Solved Paper 2020

Mathematics (Standard)

CLASS-X

Time : 3 Hours **Max. Marks : 80**

General Instructions:

(i) *This question paper comprises four sections – A, B, C and D. This question paper carries 40 questions. All questions are compulsory.*

(ii) *Section **A** : Q. No. **1** to **20** question of one mark each.*

(iii) *Section **B** : Q. No. **21** to **26** comprises of **6** question of two mark each.*

(iv) *Section **C** : Q. No. **27** to **34** comprises of **8** questions of three marks each.*

(v) *Section **D** : Q. No. **35** to **40** comprises of **6** questions of four marks each.*

(vi) *There is no overall choice in the question paper. However, an internal choice has been provided in 2 question of one mark each. 2 questions of two marks each, 3 questions of three marks each and 3 questions of four marks each. You have to attempt only one of the choices in such questions.*

(vii) *In addition to this, separate instructions are given with each section and question, wherever necessary.*

(viii) *Use of calculator is not permitted.*

Delhi Set-I **Code No. 30/1/1**

SECTION - A

Q. No. 1 to 10 are multiple choice type questions of 1 mark each. Select the correct option.

1. If one zero of the quadratic polynomial $x^2 + 3x + k$ *is* 2, then the value of k is

(a) 10 (b) –10

(c) –7 (d) –2

Ans. Option (b) is correct.

Explanation:

Let $p(x) = x^2 + 3x + k$

$\because$ 2 is a zero of $p(x)$, then

$p(2) = 0$

$\therefore \quad (2)^2 + 3(2) + k = 0$

$\Rightarrow \quad 4 + 6 + k = 0$

$\Rightarrow \quad 10 + k = 0$

$\Rightarrow \quad k = -10$

2. The total number of factors of prime number is

(a) 1 (b) 0

(c) 2 (d) 3

Ans. Option (c) is correct.

Explanation: We have only two factors (1 and number itself) of any prime number.

3. The quadratic polynomial, the sum of whose zeroes is –5 and their product is 6, is

(a) $x^2 + 5x + 6$ (b) $x^2 - 5x + 6$

(c) $x^2 - 5x - 6$ (d) $-x^2 + 5x + 6$

Ans. Option (a) is correct.

Explanation: Let α and β be the zeroes of the quadratic polynomial, then

$\alpha + \beta = -5$

and $\alpha\beta = 6$

So, required polynomial is

$$x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - (-5)x + 6$$
$$= x^2 + 5x + 6$$

4. The value of k for which the system of equations | $x + y - 4 = 0$ and $2x + ky = 3$, has no solution, is

(a) – 2 (b) ≠ 2

(c) 3 (d) 2

Ans. Option (d) is correct.

Explanation: Given equations:

$x + y - 4 = 0$

and $2x + ky - 3 = 0$

Here, $\frac{a_1}{a_1} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{1}{k}$ and $\frac{c_1}{c_2} = \frac{-4}{-3} = \frac{4}{3}$

$\because$ System has no solution

$$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{1}{k} \neq \frac{4}{3}$$

$$\Rightarrow \quad k = 2 \text{ or } k \neq \frac{3}{4}$$

5. The HCF and the LCM of 12, 21, 15 respectively are

(a) 3, 140 (b) 12, 420

(c) 3, 420 (d) 420, 3

Ans. Option (c) is correct.

Explanation: $12 = 2 \times 2 \times 3$

$21 = 3 \times 7$

and $15 = 3 \times 5$

$\therefore$ HCF = 3

and LCM = $2 \times 2 \times 3 \times 5 \times 7 = 420$

6. The value of x for which $2x$, $(x + 10)$ and $(3x + 2)$ are the three consecutive terms of an A.P., is

(a) 6 (b) – 6

(c) 18 (d) –18

Ans. Option (a) is correct.

Explanation: $2x$, $(x + 10)$ and $(3x + 2)$ are in A.P.

$$\therefore \quad (x + 10) - 2x = (3x + 2) - (x + 10)$$
$$\Rightarrow \quad -x + 10 = 2x - 8$$
$$\Rightarrow \quad -x - 2x = -8 - 10$$
$$\Rightarrow \quad -3x = -18$$
$$\Rightarrow \quad x = 6$$

7. The first term of A.P. is p and the common difference is q, then its 10th term is

(a) $q + 9p$ **(b)** $p - 9q$

(c) $p + 9q$ **(d)** $2p + 9q$

Ans. Option (c) is correct.

Explanation: $a = p$ and $d = q$ (given)

$$\therefore \quad 10^{th} \text{ term} = a + (10 - 1)d$$
$$= p + 9q$$

8. The distance between the points ($a \cos\theta + b \sin\theta$, 0) and (0, $a \sin\theta - b \cos\theta$), is

(a) $a^2 + b^2$ **(b)** $a^2 - b^2$

(c) $\sqrt{a^2+b^2}$ **(d)** $\sqrt{a^2-b^2}$

Ans. Option (c) is correct.

Explanation: Here,

$x_1 = a \cos\theta + b \sin\theta$, $y_1 = 0$

and $x_2 = 0$, $y_2 = a \sin\theta - b \cos\theta$

$\therefore$ Distance

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(0 - a\cos\theta - b\sin\theta)^2 + (a\sin\theta - b\cos\theta - 0)^2}$$
$$= \sqrt{(-1)^2(a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2}$$
$$= \sqrt{\begin{aligned}&a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta + \\ &a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta\end{aligned}}$$
$$= \sqrt{a^2(\sin^2\theta + \cos^2\theta) + b^2(\sin^2\theta + \cos^2\theta)}$$
$$= \sqrt{a^2 \times 1 + b^2 \times 1} = \sqrt{a^2 + b^2}$$

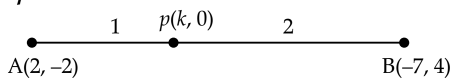
9. If the point P(k, 0) divides the line segment joining the points A(2, –2) and B(–7, 4) in the ratio 1 : 2, then the value of k is

(a) 1 **(b)** 2

(c) – 2 **(d)** – 1

Ans. Option (d) is correct.

Explanation:



$$\therefore \quad k = \frac{1(-7) + 2(2)}{1 + 2} \quad \left[\because x = \frac{mx_2 + nx_1}{m + n}\right]$$
$$\Rightarrow \quad k = \frac{-7 + 4}{3}$$
$$\Rightarrow \quad k = -1$$

* **10. The value of p, for which the points A(3, 1), B(5, p) and C(7, – 5) are collinear, is**

(a) – 2 **(b)** 2

(c) – 1 **(d)** 1

In Q. Nos. 11 to 15, fill in the blanks. Each question is of 1 marks.

11. In Fig. 1, ΔABC is circumscribing a circle, the length of BC is_______ cm.

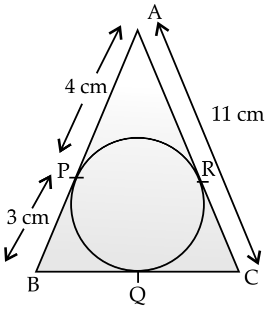
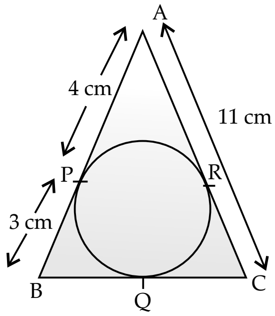


Fig.1

Sol. 10

Explanation: $\because$ AP and AR are tangents to the circle from external point A.

$$\therefore \quad AP = AR \text{ i.e., } AR = 4 \text{ cm}$$



Similarly, PB and BQ are tangents.

$\therefore$ $BP = BQ$ *i.e.*, $BQ = 3$ cm

Now, $CR = AC - AR = 11 - 4 = 7$cm

Similarly, CR and CQ are tangents.

$\therefore$ $CR = CQ$ *i.e.*, $CQ = 7$ cm

Now, $BC = BQ + CQ = 3 + 7 = 10$ cm.

Hence, the length of BC is 10 cm.

* **12. Given $\Delta\ ABC \sim \Delta\ PQR$, if $\frac{AB}{PQ} = \frac{1}{3}$, then**

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \_\_\_\_\_\_\_\_.$$

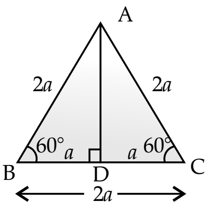
13. ΔABC is an equilateral triangle of side $2a$, then length of one of its altitude is _______.

Sol. $a\sqrt{3}$

Explanation: ABC is an equilateral triangle in which $AD \perp BC$.

From $\Delta\ ABC$,

* *Out of Syllabus*



$$AB^2 = (AD)^2 + (BD)^2$$

(By using Pythogoras Theorem)

$$\Rightarrow \quad (2a)^2 = (AD)^2 + (a)^2$$

$$\Rightarrow \quad 4a^2 - a^2 = (AD)^2$$

$$\Rightarrow \quad (AD)^2 = 3a^2$$

$$\Rightarrow \quad AD = a\sqrt{3}$$

Hence, the length of attitude is $a\sqrt{3}$

*** 14.** $\dfrac{\cos 80°}{\sin 10°}$ **+ cos 59° cosec 31°=———.**

15. The value of $\left(\sin^2\theta + \dfrac{1}{1+\tan^2\theta}\right)$ **=———.**

OR

The value of (1 + tan²θ) (1 – sinθ) (1 + sinθ) = ———.

Sol. 1

Explanation : $\sin^2\theta + \dfrac{1}{1+\tan^2\theta} = \sin^2\theta + \dfrac{1}{\sec^2\theta}$

$$= \sin^2\theta + \cos^2\theta$$

$$= 1$$

OR

1

Explanation : $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$

$$= \sec^2\theta(1 - \sin\theta)(1 + \sin\theta)$$

$$= \sec^2\theta(1 - \sin^2\theta)$$

$$[\because (a-b)(a+b) = a^2 - b^2]$$

$$= \sec^2\theta \times \cos^2\theta$$

$$= \frac{1}{\cos^2\theta} \times \cos^2\theta$$

$$= 1$$

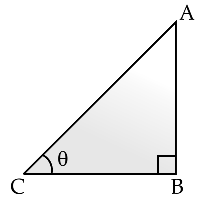
Q. Nos. 16 to 20 are short answer type questions of 1 mark each.

16. The ratio of the length of a vertical rod and the length its shadow is 1 : $\sqrt{3}$. Find the angle of elevation of the sun at that moment ?

Sol. Let AB be a vertical rod and BC be its shadow.

From the figure, $\angle ACB = \theta$.

In ΔABC,



$$\tan\theta = \frac{AB}{BC}$$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}} \qquad \left[\because \frac{AB}{BC} = \frac{1}{\sqrt{3}}\text{(given)}\right]$$

$$\Rightarrow \quad \tan\theta = \tan 30° \qquad \left(\because \tan 30° = \frac{1}{\sqrt{3}}\right)$$

$$\Rightarrow \quad \theta = 30°$$

Here the angle of elevation of the sun is 30°.

17. Two cones have their heights in the ratio 1 : 3 and radii in the ratio 3 : 1. What is the ratio of their volumes ?

Sol. Let h_1 and h_2 be height and r_1, r_2 be radii of two cones, then ratio of their volumes

$$= \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2}$$

Given : $\dfrac{h_1}{h_2} = \dfrac{1}{3}$ and $\dfrac{r_1}{r_2} = \dfrac{3}{1}$

$$= \left(\frac{r_1}{r_2}\right)^2\left(\frac{h_1}{h_2}\right)$$

$$= \left(\frac{3}{1}\right)^2\left(\frac{1}{3}\right) = \frac{3}{1}$$

Hence, ratio of their volumes is 3 : 1.

18. A letter of English alphabet is chosen at random. What is the probability that the chosen letter is a consonant.

Sol. In the English language, there are 26 alphabets. Consonants are 21.

