Solved Paper 2022

Mathematics (Standard) (TERM-II)

CLASS-X

Time : 2 Hours **Max. Marks : 40**

General Instructions :

(i) This question paper consists of 14 questions. All questions are compulsory.

(ii) This question paper is divided into four sections – A, B, C and D.

(iii) Section A contains 6 questions (Q No. 1 to 6) of 2 marks each. Internal choice has been provided in two questions.

(iv) Section B contains 4 questions (Q No. 7 to 10) of 3 marks each. Internal choice has been provided in one question.

(v) Section C contains 4 questions (Q No. 11 to 14) of 4 marks each. An internal choice has been provided in one question. It also contains two case study board questions.

(vii) Use of calculator is not permitted.

Term-II, Delhi Set-I—SERIES: PPQQC/2 *Code No. 30/2/1*

SECTION - A

Question Numbers 1 to 6 carry 2 marks each.

1. Solve the quadratic equation: $x^2 + 2\sqrt{2}x - 6 = 0$ for x.

Ans. Given quadratic equation is:

$$x^2 + 2\sqrt{2}x - 6 = 0$$

$\Rightarrow$ $x^2 + 3\sqrt{2}\,x - \sqrt{2}\,x - 6 = 0$

$\Rightarrow$ $x(x + 3\sqrt{2}) - \sqrt{2}\,(x + 3\sqrt{2}) = 0$

$\Rightarrow$ $(x + 3\sqrt{2})(x - \sqrt{2}) = 0$

$\Rightarrow$ $x + 3\sqrt{2} = 0$ or $x - \sqrt{2} = 0$

$\Rightarrow$ $x = -3\sqrt{2}$ or $x = \sqrt{2}$

2. (a) Which term of the *A.P.* $-\frac{11}{2}, -3, -\frac{1}{2}$ .... is $\frac{49}{2}$?

OR

(b) Find a and b so that the numbers $a, 7, b, 23$ are in *A.P.*

Ans. (a) Given A.P. is:

$$-\frac{11}{2}, -3, -\frac{1}{2}, ...$$

Here, first term, $a = -\frac{11}{2}$

common difference,

$$d = -3 - \left(-\frac{11}{2}\right)$$

$$= -3 + \frac{11}{2} = \frac{5}{2}$$

According to question,

$$a_n = \frac{49}{2}$$

$\Rightarrow$ $\frac{49}{2} = a + (n-1)d$

[since, $a_n = a + (n-1)d$]

or, $\frac{49}{2} = -\frac{11}{2} + (n-1)\frac{5}{2}$

or, $\frac{49}{2} + \frac{11}{2} = (n-1)\frac{5}{2}$

or, $30 = (n-1)\frac{5}{2}$

or, $n - 1 = \frac{60}{5}$

or, $n = 12 + 1$

$= 13$

Hence, 13th term of *A.P.* is $\frac{49}{2}$.

OR

(b) Given, numbers $a, 7, b, 23$ are in A.P. .

$\therefore$ $7 - a = b - 7 = 23 - b$

[*A.P.* has equal common difference]

By equating, $b - 7 = 23 - b$

$\Rightarrow$ $2b = 30$

$\Rightarrow$ $b = 15$

Now, equating $7 - a = b - 7$

$\Rightarrow$ $7 - a = 15 - 7$

[Putting the value of a]

$\Rightarrow$ $-a = 1$

$\Rightarrow$ $a = -1$

Hence, $a = -1$ and $b = 15$.

3. A solid piece of metal in the form of a cuboid of dimensions 11 cm × 7 cm × 7 cm is melted to form 'n' number of solid spheres of radii $\frac{7}{2}$ cm each. Find the value of n.

Ans. We know that, volume of cuboid $= l \times b \times h$

volume of sphere $= \dfrac{4}{3}\pi r^3$

Given, $l = 11$ cm

$b = 7$ cm,

$h = 7$ cm and $r = \dfrac{7}{2}$ cm

Here,

volume of cuboid $= n \times$ volume of sphere

or, $11 \times 7 \times 7 = n \times \dfrac{4}{3}\pi\left(\dfrac{7}{2}\right)^3$

or, $11 \times 7 \times 7 = n \times \dfrac{4}{3} \times \dfrac{22}{7} \times \dfrac{7}{2} \times \dfrac{7}{2} \times \dfrac{7}{2}$

or, $n = \dfrac{11 \times 7 \times 7 \times 3 \times 7 \times 2 \times 2 \times 2}{4 \times 22 \times 7 \times 7 \times 7}$

or, $n = 3$

4. (a) In Fig. 1, *AB* is diameter of a circle centred at *O*. *BC* is tangent to the circle at *B*. If *OP* bisects the chord *AD* and $\angle AOP = 60°$, then find $\angle C$.

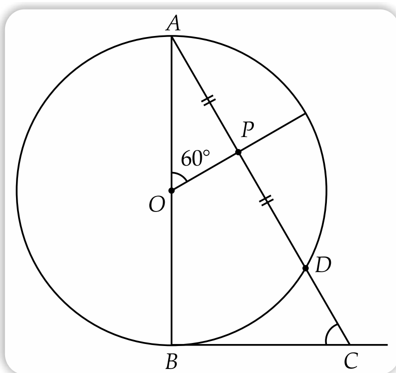


Fig. 1

OR

(b) In Fig. 2, *XAY* is a tangent to the circle centred at *O*. If $\angle ABO = 40°$, then find $\angle BAY$ and $\angle AOB$.

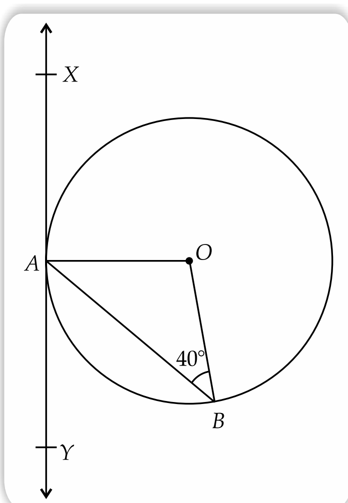
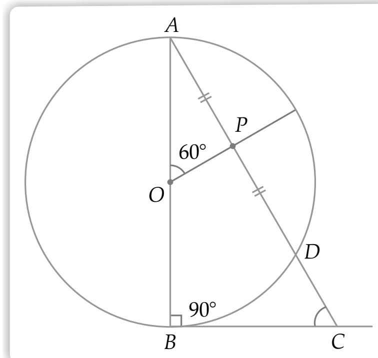


Fig. 2

Ans. (a) Given, OP bisect the chord AD.

$\therefore$ $OP \perp AD$

$\angle P = 90°$ and $\angle B = 90°$



$\angle BOP = 180° - 60° = 120°$

$\angle P = 90°$

$\therefore$ *OP* bisect the chord *AD*, as radius bisect the chord at 90°.

Now, in quad. *BOPC*, applying angle sum property

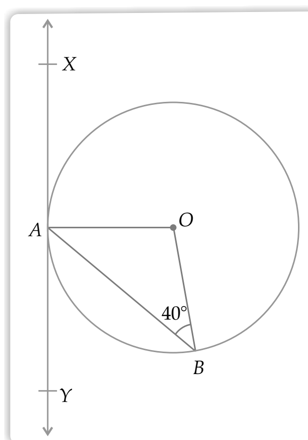
$\angle P + \angle B + \angle O + \angle C = 360°$

or, $90° + 90° + 120° + \angle C = 360°$

or, $\angle C = 360° - 300° = 60°$

OR

(b)



Given, $\angle ABO = 40°$

$\angle XAO = 90°$

(Angle between radius and tangent)

$OA = OB$ (Radii of same circle)

$\Rightarrow$ $\angle OAB = \angle OBA$

$\therefore$ $\angle OAB = 40°$

Now, applying linear pair of angles property, we get

$\angle BAY + \angle OAB + \angle XAO = 180°$

$\Rightarrow$ $\angle BAY + 40° + 90° = 180°$

$\Rightarrow$ $\angle BAY + 130° = 180°$

$\Rightarrow$ $\angle BAY = 180° - 130° = 50°$

Now, in ΔAOB,

$\angle AOB + \angle OAB + \angle OBA = 180°$

or, $\angle AOB + 40° + 40° = 180°$

or, $\angle AOB = 180° - 80° = 100°$

5. If mode of the following frequency distribution is 55, then find the value of x.

Class	0 - 15	15 - 30	30 - 45	45 - 60	60 - 75	75 - 90
Frequency	10	7	x	15	10	12

Ans. Given:

Mode of frequency distribution = 55

So, modal class is 45 – 60.

