Solved Paper 2016
Mathematics
Class-XII

Time : 3 Hours **Max. Marks : 100**

General Instructions:

1. *All questions are compulsory.*
2. *Please check that this question paper contains 26 questions.*
3. *Question 1 to 6 in Section A are very short – answer type questions carrying 1 mark each.*
4. *Questions 7 to 19 in Section B are long – answer I type question carrying 4 marks each.*
5. *Questions 20 to 26 in Section C are long – answer II type question carrying 6 marks each.*
6. *Please write down the serial number of the question before attempting it.*

Delhi Set-I **Code No : 65/1/1/D**

SECTION - A

1. Find the maximum value of

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}.$$

Ans. $\Delta = \begin{vmatrix} 1 & 0 & 0 \\ 1 & \sin\theta & 0 \\ 1 & 0 & \cos\theta \end{vmatrix} = \sin\theta\cos\theta$ ½

$= \frac{1}{2}\sin 2\theta \therefore$ Max value $= \frac{1}{2}$ ½

[CBSE Marking Scheme, 2016]

2. If A is a square matrix such that $A^2 = I$, then find the simplified value of $(A - I)^3 + (A + I)^3 - 7A$.

Ans. $(A - I)^3 + (A + I)^3 - 7A$, $A^2 = I \Rightarrow A^3 = A$ ½

$= 2A - A = A$ ½

[CBSE Marking Scheme, 2016]

3. Matrix $A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ is given to be symmetric, find values of a and b.

Ans. $\left.\begin{array}{l} 2b = 3 \text{ and } 3a = -2 \\ b = \frac{3}{2} \text{ and } a = -\frac{2}{3} \end{array}\right\}$ ½+ ½

[CBSE Marking Scheme, 2016]

4. Find the position vector of a point which divides the join of points with position vectors $\vec{a} - 2\vec{b}$ and $2\vec{a} + \vec{b}$ externally in the ratio 2 : 1.

Ans. Getting position vector as $2(2\vec{a} + \vec{b}) - 1(\vec{a} - 2\vec{b})$ ½

$= 3\vec{a} + 4\vec{b}$ ½

[CBSE Marking Scheme, 2016]

5. The two vectors $\hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ represent the two sides AB and AC, respectively of ΔABC. Find the length of the median through A.

Ans. $\overrightarrow{AD} = \frac{1}{2}[\overrightarrow{AB} + \overrightarrow{AC}]$ ½

$|\overrightarrow{AD}| = \frac{1}{2}|3\hat{i} + 5\hat{k}| = \frac{1}{2}\sqrt{34}$ ½

[CBSE Marking Scheme, 2016]

* **6. Find the vector equation of a plane which is at a distance of 5 units from the origin and its normal vector is $2\hat{i} - 3\hat{j} + 6\hat{k}$.**

SECTION - B

7. Prove that:

$$\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{8} = \frac{\neq}{4}$$

OR

Solve for the x : 2 $\tan^{-1}(\cos x) = \tan^{-1}$ (2cosec)

* *Out of Syllabus*

Ans. LHS $= \tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{7}}{1-\frac{1}{5}.\frac{1}{7}}\right)+\tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{8}}{1-\frac{1}{3}.\frac{1}{8}}\right)$ **1**

$= \tan^{-1}\left(\frac{6}{17}\right)+\tan^{-1}\left(\frac{11}{23}\right)$ **1**

$= \tan^{-1}\left(\frac{\frac{6}{17}+\frac{11}{23}}{1-\frac{6}{17}.\frac{11}{23}}\right) = \tan^{-1}\left(\frac{325}{325}\right)$ **1**

$= \tan^{-1}(1) = \frac{\pi}{4}$ **1**

[CBSE Marking Scheme, 2016]

OR

$\Rightarrow = \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}\left(\frac{2}{\sin x}\right)$ **2**

$\Rightarrow \sin x(\sin x - \cos x) = 0$ **1**

$\Rightarrow \sin x = \cos x$ ½

$\Rightarrow$ the solution is $x = \frac{\pi}{4}$ ½

[CBSE Marking Scheme, 2016]

8. The monthly incomes of Aryan and Babban are in the ratio 3 : 4 and their monthly expenditures are in the ratio 5 : 7. If each saves ₹ 15,000 per month, find their monthly incomes using matrix method. This problem reflects which value ?

Ans. Let the income be $3x$, $4x$ and expenditures, $5y$, $7y$

$\therefore \left.\begin{array}{l}3x-5y=15000\\4x-7y=15000\end{array}\right\}$ **1**

$$\begin{pmatrix}3 & -5\\4 & -7\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}15000\\15000\end{pmatrix}$$

$$\begin{pmatrix}x\\y\end{pmatrix} = -1\begin{pmatrix}-7 & 5\\-4 & 3\end{pmatrix}\begin{pmatrix}15000\\15000\end{pmatrix}$$

$x = 30000,\ y = 15000$ **1½**

$\therefore$ Incomes are ₹ 90000 and ₹ 120000 respectively ½

"Expenditure must be less than income"

(or any other relevant answer) **1**

[CBSE Marking Scheme, 2016]

9. If $x = a \sin 2t\ (1 + \cos 2t)$ and $y = b \cos 2t\ (1 - \cos 2t)$, find the value of $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$ and $t = \frac{\pi}{3}$.

OR

If $y = x^x$, prove that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$.

Ans. Here $x = a\left(\sin 2t + \frac{1}{2}\sin 4t\right)$,

$y = b\,(\cos 2t - \cos^2 2t)$

$\frac{dx}{dt} = 2a[\cos 2t + \cos 4t]$,

$\frac{dy}{dt} = 2b[-\sin 2t + 2\cos 2t \sin 2t]$

$= 2b[\sin 4t - \sin 2t]$ **1+1**

$\frac{dy}{dx} = \frac{b}{a}\left[\frac{\sin 4t - \sin 2t}{\cos 4t + \cos 2t}\right] = \frac{b}{a}\tan t$ **1**

$\left.\frac{dy}{dx}\right]_{t=\frac{\pi}{4}} = \frac{b}{a}\tan\frac{\pi}{4} = \frac{b}{a}$ ½

and $\left.\frac{dy}{dx}\right]_{t=\frac{\pi}{3}} = \sqrt{3}\frac{b}{a}$ ½

[CBSE Marking Scheme, 2016]

OR

$y = x^x \Rightarrow \log y = x.\log x$ ½

$\frac{1}{y}\frac{dy}{dx} = (1+\log x)$ **1½**

$\Rightarrow \frac{1}{y}\frac{d^2y}{dx^2} - \frac{1}{y^2}\left(\frac{dy}{dx}\right)^2 = \frac{1}{x}$ **1½**

$\Rightarrow \frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$ ½

[CBSE Marking Scheme, 2016]

10. Find the values of p and q, for which

$$f(x) = \begin{cases} \dfrac{1-\sin^3 x}{3\cos^2 x}, & \text{if } x < \dfrac{\pi}{2} \\ p, & \text{if } x = \dfrac{\pi}{2} \\ \dfrac{q(1-\sin x)}{(\pi - 2x)^2}, & \text{if } x > \dfrac{\pi}{2} \end{cases}$$

is continuous at $x = \frac{\pi}{2}$.