$\therefore$ The probability of choosing a consonant = $\dfrac{21}{26}$

19. A die is thrown once. What is the probability of getting a number less than 3 ?

OR

If the probability of wining a game is 0.07, what is the probability of losing it ?

Sol. Total possible outcomes = 6

$$\therefore \quad P(\text{number less than 3}) = \frac{2}{6} = \frac{1}{3}$$

OR

$$P(\text{winning the game}) = 0.07$$

$$P(\text{losing the game}) = 1 - 0.07$$

$$= 0.93$$

* *Out of Syllabus*

20. If the mean of the first n natural number is 15, then find n.

Sol. Given : 1, 2, 3, 4, ... to n terms.

$\therefore$ The sum of first n natural numbers $= \frac{n(n+1)}{2}$

So, $\text{mean} = \frac{n(n+1)}{2 \times n}$

$\Rightarrow \frac{n+1}{2} = 15$

$\Rightarrow n + 1 = 30$

$\Rightarrow n = 29$

SECTION - B

Q. Nos. 21 to 26 carry 2 marks each.

21. Show that $(a-b)^2$, (a^2+b^2) and $(a+b)^2$ are in A.P.

Sol. Given : $(a-b)^2$, (a^2+b^2) and $(a+b)^2$

Common difference,

$$d_1 = (a^2+b^2) - (a-b)^2$$
$$= (a^2+b^2) - (a^2+b^2-2ab)$$
$$= a^2 + b^2 - a^2 - b^2 + 2ab$$
$$= 2ab$$

and $d_2 = (a+b)^2 - (a^2+b^2)$
$$= a^2 + b^2 + 2ab - a^2 - b^2$$
$$= 2ab$$

Since, $d_1 = d_2$

Thus, $(a-b)^2$, (a^2+b^2) and $(a+b)^2$ are in $A.P.$

Hence Proved.

22. In Fig. 2, $DE \parallel AC$ and $DC \parallel AP$. Prove that $\frac{BE}{EC} = \frac{BC}{CP}$

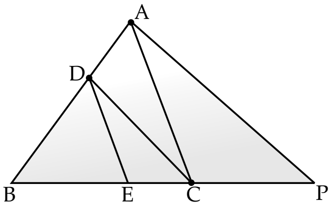


Fig.2

OR

In Fig. 3, two tangents TP and TQ are drawn to circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.

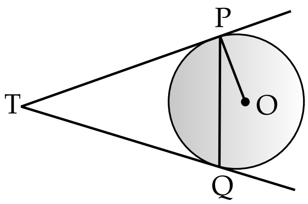
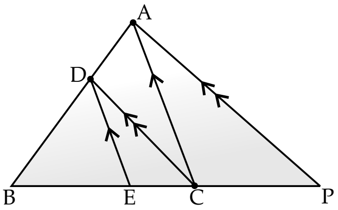


Fig.3

Sol. In ΔABP,

$DC \parallel AP$ (Given)

$\therefore \frac{BD}{DA} = \frac{BC}{CP}$ (From BPT) ....(i)



In ΔABC,

$DE \parallel AC$ (Given)

$\therefore \frac{BD}{DA} = \frac{BE}{EC}$ (From BPT) ....(ii)

From equations (i) and (ii), we have

$\frac{BE}{EC} = \frac{BC}{CP}$ **Hence Proved.**

OR

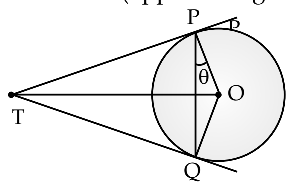
Let $\angle OPQ$ be θ, then

$\angle TPQ = 90° - \theta$

Since, $TP = TQ$

$\therefore \angle TQP = 90° - \theta$

(opposite angles of equal sides)



Now, $\angle TPQ + \angle TQP + \angle PTQ = 180°$

(Angle sum property of a Triangle)

$\Rightarrow 90° - \theta + 90° - \theta + \angle PTQ = 180°$

$\Rightarrow \angle PTQ = 180° - 180° + 2\theta$

$\Rightarrow \angle PTQ = 2\theta$

Hence, $\angle PTQ = 2\angle OPQ$

Hence Proved.

23. The rod AC of TV disc antenna is fixed at right angles to wall AB and a rod CD is supporting the disc as shown in Fig. 4. If AC=1.5 m long and CD = 3 m, find (i) tan θ (ii) sec θ + cosec θ.

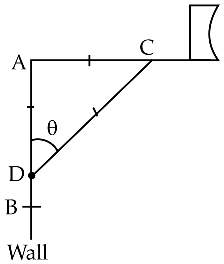
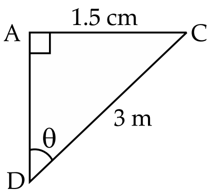


Fig.4

Sol. Given, $AC = 1.5$ m

and $CD = 3$ m



In right angle triangle CAD,

$$AD^2 + AC^2 = DC^2$$

(Using Pythagoras theorem)

$$\Rightarrow \quad AD^2 + (1.5)^2 = (3)^2$$

$$\Rightarrow \quad AD^2 = 9 - 2.25 = 6.75$$

$$\Rightarrow \quad AD = \sqrt{6.75} = 2.6\text{m} \quad \textbf{(Approx)}$$

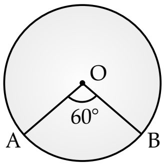
(i) $$\tan\theta = \frac{AC}{AD} = \frac{1.5}{2.6} = \frac{15}{26}$$

(ii) $$\sec\theta + \operatorname{cosec}\theta = \frac{CD}{AD} + \frac{CD}{AC}$$

$$= \frac{3}{2.6} + \frac{3}{1.5} = \frac{41}{13}$$

24. A piece of wire 22 cm long is bent into the form an arc of a circle subtending an angle of 60° at its centre. Find the radius of the circle. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Sol. AB is an arc of a circle.



i.e., $AB = 22$ cm

and $\theta = 60°$

$$\therefore \quad \text{Length of an arc} = \frac{2\pi r\theta}{360°}$$

$$\Rightarrow \quad 22 = \frac{2\times 22\times r\times 60°}{7\times 360°}$$

$$\Rightarrow \quad 22 = \frac{22\times r}{21}$$

$$\Rightarrow \quad 22 \times r = 22 \times 21$$

$$\Rightarrow \quad r = 21$$

Hence, The radius of the circle (r) is 21 cm.

25. If a number x is chosen at random from the number – 3, – 2, – 1, 0, 1, 2, 3. What is probability that $x^2 \leq 4$?

Sol. Total number of outcomes = 7

Favourable outcomes = 5(–2, –1, 0, 1, 2)

$$\therefore \quad P(x^2 \leq 4) = \frac{\text{Favourable outcomes}}{\text{Total outcomes}}$$

$$= \frac{5}{7}$$

26. Find the mean the following distribution :

Class :	Frequency :
3 – 5	5
5 – 7	10
7 – 9	10
9 – 11	7
11 – 13	8

OR

Find the mode of the following data:

Class :	Frequency :
0 – 20	6
20 – 40	8
40 – 60	10
60 – 80	12
80 – 100	6
100 – 120	5
120 – 140	3

Sol.

Class	Frequency (f)	Mid-Value (x)	$f \times x$
3 – 5	5	4	20
5 – 7	10	6	60
7 – 9	10	8	80
9 – 11	7	10	70
11 – 13	8	12	96
	$\Sigma f = 40$		$\Sigma fx = 326$

$$\therefore \quad \text{mean} = \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{326}{40} = 8.15$$

OR

Modal class = 60 – 80

$$\therefore \quad \text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

Hence, $l = 60, f_1 = 12, f_0 = 10, f_2 = 6$ and $h = 20$

$$\text{Mode} = 60 + \frac{12-10}{2\times 12-10-6} \times 20$$

$$= 60 + \frac{2\times 20}{24-16}$$

$$= 60 + \frac{40}{8} = 60 + 5$$

$$= 65.$$

SECTION - C

Q. Nos. 27 to 34 carry 3 marks each.

27. Find a quadratic polynomial whose zeroes are reciprocals of the zeroes of the polynomial $f(x) = ax^2 + bx + c, a \neq 0, c \neq 0$.

OR

*** Divide the polynomial $f(x) = 3x^2 - x^3 - 3x + 5$ by the polynomial $g(x) = x - 1 - x^2$ and verify the division algorithm.**

Sol. Let α and β be zero of the given polynomial $ax^2 + bx + c$

* *Out of Syllabus*

$\therefore \qquad \alpha + \beta = -\dfrac{b}{a}$ and $\alpha\beta = \dfrac{c}{a}$

$$s = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{-\frac{b}{a}}{\frac{c}{a}} = \frac{-b}{c}$$

$$p = \frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{a}{c}$$

The required polynomial, $x^2 - sx + p$ or $cx^2 + bx + a$

28. Determine graphically the coordinates of the vertices of triangle, the equations of whose sides are given by $2y - x = 8$, $5y - x = 14$ and $y - 2x = 1$.

OR

*** If 4 is zero of the cubic polynomial $x^3 - 3x^2 - 10x + 24$, find its other two zeroes.**

Sol. Given, $\qquad 2y - x = 8$

$\Rightarrow \qquad x = 2y - 8$

y	0	4	5
$x = 2y - 8$	– 8	0	2

$5y - x = 14$

$\Rightarrow \qquad x = 5y - 14$

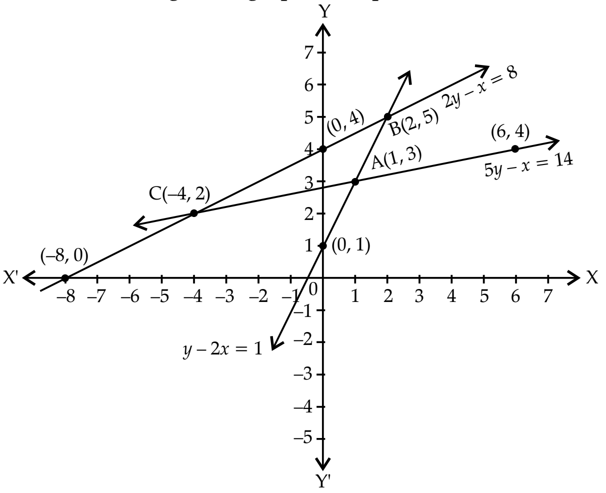
y	3	4	2
$x = 5y - 14$	1	6	– 4

and $\qquad y - 2x = 1$

$\Rightarrow \qquad y = 1 + 2x$

x	0	1	2
$y = 1 + 2x$	1	3	5

Plotting the above points and drawing lines joining them, we get the graphical representation:



Hence, the coordinates of the vertices of a triangle ABC are $A(1, 3)$, $B(2, 5)$ and $C(-4, 2)$.

29. In a flight of 600 km, an aircraft was slowed due to bad weather. Its average speed for the trip was reduce by 200 km/h and time of flight increased by 30 minutes. Find the original duration of flight.

Sol. Let original speed of flight be x km/h, then according to question,

$$\frac{600}{x - 200} - \frac{600}{x} = 30 \text{ minutes}$$

$$\left[\because \text{Time} = \frac{\text{Distance}}{\text{Speed}}\right]$$

$$\Rightarrow \quad 600\left[\frac{1}{x-200} - \frac{1}{x}\right] = \frac{30}{60}$$

$$\Rightarrow \quad \frac{x - x + 200}{x(x - 200)} = \frac{1}{2 \times 600}$$

$$\Rightarrow \quad \frac{200}{x^2 - 200x} = \frac{1}{1200}$$

$$\Rightarrow \quad x^2 - 200x = 240000$$

$$\Rightarrow \quad x^2 - 200x - 240000 = 0$$

Here, $a = 1$, $b = -200$ and $c = -240000$

$$\therefore \quad x = \frac{200 \pm \sqrt{40000 + 960000}}{2 \times 1}$$

$$= \frac{200 \pm \sqrt{1000000}}{2}$$

$$= \frac{200 \pm 1000}{2}$$

$$= \frac{200 + 1000}{2}, \frac{200 - 1000}{2}$$

$$= 600, -400$$

Since, speed cannot be negative, therefore

original speed $= 600$ km/h

and original distance $= 600$ km

$$\therefore \quad \text{Time} = \frac{\text{original distance}}{\text{original speed}}$$

$$= \frac{600 \text{ km}}{600 \text{ km/hr}} = 1 \text{ h}$$

Hence, the original duration of flight is 1 h.