Lower limit (l) = 45

Class interval (h) = 15

Also, $f_0 = 15, f_1 = x$ and $f_2 = 10$

$$\text{Mode} = l + \left(\frac{f_0 - f_1}{2f_0 - f_1 - f_2}\right) \times h$$

$$\Rightarrow \quad 55 = 45 + \left(\frac{15 - x}{30 - x - 10}\right) \times 15$$

$$\Rightarrow \quad 55 - 45 = \frac{15(15 - x)}{30 - x - 10}$$

$\Rightarrow \quad 10(30 - x - 10) = 225 - 15x$

$\Rightarrow \quad 300 - 10x - 100 = 225 - 15x$

$\Rightarrow \quad 5x = 25$

$\Rightarrow \quad x = 5$

6. Find the sum of first 20 terms of an $A.P.$ whose n^{th} term is given as $a_n = 5 - 2n$.

Ans. Given, $a_n = 5 - 2n$

for $n = 1, a_1 = 5 - 2(1) = 3$

$n = 2, a_2 = 5 - 2(2) = 1$

$\therefore$ Common difference $= 1 - (3) = -2$

Sum of first n terms :

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$\therefore$ Sum of first 20 terms is :

$$S_n = \frac{20}{2}[2(3) + (20 - 1)(-2)]$$

$= 10(6 - 38)$

$= 10 \times (-32) = -320$

Hence, sum of first 20 terms is – 320.

SECTION - B

Question Numbers 7 to 10 carry 3 marks each.

*** 7. Draw two concentric circles of radii 2 cm and 3 cm. From a point on the outer circle, construct a pair of tangents to the inner circle.**

8. In Fig. 3. AB is tower of height 50 m. A man standing on its top, observes two cars on the opposite sides of the tower with angles of depression 30° and 45° respectively. Find the distance between the two cars.

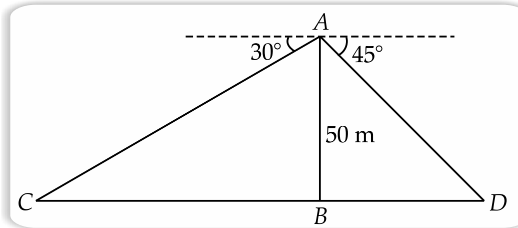


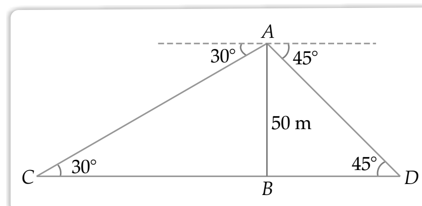
Fig. 3

Ans. In ΔABC, $\angle B = 90°$

$$\tan 30° = \frac{AB}{CB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{50}{CB}$$

$$\Rightarrow \quad CB = 50\sqrt{3} \text{ m}$$



In ΔABD,

$$\tan 45° = \frac{AB}{BD}$$

$$\Rightarrow \quad 1 = \frac{50}{BD}$$

$\Rightarrow \quad BD = 50$ m

$\therefore \quad CD = CB + BD = 50\sqrt{3} + 50$

or, $CD = 50(\sqrt{3} + 1)$

or, $CD = 50(1.732 + 1)$

or, $CD = 50 \times 2.732$

or, $CD = 136.6$ m

9. (a) The mean of the following frequency distribution is 25. Find the value of f.

Class	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Frequency	5	18	15	f	6

OR

(b) Find the mean of the following data using assumed mean method:

Class	0 - 5	5 - 10	10 - 15	15 - 20	20 - 25
Frequency	8	7	10	13	12

* *Out of Syllabus*

Ans. (a) Given, mean = 25

Class Interval	Mid-point x_i	Frequency f_i	$f_i x_i$
0–10	5	5	25
10–20	15	18	270
20–30	25	15	375
30–40	35	f	$35f$
40–50	45	6	270
		$\Sigma f_i = 44 + f$	$\Sigma f_i x_i = 940 + 35f$

$$\therefore \quad \text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\Rightarrow \quad 25 = \frac{940 + 35f}{44 + f}$$

$$\Rightarrow \quad 25 \times 44 + 25f = 940 + 35f$$

$$\Rightarrow \quad 10f = 1100 - 940$$

$$\Rightarrow \quad 10f = 160$$

$$\Rightarrow \quad f = 16$$

OR

(b)

Class Interval	Mid-point (x)	Frequency (f)	$d = x - A$	fd
0–5	2.5	8	– 10	– 80
5–10	7.5	7	– 5	– 35
10–15	12.5 = A	10	0	0
15–20	17.5	13	5	65
20–25	22.5	12	10	120
		$\Sigma f = 50$		$\Sigma fd = 70$

Here, assumed mean, $A = 12.5$

$$\text{Now,} \quad \text{Mean} = A + \frac{\Sigma fd}{\Sigma f}$$

$$= 12.5 + \frac{70}{50}$$

$$= 12.5 + 1.4 = 13.9$$

10. Heights of 50 students of class X of a school are recorded and following data is obtained:

Height (in cm)	130 - 135	135 - 140	140 - 145	145 - 150	150 - 155	155 - 160
Number of Students	4	11	12	7	10	6

Find the median height of the students.

Ans.

Height (in cm)	No. of Students (f)	Cumulative Frequency (cf)
130–135	4	4
135–140	11	15
140–145	12	27 → Median Class
145–150	7	34
150–155	10	44
155–160	6	50
	$N = \Sigma f = 50$	

Since, $N = 50$ is an even number.

So, $\frac{N}{2} = \frac{50}{2} = 25$ and median class is 140 – 145.

$l = 140, h = 5, c = 15, f = 12$ (given)

$$\text{Now,} \quad \text{Median} = l + h\left(\frac{\frac{N}{2} - c}{f}\right)$$

$$= 140 + 5\left(\frac{25 - 15}{12}\right)$$

$$= 140 + \left(\frac{5 \times 10}{12}\right)$$

$$= 140 + 4.167 = 144.167$$

Hence, median height of the students is 144.167 cm.

SECTION - C

Question Numbers 11 to 14 carry 4 marks each.

11. In Fig. 4. PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q meet at a point T. Find the length of TP.

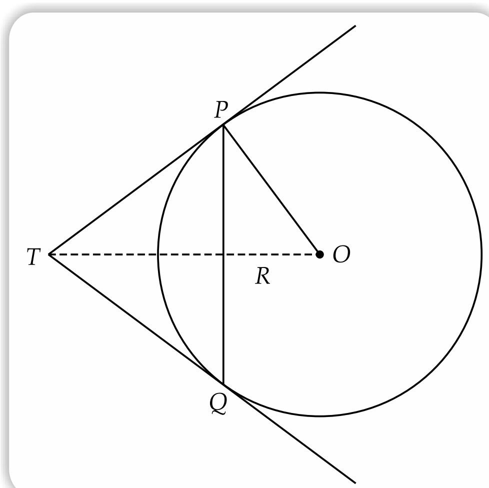
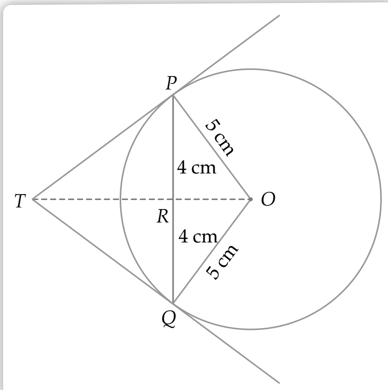


Fig. 4

Ans. Here, TP and TQ are the tangents from point T upon the circle. So, ΔTPQ is an isosceles triangle and TO is the angle bisector of $\angle PTO$.



$\therefore \quad OT \perp PQ$

$\therefore$ OT bisects PQ

$PR = RQ = 4$ cm

Now,
$$OR = \sqrt{OP^2 - PR^2}$$
$$= \sqrt{5^2 - 4^2} = 3 \text{ cm}$$

Now, $\angle TPR + \angle RPO = 90°$ ...(i)

($\because \angle TPO = 90°$ angle between radius and tangent)

and $\angle TPR + \angle PTR = 90°$ ...(ii)

from eqs (i) and (ii), we get

$$\angle RPO = \angle PTR$$

Thus, Right ΔTRP ~ Right ΔPRO

(By AA rule of similarity)

$$\therefore \quad \frac{TP}{PO} = \frac{RP}{RO}$$
$$\Rightarrow \quad \frac{TP}{5} = \frac{4}{3}$$
$$\Rightarrow \quad TP = \frac{20}{3} \text{ cm} = 6.67 \text{ cm}.$$

12. (a) A 2-digit number is such that the product of its digits is 24. If 18 is subtracted from the number, the digits interchange their places. Find the number.

OR

(b) The difference of the squares of two numbers is 180. The square of the smaller number is 8 times the greater number. Find the two numbers.

Ans. (a) Let the ten's digit be x and one's digit be y.

The number will be $10x + y$.

