Solved Paper 2013
Mathematics
Class-XII

Time : 3 Hours **Max. Marks : 100**

General Instructions:

(i) All questions are compulsory.
(ii) The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.
(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
(iv) There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

Delhi Set I **Code No. 2/1/1**

SECTION - A

Question numbers 1 to 10 carry 1 mark each.

1. Write the principal value of $\left[\cos^{-1}\frac{\sqrt{3}}{2}+\cos^{-1}\left(-\frac{1}{2}\right)\right]$

Sol. $\left[\cos^{-1}\frac{\sqrt{3}}{2}+\cos^{-1}\left(-\frac{1}{2}\right)\right]$

$$= \frac{\pi}{6}+\pi-\frac{\pi}{3}$$
$$= \frac{5\pi}{6}$$

2. Write the value of the following:

$$\tan^{-1}\left(\frac{a}{b}\right)-\tan^{-1}\left(\frac{a-b}{a+b}\right)$$

Sol. $\tan^{-1}\left(\frac{a}{b}\right)-\tan^{-1}\left(\frac{a-b}{a+b}\right)$

$$= \tan^{-1}\left(\frac{a}{b}\right)-\tan^{-1}\left(\frac{1-\frac{b}{a}}{1+\frac{b}{a}}\right)$$
$$= \tan^{-1}\left(\frac{a}{b}\right)-\left[\tan^{-1}1-\tan^{-1}\left(\frac{b}{a}\right)\right]$$
$$= \tan^{-1}\left(\frac{a}{b}\right)-\left[\frac{\pi}{4}-\cot^{-1}\left(\frac{a}{b}\right)\right]$$
$$= \tan^{-1}\left(\frac{a}{b}\right)+\cot^{-1}\left(\frac{a}{b}\right)-\frac{\pi}{4}$$
$$= \frac{\pi}{2}-\frac{\pi}{4} = \frac{\pi}{4}$$

3. If A is a square matrix of order 3 such that |adj A| = 64, find |A|.

Sol. $|adjA| = 64$

$|adjA| = |A|^{n-1}$

where n is the order of matrix

$|A|^{3-1} = 64$

$\Rightarrow \quad |A| = 8$

4. If $\begin{vmatrix} 2x & x+3 \\ 2(x+1) & x+1 \end{vmatrix} = \begin{vmatrix} 1 & 5 \\ 3 & 3 \end{vmatrix}$, then write the value of x.

Sol. $\begin{vmatrix} 2x & x+3 \\ 2(x+1) & x+1 \end{vmatrix} = \begin{vmatrix} 1 & 5 \\ 3 & 3 \end{vmatrix}$

$= 2x(x + 1) - 2(x + 1)(x + 3)$
$= 3 - 15 = -12$
$= 2x^2 + 2x - 2x^2 - 8x - 6 + 12 = 0$
$= -6x + 6 = 0$
$= x = 1$

5. If $2\begin{vmatrix} 1 & 3 \\ 0 & x \end{vmatrix} + \begin{vmatrix} y & 0 \\ 1 & 2 \end{vmatrix} = \begin{vmatrix} 5 & 6 \\ 1 & 8 \end{vmatrix}$, then write the value of $(x + y)$.

Sol. $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix}+\begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$\begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix}+\begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$\begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$\therefore \quad 2 + y = 5$
$\Rightarrow \quad y = 3$
$2x + 2 = 8$
$\Rightarrow \quad x = 3$
$x + y = 3 + 3 = 6$
$\therefore \quad x + y = 6$

6. The total cost C(x) associated with provision of free mid-day meals to x students of a school in primary classes is given by
$C(x) = 0.005x^3 - 0.02x^2 + 30x + 50$
If the marginal cost is given by rate of change $\frac{dC}{dx}$ of total cost, write the marginal cost of food for 300 students. What value is shown here ?

Sol. $C(x) = 0.005x^3 - 0.02x^2 + 30x + 50$

$$\text{Marginal cost} = \frac{dC}{dx}$$
$$= 0.015x^2 - 0.04x + 30 + 0$$
$$\frac{dC}{dx} = 0.015x^2 - 0.04x + 30$$
$$\left(\frac{dC}{dx}\right)_{x=300} = 0.015(300)^2 - 0.04\,(300) + 30$$
$$= 1350 - 12 + 30$$
$$= 1368$$

7. Write the degree of the differential equation :

$$\left(\frac{dy}{dx}\right)^4 + 3y\frac{d^2y}{dx^2} = 0$$

Sol. $$\left(\frac{dy}{dx}\right)^4 + 3y\frac{d^2y}{dx^2} = 0$$

Degree of the differential
derivative i.e., = 1 equation is power of its highest

8. Write the value of λ so that the vectors $\vec{a} = 2\hat{i}+\lambda\hat{j}+\hat{k}$ and $\vec{b} = \hat{i}-2\hat{j}+3\hat{k}$ are perpendicular to each other.

