Solved Paper 2016

Mathematics (Standard)

CLASS-X

Time : 3 Hours **Max. Marks : 90**

General Instructions :

(i) All questions are compulsory.

*(ii) This question paper consists of **31** questions divided into four sections – A, B, C and D.*

*(iii) Section A contains **4** questions of **1** mark each. Section B contains **6** questions of **2** marks each. Section C contains **10** questions of **3** marks each. Section D contains **11** questions of **4** marks each.*

*(v) Use of calculator is **not** permitted.*

Delhi Set-I **Code No. 30/1/1**

SECTION - A

Question numbers 1 to 4 carry 1 mark each.

1. From an external point P, tangents PA and PB are drawn to a circle with centre O. If $\angle PAB = 50°$, then find $\angle AOB$.

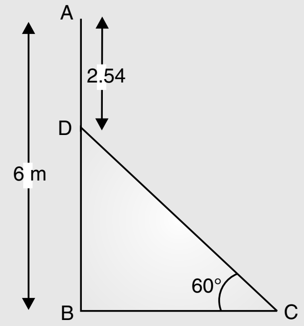
Sol. $\angle APB = 80°$ ½

$\therefore$ $\angle AOB = 100°$ ½

(CBSE Marking Scheme, 2016)

2. In Fig., AB is a 6 m high pole and CD is a ladder inclined at an angle of 60° to the horizontal and reaches up to a point D of pole. If $AD = 2.54$ m, find the length of the ladder. (use $\sqrt{3} = 1.73$)

Sol.



$$AB = 6 \text{ m}$$
$$AD = 2.54 \text{ m}$$
$$DB = AB - AD$$
$$= 6 - 2.54 = 3.46 \text{ m}$$
$$\sin 60° = \frac{DB}{DC}$$
$$DC = \frac{DB}{\sin 60°}$$
$$DC = \frac{3.46}{\frac{\sqrt{3}}{2}}$$
$$DC = \frac{3.46}{1.73} \times 2 = 2 \times 2 = 4 \text{ m} \quad 1$$

(CBSE Marking Scheme, 2016)

3. Find the 9th term from the end (towards the first term) of the A.P. 5, 9, 13, ..., 185.

Sol.
$l = 185, d = -4$ ½
$$l_9 = d + (n-1)\,d$$
$$= 185 - 8 \times 4$$
$l_9 = 153$ ½

(CBSE Marking Scheme, 2016)

4. Cards marked with number and 3, 4, 5, ..., 50 are placed in a box and mixed thoroughly. A card is drawn at random from the box. Find the probability that the selected card bears a perfect square number.

Sol. Possible outcomes are 4, 9, 16, 25, 36, 49, i.e., 6. **1**

$\therefore$ P(perfect square number) $= \frac{6}{48}$ or $\frac{1}{8}$ **1**

(CBSE Marking Scheme, 2016)

SECTION - B

Question numbers 5 to 10 carry 2 marks each.

5. If $x = \frac{2}{3}$ and $x = -3$ are roots of the quadratic equation $ax^2 + 7x + b = 0$, find the values of a and b.

Sol. $\frac{-7}{a} = \frac{2}{3} - 3$ ½

$\Rightarrow$ $a = 3$

and $\frac{b}{a} = \frac{2}{3} \times (-3)$

$\Rightarrow$ $b = -6$ ½

(CBSE Marking Scheme, 2016)

6. Find the ratio in which y-axis divides the line segment joining the points A(5, −6) and B(−1, −4). Also, find the coordinates of the point of division.

Sol. Let the point on y-axis be $(0, y)$ and $AP : PB = k : 1$ ½

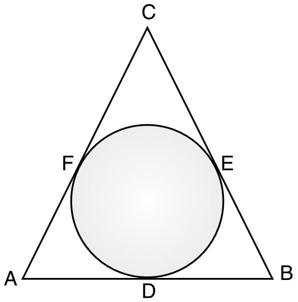
Therefore, $\frac{5-k}{k+1} = 0$ gives $k = 5$ ½

Hence, required ratio is 5 : 1

$$y = \frac{-4(5)-6}{6} = \frac{-13}{3} \quad \frac{1}{2}$$

Hence, point on y-axis is $\left(0, \frac{-13}{3}\right)$. ½

7. In fig., a circle is inscribed in a ΔABC, such that it touches the sides AB, BC and CA at points D, E and F respectively. If the lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively, find the lengths of AD, BE and CF.



Sol. Let $AD = AF = x$

$\therefore$ $DB = BE = 12 - x$

and $CF = CE = 10 - x$

$BC = BE + EC$

$\Rightarrow$ $8 = 12 - x + 10 - x$

$\Rightarrow$ $x = 7$ **1**

$\therefore AD = 7$ cm, $BE = 5$ cm, $CF = 3$ cm **1**

8. The x-co-ordinate of a point P is twice its y-co-ordinate. If P is equidistant from $Q(2, -5)$ and $R(-3, 6)$, find the co-ordinates of P.

Sol. Let the point P be $(2y, y)$

$PQ = PR$ ½

$\Rightarrow \sqrt{(2y-2)^2+(y+5)^2} = \sqrt{(2y+3)^2+(y-6)^2}$ ½

$4y^2 - 8y + 4 + y^2 + 10y + 25$

$= 4y^2 + 12y + 9 + y^2 - 12y + 36$

Solving to get $y = 8$ ½

Hence, co-ordinates of points P are (16, 8). ½

9. How many terms of the A.P. 18, 16, 14, ... be taken so that their sum is zero?

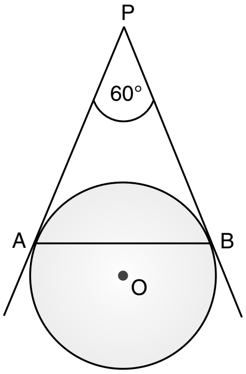
Sol. Here, $a = 18$, $d = -2$, $S_n = 0$ ½

Therefore, $\frac{n}{2}[36 + (n-1)(-2)] = 0$ **1**

$n(-2n + 38) = 0$

$\Rightarrow$ $n = 19$ ½

10. In fig., AP and BP are tangents to a circle with centre O, such that AP = 5 cm and $\angle APB$ = 60°. Find the length of chord AB.



Sol. $PA = PB$ (tangents) ½

$\Rightarrow$ $\angle PAB = \angle PBA = 60°$ ½

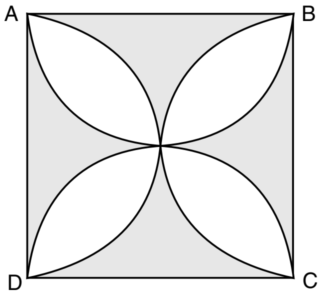
$\therefore \Delta PAB$ is an equilateral triangle. ½

Hence, $AB = PA = 5$ cm. ½

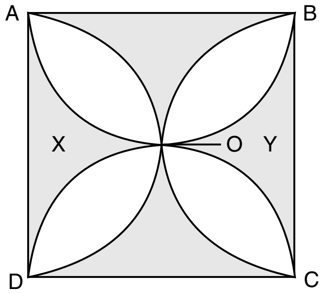
SECTION - C

Question numbers 11 to 20 carry 3 marks each.

11. In fig., ABCD is a square of side 14 cm. Semi-circles are drawn with each side of square as diameter. Find the area of the shaded region. $\left(\text{use}\,\pi = \frac{22}{7}\right)$



Sol.



Area of square $= 196$ cm^2 **1**

Area of semicircles $AOB + DOC = \frac{22}{7} \times 49$

$= 154$ cm^2 **1**

Hence, area of two shaded parts $(X + Y)$

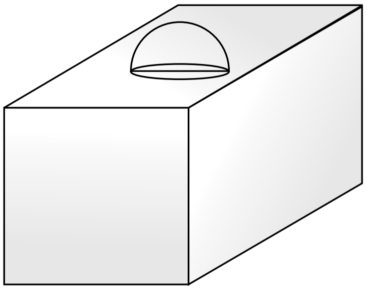
Area of square – Area of semicircles AOB and DOC

$= 196 - 154$

$= 42$ cm^2 **1**

Therefore, area of four shaded parts = 84 cm^2.