Given, product of digits is 24

$$\therefore \quad xy = 24$$
$$\text{or,} \quad y = \frac{24}{x} \quad \text{...(i)}$$

Given that when 18 is subtracted from the number, the digits interchange their places.

$$\therefore \quad 10x + y - 18 = 10y + x$$
$$\text{or,} \quad 9x - 9y = 18 \quad \text{...(ii)}$$

Substituting y from eq (i) in eq (ii), we get

$$9x - 9\left(\frac{24}{x}\right) = 18$$
$$\text{or,} \quad x - \frac{24}{x} = 2$$
$$\text{or,} \quad x^2 - 24 - 2x = 0$$
$$\text{or,} \quad x^2 - 2x - 24 = 0$$
$$\text{or,} \quad x^2 - 6x + 4x - 24 = 0$$
$$\text{or,} \quad x(x-6) + 4(x-6) = 0$$
$$\text{or,} \quad (x-6)(x+4) = 0$$
$$\text{or,} \quad x - 6 = 0 \text{ and } x + 4 = 0$$
$$\text{or,} \quad x = 6 \text{ and } x = -4$$

Since, the digit cannot be negative, so, $x = 6$

Substiting $x = 6$ in eq (i), we get

$$y = \frac{24}{6} = 4$$
$$\therefore \quad \text{The number} = 10(6) + 4 = 60 + 4$$
$$= 64$$

OR

(b) Let the greater number be x.

The square of the smaller number is 8 times of the greater number $= 8x$

Given, the difference of squares of two numbers is 180.

$$\therefore \quad x^2 - 8x = 180$$
$$\Rightarrow \quad x^2 - 8x - 180 = 0$$
$$\Rightarrow \quad x^2 - 18x + 10x - 180 = 0$$
$$\Rightarrow \quad x(x-18) + 10(x-18) = 0$$
$$\Rightarrow \quad (x-18)(x+10) = 0$$
$$\Rightarrow \quad (x-18) = 0 \text{ or } (x+10) = 0$$
$$\Rightarrow \quad x = 18 \text{ or } x = -10$$

Since, number cannot be negative. So, $x = 18$

Now, square of smaller number

$$= 8x$$
$$= 8 \times 18$$
$$= 144$$
$$\therefore \quad \text{smaller number} = \sqrt{144} = 12$$

Hence, smaller number is 12 and greater number is 18.

13. Case Study-1:

Kite Festival

Kite festival is celebrated in many countries at different times of the year. In India, every year 14th January is celebrated as International Kite Day. On this day many people visit India and participate in the festival by flying various kinds of kites.

The picture given below, show three kites flying together.

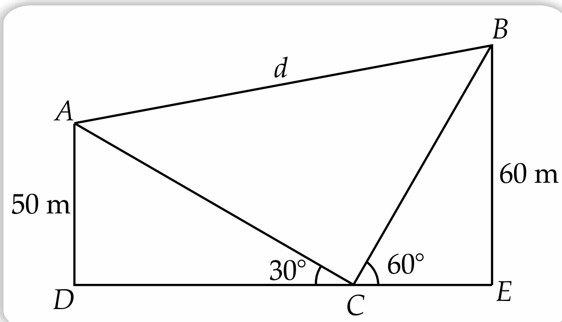


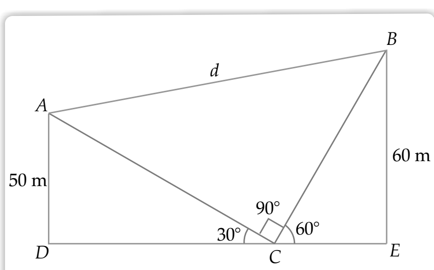
Fig. 5

In Fig. 5, the angles of elevation of two kites (Points A and B) from the hands of a man (Point C) are found to be 30° and 60° respectively. Taking $AD = 50$ m and $BE = 60$ m, find.

(1) the lengths of strings used (take them straight) for kites A and B as shown in the figure. 2

(2) the distance 'd' between these two kites. 2

Ans. Case study-1



(1) In ΔADC, ∠D = 90°

$$\sin 30° = \frac{AD}{AC}$$

$$\therefore \quad \frac{1}{2} = \frac{50}{AC}$$

or, $\quad AC = 100$ m ...(i)

In ΔBEC, $\quad \angle E = 90°$

$$\sin 60° = \frac{BE}{BC}$$

$$\therefore \quad \frac{\sqrt{3}}{2} = \frac{60}{BC}$$

or, $\quad BC = \dfrac{120}{\sqrt{3}} = 40\sqrt{3}$ m ...(ii)

Hence, the length of strings used for kites A and B are 100 m and $40\sqrt{3}$ m, respectively.

(2) Here, $\angle DCA + \angle ACB + \angle BCE = 180°$

(Angles in straight line)

$\therefore \quad 30° + \angle ACB + 60° = 180°$

or, $\quad \angle ACB = 180° - 90° = 90°$

Now, in right ΔACB,

$$AB^2 = AC^2 + BC^2$$

$\Rightarrow \quad d^2 = (100)^2 + (40\sqrt{3})^2$

[from eq (i) and eq (ii)]

$\Rightarrow \quad d^2 = 10{,}000 + 4{,}800$

$\Rightarrow \quad d^2 = 14800$

$\Rightarrow \quad d = 20\sqrt{37}$ cm

Hence, distance between two kites A and B is $20\sqrt{37}$ cm.

14. Case Study-2

A 'circus' is a company of performers who put on shows of acrobats, clowns etc. to entertain people started around 250 years back, in open fields, now generally performed in tents.

One such 'Circus Tent' is shown below.

Fig. 6

The tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of cylindrical part are 9 m and 30 m respectively and height of conical part is 8 m with same diameter as that of the cylindrical part, then find

(1) the area of the canvas used in making the tent; 3

(2) the cost of the canvas bought for the tent at the rate ₹ 200 per sq m, if 30 sq m canvas was wasted during stitching. 1

Ans. Case study-2

(1) For cylinder,

height = 9 m, diameter = 30 m ⇒ radius = $\dfrac{30}{2}$ = 15 m.

For cone,

height = 8 m, radius = 15 m

$\therefore$ slant height,

$l = \sqrt{(8)^2 + (15)^2}$

$= \sqrt{64 + 225}$

$= \sqrt{289}$

$= 17$ m

Area of canvas required = *C.S.A* of cylinder + *C.S.A.* of cone

$= 2\pi rh + \pi rl$

$= \pi r\,(2h + l)$

$= \frac{22}{7} \times 15\ (2 \times 9 + 17)$

$= \frac{22}{7} \times 15 \times 35$

$= 22 \times 15 \times 5$

$= 1650$ sq m.

(2) The cost of the canvas = (Area of canvas required + area of canvas wasted during stitching) × 200

$= (1650 + 30) \times 200$

$= 1680 \times 200$

$=$ ₹ 3,36,000

Term-II, Delhi Set-II—SERIES: PPQQC/2 **Code No. 30/2/2**

Note: Except these, all other questions are from Delhi Set-I

SECTION - A

5. Solve the quadratic equation :

$x^2 - 2ax + (a^2 - b^2) = 0$ for x.

Ans. $x^2 - 2ax + (a^2 - b^2) = 0$

$\Rightarrow (x^2 - 2ax + a^2) - b^2 = 0$

$\Rightarrow (x - a)^2 - b^2 = 0$

$\Rightarrow (x - a + b)(x - a - b) = 0$

$\Rightarrow x - a + b = 0$ or $x - a - b = 0$

$\Rightarrow x = -(-a + b)$ or $x = -(-a - b)$

$\Rightarrow x = a - b$ or $x = a + b$

SECTION - B

9. Two men on either side of a cliff 75 m high observe the angles of elevation of the top of the cliff to be 30° and 60°. Find the distance between the two men.

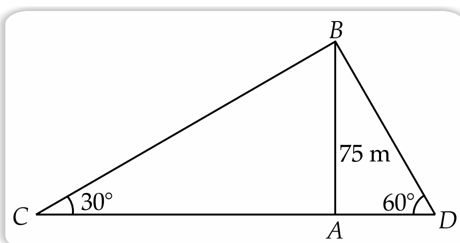
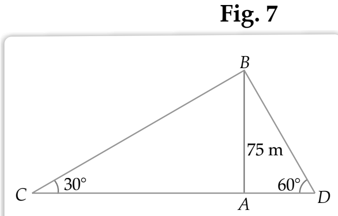


Fig. 7

Ans.



In ΔABC, $\angle A = 90°$

$\tan 30° = \frac{AB}{CA}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{CA}$

$\Rightarrow CA = 75\sqrt{3}$ m

In ΔABD, $\angle A = 90°$

$\tan 60° = \frac{AB}{AD}$

$\Rightarrow \sqrt{3} = \frac{75}{AD}$

$\Rightarrow AD = \frac{75}{\sqrt{3}} = 25\sqrt{3}$

$\therefore CD = CA + AD$

$= 75\sqrt{3} + 25\sqrt{3}$

$= 100\sqrt{3}$

$= 100 \times 1.732$

$= 173.2$ m

*** 10. Construct a pair of tangents to a circle of radius 3 cm which are inclined to each other at an angle of 60°.**

11. (a) The sum of two numbers is 34. If 3 is subtracted from one number and 2 is added to another, the product of these two numbers becomes 260. Find the numbers.