30. * Find the area of triangle PQR formed by the points $P(-5, 7)$, $Q(-4, -5)$ and $R(4, 5)$.

OR

If the point $C(-1, 2)$ divides internally the line segment joining $A(2, 5)$ and $B(x, y)$ in the ratio 3 : 4, find the coordinates of B.

Sol. OR

By using section formula,

A(2, 5) —3— C(–1, 2) —4— B(x, y)

$$-1 = \frac{mx_2 + nx_1}{m + n}$$

$$-1 = \frac{3 \times x + 4 \times 2}{3 + 4} = \frac{3x + 8}{7}$$

$$\Rightarrow \quad 3x + 8 = -7$$

$$\Rightarrow \quad 3x = -15$$

$$\Rightarrow \quad x = -5$$

* *Out of Syllabus*

and $2 = \dfrac{my_2 + ny_1}{m+n}$

$= \dfrac{3 \times y + 4 \times 5}{3+4} = \dfrac{3y+20}{7}$

$\Rightarrow 3y + 20 = 14$

$\Rightarrow 3y = 14 - 20 = -6$

$\Rightarrow y = -2$

Hence, the coordinates of $B(x, y)$ is $(-5, -2)$.

31. In Fig. 5, $\angle D = \angle E$ and $\dfrac{AD}{DB} = \dfrac{AE}{EC}$, Prove that ΔBAC is an isosceles triangle.

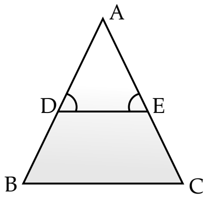
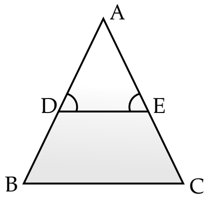


Fig.5

Sol. Given : $\angle D = \angle E$ and $\dfrac{AD}{DB} = \dfrac{AE}{EC}$



To prove : ΔBAC is an isosceles triangle.

Proof : $\dfrac{AD}{DB} = \dfrac{AE}{EC}$

By converse of BPT, $DE \parallel BC$

$\therefore \angle ADE = \angle ABC$ (Corresponding angles)

and $\angle AED = \angle ACB$ (Corresponding angles)

$\because \angle ADE = \angle AED$ (Given)

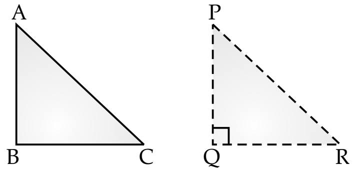
$\therefore \angle ABC = \angle ACB$

So, BAC is an isosceles triangle. Hence Proved.

32. In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then prove that the angle opposite to the first is a right angle.

Sol. Given : $AC^2 = AB^2 + BC^2$

To prove : Angle opposite to the first side is a right angle.



Construction : Draw ΔPQR, where $AB = PQ$, $BC = QR$ and $\angle Q = 90°$.

Proof : In ΔPQR,

$PR^2 = PQ^2 + QR^2$

(By using Pythagoreas Theorem)

$\because AB = PQ, BC = QR$

[From construction]

$\therefore PR^2 = AB^2 + BC^2$

Now, $AC^2 = AB^2 + BC^2$ (given)

$\therefore AC^2 = PR^2$

$\Rightarrow AC = PR$

In ΔABC and ΔPQR,

$AB = PQ$ (By construction)

$BC = QR$ (By construction)

and $AC = PR$ (Proved above)

$\therefore \Delta ABC \cong \Delta PQR$

(By SSS congruency rule)

So, $\angle B = \angle Q$ (By CPCT)

But $\angle Q = 90°$ (by construction)

Hence, $\angle B = 90°$ **Hence Proved.**

33. If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$.

Sol. Given, $\sin\theta + \cos\theta = \sqrt{3}$

On squaring both the sides, we get

$(\sin\theta + \cos\theta)^2 = (\sqrt{3})^2$

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$

$\Rightarrow 1 + 2\sin\theta\cos\theta = 3$

$\Rightarrow 2\sin\theta\cos\theta = 3 - 1 = 2$

$\Rightarrow \sin\theta\cos\theta = 1$ ...(i)

Now taking LHS,

$\tan\theta + \cot\theta = \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$= \dfrac{1}{\sin\theta\cos\theta}$

$= \dfrac{1}{1} = 1$ [Form eq. (i)]

Hence, $\tan\theta + \cot\theta = 1$ **Hence Proved.**

*** 34. A cone of base radius 4 cm is divided into two parts by drawing a plane through the mid-point of its height and parallel to its. Compare the volume of the two parts.**

SECTION - D

Q. Nos. 35 to 40 carry 4 marks each.

*** 35. Show that the square of any positive integer cannot be of the from $(5q + 2)$ or $(5q + 3)$ for any integer q.**

OR

Prove that one of every three consecutive positive integers is divisible by 3.

36. The sum of four consecutive number in A.P. is 32 and the ratio of the product of the first and last and term to the product of two middle terms is 7: 15. Find the numbers.

OR

Solve : $1 + 4 + 7 + 10 + ... + x = 287$

* *Out of Syllabus*

Sol. Let the four consecutive terms of A.P. be $(a - 3d)$, $(a - d)$, $(a + d)$ and $(a + 3d)$

By given conditions,

$a - 3d + a - d + a + d + a + 3d = 32$

$\Rightarrow \quad 4a = 32 \Rightarrow a = 8$

and $\dfrac{(a-3d)(a+3d)}{(a-d)(a+d)} = \dfrac{7}{15}$

$\Rightarrow \quad \dfrac{a^2 - 9d^2}{a^2 - d^2} = \dfrac{7}{15}$

$\Rightarrow \quad \dfrac{(8)^2 - 9d^2}{(8)^2 - d^2} = \dfrac{7}{15}$

$\Rightarrow \quad \dfrac{64 - 9d^2}{64 - d^2} = \dfrac{7}{15}$

$\Rightarrow \quad 960 - 135d^2 = 448 - 7d^2$

$\Rightarrow \quad 7d^2 - 135d^2 = 448 - 960$

$\Rightarrow \quad -128d^2 = -512$

$\Rightarrow \quad d^2 = 4 \Rightarrow d = \pm 2$

Hence, the number are 2, 6, 10 and 14 or 14, 10, 6 and 2.

OR

Given, $a = 1$ and $d = 3$.

Let number of terms in the series be n,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$\therefore \quad \dfrac{n}{2}[2 \times 1 + (n-1)3] = 287$

$\Rightarrow \quad \dfrac{n}{2}[2 + 3n - 3] = 287$

$\Rightarrow \quad \dfrac{n}{2}[3n - 1] = 287$

$\Rightarrow \quad 3n^2 - n = 574$

$\Rightarrow \quad 3n^2 - n - 574 = 0$

$\Rightarrow \quad 3n^2 - 42n + 41n - 574 = 0$

$\Rightarrow \quad 3n(n - 14) + 41(n - 14) = 0$

$\Rightarrow \quad (n - 14)(3n + 41) = 0$

i.e., $n = 14$ or $n = -\dfrac{41}{3}$, it is not possible

Thus, the 14th term is x,

$a + (n-1)d = x$

$\Rightarrow \quad x = 1 + (14 - 1)\,3$

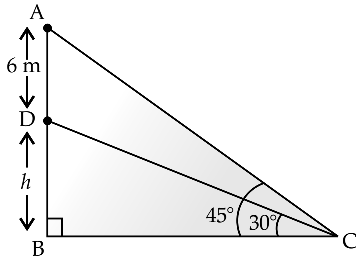
Hence, $\quad x = 40$

*** 37. Draw a line segment *AB* of length 7 cm. Taking A as centre, draw a circle of radius 3 cm and taking *B* as centre, draw another circle of radius 2 cm. Construct tangents to each circle from the centre of the other circle.**

38. A vertical tower stands on horizontal plane and is surmounted by a vertical flag-staff of height 6 m. At a point on the bottom and top of the flag-staff are 30° and 45° respectively. Find the height of the tower. (Take $\sqrt{3}$ = 1.73)

* *Out of Syllabus*

Sol. According to question,



AD is a flagstaff and BD is a tower.

In ΔABC,

$$\tan 45° = \frac{AB}{BC}$$

$\Rightarrow \quad 1 = \dfrac{h+6}{BC}$

$\Rightarrow \quad BC = h + 6 \qquad$ ...(i)

In ΔDBC, $\quad \tan 30° = \dfrac{DB}{BC} \qquad$ [from (i)]

$\Rightarrow \quad \dfrac{1}{\sqrt{3}} = \dfrac{h}{h+6}$

$\Rightarrow \quad h\sqrt{3} = h + 6$

$\Rightarrow \quad h\sqrt{3} - h = 6$

$\Rightarrow \quad h(\sqrt{3} - 1) = 6$

$\Rightarrow \quad h = \dfrac{6}{\sqrt{3}-1} \times \dfrac{\sqrt{3}+1}{\sqrt{3}+1}$

$\Rightarrow \quad h = \dfrac{6(\sqrt{3}+1)}{2}$

$\Rightarrow \quad h = 3(\sqrt{3}+1) = 3(1.73 + 1)$

$\Rightarrow \quad h = 3 \times 2.73$

$\Rightarrow \quad h = 8.19$ m.

*** 39. A bucket in the from of a frustum of a cone of height 30 cm with radii of its lower and upper ends as 10 cm and 20 cm, respectively. Find the capacity of the bucket. Also find the cost of milk which can completely fill the bucket at the rate of ₹ 40 per litre. $\left(\text{Use } \pi = \frac{22}{7}\right)$**

*** 40. The following table gives production yield per hectare (in quintals) of wheat of 100 farms of a village :**

Production yield/hect.	No. of farms
40 – 45	4
45 – 50	6
50 – 55	16
55 – 60	20
60 – 65	30
65 – 70	24

Change the distribution to 'a more than' type distribution and draw its ogive

OR

The median of the following data is 525. Find the values of x and y, if total frequency is 100 :

Class	Frequency
0 – 100	2
100-200	5
200-300	x
300-400	12
400-500	17
500-600	20
600-700	y
700-800	9
800-900	7
900-1000	4

Sol.

OR

Class Interval	Frequency	Cumulative frequency
0 – 100	2	2
100 – 200	5	7
200 – 300	x	$7 + x$
300 – 400	12	$19 + x$
400 – 500	17	$36 + x$
500 – 600	20	$56 + x$
600 – 700	y	$56 + x + y$
700 – 800	9	$65 + x + y$
800 – 900	7	$72 + x + y$
900 – 1000	4	$76 + x + y$
	***N* = 100**	

Also, $76 + x + y = 100$

$\Rightarrow \quad x + y = 100 - 76 = 24 \quad$ ...(i)

Given, Median = 525, which lies between class 500 – 600.

$\Rightarrow$ Median class = 500 – 600

Now, $\quad \text{Median} = l + \dfrac{\frac{n}{2} - c.f.}{f} \times h$

$$\Rightarrow \quad 525 = 500 + \left[\dfrac{\frac{100}{2} - (36 + x)}{20}\right] \times 100$$

$\Rightarrow \quad 25 = (50 - 36 - x)\, 5$

$\Rightarrow \quad 14 - x = \dfrac{25}{5} = 5$

$\Rightarrow \quad x = 14 - 5 = 9$

Putting the value of x is eq. (i), we get

$y = 24 - 9 = 15$

Hence, $x = 9$ and $y = 15$.