Sol. $$\vec{a} = 2\hat{i}+\lambda\hat{j}+\hat{k}$$
$$\vec{b} = \hat{i}-2\hat{j}+3\hat{k}$$

$\vec{a} \perp \vec{b}$

$\therefore$ $$\vec{a}.\vec{b} = 0$$
$$(2\hat{i}+\lambda\hat{j}+\hat{k}).(\hat{i}-2\hat{j}+3\hat{k}) = 0$$
$$2 - 2\lambda + 3 = 0$$
$$\lambda = \frac{5}{2}$$

9. Write the projection of the vector $7\hat{i}+\hat{j}-4\hat{k}$ on the vector $2\hat{i}+6\hat{j}+3\hat{k}$.

Sol. $$\vec{a} = 7\hat{i}+\hat{j}-4\hat{k}$$
$$\vec{b} = 2\hat{i}+6\hat{j}+3\hat{k}$$

Projection of the vector $\vec{a}$ on $\vec{b} = \dfrac{\vec{a}.\vec{b}}{|\vec{b}|}$

$$= \frac{(7\hat{i}+\hat{j}-4\hat{k}).(2\hat{i}+6\hat{j}+3\hat{k})}{|\sqrt{2^2+6^2+3^2}|}$$
$$= \frac{14+6-12}{|\sqrt{4+36+9}|}$$
$$= \frac{8}{7} \text{ units}$$

10. Find the direction cosines of the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$.

Sol. Equation of the line

$$\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$$
$$\frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3}$$

direction ratio of the line (–2, 6, –3)
∴ Direction cosine of the line is

$$= \frac{-2}{\sqrt{(-2)^2+(6)^2+(-3)^2}}, \frac{6}{\sqrt{(-2)^2+(6)^2+(-3)^2}}, \frac{-3}{\sqrt{(-2)^2+(6)^2+(-3)^2}}$$
$$= \frac{-2}{7}, \frac{6}{7}, \frac{-3}{7}$$

SECTION - B

Question numbers 11 to 22 carry 4 marks each.

*** 11. Let A = R – {2} and B = R – {1}. If f : A → B is a function defined by $f(x) = \frac{x-1}{x-2}$, show that f is one-one and onto. Hence find f^{-1}.**

*** 12. Prove that:**

$$\sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{77}{36}$$

OR

Solve for x:

$$\tan^{-1}3x + \tan^{-1}2x = \frac{\pi}{4}$$

*** 13. Using properties of determinants, prove that**

$$\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$

14. If $x = 2\cos\theta - \cos 2\theta$ and $y = 2\sin\theta - \sin 2\theta$, then prove that

$$\frac{dy}{dx} = \tan\left(\frac{3\theta}{2}\right)$$

OR

If $y = (\sin x)^x + \sin^{-1}\sqrt{x}$, then find $\frac{dy}{dx}$.

Sol. $$x = 2\cos\theta - \cos 2\theta$$
$$y = 2\sin\theta - \sin 2\theta$$
$$\frac{dx}{d\theta} = -2\sin\theta + 2\sin 2\theta$$
$$\frac{dy}{d\theta} = 2\cos\theta - 2\cos 2\theta$$

* *Out of Syllabus*

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$= \frac{2(\cos\theta - \cos 2\theta)}{2(\sin 2\theta - \sin\theta)}$$

$$\frac{dy}{dx} = \frac{2\sin\frac{3\theta}{2}\sin\frac{\theta}{2}}{2\cos\frac{3\theta}{2}\sin\frac{\theta}{2}}$$

$$\frac{dy}{dx} = \tan\frac{3\theta}{2} = \text{R.H.S}$$

Hence Proved

OR

$$y = (\sin x)^x + \sin^{-1}\sqrt{x}$$

$$\frac{dy}{dx} = \frac{d}{dx}(\sin x)^x + \frac{d}{dx}\sin^{-1}\sqrt{x}$$

$$\frac{dy}{dx} = \frac{d}{dx}(\sin x)^x + \frac{1}{\sqrt{1-x}}\frac{d}{dx}\sqrt{x}$$

$$\frac{dy}{dx} = \frac{d}{dx}(\sin x)^x + \frac{1}{2\sqrt{x}\sqrt{1-x}}$$

$$\frac{dy}{dx} = \frac{d}{dx}(\sin x)^x + \frac{1}{2\sqrt{x-x^2}} \quad \text{...(i)}$$