OR

(b) The hypotenuse (in cm) of a right angled triangle is 6 cm more than twice the length of the shortest side. If the length of third side is 6 cm less than thrice the length of shortest side, then find the dimensions of the triangle.

Ans. (a) Let the first number be x and second number be y.

According to question,

$x + y = 34$

$\Rightarrow y = 34 - x$ ...(i)

and $(x - 3)(y + 2) = 260$ ...(ii)

Substituting value of y from eq (i), in eq (ii), we get

$(x - 3)(34 - x + 2) = 260$

$\Rightarrow (x - 3)(36 - x) = 260$

* *Out of Syllabus*

$\Rightarrow \quad 36x - x^2 - 108 + 3x = 260$

$\Rightarrow \quad x^2 - 39x + 368 = 0$

On comparing the above quadratic equation with $ax^2 + bx + c = 0$, we get

$$a = 1, b = -39 \text{ and } c = 368$$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{39 \pm \sqrt{(-39)^2 - 4(1)(368)}}{2 \times 1}$$

$$x = \frac{39 \pm \sqrt{1521 - 1472}}{2}$$

$$= \frac{39 \pm \sqrt{49}}{2} = \frac{39 \pm 7}{2}$$

$$= \frac{39 + 7}{2} \text{ or } \frac{39 - 7}{2}$$

$$x = \frac{46}{2} \text{ or } x = \frac{32}{2}$$

$x = 23$ or $x = 16$

When $x = 23, y = 34 - 23 = 11$

When $x = 16, y = 34 - 16 = 18$

Hence, the numbers will be either 23 and 11 or 16 and 18.

OR

(b) Let the length of the shortest side be x cm.

Then, hypotenuse $= (2x + 6)$ cm

and third side $= (3x - 6)$ cm

By pythagoras theorem, we have

$$(\text{hypotenuse})^2 = (\text{shortest side})^2 + (\text{third side})^2$$

$\Rightarrow \quad (2x + 6)^2 = x^2 + (3x - 6)^2$

$\Rightarrow \quad 4x^2 + 36 + 24x = x^2 + 9x^2 + 36 - 36x$

$\Rightarrow \quad 6x^2 = 60x$

$\Rightarrow \quad 6x^2 - 60x = 0$

$\Rightarrow \quad x^2 - 10x = 0$

$\Rightarrow \quad x(x - 10) = 0$

$\Rightarrow \quad x = 0$ and $x = 10$

Length of the shortest side can't be zero.

So, $x = 10$

i.e., Shortest side $= 10$ cm

hypotenuse $= (2 \times 10 + 6) = 26$ cm

and third side $= (3 \times 10 - 6) = 24$ cm

Term-II, Delhi Set-III—SERIES: PPQQC/2 — *Code No. 30/2/3*

Note: Except these all other questions are from Delhi Set-II

SECTION - A

3. (a) In an *A.P.* if the sum of third and seventh term is zero. Find its 5^{th} term.

OR

(b) Determine the *A.P.* whose third term is 5 and seventh term is 9.

Ans. (a) Given, sum of third and seventh term of *A.P.* is zero.

We know that, n^{th} term of an *A.P.* is

$$T_n = a + (n - 1)d$$

$\therefore \quad T_3 + T_7 = 0$

$\Rightarrow \quad a + 2d + a + 6d = 0$

$\Rightarrow \quad 2a + 8d = 0$

$\Rightarrow \quad a + 4d = 0$

Now, $T_5 = a + (5 - 1)d$

$= a + 4d$

$= 0$

Hence, 5^{th} term of *A.P.* is zero.

OR

(b) Given, $T_3 = 5$ and $T_7 = 9$

We know that, n^{th} term of an *A.P.* is

$$T_n = a + (n - 1)d$$

$\therefore \quad 5 = a + 2d \quad$ ...(i)

and $\quad 9 = a + 6d \quad$ ...(ii)

Eq (i) – Eq (ii), $\quad -4 = -4d \Rightarrow d = 1$

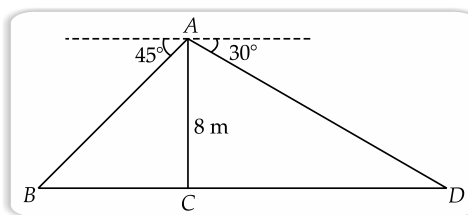
From eq (i), $\quad 5 = a + 2(1)$

$\Rightarrow \quad a = 5 - 2 = 3$

So, required *A.P.* is 3, 5, 7, 9, ...

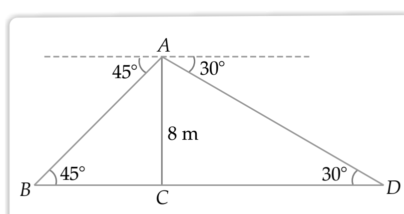
SECTION - B

8. From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45°. If the bridge is at a height of 8 m from the banks, then find the width of the river.



Ans. In ΔABC, $\quad \tan 45° = \dfrac{AC}{BC}$

$\Rightarrow \quad 1 = \dfrac{8}{BC} \Rightarrow BC = 8$ m



In ΔACD, $\quad \tan 30° = \dfrac{AC}{CD}$

$\Rightarrow \quad \dfrac{1}{\sqrt{3}} = \dfrac{8}{CD}$

$\Rightarrow$ $CD = 8\sqrt{3}$ m

Now, $BD = BC + CD$

$= 8 + 8\sqrt{3}$

$= 8(1+\sqrt{3}) = 8(1 + 1.732)$

$= 8 \times 2.732$

$= 21.856$ m

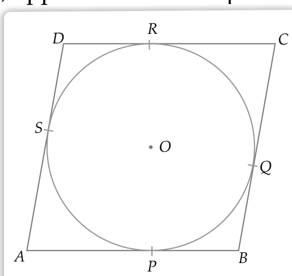
*** 10. Construct a pair of tangents to a circle of radius 4 cm from a point *P* lying outside the circle at a distance of 6 cm from the centre.**

SECTION - C

12. Prove that a parallelogram circumscribing a circle is a rhombus.

Ans. Let *ABCD* be a parallelogram.

Therefore, opposite sides are equal.



$\therefore$ $AB = CD$ ...(i)

$BC = AD$ ...(ii)

Now, $BP = BQ$ (Tangents from point B) ...(iii)

$CR = CQ$ (Tangents from point C) ...(iv)

$DR = DS$ (Tangents from Point D) ...(v)

$AP = AS$ (Tangents from point A) ...(vi)

On adding eqs. (iii), (iv), (v) and (vi), we get

$BP + CR + DR + AP$

$= BQ + CQ + DS + AS$

On re-grouping, we get

$(BP + AP) + (CR + DR)$

$= (BQ + CQ) + (DS + AS)$

$\Rightarrow$ $AB + CD = BC + AD$

$\Rightarrow$ $AB + AB = BC + BC$ [from eqs. (i) and (ii)]

$\Rightarrow$ $2AB = 2BC$

$\Rightarrow$ $AB = BC$

Thus, $AB = BC = CD = DA$

This implies that all the four sides are equal.

Therefore, the parallelogram cicumscribing a circle is a rhombus.

Term-II, Outside Delhi Set-I—SERIES: PPQQD/4 — Code No. 30/4/1

SECTION - A

1. The mode of a grouped frequency distribution is 75 and the modal class is 65-80. The frequency of the class preceding the modal class is 6 and the frequency of the class succeeding the modal class is 8. Find the frequency of the modal class. 2

Ans. Given, Mode = 75

Modal class = 65 – 80

Frequency of the class preceding the modal class, $f_0 = 6$

Frequency of class succeeding the modal class, $f_2 = 8$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$\Rightarrow \quad 75 = 65 + \left(\frac{f_1 - 6}{2f_1 - 6 - 8}\right) \times 15$$

[Here, lower limit of modal class, $l = 65$ and class size = 15]

$$\Rightarrow \quad 10 = \frac{f_1 - 6}{2f_1 - 14} \times 15$$

$\Rightarrow$ $20f_1 - 140 = 15f_1 - 90$

$\Rightarrow$ $5f_1 = 50$

$\Rightarrow$ $f_1 = 10$

Hence, frequency of modal class (f_1) is 10.

2. How many natural numbers are there between 1 and 1000 which are divisible by 5 but not by 2 ? 2

Ans. The natural numbers between 1 and 1000, which are divisible by 5 but not by 2, are :

5, 15, 25, 35, ...995

The above sequence is an *A.P.* with common difference 10.

Using formula, $l = a + (n - 1)d$

$995 = 5 + (n - 1)10$

$$\Rightarrow \quad \frac{990}{10} = n - 1$$

$\Rightarrow$ $n - 1 = 99$

$\Rightarrow$ $n = 100$

Thus, there are 100 terms between 1 and 1000, which are divisible by 5 but not by 2.

