Solved Paper 2019
Mathematics (Standard)
CLASS-X

Time : 3 Hours **Max. Marks : 80**

General Instructions :

(i) *All questions are **compulsory**.*

(ii) *The question paper consists of **30** questions divided into four sections **A, B, C** and **D**.*

(iii) *Section **A** contains **6** questions of **1** mark each. Section **B** contains **6** questions of **2** marks each; Section **C** contains **10** questions of **3** marks each. Section **D** contains **8** questions of **4** marks each.*

(iv) *there is no overall choice. However, an internal choice has been provided in **two** question of **1** mark, **two** questions of **2** marks, **four** questions of **3** marks each and **three** questions of **4** marks each. You have to attempt only **one** of the alternative in all such questions.*

(v) *Use of calculator is not permitted.*

Delhi Set-I **Code No. 30/1/1**

SECTION - A

Question numbers 1 to 6 carry 1 mark each.

1. Find the coordinates of a point A, where AB is diameter of a circle whose centre is (2, –3) and B is the point (1, 4).

Sol. Let the point A be (x, y)

$\therefore \quad \frac{x+1}{2} = 2$ and $\frac{4+y}{2} = -3$ ½

$\Rightarrow \quad x = 3$ and $y = -10$ ½

$\therefore$ Point A is (3, –10)

(CBSE Marking Scheme, 2019)

2. For what values of k, the roots of the equation $x^2 + 4x + k = 0$ are real?

OR

Find the value of k for which the roots of the equation $3x^2 - 10x + k = 0$ are reciprocal of each other.

Sol. Since roots of the equation $x^2 + 4x + k = 0$ are real

$\Rightarrow \quad 16 - 4k \geq 0$ ½

$\Rightarrow \quad k \leq 4$ ½

OR

Roots of the equation $3x^2 - 10x + k = 0$ are reciprocal of each other

$\Rightarrow$ Product of roots = 1 ½

$\Rightarrow \quad \frac{k}{3} = 1 \Rightarrow k = 3$ ½

(CBSE Marking Scheme, 2019)

*** 3. Find A if tan 2A = cot (A – 24°)**

OR

Find the value of ($\sin^2 33° + \sin^2 57°$)

4. How many two digits numbers are divisible by 3 ?

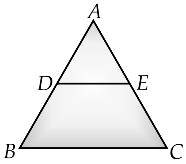
Sol. Numbers are 12, 15, 18, ..., 99 ½

$\therefore \quad 99 = 12 + (n-1) \times 3$ ½

$\Rightarrow \quad n = 30$

(CBSE Marking Scheme, 2019)

*** 5. In Figure, $DE \parallel BC$, AD = 1 cm and BD = 2 cm. What is the ratio of the ar (ΔABC) to the ar (ΔADE) ?**



6. Find a rational number between $\sqrt{2}$ and $\sqrt{3}$.

Sol. Any one rational number between $\sqrt{2}$ (1.41 approx.) and $\sqrt{3}$ (1.73 approx.)

e.g., 1.5, 1.6, 1.63 etc. **1**

Rational No. = 1.416893

(CBSE Marking Scheme, 2019)

SECTION - B

Question numbers 7 to 12 carry 2 marks each.

*** 7. Find the HCF of 1260 and 7344 using Euclid's algorithm ?**

OR

*** Show that every positive odd integer is of the form ($4q$ + 1) or ($4q$ + 3), where q is some integer.**

8. Which term of the A.P. 3, 15, 27, 39, ... will be 120 more than its 21st term ?

OR

If S_n, the sum of first n terms of an A.P. is given by $S_n = 3n^2 - 4n$. Find the n^{th} term.

* *Out of Syllabus*

Sol. $a_n = a_{21} + 120$
$= (3 + 20 \times 12) + 120$
$= 363$ **1**

$\therefore \quad 363 = 3 + (n-1) \times 12$

$\Rightarrow \quad n = 31$ **1**

or 31^{st} term is 120 more than a_{21}.

OR

$a_1 = S_1 = 3 - 4 = -1$ ½

$a_2 = S_2 - S_1$
$= [3(2)^2 - 4(2)] - (-1) = 5$ ½

$\therefore \quad d = a_2 - a_1 = 6$ ½

Hence $a_n = -1 + (n-1) \times 6 = 6n - 7$ ½

Alternate method :

$S_n = 3n^2 - 4n$ **1**

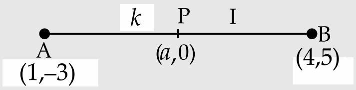
$\therefore \quad S_{n-1} = 3(n-1)^2 - 4(n-1)$
$= 3n^2 - 10n + 7$ ½

Hence $a_n = S_n - S_{n-1}$
$= (3n^2 - 4n) - (3n^2 - 10n + 7)$
$= 6n - 7$ ½

(CBSE Marking Scheme, 2019)

9. Find the ratio in which the segment joining the points (1, – 3) and (4, 5) is divided by *x*-axis ? Also find the coordinates of this point on *x*-axis.

Sol. Let the required point be $(a, 0)$ and required ratio $AP : PB = k : 1$ ½



$\therefore \quad a = \dfrac{4k+1}{k+1}$

$0 = \dfrac{5k-3}{k+1}$

$\Rightarrow \quad k = \dfrac{3}{5}$ or required ratio is 3 : 5 **1**

Point P is $\left(\dfrac{17}{8}, 0\right)$ ½

(CBSE Marking Scheme, 2019)

10. A game consists of tossing a coin 3 times and noting the outcome each time. If getting the same result in all the tosses is a success, find the probability of losing the game.

Sol. Total number of outcomes = 8 ½

Favourable number of outcomes
(*HHH, TTT*) = 2 ½

Prob. (getting success) $= \dfrac{2}{8}$ or $\dfrac{1}{4}$ ½

$\therefore$ Prob. (losing the game) $= 1 - \dfrac{1}{4} = \dfrac{3}{4}$ ½

(CBSE Marking Scheme, 2019)

11. A die is thrown once. Find the probability of getting a number which (i) is a prime number (ii) lies between 2 and 6.

Sol. Total number of outcomes = 6.

(i) Prob. (getting a prime number (2, 3, 5))

$= \dfrac{3}{6}$ or $\dfrac{1}{2}$ **1**

(ii) Prob. (getting a number between 2 and 6 (3, 4, 5)

$= \dfrac{3}{6}$ or $\dfrac{1}{2}$. **1**

(CBSE Marking Scheme, 2019)

12. Find *c* if the system of equations $cx + 3y + (3 - c) = 0$; $12x + cy - c = 0$ has infinitely many solutions ?

Sol. System of equations has infinitely many solutions

$\therefore \quad \dfrac{c}{12} = \dfrac{3}{c} = \dfrac{3-c}{-c}$ ½

$\Rightarrow \quad c^2 = 36 \Rightarrow c = 6$ or $c = -6$ ...(1) ½

Also, $-3c = 3c - c^2 \Rightarrow c = 6$ or $c = 0$ ...(2) ½

From equations (1) and (2), ½

$c = 6$.

(CBSE Marking Scheme, 2019)

SECTION - C

Question numbers 13 to 22 carry 3 marks each.

13. Prove that $\sqrt{2}$ is an irrational number ?

Sol. Let us assume $\sqrt{2}$ be a rational number and its simplest form be $\dfrac{a}{b}$, a and b are co-prime positive integers and $b \neq 0$. ½

So $\quad \sqrt{2} = \dfrac{a}{b}$

$\Rightarrow \quad a^2 = 2b^2$ **1**

Thus a^2 is a multiple of 2

$\Rightarrow a$ is a multiple of 2. ½

Let $a = 2m$ for some integer m

$\therefore b^2 = 2m^2$ ½

Thus b^2 is a multiple of 2

$\Rightarrow b$ is a multiple of 2

Hence 2 is a common factor of a and b.

This contradicts the fact that a and b are co-primes ½

Hence $\sqrt{2}$ is an irrational number.

(CBSE Marking Scheme, 2019)

14. Find the value of *k* such that the polynomial $x^2 - (k+6)x + 2(2k-1)$ has sum of its zeros equal to half of their product.

Sol. Sum of zeroes = $k + 6$ **1**

Product of zeroes = $2(2k - 1)$ **1**

Hence $k + 6 = \frac{1}{2} \times 2\,(2k - 1)$ **1**

$\Rightarrow k = 7$

(CBSE Marking Scheme, 2019)

15. A father's age is three times the sum of the ages of his two children. After 5 years his age will be two times the sum of their ages. Find the present age of the father.

OR

A fraction becomes $\frac{1}{3}$ when 2 is subtracted from the numerator and it becomes $\frac{1}{2}$ when 1 is subtracted from the denominator- Find the fraction.

Sol. Let sum of the ages of two children be x yrs and father's age be y yrs.

$\therefore \quad y = 3x$ ...(1) **1**

and $\quad y + 5 = 2(x + 10)$ ...(2) **1**

Solving equations (1) and (2),

$x = 15$

and $\quad y = 45$

Father's present age is 45 years. **1**

(CBSE Marking Scheme, 2019)

OR

Let the fraction be $\frac{x}{y}$

$\therefore \quad \frac{x-2}{y} = \frac{1}{3}$ ...(1) **1**

and $\quad \frac{x}{y-1} = \frac{1}{2}$ ...(2) **1**

Solving (1) and (2) to get $x = 7, y = 15$.