12. In fig., a decorative block, made up of two solids – a cube and a hemisphere. The base of the block is a cube of side 6 cm and the hemisphere fixed on the top has a diameter of 3.5 cm. Find the total surface area of the block. $\left(\text{use}\,\pi = \frac{22}{7}\right)$



Sol. Surface area of block

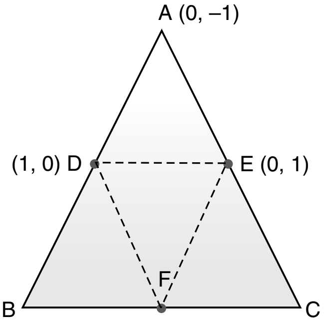
$= 216 - \frac{22}{7}\times\frac{3.5}{2}\times\frac{3.5}{2} + 2\times\frac{22}{7}\times\frac{3.5}{2}\times\frac{3.5}{2}$ **2**

$= 216 + \frac{22}{7}\times\frac{3.5}{2}\times\frac{3.5}{2}$

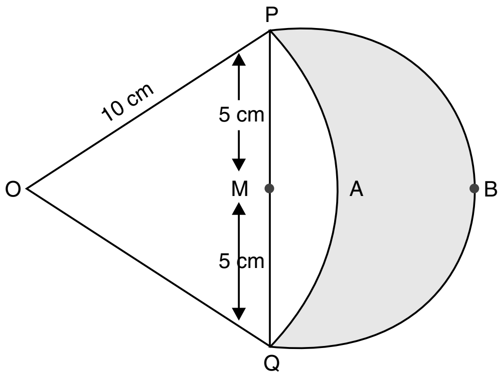
$= 216 + 9.42.$

$= 225.42 \text{ cm}^2$ **1**

*** 13. In fig., *ABC* is a triangle, co-ordinates of whose vertex *A* is (0, –1). *D* and *E* respectively are the mid-points of the sides *AB* and *AC* and their co-ordinates are (1, 0) and (0, 1) respectively. If *F* is the mid-point of *BC*, find the areas of ΔABC and ΔDEF.**



14. In fig., are shown two arcs *PAQ* and *PBQ*. Arc *PAQ* is a part of circle with centre *O* and radius *OP* while arc *PBQ* is a semi-circle drawn on *PQ* as diameter with centre *M*. If *OP* = *PQ* = 10 cm, show that area of shaded region is $25\left(\sqrt{3}-\frac{\pi}{6}\right)$ cm².



Sol. $\angle POQ = 60°$

($\because$ ΔPOQ is equilateral triangle)

Area of segment $PAQM = \left(\frac{100\pi}{6} - \frac{100\sqrt{3}}{4}\right) \text{cm}^2$ **1**

Area of semicircle $= \frac{25\pi}{2} \text{ cm}^2$ ½

Area of shaded region $= \frac{25\pi}{2} - \left(\frac{50\pi}{3} - 25\sqrt{3}\right)$ **1**

$= 25\left(\sqrt{3} - \frac{\pi}{6}\right) \text{cm}^2$ ½

15. If the sum of first 7 terms of an A.P. is 49 and that of its first 17 terms is 289, find the sum of first *n* terms of the A.P.

Sol. $S_7 = 49$

$\frac{7}{2}(2a + 6d) = 49$ ½

$\Rightarrow$ $2a + 6d = 14$

and $S_{17} = 289$

$\frac{17}{2}(2a + 16d) = 289$ **1**

$\Rightarrow 2a + 16d = 34$ ½

Solving equations to get $a = 1$ and $d = 2$

Hence $S_n = \frac{n}{2}[2 + (n-1)2] = n^2$ **1**

16. Solve for x :

$$\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0, x \neq 3, \frac{-3}{2}$$

Sol. $2x(2x + 3) + (x - 3) + (3x + 9) = 0$ 1

$\Rightarrow$ $2x^2 + 5x + 3 = 0$ **1**

$2x^2 + 2x + 3x + 3 = 0$

$\Rightarrow$ $(x + 1)(2x + 3) = 0$ ½

$\Rightarrow$ $x = -1, x = -\frac{3}{2}$ ½

17. A well of diameter 4 m is dug 21 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 3 m to form an embankment. Find the height of the embankment.

Sol. Volume of earth dug out

$= \pi \times 2 \times 2 \times 21 = 264 \text{ m}^3$ **1**

As, Volume of earth dug out

= Volume of embankment

And, Volume of embankment

$= \pi(25 - 4) \times h = 66h \text{ m}^3$ **1**

$\therefore$ $66h = 264$ ½

$\Rightarrow$ $h = 4$ m ½

18. The sum of the radius of base and height of a solid right circular cylinder is 37 cm. If the total surface area of the solid cylinder is 1628 sq. cm, find the volume of the cylinder.

Sol. Here $r + h = 37$ and $2\pi r(r + h) = 1628$ ½+½

$\Rightarrow$ $2\pi r = \frac{1628}{37}$

* *Out of Syllabus*

$$r = \frac{1628 \times 7}{2 \times 22 \times 37}$$

$\Rightarrow$ $r = 7$ cm ½

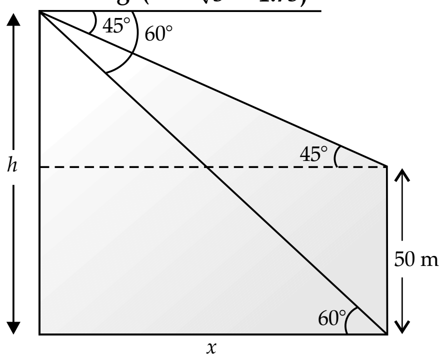
and $h = 30$ cm ½

Hence, volume of cylinder $= \frac{22}{7} \times 7 \times 7 \times 30$

$= 4620$ cm^3 **1**

19. The angles of depression of the top and bottom of a 50 m high building from the top of a tower are 45° and 60° respectively. Find the height of the tower and the horizontal distance between the tower and the building. (use $\sqrt{3}$ =1.73)

Sol.



1

$$\tan 45° = \frac{h-50}{x}$$

$\Rightarrow$ $x = h - 50$ ½

$$\tan 60° = \frac{h}{x} \Rightarrow x = \frac{h}{\sqrt{3}}$$

$$\tan 45° = \frac{h-50}{x}$$

$x = h - 50$ – (i) ($\because$ tan 45° = 1)

$\tan 60° = \frac{h}{x}$ ($\because \tan 60° = \sqrt{3}$)

$\sqrt{3}x = h$ ...(ii)

By substituting value of x we get

$$\sqrt{3}(h-50) = h$$

$$\sqrt{3}h - 50\sqrt{3} = h$$

$$\sqrt{3}h - h = 50\sqrt{3}$$

$$(\sqrt{3}-1)h = 50\sqrt{3}$$

$$h = \frac{50\sqrt{3}}{\sqrt{3}-1}$$

$$h = \frac{50\sqrt{3}(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$= \frac{50(3+1.732)}{2}$$

$$h = \frac{50 \times 4.732}{2}$$

$\therefore$ $h = 118.30$

Now put value of 'h' is equation (i)

$x = 118.30 - 50$

$x = 68.30$ m

Hence height of tower = 118.30 m

Distance between tower and building

= 68.30 m

20. In a single throw of a pair of different dice, what is the probability of getting (i) a prime number on each dice ? (ii) a total of 9 or 11 ?

Sol. **(i)** Favourable outcomes are (2, 2) (2, 3) (2, 5) (3, 2) (3, 3) (3, 5) (5, 2) (5, 3) (5, 5) i.e., 9 outcomes. ½

P(a prime number on each die) $= \frac{9}{36}$ or $\frac{1}{4}$. ½

(ii) Favourable outcomes are (3, 6), (4, 5), (5, 4), (6, 3) (5, 6), (6, 5) i.e., 6 outcomes. **1**

P(a total of 9 or 11) $= \frac{6}{36}$ or $\frac{1}{6}$. **1**

SECTION - D

Question numbers 21 to 31 carry 4 marks each.

21. A passenger, while boarding the plane, slipped from the stairs and got hurt. The pilot took the passenger in the emergency clinic at the airport for treatment. Due to this, the plane got delayed by half an hour. To reach the destination 1500 km away in time, so that the passengers could catch the connecting flight, the speed of the plane was increased by 250 km/hour than the usual speed. Find the usual speed of the plane. What value is depicted in this question?

Sol. Let the usual speed of the plane is = s km/h

and the usual time it takes to reach the destination = t h

Distance , $d = 1500$ km

Now, speed $= \frac{\text{distance}}{\text{time}}$

$$s = \frac{1500}{t}$$

$$t = \frac{1500}{s} \quad \text{...(1)}$$

The plane got delayed by half an hour and speed was increased by 250 km/h to reach the destination on time.

Now, speed = s + 250 km/h

and time $= t - \frac{1}{2}$ hr

Now, speed × time = distance

$$(s + 250)\left(t - \frac{1}{2}\right) = 1500$$

$$(s + 250)\left(\frac{1500}{s} - \frac{1}{2}\right) = 1500 \text{ (from (1) } t = 1500/s\text{)}$$

$$(s + 250)\left(\frac{3000 - s}{2s}\right) = 1500$$

$$3000s - s^2 + 250 \times 3000 - 250s = 3000s$$

$$-s^2 + 250 \times 3000 - 250s = 0$$

$$s^2 + 250s - 250 \times 3000 = 0$$

$$s^2 + 1000s - 750s - 250 \times 3000 = 0$$

$$(s + 1000)(s - 750) = 0$$

$$s = -1000$$

or $\quad s = 750$

Speed cannot be negative.