Ans. LHL $= \lim_{x\to\frac{\pi}{2}^-} \frac{(1-\sin x)(1+\sin x+\sin^2 x)}{3(1-\sin x)(1+\sin x)}$ **1**

$= \frac{1}{2}$ ½

$\therefore\ p = \frac{1}{2}$ ½

RHL $= \lim_{x\to\frac{\pi}{2}^+} \frac{q(1-\sin x)}{(\pi-2x)^2} = \lim_{h\to 0}\frac{q(1-\cos h)}{(-2h)^2}$, where $x - \frac{\pi}{2} = h$ **1**

$$= \lim_{h \to 0} \frac{2q \sin^2 \frac{h}{2}}{4.4.\frac{h^2}{4}} = \frac{q}{8}$$ ½

$$\therefore \frac{q}{8} = \frac{1}{2} \Rightarrow q = 4$$ ½

[CBSE Marking Scheme, 2016]

* **11. Show that the equation of normal at any point t on the curve $x = 3 \cos t - \cos^3 t$ and $y = 3 \sin t - \sin^3 t$ is $4(y \cos^3 t - x \sin^3 t) = 3 \sin 4t$.**

12. Find: $\int \frac{(3 \sin\theta - 2)\cos\theta}{5 - \cos^2\theta - 4\sin\theta} d\theta$

OR

Evaluate: $\int_0^{\pi} e^{2x} . \sin\left(\frac{\pi}{4} + x\right) dx$

Ans. $$I = \int \frac{(3\sin\theta - 2)\cos\theta}{5 - (1 - \sin^2\theta) - 4\sin\theta} d\theta$$ ½

$\sin\theta = t \Rightarrow \cos\theta \, d\theta = dt$

$$\therefore \quad I = \int \frac{3t - 2}{t^2 - 4t + 4} dt = \int \frac{3t - 2}{(t - 2)^2} dt$$ **1**

$$= 3\int \frac{dt}{t - 2} + 4\int \frac{dt}{(t - 2)^2}$$ **1**

$$= 3\log|t - 2| - \frac{4}{(t - 2)} + C$$ **1**

$$= 3\log|\sin\theta - 2| - \frac{4}{(\sin\theta - 2)} + C$$ ½

[CBSE Marking Scheme, 2016]

OR

Let $I = {}_0^{\pi} \sin \ \frac{\pi}{4} + x \ \ e^{2x} \ dx$

$$= \sin \ \frac{\pi}{4} + x \ \ \frac{e^{2x}}{2} \Big]_0^{\pi} - {}_0^{\pi} \cos \ \frac{\pi}{4} + x \ \ \frac{e^{2x}}{2} dx$$ **1**

$$I = \left[\sin\left(\frac{\pi}{4} + x\right)\frac{e^{2x}}{2} - \frac{1}{2}\left\{\cos\left(\frac{\pi}{4} + x\right)\frac{e^{2x}}{2}\right\}\right]_0^{\pi}$$

$$+ \frac{1}{2}\int_0^{\pi} -\sin\left(\frac{\pi}{4} + x\right)\frac{e^{2x}}{2} dx$$ 1

$$\frac{5}{\not{4}} I = \left\{\frac{1}{\not{4}}\left[2\sin\left(\frac{\pi}{4} + x\right) - \cos\left(\frac{\pi}{4} + x\right)\right]e^{2x}\right\}_0^{\pi}$$ **1**

$$I = \frac{1}{5}\left[\left\{2\left(-\frac{1}{\sqrt{2}}\right) + \frac{1}{\sqrt{2}}\right\}e^{2\pi} - \left\{2\left(\frac{1}{\sqrt{2}}\right) - \frac{1}{\sqrt{2}}\right\}\right]$$

$$= \frac{-1}{5\sqrt{2}}(e^{2\pi} + 1)$$ **1**

[CBSE Marking Scheme, 2016]

13. Find: $\int \sqrt{\frac{x}{a^3 - x^3}} dx$

Ans. $$I = \int \frac{\sqrt{x}}{\sqrt{a^3 - x^3}} dx$$

Put $x^{3/2} = t \Rightarrow \frac{3}{2}.x^{1/2} dx = dt \ or \ \sqrt{x} \ dx = \frac{2}{3} dt$ 1½

$$I = \frac{2}{3}\int \frac{dt}{\sqrt{(a^{3/2})^2 - t^2}}$$ **1**

$$= \frac{2}{3}.\sin^{-1}\left(\frac{t}{a^{3/2}}\right) + C$$ ½

$$= \frac{2}{3}\sin^{-1}\left(\frac{x^{3/2}}{a^{3/2}}\right) + C$$ ½

[CBSE Marking Scheme, 2016]

14. Evaluate: $\int_{-1}^{2} |x^3 - x| dx$

Ans. $$I = \int_{-1}^{2} \left|x^3 - x\right| dx$$

$$= \int_{-1}^{0} (x^3 - x) \, dx + \int_0^1 -(x^3 - x) dx + \int_1^2 (x^3 - x) \, dx$$

1½

$$= \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 + \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_1^2$$ **1½**

$$= -\left(\frac{1}{4} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + (4 - 2) - \left(\frac{1}{4} - \frac{1}{2}\right)$$

$$= \frac{1}{4} + \frac{1}{4} + 2 + \frac{1}{4} = \frac{11}{4}$$ **1**

[CBSE Marking Scheme, 2016]

15. Find the particular solution of the differential equation $(1 - y^2)$ $(1 + \log x)dx + 2xy \, dy = 0$, given that $y = 0$ when $x = 1$.

Ans. Given differential equation can be written as

$$\frac{(1 + \log x)}{x} dx + \frac{2y}{1 - y^2} dy = 0$$ **1**

integrating to get,

$$\frac{1}{2}(1 + \log x)^2 - \log|1 - y^2| = C$$ **2**

* *Out of Syllabus*

$x = 1, y = 0 \Rightarrow C = \frac{1}{2}$ ½

$\Rightarrow (1 + \log x)^2 - 2 \log |1 - y^2| = 1$ ½

[CBSE Marking Scheme, 2016]

16. Find the general solution of the following differential equation:

$$(1 + y^2) + (x - e^{\tan^{-1} y}) \frac{dy}{dx} = 0$$

Ans. Given differential equation can be written as

$$\frac{dx}{dy} + \frac{1}{1+y^2} x = \frac{e^{\tan^{-1} y}}{1+y^2}$$ 1

Integrating factor is $e^{\tan^{-1} y}$ 1

$\therefore$ Solution is $x.e^{\tan-1 y} = \int e^{2\tan^{-1} y} \frac{1}{1+y^2} dy$ 1

$\therefore \; x e^{\tan-1 y} = \frac{1}{2} e^{2\tan-1 y} + C$ 1

[CBSE Marking Scheme, 2016]

* **17. Show that the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar if $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ are coplanar.**

18. Find the vector and cartesian equations of the line through the point (1, 2, –4) and perpendicular to the two lines

$\vec{a} = (8\hat{i} - 9\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$ and

$\vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$

Ans. Vector equation of the required line is

$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \mu[(3\hat{i} - 16\hat{j} + 7\hat{k}) \times (3\hat{i} + 8\hat{j} - 5\hat{k})]$ 1

$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda[(2\hat{i} + 3\hat{j} + 6\hat{k})$ 2

in cartesian form, $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$ 1

[CBSE Marking Scheme, 2016]

19. Three persons *A, B* and *C* apply for a job of Manager in a Private company. Chance of their selection (*A, B* and *C*) are in the ratio 1 : 2 : 4. The probabilities that *A, B* and *C* can introduce changes to improve profits of the company are 0.8, 0.5 and 0.3 respectively. If the changes does not take place, find the probability that it is due to the appointment of *C*.