3. (a) If the sum of the roots of the quadratic equation $ky^2 - 11y + (k - 23) = 0$ is $\frac{13}{21}$ more than the product of the roots, then find the value of *k*. 2

OR

(b) If $x = -2$ is the common solution of quadratic equations $ax^2 + x - 3a = 0$ and $x^2 + bx + b = 0$, then find the value of a^2b. 2

* *Out of Syllabus*

Ans. **(a)** Given, quadratic equation is $ky^2 - 11y + (k - 23) = 0$

Let the roots of the above quadratic equation be α and β.

Now, Sum of roots,

$$\alpha + \beta = \frac{-(-11)}{k} = \frac{11}{k} \quad \text{...(i)}$$

and Product of roots, $$\alpha\beta = \frac{k-23}{k} \quad \text{...(ii)}$$

According to question,

$$\alpha + \beta = \alpha\beta + \frac{13}{21}$$

$$\therefore \quad \frac{11}{k} = \frac{k-23}{k} + \frac{13}{21}$$

[from eqs. (i) & (ii)]

$$\Rightarrow \quad \frac{11}{k} - \frac{(k-23)}{k} = \frac{13}{21}$$

$$\Rightarrow \quad \frac{11-k+23}{k} = \frac{13}{21}$$

$$\Rightarrow \quad 21(34 - k) = 13k$$

$$\Rightarrow \quad 34k = 714$$

$$\Rightarrow \quad k = 21$$

OR

(b) Given quadratic equations are

$$ax^2 + x - 3a = 0 \quad \text{...(i)}$$

$$x^2 + bx + b = 0 \quad \text{...(ii)}$$

Since, given $x = -2$ is the common solution of above quadratic equation.

$\therefore$ from eq (i),

$$a(-2)^2 + (-2) - 3a = 0$$

$$\Rightarrow \quad 4a - 2 - 3a = 0$$

$$\Rightarrow \quad a = 2$$

From eq (ii),

$$(-2)^2 + b(-2) + b = 0$$

$$\Rightarrow \quad 4 - 2b + b = 0$$

$$\Rightarrow \quad b = 4$$

Now, $$a^2b = (2)^2 \times 4 = 4 \times 4 = 16$$

4. Find the mean of the following frequency distribution: **2**

Class	1 - 5	5 - 9	9 - 13	13 - 17
Frequency	4	8	7	6

Ans.

Class	Frequency	Mid point (x_i)	$f_i x_i$
1 – 5	4	3	12
5 – 9	8	7	56
9 – 13	7	11	77
13 – 17	6	15	90
	$\Sigma f_i = 25$		$\Sigma f_i x_i = 235$

$$\therefore \quad \text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{235}{25} = 9.4$$

5. In Fig. 1. there are two concentric circles with centre *O*. If *ARC* and *AQB* are tangents to the smaller circle from the point *A* lying on the larger circle, find the length of *AC*, if *AQ* = 5 cm. **2**

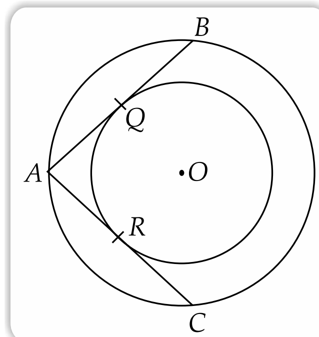
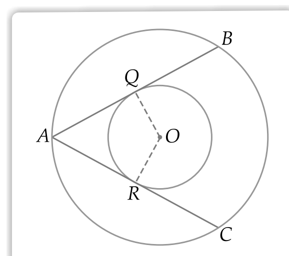


Fig. 1

Ans. Here, *AC* and *AB* are the tangents from external point A to smaller circle.



$$\therefore \quad AC = AB$$

Now, AB is the chord of bigger circle and *OQ* is the perpendicular bisector of chord AB.

$$\therefore \quad AQ = QB$$

or, $$AB = 2AQ$$

or, $$AB = 2(5) = 10 \text{ cm}$$

[$\because$ Given AQ = 5 cm]

$$\therefore \quad AC = 10 \text{ cm}$$

6. (a) The curved surface area of a right circular cylinder is 176 sq cm and its volume is 1232 cu cm. What is the height of the cylinder ? **2**

OR

(b) The largest sphere is carved out of a solid cube of side 21 cm. Find the volume of the sphere. **2**

Ans. **(a)** Given, C.S.A of cylinder = 176 cm^2

$$\therefore \quad 2\pi rh = 176 \quad \text{...(i)}$$

and volume of cylinder = 1232

$$\therefore \quad \pi r^2 h = 1232 \quad \text{...(ii)}$$

On dividing eq (ii) by eq (i), we get

$$\frac{\pi r^2 h}{2\pi rh} = \frac{1232}{176}$$

$$\Rightarrow \quad \frac{r}{2} = \frac{1232}{176}$$

$\Rightarrow \quad r = \frac{1232 \times 2}{176}$

$\Rightarrow \quad r = \frac{2464}{176} = 14$ cm

Now, from eq (i),

$$2\pi(14)\,h = 176$$

$$h = \frac{176 \times 7}{2 \times 22 \times 14}$$

$$= \frac{1232}{616} = 2 \text{ cm}$$

Hence, height of right circular cylinder = 2 cm

OR

(b) The largest sphere that can be carved out of a solid cube of side 21 cm means diameter of sphere will be 21 cm.

Therefore, radius of sphere, $r = \frac{21}{2}$ cm

Now, Volume of sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times \left(\frac{21}{2}\right)^3$$

$$= \frac{4 \times 22 \times 21 \times 21 \times 21}{7 \times 3 \times 2 \times 2 \times 2}$$

$$= 11 \times 21 \times 21$$

$$= 4851 \text{ cm}^3$$

SECTION - B

*** 7. Construct a pair of tangents to a circle of radius 4 cm which are inclined to each other at an angle of 60°. 3**

8. (a) Find the value of 'p' for which the quadratic equation $p(x-4)(x-2) + (x-1)^2 = 0$ has real and equal roots. 3

OR

(b) Had Aarush scored 8 more marks in a Mathematics test, out of 35 marks, 7 times these marks would have been 4 less than square of his actual marks. How many marks did he get in the test ? 3

Ans. (a) Given quadratic equation is

$$p(x-4)(x-2) + (x-1)^2 = 0$$

$\Rightarrow \quad p(x^2 - 4x - 2x + 8) + (x^2 + 1 - 2x) = 0$

$\Rightarrow \quad px^2 - 6px + 8p + x^2 + 1 - 2x = 0$

$\Rightarrow \quad x^2(p+1) - 2x(3p+1) + (8p+1) = 0$

Comparing the above equation with $ax^2 + bx + c = 0$, we get

$a = p + 1$, $b = -2(3p+1)$ and $c = 8p + 1$

For real and equal roots

$$D = 0 \text{ i.e., } b^2 - 4ac = 0$$

$\therefore \quad [-2(3p+1)]^2 - 4(p+1)(8p+1) = 0$

$\Rightarrow \quad 4(3p+1)^2 - 4(8p^2 + 9p + 1) = 0$

$\Rightarrow \quad 4(9p^2 + 1 + 6p) - 32p^2 - 36p - 4 = 0$

$\Rightarrow \quad 36p^2 + 4 + 24p - 32p^2 - 36p - 4 = 0$

$\Rightarrow \quad 4p^2 - 12p = 0$

$\Rightarrow \quad 4p(p-3) = 0$

$\Rightarrow \quad p = 0$ or $p = 3$

Hence, for $p = 0$ or $p = 3$, the given quadratic equation has real and equal roots.

OR

(b) Let the actual marks be x.

According to question,

$$7(x+8) = x^2 - 4$$

$\Rightarrow \quad 7x + 56 = x^2 - 4$

$\Rightarrow \quad x^2 - 7x - 60 = 0$

$\Rightarrow \quad x^2 - 12x + 5x - 60 = 0$

$\Rightarrow \quad x(x-12) + 5(x-12) = 0$

$\Rightarrow \quad (x-12)(x+5) = 0$

$\Rightarrow \quad x - 12 = 0$ or $x + 5 = 0$

$\Rightarrow \quad x = 12$ or $x = -5$

$\Rightarrow \quad x = 12$

[$\because$ Marks can't be negative]

Hence, Aarush scored 12 marks in Mathematics test.

