

Solved Paper 2015

Mathematics

Class-XII

Time : 3 Hours

Max. Marks : 100

General Instructions:

- (i) All questions are compulsory.
- (ii) Please check that this Question Paper contains 26 Questions.
- (iii) Marks for each question are indicated against it.
- (iv) Questions 1 to 6 in Section-A are Very Short Answer Type Questions carrying one mark each.
- (v) Questions 7 to 19 in Section-B are Long Answer I Type Questions carrying 4 marks each.
- (vi) Questions 20 to 26 in Section-C are Long Answer II Type Questions carrying 6 marks each
- (vii) Please write down the serial number of the Question before attempting it.

Delhi Set-I

Code No : 65/1/D

SECTION - A

Question numbers 1 to 10 carry 1 mark each.

1. If $\vec{a} = 7\hat{i} + \hat{j} - 4\hat{k}$ and $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$, then
find the projection of \vec{a} on \vec{b} . 1

Sol.
$$p = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

$$= \frac{(7\hat{i} + \hat{j} - 4\hat{k}) \cdot (2\hat{i} + 6\hat{j} + 3\hat{k})}{|2\hat{i} + 6\hat{j} + 3\hat{k}|}$$

$$= \frac{14 + 6 - 12}{\sqrt{4 + 36 + 9}}$$

$$= \frac{8}{7}$$

- * 2. Find λ , if the vectors $\vec{a} = \hat{i} + 3\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} - \hat{k}$
and $\vec{c} = \lambda\hat{j} + 3\hat{k}$ are coplanar. 1

3. If a line makes angles 90° , 60° and θ with X, Y Z-axis respectively, when θ is acute, then find θ .

Sol. $\cos^2 \frac{\pi}{2} + \cos^2 \frac{\pi}{3} + \cos^2 \theta = 1$
 $\Rightarrow \theta = \frac{\pi}{6}$

4. Write the element a_{23} of a 3×3 matrix $A = (a_{ij})$
whose elements are given by $a_{ij} = \frac{|i-j|}{2}$

Sol. $a_{23} = \frac{|2-3|}{2} = \frac{1}{2}$

5. Find the differential equation representing the family of curves $v = \frac{A}{r} + B$, where A and B are arbitrary constants.

Sol.
$$\frac{dv}{dr} = \frac{A}{r^2},$$

$$\Rightarrow r^2 \frac{d^2v}{dr^2} + 2r \frac{dv}{dr} = 0$$

6. Find the integrating factor of the differential equation $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}} \right) dx - dy = 1$.

Sol. I.F. $= e^{\int \frac{1}{\sqrt{x}} dx} = e^{2\sqrt{x}}$

SECTION - B

Question numbers 7 to 19 carry 4 marks each.

7. If $A = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}$ find $A^2 - 5A + 4I$ and hence find a matrix X such that $A^2 - 5A + 4I + X = 0$

OR
If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$, find $(A')^{-1}$.

Sol. Getting $A^2 = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}$
 $= \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & 2 \end{pmatrix}$

* Out of Syllabus

$$A^2 - 5A + 4I = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix} + \begin{pmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{pmatrix} + \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{pmatrix}$$

$$\therefore X = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & -4 & -2 \end{pmatrix}$$

OR

$$A' = \begin{pmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{pmatrix}$$

$$|A'| = 1(-9) - 2(-5) = -9 + 10 = 1 \neq 0$$

$$\text{Adj } A' = \begin{pmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{pmatrix} \quad \left[A^{-1} = \frac{\text{Adj } A}{|A|} \right]$$

$$\therefore (A')^{-1} = \begin{pmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{pmatrix}$$

$$8. \text{ If } f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}, \text{ using properties of determinants find the value of } f(2x) - f(x).$$

$$\text{Sol. } f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$$

$$R_2 \rightarrow R_2 - x R_1 \text{ and } R_3 \rightarrow R_3 - x^2 R_1$$

$$f(x) = \begin{vmatrix} a & -1 & 0 \\ 0 & a+x & -1 \\ 0 & ax+x^2 & a \end{vmatrix}$$