OR

***A* and *B* throw a pair of dice alternately. *A* wins the game if he gets a total of 7 and *B* wins the game if he gets a total of 10. If *A* starts the game, then find the probability that *B* wins.**

Ans. Let events are:

E_1 : *A is selected*
E_2 : *B is selected*
E_3 : *C is selected*
A : *Change is not introduced*

$P(E_1) = \frac{1}{7}, P(E_2) = \frac{2}{7}, P(E_3) = \frac{4}{7}$ 1

$P(A/E_1) = 0.2, P(A/E_2) = 0.5, P(A/E_3) = 0.7$ 1

$$\therefore \quad P(E_3 / A) = \frac{\frac{4}{7} \times \frac{7}{10}}{\frac{1}{7} \times \frac{2}{10} + \frac{2}{7} \times \frac{5}{10} + \frac{4}{7} \times \frac{7}{10}}$$ 1

$$= \frac{28}{40} = \frac{7}{10}$$ 1

[CBSE Marking Scheme, 2016]

OR

Prob. of success for $A = \frac{1}{6}$
Prob. of failure for $A = \frac{5}{6}$
Prob. of success for $B = \frac{1}{12}$
Prob. of failure for $B = \frac{11}{12}$ 1

B can win in 2nd or 4th or 6th or....throw 1

$$\therefore P(B) = \left(\frac{5}{6}.\frac{1}{12}\right) + \left(\frac{5}{6}.\frac{11}{12}.\frac{5}{6}.\frac{1}{12}\right) + \left(\frac{5}{6}.\frac{11}{12}.\frac{5}{6}.\frac{11}{12}.\frac{5}{6}.\frac{1}{12}\right) + ...$$ 1

$$= \frac{5}{72} \; 1 + \frac{55}{72} + \frac{55}{72}^{2} + ...$$

$$= \frac{5}{72} \times \frac{1}{1 - \frac{55}{72}} = \frac{5}{72} \times \frac{72}{17} = \frac{5}{17}$$ 1

[CBSE Marking Scheme, 2016]

SECTION - C

20. Let $f : N \to N$ be a function defined as $f(x) = 9x^2 + 6x - 5$. Show that $f : N \to S$, where S is the range of f. Find the inverse of f and hence find f^{-1} (43) and f^{-1} (163).

Ans. Let $x_1, x_2 \in$ N and $f(x_1) = f(x_2)$

$9x_1^2 + 6x_1 - 5 = 9x_2^2 + 6x_2 - 5$

$\Rightarrow 9(x_1^2 - x_2^2) + 6(x_1 - x_2) = 0$

$\Rightarrow (x_1 - x_2)(9x_1 + 9x_2 + 6) = 0$

* *Out of Syllabus*

$\Rightarrow$ $x_1 - x_2 = 0$ or $x_1 = x_2$ as $(9x_1 + 9x_2 + 6) \neq 0$, $x_1, x_2 \in N$

$\therefore$ f is a one-one function **2**

f: N $\to$ S is ONTO as co-domain = Range **1**

Hence f is invertible

$$y = 9x^2 + 6x - 5 = (3x+1)^2 - 6 \Rightarrow x = \frac{\sqrt{y+6}-1}{3}$$

$$\therefore\ f^{-1}(y) = \frac{\sqrt{y+6}-1}{3},\ \ y \in S$$ **2**

$$f^{-1}(43) = \frac{\sqrt{49}-1}{3} = 2$$ **½**

$$f^{-1}(163) = \frac{\sqrt{169}-1}{3} = 4$$ **½**

[CBSE Marking Scheme, 2016]

21. Prove that $\begin{vmatrix} yz-x^2 & zx-y^2 & xy-z^2 \\ zx-y^2 & xy-z^2 & yz-x^2 \\ xy-z^2 & yz-x^2 & zx-y^2 \end{vmatrix}$ **is divisible by $(x + y + z)$, and hence find the quotient.**

OR

* **Using elementary transformations, find the inverse of the matrix** $A = \begin{pmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ **and use it to solve the following system of linear equation:**

$$8x + 4y + 3z = 19$$
$$2x + y + z = 5$$
$$x + 2y + 2z = 7$$

Ans. Using $C_1 \to C_1 - C_3$ and $C_2 \to C_2 - C_3$ we get

$$\Delta = \begin{vmatrix} y(z-x)+z^2-x^2 & x(z-y)+z^2-y^2 & xy-z^2 \\ z(x-y)+x^2-y^2 & y(x-z)+x^2-z^2 & yz-x^2 \\ x(y-z)+y^2-z^2 & z(y-x)+y^2-x^2 & zx-y^2 \end{vmatrix}$$ **2**

Taking $(x + y + z)$ common from C_1 & C_2

$$\Rightarrow \Delta = (x+y+z)^2 \begin{vmatrix} z-x & z-y & xy-z^2 \\ x-y & x-z & yz-x^2 \\ y-z & y-x & zx-y^2 \end{vmatrix}$$ **1**

$R_1 \to R_1 + R_2 + R_3$

$$\Delta = (x+y+z)^2 \begin{vmatrix} 0 & 0 & xy+yz+zx-x^2-y^2-z^2 \\ x-y & x-z & yz-x^2 \\ y-z & y-x & zx-y^2 \end{vmatrix}$$ **1**

Expanding to get

$\Delta = (x + y + z)^2 (xy + zy + zx - x^2 - y^2 - z^2)^2$ **1**

Hence Δ is divisible by $(x + y + z)$ and the quotient is

$(x + y + z)(xy + yz + zx - x^2 - y^2 - z^2)^2$ **1**

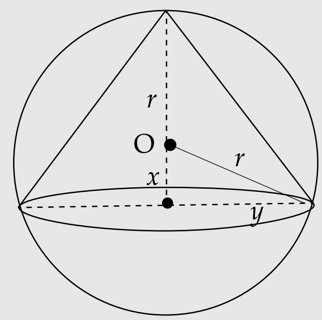
[CBSE Marking Scheme, 2016]

22. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$. Also find maximum volume in terms of volume of the sphere.

OR

Find the intervals in which $f(x) = \sin 3x - \cos 3x$, $0 < x < \pi$, is strictly increasing or strictly decreasing.

Ans.