Let $u = (\sin x)^x$

taking log both sides

$$\log 4 = \log(\sin x)^x$$

$$= x\log(\sin x)$$

$$\frac{d}{dx}\log u = \frac{d}{dx}x\log\sin x$$

$$\frac{1}{u}\frac{du}{dx} = x\frac{d}{dx}\log\sin x + \log\sin x\frac{d}{dx}x$$

$$\frac{1}{u}\frac{du}{dx} = \frac{x\cos x}{\sin x} + \log\sin x \times 1$$

$$\frac{d}{dx}(\sin x)^x = (\sin x)^x(x\cot x + \log\sin x) \quad \text{...(ii)}$$

from (i) & (ii)

$$\frac{dy}{dx} = (\sin x)^x(x\cot x + \log\sin x) + \frac{1}{2\sqrt{x-x^2}}$$

15. If $y = x\log\left(\frac{x}{a+bx}\right)$, then prove that

$$x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2.$$

Sol.

$$y = x\log\frac{x}{a+bx}$$

$$\frac{dy}{dx} = x\frac{d}{dx}\log\frac{x}{a+bx} + \log\frac{x}{a+bx}\frac{d}{dx}x$$

$$\frac{dy}{dx} = x.\frac{a+bx}{x}\frac{d}{dx}\frac{x}{a+bx} + \log\frac{x}{a+bx}$$

$$\frac{dy}{dx} = (a+bx)\left\{\frac{(a+bx).1 - x(0+b)}{(a+bx)^2}\right\} + \log\frac{x}{a+bx}$$

$$\frac{dy}{dx} = \frac{a}{a+bx} + \log\frac{x}{a+bx}$$

$$x\frac{dy}{dx} = \frac{ax}{a+bx} + x\log\frac{x}{a+bx}$$

$$x\frac{dy}{dx} = \frac{ax}{a+bx} + y \quad \text{...(i)}$$

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} = \frac{(a+bx).a - ax(0+b)}{(a+bx)^2} + \frac{dy}{dx}$$

$$x\frac{d^2y}{dx^2} = \frac{a^2}{(a+bx)^2}$$

$$x^3\frac{d^2y}{dx^2} = \frac{a^2x^2}{(a+bx)^2} = \left(\frac{ax}{a+bx}\right)^2$$

$$x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2 \quad \text{[from (i)]}$$

Hence Proved

16. If $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2}, & \text{when } x < 0 \\ a, & \text{when } x = 0 \\ \frac{\sqrt{x}}{(\sqrt{16+\sqrt{x}})-4}, & \text{when } x > 0 \end{cases}$

and f is continuous at $x = 0$, find the value of a.

Sol. L.H.L. $f(x) = \frac{1-\cos 4x}{x^2}$, $x < 0$

L.H.L. $\lim_{x\to 0^-} f(x)$

$$= \lim_{x\to 0^-}\frac{1-\cos 4x}{x^2}$$

$$= \lim_{x\to 0^-}\frac{2\sin^2 2x}{x^2}$$

$$= \lim_{x\to 0^-}\frac{2\times 4\sin^2 2x}{4x^2}$$

$$= 8\lim_{x\to 0^-}\left(\frac{\sin 2x}{2x}\right)^2$$

$$= 8 \quad \left[\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1\right]$$

R.H.L. $f(x) = \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}} - 4}$, $x > 0$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}\frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}\times\frac{\sqrt{16+\sqrt{x}}+4}{\sqrt{16+\sqrt{x}}+4}$$