(For bringing 2 zeroes in any row/column)

$$\therefore f(x) = a(a^2 + 2ax + x^2) = a(x+a)^2$$

$$\therefore f(2x) - f(x) = a[2x+a]^2 - a(x+a)^2 \\ = ax(3x+2a)$$

$$9. \text{ Find: } \int \frac{dx}{\sin x + \sin 2x}$$

OR

Integrate the following w.r.t. x

$$\frac{x^2 - 3x + 1}{\sqrt{1-x^2}}$$

$$\text{Sol. } \int \frac{dx}{\sin x + \sin 2x} = \int \frac{dx}{\sin x(1+2\cos x)}$$

$$= \int \frac{\sin x \cdot dx}{(1-\cos x)(1+\cos x)(1+2\cos x)}$$

$$= - \int \frac{dt}{(1-t)(1+t)(1+2t)} \text{ where } \cos x = t$$

$$= \int \left(\frac{-\frac{1}{6}}{1-t} + \frac{\frac{1}{2}}{1+t} - \frac{\frac{4}{3}}{1+2t} \right) dt$$

$$= +\frac{1}{6} \log |1-t| + \frac{1}{2} \log |1+t| - \frac{2}{3} \log |1+2t| + c$$

$$= \frac{1}{6} \log |1-\cos x| + \frac{1}{2} \log |1+\cos x|$$

$$- \frac{2}{3} \log |1+2\cos x| + c$$

OR

$$\int \frac{x^2 - 3x + 1}{\sqrt{1-x^2}} dx = \int \frac{2-3x-(1-x^2)}{\sqrt{1-x^2}} dx$$

$$= 2 \int \frac{1}{\sqrt{1-x^2}} dx - 3 \int \frac{x}{\sqrt{1-x^2}} dx - \int \sqrt{1-x^2} dx$$

$$= 2 \sin^{-1} x + 3 \sqrt{1-x^2} - \frac{x}{2} \sqrt{1-x^2} - \frac{1}{2} \sin^{-1} x + c$$

$$= \frac{3}{2} \sin^{-1} x + \frac{1}{2}(6-x)\sqrt{1-x^2} + c$$

$$10. \text{ Evaluate: } \int_{-\pi}^{\pi} (\cos ax - \sin bx)^2 dx$$

$$\text{Sol. } I = \int_{-\pi}^{\pi} (\cos ax - \sin bx)^2 dx$$

$$= \int_{-\pi}^{\pi} (\cos^2 ax + \sin^2 bx) dx - \int_{-\pi}^{\pi} 2 \cos ax \sin bx dx$$

$$= I_1 - I_2$$

$$I_1 = 2 \int_0^{\pi} (\cos^2 ax + \sin^2 bx) dx \text{ (being an even fun)}$$

$$I_2 = 0 \text{ (being an odd fun.)}$$

$$\therefore I = I_1 = \int_0^{\pi} (1 + \cos 2ax + 1 - \cos 2bx) dx$$

$$= \left[2x + \frac{\sin 2ax}{2a} - \frac{\sin 2bx}{2b} \right]_0^{\pi}$$

$$= \left[2x + \frac{1}{2a} \cdot \sin 2a\pi - \frac{\sin 2b\pi}{2b} \right] \text{ or } 2\pi$$

11. A bag A contains 4 black and 6 red balls and bag B contains 7 black and 3 red balls. A die is thrown. If 1 or 2 appears on it, then bag A is chosen, otherwise bag B. If two balls are drawn at random (without replacement) from the selected bag, find the probability of one of them being red and another black.

OR

- * An unbiased coin is tossed 4 times. Find the mean and variance of the number of heads obtained.

Sol. Let E_1 : selecting bag A, and E_2 : selecting bag B.

$$\therefore P(E_1) = \frac{1}{3}, P(E_2) = \frac{2}{3}$$

Let A : Getting one Red and one black ball

$$\therefore P(A/E_1) = \frac{^4C_1 \cdot ^6C_1}{^{10}C_2} = \frac{8}{15},$$

$$P(A/E_2) = \frac{^7C_1 \cdot ^3C_1}{^{10}C_2} = \frac{7}{15}$$

$$\begin{aligned} P(A) &= P(E_1).P(A|E_1) + P(E_2).P(A|E_2) \\ &= \frac{1}{3} \cdot \frac{8}{15} + \frac{2}{3} \cdot \frac{7}{15} = \frac{22}{45} \end{aligned}$$

12. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, find $(\vec{r} \times \hat{i}) \cdot (\vec{r} \times \hat{j}) + xy$

$$\text{Sol. } \vec{r} \times \vec{i} = (x\hat{i} + y\hat{j} + z\hat{k}) \times \hat{i} = -y\hat{k} + z\hat{j}$$

$$\vec{r} \times \vec{j} = (x\hat{i} + y\hat{j} + z\hat{k})\hat{j} = x\hat{k} - z\hat{i}$$

$$\begin{aligned} (\vec{r} \times \hat{i}) \cdot (\vec{r} \times \hat{j}) &= (o\hat{i} + z\hat{j} - y\hat{k}) \cdot (-z\hat{i} + o\hat{j} + x\hat{k}) \\ &= -xy \end{aligned}$$

$$(\vec{r} \times \hat{i}) \cdot (\vec{r} \times \hat{j}) + xy = -xy + xy = 0$$

- * 13. Find the distance between the point $(-1, -5, -10)$

and the point of intersection of the line $\frac{x-2}{3} =$

$\frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 5$.