Let radius of cone be y and the altitude be $r + x$

$\therefore x^2 + y^2 = r^2$ ...(i) **½**

Volume $V = \frac{1}{3}\pi y^2 (r+x)$

$$= \frac{1}{3}\pi (r^2 - x^2)(r+x)$$ **1**

$$\frac{dV}{dx} = \frac{\pi}{3}\left[(r^2 - x^2)1 + (r+x)(-2x)\right] = \frac{\pi}{3}(r+x)(r-3x)$$ **1**

$$\frac{dV}{dx} = 0 \Rightarrow x = \frac{r}{3}$$ **½**

$\therefore$ Altitude $= r + \frac{r}{3} = \frac{4r}{3}$ **½**

and

$$\frac{d^2V}{dx^2} = \frac{\pi}{3}\left[(r+x)(-3) + (r-3x)\right] = \frac{\pi}{3}[-2r - 6x] < 0$$ **1**

$$\therefore \text{Max. Volume} = \frac{\pi}{3}\left(r^2 - \frac{r^2}{9}\right)\left(r + \frac{r}{3}\right) = \frac{8}{27}\left(\frac{4}{3}\pi r^3\right)$$ **½**

$= \frac{8}{27}$ (Vol. of sphere)

[CBSE Marking Scheme, 2016]

* *Out of Syllabus*

OR

$f(x) = \sin 3x - \cos 3x,\ 0 < x < \pi$

$f'(x) = 3\cos 3x + 3\sin 3x$ **1**

$f'(x) = 0 \Rightarrow \tan 3x = -1 = \tan\dfrac{3\pi}{4}$ **½**

$\Rightarrow x = \dfrac{n\pi}{3} + \dfrac{\pi}{4}, n \in Z$

$\Rightarrow x = \dfrac{\pi}{4}, \dfrac{7\pi}{12}, \dfrac{11\pi}{12}$ **1½**

Intervals are:

$\left(0, \dfrac{\pi}{4}\right), \left(\dfrac{\pi}{4}, \dfrac{7\pi}{12}\right), \left(\dfrac{7\pi}{12}, \dfrac{11\pi}{12}\right), \left(\dfrac{11\pi}{12}, \pi\right)$ **1**

$f(x)$ is strictly increasing in

$$\left(0, \frac{\pi}{4}\right) \cup \left(\frac{7\pi}{12}, \frac{11\pi}{12}\right)$$ **1**

and strictly decreasing in

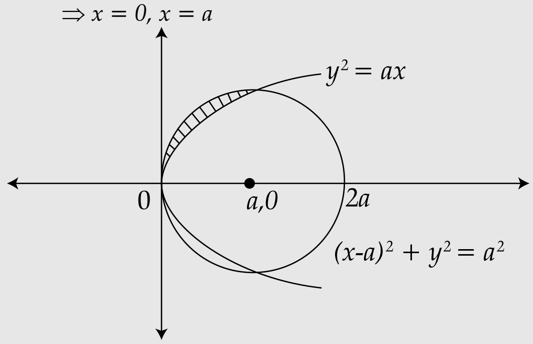
$$\left(\frac{\pi}{4}, \frac{7\pi}{12}\right) \cup \left(\frac{11\pi}{12}, \pi\right)$$ **1**

[CBSE Marking Scheme, 2016]

23. Using integration find the area of the region $\{(x, y) : x^2 + y^2 \le 2ax, y^2 \ge ax, x, y \ge 0\}$.

Ans. $y^2 = ax,\ x^2 + y^2 = 2ax \Rightarrow x^2 - ax = 0$

$\Rightarrow x = 0, x = a$ **1**



1

Shaded area

$= \left[\int_0^a [\sqrt{a^2 - (x-a)^2} - \sqrt{a}\sqrt{x}]\,dx\right.$ 1

$$A = \left[\frac{x-a}{2}\sqrt{a^2 - (x-a)^2} + \frac{a^2}{2}\sin^{-1}\frac{x-a}{a} - \sqrt{a}\,\frac{2}{3}x^{3/2}\right]_0^a$$ **2**

$$= \left[-\frac{2}{3}a^2 + \frac{a^2}{2}\frac{\pi}{2}\right] = \frac{\pi a^2}{4} - \frac{2a^2}{3} \text{ sq. units}$$ **1**

[CBSE Marking Scheme, 2016]

* **24. Find the coordinate of the point P where the line through $A(3, -4, -5)$ and $B(2, -3, 1)$ crosses the plane passing through three points $L(2, 2, 1)$, $M(3, 0, 1)$ and $N(4, -1, 0)$. Also, find the ratio in which P divides the line segment AB.**

* **25. An urn contains 3 white and 6 red balls. Four balls are drawn one by one with replacement from the urn. Find the probability distribution of the number of red balls drawn. Also find mean and variance of the distribution.**

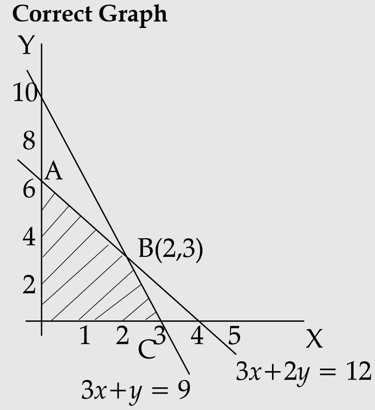
26. A manufacturer produces two products A and B. Both the products are processed on two different machines. The available capacity of first machine is 12 hours and that of second machine is 9 hours per day. Each unit of product A requires 3 hours on both machines and each unit of product B requires 2 hours on first machine and 1 hour on second machine. Each unit of product A is sold at ₹ 7 profit and that of B at a profit of ₹ 4. Find the production level per day for maximum profit graphically.

Ans. Let production of A, B (per day) be x, y respectively

Maximise $P = 7x + 4y$ **1**

Subject to
$$\left.\begin{array}{l} 3x + 2y \le 12 \\ 3x + y \le 9 \\ x \ge 0, y \ge 0 \end{array}\right\}$$ **2**

Correct Graph



2

P(A) = 24

P(B) = 26

P(C) = 21

∴ 2 units of product A and 3 units of product B for maximum profit **1**

[CBSE Marking Scheme, 2016]

Delhi Set-II & Set-III — Code No : 65/1/2/D

Note : All questions are from Set I.

* *Out of Syllabus*

Outside Delhi Set-I — Code No : 65/1/C

SECTION - A

1. **If $x \in N$ and $\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$, then find the value of x.**

Ans. $(x+3)2x-(-2)(-3x)=8$ ½

$2x^2 = 8$

$\therefore x = 2$ ½

[CBSE Marking Scheme, 2016]

* 2. **Use elementary column operation $C_2 \to C_2 + 2C_1$ in the following matrix equation :**

$$\begin{pmatrix} 2 & 1 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1 & +1 \end{pmatrix}$$

3. **Write the number of all possible matrices of order 2 × 2 with each entry 1, 2 or 3.**

Ans. No. of possible matrices $= 3^4$ or 81 1

[CBSE Marking Scheme, 2016]

4. **Write the position vector of the point which divides the join of points with position vectors $3\vec{a}-2\vec{b}$ and $2\vec{a}+3\vec{b}$ in the ratio 2 : 1.**

Ans. $\dfrac{2(2\vec{a}+3\vec{b})+1(3\vec{a}-2\vec{b})}{2+1}$ ½

$= \dfrac{7}{3}\vec{a}+\dfrac{4}{3}\vec{b}$

(or enternal division may also beconsidered)

½

[CBSE Marking Scheme, 2019]

5. **Write the number of vectors of unit length perpendicular to both the vectors $\vec{a} = 2\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b} = \hat{j}+\hat{k}$.**

Ans. 2 1

[CBSE Marking Scheme, 2016]

* 6. **Find the vector equation of the plane with intercepts 3, – 4 and 2 on *X*, *Y* and *X*-axis respectively.**