$\therefore$ Required fraction is $\frac{7}{15}$ **1**

(CBSE Marking Scheme, 2019)

16. Find the point on y-axis which is equidistant from the points (5, –2) and (–3, 2).

OR

The line segment joining the points $A(2, 1)$ and $B(5, -8)$ is trisected at the points P and Q such that P is nearer to A. If P also lies on the line given by $2x - y + k = 0$, find the value of k.

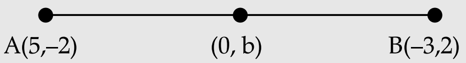
Sol. Let the required point on y-axis be $(0, b)$ ½

$\therefore (5 - 0)^2 + (-2 - b)^2 = (-3 - 0)^2 + (2 - b)^2$ **1**

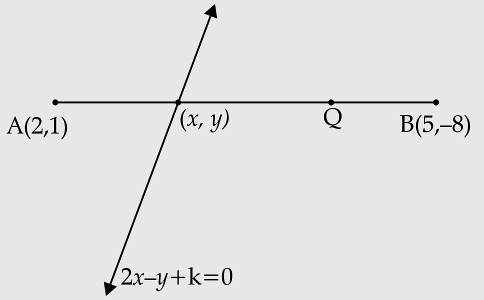
$\Rightarrow 29 + 4b + b^2 = 13 + b^2 - 4b$

$\Rightarrow b = -2$ **1**

$\therefore$ Required point is (0, –2). ½



OR



1

$AP : PB = 1 : 2$

$x = \frac{4+5}{3} = 3$ and $y = \frac{2-8}{3} = -2$ ½ + ½

Thus point P is (3, –2).

Point (3, –2) lies on $2x - y + k = 0$

$\Rightarrow \quad 6 + 2 + k = 0$

$\Rightarrow \quad k = -8.$ **1**

(CBSE Marking Scheme, 2019)

17. Prove that

$(\sin\theta + \text{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$.

OR

Prove that

$(1 + \cot A - \text{cosec}\,A)(1 + \tan A + \sec A) = 2$

Sol. LHS $= \sin^2\theta + \text{cosec}^2\theta + 2\sin\theta\,\text{cosec}\,\theta + \cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta$ **1**

$= (\sin^2\theta + \cos^2\theta) + \text{cosec}^2\theta + \sec^2\theta + \frac{2\sin\theta}{\sin\theta} + 2\frac{\cos\theta}{\cos\theta}$

$= 1 + 1 + \cot^2\theta + 1 + \tan^2\theta + 2 + 2$ **1½**

$= 7 + \cot^2\theta + \tan^2\theta =$ RHS ½

OR

LHS $= \left(1 + \frac{1}{\tan A} - \text{cosec}\,A\right)(1 + \tan A + \sec A)$ **1**

$= \frac{1}{\tan A}(\tan A + 1 - \sec A)(1 + \tan A + \sec A)$ **1**

$= \frac{1}{\tan A}[(1 + \tan A)^2 - \sec^2 A]$

$= \frac{1}{\tan A}[1 + \tan^2 A + 2\tan A - 1 - \tan^2 A]$

$= 2 =$ RHS **1**

Alternate method :

LHS $= \left(1 + \frac{\cos A}{\sin A} - \frac{1}{\sin A}\right)\left(1 + \frac{\sin A}{\cos A} + \frac{1}{\cos A}\right)$ **1**

$= (\sin A + \cos A - 1)(\cos A + \sin A + 1).\frac{1}{\cos A \sin A}$

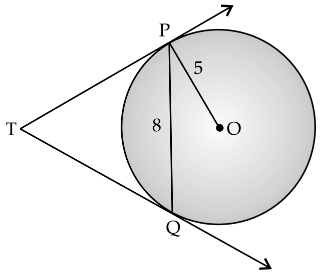
$= [(\sin A + \cos A)^2 - 1)] \times \frac{1}{\sin A \cos A}$ **1**

$= (1 + 2\sin A\cos A - 1) \times \frac{1}{\sin A \cos A}$ ½

$= 2 =$ RHS ½

(CBSE Marking Scheme, 2019)

18. In Figure PQ is a chord of length 8 cm of a circle of radius 5 cm and centre O. The tangents at P and Q intersect at point T. Find the length of TP.

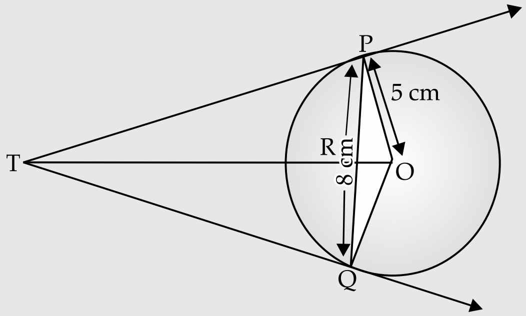


Sol. Join OT and OQ.

$TP = TQ$

$\therefore TR \perp PQ$ and bisects PQ

Hence $PR = 4$ cm



Therefore $OR = \sqrt{25-16} = \sqrt{9} = 3$ cm. ½

Let $TR = x$

In ΔPRT, $PT^2 = x^2 + 16$ (Pythagoras Theorem)

In ΔPOT, $PT^2 = (x + 3)^2 - 25$ ½

Hence $x^2 + 16 = x^2 + 9 + 6x - 25$

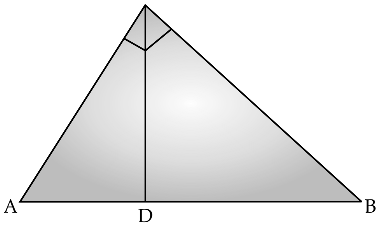
$\Rightarrow 6x = 32 \Rightarrow x = \frac{16}{3}$ ½

Hence $PT^2 = \frac{256}{9} + 16 = \frac{400}{9}$ ½

$\therefore PT = \frac{20}{3}$ cm 1

(CBSE Marking Scheme, 2019)

19. In Figure $\angle ACB = 90°$ and $CD \perp AB$, prove that $CD^2 = BD \times AD$.



OR

If P and Q are the points on side CA and CB respectively of ΔABC, right angled at C. Prove that $(AQ^2 + BP^2) = (AB^2 + PQ^2)$

Sol. $\Delta ACB \sim \Delta ADC$ (AA similarity)

$\Rightarrow \frac{AC}{BC} = \frac{AD}{CD}$ ...(1)

Also $\Delta ACB \sim \Delta CDB$ (AA similarity)

$\Rightarrow \frac{AC}{BC} = \frac{CD}{BD}$ ...(2)

Using equations (1) and (2)

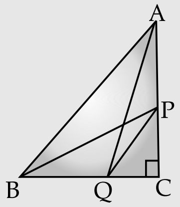
$\frac{AD}{CD} = \frac{CD}{BD}$

$\Rightarrow CD^2 = AD \times BD$

OR

Correct Figure ½

$AQ^2 = CQ^2 + AC^2$ 1



$BP^2 = CP^2 + BC^2$ ½

$\therefore AQ^2 + BP^2 = (CQ^2 + CP^2) + (AC^2 + BC^2)$

$= PQ^2 + AB^2$. 1

(CBSE Marking Scheme, 2019)

20. Find the area of the shaded region in Figure 4, if $ABCD$ is a rectangle with sides 8 cm and 6 cm and O is the centre of circle. (Take $\pi = 3.14$)

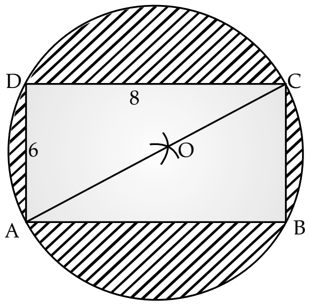


Figure 4

Sol. $AC = \sqrt{64+36} = 10$ cm.

$\therefore$ Radius of the circle (r) = 5 cm. 1

Area of shaded region = Area of circle − Ar($ABCD$) ½

$= 3.14 \times 25 - 6 \times 8$ 1

$= 78.5 - 48$

$= 30.5$ cm^2. ½

(CBSE Marking Scheme, 2019)

21. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/hour. How much area will it irrigate in 30 minutes; if 8 cm standing water is needed ?

Sol. Refer to 2020 year OD Set-III Q.33 on page 24

22. Find the mode of the following frequency distribution.

Class	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70
Frequency	8	10	10	16	12	6	7

Sol. Modal class is 30 – 40

$\therefore$ Mode $= l+\left(\dfrac{f_1-f_0}{2f_1-f_0-f_2}\right)\times h$ ½

$= 30+\left(\dfrac{16-10}{32-10-12}\right)\times 10$ **2**

$= 36.$ ½

(CBSE Marking Scheme, 2019)

SECTION - D

Question numbers 23 to 30 carry 4 marks each.

23. Two water taps together can fill a tank in $1\frac{7}{8}$ hours. The tap with longer diameter takes 2 hours less than the tap with smaller one to fill the tank separately. Find the time in which each tap can fill the tank separately.

OR

A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km downstream. Determine the speed of the stream and that of the boat in still water.

Sol. Let the smaller tap fills the tank in x hrs

$\therefore$ the larger tap fills the tank in $(x-2)$ hrs.

Time taken by both the taps together $= \dfrac{15}{8}$ hrs.

Therefore $\dfrac{1}{x}+\dfrac{1}{x-2}=\dfrac{8}{15}$ **2**

$\Rightarrow \quad 4x^2-23x+15=0$ ½

$\Rightarrow \quad (4x-3)(x-5)=0$

$x \neq \dfrac{3}{4} \quad \therefore x=5$ **1**

Smaller and larger taps can fill the tank separately in 5 hrs and 3 hrs respectively. ½

OR

Let the speed of the boat in still water be x km/h and speed of the stream be y km/hr.

