipe of smaller diameter for 9 hours, only half the pool can be filled. Find, how long it would take for each pipe to fill the pool separately, if the pipe of smaller diameter takes 10 hours more than the pipe of larger diameter to fill the pool.

Sol. Let the bigger pipe fills the tank in x hours

$\therefore$ the smaller pipe fills the tanks in $(x + 10)$ hours

$$\therefore \quad \frac{4}{x} + \frac{9}{x+10} = \frac{1}{2}$$

$$\Rightarrow \quad 2(13x + 40) = x^2 + 10x$$

$$\text{or} \quad x^2 - 16x - 80 = 0$$

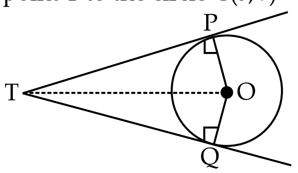
$$\Rightarrow \quad (x - 20)(x + 4) = 0$$

$$\Rightarrow \quad x = 20$$

the pipe with larger diameter fills the tank in 20 hours and the pipe with smaller diameter fills the tank in 30 hour

29. Prove that the lengths of tangents drawn from an external point to a circle are equal.

Sol. Given: PT and TQ are two tangents drawn from an external point T to the circle C(o, r)



To prove: $PT = TQ$

Proof: We know that a tangent to the circle is $\perp$ to the radius through the point of contact.

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* *Out of Syllabus*

So, In ΔOPT and $\angle$OQT,

$$OT = OT \quad \text{(common)}$$

$$\angle OPT = \angle OQT = 90°$$

(Tangent and radius are perpendicular at point of contact)

$$OP = OQ = \text{radius}$$

$\therefore$ $\Delta OPT \cong \Delta OQT$ (RHS congruence)

$\therefore$ $PT = TQ$ (by c.p.c.t)

So, length of the tangents drawn from an external point to circle are equal.

*** 30. Diameter takes 10 hours more than the pipe of separately, if the pipe of smaller larger diameter to fill the pool.**

*** 31. If P(–5, –3), Q(–4, –6), R(2, –3) and S(1, 2) are the vertices of a quadrilateral PQRS, find its area.**

Delhi Set III — Code No. 30/1/3

SECTION - B

10. Solve the following quadratic equation for x:
$9x^2 - 6b^2x - (a^4 - b^4) = 0$

Sol. The given quadratic equation can be written as

$$(9x^2 - 6b^2x + b^4) - a^4 = 0$$

$$\text{or } (3x - b^2)^2 - (a^2)^2 = 0$$

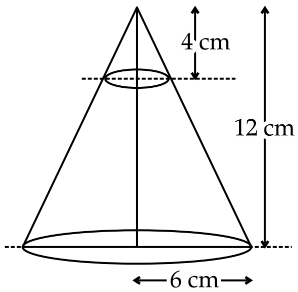
or $\quad (3x - b^2 + a^2)(3x - b^2 - a^2) = 0$

$$\Rightarrow \quad x = \frac{b^2 - a^2}{3}, \frac{b^2 + a^2}{3}$$

SECTION - C

*** 13. In Fig. 4, from the top of a solid cone of height 12 cm and base radius 6 cm, a cone of height 4 cm is removed by a plane parallel to the base. Find the total surface area of the remaining solid.**

$\left(\text{Use } \pi = \frac{22}{7} \text{ and } \sqrt{5} = 2.236\right)$



18. All red face cards are removed from a pack of playing cards. The remaining cards were well shuffled and then a card is drawn at random from them. Find the probability that the drawn card is

(i) a red card

(ii) a face card

(iii) a card of clubs

Sol. Number of redface cards removed = 6

$\therefore$ Remaining cards = 46

Total Red cards = 26

Remaining Red cards = 20

(i) $$P(\text{a redcard}) = \frac{20}{46} \text{ or } \frac{10}{23}$$

(ii) Remaining Face card = 12 – 6 = 6

$$P(\text{a facecard}) = \frac{6}{46} \text{ or } \frac{3}{23}$$

(iii) $$P(\text{a card of clubs}) = \frac{13}{46}$$

20. If S_n denotes the sum of first n terms of an A.P., prove that $S_{30} = 3[S_{20} - S_{10}]$.

Sol. Let a be the first term and d the common difference of the A.P.

$$S_{30} = 15[2a + 29d] = 30a + 435d$$

$$S_{20} = 10[2a + 19d] = 20a + 190d$$

$$S_{10} = 5[2a + 9d] = 10a + 45d$$

$$3(S_{20} - S_{10}) = 3(10a + 145d) = 30a + 435d = S_{30}$$

SECTION - D

28. A 21 m deep well with diameter 6 m is dug and the earth from digging is evenly spread to from a platform 27 m × 11 m. Find the height of the platform. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Sol. Volume of earth taken out after digging the well

$$= \left(\frac{22}{7} \times 3 \times 3 \times 21\right) \text{ cu. m}$$

$$= 594 \text{ cu. m}$$

Let h be the height of the platform

Volume of platform = Volume of earth

$$11 \times 27 \times h = 594$$

$$\therefore \quad h = \frac{594}{27 \times 11}$$

$\therefore$ Height of platform = 2 m

29. A bag contains 25 cards numbered from 1 to 25. A card is drawn at random from the bag. Find the probability that the number on the drawn card is:

