Solved Paper 2018
Mathematics (Standard)
CLASS-X

Time : 3 Hours **Max. Marks : 80**

General Instructions :

(i) ***All*** *questions are compulsory.*

(ii) This question paper consists of ***30*** *questions divided into four sections – A, B, C and D.*

(iii) Section A contains ***6*** *questions* ***1*** *mark each. Section B contains* ***6*** *questions of* ***2*** *marks each. Section C contains* ***10*** *questions of* ***3*** *marks each. Section D contains* ***8*** *questions of* ***4*** *marks each.*

(iv) There is no overall choice. However, an internal choice has been provided in ***four*** *questions of* ***3*** *marks each and* ***3*** *questions of* ***4*** *marks each. You have to attempt only* ***one*** *of the alternatives in all such questions.*

(v) Use of calculator is ***not*** *permitted.*

Delhi/Outside Delhi Set-I **Code No. 30/1/1**

SECTION - A

Question numbers 1 to 6 carry 1 mark each.

1. if $x = 3$ is one root of the quadratic equation $x^2 - 2kx - 6 = 0$, then find the value of k.

Sol. $x = 3$ is one root of the equation

$\therefore \quad 9 - 6k - 6 = 0$ ½

$k = \frac{1}{2}$ ½

(CBSE Marking Scheme, 2017)

2. What is the HCF of smallest prime number and the smallest composite number ?

Sol. The required numbers are 2 and 4.

HCF of 2 and 4 is 2. ½ + ½

(CBSE Marking Scheme, 2017)

3. Find the distance of a point $P(x, y)$ from the origin.

Sol. $OP = \sqrt{x^2 + y^2}$ **1**

(CBSE Marking Scheme, 2017)

4. In an *A.P.*, if the common difference (d) = – 4, and the seventh term (a_7) is 4, then find the first term.

Sol. $a + 6(-4) = 4$ ½

$\Rightarrow \quad a = 28$ ½

(CBSE Marking Scheme, 2017)

*** 5. What is the value of ($\cos^2 67° - \sin^2 23°$) ?**

*** 6. Given $\Delta ABC \sim \Delta PQR$, if $\frac{AB}{PQ} = \frac{1}{3}$, then find $\frac{ar\ \Delta ABC}{ar\ \Delta PQR}$.**

SECTION - B

Question numbers 7 to 12 carry 2 marks each.

7. Given that $\sqrt{2}$ is irrational, prove that $(5 + 3\sqrt{2})$ is an irrational number.

Sol. Let us assume $(5 + 3\sqrt{2})$ is a rational number

$\therefore \quad 5 + 3\sqrt{2} = \frac{p}{q}$ ½

(where $q \neq 0$ and p and q are integers)

$\Rightarrow \quad \sqrt{2} = \frac{p - 5q}{3q}$ ½

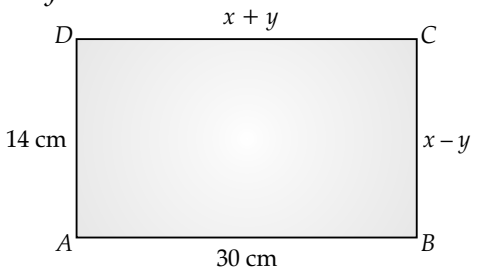
$\Rightarrow \sqrt{2}$ is a rational number as RHS is rational. ½

This contradicts the given fact that $\sqrt{2}$ is irrational.