Given $\dfrac{30}{x-y}+\dfrac{44}{x+y}=10$ ...(i) **1**

and $\dfrac{40}{x-y}+\dfrac{55}{x+y}=13$ ...(ii) **1**

Solving (i) and (ii) to get

$x+y=11$ ...(iii)

and $x-y=5$ ...(iv)

Solving (iii) and (iv) to get $x=8, y=3.$ **1**

Speed of boat = 8 km/h & speed of stream = 3 km/h. **1**

(CBSE Marking Scheme, 2019)

24. If the sum of first four terms of an A.P. is 40 and that of first 14 terms is 280. Find the sum of its first *n* terms.

Sol. $S_4 = 40 \Rightarrow 2(2a+3d)=40 \Rightarrow 2a+3d=20$ ½

$S_{14}=280 \Rightarrow 7(2a+13d)=280 \Rightarrow 2a+13d=40$ ½

Solving to get $d=2$ ½

and $a=7$ ½

$\therefore \quad S_n = \dfrac{n}{2}[14+(n-1)\times 2]$ **1**

$= n(n+6)$ or (n^2+6n) **1**

(CBSE Marking Scheme, 2019)

25. Prove that $\dfrac{\sin A-\cos A+1}{\sin A+\cos A-1}=\dfrac{1}{\sec A-\tan A}$

Sol. LHS $= \dfrac{\sin A-\cos A+1}{\sin A+\cos A-1}$

Dividing num. & deno. by cos A

$= \dfrac{\tan A-1+\sec A}{\tan A+1-\sec A}$ **1**

$= \dfrac{\tan A-1+\sec A}{(\tan A-\sec A)+(\sec^2 A-\tan^2 A)}$ **1**

$= \dfrac{\tan A-1+\sec A}{(\tan A-\sec A)(1-\sec A-\tan A)}$ **1**

$= \dfrac{-1}{\tan A-\sec A}=\dfrac{1}{\sec A-\tan A}$ = RHS **1**

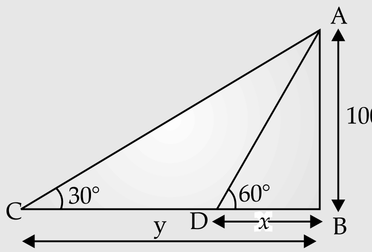
(CBSE Marking Scheme, 2019)

26. A man in a boat rowing away from a light house 100 m high takes 2 minutes to change the angle of elevation of the top of the light house from 60° to 30°. Find the speed of the boat in metres per minute. [Use $\sqrt{3}$ = 1.732]

OR

Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30° respectively. Find the height of the poles and the distances of the point from the poles.

Sol.



1

Let the speed of the boat be y m/min

$\therefore CD = 2y$

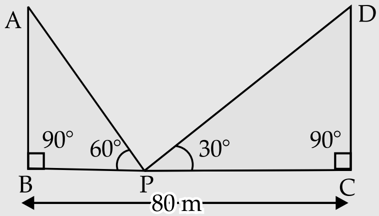
$\tan 60° = \sqrt{3} = \frac{100}{x} \Rightarrow x = \frac{100}{\sqrt{3}}$ 1

$\tan 30° = \frac{1}{\sqrt{3}} = \frac{100}{x+2y} \Rightarrow x + 2y = 100\sqrt{3}$ ½

$\therefore y = \frac{100\sqrt{3}}{3} = 57.73$ 1

or speed of boat = 57.74 m/min. ½

OR



1

Let $BP = x$ so $CP = 80 - x$ 1

where BC is the road.

$\tan 60° = \sqrt{3} = \frac{h}{x} \Rightarrow h = x\sqrt{3}$

and $\tan 30° = \frac{1}{\sqrt{3}} = \frac{100}{x+2y} \Rightarrow h\sqrt{3} = 80 - x$

Solving equation to get 1

$x = 20, h = 20\sqrt{3}$

$\therefore CP = 60$ m, BP = 20 m and $h = 20\sqrt{3}$ m. 1

(CBSE Marking Scheme, 2019)

*** 27. Construct a ΔABC in which $CA = 6$ cm, $AB = 5$ cm and $\angle BAC = 45°$. Then construct a triangle whose sides are $\frac{3}{5}$ of the corresponding sides of ΔABC.**

*** 28. A bucket open at the top is in the form of a frustum of a cone with a capacity of 12308.8 cm^3. The radii of the top and bottom of circular ends of the bucket are 20 cm and 12 cm respectively. Find the height of the bucket and also the area of the metal sheet used in making it. (Use $\pi = 3.14$)**

*** 29. Prove that in a right angle triangle, the square of the hypotenuse is equal to the sum of squares of the other two sides.**

30. If the median of the following frequency distribution is 32.5. Find the values of f_1 and f_2.

Class	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	Total
Frequency	f_1	5	9	12	f_2	3	2	40

OR

*** The marks obtained by 100 students of a class is an examination are given below :**

Marks	0 - 5	5 - 10	10 - 15	15 - 20	20 - 25	25 - 30	30 - 35	35 - 40	40 - 45	45 - 50
No. of Students	2	5	6	8	10	25	20	18	4	2

Draw 'a less than' type cumulative frequency curves (ogive). Hence find median.

Sol.

Class	Frequency	Cumulative freq.
0-10	f_1	f_1
10-20	5	$5 + f_1$
20-30	9	$14 + f_1$
30-40	12	$26 + f_1$
40-50	f_2	$26 + f_1 + f_2$
50-60	3	$29 + f_1 + f_2$
60-70	2	$31 + f_1 + f_2$
	40	

1

Median = 32.5 ⇒ median class is 30-40. ½

Now $32.5 = 30 + \frac{10}{12}(20 - 14 - f_1)$ 1

$\Rightarrow f_1 = 3$ 1

Also $31 + f_1 + f_2 = 40$

$\Rightarrow f_2 = 6$ ½

Delta Set-II Code No. 30/1/2

SECTION - A

Question numbers 1 to 6 carry 1 mark each.

6. Find the coordinates of a point A, where AB is diameter of the circle whose centre is (2, –3) and B is the point (3, 4).

Sol. Let the point A be (x, y) ½

$2 = \frac{x+3}{2}$ and $\frac{y+4}{2} = -3$

$x = 1, y = -10$

Point A is (1, –10) ½

(CBSE Marking Scheme, 2019)

* *Out of Syllabus*

SECTION - B

Question numbers 7 to 12 carry 2 marks each.

7. Find the value of, k for which the following pair of linear equations have infinitely many solutions :
$2x + 3y = 7$ and $(k + 1)x + (2k - 1)y = 4k + 1$

Sol. System of equations has infinitely many solutions.

$\therefore \quad \frac{2}{k+1} = \frac{3}{2k-1} = \frac{7}{4k+1}$ ½

$\Rightarrow \quad 4k - 2 = 3k + 3$ ½

$\Rightarrow \quad k = 5$

Also $\quad 12k + 3 = 14k - 7$ ½

$\Rightarrow \quad k = 5$

Hence $k = 5$. ½

(CBSE Marking Scheme, 2019)

SECTION - C

Question numbers 13 to 22 carry 3 marks each.

13. The arithmetic mean of the following frequency distribution is 53. Find the value of k.

Class	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100
Frequency	12	15	32	k	13

Sol.

Class	x	**Freq (f)**	$u = \frac{x-50}{20}$	fu
0-20	10	12	–2	–24
20-40	30	15	–1	–15
40-60	50 = A	32	0	0
60-80	70	k	1	k
80-100	90	13	2	26
		$72 + k$		$-13 + k$

2

$$(\text{Mean})\bar{X} = 53 = 50 + 20 \times \frac{-13+k}{72+k}$$

$\Rightarrow \quad 3k + 216 = 20k - 260$ **1**

$\Rightarrow \quad k = 28$

(CBSE Marking Scheme, 2019)

14. Find the area of the segment shown in Figure 2, if radius of the circle is 21 cm and $\angle AOB = 120°$ $\left(\text{Use } \pi = \frac{22}{7}\right)$

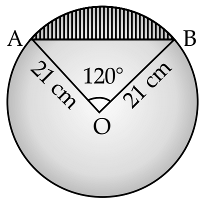
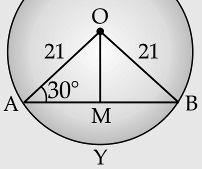


Fig. 2

* *Out of Syllabus*

Sol. Draw $OM \perp AB$

$\angle OAB = \angle OBA = 30°$



$$\sin 30° = \frac{1}{2} = \frac{OM}{21}$$

$\Rightarrow \quad OM = \frac{21}{2}$ ½

$$\cos 30° = \frac{\sqrt{3}}{2} = \frac{AM}{21}$$

$\Rightarrow \quad AM = \frac{21}{2}\sqrt{3}$ ½

Area of $\Delta OAB = \frac{1}{2} \times AB \times OM$

$= \frac{1}{2} \times 21\sqrt{3} \times \frac{21}{2}$ ½

$= \frac{441}{4}\sqrt{3}\text{cm}^2.$

$\therefore$ Area of shaded region

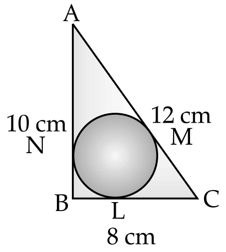
= Area (sector $OAYB$) – Area (ΔOAB) ½

$= \frac{22}{7} \times 21 \times 21 \times \frac{120}{360} - \frac{441}{4}\sqrt{3}$ ½

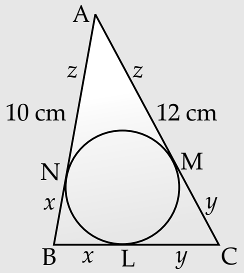
$= \left(462 - 441\frac{\sqrt{3}}{4}\right)$ cm^2 or 271.3 cm^2 (approx.) ½

(CBSE Marking Scheme, 2019)

16. In Figure 4, a circle is inscribed in a ΔABC having sides $BC = 8$ cm, $AB = 10$ cm and $AC = 12$ cm. Find the length BL, CM and AN.