(i) divisible by 3 or 5

(ii) a perfect square number

Sol. (i) Number of numbers divisible by 3 or 5 in numbers 1 to 25

(3, 6, 9, 12, 15, 18, 21, 24, 5, 10, 20, 25) : their number is 12

* *Out of Syllabus*

$$P(\text{divisible by 3 or 5}) = \frac{12}{25}$$

(ii) No. of favourable outcomes = 5

p(a Perfect square number)

$$= \frac{5}{25} = \frac{1}{5} \quad (1, 4, 9, 16, 25)$$

*** 30. Draw a line segment AB of length 7 cm. Taking A as centre, draw a circle of radius 3 cm and taking B as centre, draw another circle of radius 2 cm. Construct tangents to each circle from the centre of the other circle.**

31. Solve for x:

$$\frac{3}{x+1} + \frac{4}{x-1} = \frac{29}{4x-1}; \; x \neq 1, \; -1, \; \frac{1}{4}$$

Sol. $$\frac{3}{x+1} + \frac{4}{x-1} = \frac{29}{4x-1}$$

$$[3(x-1) + 4(x+1)][4x-1] = 29(x^2-1)$$

$$(7x+1)(4x-1) = 29x^2 - 29$$

$$28x^2 - 3x - 1 = 29x^2 - 29$$

or $$x^2 + 3x - 28 = 0$$

$$(x+7)(x-4) = 0$$

$\Rightarrow$ $$x = -7, 4$$

Outside Delhi Set I — Code No. 30/1

SECTION - A

Question number 1 to 4 carry 1 mark each.

1. If the quadratic equation $px^2 - 2\sqrt{5}px + 15 = 0$ has two equal roots, then find the value of p.

Sol. We know that,

For equal roots,

Discriminant = 0

$\Rightarrow$ $20p^2 - 4(p)(15) = 0$

$\Rightarrow$ $20p^2 = 60p$

$\Rightarrow$ $p = 3$

2. In Figure 1, a tower AB is 20 m high and BC, its shadow on the ground, is $20\sqrt{3}$ m long. Find the Sun's altitude.

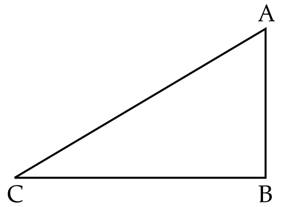


Fig. 1

Sol. Given that,

$$\tan C = \frac{AB}{BC}$$

$$\tan C = \frac{20}{20\sqrt{3}}$$

$$\tan C = \frac{1}{\sqrt{3}}$$

$$C = 30° \quad \left(\because \tan 30° = \frac{1}{\sqrt{3}}\right)$$

So, Sun's altitude is 30°.

3. Two different dice are tossed together. Find the probability that the product of the two numbers on the top of the dice is 6.

Sol. For product of numbers being 6

favorable number of outcomes = 4

(2, 3) (3, 2) (1, 6) (6, 1)

$$\text{Required Probability} = \frac{4}{36} = \frac{1}{9}$$

4. In Figure 2, PQ is a chord of a circle with centre O and PT is a tangent. If $\angle QPT = 60°$, find $\angle PRQ$.

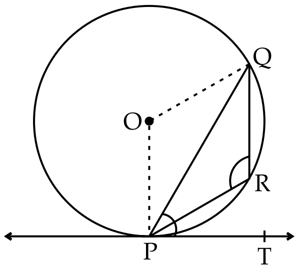


Fig. 2

Sol. We have, $\angle OPT = 90°$

(Tangent and radius are perpendicular at point of contact)

So, $\angle OPQ = \angle OPT - \angle OPR$

$= 90° - 60°$

$(\because \angle QPT = 60°$ given)

$= 30°$

$OP = OQ$ (radii)

$\therefore$ $\angle OPQ = \angle OQP = 30°$

(Angle opposite to equal side)

$\Rightarrow$ $\angle POQ = 180° - (30° + 30°)$

$= 120°$

Minor arc angle $POQ = 120°$

$\Rightarrow$ Major arc $\angle POQ = 360° - 120°$

$= 240°$

Angle subtended by an arc at centre is double the angle subtended by it on remaining part of circle.

So, $$\angle PRQ = \frac{1}{2} \times 240° = 120°$$

* *Out of Syllabus*

SECTION - B

Question numbers 5 to 10 carry 2 marks each.