Hence $(5 + 3\sqrt{2})$ is an irrational number. ½

(CBSE Marking Scheme, 2017)

8. In Fig. 1, *ABCD* is a rectangle. Find the values of x and y.



(Fig. 1)

* *Out of Syllabus*

Sol. $AB = DC$ and $BC = AD$ **1**

$\Rightarrow \quad x + y = 30$

and $\quad x - y = 14$

Solving to get $x = 22$ and $y = 8$. ½+½

(CBSE Marking Scheme, 2017)

9. Find the sum of first 8 multiples of 3.

Sol. $S = 3 + 6 + 9 + 12 + ... + 24$

$= 3(1 + 2 + 3 + ... + 8)$ ½

$= 3 \times \frac{8 \times 9}{2}$ **1**

$= 108$ ½

(CBSE Marking Scheme, 2017)

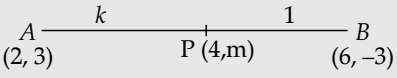
10. Find the ratio in which $P(4, m)$ divides the line segment joining the points $A(2, 3)$ and $B(6, -3)$. Hence find m.

Sol. Let $AP : PB = k : 1$

$\therefore \quad \frac{6k+2}{k+1} = 4$ **1**

$\Rightarrow \quad k = 1$, ratio is 1 : 1

Hence $\quad m = \frac{-3+3}{2} = 0$ ½



½

(CBSE Marking Scheme, 2017)

11. Two different dice are tossed together. Find the probability :

(i) of getting a doublet

(ii) of getting a sum 10, of the numbers on the two dice.

Sol. Total number of possible outcomes = 36

(i) Doublets are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) ½

Total number of doublets = 6

$\therefore$ Probability (getting a doublet) $= \frac{6}{36}$ or $\frac{1}{6}$ ½

(ii) Favourable outcomes are (4, 6), (5, 5), (6, 4) i.e., 3 ½

$\therefore$ Probability (getting a sum 10) $= \frac{3}{36}$ or $\frac{1}{12}$ ½

(CBSE Marking Scheme, 2017)

12. An integer is chosen at random between 1 and 100. Find the probability that it is :

(i) divisible by 8.

(ii) not divisible by 8.

Sol. Total number of outcomes = 98

(i) Favourable outcomes are 8,16,24,..., 96, i.e., 12 ½

$\therefore$ Probability (integer is divisible by 8)

$= \frac{12}{98}$ or $\frac{6}{49}$ **1**

(ii) Probability (integer is not divisible by 8)

$= 1 - \frac{6}{49}$

$= \frac{43}{49}$ ½

(CBSE Marking Scheme, 2017)

SECTION - C

Question numbers 13 to 22 carry 3 marks each.

13. Find HCF and LCM of 404 and 96 and verify that HCF × LCM = Product of the two given numbers.

Sol. $404 = 2 \times 2 \times 101 = 2^2 \times 101$

$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 2^5 \times 3$

$\therefore$ HCF of 404 and 96 $= 2^2 = 4$ **1**

LCM of 404 and 96 $= 101 \times 2^5 \times 3 = 9696$ **1**

HCF × LCM $= 4 \times 9696 = 38784$

Also $\quad 404 \times 96 = 38,784$ **1**

(CBSE Marking Scheme, 2017)

14. Find all zeroes of the polynomial $(2x^4 - 9x^3 + 5x^2 + 3x - 1)$ if two of its zeroes are $\left(2+\sqrt{3}\right)$ and $\left(2-\sqrt{3}\right)$.

Sol. $p(x) = 2x^4 - 9x^3 + 5x^2 + 3x - 1$

$2+\sqrt{3}$ and $2-\sqrt{3}$ are zeroes of $p(x)$

$\therefore p(x) = \left(x-2-\sqrt{3}\right)\left(x-2+\sqrt{3}\right) \times g(x)$

$= (x^2 - 4x + 1)\, g(x)$ **1**

$(2x^4 - 9x^3 + 5x^2 + 3x - 1) \div (x^2 - 4x + 1) = 2x^2 - x - 1$

$\therefore \quad g(x) = 2x^2 - x - 1$

$= (2x + 1)(x - 1)$ **1**

Therefore other zeroes are $x = -\frac{1}{2}$

and $x = 1$ **1**

$\therefore$ Therefore all zeroes are

$2+\sqrt{3}, 2-\sqrt{3}, \frac{-1}{2}$ and 1

(CBSE Marking Scheme, 2017)

15. If $A(-2, 1)$, $B(a, 0)$, $C(4, b)$ and $D(1, 2)$ are the vertices of a parallelogram $ABCD$, find the values of a and b. Hence find the lengths of its sides.