9. An aeroplane when flying at a height of 3125 m from the ground passes vertically below another plane at an instant when the angles of elevation of the two planes from the same point on the ground are 30° and 60° respectively. Find the distance between the two planes at that instant. 3

Ans. Let C and D be the two aeroplanes and A be the point of observation. Then,

$\angle CAB = 30^\circ$, $\angle DAB = 60^\circ$, $BC = 3125$ m

Let $DC = y$ m, $AB = x$ m

In right ΔABC, $\angle B = 90^\circ$

$$\tan 30^\circ = \frac{BC}{AB}$$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{3125}{AB}$

$\Rightarrow \quad AB = 3125\sqrt{3}$ m ...(i)

In right ΔABD, $\angle B = 90^\circ$

$$\tan 60^\circ = \frac{BD}{AB}$$

$\Rightarrow \quad \sqrt{3} = \frac{y + 3125}{3125\sqrt{3}}$ [from eq. (i)]

$\Rightarrow \quad 3125 \times 3 = y + 3125$

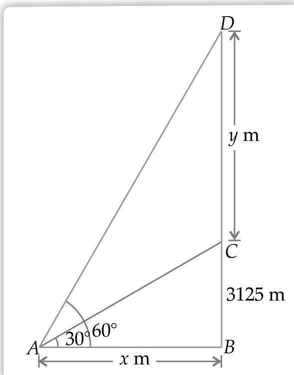
$\Rightarrow \quad y = 3125(3-1)$

$\Rightarrow \quad y = 2 \times 3125$

$\Rightarrow \quad y = 6250$ m

* *Out of Syllabus*

Therefore, the distance between two planes is 6250 m.



10. If the last term of an *A.P.* of 30 terms is 119 and the 8th term from the end (towards the first term) is 91, then find the common difference of the *A.P.* Hence, find the sum of all the terms of the A.P. **3**

Ans. Given, last term, $l = 119$

No. of terms in $AP = 30$

8^{th} term from the end = 91

Let d be the common difference and assume that the first terms of AP is 119 (from the end)

Since, n^{th} term of AP is

$$a_n = l + (n-1)\,d$$

$\therefore$ $a_8 = 119 + (8-1)\,d$

$\Rightarrow$ $91 = 119 + 7d$

$\Rightarrow$ $7d = 91 - 119$

$\Rightarrow$ $7d = -28$

$\Rightarrow$ $d = -4$

Now, this common difference is from the end of *A.P.*

So, common difference from the beginning $= -d$

$$= -(-4) = 4$$

Thus, common difference of the AP is 4.

Now, using formula

$$l = a + (n-1)\,d$$

$\Rightarrow$ $119 = a + (30-1)\,4$

$\Rightarrow$ $119 = a + 116$

$\Rightarrow$ $a = 119 - 116$

$\Rightarrow$ $a = 3$

Hence, using formula for sum of n terms of an AP.

i.e., $$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{30} = \frac{30}{2}[2 \times 3 + (30-1) \times 4]$$

$$= 15\,(6 + 29 \times 4)$$

$$= 15\,(6 + 116)$$

$$= 15 \times 122$$

$$= 1830$$

Therefore, sum of 30 terms of an AP is 1830.

SECTION - C

11. (a) In fig. 2 if a circle touches the side QR of ΔPQR at S and extended sides PQ and PR at M and N, respectively, then **4**

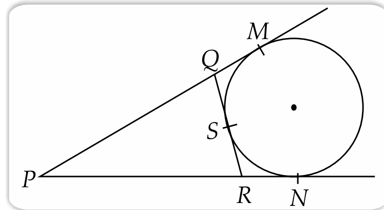


Fig. 2

Prove that $PM = \frac{1}{2}(PQ + QR + PR)$

OR

(b) In Fig. 3, a triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 6 cm and 8 cm respectively. If the area of ΔABC is 84 cm^2, find the lengths of sides AB and AC. **4**

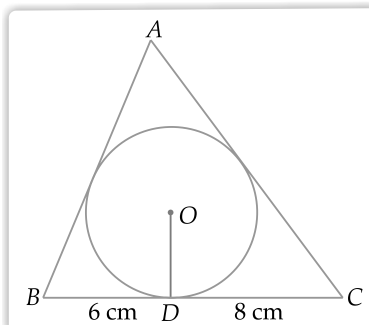
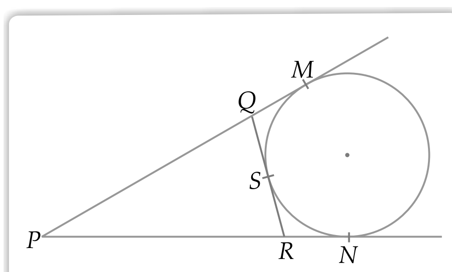


Fig. 3

Ans. (a) Given: A circle is touching a side QR of ΔPQR at point S.



PQ and PR are produced at M and N respectively.

To prove: $PM = \frac{1}{2}(PQ + QR + PR)$

Proof: $PM = PN$ ...(i)

* *Out of Syllabus*

(Tangents drawn from an external point P to a circle are equal)

$$QM = QS \quad ...(ii)$$

(Tangents drawn from an external point Q to a circle are equal)

$$RS = RN \quad ...(iii)$$

(Tangents drawn from an external point R to a circle are equal)

Now,

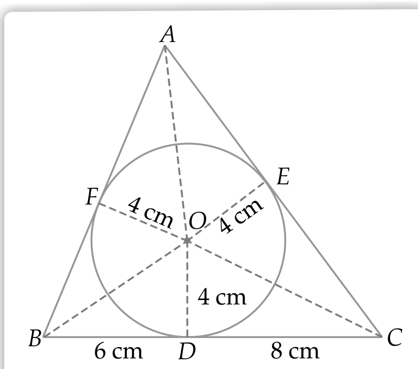
$$2\,PM = PM + PM$$
$$= PM + PN \quad \text{[from eqs. (i)]}$$
$$= (PQ + QM) + (PR + RN)$$
$$= PQ + QS + PR + RS \quad \text{[from eqs. (i) \& (ii)]}$$
$$= PQ + (QS + SR) + PR$$
$$= PQ + QR + PR$$

$$\therefore \quad PM = \frac{1}{2}(PQ + QR + PR)$$

Hence Proved

OR

(b)



Given, $BD = 6$ cm, $DC = 8$ cm

Here, $BD = BF$ and $DC = CE$

[Tangents drawn from external point to a circle are equal]

$\therefore$ $BF = 6$ cm and $CE = 8$ cm

Let $AF = x = AE$

[Tangents drawn from external point A to the circle are equal]

In ΔABC,

$$a = BC = BD + DC$$
$$= 6 + 8$$
$$= 14 \text{ cm}$$
$$b = AC = CE + AE$$
$$= (8 + x) \text{ cm}$$
$$c = AB = BF + AF$$
$$= (6 + x) \text{ cm}$$

Now,

$$s = \frac{a+b+c}{2}$$
$$= \frac{14+(8+x)+(6+x)}{2}$$
$$= \frac{28+2x}{2}$$
$$= (14 + x) \text{ cm}$$

$\therefore$ Area of ΔABC

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$84 = \sqrt{(14+x)(14+x-14)(14+x-8-x)(14+x-6-x)}$$
$$84 = \sqrt{x(14+x)(6)(8)}$$
$$84 = \sqrt{48x(x+14)} \text{ cm}^2 \quad ...(i)$$
$$\sqrt{48x(x+14)} = 84$$

On squaring both sides, we get

$$48x(x + 14) = 84 \times 84$$
$$\Rightarrow \quad 4x(x + 14) = 84 \times 7$$
$$\Rightarrow \quad x^2 + 14x - 147 = 0$$
$$\Rightarrow \quad x^2 + 21x - 7x - 147 = 0$$
$$x(x+21) - 7(x + 21) = 0$$
$$(x + 21)(x - 7) = 0$$

So, $x = 7$, or $x = -21$(rejected as – ve)

Hence, $x = 7$

Therefore,

$$AB = c = 6 + x = 6 + 7 = 13 \text{ cm}$$
$$AC = b = 8 + x = 8 + 7 = 15 \text{ cm}$$

12. From the top of an 8 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower. (Take $\sqrt{3}$ = 1.732). 4

Ans. Let $BE = h$ m

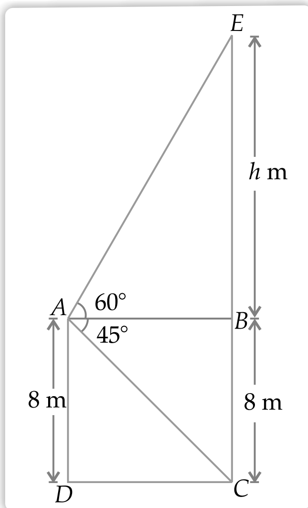
In ΔABC, $\angle B = 90°$

$$\tan 45^o = \frac{BC}{AB}$$
$$\Rightarrow \quad 1 = \frac{8}{AB}$$
$$\Rightarrow \quad AB = 8 \text{ m} \quad ...(i)$$

In ΔABE, $\angle B = 90°$

$$\tan 60^o = \frac{BE}{AB}$$
$$\Rightarrow \quad \sqrt{3} = \frac{h}{AB}$$

$\Rightarrow \qquad h = AB\sqrt{3}$ m



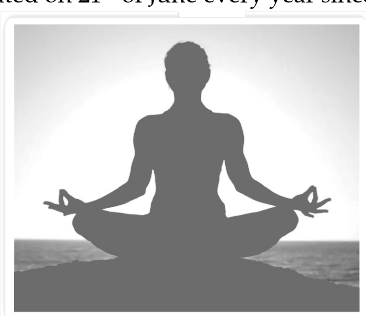
$h = 8\sqrt{3}$ m [Using eqn. (i)]

So, height of the tower

$$\begin{aligned} CE &= BC + BE \\ &= 8 + h \\ &= 8 + 8\sqrt{3} \text{ (h = 8 m)} \\ &= 8(1 + \sqrt{3}) \\ &= 8(1 + 1.732) \\ &= 8 \times 2.732 \\ &= 21.856 \text{ m} \end{aligned}$$

Case Study-I

13. Yoga is an ancient practice which is a form of meditation and exercise. By practising yoga, we not even make our body healthy but also achieve inner peace and calmness. The International Yoga Day is celebrated on 21st of June every year since 2015.