5. In Figure 3, two tangents RQ and RP are drawn from an external point R to the circle with centre O. If $\angle PRQ = 120°$, then prove that $OR = PR + RQ$.

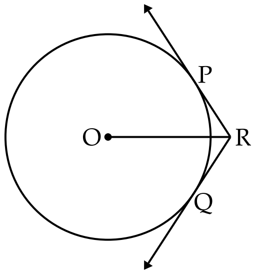


Fig. 3

Sol. Join OP and OQ

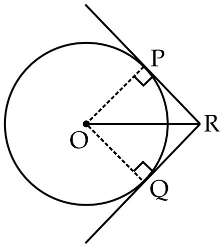
$$\angle OPR = \angle OQR = 90°$$

(Tangent and radius are ⊥ at point of contact)

$$\angle POR = 90° - 60° = 30°$$

$$\frac{PR}{OR} = \sin 30°$$

$$= \frac{1}{2}$$



$\Rightarrow$ $OR = 2\text{ PR}$

As, $PR = QR$ (tangents)

So, $OR = PR + QR$

6. In Figure 4, a triangle ABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If the area of ΔABC is 54 cm², then find the lengths of sides AB and AC.

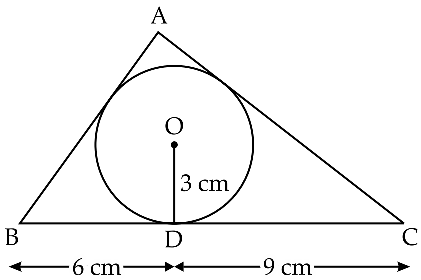


Fig. 4

Sol. Let $AF = AE = x$

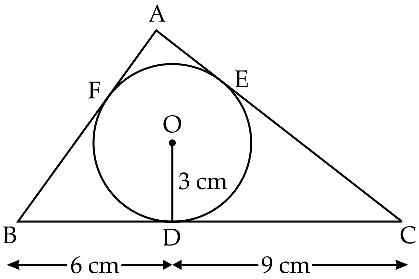
$BF = BD = 6$

$\therefore$ $AB = 6 + x$

$DC = CE = 9$

$\therefore$ $AC = 9 + x$

$BC = 15$



Area of $\Delta ABC = 54$

Area of $\Delta ABC = \text{Area}(\Delta BOC) + (\Delta AOB) + (\Delta AOC)$

$$= \frac{1}{2} \times BC \times OD + \frac{1}{2} \times AB \times OF$$

$$= \frac{1}{2} \times AC \times OE$$

$$\frac{1}{2}[15+6+x+9+x].3 = 54$$

$\Rightarrow$ $x = 3$

$\therefore$ $AB = 9$ cm,

$AC = 12$ cm

and $BC = 15$ cm

7. Solve the following quadratic equation for x:

$$4x^2 + 4bx - (a^2 - b^2) = 0$$

Sol. $4x^2 + 4bx + b^2 - a^2 = 0$

$\Rightarrow$ $(2x + b)^2 - (a)^2 = 0$

$\Rightarrow$ $(2x + b + a)(2x + b - a) = 0$

$$x = -\frac{a+b}{2}$$

$$x = \frac{a-b}{2}$$

8. In an AP, if $S_5 + S_7 = 167$ and $S_{10} = 235$, then find the AP, where S_n denotes the sum of its first n terms.

Sol. $S_5 + S_7 = 167$

$\Rightarrow$ $167 = \frac{5}{2}[2a+4d]+\frac{7}{2}[2a+6d]$

$24a + 62d = 334$

or $12a + 31d = 167$ ...(i)

$S_{10} = 235$

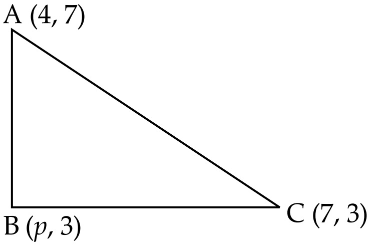
$5[2a + 9d] = 235$

or $2a + 9d = 47$ ...(ii)

Solving (i) and (ii) to get $a = 1, d = 5$. Hence AP is 1, 6, 11, .

9. The points A(4, 7), B(p, 3) and C(7, 3) are the vertices of a right triangle, right-angled at B. Find the value of p.

Sol.



Here, $AB^2 + BC^2 = AC^2$

$\Rightarrow (-4)^2 + (p-4)^2 + (7-p)^2 + (0) = (3)^2 + (-4)^2$

$\Rightarrow \quad p = 7 \text{ or } 4$

since $p \neq 7 \therefore p = 4$

*** 10. Find the relation between x and y if the points A(x, y), B(– 5, 7) and C(– 4, 5) are collinear.**

SECTION - C

Question numbers 11 to 20 carry 3 marks each.

