

Solved Paper 2018

Mathematics

Class-XII

Time : 3 Hours

Max. Marks : 100

General Instructions :

- (i) All questions are compulsory.
- (ii) The question paper consists of 29 questions divided into four sections A, B, C and D. Section A comprises of 4 questions of one mark each, Section B comprises of 8 questions of two marks each, Section C comprises of 11 questions of four marks each and Section D comprises of 6 questions of six marks each.
- (iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
- (iv) There is no overall choice. However, internal choice has been provided in 3 questions of four marks each and 3 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
- (v) Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION - A

1. Find the value of $\tan^{-1} \sqrt{3} - \cot^{-1}(-\sqrt{3})$.

Ans. $\frac{\pi}{3} - \left(\pi - \frac{\pi}{6}\right) = -\frac{\pi}{2}$ 1/2+1/2

Note : $\frac{1}{2}$ mark for any one of the two correct values and $\frac{1}{2}$ mark for final answer.

[CBSE Marking Scheme, 2018]

2. If the matrix $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is skew symmetric, find the values of 'a' and 'b'.

Ans. $a = -2, b = 3$ 1/2+1/2

[CBSE Marking Scheme, 2018]

3. Find the magnitude of each of the vectors \vec{a} and \vec{b} , having the same magnitude such that the angle between them is 60° and their scalar product is $\frac{9}{2}$.

Ans. $|\vec{a}| = |\vec{b}| = 3$ 1/2+1/2

[CBSE Marking Scheme, 2018]

- * 4. If $a * b$ denotes the large of 'a' and 'b' if $a \circ b = (a * b) + 3$, then write the value of (5) \circ (10), where * and \circ are binary operations.

SECTION - B

5. Prove that :

$$3 \sin^{-1} x = \sin^{-1} (3x - 4x^3), x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$$

Ans. In RHS, put $x = \sin \theta$ 1/2
RHS = $\sin^{-1}(3 \sin \theta - 4 \sin^3 \theta)$
= $\sin^{-1}(\sin 3\theta)$ 1
= $3\theta = 3 \sin^{-1} x = \text{LHS}$ 1/2
[CBSE Marking Scheme, 2018]

6. Given $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$, compute A^{-1} and show that $2A^{-1} = 9I - A$.

Ans. $|A| = 2,$
 $A^{-1} = \frac{1}{2} \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$ 1
LHS = $2A^{-1}$
= $\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$
RHS = $9 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$
= $\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$ 1
LHS = RHS
[CBSE Marking Scheme, 2018]

7. Differentiate $\tan^{-1}\left(\frac{1+\cos x}{\sin x}\right)$ with respect to x .

Ans. $f(x) = \tan^{-1}\left(\frac{1+\cos x}{\sin x}\right)$

$$= \tan^{-1}\left(\frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right) \quad 1$$

$$= \tan^{-1}\left(\cot\frac{x}{2}\right) = \frac{\pi}{2} - \frac{x}{2} \quad \frac{1}{2}$$

$$\therefore f'(x) = -\frac{1}{2} \quad \frac{1}{2}$$

[CBSE Marking Scheme, 2018]

8. The total cost $C(x)$ associated with the production x units of an item is given by $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$. Find the marginal cost when 3 units are produced, where by marginal cost we mean the instantaneous rate of change of total cost at any level of output.

Ans. Marginal cost = $C'(x) = 0.015x^2 - 0.04x + 30$ 1
 At $x = 3, C'(3) = 30.015$ 1
 [CBSE Marking Scheme, 2018]

9. Evaluate:

$$\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx$$

Ans. $I = \int \frac{1 - 2\sin^2 x + 2\sin^2 x}{\cos^2 x} dx$ ½
 $= \int \sec^2 x dx$ 1
 $= \tan x + C$ ½
 [CBSE Marking Scheme, 2018]

10. Find the differential equation representing the family of curves $y = a e^{bx+5}$, where a and b are arbitrary constants.

Ans. $\frac{dy}{dx} = bae^{bx+5}$ ½
 $\frac{dy}{dx} = by$ ½
 $\frac{d^2y}{dx^2} = b\frac{dy}{dx}$ ½
 \therefore The differential equation is:
 $y\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$ ½
 [CBSE Marking Scheme, 2018]

*11. If θ is the angle between two vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$, find $\sin \theta$.

Ans. $\sin \theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|} = \frac{|\sqrt{96}|}{|\sqrt{14}||\sqrt{14}|}$
 $= \frac{4\sqrt{6}}{14}$
 $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 3 & -2 & 1 \end{vmatrix}$
 $= 4\hat{i} + 8\hat{j} + 4\hat{k}$
 $\therefore \sin \theta = \frac{2}{7}\sqrt{6}$

12. A black and a red die are rolled together. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Ans. A : Getting a sum of 8,
 B : Red die resulted in no. < 4 .
 $P(A/B) = \frac{P(A \cap B)}{P(B)}$ 1
 $= \frac{2/36}{18/36} = \frac{1}{9}$ 1
 [CBSE Marking Scheme, 2018]

SECTION - C

13. Using properties of determinants, prove that

$$\begin{vmatrix} 1 & 1 & 1+3x \\ 1+3y & 1 & 1 \\ 1 & 1+3z & 1 \end{vmatrix} = 9(3xyz + xy + yz + zx)$$

Ans. LHS = $\begin{vmatrix} 1 & 1 & 1+3x \\ 1+3y & 1 & 1 \\ 1 & 1+3z & 1 \end{vmatrix}$ 1
 $= \begin{vmatrix} 1 & 0 & 3x \\ 1+3y & -3y & -3y \\ 1 & 3z & 0 \end{vmatrix}$ 1
 (Using $C_2 \rightarrow C_2 - C_1$ & $C_3 \rightarrow C_3 - C_1$)
 $= 1 \times (9yz) + 3x(3z + 9yz + 3y)$
 (Expanding along R_1) 1
 $= 9(3xyz + xy + yz + zx) = \text{RHS}$ 1
 [CBSE Marking Scheme, 2018]

14. If $(x^2 + y^2)^2 = xy$, find $\frac{dy}{dx}$.

OR

If $x = a(2\theta - \sin 2\theta)$ and $y = a(1 - \cos 2\theta)$, find $\frac{dy}{dx}$

when $\theta = \frac{\pi}{3}$.