So, usual speed s = 750 km/h

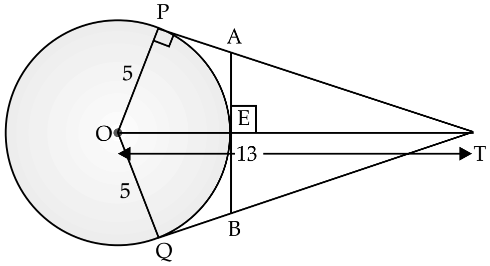
$$\text{usual time } t = \frac{15000}{750} = 2 \text{ hr.}$$

22. Prove that the lengths of tangents drawn from an external point to a circle are equal.

Sol. Solution Refer to 2018 year Delhi Set-I Q.18

*** 23. Draw two concentric circles of radii 3 cm and 5 cm. Construct a tangent to smaller circle from a point on the larger circle. Also measure its length.**

24. In fig., *O* is the centre of a circle of radius 5 cm. *T* is a point such that *OT* = 13 cm and *OT* intersects circle at *E*. If *AB* is a tangent to the circle at *E*, find the length of *AB*, where *TP* and *TQ* are two tangents to the circle.



Sol. $PT = \sqrt{169 - 25} = 12$ cm **1**

$$TE = 13 - 5 = 8 \text{ cm}$$

Let $\quad PA = AE = x$

$$TA^2 = TE^2 + EA^2 \quad \mathbf{1}$$

$$\Rightarrow \quad (12 - x)^2 = 64 + x^2 \quad \mathbf{1}$$

$$x = \frac{80}{24}$$

$$\Rightarrow \quad x = 3.3 \text{ cm.}$$

Thus $\quad AB = 6.6$ cm. **1**

25. Find x in terms of a, b and c:

$$\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c}, x \neq a, b, c$$

Sol. $a(x - b)(x - c) + b(x - a)(x - c) = 2c(x - a)(x - b)$

$\Rightarrow x^2(a + b - 2c) + x(-ab - ac - ab - bc + 2ac + 2bc) = 0$ **1½**

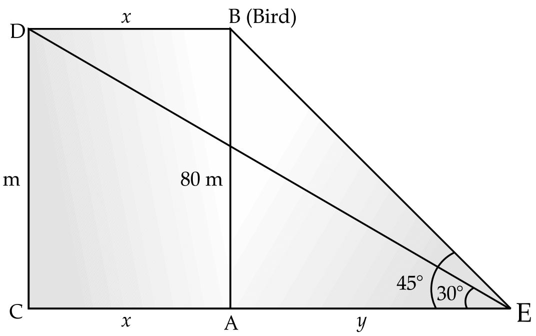
$\Rightarrow x^2(a + b - 2c) + x(-2ab + ac + bc) = 0$ **1½**

$$\Rightarrow \quad x = \frac{ac + bc - 2ab}{a + b - 2c} \quad \mathbf{1}$$

26. A bird is sitting on the top of a 80 m high tree. From a point on the ground, the angle of elevation of the bird is 45°. The bird flies away horizontally in such a way that it remained at a constant height from the ground. After 2 seconds, the angle of elevation of the bird from the same point is 30°. Find the speed of flying of the bird.

(Take $\sqrt{3}$ =1.732)

Sol.



1

In ΔABE $\quad \tan 45° = \dfrac{80}{y} \Rightarrow y = 80$ **½**

In ΔDCE $\quad \tan 30° = \dfrac{80}{x + y}$ **½**

$$\Rightarrow \quad x + y = 80\sqrt{3}$$

$$\therefore \quad x = 80(\sqrt{3} - 1) = 58.4 \text{ m} \quad \mathbf{1}$$

$$\text{Hence, speed of bird} = \frac{58.4}{2} = 29.2 \text{ m/s} \quad \mathbf{1}$$

27. A thief runs with a uniform speed of 100 m/minute. After one minute a policeman runs after the thief to catch him. He goes with a speed of 100 m/minute in the first minute and increases his speed by 10 m/ minute every succeeding minute. After how many minutes the policeman will catch the thief.

Sol. Let total time be n minutes

Total distance covered by thief = $(100n)$ metre **½**

Total distance covered by Policeman = 100 + 110 + 120 + ... + $(n - 1)$ terms **½**

$$\therefore \quad 100n = \frac{n-1}{2}[200 + (n-2)10] \quad \mathbf{1}$$

$$\Rightarrow \quad n^2 - 3n - 18 = 0 \quad \mathbf{½}$$

$$\Rightarrow \quad (n - 6)(n + 3) = 0 \quad \mathbf{½}$$

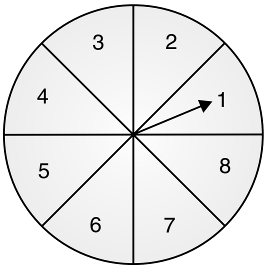
$$\Rightarrow \quad n = 6 \quad \mathbf{½}$$

Policeman took 6 minutes to catch the thief. **½**

*** 28. Prove that the area of a triangle with vertices $(t, t - 2)$, $(t + 2, t + 2)$ and $(t + 3, t)$ is independent of t.**

* *Out of Syllabus*

29. A game of chance consists of spinning an arrow on a circular board, divided into 8 equal parts, which comes to rest pointing at one of the numbers 1, 2, 3, ..., 8 (fig.), which are equally likely outcomes. What is the probability that the arrow will point at (i) an odd number (ii) a number greater than 3 (iii) a number less than 9.



Sol. (i) Favourable outcomes are 1, 3, 5, 7

i.e., 4 outcomes. **1**

$\therefore P(\text{an odd number}) = \frac{4}{8}$ or $\frac{1}{2}$ ½

(ii) Favourable outcomes are 4, 5, 6, 7, 8 *i.e.*, 5 outcomes **1**

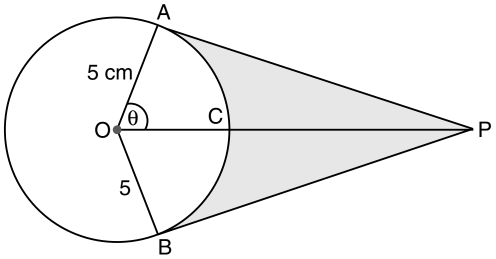
$P(\text{a number greater than 3}) = \frac{5}{8}$ ½

(iii) Favourable outcomes are 1, 2, 3, ..., 8

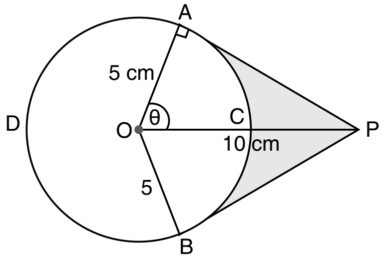
$P(\text{a number less than 9}) = \frac{8}{8} = 1$ **1**

30. An elastic belt is placed around the rim of a pulley of radius 5 cm (fig.). From one point *C* on the belt, the elastic belt is pulled directly away from the centre *O* of the pulley until it is at *P*, 10 cm from the point *O*. Find the length of the belt that is still in contact with the pulley. Also, find the shaded area.

(use π = 3.14 and $\sqrt{3}$ =1.73)



Sol.



$$\cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$$ ½

$$\text{Reflex } \angle AOB = 240°$$

$$\therefore \widehat{ADB} = \frac{2\times3.14\times5\times240}{360}$$ ½

$$= 20.93 \text{ cm}$$ **1**

Hence, length of elastic in contact = 20.93 cm

Now, $AP = 5\sqrt{3}$ cm

$$\text{Area } (\Delta OAP + \Delta OBP) = 25\sqrt{3} = 43.25 \text{ cm}^2$$ ½

$$\text{Area of sector } OACB = \frac{25\times3.14\times120}{360}$$ ½

$$= 26.16 \text{ cm}^2$$

$$\text{Shaded Area} = 43.25 - 26.16$$

$$= 17.09 \text{ cm}^2$$ **1**

*** 31. A bucket open at the top is in the form of a frustum of a cone with a capacity of 12308.8 cm³. The radii of the top and bottom circular ends are 20 cm and 12 cm respectively. Find the height of the bucket and the area of metal sheet used in making the bucket. (use π = 3.14)**

Delhi Set-II — Code No. 30/1/2

SECTION - B

Question numbers 5 to 10 carry 2 marks each.

10. How many terms of the A.P. 27, 24, 21, ... should be taken so that their sum is zero ?

Sol. Here $a = 27, d = -3, S_n = 0$ ½

$\therefore \quad 54 + (n-1)(-3) = 0$

$$\left[S_n = \frac{n}{2}[2a+(n-1)d]\right]$$ **1**

$\Rightarrow \quad n = 19$ ½

SECTION - C

Question numbers 11 to 20 carry 3 marks each.