11. The 14th term of an AP is twice its 8th term. If its 6th term is – 8, then find the sum of its first 20 terms.

Sol.

$$a_{14} = 2a_8$$
$$\Rightarrow a + 13d = 2(a + 7d)$$
$$\Rightarrow a = -d$$
$$\Rightarrow a_6 = -8$$
$$\Rightarrow a + 5d = -8$$

Solving to get $a = 2, d = -2$

$$S_{20} = 10(2a+19d) = 10(4-38) = -340$$

12. Solve for x:

$$\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$$

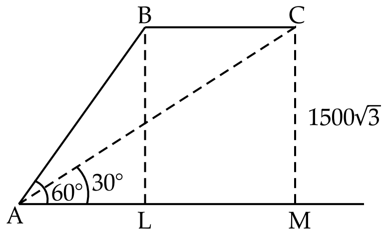
Sol.

$$\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$$
$$\Rightarrow \sqrt{3}x^2 - 3\sqrt{2}x + \sqrt{2}x - 2\sqrt{3} = 0$$
$$\Rightarrow \left(x-\sqrt{6}\right)\left(\sqrt{3}x+\sqrt{2}\right) = 0$$
$$\Rightarrow x = \sqrt{6},\ x = -\sqrt{\frac{2}{3}}$$

13. The angle of elevation of an aeroplane from a point A on the ground is 60°. After a flight of 15 seconds, the angle of elevation changes to 30°. If the aeroplane is flying at a constant height of 1500 $\sqrt{3}$ m, find the speed of the plane in km/hr.

Sol. Let $AL = x$

$$\therefore \frac{BL}{x} = \tan 60°$$



$$\Rightarrow \frac{1500\sqrt{3}}{x} = \sqrt{3}$$
$$\Rightarrow x = 1500 \text{ m}$$
$$\frac{CM}{AL+LM} = \tan 30° = \frac{1}{\sqrt{3}}$$
$$\Rightarrow 1500 + LM = 1500(3) = 4500$$
$$\Rightarrow LM = 3000 \text{ m}$$
$$\therefore \text{Speed} = \frac{3000}{15} = 200 \text{ m/s.} = 720 \text{ km/hr.}$$

14. If the coordinates of points A and B are (– 2, – 2) and (2, – 4) respectively, find the coordinates of P such that AP = $\frac{3}{7}$ AB, where P lies on the line segment AB.

Sol.

A	P(x, y)	B
(–2, –2)	3 : 4	(2, –4)

$$AP = \frac{3}{7}AB$$
$$AP : PB = 3 : 4$$
$$\therefore x = \frac{6-8}{7} = -\frac{2}{7}$$
$$y = \frac{-12-8}{7} = -\frac{20}{7}$$
$$\therefore \text{Coordinates of } P = P\left(-\frac{2}{7}, -\frac{20}{7}\right)$$

15. The probability of selecting a red ball at random from a jar that contains only red, blue and orange balls is $\frac{1}{4}$. The probability of selecting a blue ball at random from the same jar is $\frac{1}{3}$. If the jar contains 10 orange balls, find the total number of balls in the jar.

Sol.

$$P(\text{Red}) = \frac{1}{4},$$
$$P(\text{blue}) = \frac{1}{3}$$
$$\Rightarrow P\ (\text{orange}) = 1 - \frac{1}{4} - \frac{1}{3} = \frac{5}{12}$$

$\Rightarrow \quad \frac{5}{12}\times(\text{Total no. of balls}) = 10$

$\Rightarrow \quad \text{Total no. of balls} = \frac{10\times12}{5} = 24$

16. Find the area of the minor segment of a circle of radius 14 cm, when its central angle is 60°. Also find the area of the corresponding major segment. [Use $\pi = \frac{22}{7}$]

Sol. $r = 14$ cm, $\theta = 60°$

$$\text{Area of minor segment} = \pi r^2 \frac{\theta}{360} - \frac{1}{2}r^2 \sin\theta$$

$$= \frac{22}{7}\times14\times14\times\frac{60°}{360°} - \frac{1}{2}\times14\times14\times\frac{\sqrt{3}}{2}$$

$$= \left(\frac{308}{3} - 49\sqrt{3}\right)\text{cm}^2$$

or 17.89 cm^2 or 17.9 cm^2 Approx.

Area of major segment

$$= \pi r^2 - \left(\frac{308}{3} - 49\sqrt{3}\right)$$

$$= \left(\frac{1540}{3} + 49\sqrt{3}\right)\text{cm}^2$$

or 598.10 cm^2 or 598 cm^2 Approx.