14. If $\sin[\cot^{-1}(x+1)] = \cos(\tan^{-1}x)$, then find x .

OR

If $(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$, then find x .

Sol. Writing $\cos^{-1}(x+1) = \sin^{-1} \frac{1}{\sqrt{1+(x+1)^2}}$

$$\text{and } \tan^{-1}x = \cos^{-1} \frac{1}{\sqrt{1+x^2}}$$

$$\therefore \sin\left(\sin^{-1} \frac{1}{\sqrt{1+(x+1)^2}}\right) = \cos\left(\cos^{-1} \frac{1}{\sqrt{1+x^2}}\right)$$

$$1 + x^2 + 2x + 1 = 1 + x^2 \Rightarrow x = -\frac{1}{2}$$

OR

$$(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$$

$$\Rightarrow (\tan^{-1}x)^2 + \left(\frac{\pi}{2} - \tan^{-1}x\right)^2 = \frac{5\pi^2}{8}$$

$$\therefore 2(\tan^{-1}x)^2 - \pi \tan^{-1}x - \frac{3\pi^2}{8} = 0$$

$$\tan^{-1}x = \frac{\pi \pm \sqrt{\pi^2 + 3\pi^2}}{4} = \frac{3\pi}{4}, \frac{-\pi}{4}$$

$$\Rightarrow x = -1$$

15. If $y = \tan^{-1}\left(\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}}\right)$, $x^2 \leq 1$, then find $\frac{dy}{dx}$.

Sol. Putting $x^2 = \cos \theta$, we get

$$y = \tan^{-1}\left(\frac{\sqrt{1+\cos\theta} + \sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta} - \sqrt{1-\cos\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\cos\frac{\theta}{2} + \sin\frac{\theta}{2}}{\cos\frac{\theta}{2} - \sin\frac{\theta}{2}}\right)$$

$$\begin{cases} 1 + \cos 2\theta = 2\cos^2 \theta \\ 1 - \cos 2\theta = 2\sin^2 \theta \end{cases}$$

$$= \tan^{-1}\left(\frac{1 + \tan\frac{\theta}{2}}{1 - \tan\frac{\theta}{2}}\right)$$

$$\left[\tan\left(\frac{\pi}{4} + \theta\right) = \frac{1 + \tan\theta}{1 - \tan\theta} \right]$$

$$y = \frac{\pi}{4} + \frac{\theta}{2} = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x^2$$

$$\frac{dy}{dx} = -\frac{1}{2} \frac{1}{\sqrt{1-x^4}} \cdot 2x = -\frac{x}{\sqrt{1-x^4}}$$

16. If $x = a \cos \theta + b \sin \theta$, $y = a \sin \theta - b \cos \theta$, show

$$\text{that } y^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} - y = 0.$$

$$\text{Sol. } \frac{dx}{d\theta} = -a \sin \theta + b \cos \theta$$

$$\frac{dy}{d\theta} = a \cos \theta + b \sin \theta$$

$$\therefore \frac{dy}{dx} = \frac{a \cos \theta + b \sin \theta}{-a \sin \theta + b \cos \theta} = -\frac{x}{y}$$

$$\text{or } y \frac{dy}{dx} + x = 0$$

$$\therefore y \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{dy}{dx} + 1 = 0 \quad \dots(i)$$

$$\text{Using (i) we get } y \frac{d^2y}{dx^2} - \frac{x}{y} \frac{dy}{dx} + 1 = 0$$

$$\therefore y^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = 0$$

17. The side of an equilateral triangle is increasing at the rate of 2 cm/s. At what rate is its area increasing when the side of the triangle is 20 cm?