* *Out of Syllabus*

SECTION - B

* 7. **Find the coordinates of the point where the line through the points *A*(3, 4, 1) and *B*(5, 1, 6) crosses the *XZ* plane. Also find angle which this line makes with the *XZ* plane.**

8. **The two adjacent sides of a parallelogram are $2\hat{i}-4\hat{j}-5\hat{k}$ and $2\hat{i}+2\hat{j}+3\hat{k}$. Find the two unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram.**

Ans. let $\vec{d}_1$ & $\vec{d}_2$ be the two diagonal vectors:

$\therefore \vec{d}_1 = 4\hat{i}-2\hat{j}-2\hat{k},\ \ \vec{d}_2 = -6\hat{j}-8\hat{k}$ ½ + ½

or $\vec{d}_2 = 6\hat{j}+8\hat{k}$

Unit vectors parallel to the diagonals are:

$\hat{d}_1 = \dfrac{2}{\sqrt{6}}\hat{i}-\dfrac{1}{\sqrt{6}}\hat{j}-\dfrac{1}{\sqrt{6}}\hat{k}$ ½

$\hat{d}_2 = -\dfrac{3}{5}\hat{j}-\dfrac{4}{5}\hat{k}\left(\text{or } \hat{d}_2 = \dfrac{3}{5}\hat{j}+\dfrac{4}{5}\hat{k}\right)$ ½

$\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -2 & -2 \\ 0 & -6 & -8 \end{vmatrix} = 4\hat{i}+32\hat{j}-24\hat{k}$

1

Area of parallelogram

$= -\left|\vec{d}_1 \times \vec{d}_2\right| = \sqrt{404}$ or $2\sqrt{101}$ sq. units 1

[CBSE Marking Scheme, 2016]

9. **In a game, a man wins ₹ 5 for getting a number greater then 4 and loses ₹ 1 otherwise, when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a number greater than 4. Find the expected value of the amount he wins/loses.**

OR

A bag contains 4 balls. Two balls are drawn at random (without replacement) and are found to be white. What is the probability that all balls in the bag are white ?

Ans. let X = Amount he wins then x = ₹ 5, 4, 3, – 3

P = Probability of getting a no. 1

$> 4 = \dfrac{1}{3},\ q = 1-p = \dfrac{2}{3}$ ½

X:	5	4	3	-3
P(x)	$\frac{1}{3}$	$\frac{2}{3}.\frac{1}{3}=\frac{2}{9}$	$\left(\frac{2}{3}\right)^2.\frac{1}{3}=\frac{4}{27}$	$\left(\frac{2}{3}\right)^3=\frac{8}{27}$

2

Expected amount he wins

$= \Sigma XP(X) = \frac{5}{3}+\frac{8}{9}+\frac{12}{27}-\frac{24}{27}$

$=₹\ \frac{19}{9}$ or ₹ $2\frac{1}{9}$ ½

[CBSE Marking Scheme, 2016]

OR

Ans.

E_1 = Event that all balls are white,
E_2 = Event that 3 balls are white and 1 ball is non white
E_3 = Event that 2 balls are white and 2 balls are non-white
A = Event that 2 balls drawn without replacement are white
1

$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$ ½

$P(A/E_1) = 1,\ P(A/E_2) = \frac{3}{4}.\frac{2}{3} = \frac{1}{2}, P(A/E_3) = \frac{2}{4}.\frac{1}{3} = \frac{1}{6}$ 1½

$P(E_1/A) = \frac{1.\frac{1}{3}}{1.\frac{1}{3}+\frac{1}{3}.\frac{1}{2}+\frac{1}{3}.\frac{1}{6}} = \frac{3}{5}$ 1

[CBSE Marking Scheme, 2016]

10. Differentiate $x^{\sin x} + (\sin x)^{\cos x}$ with respect to x.

OR

If $y = 2\cos(\log x) + 3\sin(\log x)$, prove that $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y = 0$.

Ans. let $y = u+v, u = x^{\sin x},\ v = (\sin x)^{\cos x}$

$\log u = \sin x.\log x \Rightarrow \frac{du}{dx} = x^{\sin x}.\left\{\cos x.\log x + \frac{\sin x}{x}\right\}$ ½ +1

$\log v = \cos x.\log(\sin x) \Rightarrow \frac{dv}{dx} = (\sin x)^{\cos x}.\{\cos x.\cot x - \sin x.\log(\sin x)\}$ ½ +1

$\frac{dy}{dx} = \frac{du}{dx}+\frac{dv}{dx} = x^{\sin x}.\left\{\cos x.\log x+\frac{\sin x}{x}\right\} + (\sin x)^{\cos x}\{\cos x.\cot x - \sin x.\log(\sin x)\}$ ½ +½

[CBSE Marking Scheme, 2016]

OR

Ans. $\frac{dy}{dx} = \frac{-2\sin(\log x)}{x}+\frac{3\cos(\log x)}{x}$ 1

$\Rightarrow x\frac{dy}{dx} = -2\sin(\log x)+3\cos(\log x)$, differentiate w.r.t '$x$' ½

$\Rightarrow x\frac{d^2y}{dx^2}+\frac{dy}{dx} = \frac{-2\cos(\log x)}{x}-\frac{3\sin(\log x)}{x}$ 2

$\Rightarrow x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx} = -y \Rightarrow x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y = 0$ ½

[CBSE Marking Scheme, 2016]

11. If $x = a\sin 2t(1+\cos 2t)$ and $y = b\cos 2t(1-\cos 2t)$, find $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$.

Ans. $\frac{dx}{dt} = 2a\cos 2t(1+\cos 2t) - 2a\sin 2t.\sin 2t$ 1½

$\frac{dy}{dt} = -2b\sin 2t(1-\cos 2t)+2b\cos 2t.\sin 2t$ 1

$= \left.\frac{dy}{dx}\right]_{t=\frac{\pi}{4}} = \left.\frac{2b\cos 2t.\sin 2t - 2b\sin 2t(1-\cos 2t)}{2a\cos 2t.(1+\cos 2t)-2a\sin 2t\sin 2t}\right]_{t=\frac{\pi}{4}}$

$= \frac{b}{a}$ ½

[CBSE Marking Scheme, 2016]

* **12. The equation of tangent at (2, 3) on the curve $y^2 = ax^3 + b$ is $y = 4x - 5$. Find the values of a and b.**

13. Find: $\int\frac{x^2}{x^4+x^2-2}dx$

Ans. Let $x^2 = t \therefore \frac{x^2}{x^4+x^2-2} = \frac{x^2}{(x^2-1)(x^2+2)}$ 1

$= \frac{t}{(t-1)(t+2)} = \frac{A}{t-1}+\frac{B}{t+2}$

Solving for A and B to get, $A = \frac{1}{3}, B = \frac{2}{3}$ 1

$\int\frac{x^2}{x^4+x^2-2}dx = \frac{1}{3}\int\frac{1}{x^2-1}dx+\frac{2}{3}\int\frac{1}{x^2+2}dx$

$= \frac{1}{6}\log\left|\frac{x-1}{x+1}\right|+\frac{\sqrt{2}}{3}\tan^{-1}\frac{x}{\sqrt{2}}+c$ 1+1

[CBSE Marking Scheme, 2016]