17. Due to sudden floods, some welfare associations jointly requested the government to get 100 tents fixed immediately and offered to contribute 50% of the cost. If the lower part of each tent is of the form of a cylinder of diameter 4.2 m and height 4 m with the conical upper part of same diameter but of height 2.8 m, and the canvas to be used costs ₹ 100 per sq. m, find the amount, the associations will have to pay. What values are shown by these associations ? [Use $\pi = \frac{22}{7}$]

Sol. $\text{Slant height } (l) = \sqrt{(2.8)^2 + (2.1)^2} = 3.5 \text{ cm.}$

$\therefore$ Area of canvas for one tent

$$= 2\times\frac{22}{7}\times(2.1)\times4 + \frac{22}{7}\times2.1\times3.5$$

Area $= 6.6\ (8 + 3.5)$

$= 6.6 \times 11.5$ m^2

$\therefore$ Area of 100 tents $= 66 \times 115$ m^2

Cost of 100 tents $= 66 \times 115 \times 100$

50% Cost $= 33 \times 11500$

$=$ ₹ 379500

Value: Helping the flood victims

18. A hemispherical bowl of internal diameter 36 cm contains liquid. This liquid is filled into 72 cylindrical bottles of diameter 6 cm. Find the height of the each bottle, if 10% liquid is wasted in this transfer.

Sol. $\text{Volume of liquid in the bowl} = \frac{2}{3}.\pi.(18)^3\text{cm}^3$

$\text{Volume, after wastage} = \frac{2\pi}{3}.(18)^3.\frac{90}{100}\text{cm}^3$

$\text{Volume of liquid in 72 bottles} = \pi(3)^2.h.72 \text{ cm}^3$

$$\Rightarrow \quad h = \frac{\frac{2}{3}\pi(18)^3.\frac{9}{10}}{\pi(3)^2.72}$$

$= 5.4$ cm.

19. A cubical block of side 10 cm is surmounted by a hemisphere. What is the largest diameter that the hemisphere can have ? Find the cost of painting the total surface area of the solid so formed, at the rate of ₹ 5 per 100 sq. cm. [Use $\pi = 3.14$]

Sol. Largest possible diameter of hemisphere = 10 cm.

$\therefore$ radius $= 5$ cm

Total surface area $= 6(10)^2 + 3.14 \times (5)^2$

Cost of painting $= \frac{678.5\times5}{100}$

$= \frac{₹\ 3392.50}{100}$

$=$ ₹ 33.9250

$=$ ₹ 33.93

20. 504 cones, each of diameter 3.5 cm and height 3 cm, are melted and recast into a metallic sphere. Find the diameter of the sphere and hence find its surface area. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Sol. Volume of metal in 504 cones

$$= 504\times\frac{1}{3}\times\frac{22}{7}\times\frac{35}{20}+\frac{35}{20}\times3 \text{ cm}$$

Volume of sphere so formed

= Volume of 504 cones

$$\therefore \quad \frac{4}{3}\times\frac{22}{7}\times r^3 = 504\times\frac{1}{3}\times\frac{22}{7}\times\frac{35}{20}\times\frac{35}{20}\times3$$

$r = 10.5$ cm

$\therefore$ diameter $= 21$ cm

$\text{Surface area of sphere} = 4\times\frac{22}{7}\times\frac{21}{2}\times\frac{21}{2}$

$= 1386$ cm^2

SECTION - D

Question numbers 21 to 31 carry 4 marks each.

21. The diagonal of a rectangular field is 16 metres more than the shorter side. If the longer side is 14 metres more than the shorter side, then find the lengths of the sides of the field.

Sol. Let the length of shorter side be x m.

$\therefore$ length of diagonal $= (x + 16)$ m and, lenght of longer side = $(x + 14)$ m

$\therefore \quad x^2 + (x + 14)^2 = (x + 16)^2$

(Using Pythagoras theorem)

$\Rightarrow \quad x^2 - 4x - 6 = 0$

$\Rightarrow \quad x = 10$ m.

$\therefore$ length of sides are 10 m and 24 m.

22. Find the 60th term of the AP 8, 10, 12, ..., if it has a total of 60 terms and hence find the sum of its last 10 terms.

Sol. $t_{60} = 8 + 59(2) = 126$

Sum of last 10 terms $= (t_{51} + t_{52} + ...+ t_{60})$

$t_{51} = 8 + 50(2) = 108$

Using the formula $S_n = \frac{n}{2}[a + l]$

$\therefore$ Sum of last 10 terms $= 5[108 + 126]$

$= 1170$

23. A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 6 km/h more than the first speed. If it takes 3 hours to complete the total journey, what is its first speed ?

Sol. Let the original average speed of (first) train be x km./h.