$$= \lim_{x\to 0^+}\frac{\sqrt{x}(\sqrt{16+\sqrt{x}}+4)}{16+\sqrt{x}-16}$$

$$= \lim_{x\to 0}(\sqrt{16+\sqrt{x}}+4)$$

$$= 4 + 4 = 8$$

for continuous function

$$\text{L.H.L} = \text{R.H.L} = f(0)$$

$$8 = 8 = a$$

$$\therefore \quad a = 8$$

17. Evaluate: $\int \frac{(3\sin x - 2)\cos x}{5 - \cos^2 x - 4\sin x}dx$

OR

Evaluate: $\int e^{2x}\left(\frac{1-\sin 2x}{1-\cos 2x}\right)dx$

Sol. $\int \frac{(3\sin x - 2)\cos x}{5 - \cos^2 x - 4\sin x}dx$

$$= \int \frac{(3\sin x - 2)\cos x}{4 + \sin^2 x - 4\sin x}dx$$

Put $\sin x = t$

$\cos x\, dx = dt$

$$= \int \frac{3t-2}{4-4t+t^2}dx$$

$$= \int \frac{\frac{3}{2}(2t-4)+4}{t^2-4t+4}dt$$

$$= \frac{+3}{2}\int \frac{(-4+2t)}{4-4t+t^2}dt + 4\int \frac{dt}{4-4t+t^2}$$

$$= \frac{3}{2}\log(t-2)^2 + 4\int \frac{dt}{(t-2)^2}$$

$$= 3\log|t-2| + \frac{4}{-(t-2)} + C$$

$$= 3\log|\sin x - 2| - \frac{4}{\sin x - 2} + C$$

$$= 3\log|\sin x - 2| - \frac{4}{\sin x - 2} + C$$

OR

$\int e^{2x}\left(\frac{1-\sin 2x}{1-\cos 2x}\right)dx$

Let $2x = t$

$2dx = dt$

$$= \frac{1}{2}\int e^t\left(\frac{1-\sin t}{1-\cos t}\right)dt$$

$$= \frac{1}{2}\int e^t\left(\frac{1-2\sin\frac{t}{2}\cos\frac{t}{2}}{2\sin^2\frac{t}{2}}\right)dt$$

$$= \frac{1}{2}\int e^t\left(\frac{1}{2}\text{cosec}^2\frac{t}{2} - \cot\frac{t}{2}\right)dt$$

Let $-\cot\frac{t}{2} = f(t)$

$$\frac{1}{2}\text{cosec}^2\frac{t}{2} = f'(t) \qquad \left[\int e^t(f(t)+f'(t))dt = e^t f(t)\right]$$

$$\therefore \frac{-1}{2}e^t\cot\frac{t}{2} + C$$

$$= \frac{-1}{2}e^{2x}\cot x + C$$

18. Evaluate: $\int \frac{3x+1}{(x+1)^2(x+3)}dx$

Sol. $\int \frac{3x+1}{(x+1)^2(x+3)}dx$

$$\frac{3x+1}{(x+1)^2(x+3)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x+3}$$

$$3x + 1 = A(x+1)(x+3) + B(x+3) + C(x+1)^2$$

Put $x = -1$

$-2 = 0 + 2B + 0$

$\Rightarrow B = -1$

$x = -3$

$-8 = 0 + 0 + 4C$

$\Rightarrow C = -2$

$x = 1$

$4 = 8A + 4B + 4C$

$4 = 8A - 4 - 8$

$8A = 16$

$\Rightarrow A = 2$

$\int \frac{3x+1}{(x+1)^2(x+3)}dx$

$$= \int \frac{2}{x+1}dx - \int \frac{1}{(x+1)^2}dx - 2\int \frac{1}{x+3}dx$$

$$= 2\log|x+1| + \frac{1}{x+1} - 2\log|x+3| + C$$

$$= 2\log\left|\frac{x+1}{x+3}\right| + \frac{1}{x+1} + C$$

19. Evaluate: $\int_0^{\frac{\pi}{4}} \log(1+\tan x)dx$

Sol.

$$I = \int_0^{\pi/4}\log(1+\tan x)dx$$

$$I = \int_0^{\pi/4}\log\left(1+\tan\left(\frac{\pi}{4}-x\right)\right)dx$$

$$I = \int_0^{\pi/4}\log\left(1+\frac{1-\tan x}{1+\tan x}\right)dx$$

$$I = \int_0^{\pi/4}\log\frac{2}{1+\tan x}dx$$

$$= \int_0^{\pi/4}\log 2 - \int_0^{\pi/4}\log(1+\tan x)dx$$

$$2I = \int_0^{\pi/4} \log 2\, dx$$

$$= [x \log 2]_0^{\pi/4} = \left(\frac{\pi}{4}\log 2 - 0\right)$$

$$I = \frac{\pi}{8}\log 2$$

20. If $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular vectors of the same magnitude, prove that $\vec{a}+\vec{b}+\vec{c}$ is equally inclined with the vectors $\vec{a}, \vec{b}$ and $\vec{c}$.

Sol. $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually perpendicular vectors of same magnitude

$\therefore$ $$|\vec{a}| = |\vec{b}| = |\vec{c}|$$

and $$\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$$

$$\vec{a}.(\vec{a}+\vec{b}+\vec{c}) = |\vec{a}|.|\vec{a}+\vec{b}+\vec{c}|\cos\alpha$$

$$\vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{a}.\vec{c} = |\vec{a}|.|\vec{a}+\vec{b}+\vec{c}|\cos\alpha$$

$$|a^2| + 0 + 0 = |\vec{a}||\vec{a}+\vec{b}+\vec{c}|\cos\alpha$$

$$\cos\alpha = \frac{|\vec{a}|}{|\vec{a}+\vec{b}+\vec{c}|} \quad ...(i)$$

Similarly angle between $\vec{b}$ and $(\vec{a}+\vec{b}+\vec{c})$ be β

$$\cos\beta = \frac{|\vec{b}|}{|\vec{a}+\vec{b}+\vec{c}|} \quad ...(ii)$$

And angle between $\vec{c}$ and $(\vec{a}+\vec{b}+\vec{c})$ be γ

$$\cos\gamma = \frac{|\vec{c}|}{|\vec{a}+\vec{b}+\vec{c}|}$$

$\therefore$ $$\left|\frac{|\vec{a}|}{\vec{a}+\vec{b}+\vec{c}}\right| = \frac{|\vec{b}|}{|\vec{a}+\vec{b}+\vec{c}|} = \frac{|\vec{c}|}{|\vec{a}+\vec{b}+\vec{c}|}$$