To promote Yoga, Green park society in Pune organised a 7-day Yoga camp in their society. The number of people of different age groups who enrolled for this camp is given as follows:

Age Group	**15 - 25**	**25 - 35**	**35 - 45**	**45 - 55**	**55 - 65**	**65 - 75**	**75 - 85**
Number of People	**8**	**10**	**15**	**25**	**40**	**24**	**18**

Based on the above, find the following:

(a) Find the median age of people enrolled for the camp. 2

(b) If x more people of age group 65 – 75 had enrolled for the camp, the mean age would have been 58. Find the value of x. 2

Ans. (a)

Age Group	No. of People (f)	cf
15 – 25	8	8
25 – 35	10	18
35 – 45	15	33
45 – 55	25	58
55 – 65	40	98
65 – 75	24	122
75 – 85	18	140
	$\Sigma f = 140$	

Here,

$$N = \Sigma f = 140$$

So, $$\frac{N}{2} = 70$$

Therefore, median class = 55 – 65

Lower limit of median class, $l = 55$

Class size, $h = 10$

Cumulative frequency of preceding class, $cf = 58$

Frequency of median class, $f = 40$

$$\therefore \quad \text{Median} = l + \frac{\left(\frac{N}{2} - cf\right)}{f} \times h$$

$$= 55 + \left(\frac{70-58}{40}\right) \times 10$$

$$= 55 + \frac{12}{4}$$

$$= 55 + 3$$

$$= 58$$

Thus, the median age of people enrolled for the camp is 58.

(b)

Age Group	Mid point x_i	frequency (f_i)	$f_i x_i$
15 – 25	20	8	160
25 – 35	30	10	300
35 – 45	40	15	600
45 – 55	50	25	1250
55 – 65	60	40	2400
65 – 75	70	24 + x	1680+ 70x
75 – 85	80	18	1440
		$\Sigma f_i = 140 + x$	$\Sigma f_i x_i = 7830+70x$

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\Rightarrow \quad 58 = \frac{7830 + 70x}{140 + x}$$

$$\Rightarrow \quad 58\,(140 + x) = 7830 + 70x$$

$$\Rightarrow \quad 8120 + 58x = 7830 + 70x$$

$$\Rightarrow \quad 12x = 290$$

$$\Rightarrow \quad x = 24.16 \sim 24 \qquad \text{(Approx.)}$$

Case Study-II

14. Khurja is a city in the Indian state of Uttar Pradesh famous for the pottery. Khurja pottery is traditional Indian pottery work which has attracted Indians as well as foreigners with a variety of tea-sets, crockery and ceramic tile works. A huge portion of the ceramics used in the country is supplied by Khurja and is also refered as ''The Ceramic Town''.

One of the private schools of Bulandshahr organised an Educational Tour for class 10 students to Khurja. Students were very excited about the trip. Following are the few pottery objects of Khurja.

Students found the shapes of the objects very interesting and they could easily relate them with mathematical shapes viz sphere, hemisphere, cylinder etc. Maths teacher who was accompanying the students asked following questions :

(a) The internal radius of hemispherical bowl (filled completely with water) in I is 9 cm and radius and height of cylindrical jar in II is 1.5 cm and 4 cm respectively. If the hemispherical bowl is to be emptied in cylindrical jars, then how many cylindrical jars are required ? **2**

(b) If in the cylindrical jar full of water, a conical funnel of same height and same diameter is immersed, then how much water will flow out of the jar ? **2**

Ans. (a) Given, radius by hemispherical bowl, r_1 = 9 cm
radius of cylindrical jar, r_2 = 1.5 cm
height of cylindrical jar, h_2 = 4 cm
Now,

$$\text{Volume of hemispherical bowl} = \frac{2}{3}\pi r_1^3$$

$$= \frac{2}{3}\pi(9)^3$$

and

$$\text{Volume of cylindrical jar} = \pi r_2^2 h_2$$

$$= \pi\,(1.5)^2 \times 4$$

Required number of cylindrical jar

$$= \frac{\text{Volume of hemispherical bowl}}{\text{Volume of cylindrical jar}}$$

$$= \frac{\frac{2}{3}\pi\,(9)^3}{\pi\,(1.5)^2 \times 4}$$

$$= \frac{2 \times 9 \times 9 \times 9}{3 \times 1.5 \times 1.5 \times 4}$$

$$= \frac{3 \times 9 \times 9 \times 10 \times 10}{15 \times 15 \times 2}$$

$$= \frac{24{,}300}{450}$$

$$= 54$$

Hence, 54 cylindrical jars are required.

(b) Volume of water flow out of the jar

$$= \text{Volume of conical funnel}$$

$$= \frac{1}{3}\pi r_2^2 h_2$$

$$= \frac{1}{3} \times \frac{22}{7} \times (1.5)^2 \times 4$$

$$= \frac{1}{3} \times \frac{22}{7} \times 1.5 \times 1.5 \times 4$$

$$= \frac{22 \times 15 \times 15 \times 4}{3 \times 7 \times 10 \times 10}$$

$$= \frac{19800}{2100} = 9.43 \text{ cubic cm}$$

Therefore, water flow out of the jar is 9.43 cubic cm.

Term-II, Outside Delhi Set-II—SERIES: PPQQD/4 *Code No. 30/4/2*

Note: Except these, all other questions are from Delhi Set-I

SECTION - A

4. If the first term of an A.P. is 5, the last term is 15 and the sum of first *n* terms is 30, then find the value of *n*. **2**

Ans.

$$a = 5$$

$$T_n = l = 15$$

$$S_n = 30$$

$$n = ?$$

$$S_n = \frac{n}{2}(a + l)$$

$$\Rightarrow \quad 30 = \frac{n}{2}(5 + 15)$$

$$\Rightarrow \quad 60 = n \times 20$$

$$\Rightarrow \quad 3 = n$$

5. For the following frequency distribution, find the mode: **2**

Class	25 – 30	30 – 35	35 – 40	40 – 45	45 – 50
Frequency	12	5	14	8	9

Ans. Here,

Maximum frequency is 14. So, modal class is 35 – 40.

lower limit of modal class, $l = 35$

Modal class size, $h = 5$

frequency of class preceding the modal class, $f_0 = 5$

frequency of modal class, $f_1 = 14$

frequency of class suceeding the modal class, $f_2 = 8$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 35 + \left(\frac{14 - 5}{2 \times 14 - 5 - 8}\right) \times 5$$

$$= 35 + \frac{9 \times 5}{15}$$

$$= 35 + 3 = 38$$

6. If the mean of the following frequency distribution is 18, then find the missing frequency '*f*'. **2**

Class	11 – 13	13 – 15	15 – 17	17 – 19	19 – 21	21 – 23	23 – 25
Frequency	**3**	**6**	**9**	**13**	***f***	**5**	**4**

Ans.

Class	Mid point x_i	Frequency (f_i)	$f_i x_i$
11 – 13	12	3	36
13 – 15	14	6	84
15 – 17	16	9	144
17 – 19	18	13	234
19 – 21	20	f	$20f$
21 – 23	22	5	110
23 – 25	24	4	96
		$\Sigma f_i = 40 + f$	$\Sigma f_i x_i = 704 + 20f$

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\therefore \quad 18 = \frac{704 + 20f}{40 + f}$$

[∵ Given, mean = 18]

$$\Rightarrow \quad 18(40 + f) = 704 + 20f$$

$$\Rightarrow \quad 720 + 18f = 704 + 20f$$

$$\Rightarrow \quad 2f = 16$$

$$\Rightarrow \quad f = 8$$

So, missing frequency f is 8.

SECTION - B

9. There is a small island in the middle of a 100 m wide river and a tall tree stands on the island. *P* and *Q* are points directly opposite to each other on two banks and in line with the tree. If the angles of elevation of the top of the tree from *P* and *Q* are respectively 30° and 45°, find the height of the tree. (Use $\sqrt{3} = 1.732$) **3**

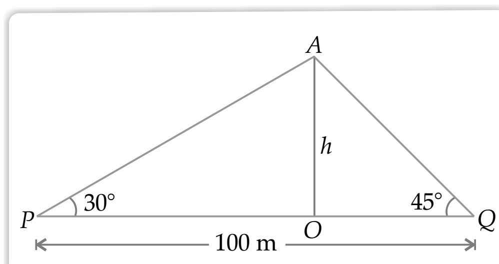
Ans. Let *OA* be the tree of height *h* m.