Sol. Let $\quad BL = x = BN$ ½

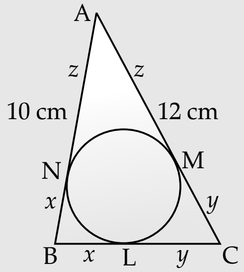


½

$\therefore$ $CL = 8 - x = CM$
$\because$ $AC = 12 \Rightarrow AM = 4 + x = AN$ **1**
Now $AB = AN + NB = 10 \Rightarrow x + 4 + x = 10$
$\Rightarrow$ $x = 3$
$\therefore$ $BL = 3$ cm, $CM = 5$ and $AN = 7$ cm

Alternate method

Let $BL = BN = x$ ½
(tangents from external points are equal)
$CL = CM = y$
$AN = AM = z$ ½



½

$\therefore$ $AB + BC + AC = 2x + 2y + 2z = 30$ ½
$\Rightarrow$ $x + y + z = 15$ ...(i)
Also $x + z = 10$, $x + y = 8$ and $y + z = 12$
Subtracting from equation (i)
$y = 5, z = 7$ and $x = 3$ $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$
$\therefore BL = 3$ cm, $CM = 5$ and $AN = 7$ cm. ½

(CBSE Marking Scheme, 2019)

SECTION - D

Question numbers 23 to 30 carry 4 marks each.

23. Prove that:

$$\frac{\tan^2 A}{\tan^2 A - 1} + \frac{\text{cosec}^2 A}{\sec^2 A - \text{cosec}^2 A} = \frac{1}{1 - 2\cos^2 A}$$

Sol. LHS $= \dfrac{\sin^2 A/\cos^2 A}{\dfrac{\sin^2 A}{\cos^2 A} - 1} + \dfrac{1/\sin^2 A}{\dfrac{1}{\cos^2 A} - \dfrac{1}{\sin^2 A}}$ **1**

$= \dfrac{\sin^2 A}{\sin^2 A - \cos^2 A} + \dfrac{\cos^2 A}{\sin^2 A - \cos^2 A}$ **1**

$= \dfrac{1}{\sin^2 A - \cos^2 A}$ **1**

$= \dfrac{1}{1 - 2\cos^2 A}$ **1**

(CBSE Marking Scheme, 2019)

24. The first term of an A.P. is 3, the last term is 83 and the sum of all its terms is 903. Find the number of terms and the common difference of the A.P.

Sol. Here $a = 3$, $a_n = 83$ and $S_n = 903$ **1**
Therefore $83 = 3 + (n-1)d$
$\Rightarrow$ $(n-1)d = 80$ ...(i) **1**
Also $903 = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(6 + 80)$
$= 43\,n$ (using (i)) **1**
$\Rightarrow$ $n = 21$
and $d = 4$ **1**

(CBSE Marking Scheme, 2019)

*** 25. Construct a triangle *ABC* with side *BC* = 6 cm, ∠*B* = 45°, ∠*A* = 105°. Then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of the Δ*ABC*.**

Delhi Set-III — Code No. 30/1/3

SECTION - A

Question numbers 1 to 6 carry 1 mark each.

1. Two positive integers *a* and *b* can be written as $a = x^3y^2$ and $b = xy^3$. *x*, *y* are prime numbers. Find LCM (*a*, *b*).

Sol. LCM $(x^3y^2, xy^3) = x^3y^3$. **1**

(CBSE Marking Scheme, 2019)

SECTION - B

Question numbers 7 to 12 carry 2 marks each.

7. Find, how many two digit natural numbers are divisible by 7.

OR

If the sum of first *n* terms of an AP is n^2, then find its 10th term.

Sol. Required numbers are **1**
14, 21, 28, 35, ..., 98.
$98 = 14 + (n-1) \times 7$ ½
$\Rightarrow$ $n = 13$ ½

OR

Given $S_n = n^2$
$S_1 = a_1 = 1$ ½
$S_2 = a_1 + a_2 = 4$
$\Rightarrow$ $a_2 = 3$ ½
$\therefore$ $d = a_2 - a_1 = 2$ ½
$a_{10} = 1 + 18 = 19$ ½

(CBSE Marking Scheme, 2019)

SECTION - C

Question numbers 13 to 22 carry 3 marks each.

*** 13. Find all zeros of the polynomial $3x^3 + 10x^2 - 9x - 4$ if one of its zero is 1.**

15. Prove that $\frac{2+\sqrt{3}}{5}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

* *Out of Syllabus*

Sol. Let us assume $\frac{2+\sqrt{3}}{5}$ be a rational number.

Let $\frac{2+\sqrt{3}}{5} = \frac{a}{b}$ ($b \neq 0$, a and b co prime number) **1**

$\Rightarrow \sqrt{3} = \frac{5a-2b}{b}$

$\Rightarrow a, b$ are integers

$\therefore \frac{5a-2b}{b}$ is a rational number **1**

i.e. $\sqrt{3}$ is a rational number

which contradicts the fact that $\sqrt{3}$ is irrational

Therefore $\frac{2+\sqrt{3}}{5}$ is an irrational number. **1**

(CBSE Marking Scheme, 2019)

SECTION - D

Question numbers 23 to 30 carry 4 marks each.

23. If $\sec\theta = x + \frac{1}{4x}, x \neq 0$, find ($\sec\theta + \tan\theta$).

Sol. $\sec^2\theta = \left(x + \frac{1}{4x}\right)^2 = x^2 + \frac{1}{16x^2} + \frac{1}{2}$ **1**

$\therefore \tan^2\theta = \sec^2\theta - 1 = x^2 + \frac{1}{16x^2} - \frac{1}{2}$ **1**

$\Rightarrow \quad \tan^2\theta = \left(x - \frac{1}{4x}\right)^2$

$\Rightarrow \quad \tan\theta = \left(x - \frac{1}{4x}\right)$ or $\left(\frac{1}{4x} - x\right)$ **1**

Hence $\sec\theta + \tan\theta = 2x$ or $\frac{1}{2x}$ **1**

(CBSE Marking Scheme, 2019)

*** 24. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.**

*** 25. The following distribution gives the daily income of 50 workers of a factory :**

Daily income (in ₹)	200 - 220	220 - 240	240 - 260	260 - 280	280 - 300
Number of workers	12	14	8	6	10

Convert the distribution above to a 'less than type' cumulative frequency distribution and draw its ogive.

OR

The table below shows the daily expenditure on food of 25 households in a locality. Find the mean daily expenditure on food.

Daily expenditure (in ₹)	100 - 150	150 - 200	200 - 250	250 - 300	300 - 350
Number of households	4	5	12	2	2

Sol. **OR**

Daily expenditure	x_i	**No. of households** (f_i)	$u_i = \frac{x-225}{50}$	f_iu_i
100 – 150	125	4	–2	–8
150 – 200	175	5	–1	–5
200 – 250	225 = A	12	0	0
250 – 300	275	2	1	2
300 – 350	325	2	2	4
		$\Sigma f_i = 25$		$\Sigma f_iu_i = -7$

2

Mean $\bar{X} = 225 + 50 \times \left(\frac{-7}{25}\right) = 211$

Mean expenditure on food is ₹ 211. **2**

(CBSE Marking Scheme, 2019)

* *Out of Syllabus*

Outside Delhi Set-I Code No. 30/2/1

SECTION - A

Question numbers 1 to 6 carry 1 marks each.

1. If HCF (336, 54) = 6, find LCM (336, 54).

Sol. $\text{LCM}(336, 54) = \frac{336 \times 54}{6}$ ½

$= 336 \times 9 = 3024$ ½

(CBSE Marking Scheme, 2019)

2. Find the nature of roots of the quadratic equation $2x^2 - 4x + 3 = 0$.

Sol. $2x^2 - 4x + 3 = 0 \Rightarrow D = 16 - 24 = -8$

$\therefore$ Equation has no real roots **1**

(CBSE Marking Scheme, 2019)

3. Find the common difference of the Arithmetic Progression (*A.P.*)

$$\frac{1}{a}, \frac{3-a}{3a}, \frac{3-2a}{3a}, \ldots (a \neq 0)$$

Sol. $\frac{3-a}{3a} - \frac{1}{a} = \frac{3-a-3}{3a} = -\frac{1}{3}$ **1**

(CBSE Marking Scheme, 2019)

4. Evaluate

$\sin^2 60° + 2\tan 45° - \cos^2 30°$

OR

If $\sin A = \frac{3}{4}$ calculate sec *A*.