Delhi Set-II — Code No. 30/1/2

SECTION - A

In Q. Nos. 11 to 15, fill in the blanks. Each question is of 1 mark.

* **14.** $\left(\dfrac{\sin 35^\circ}{\cos 55^\circ}\right)^2 + \left(\dfrac{\cos 43^\circ}{\sin 47^\circ}\right)^2 - 2\cos 60^\circ =$ ________.

* **15. ΔABC and ΔBDE are two equilateral triangle such that D is the mid-point of BC. Ratio of the areas of triangles ABC and BDE is ______.**

Q. Nos. 16 to 20 are short answer type questions of 1 mark each.

20. A die thrown once. What is the probability of getting an even prime number ?

Sol. Total possible outcomes = 6

Favourable outcomes = {2} i.e., 1

P (getting an even prime number) = $\dfrac{1}{6}$

SECTION - B

Q. Nos. 21 to 26 carry 2 marks each.

25. Find the sum of first 20 terms of the following A.P. :

1, 4, 7, 10, ...

Sol. Given A.P. : 1, 4, ,7, 10,, ...

Here, $\quad a = 1, d = 4 - 1 = 3$ and $n = 20$

$\therefore \quad S_{20} = \dfrac{n}{2}[2a + (n - 1)\, d]$

$= \dfrac{20}{2}[2 \times 1 + (20 - 1)\, 3]$

$= 10\,(2 + 57)$

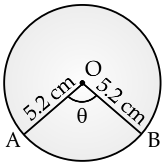
$= 10 \times 59 = 590$

26. The perimeter of a sector of a circle of radius 5.2 cm is 16.4 cm. Find the area of the sector.

* *Out of Syllabus*

Sol. Perimeter of the sector $= 2r + \frac{2\pi r\theta}{360°}$

$$\Rightarrow \quad 16.4 = 2 \times 5.2 + \frac{2\pi \times 5.2 \times \theta}{360°}$$



$$\Rightarrow \quad \frac{2\pi \times 5.2 \times \theta}{360°} = 6$$

$$\Rightarrow \quad \theta = \frac{6 \times 360°}{2\pi \times 5.2}$$

Now, area of sector $= \frac{\theta}{360°} \times \pi r^2$

$$= \frac{6 \times 360°}{2\pi \times 5.2 \times 360°} \times \pi \times (5.2)^2$$

$= 15.6$ sq. units.

SECTION - C

Q. Nos. 27 to 34 carry 3 marks each.

32. A train covers a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. Find the original speed of the train.

Sol. Let the speed of the train $= x$ km/h

Total distance covered by the train = 480 km

$\therefore$ Time taken cover the distance 480 km $= \frac{480}{x}$ h

If the speed has decreased 8 km/h, *i.e.*, $(x - 8)$ km/h

Then, time taken to cover the distance 480 km $= \frac{480}{x-8}$ h.

According to question,

$$\frac{480}{x-8} - \frac{480}{x} = 3$$

$$\Rightarrow \quad 480\left[\frac{x - x + 8}{x(x-8)}\right] = 3$$

$$\Rightarrow \quad \frac{8}{x^2 - 8x} = \frac{3}{480} = \frac{1}{160}$$

$$\Rightarrow \quad x^2 - 8x - 1280 = 0$$

Compare with $ax^2 + bx + c = 0$, we get $a = 1, b = -8$ and $c = -1280$

$$\therefore \quad x = \frac{8 \pm \sqrt{64 + 4 \times 1280}}{2 \times 1}$$

$$= \frac{8 \pm \sqrt{5184}}{2}$$

$$= \frac{8 \pm 72}{2} = \frac{8+72}{2}, \frac{8-72}{2}$$

$$= \frac{80}{2}, \frac{-64}{2} = 40, -32$$

Since, negative speed cannot be possible.

Hence, the original speed of the train = 40 km/h.

33. Prove that the parallelogram circumscribing a circle is a rhombus.

Sol. Refer to 2022 year Delhi Set-III Q.12.

34. Prove that : $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$.

Sol. L.H.S.

$= 2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$

$= 2[(\sin^2\theta)^3 + (\cos^2\theta)^3] - 3(\sin^4\theta + \cos^4\theta) + 1$

$= 2[(\sin^2\theta + \cos^2\theta)(\sin^4\theta - \sin^2\theta\cos^2\theta + \cos^4\theta] - 3(\sin^4\theta + \cos^4\theta) + 1$

$[\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)]$

$= 2(\sin^4\theta - \sin^2\theta\cos^2\theta + \cos^4\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$= -\sin^4\theta - \cos^4\theta - 2\sin^2\theta\cos^2\theta + 1$

$= -(\sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta) + 1$

$= -(\sin^2\theta + \cos^2\theta)^2 + 1$ $[\because (a+b)^2 = a^2 + b^2 + 2ab]$

$= -1 + 1$

$= 0 =$ R.H.S. **Hence Proved.**

SECTION - D

Q. Nos. 35 to 40 carry 4 marks each.

*** 39. A bucket is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper circular ends as 8 cm and 20 cm respectively. Find the cost of milk which can completely fill the bucket, at the rate of ₹ 40 per litre.**

(Use $\pi = 3.14$)

*** 40. Construct a triangle with sides 4 cm, 5 cm and 6 cm. Then construct another triangle whose sides are $\frac{2}{3}$ times the corresponding sides of the first triangle.**

Delhi Set-III — Code No. 30/1/3

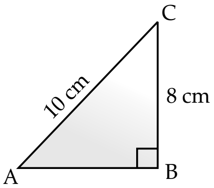
SECTION - A

In Q. Nos. 11 to 15, fill in the blanks. Each question is of 1 mark.

14. A ladder 10 m long reaches a window 8 m above the ground. The distance of the foot of the ladder from the base of the wall is _________ m.

Sol. 6 m

Explanation : Let BC be the height of the window above the ground and AC be a ladder.



* *Out of Syllabus*

Here, $BC = 8$ cm and $AC = 10$ cm

$\therefore$ In right angled triangle ABC,

$$AC^2 = AB^2 + BC^2$$

(By using pythogoras Theorem)

$\Rightarrow$ $(10)^2 = AB^2 + (8)^2$

$\Rightarrow$ $AB^2 = 100 - 64 = 36$

$\Rightarrow$ $AB = 6$ m

*** 15.** $\frac{2\cos 67^\circ}{\sin 23^\circ} - \frac{\tan 40^\circ}{\cot 50^\circ} - \cos 0^\circ =$ __________.

Q. No. 16 to 20 are short answer type questions of 1 mark each.

20. A pair of dice is thrown once. What is the probability of getting a doublet ?

Sol. Total possible outcomes $= 6 \times 6 = 36$

Favourable outcomes $= \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$

i.e.,

$\therefore$ $P(\text{getting doublet}) = \frac{6}{36} = \frac{1}{6}$

SECTION - B

Q. Nos. 21 to 26 carry 2 marks each.

25. The minute hand of a clock is 12 cm long. Find the area of the face of the clock described by the minute hand in 35 minutes.

Sol. $\because$ Angle subtended in 1 minutes $= 6^\circ$

$\therefore$ Angle subtended in 35 minutes $= 6^\circ \times 35 = 210^\circ$

Area of the face of the clock by the minute hand

= Area of the sector

$$= \frac{\pi r^2\theta}{360^\circ}$$

$$= \frac{22}{7} \times \frac{12 \times 12 \times 210^\circ}{360^\circ}$$

$$= \frac{665280}{2520} = 264 \text{ cm}^2$$

26. The sum of the first 7 terms of an A.P. is 63 and that of its next 7 terms is 161. Find the A.P.

Sol. Since, $S_n = \frac{n}{2}[2a + (n-1)d]$

Given, $S_7 = 63$

So, $S_7 = \frac{7}{2}[2a + 6d] = 63$

or, $2a + 6d = 18$ ...(i)

Now, sum of 14 terms is :

$$S_{14} = S_{\text{first 7 terms}} + S_{\text{next 7 terms}}$$
$$= 63 + 161 = 224$$

$\therefore$ $\frac{14}{2}[2a + 13d] = 224$

$\Rightarrow$ $2a + 13d = 32$ ...(ii)

On subtracting (i) from (ii), we get

$(2a + 13d) - (2a + 6d) = 32 - 18$

$\Rightarrow$ $7d = 14$

$\Rightarrow$ $d = 2$

Putting the value of d in (i), we get

$a = 3$

Hence, the A. P. will be : 3, 5, 7, 9, ...

SECTION - C

Q. Nos. 27 to 34 carry 3 marks each.

32. A man can row a boat downstream 20 km in 2 hours and upstream 4 km in 2 hours. Find his speed of rowing in still water. Also find the speed of the stream.

Sol. Let the speed of the boat in still water be x km/h and speed of the stream be y km/h.

$\therefore$ Relative Speed of boat in upstream $= (x - y)$ km/h and Relative speed of boat in downstream $= (x + y)$ km/h

According to question, $\frac{20}{x+y} = 2$

$\Rightarrow$ $x + y = 10$ ...(i)

and $\frac{4}{x-y} = 2$

$\Rightarrow$ $x - y = 2$ ...(ii)

On adding eq. (i) and (ii), we get

$2x = 12$

$\Rightarrow$ $x = 6$

Putting the value of x is eq. (i),

$6 + y = 10$

$\Rightarrow$ $y = 10 - 6 = 4$

Speed of a boat in still water $= 6$ km/h

and speed of the stream $= 4$ km/h.

33. In given Fig. 5, two circles touch each other at the point *C*. Prove that the common tangent to the circles at *C*, bisects the common tangent at *P* and *Q*.

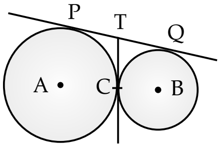
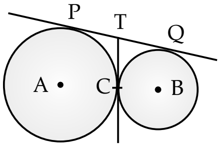


Fig.5

Sol. Since, $PT = TC$ (tangents of circle)

and $QT = TC$

(tangents of circle from extended point)



So, $PT = QT$

Now $PQ = PT + TQ$

$\Rightarrow$ $PQ = PT + PT$

* *Out of Syllabus*

$$\Rightarrow \quad PQ = 2PT$$

$$\Rightarrow \quad \frac{1}{2}PQ = PT$$

Hence, the common tangent to the circle at C, bisects the common tangents at P and Q.

34. Prove that : $\dfrac{\cot\theta + \text{cosec}\,\theta - 1}{\cot\theta - \text{cosec}\,\theta + 1} = \dfrac{1+\cos\theta}{\sin\theta}$

Sol. L.H.S. $= \dfrac{\cot\theta + \text{cosec}\,\theta - 1}{\cot\theta - \text{cosec}\,\theta + 1}$

$$= \frac{\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta} - 1}{\frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta} + 1}$$

$$= \frac{\sin\theta(\cos\theta - \sin\theta + 1)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{\sin\theta\cos\theta - \sin^2\theta + \sin\theta}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{\sin\theta\cos\theta + \sin\theta - (1-\cos^2\theta)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{\sin\theta(\cos\theta + 1) - [(1-\cos\theta)(1+\cos\theta)]}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{(1+\cos\theta)(\sin\theta - 1 + \cos\theta)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{1+\cos\theta}{\sin\theta}$$

= R.H.S. **Hence Proved**

SECTION - D

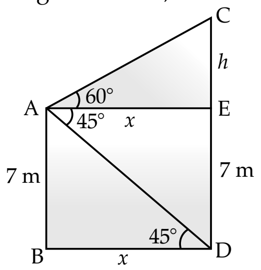
Q. Nos. 35 to 40 carry 4 marks each.