In Δ*POA*, $\angle O = 90°$

$$\tan 30^\circ = \frac{OA}{OP}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{OP}$$

$$\Rightarrow \quad OP = \sqrt{3}\,h \qquad \text{...(i)}$$



In Δ*QOA*, $\angle O = 90°$

$$\tan 45^\circ = \frac{OA}{OQ}$$

$$\Rightarrow \quad 1 = \frac{h}{OQ}$$

$$\Rightarrow \quad OQ = h \qquad \text{...(ii)}$$

Adding eq (i) and (ii), we get

$$OP + OQ = \sqrt{3}\,h + h$$

$$\Rightarrow \quad PQ = h(\sqrt{3} + 1)$$

$$\Rightarrow \quad 100 = h(\sqrt{3} + 1)$$

$$\Rightarrow \quad h = \frac{100}{\sqrt{3} + 1}$$

$$\Rightarrow \quad h = \frac{100(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$$

$$\Rightarrow \quad h = \frac{100(\sqrt{3}-1)}{2}$$

$$\Rightarrow \quad h = 50(1.732 - 1)$$

$$\Rightarrow \quad h = 50 \times 0.732$$

$$\Rightarrow \quad h = 36.6 \text{ m}$$

Thus, height of the tree is 36.6 m.

10. In an *A.P.*, the sum of first n terms is $\frac{n}{2}(3n + 5)$. Find the 25th term of the *A.P.* **3**

Ans. Given,

$$S_n = \frac{n}{2}(3n+5)$$

$$\therefore \quad S_{n-1} = \frac{n-1}{2}[3(n-1)+5]$$

or $$S_{n-1} = \frac{n-1}{2}(3n+2)$$

Since,

$$a_n = S_n - S_{n-1}$$

$$= \frac{n}{2}(3n+5) - \frac{n-1}{2}(3n+2)$$

$$= \frac{3n^2}{2} + \frac{5n}{2} - \frac{3n(n-1)}{2} - \frac{2(n-1)}{2}$$

$$= \frac{3n^2}{2} + \frac{5n}{2} - \frac{3n^2}{2} + \frac{3n}{2} - n + 1$$

$$= \frac{8n}{2} - n + 1$$

$$= 4n - n + 1$$

$$= 3n + 1$$

Now, $a_{25} = 3(25) + 1$

or, $a_{25} = 75 + 1 = 76$

Thus, 25th term of *A.P.* is 76.

Term-II, Outside Delhi Set-III—SERIES: PPQQD/4 ***Code No. 30/4/3***

Note: Except these, all other questions are from Delhi Set-II

SECTION - A

1. (a) Find the value of 'k' for which the quadratic equation $2kx^2 - 40x + 25 = 0$ has real and equal roots. **2**

OR

(b) Solve for x : $\frac{5}{2}x^2 + \frac{2}{5} = 1 - 2x$.

Ans. (a) Given quadratic equation is

$$2kx^2 - 40x + 25 = 0$$

On comparing the above equation with

$ax^2 + bx + c = 0$, we get

We get,

$$a = 2k, b = -40, c = 25$$

For real and equal roots, $D = 0$

i.e., $b^2 - 4ac = 0$

or, $(-40)^2 - 4(2k)(25) = 0$

$\Rightarrow$ $1600 - 200k = 0$

$\Rightarrow$ $200k = 1600$

$\Rightarrow$ $k = 8$

OR

(b) Given, quadratic equation is

$$\frac{5}{2}x^2 + \frac{2}{5} = 1 - 2x$$

$\Rightarrow$ $25x^2 + 4 = 10(1 - 2x)$

$\Rightarrow$ $25x^2 + 20x - 6 = 0$

By using quadratic formula,

i.e., $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here, $a = 25, b = 20$ and $c = -6$

$$\therefore \quad x = \frac{-20 \pm \sqrt{(20)^2 - 4(25)(-6)}}{2 \times 25}$$

$$= \frac{-20 \pm \sqrt{400 + 600}}{50}$$

$$= \frac{-20 \pm 10\sqrt{10}}{50}$$

$$x = \frac{-2 \pm \sqrt{10}}{5}$$

4. Find the sum of all 11 terms of an *A.P.* whose 6th term is 30. **2**

Ans. Given,

6th term of $A.P = 30$

or, $a_6 = 30$

or, $a + (6-1)d = 30$

or, $a + 5d = 30$ ...(i)

Since,

Sum of n terms of A.P. is

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore \quad S_{11} = \frac{11}{2}[2a + (11-1)d]$$

$$= \frac{11}{2}(2a + 10d)$$

$$= \frac{11 \times 2}{2}(a + 5d)$$

$= 11 \times 30$ [from eq (i)]

$= 330$

5. Find the median of the following distribution : 2

Marks	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
Number of students	5	8	20	15	7	5

Ans.

Marks	No. of students (f)	cf
0 – 10	5	5
10 – 20	8	13
20 – 30	20	33
30 – 40	15	48
40 – 50	7	55
50 – 60	5	60
	$\Sigma f = 60$	

Here, $N = \Sigma f = 60$

$$\therefore \quad \frac{N}{2} = \frac{60}{2} = 30$$

So, median class is 20 – 30.

lower limit of median class,

$l = 20$

Class size, $h = 10$

cumulative frequency of preceding class,

$cf = 13$

frequency of median class,

$f = 20$

$$\therefore \quad \text{Median} = l + \frac{\left(\frac{N}{2} - cf\right)}{f} \times h$$

$$= 20 + \left(\frac{\frac{60}{2} - 13}{20}\right) \times 10$$

$$= 20 + \frac{17}{2}$$

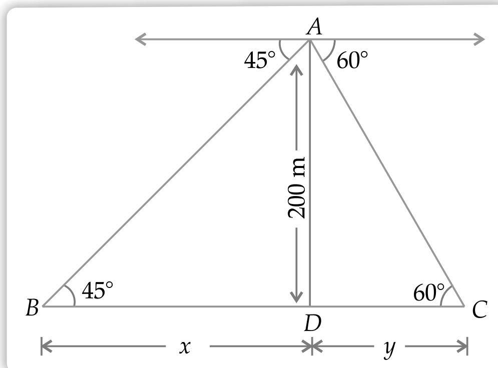
$= 20 + 8.5$

$= 28.5$

SECTION - B

7. An aeroplane at an altitude of 200 metres observes the angles of depression of opposite points on the two banks of a river to be 45° and 60°. Find the width of the river (Use $\sqrt{3}$ = 1.732) 3

Ans.



Let the position of aeroplane be A; B and C be two points on the two banks of a river such that the angles of depression at B and C are 45^o and 60^o respectively.

Let $BD = x$ m, $CD = y$ m

Given, $AD = 200$ m

In ΔADB, $\angle D = 90°$

$$\tan 45^o = \frac{AD}{BD}$$

$$\Rightarrow \quad 1 = \frac{200}{x}$$

$$\Rightarrow \quad x = 200 \text{ m} \quad \text{...(i)}$$

In ΔADC, $\angle D = 90°$

$$\tan 60^o = \frac{AD}{CD}$$

$$\Rightarrow \quad \sqrt{3} = \frac{200}{y}$$

$$\Rightarrow \quad y = \frac{200}{\sqrt{3}}$$

$$\Rightarrow \quad y = \frac{200\sqrt{3}}{3} \quad \text{...(ii)}$$

On adding eqs. (i) & (ii), we get

$$x + y = 200 + \frac{200\sqrt{3}}{3}$$

$$= \frac{600 + 200\sqrt{3}}{3}$$

$$= \frac{200(3 + \sqrt{3})}{3}$$

$$= \frac{200(3 + 1.732)}{3}$$

$$= \frac{200 \times 4.732}{3}$$

$$= \frac{946.4}{3} = 315.4 \text{ m}$$

Hence, width of the river is 315.4 m.

8. The sum of the first three terms of an *A.P.* is 33. If the product of first and third term exceeds the second term by 29, find the *A.P.* **3**

Ans. Let first three terms of *A.P.* be $a - d, a, a + d$.

Given, $a - d + a + a + d = 33$

$\Rightarrow \quad 3a = 33$

$\Rightarrow \quad a = 11 \quad$ ...(i)

and $\quad (a - d)(a + d) = a + 29$

$\Rightarrow \quad a^2 - d^2 = a + 29$

$\Rightarrow \quad (11)^2 - d^2 = 11 + 29 \quad$ [from eq (i)]

$\Rightarrow \quad 121 - d^2 = 40$

$\Rightarrow \quad d^2 = 81$

$\Rightarrow \quad d = \pm 9$

When, $a = 11$ and $d = 9$

Then, *A.P.* is 2, 11, 20.....

When, $a = 11$ and $d = -9$

Then, *A.P.* is 20, 11, 2.....

*** 10. Construct a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60°.** **3**

❑❑

* *Out of Syllabus*