Sol. $\sin^2 60° + 2\tan 45° - \cos^2 30°$

$= \left(\frac{\sqrt{3}}{2}\right)^2 + 2(1) - \left(\frac{\sqrt{3}}{2}\right)^2$ ½

$= 2$ ½

OR

$\sin A = \frac{3}{4} \Rightarrow \cos A = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}$ ½

$\sec A = \frac{4}{\sqrt{7}}$ ½

(CBSE Marking Scheme, 2019)

5. Write the coordinates of a point *P* on x-axis which is equidistant from the points *A*(–2, 0) and *B*(6, 0).

Sol. Point on x-axis is (2, 0) **1**

(CBSE Marking Scheme, 2019)

Explanation: The point P will lie at the midpoint of line segment joining A(– 2, 0) and B(6, 0)

So, $x = \frac{-2+6}{2} = 2$

$y = 0$

So, coordinates of P are (2, 0). **1**

6. In Figure 1, *ABC* is an isosceles triangle right angled at C with *AC* = 4 cm. Find the length of *AB*.

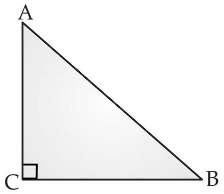
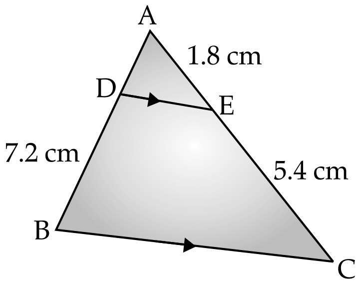


Figure 1

OR

In Figure 2, *DE* || *BC*. Find the length of side *AD*, given that *AE* = 1.8 cm, *BD* = 7.2 cm and *CE* = 5.4 cm.



Sol. ΔABC : Isosceles $\Delta \Rightarrow AC = BC = 4$ cm. ½

$AB = \sqrt{4^2 + 4^2} = 4\sqrt{2}$ cm ½

OR

$\frac{AD}{BD} = \frac{AE}{CE} \Rightarrow \frac{AD}{7.2} = \frac{1.8}{5.4}$ ½

$\therefore AD = \frac{7.2 \times 1.8}{5.4} = 2.4$ cm. ½

(CBSE Marking Scheme, 2019)

SECTION - B

Question numbers 7 to 12 carry 2 marks each.

7. Write the smallest number which is divisible by both 306 and 657.

Sol. Smallest number divisible by 306 and 657

$= LCM\ (306, 657)$ **1**

$LCM\ (306, 657) = 22338$ **1**

(CBSE Marking Scheme, 2019)

*** 8. Find a relation between *x* and *y* if the points *A*(*x*, *y*), *B*(–4, 6) and *C*(–2, 3) are collinear.**

OR

Find the area of a triangle whose vertices are given as (1, –1) (–4, 6) and (–3, –5).

9. The probability of selecting a blue marble at random from a jar that contains only blue, black

* *Out of Syllabus*

and green marbles is $\frac{1}{5}$. The probability of selecting a black marble at random from the same jar is $\frac{1}{4}$. If the jar contains 11 green marbles, find the total number of marbles in the jar.

Sol. $P(\text{blue marble}) = \frac{1}{5}$, $P(\text{black marble}) = \frac{1}{4}$

$\therefore \quad P(\text{green marble}) = 1 - \left(\frac{1}{5}+\frac{1}{4}\right) = \frac{11}{20}$ **1**

Let total number of marbles be x

then $\frac{11}{20} \times x = 11 \Rightarrow x = 20$ **1**

(CBSE Marking Scheme, 2019)

10. Find the value(s) of k so that the pair of equations $x + 2y = 5$ and $3x + ky + 15 = 0$ has a unique solution.

Sol. For unique solution $\frac{1}{3} \neq \frac{2}{k}$ **1**

$\Rightarrow \quad k \neq 6$ **1**

(CBSE Marking Scheme, 2019)

11. The larger of two supplementary angles exceed the smaller by 18°. Find the angles.

OR

Sumit is 3 times as old as his son. Five years later, he shall be two and a half times as old as his son. How old is Sumit at present?

Sol. Let larger angle be $x°$

$\therefore$ Smaller angle $= 180° - x°$ ½

$\therefore \quad (x) - (180 - x) = 18$ ½

$2x = 180+18 = 198 \Rightarrow x = 99$ ½

$\therefore$ The two angles are 99°, 81° ½

OR

Let Son's present age be x years

Then Sumit's present age $= 3x$ years. ½

$\therefore$ 5 Years later, we have, $3x + 5 = \frac{5}{2}(x + 5)$ ½

$6x + 10 = 5x + 25 \Rightarrow x = 15$ ½

$\therefore$ Sumit's present age = 45 years ½

(CBSE Marking Scheme, 2019)

12. Find the mode of the following frequency distribution

Class Interval	**25 - 30**	**30 - 35**	**35 - 40**	**40 - 45**	**45 - 50**	**50 - 55**
Frequency	**25**	**34**	**50**	**42**	**38**	**14**

Sol. Maximum frequency = 50, class (modal) = 35 – 40. ½

$$\text{Mode} = L + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 35 + \frac{50-34}{100-34-42} \times 5 \quad \textbf{1}$$

$$= 35 + \frac{16}{24} \times 5 = 38.33 \quad ½$$

(CBSE Marking Scheme, 2019)

SECTION - C

Question numbers 13 to 22 carry 3 marks each.

13. Prove that $2 + 5\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

OR

*** Using Euclid's Algorithm, find the HCF of 2048 and 960.**

Sol. Let $2 + 5\sqrt{3} = a$, where 'a' is a rational number. **1**

then $\sqrt{3} = \frac{a-2}{5}$ **1**

Which is a contradiction as LHS is irrational and RHS is rational **1**

$\therefore 2 + 5\sqrt{3}$ can not be rational

Hence $2 + 5\sqrt{3}$ is irrational.

Alternate method :

Let $2 + 5\sqrt{3}$ be rational ½

$\therefore \quad 2 + 5\sqrt{3} = \frac{p}{q}$, p, q are integers, $q \neq 0$

$\Rightarrow \quad \sqrt{3} = \left(\frac{p}{q} - 2\right) \div 5 = \frac{p-2q}{5q}$ **1**

LHS is irrational and RHS is rational which is a contradiction. ½

$\therefore 2 + 5\sqrt{3}$ is irrational. **1**

(CBSE Marking Scheme, 2019)

14. Two right triangles *ABC* and *DBC* are drawn on the same hypotenuse *BC* and on the same side of *BC*. If *AC* and *BD* intersect at *P*, prove that $AP \times PC = BP \times DP$.

OR

*** Diagonals of a trapezium *PQRS* intersect each other at the point *O*, $PQ \parallel RS$ and $PQ = 3RS$. Find the ratio of the areas of triangles *POQ* and *ROS*.**

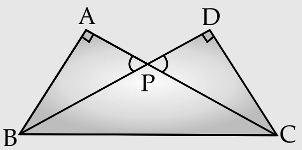
* *Out of Syllabus*

Sol. $\Delta APB \sim \Delta DPC$ [*AA* similarity] **1**

$\angle A = \angle D = 90°$

$$\frac{AP}{DP} = \frac{BP}{PC}$$ **1**

$\Rightarrow$ $AP \times PC = BP \times DP$ ½



½

15. In Figure 3, *PQ* and *RS* are two parallel tangents to a circle with centre *O* and another tangent *AB* with point of contact *C* intersecting *PQ* at *A* and *RS* at *B*. Prove that $\angle AOB = 90°$.

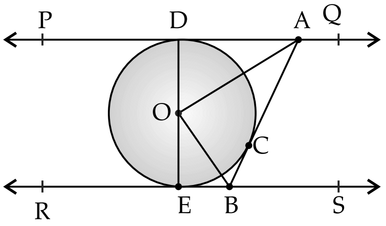


Figure 3

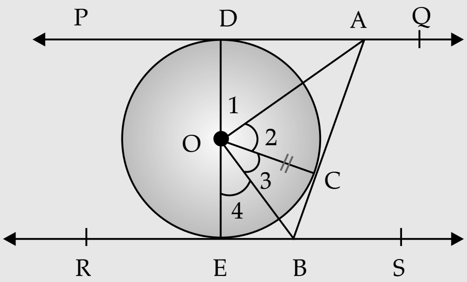
Sol. $\Delta AOD \cong \Delta AOC$ [*RHS*] **1**

$\Rightarrow$ $\angle 1 = \angle 2$ ½

Similarly $\angle 4 = \angle 3$ ½

$\Rightarrow$ $\angle 1 + \angle 4 = \angle 2 + \angle 3 = \frac{1}{2}(180°)$

$\Rightarrow$ $\angle 2 + \angle 3 = 90°$ or $\angle AOB = 90°$ ½



½

Alternate method:

$\Delta OAD \cong \Delta AOC$ [RHS]

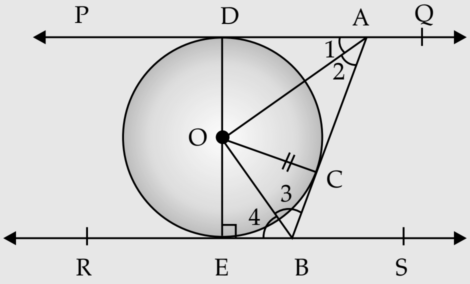
$\Rightarrow$ $\angle 1 = \angle 2$ **1**

Similarly $\angle 4 = \angle 3$ **1**

But $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180°$ [$\because PQ \,||\, RS$]

$\Rightarrow$ $\angle 2 + \angle 3 = \angle 1 + \angle 4 = \frac{1}{2}(180°) = 90°$

$\therefore$ In ΔAOB, $\angle AOB = 180° - (\angle 2 + \angle 3) = 90°$

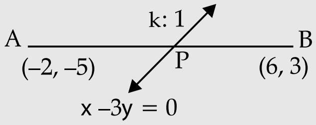


1

(CBSE Marking Scheme, 2019)

16. Find the ratio in which the line $x - 3y = 0$ divides the line segment joining the points (–2, – 5) and (6, 3). Find the coordinates of the point of intersection.