* **39. Draw a $\triangle ABC$ with BC = 7 cm, $\angle B$ = 45° and $\angle A$ = 105°. Then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of $\triangle ABC$.**

40. From the top of a 7 m high building the angle of elevation of the top of a tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

Sol. Let AB be a building, then AB = 7 cm

CD be the height of tower, so



$CD = (7 + h)$m

$BD = AE = x$ m

In ΔABD,

$$\tan 45° = \frac{AB}{BD}$$

$$\Rightarrow \quad 1 = \frac{7}{x}$$

$$\Rightarrow \quad x = 7 \text{ m}$$

In ΔCEA,

$$\tan 60° = \frac{CE}{AE}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \quad h = x\sqrt{3}$$

putting the value of x, we get

$$h = 7\sqrt{3}$$

Now, $CD = CE + ED$

$= (7 + 7\sqrt{3})$ m

Hence, height of tower $= 7(1 + \sqrt{3})$ m

$= 7(1 + 1.732)$ m

$= 7 \times 2.732$ m

$= 19.124$ m

$= 19.12$ m (Approx)

Outside Delhi Set-I — Code No. 30/2/1

SECTION - A

Question numbers 1 to 10 are Multiple Choice Questions of 1 mark each. Select the correct option.

1. The sum of exponents of prime factors in the prime-factorisation of 196 is

(a) 3 **(b)** 4

(c) 5 **(d)** 2

Sol. Option (b) is correct.

Explanation:

Prime factors of 196 $= 2^2 \times 7^2$

2	196
2	98
7	49
7	7
	1

$\therefore$ The sum of exponents of prime factor

$= 2 + 2 = 4.$

* **2. Euclid's division Lemma states that for two positive integers a and b, there exists unique integer q and r satisfying $a = bq + r$, and**

* *Out of Syllabus*

(a) $0 < r < b$ (b) $0 < r \leq b$
(c) $0 \leq r < b$ (d) $0 \leq r \leq b$

3. The zeroes of the polynomial $x^2 - 3x - m(m+3)$ are

(a) $m, m+3$ (b) $-m, m+3$
(c) $m, -(m+3)$ (d) $-m, -(m+3)$

Sol. Option (b) is correct.

Explanation: Given, $x^2 - 3x - m(m+3)$

putting $x = -m$, we get

$$= (-m)^2 - 3(-m) - m(m+3)$$
$$= m^2 + 3m - m^2 - 3m = 0,$$

putting $x = m + 3$, we get

$$= (m+3)^2 - 3(m+3) - m(m+3)$$
$$= (m+3)[m+3-3-m]$$
$$= (m+3)[0] = 0.$$

Hence, $-m$ and $m+3$ are the zeroes of given polynomial.

4. The value of k for which the system of linear equations $x + 2y = 3, 5x + ky + 7 = 0$ is inconsistent is

(a) $-\frac{14}{3}$ (B) $\frac{2}{5}$
(c) 5 (d) 10

Sol. Option (d) is correct.

Explanation:

$$x + 2y - 3 = 0$$

and $$5x + ky + 7 = 0$$

If system is inconsistent, then

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Taking first two orders, we have

$$\frac{1}{5} = \frac{2}{k} \Rightarrow k = 10.$$

5. The roots of the quadratic equation $x^2 - 0.04 = 0$ are

(a) ± 0.2 (b) ± 0.02
(c) 0.4 (d) 2

Sol. Option (a) is correct.

Explanation:

$$x^2 - 0.04 = 0$$
$$\Rightarrow x^2 = 0.04$$
$$\Rightarrow x = \pm\sqrt{0.04}.$$
$$\Rightarrow x = \pm 0.2.$$

6. The common difference of the A.P. $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}$, ... is

(a) 1 (b) $\frac{1}{p}$
(c) -1 (d) $-\frac{1}{p}$

Sol. Option (c) is correct.

Explanation:

Given A.P. $= \frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}$ ...

$$\therefore \text{ Common difference } = \frac{1-p}{p} - \frac{1}{p}$$
$$= \frac{1-p-1}{p} = \frac{-p}{p}$$
$$= -1.$$

7. The n^{th} term of the A.P. $a, 3a, 5a$, ... is

(a) na (b) $(2n-1)a$
(c) $(2n+1)a$ (d) $2na$

Sol. Option (b) is correct.

Explanation:

Given A.P. $= a, 3a, 5a, \ldots$

Here first term, $a = a$ and $d = 3a - a = 2a$

$$\therefore \quad n^{th} \text{ term} = a + (n-1)d$$
$$= a + (n-1)2a$$
$$= a + 2na - 2a$$
$$= 2na - a = (2n-1)a.$$

8. The point P on x-axis equidistant from the points $A(-1, 0)$ and $B(5, 0)$ is

(a) (2, 0) (b) (0, 2)
(c) (3, 0) (d) (2, 2)

Sol. Option (a) is correct.

Explanation: Let the position of the point P on x-axis be $(x, 0)$, then

$$PA^2 = PB^2$$
$$\Rightarrow (x+1)^2 + (0)^2 = (5-x)^2 + (0)^2$$
$$\Rightarrow x^2 + 2x + 1 = 25 + x^2 - 10x$$
$$\Rightarrow 2x + 10x = 25 - 1$$
$$\Rightarrow 12x = 24 \Rightarrow x = 2$$

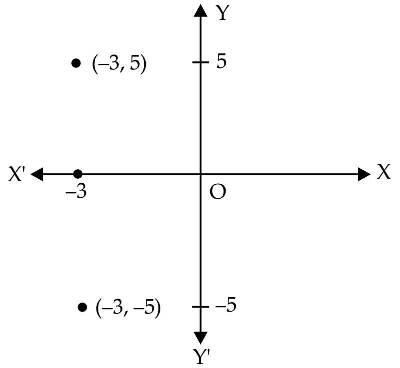
Hence, the point $P(x, 0)$ is (2, 0).

9. The co-ordinates of the point which is reflection of point (–3, 5) in x-axis are

(a) (3, 5) (b) (3, –5)
(c) (–3, –5) (d) (–3, 5)

Sol. Option (c) is correct.

Explanation: By using the graph of coordinate plane, we have the reflection of point (– 3, 5) is x-axis is (– 3, – 5).



10. If the point $P(6, 2)$ divides the line segment joining $A(6, 5)$ and $B(4, y)$ in the ratio 3 : 1, then the value of y is

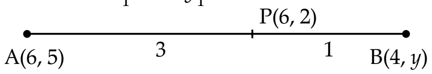
(a) 4 **(b)** 3

(c) 2 **(d)** 1

Sol. Option (d) is correct.

Explanation:

Here, $x_1 = 6, y_1 = 5$



and $x_2 = 4, y_2 = y$

Then $x = \frac{mx_2 + nx_1}{m+n}$ and $y = \frac{my_2 + ny_1}{m+n}$

$\therefore \quad 2 = \frac{3 \times y + 1 \times 5}{3+1} = \frac{3y+5}{4}$

$\Rightarrow \quad 3y + 5 = 8$

$\Rightarrow \quad 3y = 8 - 5 = 3$

$\Rightarrow \quad y = 1.$

In Q. Nos. 11 to 15, fill in the blanks. Each question is of 1 mark.

* **11. In fig. 1, $MN \parallel BC$ and $AM : MB = 1 : 2$, then $\frac{ar(\Delta AMN)}{ar(\Delta ABC)}$ = ...**

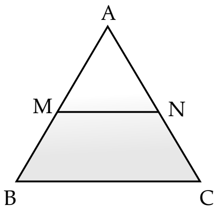


Fig. 1

12. In given Fig. 2, the length PB = ... cm.

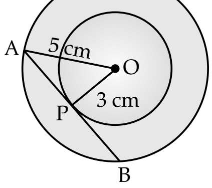
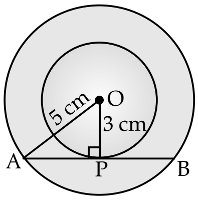


Fig. 2

Sol. 4

Explanation: Since AB is a tangent at P and OP is radius.

$\therefore \quad \angle APO = 90°, AO = 5$ cm and $OP = 3$ cm



In right angled ΔOPA,

$AP^2 = AO^2 - OP^2$

(By pythagoras theorem)

$AP^2 = (5)^2 - (3)^2 = 25 - 9 = 16$

$\Rightarrow \quad AP = 4$ cm

$\because$ Perpendicular from centre to chord bisect the chord

$\Rightarrow \quad AP = BP = 4$ cm.

13. In ΔABC, $AB = 6\sqrt{3}$ cm, $AC = 12$ cm and $BC = 6$ cm, then $\angle B$ = ...

OR

Two triangles are similar if their corresponding sides are ....

Sol. 90° or in same ratio

Explanation: Given that $AB = 6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm.

It can be observed that

$AB^2 = 108$ cm, $AC^2 = 144$ cm and $BC^2 = 36$ cm

Now $AB^2 + BC^2 = 108 + 36 = 144$ cm and $AC^2 = 144$ cm

i.e., $AB^2 + BC^2 = AC^2$, which satisfies Pythagoras theorem

So, $\angle B = 90°$.

OR

Explanation: Two triangles are similar if their corresponding sides are in the same ratio.

* **14. The value of (tan 1° tan 2° ...... tan 89°) is equal to ....**

15. In fig. 3, the angles of depressions from the observing positions O_1 and O_2 respectively of the object A are ....

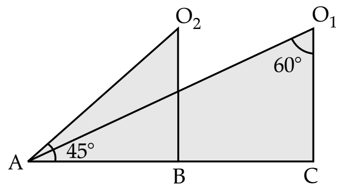


Fig. 3

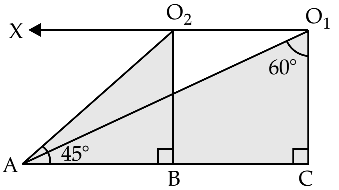
Sol. 30°, 45°

Explanation:

Draw $AC \parallel O_1X$

$\therefore \quad \angle AO_1X = 90° - 60° = 30°$

and $\angle AO_2X = \angle BAO_2 = 45°$.



* *Out of Syllabus*

Q. Nos. 16 to 20 are Short Answer Type Questions of 1 mark each.

16. If $\sin A + \sin^2 A = 1$, then find the value of the expression $(\cos^2 A + \cos^4 A)$.

Sol. Given, $\sin A + \sin^2 A = 1$

$\Rightarrow \sin A = 1 - \sin^2 A = \cos^2 A$

On squaring both sides, we get

$\sin^2 A = \cos^4 A$

$\Rightarrow 1 - \cos^2 A = \cos^4 A$

$\Rightarrow \cos^2 A + \cos^4 A = 1.$

17. In fig. 4 is a sector of circle of radius 10.5 cm. Find the perimeter of the sector. $\left(\text{Take } \pi = \frac{22}{7}\right)$

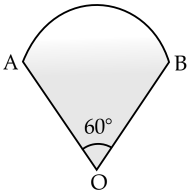
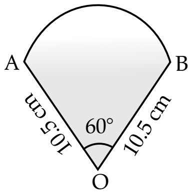


Fig. 4

Sol. Perimeter of the sector

$= 2r + \frac{2\pi r\theta}{360°}$



$= 2 \times 10.5 + \frac{2 \times 22 \times 10.5 \times 60°}{7 \times 360°}$

$= 21 + 11$

$= 32$ cm.

18. If a number x is chosen at random from the numbers –3, –2, –1, 0, 1, 2, 3, then find the probability of $x^2 < 4$.

OR

What is the probability that a randomly taken leap year has 52 Sundays ?

Sol.

x	–3	–2	–1	0	1	2	3
x^2	9	4	1	0	1	4	9

Total possible outcomes = 7

Favourable outcomes = $x^2 < 4$ *i.e.*, $x = -1, 0, 1$
= 3

$P(x^2 < 4) = \frac{3}{7}$

OR

Number of days in a leap year = 366

Number of weeks $= \frac{366}{7} = 52.28$

So, there will be 52 weeks and 2 days

So, every leap year has 52 Sundays

Now, the probability depends on remaining 2 days

The possible pairing of days are

Sunday	—	Monday
Monday	—	Tuesday
Tuesday	—	Wednesday
Wednesday	—	Thursday
Thursday	—	Friday
Friday	—	Saturday
Saturday	—	Sunday

There are total 7 pairs and out of 7 pairs, only 2 pairs have Sunday. The remaining 5 pairs does not include Sunday.