$\therefore \quad \frac{54}{x} + \frac{63}{x+6} = 3$

$\Rightarrow \quad 54x + 324 + 63x = 3x(x + 6)$

$\Rightarrow \quad x^2 - 33x - 108 = 0$

Solving to get $x = 36$

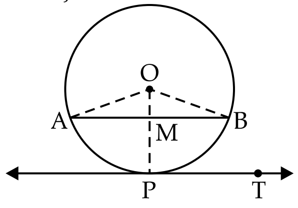
$\therefore$ First speed of train = 36 km/h.

* **24. Prove that the lengths of the tangents drawn from an external point to a circle are equal.**

25. Prove that the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc.

Sol. To prove: $AB \parallel PT$

Construction: join OA, OB, & OP



Proof:

$\Rightarrow \quad OP \perp PT$ [Radius is $\perp$ to tangent through a point of contact]

$\Rightarrow \quad \angle OPT = 90°$

Since P is the midpoint of Arc APB

$\Rightarrow \quad \text{Arc}(AP) = \text{arc}(BP)$

$\Rightarrow \quad \angle AOP = \angle BOP$

$\Rightarrow \quad \angle AOM = \angle BOM$

$\Rightarrow$ In ΔAOM & ΔBOM

$\Rightarrow \quad OA = OB = r$

$\Rightarrow \quad OM = OM$ (Common)

$\Rightarrow \quad \angle AOM = \angle BOM$ (proved above)

$\Rightarrow \quad \Delta AOM \cong \Delta BOM$

(by SAS congruence axiom)

$\Rightarrow \quad \angle AMO = \angle BMO$ (C.P.C.T)

$\Rightarrow \quad \angle AMO + \angle BMO = 180°$ (Linear pair)

$\Rightarrow \quad \angle AMO = \angle BMO = 90°$

$\Rightarrow \quad \angle BMO = \angle OPT = 90°$

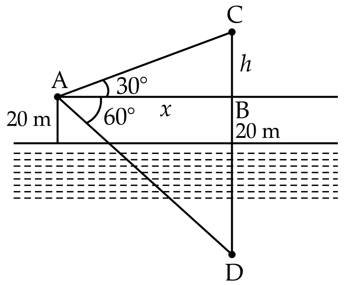
As, they are corresponding angles.

Hence, $AB \parallel PT$

* **26. Construct a ΔABC in which $AB = 6$ cm, $\angle A = 30°$ and $\angle B = 60°$. Construct another $\angle$AB'C' similar to $\angle$ABC with base $AB' = 8$ cm.**

27. At a point A, 20 metres above the level of water in a lake, the angle of elevation of a cloud is 30°. The angle of depression of the reflection of the cloud in the lake, at A is 60°. Find the distance of the cloud from A.

Sol.



$\frac{h}{x} = \tan 30°$

$= \frac{1}{\sqrt{3}}$

$\Rightarrow \quad x = \sqrt{3}\,h$

$\frac{40 + h}{x} = \tan 60°$

$= \sqrt{3}$

$\Rightarrow \quad x = \frac{40 + h}{\sqrt{3}}$

$\therefore \quad \sqrt{3}h = \frac{40 + h}{\sqrt{3}}$

$\Rightarrow \quad h = 20$ m

$\therefore \quad x = 20\sqrt{3}$ m

$\therefore \quad AC = \sqrt{(20)^2 + (20\sqrt{3})^2}$

$= 40$ m

Distance of Cloud from A = 40 m

28. A card is drawn at random from a well-shuffled deck of playing cards.

Find the probability that the card drawn is

(i) a card of spade or an ace.

(ii) a black king.

(iii) neither a jack nor a king.

(iv) either a king or a queen.

* *Out of Syllabus*

Sol. (i) P(spade or an ace) $= \dfrac{13+3}{52} = \dfrac{4}{13}$

(ii) P(a black king) $= \dfrac{2}{52} = \dfrac{1}{26}$

(iii) P(neither a jack nor a king) $= \dfrac{52-8}{52} = \dfrac{44}{52} = \dfrac{11}{13}$

(iv) P(either a king or a queen) $= \dfrac{4+4}{52} = \dfrac{8}{52} = \dfrac{2}{13}$

*** 29. Find the values of k so that the area of the triangle with vertices (1, – 1), (– 4, 2k) and (– k, – 5) is 24 sq. units.**

30. In Figure 5, PQRS is a square lawn with side PQ = 42 metres. Two circular flower beds are there on the sides PS and QR with centre at O, the intersection of its diagonals. Find the total area of the two flower beds (shaded parts).