Sol. Let x be the side of an equilateral triangle

$$\therefore \frac{dx}{dt} = 2 \text{ cm}^2/\text{s.}$$

$$\text{Area (A)} = \frac{\sqrt{3}}{4} x^2$$

$$\Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}}{2} \times \frac{dx}{dt}$$

$$\Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}}{2} \cdot (20) \cdot (2) \\ = 20\sqrt{3} \text{ cm}^2/\text{s}$$

$$18. \text{Find: } \int (x+3) \sqrt{3-4x-x^2} dx .$$

Sol. Writing $x+3 = -\frac{1}{2}(-4-2x)+1$

$$\begin{aligned} \therefore \int (x+3) \sqrt{3-4x-x^2} dx \\ &= -\frac{1}{2} \int (-4-2x) \sqrt{3-4x-x^2} dx + \int \sqrt{7-(x+2)^2} dx \\ &= -\frac{1}{3} (3-4x-x^2)^{3/2} + \frac{x+2}{2} \sqrt{3-4x-x^2} \\ &\quad + \frac{7}{2} \sin^{-1} \left(\frac{x+2}{\sqrt{7}} \right) + C \end{aligned}$$

19. Three schools A, B and C organized a mela for collecting funds for helping the rehabilitation of flood victims. They sold hand made fans, mats and plates from recycled material at a cost of ₹ 25, ₹ 100 and ₹ 50 each. The number of articles sold are given below :

<i>School</i> <i>Article</i>	<i>A</i>	<i>B</i>	<i>C</i>
<i>Hand-fans</i>	40	25	35
<i>Mats</i>	50	40	50
<i>Plants</i>	20	30	40

Find the funds collected by each school separately by selling the above articles. Also find the total funds collected for the purpose.

Write one value generated by the above situation.

Sol. HF. M P

$$\begin{array}{l} A \\ B \\ C \end{array} \begin{pmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{pmatrix} \begin{pmatrix} 25 \\ 100 \\ 50 \end{pmatrix} = \begin{pmatrix} 7000 \\ 6125 \\ 7875 \end{pmatrix}$$

Funds collected by school A : ₹ 7000,

School B : ₹ 6125, School C : ₹ 7875

Total collected : ₹ 21000

Helping the flood victims.

SECTION - C

Question numbers 20 to 26 carry 6 mark each.

20. Let N denote the set of all natural numbers and R be the relation on $N \times N$ defined by $(a, b) R (c, d)$ if $ad(b+c) = bc(a+d)$. Show that R is an equivalence relation.

Sol. $\forall a, b \in N, (a, b) R (a, b)$ as $ab(b+a) = ba(a+b)$

$\therefore R$ is reflexive ... (i)

Let $(a, b) R (c, d)$ for $(a, b), (c, d) \in N \times N$

$\therefore ab(b+c) = bc(a+d)$... (ii)

Also $(c, d) R (a, b) \because cb(d+a) = da(c+b)$ (using ii)

$\therefore R$ is symmetric ... (iii)

Let $(a, b) R (c, d)$ and $(c, d) R (e, f)$, for $a, b, c, d, e, f \in N$

$\therefore ad(b+c) = bc(a+d)$ and $cf(d+e) = de(c+f)$

$$\therefore \frac{b+c}{bc} = \frac{a+d}{ad} \text{ and } \frac{d+e}{de} = \frac{c+f}{cf}$$

$$\text{i.e., } \frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a} \text{ and } \frac{1}{e} + \frac{1}{d} = \frac{1}{f} + \frac{1}{c}$$

$$\text{adding we get } \frac{1}{c} + \frac{1}{b} + \frac{1}{e} + \frac{1}{d} = \frac{1}{d} + \frac{1}{a} + \frac{1}{f} + \frac{1}{c}$$

$$\Rightarrow af(b+e) = be(a+f)$$

Hence $(a, b) R (e, f) \therefore R$ is transitive ... (iv)

From (i), (iii) and (iv) R is an equivalence relation.

* 21. Using integration find the area of the triangle formed by position tangent and normal to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$.

OR

* Evaluate $\int_1^3 (e^{2-3x} + x^2 + 1) dx$ as a limit of a sum.

22. Solve the differential equation:

$$(\tan^{-1}y - x) dy = (1 + y^2) dx.$$

OR

Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$ **given that** $y = 1$, **when** $x = 0$.

Sol. Given differential equation can be written as

$$\frac{dx}{dy} + \frac{1}{1+y^2}x = \frac{\tan^{-1}y}{1+y^2}$$

\therefore Integrating factor is $e^{\tan^{-1}y}$

$$\therefore \text{Solution is } x.e^{\tan^{-1}y} = \int \frac{\tan^{-1}y \cdot e^{\tan^{-1}y}}{1+y^2} dy$$

$$\Rightarrow x.e^{\tan^{-1}y} = \int t e^t dt \text{ where } \tan^{-1}y = t \\ = t e^t - e^t + c = e^{\tan^{-1}y} (\tan^{-1}y - 1) + c$$