OR

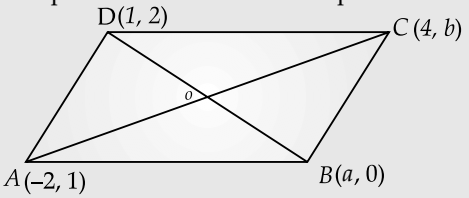
*** If $A(-5, 7)$, $B(-4, -5)$, $C(-1, -6)$ and $D(4, 5)$ are the vertices of a quadrilateral, find the area of the quadrilateral $ABCD$.**

Sol. $ABCD$ is a parallelogram

$\therefore$ Diagonals AC and BD bisect each other

Therefore

Mid point of BD is same as mid point of AC ½



* *Out of Syllabus*

$$\Rightarrow \quad \left(\frac{a+1}{2}, \frac{2}{2}\right) = \left(\frac{-2+4}{2}, \frac{b+1}{2}\right) \qquad \textbf{1}$$

$$\Rightarrow \quad \frac{a+1}{2} = 1 \text{ and } \frac{b+1}{2} = 1$$

$\Rightarrow a = 1, b = 1.$

Lengths of side

$$AB = \sqrt{(-1-2)^2 + (0-2)^2} = \sqrt{10}$$

$$CD = \sqrt{(4-1)^2 + (1-0)^2} = \sqrt{10}$$

$$DA = \sqrt{(1+2)^2 + (2-1)^2} = \sqrt{10}$$

(CBSE Marking Scheme, 2017)

16. A plane left 30 minutes late than its scheduled time and in order to reach of destination 1500 km away in time, it had to increase its speed by 100 km/h from the usual speed. Find its usual speed.

Sol. Let usual speed of the plane be x km/h.

$$\therefore \quad \frac{1500}{x} - \frac{1500}{x+100} = \frac{30}{60} \qquad \textbf{1}$$

$$\Rightarrow \quad x^2 + 100x - 300000 = 0$$

$$\Rightarrow \quad x^2 + 600x - 500x - 300000 = 0 \qquad \textbf{1}$$

$$\Rightarrow \quad (x + 600)(x - 500) = 0$$

$x \neq -600, \therefore x = 500$ ½

Speed of plane = 500 km/h. ½

(CBSE Marking Scheme, 2017)

17. Prove that the area of an equilateral triangle described on one side of the square is equal to half the area of the equilateral triangle described on one of its diagonal.

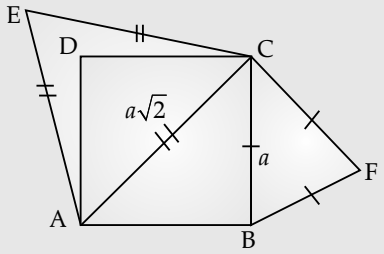
OR

*** If the area of two similar triangles are equal, prove that they are congruent.**

Sol. Let the side of the square be 'a' units

$\therefore \ AC^2 = a^2 + a^2 = 2a^2$

$$\Rightarrow \quad AC = \sqrt{2}a \text{ units.} \qquad \textbf{1}$$



Area of equilateral triangle $\Delta BCF = \frac{\sqrt{3}}{4}a^2$ sq.u ½

Area of equilateral triangle

$$\Delta ACE = \frac{\sqrt{3}}{4}(\sqrt{2}a)^2 = \frac{\sqrt{3}}{2}a^2 \text{ sq.u} \qquad \textbf{1}$$

$$\text{Area of } \Delta BCF = \frac{1}{2} \text{ Ar } \Delta ACE \qquad ½$$

(CBSE Marking Scheme, 2017)