Therefore, the probability of only 52 Sundays in a Leap year is $\frac{5}{7}$

19. Find the class-marks of the classes 10–25 and 35–55.

Sol. Class mark of $10 - 25 = \frac{10+25}{2}$

$= \frac{35}{2} = 17.5$

and class mark of $35 - 55 = \frac{35+55}{2}$

$= \frac{90}{2} = 45.$

20. A die is thrown once. What is the probability of getting a prime number.

Sol. Total possible outcomes = 6

Favourable outcomes = {2, 3, 5} *i.e.*, 3

$\therefore$ Probability $= \frac{3}{6} = \frac{1}{2}.$

SECTION - B

Q. Nos. 21 to 26 carry 2 marks each.

21. A teacher asked 10 of his students to write a polynomial in one variable on a paper and then to handover the paper. The following were the answers given by the students :

$2x + 3$, $3x^2 + 7x + 2$, $4x^3 + 3x^2 + 2$, $x^3 + \sqrt{3x} + 7$, $7x + \sqrt{7}$, $5x^3 - 7x + 2$, $2x^2 + 3 - \frac{5}{x}$, $5x - \frac{1}{2}$, $ax^3 + bx^2 + cx + d$, $x + \frac{1}{x}$.

Answer the following question :

(i) How many of the above ten, are not polynomials ?

(ii) How many of the above ten, are quadratic polynomials ?

Sol. (i) $x^3 + \sqrt{3x} + 7, 2x^2 + 3 - \frac{5}{x}$ and $x + \frac{1}{x}$ are not polynomials.

(ii) $3x^2 + 7x + 2$ is only one quadratic polynomial.

22. * In fig. 5, *ABC* and *DBC* are two triangles on the same base *BC*. If *AD* intersects *B* at *O*, show that

$$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$$

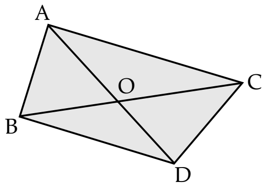


Fig. 5

OR

In fig. 6, if $AD \perp BC$, then prove that $AB^2 + CD^2 = BD^2 + AC^2$.

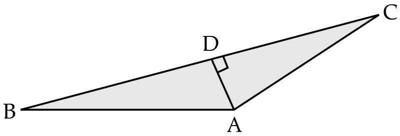
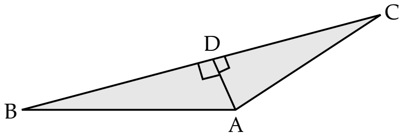


Fig. 6

Sol. **OR**

In right ΔADC,

$$AC^2 = AD^2 + CD^2 \quad ...(i)$$



In right ΔADB,

$$AB^2 = AD^2 + BD^2 \quad ...(ii)$$

Subtracting eq. (i) from eq. (ii),

$$AB^2 - AC^2 = BD^2 - CD^2$$

Hence $AB^2 + CD^2 = AC^2 + BD^2$ **Proved**

23. Prove that $1 + \frac{\cot^2 \alpha}{1 + \operatorname{cosec} \alpha} = \operatorname{cosec} \alpha$

OR

Show that $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$

Sol.

$$\text{L.H.S} = 1 + \frac{\cot^2\alpha}{1 + \text{cosse } \alpha}$$

$$= 1 + \frac{\text{cosec}^2\alpha - 1}{1 + \text{cosse } \alpha}$$

$$= 1 + \frac{(1 + \text{cosec } \alpha)(\text{cosec } \alpha - 1)}{1 + \text{cosec } \alpha}$$

$$= 1 + \text{cosec } \alpha - 1$$

$= \text{cosec } \alpha =$ R.H.S. **Hence Proved**

OR

$$\text{L.H.S.} = \tan^4\theta + \tan^2\theta$$

$$= \tan^2\theta (1 + \tan^2\theta)$$

$$= \tan^2\theta \times \sec^2\theta$$

$$= (\sec^2\theta - 1) \sec^2\theta$$

$$= \sec^4\theta - \sec^2\theta = \text{R.H.S.}$$

Hence Proved

24. The volume of a right circular cylinder with its height equal to the radius is $25\frac{1}{7}$ cm^3. Find the height of the cylinder. $\left(\text{Use } \pi = \frac{22}{7}\right)$

Sol. Given,

Volume of a right circular cylinder $= 25\frac{1}{7}$ cm

i.e., $\pi r^2 h = \frac{176}{7}$

where height, h = radius r, then

$$\Rightarrow \quad \frac{22}{7} \times h^2 \times h = \frac{176}{7}$$

$$\Rightarrow \quad h^3 = \frac{176}{22} = 8 = 2^3.$$

Hence, height of the cylinder = 2 cm.

25. 'A child has a die whose six faces show the letters as shown below :

A	B	C	D	E	A

The die is thrown once. What is the probability of getting (i) A, (ii) D ?

Sol. Total possible outcomes, $n(S) = 6$

(i) Let E_1 = getting event letter A, then

$$n(E_1) = 2$$

$$\therefore \quad \text{Probability} = \frac{n(E_1)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

(ii) Let E_2 = getting event letter D, then

$$n(E_2) = 1$$

$$\therefore \quad \text{Probability} = \frac{n(E_2)}{n(S)} = \frac{1}{6}.$$

26. compute the mode for the following frequency distribution :

Size of items (in cm)	Frequency
0 – 4	5
4 – 8	7
8 – 12	9
12 – 16	17
16 – 20	12
20 – 24	10
24 – 28	6

* *Out of Syllabus*

Sol. Here, Modal class = 12 – 16

$\therefore l = 12, f_1 = 17, f_0 = 9, f_2 = 12$ and $h = 4$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 12 + \left(\frac{17-9}{2\times17-9-12}\right) \times 4$$

$$= 12 + \frac{8\times4}{13}$$

$$= 12 + 2.46 = 14.46.$$

SECTION - C

Q. Nos. 27 to 34 carry 3 marks each.

27. If $2x + y = 23$ and $4x - y = 19$, find the value of $(5y - 2x)$ and $\left(\frac{y}{x} - 2\right)$.

OR

Solve for x : $\frac{1}{x+4} - \frac{1}{x+7} = \frac{11}{30}, x \neq -4, 7.$

Sol. Given,

$$2x + y = 23 \quad ...(i)$$

and $$4x - y = 19 \quad ...(ii)$$

On adding eq. (i) and (ii), we get

$$6x = 42 \Rightarrow x = 7$$

Putting the value of x in eq. (i), we get

$$14 + y = 23$$

$$\Rightarrow y = 23 - 14 = 9$$

Hence, $5y - 2x = 5 \times 9 - 2 \times 7 = 45 - 14$

$= 31$

and $\frac{y}{x} - 2 = \frac{9}{7} - 2 = \frac{9-14}{7} = \frac{-5}{7}$

OR

Given, $$\frac{1}{x+4} - \frac{1}{x+7} = \frac{11}{30}$$

$$\Rightarrow \frac{x+7-x-4}{(x+4)(x+7)} = \frac{11}{30}$$

$$\Rightarrow \frac{3}{x^2+4x+7x+28} = \frac{11}{30}$$

$$\Rightarrow \frac{3}{x^2+11x+28} = \frac{11}{30}$$

$\Rightarrow 11x^2 + 121x + 308 = 90$

$\Rightarrow 11x^2 + 121x + 218 = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$a = 11, b = 121$ and $c = 218$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-121 \pm \sqrt{14641 - 9592}}{22}$$

$$\Rightarrow x = \frac{-121 \pm \sqrt{5049}}{22}$$

$$= \frac{-121 \pm 71.06}{22}$$

$$\Rightarrow x = \frac{-121+71.06}{22}, \frac{-121-71.06}{22}$$

$$\Rightarrow x = \frac{-49.94}{22}, \frac{-192.06}{22}$$

$$\Rightarrow x = -2.27, -8.73.$$

28. Show that the sum of all terms of an A.P. whose first term is a, the second term is b and the last term is c is equal to $\frac{(a+c)(b+c-2a)}{2(b-a)}$

OR

Solve the equation :

1 + 4 + 7 + 10 + ... + x = 287.

Sol. Given, first term, $A = a$

and second term $= b$

$\Rightarrow$ common difference,

$$d = b - a$$

Last term, $l = c$

$\Rightarrow A + (n-1)\, d = c$

$\Rightarrow a + (n-1)\, d = c$

$a + (n-1)(b-a) = c$

$\Rightarrow (b-a)(n-1) = c - a$

$$\Rightarrow n - 1 = \frac{c-a}{b-a}$$

$$\Rightarrow n = \frac{c-a}{b-a} + 1 = \frac{c-a+b-a}{b-a}$$

$$\Rightarrow n = \frac{b+c-2a}{b-a}$$

Now $$\text{sum} = \frac{n}{2}[A + l]$$

$$= \frac{(b+c-2a)}{2(b-a)}[a+c]$$

$$= \frac{(a+c)(b+c-2a)}{2(b-a)}$$

Hence Proved.

OR

Given, $a = 1$ and $d = 4 - 1 = 3$

Let number of terms is the series be n, the

$$\Rightarrow S_n = \frac{n}{2}[2a + (n-1)\, d]$$

$$\Rightarrow \frac{n}{2}[2 \times 1 + (n-1)\, 3] = 287$$

$\Rightarrow \quad \frac{n}{2}[2 + 3n - 3] = 287$

$\Rightarrow \quad 3n^2 - n - 574 = 0$

$\Rightarrow \quad 3n^2 - 42n + 41n - 574 = 0$

$\Rightarrow \quad 3n(n - 14) + 41(n - 14) = 0$

$\Rightarrow \quad (n - 14)(3n + 41) = 0$

Either $n = 14$ or $n = -\frac{41}{3}$, it is not possible

Thus 14^{th} term is x

$\therefore \quad a + (n - 1)d = x$

$\Rightarrow x = 1 + 13 \times 3 = 40.$

29. In a flight of 600 km, an aircraft was slowed down due to bad weather. The average speed of the trip was reduced by 200 km/h and the time of flight increased by 30 minutes. Find the duration of flight.

Sol. Please see the solution of Question No. 29 of Delhi Set - I on page 6

30. If the mid-point of the line segment joining the points $A(3, 4)$ and $B(k, 6)$ is $P(x, y)$ and $x + y - 10 = 0$, find the value of k.

OR

*** Find the area of triangle ABC with $A(1, -4)$ and the mid-points of sides through A being (2, –1) and (0, –1).**

Sol. Here, $\frac{3+k}{2} = x$

and $y = \frac{4+6}{2} = \frac{10}{2} = 5$

Given, $x + y - 10 = 0$

$\Rightarrow \quad \frac{3+k}{2} + 5 - 10 = 0$

$\Rightarrow \quad \frac{3+k}{2} = 5$

$\Rightarrow \quad 3 + k = 10$

$\Rightarrow \quad k = 10 - 3 = 7.$

31. In Fig. 7, if $\Delta ABC \sim DEF$ and their sides of lengths (in cm) are marked along them, then find the lengths of sides of each triangle.