18. Solve for x :

$$\frac{x+1}{x-1}+\frac{x-2}{x+2} = 4-\frac{2x+3}{x-2};\ x \neq 1, -2, 2$$

Sol.
$$\frac{x^2+3x+2+x^2-3x+2}{x^2+x-2} = \frac{4x-8-2x-3}{x-2}$$ **1**

$$(2x^2+4)(x-2) = (2x-11)(x^2+x-2)$$

$$2x^3+4x-4x^2-8 = 2x^3-11x^2+2x^2 - 4x - 11x + 22$$

* *Out of Syllabus*

$\Rightarrow \quad 5x^2 + 19x - 30 = 0$ **1**

$\Rightarrow \quad 5x^2 + 25x - 6x - 30$

$\Rightarrow \quad (5x - 6)(x + 5) = 0$ ½

$\Rightarrow \quad x = -5, \frac{6}{5}$ ½

19. Two different dice are thrown together. Find the probability of:

(i) getting a number greater than 3 on each die

(ii) getting a total of 6 or 7 of the numbers on two dice

Sol. **(i)** Favourable outcomes are (4, 5) (4, 4) (4, 6) (5, 4) (5, 5) (5, 6) (6, 4) (6, 5) (6, 6) *i.e.*, 9 outcomes. **1**

$P(\text{a number} > 3 \text{ on each die}) = \frac{9}{36} \text{ or } \frac{1}{4}.$ ½

(ii) Favourable outcomes are (1, 5) (2, 4) (3, 3) (4, 2) (5, 1) (1, 6) (2, 5) (3, 4) (4, 3) (5, 2) (6, 1) i.e., 11 outcomes. **1**

$P(\text{a total of 6 to 7}) = \frac{11}{36}$ ½

20. A right circular cone of radius 3 cm has a curved surface area of 47.1 cm^2. Find the volume of the cone.

use π = 3.14

Sol. Here $r = 3$, $\pi rl = 47.1$

$\therefore \quad l = \frac{47.1}{3 \times 3.14} = 5 \text{ cm}$ **1**

$h = \sqrt{5^2 - 3^2} = 4 \text{ cm}$ ½

$\text{Volume of cone} = \frac{1}{3} \times 3.14 \times 3 \times 3 \times 4$ ½

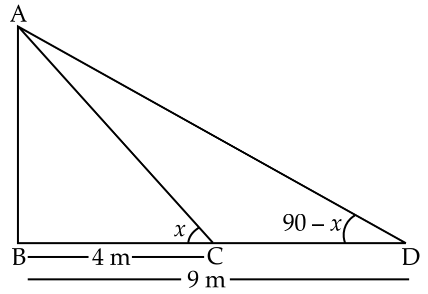
$= 37.68 \text{ cm}^3$ **1**

SECTION - D

Question numbers 21 to 31 carry 4 marks each.

28. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are x and 90 – x respectively. Find the height of the tower.

Sol. Let AB be the tower. C and D be the two points with distance 4 m and 9 m from the base respectively. As per question,



In right ΔABC,

$$\tan x = \frac{AB}{BC}$$

$$\tan x = \frac{AB}{4}$$

$$AB = 4 \tan x \qquad \ldots\text{(i)}$$

Again, from right ΔABD,

$$\tan (90° - x) = \frac{AB}{BD}$$

$$\cot x = \frac{AB}{9}$$

$$AB = 9 \cot x \qquad \ldots\text{(ii)}$$

Multiplying equation (i) and (ii)

$$AB^2 = 9 \cot x \times 4 \tan x$$

$\Rightarrow \quad AB^2 = 36$

$$\left(\text{because } \cot x = \frac{1}{\tan x}\right)$$

$\Rightarrow \quad AB = \pm 6$

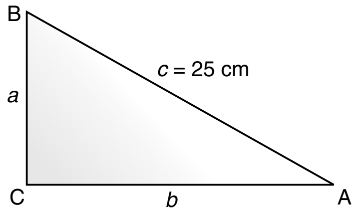
Since height cannot be negative.

Therefore, the height of the tower is 6 m.

*** 29. Construct a triangle *ABC* in which *BC* = 6 cm, *AB* = 5 cm and $\angle ABC$ = 60°. Then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of Δ*ABC*.**

30. The perimeter of a right triangle is 60 cm. Its hypotenuse is 25 cm. Find the area of the triangle.

Sol.



Here $a + b + c = 60$, $c = 25$

$\therefore \quad a + b = 35$ **1**

Using Pythagoras theorem

$$a^2 + b^2 = 625$$

Using identity $\quad (a + b)^2 = a^2 + b^2 + 2ab$ **1**

$(35)^2 = 625 + 2ab$ **1**

$\Rightarrow \quad ab = 300$

$\text{Area of } \Delta ABC = \frac{1}{2} ab = 150 \text{ cm}^2$ **1**

31. A thief, after committing a theft, runs at a uniform speed of 50 m/ minute. After 2 minutes, a policeman runs to catch him. He goes 60 m in first minute and increases his speed by 5 m/ minute every succeeding minute. After how many minutes, the policeman will catch the thief ?

Sol. Let total time be n minutes

Total distance covered by thief = $(50n)$ meter ½

Total distance covered by policeman

* *Out of Syllabus*

$= 60 + 65 + 70 + ... + (n - 2)$ terms ½

$\therefore \quad 50n = \frac{n-2}{2}[120+(n-3)5]$ **1**

$100n = (n - 2)(105 + 5n)$

$\Rightarrow \quad n^2 - n - 42 = 0$ ½

$n^2 - 7n + 6n - 12 = 0$

$(n - 7)(n + 6) = 0$ ½

$\therefore \quad n = 7$ ½

Policeman took 7 minutes to catch the thief. ½

Delhi Set-III — Code No. 30/1/3

Note: Except these, all other questions are from Set-I & II.

SECTION - B

Question numbers 5 to 10 carry 2 marks each.

10. How many terms of the A.P. 65, 60, 55, ... be taken so that their sum is zero ?

Sol. Here $a = 65$, $d = -5$, $S_n = 0$ ½

$\therefore \quad S_n = \frac{n}{2}[2a + (n-1)d]$

$O = \frac{\pi}{2}[130 + (n - 1) - 5]$

$\Rightarrow \quad n = 27$ ½

SECTION - C

Question numbers 11 to 20 carry 3 marks each.

18. A box consists of 100 shirts of which 88 are good, 8 have minor defects and 4 have major defects. Ramesh, a shopkeeper will buy only those shirts which are good but Kewal another shopkeeper will not buy shirts with major defects. A shirt is taken out of the box at random. What is the probability that

(i) Ramesh will buy the selected shirt?

(ii) Kewal will buy the selected shirt?

Sol. (i) Number of good shirts = 88 **1**

$P(\text{Ramesh buys the shirt}) = \frac{88}{100}$ or $\frac{22}{25}$ ½

(ii) Number of shirts without major defect = 96 **1**

$P(\text{Kewal buys a shirt}) = \frac{96}{100}$ or $\frac{24}{25}$ ½

19. Solve the following quadratic equation for x :

$$x^2 + \left(\frac{a}{a+b} + \frac{a+b}{a}\right)x + 1 = 0$$

Sol. $x^2 + \frac{a}{a+b}x + \frac{a+b}{a}x + 1 = 0$

$x\left(x + \frac{a}{a+b}\right) + \frac{a+b}{a}\left(x + \frac{a}{a+b}\right) = 0$ **1**

$\Rightarrow \quad \left(x + \frac{a}{a+b}\right)\left(x + \frac{a+b}{a}\right) = 0$ **1**

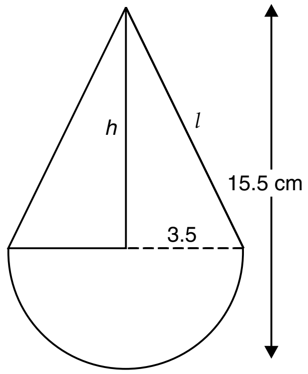
$\Rightarrow \quad x = \frac{-a}{a+b}, \frac{-(a+b)}{a}$ **1**

20. A toy is in the form of a cone of base radius 3.5 cm mounted on a hemisphere of base diameter 7 cm. If the total height of the toy is 15.5 cm, find the total surface area of the toy.

Sol. In cone $h = 15.5 - 3.5 = 12$ cm ½

$l = \sqrt{144 + 12.25} = 12.5$ cm ½

$\text{TSA} = \pi rl + 2\pi r^2$



$= \frac{22}{7} \times 3.5 \times 12.5 + 2 \times \frac{22}{7} \times 3.5 \times 3.5$ **1**

$= 137.5 + 77 = 214.5 \text{ cm}^2$ **1**

SECTION - D

Question numbers 21 to 31 carry 4 marks each.

28. The sum of three numbers in A.P. is 12 and sum of their cubes is 288. Find the numbers.

Sol. Let the three numbers in A.P. be $a - d, a, a + d$.

$3a = 12$ **1**

$\Rightarrow \quad a = 4.$

Also $(4 - d)^3 + 4^3 + (4 + d)^3 = 288$ **1**

$\Rightarrow 64 - 48d + 12d^2 - d^3 + 64 + 64 + 48d + 12d^2 + d^3 = 288$

$\Rightarrow \quad 24d^2 + 192 = 288$

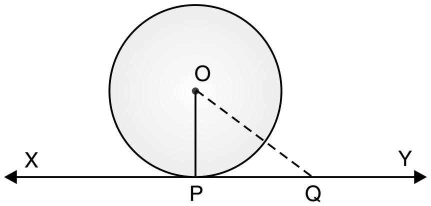
$\Rightarrow \quad d^2 = 4$

$d = \pm 2$ **1**

The numbers are 2, 4, 6, or 6, 4, 2. **1**

29. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

Sol. **Proof :** We are given a circle with centre O and a tangent XY to the circle at a point P. We need to prove that OP is perpendicular to XY.



1

Take a point Q on XY other than P and join OQ (See fig.)