18. Prove that the lengths of tangents drawn from an external point to a circle are equal.

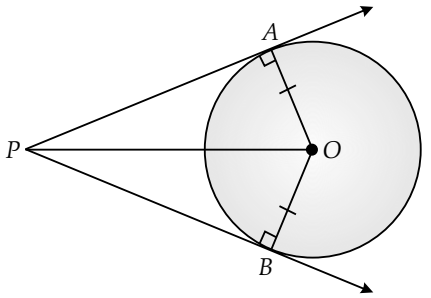
Sol. Correct given, to prove,figure, ½ × 4 = **2**

construction correct proof. **1**

(CBSE Marking Scheme, 2017)

Detailed Solution:

Given : AP and BP are tangents of circle having centre O.



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To prove : $AP = BP$

Construction Join OP, AO and BO

Proof : ΔOAP and ΔOBP

$OA = OB$ (Radius of circle)

$OP = OP$ (Common side)

$\angle OAP = \angle OBP = 90°$

(Radius – tangent angle)

$\therefore \quad \Delta OAP \cong \Delta OBP$

(*RHS* congruency rule)

$\therefore \quad AP = BP$ (CPCT)

Hence Proved.

19. If 4 tan θ = 3, evaluate $\left(\frac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1}\right)$

OR

*** If tan 2A = cot(A – 18°), where 2A is an acute angle, find the value of A.**

Sol.

$$4 \tan\theta = 3$$

$$\Rightarrow \quad \tan\theta = \frac{3}{4}$$

$$\Rightarrow \quad \sin\theta = \frac{3}{5} \text{ and } \cos\theta = \frac{4}{5} \qquad ½ + ½$$

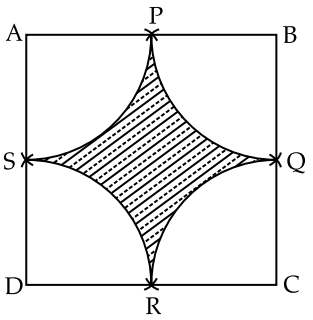
$$\therefore \quad \frac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1} = \frac{4 \times \frac{3}{5} - \frac{4}{5} + 1}{4 \times \frac{3}{5} + \frac{4}{5} - 1} \qquad \textbf{1}$$

$$= \frac{13}{11} \qquad \textbf{1}$$

(CBSE Marking Scheme, 2017)

20. Find the area of the shaded region in Fig. 2, where arcs drawn with centres A, B, C and D intersect in pairs at mid-points P, Q, R and S of the sides AB, BC, CD and DA respectively of a square $ABCD$ of side 12 cm. [Use π = 3.14]

* *Out of Syllabus*



(Fig. 2)

Sol. Radius of each arc drawn = 6 cm ½

$$\text{Area of one quadrant} = (3.14)\times\frac{36}{4}$$

$$\text{Area of four quadrants} = 3.14 \times 36 \quad \textbf{1}$$

$$= 113.04 \text{ cm}^2 \quad \textbf{1}$$

$$\text{Area of square } ABCD = 12\times12$$

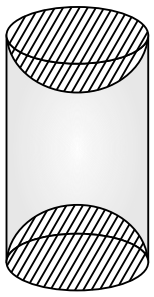
$$= 144 \text{ cm}^2$$

$$\text{Hence Area of shaded region} = 144 - 113.04$$

$$= 30.96 \text{ cm}^2 \quad ½$$

(CBSE Marking Scheme, 2017)

21. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in Fig. 3. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm. Find the total surface area of the article.



(Fig. 3)

OR

A heap of rice is in the form of a cone of base diameter 24 m and height 3.5 m. Find the volume of the rice. How much canvas cloth is required to just cover the heap ?