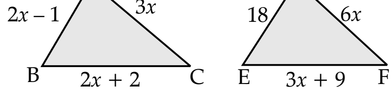


Fig. 7

Sol. Given, $\Delta ABC \sim \Delta DEF$

Then according to question,

$$\frac{AB}{BC} = \frac{DE}{EF}$$

$\Rightarrow \quad \frac{2x-1}{2x+2} = \frac{18}{3x+9}$

$\Rightarrow \quad (2x - 1)(3x + 9) = 18(2x + 2)$

$\Rightarrow \quad (2x - 1)(x + 3) = 6(2x + 2)$

$\Rightarrow \quad 2x^2 - x + 6x - 3 = 12x + 12$

$\Rightarrow \quad 2x^2 + 5x - 12x - 15 = 0$

$\Rightarrow \quad 2x^2 - 7x - 15 = 0$

$\Rightarrow \quad 2x^2 - 10x + 3x - 15 = 0$

$\Rightarrow \quad 2x(x - 5) + 3(x - 5) = 0$

$\Rightarrow \quad (x - 5)(2x + 3) = 0$

Either $x = 5$ or $x = \frac{-3}{2}$, it is not possible

So, $x = 5$

Then in ΔABC, we have

$AB = 2x - 1 = 2 \times 5 - 1 = 9$

$BC = 2x + 2 = 2 \times 5 + 2 = 12$

$AC = 3x = 3 \times 5 = 15$

and in ΔDEF, we have

$DE = 18$

$EF = 3x + 9 = 3 \times 5 + 9 = 24$

$DE = 6x = 6 \times 5 = 30.$

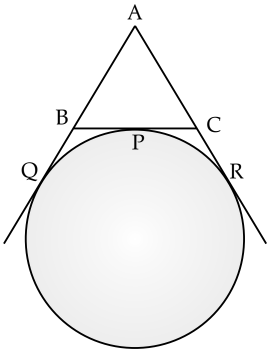
32. If a circle touches the side BC of a triangle ABC at P and extended sides AB and AC at Q and R, respectively, prove that

$AQ = \frac{1}{2}(BC + CA + AB)$

Sol. $BC + CA + AB$

$= (BP + PC) + (AR - CR) + (AQ - BQ)$

$= AQ + AR - BQ + BP + PC - CR$



$\because$ From the same external point, the tangent segments drawn to a circle are equal.

So, From the point B, $BQ = BP$

From the point A, $AQ = AR$

From the point C, $CP = CR$

$\therefore$ Perimeter of ΔABC, i.e.,

* *Out of Syllabus*

$$AB + BC + CA = 2AQ - BQ + BQ + CR - CR$$

$$\Rightarrow \quad 2AQ = AB + BC + CA$$

$$\Rightarrow \quad AQ = \frac{1}{2}(BC + CA + AB)$$

Hence proved.

33. If $\sin\theta + \cos\theta = \sqrt{2}$, prove that $\tan\theta + \cot\theta = 2$.

Sol. Given, $\sin\theta + \cos\theta = \sqrt{2}$

On squaring both the sides, we get

$$(\sin\theta + \cos\theta)^2 = (\sqrt{2})^2$$

$$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2$$

$$\Rightarrow \quad 1 + 2\sin\theta\cos\theta = 2$$

$$\Rightarrow \quad 2\sin\theta\cos\theta = 1$$

$$\Rightarrow \quad \sin\theta\cos\theta = \frac{1}{2} \quad \text{...(i)}$$

Now taking L.H.S.,

$$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\ \sin\theta}$$

$$= \frac{1}{\sin\theta\ \cos\theta} = \frac{1}{1/2}$$

[From eq. (i)]

$$\Rightarrow \quad \tan\theta + \cot\theta = 2 = \text{R.H.S.}$$

Hence Proved

34. The area of a circular play ground is 22176 cm^2. Find the cost of fencing this ground at the rate of ₹ 50 per metre.

Sol. Area of a circular play ground = 22176 cm^2

i.e., $\pi r^2 = 22176\ cm^2$

$$\Rightarrow \quad r^2 = 22176 \times \frac{7}{22} = 7056$$

$$\Rightarrow \quad r = 84 \text{ cm} = 0.84 \text{ m}$$

Cost of fencing this ground = ₹ 50 $\times 2\pi r$

$$= ₹\,50 \times 2 \times \frac{22}{7} \times 0.84$$

$$= ₹\,264.$$

SECTION - D

Q. Nos. 35 to 40 carry 4 marks each.

35. Prove that $\sqrt{5}$ is an irrational number.

Sol. Let $\sqrt{5}$ be a rational number.

$$\therefore \quad \sqrt{5} = \frac{p}{q},$$

where p and q are co-prime integers and $q \neq 0$

On squaring both the sides, we get

$$5 = \frac{p^2}{q^2}$$

or $\quad p^2 = 5q^2$

$\because p^2$ is divisible by 5

$\therefore p$ is divisible by 5

Let $p = 5r$ for some positive integer r,

$$p^2 = 25r^2$$

$$\therefore \quad 5q^2 = 25r^2$$

or $\quad q^2 = 5r^2$

$\because q^2$ is divisible by 5

$\therefore q$ is divisible by 5.

Here p and q are divisible by 5, which contradicts the fact that p and q are co-prime.

Hence, our assumption is false

$\therefore \sqrt{5}$ is an irrational number.

* **36. It can take 12 hours to fill a swimming pool using two pipes. If the pipe of larger diameter is used for 4 hours and the pipe of smaller diameter for 9 hours, only half of the pool can be filled. How long would it take for each pipe to fill the pool separately ?**

* **37. Draw a circle of radius 2 cm with centre *O* and take a point *P* outside the circle such that *OP* = 6.5 cm. From *P*, draw two tangents to the circle.**

OR

* **Construct a triangle with sides 5 cm, 6 cm and 7 cm and then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of the first triangle.**

38. From a point on the ground, the angles of elevation of the bottom and the top of a tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

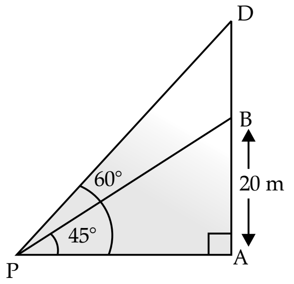
Sol. Let the height of the tower be BD

In ΔPAB,

$$\tan 45° = \frac{AB}{AP}$$

$$\Rightarrow \quad 1 = \frac{20}{AP}$$

$$\Rightarrow \quad AP = 20\text{m}$$



* *Out of Syllabus*

In ΔPAD,

$$\tan 60° = \frac{AD}{AP} = \frac{20 + BD}{20}$$

$$\Rightarrow \quad \sqrt{3} = \frac{20 + BD}{20}$$

$$\Rightarrow \quad 20 + BD = 20\sqrt{3}$$

$$\Rightarrow \quad BD = 20\sqrt{3} - 20 = 20\,(\sqrt{3} - 1)$$

$$= 20\,(1.732 - 1) = 20 \times 0.732$$

$$= 14.64 \text{ m.}$$

39. Find the area of the shaded region in fig. 8, if PQ = 24 cm, PR = 7 cm and O is the centre of the circle.

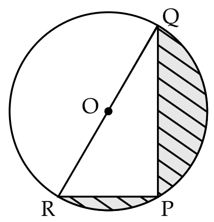


Fig. 8

OR

*** Find the curved surface area of the frustum of a cone, the diameters of whose circular ends are 20 m and 6 m and its height is 24 m.**

Sol. Given, $PQ = 24$ cm, $PR = 7$ cm

We know that the angle in the semicircle is right angle.

Here, $\angle RPQ = 90°$

In right angle ΔRPQ,

$$RQ^2 = PR^2 + PQ^2$$

(By Pythagoras theorem)

$$\Rightarrow \quad RQ^2 = (7)^2 + (24)^2 = 49 + 576 = 625$$

$$\therefore \quad RQ = 25 \text{ cm}$$

$\therefore$ Area of ΔRPQ

$$= \frac{1}{2} \times RP \times PQ$$

$$= \frac{1}{2} \times 7 \times 24 = 84 \text{ cm}^2$$

and area of semi-circle $= \frac{1}{2} \times \pi r^2$

$$= \frac{1}{2} \times \frac{22}{7} \times \left(\frac{25}{2}\right)^2$$

$$= \frac{11 \times 625}{7 \times 4} = \frac{6875}{28} \text{ cm}$$

Now, area of shaded region

= area of sem-circle – area of ΔRPQ

$$= \frac{6875}{28} - 84 = \frac{6875 - 2352}{28}$$

$$= \frac{4523}{28} = 161.54 \text{ cm}^2.$$

40. The mean of the following frequency distribution is 18. The frequency *f* in the class interval 19 – 21 is missing. Determine *f*.

Class interval	Frequency
11 – 13	3
13 – 15	6
15 – 17	9
17 – 19	13
19 – 21	f
21 – 23	5
23 – 25	4

OR

*** The following table gives production yield per hectare of wheat of 100 farms of a village :**

Production yield	Frequency
40 – 45	4
45 – 50	6
50 – 55	16
55 – 60	20
60 – 65	20
65 – 70	24

Change the distribution to a 'more than' type distribution and draw its ogive.

Sol.

Class	Class mark (x)	Frequency (f)	fx
11 – 13	12	3	36
13 – 15	14	6	84
15 – 17	16	9	144
17 – 19	18	13	234
19 – 21	20	f	$20f$
21 – 23	22	5	110
23 – 25	24	4	96
		$\Sigma f = 40 + f$	$\Sigma fx = 704 + 20f$

$$\Sigma f = 40 + f$$

$$\Sigma fx = 704 + 20f$$

$$\text{Mean} = 18 = \frac{704 + 20f}{40 + f}$$

$$\Rightarrow \quad 720 + 18f = 704 + 20f$$

$$\Rightarrow \quad f = 8.$$

* *Out of Syllabus*

Outside Delhi Set-II Code No. 30/2/2

Note : All other Questions are from Set I

SECTION - A

In Q. Nos. 11 to 15, fill in the blanks. Each question is of 1 mark.

*** 15. The value of sin 23° cos 67° + cos 23° sin 67° is ....**

Q. Nos. 16 to 20 are short answer type questions of 1 mark each.

*** 19. If tan A = cot B, then find the value of (A + B).**

20. Find the class marks of the classes 15 – 35 and 45 – 60.

Sol. Class mark of 15 – 35 $= \frac{15+35}{2} = \frac{50}{2} = 25$

and class mark of 45 – 60 $= \frac{45+60}{2} = \frac{105}{2} = 52.5.$

SECTION - B

Q. Nos. 21 to 26 carry 2 marks each.

25. A child has a die whose six faces show the letters as shown below :

A	**A**	**B**	**C**	**C**	**C**

The die is thrown once. What is the probability of getting (i) A, (ii) C ?

Sol. Total possible outcomes $n(s) = 6$

(i) Let E_1 = getting event letter A, then $n(E_1) = 2$

$\therefore$ Probability $= \frac{n(E_1)}{n(s)} = \frac{2}{6} = \frac{1}{3}$

(ii) Let E_2 = Getting event letter C, then $n(E_2) = 3$

$\therefore$ Probability $= \frac{n(E_2)}{n(s)} = \frac{3}{6} = \frac{1}{2}.$

26. A solid is in the shape of a cone mounted on a hemisphere of same base radius. If the curved surface areas of the hemispherical part and the conical part are equal, then find the ratio of the radius and the height of the conical part.

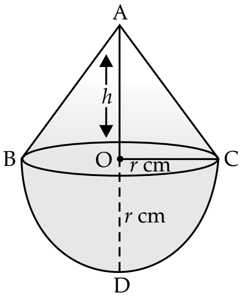
Sol. Let ABC be a cone, which is mounted on a hemisphere.

Given : $OC = OD = r$ cm

Curved surface area of the hemispherical part

$= \frac{1}{2}(4\pi r^2)$

$= 2\pi r^2$



Slant height of a cone,

$$l = \sqrt{r^2+h^2}$$

So, curved surface area of a cone $= \pi r l$

$$= \pi r\sqrt{h^2+r^2}$$

i.e., $2\pi r^2 = \pi r\sqrt{h^2+r^2}$ (given)

$\Rightarrow$ $2r = \sqrt{h^2+r^2}$

on squaring both of the sides, we get

$$4r^2 = h^2 + r^2$$

$\Rightarrow$ $4r^2 - r^2 = h^2$

$\Rightarrow$ $3r^2 = h^2$

$\Rightarrow$ $\frac{r^2}{h^2} = \frac{1}{3} \Rightarrow \frac{r}{h} = \frac{1}{\sqrt{3}}$

Hence, the ratio of the radius and the height

$$= 1 : \sqrt{3}.$$

SECTION - C

Q. Nos. 27 to 34 carry 3 marks each.

32. If in an A.P., the sum of first m terms is n and the sum of its first n terms is m, then prove that the sum of its first (m + n) terms is –(m + n).