The point Q must lie outside the circle. Note that if Q lies inside the circle, XY will become a secant not a tangent to the circle. **1**

Therefore, OQ is longer than the radius OP of the circle.

$\Rightarrow \qquad OQ > OP.$ **1**

Since this happens for every point on the line XY except the point P, OP is the shortest of all the distances of the point O to the points of XY.

So, OP is perpendicular to XY. **1**

30. The time taken by a person to cover 150 km was 2½ hours more than the time taken in the return journey. If he returned at a speed of 10 km/hour more than the speed while going, find the speed per hour in each direction.

Sol. Let the speed while going be x km/h

Therefore $\dfrac{150}{x} - \dfrac{150}{x+10} = \dfrac{5}{2}$ **1**

$\Rightarrow \qquad x^2 + 10x - 600 = 0$ **1**

$\Rightarrow \qquad (x + 30)(x - 20) = 0$ **1**

$\Rightarrow \qquad x = 20$

$\therefore$ Speed while going $= 20$ km/h

and speed while returning $= 30$ km/h **1**

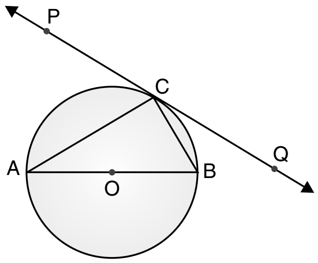
*** 31. Draw a triangle ABC with $BC = 7$ cm, $\angle B = 45°$ and $\angle A = 105°$. Then construct a triangle whose sides are 4/5 times the corresponding sides of ΔABC.**

Outside Delhi Set I — Code No. 30/1

SECTION - A

Question numbers 1 to 4 carry 1 mark each.

1. In fig., PQ is a tangent at a point C to a circle with centre O. If AB is a diameter and $\angle CAB = 30°$; find $\angle PCA$.



Sol. For $\angle ACB = 90°$

(Semicircle angle) ½

$\angle QCB = \angle CAB = 30°$

$\angle PCA + \angle ACB + \angle BCQ = 180°$

(Straight line angles)

$\therefore \qquad \angle PCA = 60°$ ½

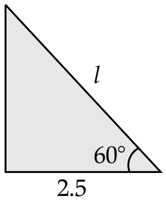
2. For what value of k will $k + 9$, $2k - 1$ and $2k + 7$ are the consecutive terms of an A.P. ?

Sol. $2(2k - 1) = k + 9 + 2k + 7$ ½

$\Rightarrow \qquad k = 18$ ½

3. A ladder, leaning against a wall, makes an angle of 60° with the horizontal. If the foot of the ladder is 2.5 m away from the wall, find the length of the ladder.

Sol. $\sec 60° = \dfrac{l}{2.5} = 2$ ½



$\Rightarrow \qquad l = 5$ m ½

4. A card is drawn at random from a well shuffled pack of 52 playing cards. Find the probability of getting neither a red card nor a queen.

Sol. No. of red cards and queens : 28 ½

No. of neither red cards nor a queen $52 - 28 = 24$

Required Probability : $\dfrac{24}{52}$ or $\dfrac{6}{13}$ ½

SECTION - B

Question numbers 5 to 10 carry 2 marks each.

5. If -5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, then find the value of k.

Sol. $2(-5)^2 + p(-5) - 15 = 0$

$\Rightarrow \qquad p = 7$ **1**

$7x^2 + 7x + k = 0$

Quadratic equations has equal roots

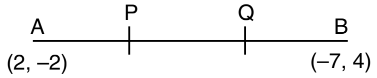
$\therefore \qquad b^2 - 4ac = 0$

gives $\qquad 49 - 28k = 0$

$\Rightarrow \qquad k = \dfrac{7}{4}$ **1**

6. Let **P and Q be the points of trisection of the line segment joining the points $A(2, -2)$ and $B(-7, 4)$ such that P is nearer to A. Find the co-ordinates of P and Q.**

* *Out of Syllabus*

Sol. 

P divides line segment in the ratio 1 : 2.

So, Coordinates of $P = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

$$\Rightarrow \left(\frac{1\times(-7)+2\times 2}{1+2}, \frac{1\times 4+2\times -2}{1+2}\right)$$

$$\Rightarrow \left(\frac{-7+4}{3}, \frac{4-4}{3}\right)$$

$$= \left(\frac{-3}{3}, \frac{0}{3}\right)$$

$$= (-1, 0)$$

Q divides line segment in the ratio 2 : 1.

So, Coordinates of $Q = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

$$\Rightarrow \left(\frac{2\times -7+12}{2+1}, \frac{2\times 4+1\times -2}{2+1}\right)$$

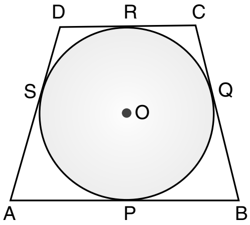
$$= \left(\frac{-14+2}{3}, \frac{8-2}{3}\right)$$

$$= \left(\frac{-12}{3}, \frac{6}{3}\right) = (-4, 2)$$

Thus, Coordinates of $P = (-1, 0)$

Coordinates of $Q = (-4, 2)$

7. In fig., a quadrilateral *ABCD* is drawn to circumscribe a circle, with centre *O*, in such a way that the sides *AB*, *BC*, *CD* and *DA* touch the circle at the points *P*, *Q*, *R* and *S* respectively. Prove that : *AB* + *CD* = *BC* + *DA*.



Sol. $AP = AS, BP = BQ, CR = CQ$ and $DR = DS$ **1**

$$AP + BP + CR + DR = AS + BQ + CQ + DS$$

$\Rightarrow$ $AB + CD = AD + BC$ **1**

8. Prove that the points (3, 0) (6, 4) and (– 1, 3) are the vertices of a right angled isosceles triangle.

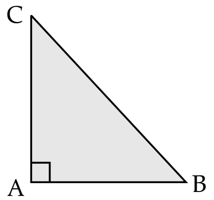
Sol. Let the point be $A(3, 0)$, $B(6, 4)$, $C(-1, 3)$

$AB = \left|\sqrt{9+16}\right| = 5,$ ½

$BC = \left|\sqrt{49+1}\right| = 5\sqrt{2}$ ½

and $AC = \left|\sqrt{16+9}\right| = 5$ ½

$AB = AC$ and $AB^2 + AC^2 = BC^2$:



$\therefore$ ΔABC is a right angled isosceles triangle. ½

9. The 4th term of an A.P. is zero. Prove that the 25th term of the A.P. is three times its 11th term.

Sol. $a + 3d = 0$ **1**

$\Rightarrow$ $a = -3d$

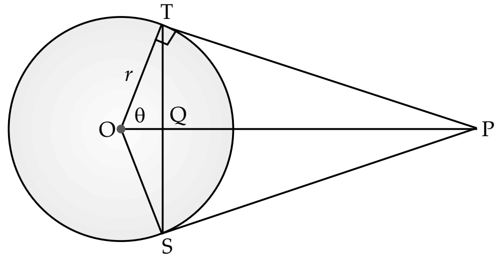
$a_{25} = a + 24d = 21d$

$3a_{11} = 3(a + 10d)$ **1**

$= 3(7d) = 21d$

Thus, $a_{25} = 3a_{11}$ **Hence Proved**

10. In fig., from an external point *P*, two tangents *PT* and *PS* are drawn to a circle with centre *O* and radius *r*. If *OP* = 2*r*, show that ∠*OTS* = ∠*OST* = 30°.



Sol. Let $\angle TOP = \theta$

$\therefore$ $\cos\theta = \frac{OT}{OP} = \frac{r}{2r} = \frac{1}{2}$

$\therefore$ $\theta = 60°$

Hence $\angle TOS = 120°$ **1**

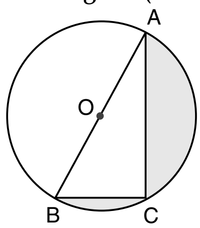
In ΔOTS, $OT = OS$ (radii)

$\Rightarrow$ $\angle OTS = \angle OST = 30°$ **1**

SECTION - C

Question numbers 11 to 20 carry 3 marks each.

11. If fig., *O* is the centre of a circle such that diameter *AB* = 13 cm and *AC* = 12 cm. *BC* is joined. Find the area of the shaded region. (Take π = 3.14)



Sol. $BC^2 = AB^2 - AC^2$

$= 169 - 144 = 25$

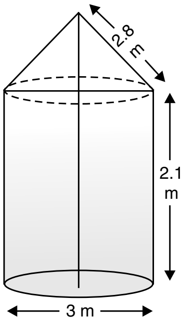
$\therefore$ BC = 5 cm **1**

Area of the shaded region = Area of semicircle – area of rt. ΔABC

$$= \frac{1}{2}\times(3.14)\times\left(\frac{13}{2}\right)^2 - \frac{1}{2}\times 12\times 5$$ **1**

$= 66.33 - 30 = 36.33$ cm^2 **1**

12. In fig., a tent is in the shape of a cylinder surmounted by a conical top of same diameter. If the height and diameter of cylindrical part are 2.1 m and 3 m respectively and the slant height of conical part is 2.8 m, find the cost of canvas needed to make the tent if the canvas is available at the rate of ₹ 500/sq. metre.