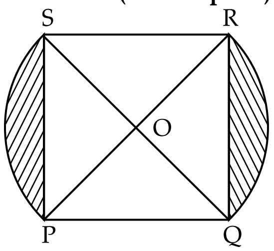


Fig. 5

Sol. Radius of circle with centre O is OR

Let $OR = x$

$\therefore \quad x^2 + x^2 = (42)^2$

$\Rightarrow \quad x = 21\sqrt{2}$ m

Area of one flower bed = Area of segment of circle with centre angle 90°

$= \dfrac{22}{7} \times 21\sqrt{2} \times 21\sqrt{2} \times \dfrac{90}{360} - \dfrac{1}{2} \times 21\sqrt{2} \times 21\sqrt{2}$

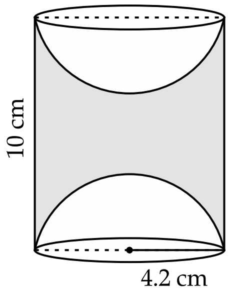
$= 693 - 441 = 252$ m^2

$\therefore$ Area of two flower beds = 2 × 252 = 504 m^2

31. From each end of a solid metal cylinder, metal was scooped out in hemispherical form of same diameter. The height of the cylinder is 10 cm and its base is of radius 4.2 cm. The rest of the cylinder is melted and converted into a cylindrical wire of 1.4 cm thickness. Find the length of the wire.

$\left[\textbf{Use } \pi = \dfrac{22}{7}\right]$

Sol.



Total volume of cylinder $= \dfrac{22}{7} \times \dfrac{42}{10} \times \dfrac{42}{10} \times 10$ cm^3

$= 554.40$ cm^3

Volume of metal scooped out $= \dfrac{4}{3} \times \dfrac{22}{7} \times \left(\dfrac{42}{10}\right)^3$

$= 310.46$ cm^3

$\therefore$ Volume of rest of cylinder = 554.40 – 310.46

$= 243.94$ cm^3

If l is the length of were, then

$\dfrac{22}{7} \times \dfrac{7}{10} \times \dfrac{7}{10} \times l = \dfrac{24394}{100}$

$\Rightarrow \quad l = 158.4$ cm

Outside Delhi Set II — Code No. 30/2

SECTION - B

Question numbers 5 to 10 carry 2 marks each.

10. If A(4, 3), B(–1, y) and C(3, 4) are the vertices of a right triangle ABC, right-angled at A, then find the value of y.

Sol. Here $AB^2 + AC^2 = BC^2$

(Using Pythagoras Theorem)

$(-1-4)^2 + (y-3)^2 + (3-4)^2 + (4-3)^2 = (3+1)^2 + (4-y)^2$

$-5^2 + y^2 + 9 - 6y + 1 + 1 = 16 + 16 + y^2 - 8y$

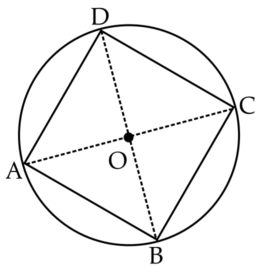
$\Rightarrow \quad y = -2$

SECTION - C

Question numbers 11 to 20 carry 3 marks each.

18. All the vertices of a rhombus lie on a circle. Find the area of the rhombus, if the area of the circle is 1256 cm^2. [Use π = 3.14]

Sol. $AB = BC = CD = AD$



$\Rightarrow \quad AC = BD = 2r$

$3.14\, r^2 = 1256$

$r = 20$ cm

Area of rhombus $= \dfrac{1}{2} \times 40 \times 40$

$= 800$ cm^2

* *Out of Syllabus*

19. Solve for x:

$$2x^2 + 6\sqrt{3}x - 60 = 0$$

Sol. Given equation can be written as

$$x^2 + 3\sqrt{3}x - 30 = 0$$

$$\Rightarrow \quad x^2 + 5\sqrt{3}x - 2\sqrt{3}x - 30 = 0$$

$$\Rightarrow \quad (x + 5\sqrt{3})(x - 2\sqrt{3}) = 0$$

$$\Rightarrow \quad x = -5\sqrt{3}, 2\sqrt{3}$$

20. The 16th term of an AP is five times its third term. If its 10th term is 41, then find the sum of its first fifteen terms.

Sol. $a_{16} = 5a_3$

$\Rightarrow \quad a + 15d = 5(a + 2d)$

$\Rightarrow \quad 4a = 5d \quad$ ...(i)

$\Rightarrow \quad a_{10} = 41$

$\Rightarrow \quad a + 9d = 41 \quad$ ...(ii)

Solving (i) and (ii), we get $a = 5, d = 4$

$$S_{15} = \frac{15}{2}(10 + 14 \times 4)$$

$$= 495$$

SECTION - D

Question numbers 21 to 31 carry 4 marks each.

28. A bus travels at a certain average speed for a distance of 75 km and then travels a distance of 90 km at an average speed of 10 km/h more than the first speed. If it takes 3 hours to complete the total journey, find its first speed.

Sol. Let the first average speed of the bus be x km./h.

$\therefore \quad \frac{75}{x} + \frac{90}{x+10} = 3$

$\Rightarrow \quad 75x + 750 + 90x = 3(x^2 + 10x)$

$\Rightarrow \quad x^2 - 45x - 250 = 0$

Solving to get $x = 50$

$\therefore \quad$ Speed $= 50$ km/h.