Sol. Let the line $x - 3y = 0$ intersect the segment



joining $A(-2, -5)$ and $B(6, 3)$ in the ratio $k : 1$ ½

$\therefore$ Coordinates of P are $\left(\frac{6k-2}{k+1}, \frac{3k-5}{k+1}\right)$ **1**

P lies on $x - 3y = 0 \Rightarrow \frac{6k-2}{k+1} = 3\left(\frac{3k-5}{k+1}\right)$

$\Rightarrow$ $k = \frac{13}{3}$

$\therefore$ Ratio is 13 : 3 ½

$\Rightarrow$ Coordinates of P are $\left(\frac{9}{2}, \frac{3}{2}\right)$ **1**

(CBSE Marking Scheme, 2019)

*** 17. Evaluate:**

$$\left(\frac{3\sin 43^\circ}{\cos 47^\circ}\right)^2 - \frac{\cos 37^\circ \operatorname{cosec} 53^\circ}{\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ}$$

18. In Figure 4, a square *OABC* is inscribed in a quadrant *OPBQ*. If *OA* = 15 cm, find the area of the shaded region. (Use π = 3.14)

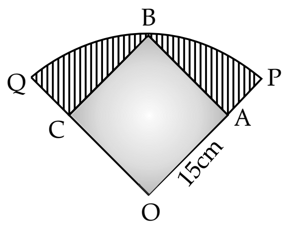


Figure 4

* *Out of Syllabus*

OR

In Figure 5, *ABCD* is a square with side $2\sqrt{2}$ cm and inscribed in a circle. Find the area of the shaded region.
(Use $\pi = 3.14$)

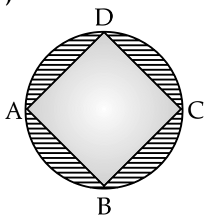


Figure 5

Sol. Radius of quadrant = $OB = \sqrt{15^2 + 15^2}$

$= 15\sqrt{2}$ cm. **1**

Shaded area = Area of quadrant – Area of square ½

$= \frac{1}{4}(3.14)\,[(5\sqrt{2})^2 - (15)^2]$ **1**

$= (15)^2(1.57-1) = 128.25\text{cm}^2$ ½

OR

$BD = \sqrt{(2\sqrt{2})^2 + (2\sqrt{2})^2} = \sqrt{16} = 4$ cm **1**

$\therefore$ Radius of circle = 2 cm ½

$\therefore$ Shaded area = Area of circle – Area of square ½

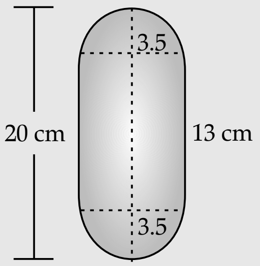
$= 3.14 \times 2^2 - \left(2\sqrt{2}\right)^2$

$= 12.56 - 8 = 4.56\text{ cm}^2$ **1**

(CBSE Marking Scheme, 2019)

19. A solid is in the form of a cylinder with hemispherical end. The total height of the solid is 20 cm and the diameter of the cylinder is 7 cm. Find the total volume of the solid. $\left(\text{Use } \pi = \frac{22}{7}\right)$

Sol. Height of cylinder = 20 – 7 = 13 cm. **1**



$$\text{Total volume} = \pi\left(\frac{7}{2}\right)^2 \times 13 + \frac{4}{3}\pi\left(\frac{7}{2}\right)^3 \text{ cm}^3 \quad \mathbf{1}$$

$$= \frac{22}{7}\times\frac{49}{4}\left(13 + \frac{4}{3}.\frac{7}{2}\right) \text{ cm}^3$$

$$= \frac{77\times 53}{6} = 680.17 \text{ cm}^3 \quad \mathbf{1}$$

(CBSE Marking Scheme, 2019)

20. The marks obtained by 110 students in an examination are given below :

Marks	Number of Students
30 - 35	14
35 - 40	16
40 - 45	28
45 - 50	23
50 - 55	18
55 - 60	8
60 - 65	3

Find the mean marks of the students.

Sol.

x_i :	32.5	37.5	42.5	47.5	52.5	57.5	62.5
f_i :	14	16	28	23	18	8	3

½

$\Sigma fi = 110$ ½

u_i :	–3	–2	–1	0	1	2	3
f_iu_i :	–42	–32	–28	0	18	16	9

$\Sigma f_iu_i = -59$ **1**

$\text{Mean} = 47.5 - \frac{59\times 5}{110} = 47.5 - 2.68 = 44.82$ **1**

(CBSE Marking Scheme, 2019)

21. * For what value of k, is the polynomial $f(x) = 3x^4 - 9x^3 + x^2 + 15x + k$ completely divisible by $3x^2 - 5$

OR

Find the zeroes of the quadratic polynomial $7y^2 - \frac{11}{3}y - \frac{2}{3}$ and verify the relationship between the zeroes and the coefficients.

Sol. **OR**

$$p(y) = 7y^2 - \frac{11}{3}y - \frac{2}{3} = \frac{1}{3}(21y^2 - 11y - 2)$$

$$= \frac{1}{3}[(7y+1)(3y-2)] \quad \mathbf{1}$$

$\therefore$ Zeroes are $\frac{2}{3}, \frac{-1}{7}$ ½

$\text{Sum of zeroes} = \frac{2}{3} - \frac{1}{7} = \frac{11}{21}$

$\frac{-b}{a} = \frac{11}{21} \quad \therefore \text{sum of zeroes} = \frac{-b}{a}$ **1**

$\text{Product of zeroes} = \left(\frac{2}{3}\right)\left(-\frac{1}{7}\right) = -\frac{2}{21}$

$\frac{c}{a} = -\frac{2}{3}\left(\frac{1}{7}\right) = -\frac{2}{21} \quad \therefore \text{Product} = \frac{c}{a}$ ½

(CBSE Marking Scheme, 2019)

* *Out of Syllabus*

22. Write all the values of p for which the quadratic equation $x^2 + px + 16 = 0$ has equal roots. Find the roots of the equation so obtained.

Sol. $x^2 + px + 16 = 0$ have equal roots if

$D = p^2 - 4(16)(1) = 0$ **1**

$p^2 = 64 \Rightarrow p = \pm 8$ ½

$\therefore \quad x^2 \pm 8x + 16 = 0 \Rightarrow (x \pm 4)^2 = 0$ **1**

$x \pm 4 = 0$

$\therefore$ Roots are $x = -4$ and $x = 4$ ½

(CBSE Marking Scheme, 2019)

SECTION - D

Question numbers 23 to 30 carry 4 marks each.

23. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio.

Sol. For correct, given, to prove, construction and figure ½ × 4 = 2

For correct proof. **2**

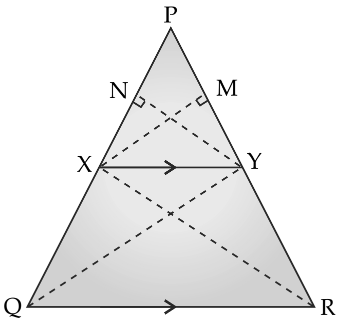
(CBSE Marking Scheme, 2019)

Detailed Solution :

Given : ΔPQR, in which $XY \parallel QR$, XY intersects PQ and PR at X and Y, respectively.

To prove : $\dfrac{PY}{YR} = \dfrac{PY}{YR}$

Construction : Join RX and QY and draw YN perpendicular to PQ and XM perpendicular to PR.



Proof : Since area of a triangle

$$= \frac{1}{2} \times \text{base} \times \text{height}$$

Therefore, $\text{ar}(\Delta PXY) = \frac{1}{2} \times PX \times YN$ ...(i)

Also, $\text{ar}(\Delta PXY) = \frac{1}{2} \times PY \times XM$ ...(ii)

Similarly, $\text{ar}(\Delta QXY) = \frac{1}{2} \times QX \times NY$ ...(iii)

and $\text{ar}(\Delta RXY) = \frac{1}{2} \times YR \times XM$ ...(iv)

Dividing (i) by (iii), we get

$$\therefore \quad \frac{\text{ar}(\Delta PXY)}{\text{ar}(\Delta QXY)} = \frac{\frac{1}{2} \times PX \times YN}{\frac{1}{2} \times QX \times YN} = \frac{PX}{QX} \quad \text{...(v)}$$

Again, dividing (ii) by (iv), we get

$$\frac{\text{ar}(\Delta PXY)}{\text{ar}(\Delta RXY)} = \frac{\frac{1}{2} \times PY \times XM}{\frac{1}{2} \times YR \times XM} = \frac{PY}{YR} \quad \text{...(vi)}$$

Since the area of triangles with same base and between same parallel lines are equal, so

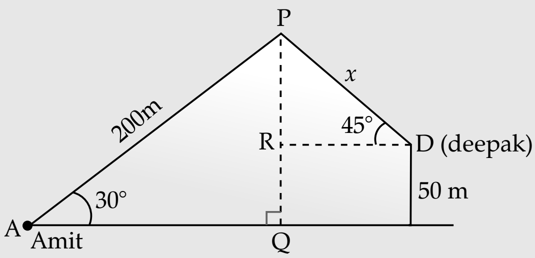
$\text{ar}(\Delta QXY) = \text{ar}(\Delta RXY)$ ...(vii)

Therefore, from (v), (vi) and (vii), we get

$$\frac{PX}{XQ} = \frac{PY}{YR}$$ **Hence proved.**

24. Amit, standing on a horizontal plane, find a bird flying at a distance of 200 m from him at an elevation of 30°. Deepak standing on the roof of a 50 m high building, find the angle of elevation of the same bird to be 45°. Amit and Deepak are on opposite sides of the bird. Find the distance of the bird from Deepak.