Sol. Area of canvas needed $= 2 \times \frac{22}{7} \times (1.5) \times (2.1) + \frac{22}{7} \times 1.5 \times 2.8$ **1½**

$$= \frac{22}{7}[6.3 + 4.2]$$

$$= \frac{22}{7} \times 10.5 = 33 \text{ m}^2$$ **1**

Cost of canvass $= 33 \times 500$

$= ₹\ 16{,}500$ **½**

13. If the point $P(x, y)$ is equidistant from the points $A(a + b, b - a)$ and $B(a - b, a + b)$. Prove that $bx = ay$.

Sol. $PA = PB$

or $(PA)^2 = (PB)^2$ **1**

$(a + b - x)^2 + (b - a - y)^2 = (a - b - x)^2 + (a + b - y)^2$ **1**

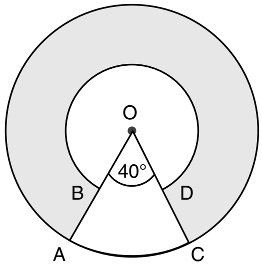
$(a + b)^2 + x^2 - 2ax - 2bx + (b - a)^2 + y^2 - 2by + 2ay$

$= (a - b)^2 + x^2 - 2ax + 2bx + (a + b)^2 + y^2 - 2ay - 2by$

$\Rightarrow \quad 4ay = 4bx$

or $\quad bx = ay$ **1**

14. In fig., find the area of the shaded region, enclosed between two concentric circles of radii 7 cm and 14 cm where $\angle AOC = 40°$.



Sol. Shaded area $= \pi(14^2 - 7^2) \times \frac{320}{360}$ **2**

$$= \frac{22}{7} \times 147 \times \frac{8}{9}$$ **½**

$$= \frac{1232}{3}$$

$= 410.67 \text{ cm}^2$ **½**

15. If the ratio of the sum of first n terms of two A.P's is $(7n + 1) : (4n + 27)$, find the ratio of their m^{th} terms.

Sol. $$\frac{S_n}{S'_n} = \frac{n/2[2a + (n-1)d]}{n/2[2a' + (n-1)d']}$$ **1**

$$= \frac{7n+1}{4n+27}$$

$$\Rightarrow \quad \frac{a + \frac{n-1}{2}d}{a' + \frac{n-1}{2}d'} = \frac{7n+1}{4n+27}$$ ...(i) ½

Since, $\frac{t_m}{t'_m} = \frac{a + (m-1)d}{a' + (m-1)d'}$,

So replacing $\frac{n-1}{2}$ by $m - 1$ *i.e.*, $n = 2m - 1$ in (i) **1**

$$\Rightarrow \quad \frac{t_m}{t'_m} = \frac{7(2m-1)+1}{4(2m-1)+27} = \frac{14m-6}{8m+23}$$ ½

16. Solve for x : $\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} = \frac{2}{3}$, $x \neq 1, 2, 3$

Sol. Here $\quad 3(x - 3 + x - 1) = 2(x - 1)(x - 2)(x - 3)$ **1½**

$\Rightarrow \quad 3(2x - 4) = 2(x - 1)(x - 2)(x - 3)$ **½**

$\Rightarrow \quad 3 = (x - 1)(x - 3)$

i.e., $\quad x^2 - 4x = 0$

$\therefore \quad x = 0, x = 4$ **1**

17. A conical vessel, with base radius 5 cm and height 24 cm is full of water. This water is emptied into a cylindrical vessel of base radius 10 cm. Find the height to which the water will rise in the cylindrical vessel.

Sol. Volume of water in conical vessel

Volume of cylinder

$$\therefore \quad \frac{1}{3}\pi r^2 h = \pi r^2 h$$ **1**

$$\Rightarrow \quad \frac{1}{3} \times \frac{22}{7} \times 25 \times 24 = \frac{22}{7} \times 10 \times 10 \times h$$ **1½**

$\Rightarrow \quad h = 2 \text{ cm}$ **½**

18. A sphere of diameter 12 cm is dropped in a right circular cylindrical vessel, partly filled with water. If the sphere is completely submerged in water, the water level in the cylindrical vessel rises by $3\frac{5}{9}$ cm. Find the diameter of the cylindrical vessel.

Sol. Volume of sphere = Volume of cylinder **1**

$$\therefore \quad \pi r^2 \frac{32}{9} = \frac{4}{3}\pi(6)^3$$ **1½**

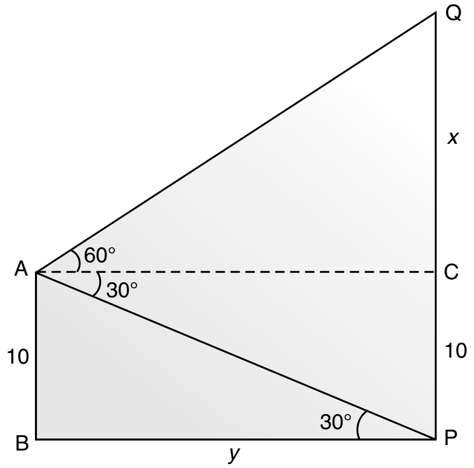
$$r^2 = \frac{4\times216\times9}{3\times32}$$

$\Rightarrow$ $r = 9$ cm ½

So, the diameter of the cylindrical vessel is 18 cm.

19. A man standing on the deck of a ship, which is 10 m above water level, observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of hill as 30°. Find the distance of the hill from the ship and the height of the hill.

Sol.



In ΔABP, $\frac{y}{10} = \cot 30° = \sqrt{3}$

$\therefore$ $y = 10\sqrt{3}$ m 1

0 So, the distance of hill from the ship is

$10 \times 1.732 = 17.32$ m

In ΔACQ, $\frac{x}{y} = \tan 60° = \sqrt{3}$

$x = \sqrt{3}(10\sqrt{3}) = 30$ m 1

$\therefore$ Height of hill = 30 + 10 = 40 m 1

20. Three different coins are tossed together. Find the probability of getting (i) exactly two heads (ii) at least two heads (iii) at least two tails.

Sol. Set of possible outcomes is

{*HHH, HHT, HTH, THH, HTT, THT, TTH, TTT*}

(i) P(exactly 2 heads) = 3/8 1

(ii) P(at least 2 heads) = 4/8 or 1/2 1

(iii) P(at least 2 tails) = 4/8 or 1/2 1

SECTION - D

Question numbers 21 to 31 carry 4 marks each.

21. Due to heavy floods in a state, thousands were rendered homeless. 50 schools collectively offered to the state government to provide place and the canvas for 1500 tents to be fixed by the government and decided to share the whole expenditure equally. The lower part of each tent is cylindrical of base radius 2.8 m and height 3.5 m, with conical upper part of same base radius but of height 2.1 m. If the canvas used to make the tents costs ₹ 120 per sq. m, find the amount shared by each school to set up the tents. What value is generated by the above problem ?

Sol. Radius of the base of cylinder (r) = 2.8 m

Radius of the base of the cone (r) = 2.8 m

Height of the cylinder (h) = 3.5 m

Height of the cone (H) = 2.1 m.

Slant height of conical part (l)

$= \sqrt{r^2 + H^2}$

$= \sqrt{(2.8)^2 + (2.1)^2}$

$= \sqrt{7.84 + 4.41}$

$= \sqrt{12.25}$

= 3.5 m

Area of canvas used to make tent

= CSA of cylinder + CSA of cone

$= 2 \times \pi \times 2.8 \times 3.5 + \pi \times 2.8 \times 3.5$

= 61.6 + 30.8

= 92.4 m^2

Cost of 1500 tents at ₹ 120 per sq.m

$= 1500 \times 120 \times 92.4$

= ₹ 16,632,000

Share of each school to set up the tents

$= \frac{16632000}{50}$

= ₹ 332,640

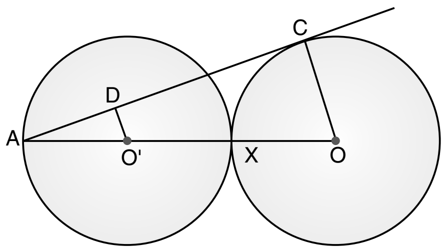
Value – Be kind and help others in need.

22. Prove that the lengths of the tangents drawn from an external point to a circle are equal.

Sol. Refer to Delhi Set-I 2012 year Q. 22

*** 23. Draw a circle of radius 4 cm. Draw two tangents to the circle inclined at an angle of 60° to each other.**

24. In fig., two equal circles, with centres *O* and *O′*; touch each other at *X*. *OO′* produced meets the circle with centre *O′* at *A*. *AC* is tangent to the circle with centre *O*, at the point *C*. *O′D* is perpendicular to *AC*. Find the value of $\frac{DO'}{CO}$.