14. Evaluate: $\int_0^{\frac{\pi}{2}}\frac{\sin^2 x}{\sin x+\cos x}dx$

OR

Evaluate: $\int_0^{\frac{3}{2}}|x\cos\pi x|dx$

* *Out of Syllabus*

Ans. Let $I = \int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x} dx,$

Also

$$I = \int_0^{\pi/2} \frac{\sin^2\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx = \int_0^{\pi/2} \frac{\cos^2 x}{\cos x + \sin x} dx \qquad 1$$

Adding to get,

$$2I = \int_0^{\pi/2} \frac{1}{\sin x + \cos x} dx \qquad ½ + 1$$

$$2I = \frac{1}{\sqrt{2}} \int_0^{\pi/2} \sec(x - \pi/4) dx = \frac{1}{\sqrt{2}}$$

$$\log\left|\sec\left(x - \frac{\pi}{4}\right) + \tan\left(x - \frac{\pi}{4}\right)\right|_0^{\pi/2} \qquad 1$$

$$2I = \frac{1}{\sqrt{2}}\left\{\log\left|\sqrt{2}+1\right| - \log\left|\sqrt{2}-1\right|\right\}$$

$$I = \frac{1}{2\sqrt{2}}\left\{\log\left|\sqrt{2}+1\right| - \log\left|\sqrt{2}-1\right| \; or\right.$$

$$\frac{1}{2\sqrt{2}} \log\left|\frac{\sqrt{2}+1}{\sqrt{2}-1}\right| \qquad ½$$

[CBSE Marking Scheme, 2016]

OR

Ans. $\int_0^{3/2} |x \cos \pi x| dx = \int_0^{1/2} x \cos \pi x \, dx - \int_{1/2}^{3/2} x \cos \pi x \, dx$

1½

$$= \left\{\frac{x \sin \pi x}{\pi} + \frac{\cos \pi x}{\pi^2}\right\}_0^{1/2} - \left\{\frac{x \sin \pi x}{\pi} + \frac{\cos \pi x}{\pi^2}\right\}_{1/2}^{3/2}$$

1½

$$= \frac{1}{2\pi} - \frac{1}{\pi^2} - \left(-\frac{3}{2\pi} - \frac{1}{2\pi}\right) = \frac{5}{2\pi} - \frac{1}{\pi^2} \qquad 1$$

[CBSE Marking Scheme, 2016]

15. Find: $\int (3x+1)\sqrt{4 - 3x - 2x^2} dx$

Ans.

$$\int (3x+1)\sqrt{4-3x-2x^2} dx \qquad 1$$

$$= -\frac{3}{4}\int(-4x-3)\sqrt{4-3x-2x^2} dx - \frac{5}{4}\int\sqrt{4-3x-2x^2} dx$$

$$= -\frac{1}{2}(4-3x-2x^2)^{3/2} - \frac{5}{4}\sqrt{2}\int\sqrt{\left(\frac{\sqrt{41}}{4}\right)^2 - \left(x+\frac{3}{4}\right)^2} dx$$

1 + 1

$$= -\frac{1}{2}(4-3x-2x^2)^{3/2} - \frac{5}{4}\sqrt{2}$$
$$\left\{\frac{4x+3}{8}\sqrt{\frac{41}{16} - \left(x+\frac{3}{4}\right)^2} + \frac{41}{32}.\sin^{-1}\left(\frac{4x+3}{\sqrt{41}}\right)\right\} + C$$

$$= -\frac{1}{2}(4-3x-2x^2)^{3/2} - \frac{5}{4}$$
$$\left\{\frac{4x+3}{8}\sqrt{4-3x-2x^2} + \frac{41}{32}.\sin^{-1}\left(\frac{4x+3}{\sqrt{41}}\right)\right\} + C \qquad 1$$

[CBSE Marking Scheme, 2016]

16. Solve the differential equation:

$$y + x\frac{dy}{dx} = x - y\frac{dy}{dx}$$

Ans. The differential equation can be re-written as:

$\frac{dy}{dx} = \frac{x-y}{x+y}$, put $y = vx$, $\frac{dy}{dx} = v + x\frac{dv}{dx}$ ½ + ½

$$\Rightarrow v + x\frac{dv}{dx} = \frac{1-v}{1+v} \Rightarrow \frac{1+v}{1-2v-v^2} dv = \frac{1}{x} dx \qquad 1$$

integrating we get

$$\Rightarrow \frac{1}{2}\int \frac{2V+2}{V^2+2V-1} dv$$
$$= -\int \frac{1}{x} dx = \frac{1}{2}\log\left|V^2+2V-1\right|$$
$$= -\log x + \log C \qquad 1½$$

∴ Solution of the differential equation is:

$$\frac{1}{2}\log\left|\frac{y^2}{x^2} + \frac{2y}{x} - 1\right| = \log C - \log x \text{ or,}$$

$y^2 + 2xy - x^2 = C^2$ ½

[CBSE Marking Scheme, 2016]

* **17. Form the differential equation of the family of circles in the second quadrant and touching the coordinate axes.**

18. Solve the equation for x : $\sin^{-1} x + \sin^{-1}(1-x) = \cos^{-1} x$

OR

If $\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = \alpha$, prove that

$$\frac{x^2}{a^2} - 2\frac{xy}{ab}\cos\alpha + \frac{y^2}{b^2} = \sin^2\alpha$$

Ans. $\sin^{-1} x + \sin^{-1}(1-x) = \cos^{-1} x$ 1

$$\Rightarrow \sin^{-1}(1-x) = \frac{\pi}{2} - 2\sin^{-1} x$$

$$1 - x = \sin\left(\frac{\pi}{2} - 2\sin^{-1} x\right)$$

$$\Rightarrow 1 - x = \cos\left(2\sin^{-1} x\right)$$

* *Out of Syllabus*

$\Rightarrow 1-x=1-2\sin^2\left(\sin^{-1}x\right)$ **1**

$1-x = 1-2x^2$

$2x^2 - x = 0$

Solving we get, $x = 0$ or $x = \dfrac{1}{2}$ **1**

[CBSE Marking Scheme, 2016]

OR

Ans. From the equation: $\left\{\begin{array}{l}\cos^{-1}\dfrac{x}{a}+\cos^{-1}\dfrac{y}{b}=\alpha \\ \therefore \cos^{-1}\dfrac{x}{a}=\alpha-\cos^{-1}\dfrac{y}{b}\end{array}\right\}$

$$\frac{x}{a}=\cos\left(\alpha-\cos^{-1}\frac{y}{b}\right)\Rightarrow\frac{x}{a}=\cos\alpha\cdot\cos\left(\cos^{-1}\frac{y}{b}\right)+\sin\alpha\cdot\sin\left(\cos^{-1}\frac{y}{b}\right)$$

1 + 1

$$\Rightarrow\frac{x}{a}=\frac{y\cdot\cos\alpha}{b}+\sin\,\alpha\sqrt{1-\frac{y^2}{b^2}}$$

$$\Rightarrow\frac{x}{a}-\frac{y}{b}\cos\alpha=\sin\alpha\sqrt{1-\frac{y^2}{b^2}}$$ **1**

Squaring both sides,

$$\Rightarrow\left(\frac{x}{a}-y\frac{\cos\alpha}{b}\right)^2=\left(\sin\alpha\sqrt{1-\frac{y^2}{b^2}}\right)^2$$

$$\Rightarrow\frac{x^2}{a^2}-\frac{2xy}{ab}\cdot\cos\alpha+\frac{y^2}{b^2}=\sin^2\alpha.$$ **½**

[CBSE Marking Scheme, 2016]

19. A trust invested some money in two type of bonds. The first bond pays 10% interest and second bond pays 12% interest. The trust received ₹ 2,800 as interest. However, if trust had interchanged money in bonds, they would have got ₹ 100 less as interest. Using matrix method, find the amount invested by the trust. Interest received on this amount will be given Helpage India as donation. Which is reflected in the question ?