Sol.



1

In ΔAPQ

$\dfrac{PQ}{AP} = \sin 30° \; \dfrac{1}{2}$ ½

$PQ = (200)\left(\dfrac{1}{2}\right) = 100$ m **1**

$PQ = 100 - 50 = 50$ m ½

In ΔPRD, $\dfrac{PR}{PD} = \sin 45° = \dfrac{1}{\sqrt{2}}$

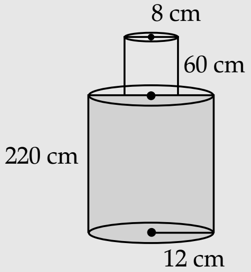
$PD = (PR)(\sqrt{2}) = 50(\sqrt{2})$ m **1**

(CBSE Marking Scheme, 2019)

25. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm³ of iron has approximately

8 gm mass. (Use π = 3.14)

Sol.



Total volume = $3.14\ (12)^2\ (220)\ \ 3.14(8)^2(60)\ cm^3$ **1**

$= 99475.2 + 12057.6 = 111532.8\ cm^3$ **1**

$$\text{Mass} = \frac{111532.8 \times 8}{1000} \text{kg}$$ **1**

$= 892.262$ kg **1**

(CBSE Marking Scheme, 2019)

*** 26. Construct an equilateral ΔABC with each side 5 cm. Then construct another triangle whose sides are $\frac{2}{3}$ times the corresponding sides of ΔABC.**

OR

*** Draw two concentric circles of radii 2 cm and 5 cm. Take a point P on the outer circle and construct a pair of tangents PA and PB to the smaller circle. Measure PA.**

*** 27. Change the following data into 'less than type' distribution and draw its ogive :**

Class Interval	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80	80 - 90	90 - 100
Frequency	7	5	8	10	6	6	8

28. Prove that:

$$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\,\text{cosec}\,\theta$$

OR

Prove that:

$$\frac{\sin\theta}{\cot\theta + \text{cosec}\,\theta} = 2 + \frac{\sin\theta}{\cot\theta - \text{cosec}\,\theta}$$

Sol. LHS $= \dfrac{\tan\theta}{1-\dfrac{1}{\tan\theta}} + \dfrac{\dfrac{1}{\tan\theta}}{1-\tan\theta}$

$$= \frac{\tan^2\theta}{\tan\theta - 1} - \frac{1}{\tan\theta\,(\tan\theta - 1)}$$ **1**

$$= \frac{\tan^3\theta - 1}{\tan\theta(\tan\theta - 1)}$$

$$= \frac{(\tan\theta - 1)(\tan^2\theta + \tan\theta + 1)}{\tan\theta(\tan\theta - 1)}$$ **1**

$= \tan\theta + 1 + \cot\theta$

$$= 1 + \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 1 + \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$ **1**

$$= 1 + \frac{1}{\sin\theta\cos\theta}$$

$= 1 + \text{cosec}\,\theta\sec\theta$ = RHS **1**

OR

Consider

$$= \frac{\sin\theta}{\text{cosec}\,\theta + \cot\theta} - \frac{\sin\theta}{\cot\theta - \text{cosec}\,\theta}$$

$$= \frac{\sin\theta}{\text{cosec}\,\theta + \cot\theta} + \frac{\sin\theta}{\text{cosec}\,\theta - \cot\theta}$$ **1+1**

$$= \frac{\sin\theta[\text{cosec}\,\theta - \cot\theta + \text{cosec}\,\theta + \cot\theta]}{\text{cosec}^2\theta - \cot^2\theta}$$

$$= \frac{\sin\theta(2\text{cosec}\,\theta)}{1} = 2$$ **1½**

Hence $\dfrac{\sin\theta}{\text{cosec}\,\theta + \cot\theta} = 2 + \dfrac{\sin\theta}{\cot\theta - \text{cosec}\,\theta}$ **½**

(CBSE Marking Scheme, 2019)

29. Which term of the Arithmetic Progression –7, –12, –17, –22, ... will be –82 ? Is –100 any term of the A.P. ? Give reason for your answer.

OR

How many terms of the Arithmetic Progression 45, 39, 33, ... must be taken so that their sum is 180? Explain the double answer.

Sol. Let $-82 = a_n$

$\therefore$ $-82 = -7 + (n-1)(-5)$ **1**

$\Rightarrow$ $15 = n - 1$ or $n = 16$ **1**

Again $-100 = a_m$

$= -7 + (m-1)(-5)$ **1**

$\Rightarrow$ $(m-1)(-5) = -93$

$m - 1 = \dfrac{93}{5}$ or $m = \dfrac{93}{5} + 1 \notin N$ **1**

$\therefore$ –100 is not a term of the AP.

OR

$S_n = 180 = \dfrac{n}{2}[90 + (n-1)(-6)]$ **1**

$360 = 90n - 6n^2 + 6n$

$\Rightarrow$ $6n^2 - 96n + 360 = 0$ **1**

$\Rightarrow 6[(n-6)(n-10)] = 0 \Rightarrow n = 6, n = 10$ **1**

Sum of $a_7, a_8, a_9, a_{10} = 0$ $\therefore$ $n = 6$ or $n = 10$ **1**

(CBSE Marking Scheme, 2019)

* *Out of Syllabus*

30. In a class test, the sum of Arun's marks in Hindi and English is 30. When he got 2 marks more in Hindi and 3 marks less in English, the product of the marks would have been 210. Find his marks in the two subjects.

Sol. Let marks in Hindi be x

Then marks in Eng $= 30 - x$ ½

$\therefore \quad (x+2)(30-x-3) = 210$ 1

$\Rightarrow \quad x^2 - 25x + 156 = 0$ or $(x-13)(x-12) = 0$ 1

$\Rightarrow \quad x = 13$ or $x = 12$

$\therefore \quad 30 - 13 = 17$ or $30 - 12 = 18$ 1

$\therefore$ Marks in Hindi & English are

(13, 17) or (12, 18) ½

(CBSE Marking Scheme, 2019)

Outside Delhi Set-II — Code No. 30/2/2

SECTION - A

Question numbers 1 to 6 carry 1 marks each.

6. Find the 21st term of the A.P. $-4\frac{1}{2}, -3, -1\frac{1}{2}, \ldots$

Sol. $a = -4\frac{1}{2}, d = 1\frac{1}{2},$

$\therefore \quad a_{21} = -\frac{9}{2} + 20\left(\frac{3}{2}\right)$ ½

$= \frac{51}{2} = 25\frac{1}{2}$ ½

(CBSE Marking Scheme, 2019)

SECTION - B

Question numbers 7 to 12 carry 2 marks each.

7. For what value of k, will the following pair of equations have infinitely many solutions :
$2x + 3y = 7$ and $(k+2)x - 3(1-k)y = 5k + 1$

Sol. For infinitely many solution

$$\frac{2}{k+2} = \frac{3}{-3(1-k)} = \frac{7}{5k+1} \quad 1$$

$\Rightarrow \quad 2k - 2 = k + 2$ or $5k + 1 = 7k - 7$

$\Rightarrow \quad k = 4$

$\Rightarrow \quad 2k = 8 \Rightarrow k = 4$

Hence $\quad k = 4.$ 1

(CBSE Marking Scheme, 2019)

SECTION - C

Question numbers 13 to 22 carry 3 marks each.

13. Point A lies on the line segment XY joining $X(6, -6)$ and $Y(-4, -1)$ in such a way that $\frac{XA}{XY} = \frac{2}{5}$.

If point A also lies on the line $3x + k(y+1) = 0$, find the value of k.

Sol. $\frac{XA}{XY} = \frac{2}{5} \Rightarrow \frac{XA}{AY} = \frac{2}{3}$ 1

$\therefore$ Coordinates of A are $\left(\frac{-8+18}{5}, \frac{-2-18}{5}\right)$ *i.e.*

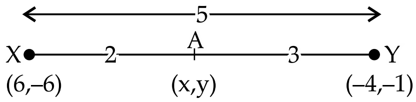
(2, –4) 1

A lies on $3x + k(y+1) = 0$

$\Rightarrow \quad 6 + k(-3) = 0 \Rightarrow k = 2$ 1

(CBSE Marking Scheme, 2019)

Detailed Solution:



Using section formula, for point $A(x, y)$

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

and $$y = \frac{m_1y_2 + m_2y_2}{m_1 + m_2}$$

Here, $m_1 = 2$, $m_2 = 3$, $x_1 = 6$, $x_2 = -4$, $y_1 = -6$ and $y_2 = -1$

Substituting values, we get

$$x = \frac{2\times(-4)+3(6)}{2+3} = \frac{-8+18}{5} = \frac{10}{5} = 2$$

and $$y = \frac{2\times(-1)+3(-6)}{2+3} = \frac{-2-18}{5} = \frac{-20}{5} = -4$$

Hence, coordinates of point A is (2, –4)

If point A also lies on the line $3x + k(y+1) = 0$, then coordinates of A satisfies the line

i.e. $\quad 3(2) + k(-4+1) = 0$

$\Rightarrow \quad 6 + (-3k) = 0$

$\Rightarrow \quad 3k = 6$

$\Rightarrow \quad k = 2$

Hence, value of k is 2.