Sol. *AC* is tangent to circle with centre *O*,

Thus $\angle ACO = 90°$ 1

O'D is $\perp$ to AC

$\therefore$ $\angle ADO' = 90°$

* *Out of Syllabus*

$$\angle A = \angle A \quad \text{(Common)}$$

$\therefore$ $\Delta AO'D \sim \Delta AOC$ (AA Similarity) **1**

$\Rightarrow$ $\dfrac{AO'}{AO} = \dfrac{DO'}{CO}$ 1

$\therefore$ $\dfrac{DO'}{CO} = \dfrac{r}{3r} = \dfrac{1}{3}$ 1

25. Solve for x : $\dfrac{1}{x+1}+\dfrac{2}{x+2} = \dfrac{4}{x+4}$, $x \neq -1, -2, -4$

Sol. $(x + 4)(x + 2 + 2x + 2) = 4(x + 1)(x + 2)$

$(x + 4)(3x + 4) = 4(x^2 + 3x + 2)$ **1**

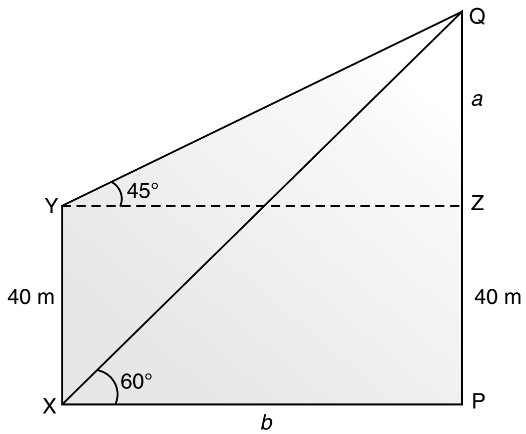
$\Rightarrow$ $x^2 - 4x - 8 = 0$ **1½**

$\Rightarrow$ $x = \dfrac{4 \pm \sqrt{16+32}}{2}$

$= 2 \pm 2\sqrt{3}$ **1½**

26. The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60°. From a point Y, 40 m vertically above X, the angle of elevation of the top Q of tower is 45°. Find the height of the tower PQ and the distance PX. (Use $\sqrt{3}$ = 1.73)

Sol.



1

In ΔYZQ, $\dfrac{a}{YZ} = \tan 45° = 1$

$\Rightarrow$ $YZ = a$ *i.e.*, $a = b$ **1**

In ΔQPX, $\dfrac{a+40}{b} = \dfrac{a+40}{a} = \tan 60° = \sqrt{3}$

$\therefore$ $(\sqrt{3}-1)a = 40$

$\Rightarrow$ $a = \dfrac{40}{\sqrt{3}-1} = 20(\sqrt{3}+1)$

$= 20\,(2.73) = 54.60$ m **1**

$\therefore$ $ZQ = 54.6$ m

So, $PQ = 54.6 + 40 = 94.6$ m **1**

and PX = 54.60 cm

27. The houses in a row are numbered consecutively from 1 to 49. Show that there exists a value of X such that sum of numbers of houses preceding the houses numbered X is equal to sum of the numbers of houses following X.

Sol. House number will form an A.P. whose first term and common difference is

Sum of numbers preceding X

$$S_{X-1} = \frac{(X-1)X}{2} \qquad 1½$$

Sum of numbers following X

$$S_{49} - S_X = \frac{(49)(50)}{2} - \frac{-(X-1)X}{2}$$

$$= \frac{2450 - X^2 - X}{2} \qquad 1½$$

Now, $S_{X-1} = S_{49} - S_X$

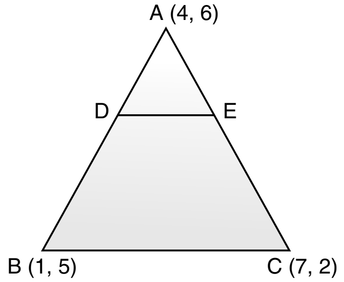
$\therefore$ $\dfrac{(X-1)X}{2} = \dfrac{2450 - X^2 - X}{2}$

$\Rightarrow$ $2X^2 = 2450$

$\Rightarrow$ $X^2 = 1225$

$\therefore$ $X = 35$ 1

*** 28. In fig., the vertices of ΔABC are $A(4, 6)$, $B(1, 5)$ and $C(7, 2)$. A line-segment DE is drawn to intersect the sides AB and AC at D and E respectively such that $\dfrac{AD}{AB} = \dfrac{AE}{AC} = \dfrac{1}{3}$. Calculate the area of ΔADE and compare it with area of ΔABC.**



29. A number x is selected at random from the numbers 1, 2, 3 and 4. Another number y is selected at random from the numbers 1, 4, 9 and 16. Find the probability that product of x and y is less than 16.

Sol. x can be any one of 1, 2, 3 or 4.

y can be any one of 1, 4, 9 or 16

Total number of cases of $xy = 16$ **1½**

Number of cases,where product is less than 16 = 8 **1½**

{1, 4, 9, 2, 8, 3, 12, 4}

$\therefore$ Required Probability $= \dfrac{8}{16}$ or $\dfrac{1}{2}$ **1**

30. In Fig., as shown a sector OAP of a circle with centre O, containing $\angle\theta$. AB is perpendicular to the radius OA and meets OP produced at B. Prove that the perimeter of shaded region is

$$r\left[\tan\theta + \sec\theta + \frac{\pi\theta}{180°} - 1\right]$$

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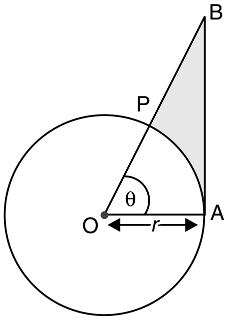

Sol. Length of arc $\widehat{AP}$. $= 2\pi r \dfrac{\theta}{360°}$ or $\dfrac{\pi r\theta}{180°}$ ...(i) **1**

$$\frac{AB}{r} = \tan\theta$$

$\Rightarrow$ $AB = r\tan\theta$ ...(ii) ½

$$\frac{OB}{r} = \sec\theta$$ ½

$\Rightarrow$ $OB = r\sec\theta$

$PB = OB - r$

$= r\sec\theta - r$ ...(iii) **1**

Perimeter $= AB + PB + \widehat{AP}$.

$= r\tan\theta + r\sec\theta - r + \dfrac{\pi r\theta}{180°}$ **1**

or $r\left[\tan\theta + \sec\theta - 1 + \dfrac{\pi\theta}{180°}\right]$

31. A motor boat whose speed is 24 km/h in still water takes 1 hour more to go 32 km upstream than to return downstream to the same spot. Find the speed of the stream.

Sol. Let x km/h be the speed of the stream

$\therefore$ $\dfrac{32}{24-x} - \dfrac{32}{24+x} = 1$ **2**

$\Rightarrow$ $32(2x) = (24-x)(24+x)$

$x^2 + 64x - 576 = 0$ **1**

$x^2 + 72x - 8x - 576 = 0$

$x(x+72) - 8(x+72) = 0$

$(x+72)(x-8) = 0$

$x + 72 = 0 \Rightarrow x = -72$

$x - 8 = 0 \Rightarrow x = 8$ **1**

$\therefore$ Speed of streams 8 km/h.

Outside Delhi Set II — Code No. 30/2

Note : Except these, all other questions are from Set-I.

SECTION - B

Question numbers 5 to 10 carry 2 marks each.

10. Solve for x : $\sqrt{2x+9} + x = 13$

Sol. $\sqrt{2x+9} = 13 - x$ ... (i)

$\Rightarrow$ $2x + 9 = 169 + x^2 - 26x$ **1**

or $x^2 - 28x + 160 = 0$

i.e., $(x-20)(x-8) = 0$

$x = 20, 8.$ ½

$x = 20$ does not satisfy (i)

$\therefore$ $x = 8$ ½

SECTION - C

Question numbers 11 to 20 carry 3 marks each.

18. The digits of a positive number of three digits are in A.P. and their sum is 15. The number obtained by reversing the digits is 594 less than the original number. Find the number.

Sol. Let the three digits be $a-d, a, a+d$ ½

$\therefore$ $a - d + a + a + d = 3a = 15$

$\Rightarrow$ $a = 5$ ½

Number is : $100(a-d) + 10(a) + (a+d)$

i.e., $111a - 99d$.

Number, on reversing the digits is :

$100(a+d) + 10a + (a-d)$

i.e., $111a + 99d$ **1**

$\therefore (111a - 99d) - (111a + 99d) = 594$ ½

$\Rightarrow$ $d = -3$

$\therefore$ Number is 852 ½

19. If the roots of the quadratic equation $(a-b)x^2 + (b-c)x + (c-a) = 0$ are equal, prove that $2a = b + c$.

Sol. Given, roots are equal. $D = 0$

$\therefore$ $(b-c)^2 - 4(c-a)(a-b) = 0$ **1**

$\Rightarrow$ $b^2 + c^2 - 2bc - 4(ac - a^2 - bc + ab) = 0$

$\Rightarrow$ $(b^2 + c^2 + 2bc) - 4a(b+c) + 4a^2 = 0$ **1**

$\Rightarrow$ $[(b+c) - 2a]^2 = 0$

$\Rightarrow$ $b + c - 2a = 0$

or $b + c = 2a$ **1**

Hence Proved.