Sol. Total surface Area of article = *CSA* of cylinder + *CSA* of 2 hemispheres

$$CSA \text{ of cylinder} = 2\pi rh$$

$$= 2\times\frac{22}{7}\times3.5\times10$$

$$= 220 \text{ cm}^2 \quad \textbf{1}$$

Surface area of two hemispherical scoops

$$= 4 \times \frac{22}{7} \times 3.5 \times 3.5$$

$$= 154 \text{ cm}^2 \quad \textbf{1}$$

Total surface area of article = 220 + 154

$$= 374 \text{ cm}^2 \quad \textbf{1}$$

OR

Radius of conical heap = 12 m ½

$$\text{Volume of rice} = \frac{1}{3}\times\frac{22}{7}\times12\times12\times3.5 \text{ m}^3$$

$$= 528 \text{ m}^3 \quad \textbf{1}$$

Area of canvas cloth required = πrl

$$l = \sqrt{12^2 + (3.5)^2} = 12.5 \text{ m} \quad ½$$

$$\therefore \text{ Area of canvas required} = \frac{22}{7} \times 12 \times 12.5$$

$$471.4 \text{ m}^2$$

(CBSE Marking Scheme, 2017)

22. The table below show the salaries of 280 persons :

Salary (In thousand ₹)	No. of Persons
5 – 10	49
10 – 15	133
15 – 20	63
20 – 25	15
25 – 30	6
30 – 35	7
35 – 40	4
40 – 45	2
45 – 50	1

Calculate the median salary of the data.

Sol.

Salary (in thousand ₹)	No. of Persons (f)	*c.f.*
5 – 10	49	49
10 – 15	133 = f	182
15 – 20	63	245
20 – 25	15	260
25 – 30	6	266
30 – 35	7	273
35 – 40	4	277
40 – 45	2	279
45 – 50	1	280

$$\frac{N}{2} = \frac{280}{2} = 140 \quad \textbf{1}$$

Median class = 10 – 15

$$\text{Median} = l + \frac{h}{f}\left(\frac{N}{2} - C\right)$$

$$= 10 + \frac{5}{133}(140 - 49) \quad \textbf{1}$$

$$= 10 + \frac{5 \times 91}{133}$$

$$= 13.42$$

Median salary is ₹ 13.42 thousand or ₹ 13420 (approx) **1**

(CBSE Marking Scheme, 2017)

SECTION - D

Question numbers 23 to 30 carry 4 marks each.

23. A motor boat whose speed is 18 km/h in still water takes 1 hr more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

OR

A train travels at a certain average speed for a distance of 63 km and then travels at a distance of 72 km at an average speed of 6 km/hr more than its original speed. If it takes 3 hours to complete the total journey, what is the original average speed ?

Sol. Let the speed of stream be x km/h.

$\therefore$ The speed of the boat upstream = $(18 - x)$ km/h

and speed of the boat downstream = $(18 + x)$ km/h **1**

As given in the question,

$$\frac{24}{18-x} - \frac{24}{18+x} = 1$$ **1**

$$\Rightarrow \quad x^2 + 48x - 324 = 0$$

$$\Rightarrow \quad (x + 54)(x - 6) = 0$$ ½

$$x \neq -54$$ **1**

$$\therefore \quad x = 6$$ ½

Speed of the stream = 6 km/h.

OR

Let the original average speed of train be x km/h.

Therefore, $\frac{63}{x} + \frac{72}{x+6} = 3$ **1½**

$$\Rightarrow x^2 - 39x - 126 = 0$$

$$\Rightarrow (x - 42)(x + 3) = 0$$ **1**

$$x \neq -3 \ \therefore \ x = 42$$

x is not equal $= -3$ $\quad \therefore x = 42$ **1**

Original speed of train is 42 km/h. ½

(CBSE Marking Scheme, 2017)

24. The sum of four consecutive numbers in an A.P. is 32 and the ratio of the product of the first and the last term to the product of two middle terms is 7 : 15. Find the numbers.