$\therefore$ $$\cos\alpha = \cos\beta = \cos\gamma$$

$\because$ (Given $|\vec{a}| = |\vec{b}| = |\vec{c}|$)

$\therefore$ $\alpha = \beta = \gamma$ **Hence Proved**

21. The cartesian equations of a line are $6x - 2 = 3y + 1 = 2z - 2$. Find the direction cosines of the line. Write down the cartesian and vector equations of a line passing through (2, – 1, – 1) which is parallel to the given line.

OR

Find the shortest distance between the two lines whose vector equations are

$$\vec{r} = (6\hat{i}+2\hat{j}+2\hat{k})+\lambda(\hat{i}-2\hat{j}+2\hat{k}) \text{ and}$$

$$\vec{r} = (-4\hat{i}-\hat{k})+\mu(3\hat{i}-2\hat{j}-2\hat{k})$$

Sol. Given line $6x - 2 = 3y + 1 = 2z - 2$

$$\frac{x-\frac{1}{3}}{\frac{1}{6}} = \frac{y+\frac{1}{3}}{\frac{1}{3}} = \frac{z-1}{\frac{1}{2}}$$

Direction ratio of the lines $\frac{1}{6}, \frac{1}{3}, \frac{1}{2}$ or 1, 2, 3

Direction cosine of the lines $\frac{1}{\sqrt{1^2+2^2+3^2}}$, $\frac{2}{\sqrt{1^2+2^2+3^2}}$, $\frac{3}{\sqrt{1^2+2^2+3^2}}$ or $\left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$

Equation of the line parallel to the given line and passing through (2, –1, –1)

$$\frac{x-2}{1} = \frac{y+1}{2} = \frac{z+1}{3}$$

In vector form $+2\hat{i}-\hat{j}-\hat{k}+\lambda(\hat{i}+2\hat{j}+3\hat{k})$

OR

Given line $$\vec{r} = (6\hat{i}+2\hat{j}+2\hat{k})+\lambda(\hat{i}-2\hat{j}+2\hat{k})$$

$$\vec{r} = (-4\hat{i}-\hat{k})+\mu(3\hat{i}-2\hat{j}-2\hat{k})$$

Given lines are not parallel

$$\vec{a}_1 = 6\hat{i}+2\hat{j}+2\hat{k},\ \vec{a}_2 = -4\hat{i}-\hat{k}$$

$$\vec{b}_1 = \hat{i}-2\hat{j}+2\hat{k},\ \vec{b}_2 = 3\hat{i}-2\hat{j}-2\hat{k}$$

$$\vec{b}_1\times\vec{b}_2 = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 2\\ 3 & -2 & -2\end{vmatrix} = 8\hat{i}+8\hat{j}+4\hat{k}$$

$$|\vec{b}_1\times\vec{b}_2| = |\sqrt{8^2+8^2+4^2}|$$

* *Out of Syllabus*

$$= |\sqrt{64+64+16}| = 12$$

$$\vec{a_2} - \vec{a_1} = -10\hat{i} - 2\hat{j} - 3\hat{k}$$

Shortest distance between lines

$$= \left|\frac{(a_2 - a_1).(\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|}\right|$$

$$= \left|\frac{(-10\hat{i} - 2\hat{j} - 3\hat{k}).(8\hat{i} + 8\hat{j} + 4\hat{k})}{12}\right|$$

$$= \left|\frac{-80 - 16 - 12}{12}\right|$$

$$= \left|\frac{-108}{12}\right|$$

$\therefore$ Shortest distance between lines

= 9 units

*** 22. Out of a group of 30 honest people, 20 always speak the truth. Two persons are selected at random from the group. Find the probability distribution of the number of selected persons who speak the truth. Also find the mean of the distribution. What values are described in this question?**

SECTION - C

Question numbers 23 to 29 carry 6 mark each.

23. Two institutions decided to award their employees for the three values of resourcefulness, competence and determination in the form of prizes at the rate of ₹ x, ₹ y and ₹ z respectively per person. The first institution decided to award respectively 4, 3 and 2 employees with a total prize money of ₹ 37,000 and the second institution decided to award respectively 5, 3 and 4 employees with a total prize money of ₹ 47,000. If all the three prizes per person together amount to ₹ 12,000, then using matrix method find the value of x, y and z.