14. Solve for x:

$x^2 + 5x - (a^2 + a - 6) = 0$

Sol. $x^2 + 5x - (a+3)(a-2) = 0$

$x^2 + (a+3)x - (a-2)x - (a+3)(a-2) = 0$ 1½

$[x + (a+3)[x - (a-2)] = 0$

$\Rightarrow \quad x = (a-2)$ or $x = -(a+3)$ 1½

Alternative Method :

$x^2 + 5x - (a^2 + a - 6) = 0$

$$x = \frac{-5 \pm \sqrt{25 + 4(a^2 + a - 6)}}{2} \quad 1$$

$$x = \frac{-5 \pm (2a+1)}{2} \quad 1$$

$x = (a-2), -(a+3)$ 1

(CBSE Marking Scheme, 2019)

Detailed Solution:

$$x^2 + 5x - (a^2 + a - 6) = 0$$
$$\Rightarrow x^2 + 5x - [a^2 + 3a - 2a - 6] = 0$$
$$\Rightarrow x^2 + 5x - [a(a + 3) - 2(a + 3)] = 0$$
$$\Rightarrow x^2 + 5x - (a + 3)(a - 2) = 0$$
$$\Rightarrow x^2 + [a + 3 - (a - 2)]x - (a + 3)(a - 2) = 0$$
$$\Rightarrow x^2 + (a + 3)x - (a - 2)x - (a + 3)(a - 2) = 0$$
$$\Rightarrow x[x + (a + 3)] - (a - 2)[x + (a + 3)] = 0$$
$$\Rightarrow [x + (a + 3)][x - (a - 2)] = 0$$

$\Rightarrow x = -(a + 3)$ or $x = (a - 2)$

Hence, roots of given equations are $x = -(a + 3)$ and $x = a - 2$.

15. Find A and B if $\sin(A + 2B) = \dfrac{\sqrt{3}}{2}$ and $\cos(A + 4B) = 0$, where A and B are acute angles.

Sol. $A + 2B = 60^\circ$ and $A + 4B = 90^\circ$ **1+1**

Solving to get $B = 15^\circ$

and $A = 30^\circ$ **1**

(CBSE Marking Scheme, 2019)

Detailed Solution:

Given, $\sin(A + 2B) = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \sin(A + 2B) = \sin 60^\circ$ $\left[\because \sin 60^\circ = \dfrac{\sqrt{3}}{2}\right]$

$\Rightarrow A + 2B = 60^\circ$ ...(i)

Also, given $\cos(A + 4B) = 0$

$\Rightarrow \cos(A + 4B) = \cos 90^\circ$ $[\because \cos 90^\circ = 0]$

$\Rightarrow A + 4B = 90^\circ$ ...(ii)

By subtracting equation (ii) from equation (i)

$A + 2B - (A + 4B) = 60^\circ - 90^\circ$

$\Rightarrow -2B = -30^\circ$

$\Rightarrow B = 15^\circ$

From eq(i),

$A + 2(15^\circ) = 60^\circ$

$\Rightarrow A = 60^\circ - 30^\circ = 30^\circ$

Hence angle $A = 30^\circ$ and angle $B = 15^\circ$.

Outsided Delhi Set-III — Code No. 30/2/3

SECTION - A

Question numbers 1 to 6 carry 1 mark each.

1. Find the nature of the roots of the quadratic equation $4x^2 + 4\sqrt{3}x + 3 = 0$.

Sol. $D = (4\sqrt{3})^2 - 4(4)(3) = 0$ ½

$\therefore$ Roots are real and equal ½

(CBSE Marking Scheme, 2019)

SECTION - B

Question numbers 7 to 12 carry 2 marks each.

7. A die is thrown twice. Find the probability that

(i) 5 will come up at least once.

(ii) 5 will not come up either time.

Sol. E_1 {(1, 5),(2, 5), (3, 5), (4, 5), (5, 5), (6, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 6)}

$\therefore P(\text{5 will come at least once}) = P(E_1) = \dfrac{11}{36}$ **1**

$P(\text{5 will not come either time}) = 1 - \dfrac{11}{36} = \dfrac{25}{36}$ **1**

(CBSE Marking Scheme, 2019)

SECTION - C

Question numbers 13 to 22 carry 3 marks each.

13. Find the ratio in which the y-axis divides the line segment joining the points (–1, –4) and (5, –6). Also find the coordinates of the point of intersection.

Sol. Any point on y-*axis* is $P(0,y)$

Let P divides AB in $k : 1$ **1**

$k : 1$

A(–1, –4) —— P(0, y) —— B(5, –6)

$\Rightarrow 0 = \dfrac{5k-1}{k+1} \Rightarrow k = \dfrac{1}{5}$ *i.e.* 1 : 5 **1**

$\Rightarrow y = \dfrac{-6k-4}{k+1} = \dfrac{-\frac{6}{5}-4}{\frac{1}{5}+1} = \dfrac{-26}{6}$

$= \dfrac{-13}{3}$ **1**

$\Rightarrow P$ is$\left(0, \dfrac{-13}{3}\right)$

(CBSE Marking Scheme, 2019)

*** 14. Evaluate:**

$$\left(\frac{3\sin 43^\circ}{\cos 47^\circ}\right)^2 - \frac{\cos 37^\circ \operatorname{cosec} 53^\circ}{\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ}$$

15. Two spheres of same metal weight 1 kg and 7 kg. The radius of the smaller sphere is 3 cm. The two spheres are melted to form a single big sphere. Find the diameter of the new sphere.

* *Out of Syllabus*

Sol. Radius of first sphere $=3\ cm$

$$\therefore \quad \frac{4}{3}\pi(3)^3 d = 1 \ \{d = \text{density}\} \quad \frac{1}{2}$$

let radius of second sphere be $r\ cm$

$$\therefore \quad \frac{4}{3}\pi(r)^3 .\ d = 7 \Rightarrow r^3 = 7(3)^3 \quad \frac{1}{2}$$

$$\Rightarrow \frac{4}{3}\pi(3)^3 + \frac{4}{3}\pi.(3)^3.7 = \frac{4}{3}\pi R^3 \quad \mathbf{1}$$

$$\Rightarrow \quad R^3 = (3)^3(1+7)$$

$$\Rightarrow \quad R = 3(2) = 6 \quad \frac{1}{2}$$

$$\therefore \quad \text{Diameter} = 12 \text{ cm} \quad \frac{1}{2}$$

(CBSE Marking Scheme, 2019)

SECTION - D

Question numbers 23 to 30 carry 4 marks each.

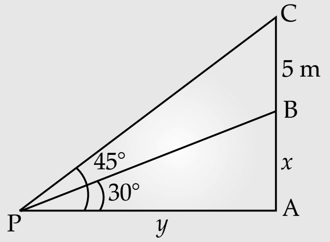
23. In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then prove that the angle opposite the first side is a right angle.

Sol. Refore to 2022 year Delhi Set-I Q. 32.

24. From a point P on the ground, the angle of elevation of the top of a tower is 30° and that of the top of the flag-staff fixed on the top of the tower is 45°. If the length of the flag-staff is 5 m, find the height of the tower.

(Use $\sqrt{3}$ = 1.732)

Sol. In ΔPAC,



$$\frac{AC}{AP} = \tan 45° = 1 \quad \mathbf{1}$$

$$\Rightarrow \quad x + 5 = y \quad \frac{1}{2}$$

$$\text{In } \Delta PAB, \quad \frac{x}{y} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\frac{x}{x+5} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad x = \frac{5}{\sqrt{3}-1}$$

$$= \frac{5(\sqrt{3}+1)}{3} = 6.83 \quad \mathbf{1\frac{1}{2}}$$

$$\therefore \quad \text{Height of tower} = 6.83 \text{ m} \quad \mathbf{1}$$

(CBSE Marking Scheme, 2019)

25. A right cylindrical container of radius 6 cm and height 15 cm if full of ice-cream, which has to be distributed to 10 children in equal cones having hemispherical shape on the top. If the height of the conical portion is four times its base radius, find the radius of the ice-cream cone.

Sol. Volume if ice-cream the cylinder

$$= \pi(6)^2.15 \text{ cm}^3 \quad \mathbf{1}$$

Volume of ice-cream in one

$$= \frac{1}{3}\pi r^2 .\ 4r + \frac{2}{3}\pi r^3 \text{cm}^3 \quad \mathbf{1}$$

(Given $h = 4r$)

$$= 2\pi r^3 \text{ cm}^3 \quad \frac{1}{2}$$

$$\Rightarrow \quad 10(2\pi r^3) = \pi(6)^2 \times 15 \quad \mathbf{1}$$

$$\Rightarrow \quad r^3 = (3)^3 \Rightarrow r = 3 \text{ cm} \quad \frac{1}{2}$$

(CBSE Marking Scheme, 2019)

❑❑