$$\text{or } x = \tan^{-1}y - 1 + c e^{-\tan^{-1}y}$$

OR

$$\text{Given differential equation is } \frac{dy}{dx} = \frac{y}{x + \left(\frac{y}{x}\right)^2}$$

$$\text{Putting } \frac{y}{x} = v \text{ to get } v + x \frac{dv}{dx} = \frac{v}{1+v^2}$$

$$\therefore x \frac{dv}{dx} = \frac{v}{1+v^2} - v = \frac{-v^3}{1+v^2}$$

$$\Rightarrow \int \frac{v^2+1}{v^3} dv = - \int \frac{dx}{x}$$

$$\Rightarrow \log|v| - \frac{1}{2v^2} = -\log|x| + c$$

$$\therefore \log y - \frac{x^2}{2y^2} = c$$

$$x = 0, y = 1 \Rightarrow c = 0 \therefore \log y - \frac{x^2}{2y^2} = 0$$

* 23. If lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then find the value of k and hence

find the equation of the plane containing these lines.

24. If A and B are two independent events such $P(\bar{A} \cap B) = \frac{2}{15}$ and $P(A \cap \bar{B}) = \frac{1}{6}$, then find P(A) and P(B).

$$\text{Sol. } P(\bar{A} \cap B) = \frac{2}{15}$$

$$\Rightarrow P(\bar{A}).P(B) = \frac{2}{15}$$

$$P(A \cap \bar{B}) = \frac{1}{6}$$

$$\Rightarrow P(A).P(\bar{B}) = \frac{1}{6}$$

$$\therefore (1 - P(A)) P(B) = \frac{2}{15}$$

$$\text{or } P(B) - P(A).P(B) = \frac{2}{15} \quad \dots(i)$$

$$P(A)(1 - P(B)) = \frac{1}{6}$$

$$\text{or } P(A) - P(A).P(B) = \frac{1}{6} \quad \dots(ii)$$

$$\text{From (i) and (ii)} P(A) - P(B) = \frac{1}{6} - \frac{2}{15} = \frac{1}{30}$$

$$\text{Let } P(A) = x, P(B) = y$$

$$\therefore x = \left(\frac{1}{30} + y\right)$$

$$(i) \Rightarrow y - \left(\frac{1}{30} + y\right)y = \frac{2}{15}$$

$$\therefore 30y^2 - 29y + 4 = 0$$

$$(6y - 1)(5y - 4) = 0$$

$$\text{Solving to get } y = \frac{1}{6} \text{ or } y = \frac{4}{5}$$

$$\therefore x = \frac{1}{5} \text{ or } x = \frac{5}{6}$$

$$\text{Hence } P(A) = \frac{1}{5}, P(B) = \frac{1}{6}$$

$$\text{OR } P(A) = \frac{5}{6}, P(B) = \frac{4}{5}$$

25. Find the local maxima and local minima, of the function $f(x) = \sin x - \cos x, 0 < x < 2\pi$. Also find the local maximum and local minimum values.

Sol. $f(x) = \sin x - \cos x, 0 < x < 2\pi$

$$f'(x) = 0 \Rightarrow \cos x + \sin x = 0 \text{ or } \tan x = -1,$$

$$\therefore x = \frac{3\pi}{4}, \frac{7\pi}{4}$$

$$f''(x) = \cos x - \sin x$$

$$f''\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}$$

i.e., -ve so, $x = \frac{3\pi}{4}$ is Local Maxima

and $f''\left(\frac{7\pi}{4}\right) = -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$

i.e., + ve so, $x = \frac{7\pi}{4}$ is Local Minima

Local Maximum value = $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$

Local Minimum value = $-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = -\sqrt{2}$

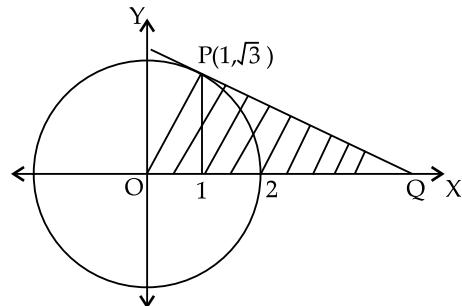
26. Find graphically, the maximum value of $z = 2x + 5y$, subject to constraints given below:

$$2x + 4y \leq 8$$

$$3x + y \leq 6$$

$$\begin{aligned} x + y &\leq 4 \\ x \geq 0, y &\geq 0 \end{aligned}$$

Sol.



Correct graph of three lines

Correctly shading

feasible region

Vertices are

$$A(0, 2), B(1.6, 1.2), C(2, 0)$$

$Z = 2x + 5y$ is maximum

at A(0, 2) and maximum value = 10