Sol. Let the four consecutive terms of A.P. be

$(a - 3d)$, $(a - d)$, $(a + d)$ and $(a + 3d)$ ½

By given conditions

$$a - 3d + a - d + d + a + 3d = 32$$ **1**

$$\Rightarrow 4a = 32 \quad \Rightarrow a = 8$$

And $\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$ **1**

$$\frac{a^2 - 9d^2}{a^2 - d^2} = \frac{7}{15}$$

$$8a^2 = 128d^2$$ **1**

$$d^2 = 4$$

$$\therefore \quad d = \pm 2$$ ½

$\therefore$ Numbers are 2, 6, 10 and 14 or 14, 10, 6 and 2.

25. * In an equilateral ΔABC, **D is a point on side BC such that $BD = \frac{1}{3}BC$. Prove that $9(AD)^2 = 7(AB)^2$.**

OR

Prove that, in a right triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

Sol. Refer to 2019 year Delhi Set-I Q. 29

*** 26. Draw a triangle ABC with $BC = 6$ cm, $AB = 5$ cm and $\angle ABC = 60°$. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the ΔABC.**

27. Prove that : $\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$.

Sol. $$\text{LHS} = \frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A}$$

$$= \frac{\sin A(1 - 2\sin^2 A)}{\cos A(2\cos^2 A - 1)}$$ **1**

$$= \frac{\sin A(1 - 2(1 - \cos^2 A))}{\cos A(2\cos^2 A - 1)}$$ **1**

$$= \tan A \cdot \frac{2\cos^2 A - 1}{2\cos^2 A - 1}$$ **1**

$$= \tan A = \text{RHS}$$ **1**

(CBSE Marking Scheme, 2017)

*** 28. The diameters of the lower and upper ends of a bucket in the from of a frustum of a cone are 10 cm and 30 cm respectively. If its height is 24 cm, find :**

(i) The area of the metal sheet used to make the bucket.

(ii) Why we should avoid the bucket made by ordinary plastic ? [Use $\pi = 3.14$]

29. As observed from the top of a 100 m high light house from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the light house, find the distance between the two ships. [Use $\sqrt{3} = 1.732$]

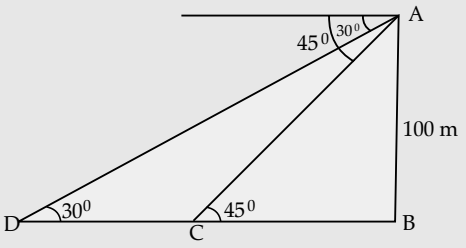
Sol. Let AB be the tower and ships are at points C and D

$$\tan 45° = \frac{AB}{BC}$$

$$\Rightarrow \quad \frac{AB}{BC} = 1$$

* *Out of Syllabus*

$\Rightarrow$ $AB = BC$ **1**



1

Also $\tan 30° = \frac{1}{\sqrt{3}} = \frac{AB}{BC + CD}$

$\Rightarrow$ $\frac{1}{\sqrt{3}} = \frac{AB}{BC + CD}$ **1**

$\Rightarrow$ $\frac{1}{\sqrt{3}} = \frac{AB}{AB + CD}$

$\Rightarrow$ $AB + CD = \sqrt{3} \cdot AB$

$\Rightarrow$ $CD = AB\ (\sqrt{3} - 1)$

$= 100 \times (1.732 - 1)$ **1**

$= 73.2$ m.

(CBSE Marking Scheme, 2017)

30. The mean of the following distribution is 18. Find the frequency *f* of the class 19–21.

Class	Frequency
11 – 13	3
13 – 15	6
15 – 17	9
17 – 19	13
19 – 21	*f*
21 – 23	5
23 – 25	4

OR

* **The following distribution gives the daily income of 50 workers of a factory :**

Daily Income (in ₹)	Number of workers
100 – 120	12
120 – 140	14
140 – 160	8
160 – 180	6
180 – 200	10

Convert the distribution above to a less than type cumulative frequency distribution and draw its ogive.

Sol. Refer to 2022 year O.D. Set-II Q. 6. on page 16

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* *Out of Syllabus*