OR

Find the sum of all 11 terms of an A.P. whose middle term is 30.

Sol. Let 1st term of series be a and common difference be d, then

$$S_m = n \text{ (given)}$$

$\Rightarrow$ $\frac{m}{2}[2a + (m-1)d] = n$

$\Rightarrow$ $m[2a + (m-1)d] = 2n$ ...(i)

and, $S_n = m$ (given)

$\Rightarrow$ $\frac{n}{2}[2a + (n-1)d] = m$

$\Rightarrow$ $n[2a + (n-1)d] = 2m$ ...(ii)

On subtracting,

$$2(n-m) = 2a(m-n) + d[m^2 - n^2 - (m-n)]$$

$\Rightarrow$ $2(n-m) = 2a(m-n) + d[(m-n)][m+n-1]$

* *Out of Syllabus*

$$\Rightarrow \quad 2(n-m) = (m-n)[2a + d(m+n-1)]$$
$$\Rightarrow \quad -2 = 2a + d(m+n-1)$$

Now, $$S_{m+n} = \frac{m+n}{2}[2a + (m+n-1)d]$$
$$= \frac{m+n}{2}(-2)$$
$$= -(m+n)$$

Hence proved.

OR

In an $A.P.$ with 11 terms, the middle term is $\left(\frac{11+1}{2}\right)$

= 6^{th} term.

Now $$t_6 = a + 5d = 30$$

Thus, $$S_{11} = \frac{11}{2}[2a + 10d]$$
$$= 11(a + 5d)$$
$$= 11 \times 30$$
$$= 330.$$

33. A fast train takes 3 hours less than a slow train for a journey of 600 km. If the speed of the slow train is 10 km/h less than that of the fast train, find the speed of each train.

Sol. Total distance of a journey = 600 km

Let speed of fast train be x km/h, then speed of slow train = $(x - 10)$ km/h

According to questions,

$$\frac{600}{x-10} - \frac{600}{x} = 3$$

$$\left[\because \text{Time} = \frac{\text{Distance}}{\text{Speed}}\right]$$

$$\Rightarrow \quad 600\left[\frac{x-x+10}{(x-10)x}\right] = 3$$
$$\Rightarrow \quad \frac{6000}{x^2-10x} = 3$$
$$\Rightarrow \quad x^2 - 10x - 2000 = 0$$
$$\Rightarrow \quad x^2 - 50x + 40x - 2000 = 0$$
$$\Rightarrow \quad x(x-50) + 40(x-50) = 0$$
$$\Rightarrow \quad (x-50)(x+40) = 0$$

Either $x = 50$ or $x = -40$

[$\because$ speed can not be possible is negative]

So, the speed of fast train = 50 km/h and the speed of slow train = 50 – 10 = 40 km/h.

34. If $1 + \sin^2\theta = 3\sin\theta\cos\theta$, prove that $\tan\theta = 1$ or $\frac{1}{2}$.

Sol. Given, $1 + \sin^2\theta = 3\sin\theta\cos\theta$

On dividing by $\sin^2\theta$ on both sides, we get

$$\frac{1}{\sin^2\theta} + 1 = 3\cot\theta$$

$$\left[\because \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$

$$\Rightarrow \quad \text{cosec}^2\theta + 1 = 3\cot\theta$$
$$\Rightarrow \quad 1 + \cot^2\theta + 1 = 3\cot\theta$$
$$\Rightarrow \quad \cot^2\theta - 3\cot\theta + 2 = 0$$
$$\Rightarrow \quad \cot^2\theta - 2\cot\theta - \cot\theta + 2 = 0$$
$$\Rightarrow \quad \cot\theta(\cot\theta - 2) - 1(\cot\theta - 2) = 0$$
$$\Rightarrow \quad (\cot\theta - 2)(\cot\theta - 1) = 0$$
$$\Rightarrow \quad \cot\theta = 1 \text{ or } 2$$
$$\tan\theta = 1 \text{ or } \frac{1}{2}.$$

Hence proved

SECTION - D

Q. Nos. 35 to 40 carry 4 marks each.

*** 39. Draw two tangents to a circle of radius 4 cm, which are inclined to each other at an angle of 60°.**

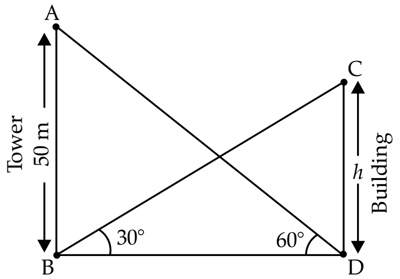
OR

*** Construct a triangle *ABC* with sides 3 cm, 4 cm and 5 cm. Now, construct another triangle whose sides are $\frac{4}{5}$ times the corresponding sides of ΔABC.**

40. The angle of elevation of the top of a building from the foot of a tower is 30° and the angle of elevation of the top of a tower from the foot of the building is 60°. If the tower is 50 m high, then find the height of the building.

Sol. According to question,

In ΔABD



$$\tan 60° = \frac{AB}{BD}$$
$$\Rightarrow \quad \sqrt{3} = \frac{50}{BD}$$
$$\Rightarrow \quad BD = \frac{50}{\sqrt{3}}$$

* *Out of Syllabus*

Now in ΔBDC,

$$\tan 30° = \frac{CD}{BD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{\frac{50}{\sqrt{3}}} = \frac{h\sqrt{3}}{50}$$

$$\Rightarrow \quad 3h = 50$$

$$\Rightarrow \quad h = \frac{50}{3} = 16.67$$

Hence, the height of the building is 16.67 m.

Outsided Delhi Set-III — Code No. 30/2/3

Note : Except these, all other questions are from Set I & Set II

SECTION - A

In Q. Nos. 11 to 15, fill in the blanks. Each question is of 1 mark.

*** 15. The value of sin 32° cos 58° + cos 32° sin 58° is ....**

OR

*** The value of $\frac{\tan 35°}{\cot 55°} + \frac{\cot 78°}{\tan 12°}$ is ....**

Q. Nos. 16 to 20 are short answer type questions of 1 mark each.

19. Find the area of the sector of a circle of radius 6 cm whose central angle is 30°. (Take π = 3.14)

Sol. Given, radius $(r) = 6$ cm

central angle $(\theta) = 30°$

$$\text{Area of the sector} = \frac{\pi r^2 \theta}{360°}$$

$$= \frac{3.14 \times 6 \times 6 \times 30°}{360°}$$

$$= 9.42 \text{ cm}^2.$$

20. Find the class marks of the classes 20 – 50 and 35 – 60.

Sol. Class mark of 20 – 50 $= \frac{20+50}{2} = \frac{70}{2} = 35$ and

class mark of 35 – 60 $= \frac{35+60}{2} = \frac{95}{2} = 47.5$.

SECTION - B

Q. Nos. 21 to 26 carry 2 marks each.

25. Find the mode of the following frequency distribution :

Class	Frequency
15 – 20	3
20 – 25	8
25 – 30	9
30 – 35	10
35 – 40	3
40 – 45	2

Sol. Here, modal class = 30 – 35

$\therefore l = 30, f_0 = 9, f_1 = 10, f_2 = 3$ and $h = 5$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 30 + \left(\frac{10-9}{2\times 10 - 9 - 3}\right) \times 5$$

$$= 30 + \frac{5}{8} = 30 + 0.625$$

$$= 30.625.$$

26. From a solid right circular cylinder of height 14 cm and base radius 6 cm, a right circular cone of same height and same base removed. Find the volume of the remaining solid.

Sol. Given, height $(h) = 14$ cm

and Base radius $(r) = 6$ cm

Volume of the remaining solid = Volume of a right circular cylinder – Volume of a right circular cone

$$= \pi r^2 h - \frac{1}{3}\pi r^2 h$$

$$= \frac{2}{3}\pi r^2 h$$

$$= \frac{2}{3} \times \frac{22}{7} \times 6 \times 6 \times 14$$

$$= 1056 \text{ cm}^3.$$

SECTION - C

Q. Nos. 27 to 34 carry 3 marks each.

* *Out of Syllabus*

32. Which term of the A.P. 20, $19\frac{1}{4}$, $18\frac{1}{2}$, $17\frac{3}{4}$, ... is the first negative term.

OR

Find the middle term of the A.P. 7, 13, 19, ..., 247.

Sol. Here, $a = 20$

and $d = \frac{77}{4} - 20 = -\frac{3}{4}$

Let $t_n <, 0$

$\because$ $t_n = a + (n-1)d$

$\therefore 20 + (n-1)\left(-\frac{3}{4}\right) < 0$

$\Rightarrow$ $80 - 3n + 3 < 0$

$\Rightarrow$ $83 - 3n < 0$

$\Rightarrow$ $n > \frac{83}{3} \Rightarrow n > 27.6$

$\Rightarrow$ $n = 28$

Hence, 28th term is the first negative term.

OR

In this A.P., $a = 7$, $d = 13 - 7 = 6$

and $t_n = 247$

$\because$ $t_n = a + (n-1)d$

$\therefore$ $247 = 7 + (n-1)\,6$

$\Rightarrow$ $6(n-1) = 240$

$\Rightarrow$ $n - 1 = 40$

$\Rightarrow$ $n = 41$

Hence, the middle term $= \frac{n+1}{2}$

$= \frac{41+1}{2} = \frac{42}{2}$

$= 21.$

33. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm standing water is required ?

Sol. Speed of water in canal = 10 km/h

In 30 min $= \frac{30}{60} = \frac{1}{2}$ h

$\therefore$ Length of water $= 10 \times \frac{1}{2} = 5$ km

$= 5000$ m.

Volume of water is canal in 30 min

= Volume of water for irrigation

$\Rightarrow$ $6 \times 1.5 \times 5000 = \frac{8}{100} \times l \times b$

$\Rightarrow$ $l \times b = \frac{6 \times 1.5 \times 5000 \times 100}{8}$

$= \frac{4500000}{8} = 562500 \text{ m}^2$

Hence, area irrigated in 30 min is 562500 m^2.

*** 34. Show that :**

$$\frac{\cos^2(45^\circ + \theta) + \cos^2(45^\circ - \theta)}{\tan(60^\circ + \theta)\tan(30^\circ - \theta)} = 1$$

SECTION - D

Q. Nos. 35 to 40 carry 4 marks each.

* **39. Draw a circle of radius 3.5 cm. From a point *P*, 6 cm from its centre, draw two tangents to the circle.**

OR

*** Construct a ΔABC with $AB = 6$ cm, $BC = 5$ cm and $\angle B = 60°$. Now construct another triangle whose sides are $\frac{2}{3}$ times the corresponding sides of ΔABC.**

40. A solid is in the shape of a hemisphere surmounted by a cone. If the radius of hemisphere and base radius of cone is 7 cm and height of cone is 3.5 cm, find the volume of the solid. $\left(\text{Take } \pi = \frac{22}{7}\right)$

Sol. Here, radius (r) = 7 cm

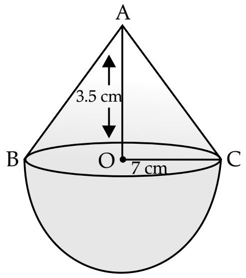
and height of a cone = 3.5 cm

$\therefore$ Volume of the solid = Volume of hemisphere

+ volume of a cone

* *Out of Syllabus*

$$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$$



$$= \frac{2}{3}\times\frac{22}{7}\times(7)^3 + \frac{1}{3}\times\frac{22}{7}\times(7)^2\times 3.5$$

$$= \frac{1}{3}[2156 + 539]$$

$$= \frac{1}{3}\times 2695$$

$$= 898.33 \text{ cm}^3.$$

❑❑