What values are described in this question ?

Sol. Problem written in algebraic equation

$$4x + 3y + 2z = 37000$$
$$5x + 3y + 4z = 47000$$
$$x + y + z = 12000$$

Equations can be arrange in matrix form

$$\begin{bmatrix} 4 & 3 & 2 \\ 5 & 3 & 4 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 37000 \\ 47000 \\ 12000 \end{bmatrix}$$

where $A = \begin{bmatrix} 4 & 3 & 2 \\ 5 & 3 & 4 \\ 1 & 1 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 37000 \\ 47000 \\ 12000 \end{bmatrix}$

$$AX = B$$
$$A^{-1}AX = A^{-1}B$$
$$IX = A^{-1}B$$
$$\Rightarrow \quad X = A^{-1}B \quad \ldots(i)$$

$$|A| = \begin{vmatrix} 4 & 3 & 2 \\ 5 & 3 & 4 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= 4\begin{vmatrix} 3 & 4 \\ 1 & 1 \end{vmatrix} - 3\begin{vmatrix} 5 & 4 \\ 1 & 1 \end{vmatrix} + 2\begin{vmatrix} 5 & 3 \\ 1 & 1 \end{vmatrix}$$

$$= 4(3 - 4) - 3(5 - 4) + 2(5 - 3)$$
$$= -4 - 3 + 4 = -3$$

$$Adj\,A = \begin{bmatrix} -1 & -1 & 2 \\ -1 & 2 & -1 \\ 6 & -6 & -3 \end{bmatrix}^t$$

$$= \begin{bmatrix} -1 & -1 & 6 \\ -1 & 2 & -6 \\ 2 & -1 & -3 \end{bmatrix}$$

$$A^{-1} = \frac{-1}{3}\begin{bmatrix} -1 & -1 & 6 \\ -1 & 2 & -6 \\ 2 & -1 & -3 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & -6 \\ 1 & -2 & 6 \\ -2 & 1 & 3 \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & -6 \\ 1 & -2 & 6 \\ -2 & 1 & 3 \end{bmatrix}\begin{bmatrix} 37000 \\ 47000 \\ 12000 \end{bmatrix}$$

$$= \begin{bmatrix} 4000 \\ 5000 \\ 3000 \end{bmatrix}$$

$\therefore$ x = ₹ 4000, y = ₹ 5000 and z = ₹ 3000

24. * For the curve $y = 4x^3 - 2x^5$, find all the points on the curve at which the tangent passes through the origin.

OR

If the length of three sides of a trapezium other than the base are each equal to 10 cm, then find the area of the trapezium when it is maximum.

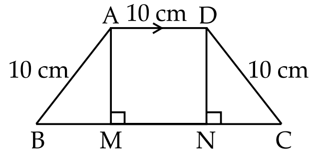
Sol. OR

Let the trapezium ABCD in which

$$AD = AB = DC = 10 \text{ cm}$$

and $\quad AD \parallel BC$

Let $\quad BC = x \text{ cm}$



$$\Delta ABM \cong \Delta DCN$$

$$\therefore \quad BM = NC = \frac{x-10}{2}$$

ΔABM, $\quad \angle M = 90°$

$$\therefore \quad AM = \sqrt{AB^2 - BM^2}$$

$$= \sqrt{100 - \left(\frac{x-10}{2}\right)^2}$$

$$= \sqrt{\frac{300 - x^2 + 20x}{4}}$$

Area of trapezium $= \frac{1}{2} AM(AD + BC)$

$$A = \frac{1}{2} \frac{\sqrt{300 - x^2 + 20x}}{2}(10 + x)$$

$$A = \frac{1}{4}(\sqrt{300 - x^2 + 20x})(10 + x)$$

$$\frac{dA}{dx} = \frac{1}{4}\left\{\frac{1}{2\sqrt{300 - x^2 + 20x}}(-2x + 20)(10 + x) + \sqrt{300 - x^2 + 20x}\ (0 + 1)\right\}$$

$$\frac{dA}{dx} = \frac{1}{4}\left\{\frac{(10 - x)(10 + x) + (300 - x^2 + 20x)}{\sqrt{300 - x^2 + 20x}}\right\}$$