20. From a pack of 52 playing cards, Jacks, Queens and Kings of red colour are removed. From the remaining, a card is drawn at random. Find the probability that drawn card is :

(i) a black King, (ii) a card of red colour, (iii) a card of black colour

Sol. (i) Remaining cards $= 52 - 6 = 46$

P(black king) $= \dfrac{2}{46}$ or $\dfrac{1}{23}$, **1**

(ii) P(a card of red colour)$= \dfrac{20}{46}$ or $\dfrac{10}{23}$ 1

(iii) P(a black card) $= \dfrac{26}{46}$ or $\dfrac{13}{23}$ **1**

SECTION - D

Question numbers 21 to 31 carry 4 marks each.

*** 28. Draw an isosceles ΔABC in which $BC = 5.5$ cm and altitude $AL = 3$ cm. Then construct another**

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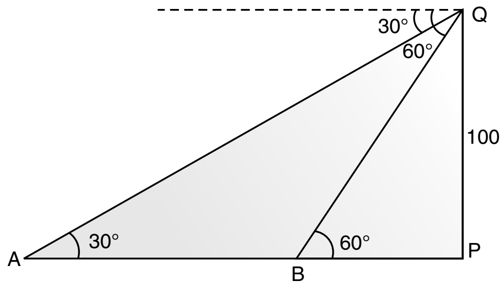
triangle whose sides are $\frac{3}{4}$ of the corresponding sides of ΔABC.

29. Prove that tangent drawn at any point of a circle is perpendicular to the radius through the point of contact.

Sol. Refer to Delhi Set-III Code No. 30/1/3 Q. 29.

30. As observed from the top of a light house, 100 m high above sea level, the angles of depression of a ship, sailing directly towards it, changes from 30° to 60°. Find the distance travelled by the ship during the period of observation. (Use $\sqrt{3}$ = 1.73)

Sol. In ΔPBQ, $\frac{PB}{100} = \cot 60° = \frac{1}{\sqrt{3}}$



1

$\Rightarrow \quad PB = \frac{100}{\sqrt{3}}$ or $\frac{100\sqrt{3}}{3}$

In ΔPAQ, $\frac{PA}{100} = \cot 30° = \sqrt{3}$

$\Rightarrow \quad PA = 100\sqrt{3}$

$\therefore \quad AB = 100\sqrt{3} - \frac{100\sqrt{3}}{3}$ **1**

$= \frac{200\sqrt{3}}{3}$

$= \frac{200(1.73)}{3} = 115.3$ m **1**

31. A rectangular park is to be designed whose breadth is 3 m less than its length. Its area is to be 4 square metres more than the area of a park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12 m. Find the length and breadth of the rectangular park.

Sol. Area of rectangle $= x(x-3)$, where x is the length ½

Area of Isosceles $\Delta = \frac{1}{2}(x-3)(12)$ ½

$\therefore \quad x(x-3) - \frac{1}{2}(x-3) \times 12 = 4$

$x^2 - 9x + 14 = 0$

or $\quad (x-7)(x-2) = 0$ **1 + 1**

$x = 7$ m (rejecting $x = 2$)

$\therefore \quad$ Length = 7 m breadth = 4 m **1**

Outside Delhi Set III — Code No. 30/3

Note : Except these, all other questions are from Set I & II.

SECTION - B

Question numbers 5 to 10 carry 2 marks each.

10. Solve for x : $\sqrt{6x+7} - (2x-7) = 0$

Sol. $\sqrt{6x+7} = (2x-7)$ ...(i)

$\Rightarrow \quad 6x + 7 = 4x^2 - 28x + 49$

$\Rightarrow \quad 2x^2 - 17x + 21 = 0$

$2x^2 - 14x - 3x + 21$

$\Rightarrow \quad (2x-3)(x-7) = 0$ **1**

$x = 3/2, x = 7$ ½

$x = \frac{3}{2}$ does not satisfy (i)

$\therefore \quad x = 7$ ½

SECTION - C

Question numbers 11 to 20 carry 3 marks each.

18. There are 100 cards in a bag on which numbers from 1 to 100 are written. A card is taken out from the bag at random. Find the probability that the number on the selected card (i) is divisible by 9 and is a perfect square (ii) is a prime number greater than 80.

Sol. (i) Number divisible by 9 and perfect squares are {9, 36, 81} i.e., 3 **1**

$\therefore \quad$ Req. prob. $= \frac{3}{100}$ ½

(ii) Prime numbers greater than 80 are 83, 89, 97 **1**

$\therefore \quad$ Req. prob. $= \frac{3}{100}$ ½

19. Three consecutive natural numbers are such that the square of the middle number exceeds the difference of the squares of the other two by 60. Find the numbers.

Sol. Let the number be x, $x + 1$, $x + 2$ ½

$\therefore \quad (x+1)^2 - [(x+2)^2 - x^2] = 60$ **1**

$x^2 - 2x - 63 = 0$

$x - 9x + 7x - 63 = 0$

$\Rightarrow \quad (x-9)(x+7) = 0$ **1**

$\Rightarrow \quad x = 9$ ½

$\therefore$ Numbers are 9, 10, 11.

20. The sums of first n terms of three arithmetic progressions are S_1, S_2 and S_3 respectively. The first term of each A.P. is 1 and their common differences are 1, 2 and 3 respectively. Prove that $S_1 + S_3 = 2S_2$.

Sol.
$$S_1 = \frac{n}{2}[2 + (n-1)1] \text{ or } \frac{n}{2}[n+1] \quad ½$$

$$S_2 = \frac{n}{2}[2 + (n-1)2] \text{ or } \frac{n}{2}(2n) = n^2 \quad ½$$

$$S_3 = \frac{n}{2}[2 + (n-1)3] \text{ or } \frac{n}{2}(3n-1) \quad ½$$

$$S_1 + S_3 = \frac{n}{2}[4n] = 2n^2 = 2.S_2 \quad ½$$

$\Rightarrow \quad S_1 + S_3 = 2.S_2$ **Hence Proved. 1**

SECTION - D

Question numbers 21 to 31 carry 4 marks each.

28. Two pipes running together can fill a tank in $11\frac{1}{9}$ minutes. If one pipe takes 5 minutes more than the other to fill the tank separately, find the time in which each pipe would fill the tank separately.

Sol. Let the time taken by the taps to fill the tank be x minutes, $x + 5$ minutes respectively

$$\therefore \quad \frac{1}{x} + \frac{1}{x+5} = \frac{9}{100} \quad 2$$

$$\Rightarrow \quad 100(2x+5) = 9x(x+5)$$

$$\Rightarrow \quad 9x^2 - 155x - 500 = 0$$

$$9x^2 - 180x + 25x - 500 = 0$$

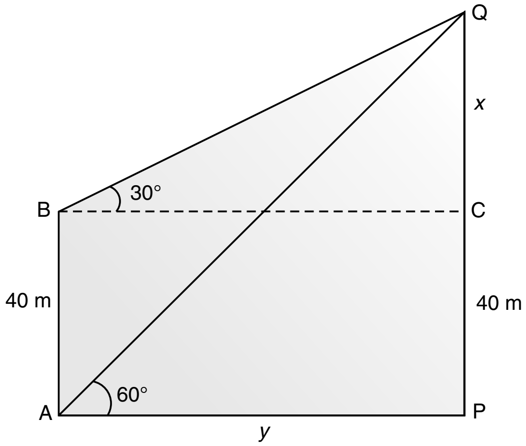
$$\Rightarrow \quad (9x+25)(x-20) = 0 \quad 1$$

$$\Rightarrow \quad x = 20$$

$\therefore$ Times are 20 min and 25 min. 1

29. From a point on the ground, the angle of elevation of the top of tower is observed top be 60°. From a point 40 m vertically above the first point of observation, the angle of elevation of the top of the tower is 30°. Find the height of the tower and its horizontal distance from the point of observation.

Sol.



1

$$\frac{x+40}{y} = \tan 60°$$

$$= \sqrt{3}$$

$$\Rightarrow \quad x + 40 = \sqrt{3}y \quad ...(i)\ 1$$

and
$$\frac{x}{y} = \tan 30°$$

$$= \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \sqrt{3}x = y \quad ...(ii)\ 1$$

$$x + 40 = 3x$$

$$\Rightarrow \quad x = 20 \text{ m}$$

$$y = 20\sqrt{3} \text{ m}$$

$\therefore$ Height of tower = 60 m

Horizontal distance = $20\sqrt{3}$ m 1

*** 30. Draw a triangle with sides 5 cm, 6 cm and 7 cm. Then draw another triangle whose sides are $\frac{4}{5}$ of the corresponding sides of first triangle.**

31. A number x is selected at random from the numbers 1, 4, 9, 16 and another number y is selected at random from the numbers 1, 2, 3, 4. Find the probability that the value of xy is more than 16.

Sol. x can be any one of 1, 4, 9, 16 ½

y can be any one of 1, 2, 3, 4 ½

Total number of cases of $xy = 16$

No. of cases where product more than 16 are 1½

{18, 27, 36, 32, 48, 64} i.e., 6 1½

$\therefore$ Required Probability $= \frac{6}{16}$ or $\frac{3}{8}$

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* *Out of Syllabus*