Ans. let ₹x be invested in first bond

and ₹**y** be invested in second bond

then the system of equations is:

$$\left.\begin{array}{l}\dfrac{10x}{100}+\dfrac{12y}{100}=2800\\ \dfrac{12x}{100}+\dfrac{10y}{100}=2700\end{array}\right\}\Rightarrow\left.\begin{array}{l}5x+6y=140000\\6x+5y=135000\end{array}\right\}$$ **1**

$$A=\begin{bmatrix}5&6\\6&5\end{bmatrix};X=\begin{bmatrix}x\\y\end{bmatrix};B\begin{bmatrix}140000\\135000\end{bmatrix}$$

$\therefore$ A·X = B

$$|A|=-11;A^{-1}=\frac{1}{-11}\begin{bmatrix}5&-6\\-6&5\end{bmatrix}$$ **1**

$\therefore$ Solution is $X=A^{-1}B\Rightarrow\begin{bmatrix}x\\y\end{bmatrix}$

$$=\frac{1}{-11}\begin{bmatrix}5&-6\\-6&5\end{bmatrix}\begin{bmatrix}140000\\135000\end{bmatrix}=\begin{bmatrix}10000\\15000\end{bmatrix}$$

$\therefore x=10000, y=15000,$

$\therefore$ Amount invested =(rupee)25000 **½ + ½**

Value: caring elders **1**

[CBSE Marking Scheme, 2016]

SECTION - C

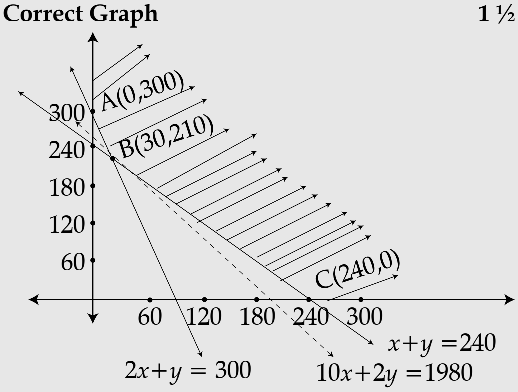
20. There are two types of fertilisers 'A' and 'B'. 'A' consists of 12% nitrogen and 5% phosphoric acid whereas 'B' consists of 4% nitrogen and 5% phosphoric acid. After testing the soil conditions, farmer finds that he needs at least 12 kg of nitrogen and 12 kg of phosphoric acid for his crops. If 'A' costs ₹ 10 per kg and 'B' cost ₹ 8 per kg, then graphically determine how much of each type of fertiliser should be used so that nutrient requirements are met at a minimum cost.

Ans. Let x kg of fertilizer A be used and y kg of fertilizer B be used then the linear programming problem is:

Minimise cost: z = $10x + 8y$ **1**

$$\left.\begin{array}{l}\text{Subject to}\dfrac{12x}{100}+\dfrac{4y}{100}\geq 12\Rightarrow 3x+y\geq 300\\ \dfrac{5x}{100}+\dfrac{5y}{100}\geq 12\Rightarrow x+y\geq 240\\ x,y\geq 0\end{array}\right\}$$ **2**

Correct Graph **1 ½**



Value of Z at corners of the unbounded region ABC:

corner	value of Z	
$A(0,300)$	(₹)2400	
$B(30,210)$	(₹)1980(Minimum)	1
$C(240,0)$	(₹)2400	

The region of $10x + 8y < 1980$ or $5x + 4y < 990$ has no point in common to the feasible region. Hence, minimum cost = ₹ 1980 at $x = 30$ and $y = 210$ ½

[CBSE Marking Scheme, 2016]

* **21. Five bad oranges are accidently mixed with 20 good ones. If four oranges are drawn one by one successively with replacement, then find the probability distribution of number of bad oranges drawn. Hence find the mean and variance of the distribution.**

* **22. Find the position vector of the foot of perpendicular and the perpendicular distance from the point P with position vector $2\hat{i}+3\hat{j}+4\hat{k}$ to the plane $\vec{r}.(2\hat{i}+\hat{j}+3\hat{k})-26=0$. Also find image of P in the plane.**

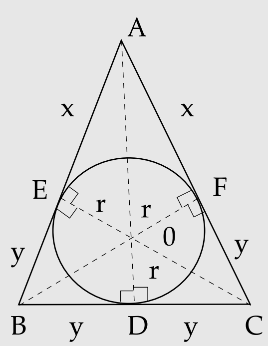
* **23. Show that the binary operation * on A = R – {– 1} defined as $a*b = a + b + ab$ for all $a, b \in$ A is commutative and associative on A. Also find the identity element of * in A and prove that every element of A is invertible.**

24. Prove that the least perimeter of an isosceles triangle in which a circle of radius r can be inscribed is $6\sqrt{3}\,r$.

OR

If the sum of lengths of hypotenuse and a side of a right angled triangle is given, show that the area of triangle is maximum, when the angle between them is $\frac{\pi}{3}$.

Ans.



Let ΔABC be isosceles with inscribed circle of radius 'r' touching sides AB, AC and BC at E, F and D respectively.

let $AE = AF = x$, $BE = BD = y$, $CF = CD = y$ then (as D is mid point of BC)

area (ΔABC) = ar(ΔAOB) + ar(ΔAOC) + ar(ΔBOC) **1**

$$\Rightarrow \frac{1}{2}.2y\left(r+\sqrt{r^2+x^2}\right)=\frac{1}{2}\{2yr+2(x+y)r\}$$

$$y\sqrt{r^2+x^2}=(x+y)r$$

$$y^2r^2+x^2y^2=x^2r^2+y^2r^2+2xyr^2$$

$$x^2(y^2-r^2)=2xyr^2$$

$$x=\frac{2r^2y}{y^2-r^2}$$

$$\Rightarrow x=\frac{2r^2y}{y^2-r^2}$$ **1**

Then,

P(Perimeter of ΔABC) = $2x + 4y$

$$=\frac{4r^2y}{y^2-r^2}+4y$$ **1**

$$\frac{dP}{dy}=\frac{-4r^2(r^2+y^2)}{(y^2-r^2)^2}+4 \text{ and } \frac{dP}{dy}=0$$

$$\Rightarrow y=\sqrt{3}r$$ **1 + ½**

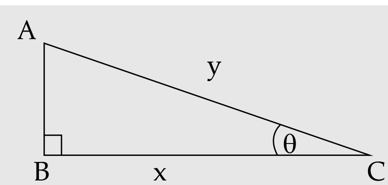
$$\left.\frac{d^2P}{dy^2}\right]_{y=\sqrt{3}r}=\frac{4r^2y(2y^2+6r^2)}{(y^2-r^2)^3}=\frac{6\sqrt{3}}{r}>0$$ ½