$$= \frac{1}{4}\left\{\frac{400 - 2x^2 + 20x}{\sqrt{300 - x^2 + 20x}}\right\}$$

$$= -\frac{1}{2}\left\{\frac{x^2 - 10x - 200}{\sqrt{300 - x^2 + 20x}}\right\}$$

$$= -\frac{1}{2}\left\{\frac{(x - 20)(x + 10)}{\sqrt{300 - x^2 + 20x}}\right\}$$

For max / minima

$$\frac{dA}{dx} = 0$$

$\therefore \ (x - 20)(x + 10) = 0$

$\Rightarrow \quad x = 20$ or $x = -10$

$$\frac{d^2A}{dx^2} = \frac{d}{dx}\left[-\frac{1}{2}\left\{\frac{x^2 - 10x - 200}{\sqrt{300 - x^2 + 20x}}\right\}\right]$$

$$= \frac{-1}{2}\left[\frac{\sqrt{300 - x^2 + 20x}\ (2x - 10) - \frac{x^2 - 10 - 200}{\sqrt{3x^2 - x^2 + 20x}}(10 - x)}{\sqrt{300 - x^2 + 20x}}\right]$$

$$\left(\frac{d^2A}{dx^2}\right)_{x=20} = \frac{-1}{2}\left[\frac{\sqrt{300}(30) - (0)}{\sqrt{300}}\right] < 0$$

Hence area is maximum at $x = 20$ cm

Area of trapezium

$$A = \frac{1}{4}\sqrt{300 - x^2 + 20x}(10 + x) \text{ cm}^2$$

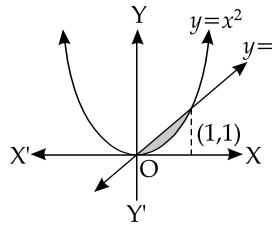
$$= \frac{1}{4}\sqrt{300 - 400 + 400}\ (10 + 20) \text{ cm}^2$$

$$= \frac{10\sqrt{3} \times 30}{4}$$

$$= 75\sqrt{3} \text{ cm}^2$$

25. Using integration, find the area of the region bounded by the curves $y = x^2$ and $y = x$.

Sol. Intersection point of the curve is (0, 0) and (1, 1)



$$\text{Required Area} = \int_0^1 (y_1 - y_2)dx$$

$$= \int_0^1 (x - x^2)dx$$

$$= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{1}{2} - \frac{1}{3}$$

$$= \frac{1}{6} \text{ unit}^2$$

26. Find the particular solution of the differential equation $(3xy + y^2)\,dx + (x^2 + xy)\,dy = 0$; for $x = 1$, $y = 1$.

Sol. $(3xy + y^2)dx + (x^2 + xy)dy = 0$

$$\frac{dy}{dx} = -\frac{3xy + y^2}{x^2 + xy}$$

Put $\quad y = Vx$

(it is homogeneous equation)

$$\frac{dy}{dx} = V + x\frac{dV}{dx}$$

$$V + x\frac{dV}{dx} = -\frac{3Vx^2 + V^2x^2}{x^2 + Vx^2}$$

$$x\frac{dV}{dx} = -\frac{3V+V^2}{1+V} - V$$

$$= \frac{-3V-V^2-V^2-V}{1+V}$$

$$\frac{1+V}{(2V^2+4V)} = -\frac{dx}{x}$$

$$\frac{1}{2}\int\frac{(1+V)dV}{(V^2+2V)} = -\int\frac{dx}{x}$$

$$\frac{1}{2}\left[\int\frac{1dV}{2V}+\int\frac{1dV}{2(V+2)}\right] = -\int\frac{dx}{x}$$

$$\frac{1}{2}\times\frac{1}{2}[\log|V|+\log|V+2|] = -\log|x| + \log C$$

$$\log|V(V+2)| = \log\frac{C_1}{x^4} \qquad (C_1 = C_4)$$

$$V(V+2) = \frac{C_1}{x^4}$$

$$\frac{y}{x}\left(\frac{y}{x}+2\right) = \frac{C_1}{x^4}$$

$$y(y+2x) = \frac{C_1}{x^2}$$

$$x^2y(y+2x) = C_1$$

where $x = 1, y = 1$

$$1 \times 1(1+2) = C_1$$

$$C_1 = 3$$

$\therefore$ Particular solution is

$$x^2(y^2+2xy) = 3$$

*** 27. Show that the lines $\vec{r} = (\hat{i}+\hat{j}-\hat{k})+\lambda(3\hat{i}-\hat{j})$ and $\vec{r} = (4\hat{i}-\hat{k})+\mu(2\hat{i}+3\hat{k})$ are coplanar. Also find the equation of the plane containing them.**

OR

Find the distance of the point (1, –2, 3) from the plane $x - y + z = 5$ measured parallel to the line $\frac{x-1}{2} = \frac{y-3}{3} = \frac{z+2}{-6}$.

28. If a young man drives his scooter at a speed of 25 km/hour, he has to spend ₹ 2 per km on petrol. If he drives the scooter at a speed of 40 km/hour, it produces more air pollution and increases his expenditure on petrol to ₹ 5 per km. He has a maximum of ₹ 100 to spend on petrol and travel a maximum distance in one hour time with less pollution. Express this problem as an LPP and solve it graphically. What value do you find here ?