* **29. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.**

* **30. Construct a right triangle ABC with $AB = 6$ cm, $BC = 8$ cm and $\angle B = 90°$. Draw BD, the perpendicular from B on AC. Draw the circle through B, C and D and construct the tangents from A to this circle.**

* **31. Find the values of k so that the area of the triangle with vertices $(k + 1, 1)$, $(4, -3)$ and $(7, -k)$ is 6 sq. units.**

Outside Delhi Set III — Code No. 30/3

SECTION - B

Question numbers 5 to 10 carry 2 marks each.

10. Solve the following quadratic equation for x:

$$x^2 - 2ax - (4b^2 - a^2) = 0$$

Sol. Given equation can be written as

$x^2 - 2ax + a^2 - 4b^2 = 0$

or $\quad (x - a)^2 - (2b)^2 = 0$

$\therefore (x - a + 2b)(x - a - 2b) = 0$

$\Rightarrow \quad x = a - 2b, x = a + 2b$

SECTION - C

Question numbers 11 to 20 carry 3 marks each.

18. The 13th term of an AP is four times its 3rd term. If its fifth term is 16, then find the sum of its first ten terms.

Sol. $a_{13} = 4a_3$

$\Rightarrow \quad a + 12d = 4[a + 2d]$

$\Rightarrow \quad 3a = 4d \quad$ ...(i)

$a_5 = 16$

$\Rightarrow \quad a + 4d = 16 \quad$ ...(ii)

Solving (i) and (ii) to get $a = 4$ and $d = 3$

$$S_{10} = 5(8 + 27) = 175$$

19. Find the coordinates of a point P on the line segment joining A(1, 2) and B(6, 7) such that $AP = \frac{2}{5}AB$.

Sol.

A	P	B
(1, 2)	2 : 3 (x, y)	(6, 7)

$$AP = \frac{2}{5}AB$$

$\Rightarrow \quad AP : PB = 2 : 3$

$\therefore \quad x = \frac{12+3}{5} = 3,$

$y = \frac{14+6}{5} = 4$

$P(x, y) = (3, 4)$

20. A bag contains, white, black and red balls only. A ball is drawn at random from the bag. If the probability of getting a white ball is $\frac{3}{10}$ and that of a black ball is $\frac{2}{5}$, then find the probability of getting a red ball. If the bag contains 20 black balls, then find the total number of balls in the bag.

* *Out of Syllabus*

Sol. $P(W) = \frac{3}{10}, \; P(B) = \frac{2}{5}$

$\therefore \quad P(R) = 1 - \frac{3}{10} - \frac{2}{5} = \frac{3}{10}$

$\frac{2}{5}$(Total no. of balls) $= 20$

$\Rightarrow$ Total no. of balls $= \frac{20 \times 5}{2} = 50$

SECTION - D

Question numbers 21 to 31 carry 4 marks each.

28. A truck covers a distance of 150 km at a certain average speed and then covers another 200 km at an average speed which is 20 km per hour more than the first speed. If the truck covers the total distance in 5 hours, find the first speed of the truck.

Sol. Let the first average speed of truck be x km/h.

$\therefore \quad \frac{150}{x} + \frac{200}{x+20} = 5$

$\Rightarrow \quad 150x + 3000 + 200x = 5(x^2 + 20x)$

$\Rightarrow \quad x^2 - 50x - 600 = 0$

Solving to get $x = 60$

$\therefore$ speed $= 60$ km/h

29. An arithmetic progression 5, 12, 19, ... has 50 terms. Find its last term. Hence find the sum of its last 15 terms.

Sol. $a_{50} = 5 + 49\,(7)$

$= 5 + 343 = 348$

$a_{36} = 5 + 35\,(7)$

$= 250$

$S_{(\text{last 15 term})} = \frac{n}{2}[a_{36} + a_{50}]$

Required Sum $= \frac{15}{2}.[250 + 348]$

$= \frac{15}{2}(598)$

$= 4485$

* **30. Construct a triangle ABC in which AB = 5 cm, BC = 6 cm and $\angle ABC$ = 60°. Now construct another triangle whose sides are $\frac{5}{7}$ times the corresponding sides of $\angle$ ABC.**

31. Find the values of k for which the points A(k + 1, 2k), B(3k, 2k + 3) and C(5k – 1, 5k) are collinear.

Sol. Here $(k + 1)(2k + 3 - 5k) + 3k(5k - 2k) + (5k - 1)(2k - 2k - 3) = 0$

$\Rightarrow \quad 6k^2 - 15k + 6 = 0$

or $\quad 2k^2 - 5k + 2 = 0$

Solving to get $k = 2$ or $k = +\frac{1}{2}$

❑❑

* *Out of Syllabus*