∴ Perimeter is least iff $y=\sqrt{3}r$ and least perimeter is

$$P=4y+\frac{4r^2y}{y^2-r^2}=4\sqrt{3}r+\frac{4r^2\sqrt{3}r}{2r^2}=6\sqrt{3}r$$ **1**

[CBSE Marking Scheme, 2016]

OR



let ABC be the right triangle with $\angle B = 90°$

$\angle ACB = \theta$, $AC = y$, $BC = x$, $x + y = k$ (constant)

A (Area of triangle)

$$=\frac{1}{2}.BC.AB=\frac{1}{2}\cdot x\sqrt{y^2-x^2}$$ **1 ½**

* *Out of Syllabus*

let $z = A^2 = \frac{1}{4}x^2\left(y^2 - x^2\right)$

$$= \frac{1}{4}x^2\left\{(k-x)^2 - x^2\right\}$$

$$= \frac{1}{4}\left(x^2k^2 - 2kx^3\right) \quad \mathbf{1}$$

$$\frac{dz}{dx} = \frac{1}{4}(2xk^2 - 6kx^2) \text{ and } \frac{dz}{dx} = 0$$

$$\Rightarrow \quad x = \frac{k}{3}, y = k - x = \frac{2k}{3} \quad \mathbf{1 + 1}$$

$$\left.\frac{d^2Z}{dx^2}\right]_{x=\frac{k}{3}} = \frac{1}{4}(2k^2 - 12kx)\Big]_{x=\frac{k}{3}} = -\frac{k^2}{2} < 0 \quad ½$$

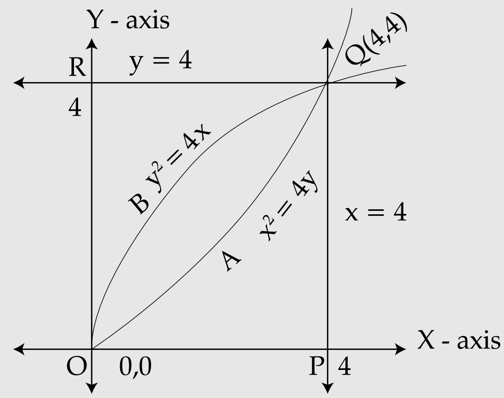
$\therefore$ z and area of ΔABC is max at $x = \frac{k}{3}$

and, $\cos\theta = \frac{x}{y} = \frac{k}{3}\cdot\frac{3}{2k} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$ **1**

[CBSE Marking Scheme, 2016]

25. Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of square bounded by $x = 0, x = 4, y = 4$ and $y = 0$ into three equal parts.

Ans. Image Point of intersection of $y^2 = 4x$ and $x^2 = 4y$ are (0, 0) and (4, 4);



1½

are $(OAQBO) = \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right)dx$ **1**

$$= \left[\left\{\frac{4}{3}x^{3/2} - \frac{x^3}{12}\right\}\right]_0^4$$

$$= \frac{32}{3} - \frac{16}{3} = \frac{16}{3} \quad ½$$

area $(OPQAO) = \int_0^4 \frac{x^2}{4}dx = \left.\frac{1}{12}x^3\right]_0^4 = \frac{16}{3}$ **1½**

area $(OBQRO) = \int_0^4 \frac{y^2}{4}dy = \left.\frac{1}{12}y^3\right]_0^4 = \frac{16}{3}$ **1½**

Hence the areas of the three regions are equal.

[CBSE Marking Scheme, 2016]

26. Using properties of determinants, show that ΔABC is isosceles if:

$$\begin{vmatrix} 1 & 1 & 1 \\ 1+\cos A & 1+\cos B & 1+\cos C \\ \cos^2 A + \cos A & \cos^2 B + \cos B & \cos^2 C + \cos C \end{vmatrix} = 0$$

OR

A shopkeeper has 3 varieties of pens '*A*', '*B*' and '*C*'. Meenu purchased 1 pen of each variety for a total of ₹ 21. Jeevan purchased 4 pens of '*A*' variety, 3 pens of '*B*' variety and 2 pens of '*C*' variety for ₹ 60. While Shikha purchased 6 pens of '*A*' variety, 2 pens of '*B*' variety and 3 pens of '*C*' variety for ₹ 70. Using matrix method, find cost of each variety of pen.

Ans.
$$\begin{vmatrix} 1 & 1 & 1 \\ 1+\cos A & 1+\cos B & 1+\cos C \\ \cos^2 A + \cos A & \cos^2 B + \cos B & \cos^2 C + \cos C \end{vmatrix} = 0$$

Apply $C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$

$$\Leftrightarrow \begin{vmatrix} 1 & 0 & 0 \\ 1+\cos A & \cos B - \cos A & \cos C - \cos A \\ \cos^2 A + \cos A & (\cos B - \cos A)(\cos B + \cos A + 1) & (\cos C - \cos A)(\cos C + \cos A + 1) \end{vmatrix} = 0 \quad \mathbf{3}$$

Taking $(\cos B - \cos A)$, $(\cos C - \cos A)$ common from C_2 & C_3

$$\Leftrightarrow (\cos B - \cos A)(\cos C - \cos A)\begin{vmatrix} 1 & 0 & 0 \\ 1+\cos A & 1 & 1 \\ \cos^2 A + \cos A & \cos B + \cos A + 1 & \cos C + \cos A + 1 \end{vmatrix} = 0 \quad \mathbf{1}$$

Expand along R_1

$\Leftrightarrow$ $(\cos B - \cos A)(\cos C - \cos A)(\cos C - \cos B) = 0$ **1**

$\Leftrightarrow$ $\cos A = \cos B$ $\Leftrightarrow A = B$ $\Leftrightarrow \Delta ABC$ is an isosceles triangle **1**

$\cos B = \cos C$ $B = C$

or

$\cos C = \cos A$ $C = A$

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OR

Ans. let the cost of one pen of variety 'A', 'B' and 'C' be ₹ x. ₹ y and ₹ z respectively then the system of equations is:

$$\left.\begin{aligned} x+y+z &= 21 \\ 4x+3y+2z &= 60 \\ 6x+2y+3z &= 70 \end{aligned}\right\}$$ **1 ½**

Matrix form of the system is:

$A \cdot X = B$, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix}; \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; \quad B = \begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}$$ ½

$|A| = (5) - 1(0) + 1(-10) = -5$ **1**

co-factors of the matrix A are:

$$\left.\begin{aligned} C_{11} &= 5; & C_{21} &= -1; & C_{31} &= -1; \\ C_{12} &= 0; & C_{22} &= -3; & C_{32} &= 2; \\ C_{13} &= -10; & C_{23} &= 4; & C_{33} &= -1; \end{aligned}\right\}$$ **2**

$$\therefore \quad A^{-1} = \frac{1}{|A|}\text{Adj}A = \frac{1}{-5}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}$$ ½

Solution of the matrix equation is $X = A^{-1}B$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}\begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

$\therefore$ $x = 5, y = 8, z = 8$ ½

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Outside Delhi Set-II & Set-III — Code No : 65/2/C, 65/3/C

Note : All questions are from Set I .

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