Sol. Let men travels x km with 25 km/hr

and y km with 40 km/hr

$\therefore$ Total distance travel $(x + y)$ km

$\therefore$ Maximise distance $z = x + y$

Constraints

Time taken to cover x km = $\frac{x}{25}$ h

Time taken to cover y km = $\frac{y}{40}$ h

$\therefore \qquad \frac{x}{25}+\frac{y}{40} \le 1$ ...(i)

$$8x + 5y \le 200$$

Expenditure on petrol

$2x + 5y \le 100$ ...(ii)

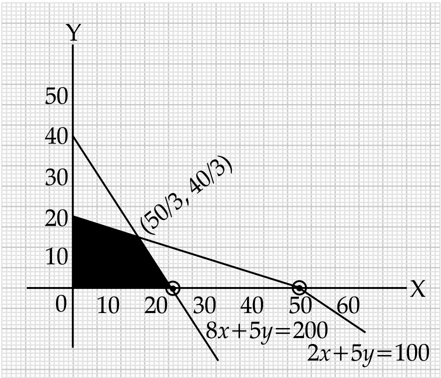
and also $x \ge 0$ and $y \ge 0$

$8x + 5y = 200$

x	0	25	20
y	40	0	8

$2x + 5y = 100$

x	0	50	25
y	20	0	10



Corner point of fiesible area	$z = x + y$
(0, 0)	$z = 0$
(0, 20)	$z = 20$
$\left(\frac{50}{3}, \frac{40}{3}\right)$	$z = 30 \rightarrow$ maximum
(25, 0)	$z = 25$

Hence distance travellatry at speed at 25 km/h is $\frac{50}{3}$ km and 50 km/h is $\frac{40}{3}$ km

29. In a group of 400 people, 160 are smokers and non-vegetarian, 100 are smokers and vegetarian and the remaining are non-smokers and vegetarian. The probabilities of getting a special chest disease are 35%, 20% and 10% respectively. A person is chosen from the group at random and is found

to be suffering from the disease. What is the probability that the selected person is a smoker and non-vegetarian ? What value is reflected in this question ?

Sol. Let A = Person suffers from the disease

E_1 = Person is a smoker and a non-vegetarian

E_2 = Person is a smoker and a vegetarian

E_3 = Person is a non smoker and a vegetarian

$$\therefore \quad P(E_1) = \frac{160}{400} = \frac{16}{40} = \frac{2}{5}$$

$$P(E_2) = \frac{100}{400} = \frac{1}{4}$$

$$P(E_3) = \frac{140}{400} = \frac{7}{20}$$

$$P\left(\frac{A}{E_1}\right) = \frac{35}{100} = \frac{7}{20}$$

$$P\left(\frac{A}{E_2}\right) = \frac{20}{100} = \frac{1}{5}$$

$$P\left(\frac{A}{E_3}\right) = \frac{10}{100} = \frac{1}{10}$$

Required probability

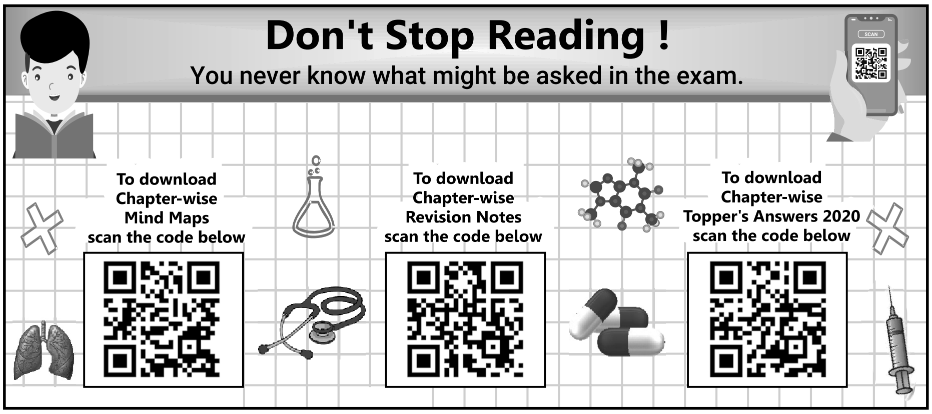
$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)+P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{2}{5}\times\frac{7}{20}}{\frac{2}{5}\times\frac{7}{20}+\frac{1}{4}\times\frac{1}{5}+\frac{7}{20}\times\frac{1}{10}}$$

$$= \frac{28}{28+10+7}$$

$$= \frac{28}{45}$$

❑❑



* *Out of Syllabus*