Ans. Differentiating with respect to 'x'

$$2(x^2 + y^2) \left(2x + 2y \frac{dy}{dx} \right) = x \frac{dy}{dx} + y \quad 2$$

$$\Rightarrow \frac{dy}{dx} = \frac{y - 4x^3 - 4xy^2}{4x^2y + 4y^3 - x} \quad 2$$

OR

$$\frac{dx}{d\theta} = a(2 - 2 \cos 2\theta) = 4a \sin^2\theta \quad 1$$

$$\frac{dy}{d\theta} = 2a \sin 2\theta = 4a \sin\theta \cos\theta \quad 1$$

$$\therefore \frac{dy}{dx} = \frac{4a \sin\theta \cos\theta}{4a \sin^2\theta} = \cot\theta \quad 1$$

$$\left. \frac{dy}{dx} \right|_{\theta=\frac{\pi}{3}} = \frac{1}{\sqrt{3}} \quad 1$$

[CBSE Marking Scheme, 2018]

15. If $y = \sin(\sin x)$, prove that

$$\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y \cos^2 x = 0.$$

Ans. $y = \sin(\sin x)$

$$\Rightarrow \frac{dy}{dx} = \cos(\sin x) \cdot \cos x \quad 1$$

$$\text{and } \frac{d^2y}{dx^2} = -\sin(\sin x) \cdot \cos^2 x - \sin x \cos(\sin x)$$

1+1

$$\text{LHS} = -\sin(\sin x) \cos^2 x - \sin x \cos(\sin x)$$

$$+ \frac{\sin x}{\cos x} \cos(\sin x) \cos x + \sin(\sin x) \cos^2 x$$

$$= 0 = \text{RHS} \quad 1$$

[CBSE Marking Scheme, 2018]

* 16. Find the equations of the tangent and the normal, to the curve $16x^2 + 9y^2 = 145$ at the point (x_1, y_1) , where $x_1 = 2$ and $y_1 > 0$.

OR

Find the intervals in which the function

$$f(x) = \frac{x^4}{4} - x^3 - 5x^2 + 24x + 12 \text{ is (a) strictly}$$

increasing, (b) strictly decreasing.

Ans.

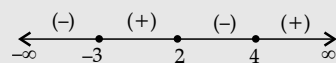
OR

$$f'(x) = x^3 - 3x^2 - 10x + 24 \quad \frac{1}{2}$$

$$= (x-2)(x-4)(x+3) \quad 1$$

$$f'(x) = 0 \Rightarrow x = -3, 2, 4. \quad \frac{1}{2}$$

sign of $f'(x)$:



$\therefore f(x)$ is strictly increasing on $(-3, 2) \cup (4, \infty)$ 1

and $f(x)$ is strictly decreasing on

$$(-\infty, -3) \cup (2, 4) \quad 1$$

[CBSE Marking Scheme, 2018]

17. An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the cost of material will be least when depth of the tank is half of its width. If the cost is to be borne by nearby settled lower income families, for whom water will be provided, what kind of value is hidden in this question?

Ans. Let side of base = x and depth of tank = y

$$V = x^2y \Rightarrow y = \frac{V}{x^2},$$

(V = Quantity of water = constant)

Cost of material is least when area of sheet used is minimum.

$$A \text{ (Surface area of tank)} = x^2 + 4xy$$

$$= x^2 + \frac{4V}{x}$$

$$\frac{dA}{dx} = 2x - \frac{4V}{x^2}, \quad \frac{dA}{dx} = 0 \quad \frac{1}{2} + \frac{1}{2}$$

$$\Rightarrow x^3 = 2V, \quad y = \frac{x^3}{2x^2} = \frac{x}{2} \quad \frac{1}{2} + \frac{1}{2}$$

$$\frac{d^2A}{dx^2} = 2 + \frac{8V}{x^3} > 0, \quad \frac{1}{2} + \frac{1}{2}$$

\therefore Area is minimum, thus cost is minimum when

$$y = \frac{x}{2}$$

Value: Any relevant value. 1

[CBSE Marking Scheme, 2018]

18. Find: $\int \frac{2 \cos x}{(1 - \sin x)(1 + \sin^2 x)} dx$

Ans. Put $\sin x = t \Rightarrow \cos x dx = dt$ 1/2

$$\text{Let } I = \int \frac{2 \cos x dx}{(1 - \sin x)(1 + \sin^2 x)}$$

$$= \int \frac{2}{(1-t)(1+t^2)} dt$$

$$\text{Let } \frac{2}{(1-t)(1+t^2)} = \frac{A}{1-t} + \frac{Bt+C}{1+t^2},$$

Solving we get

$$A = 1, B = 1, C = 1 \quad 1\frac{1}{2}$$

$$\therefore I = \int \frac{1}{1-t} dt + \frac{1}{2} \int \frac{2t}{1+t^2} dt + \int \frac{1}{1+t^2} dt$$

$$= -\log |1-t| + \frac{1}{2} \log |1+t^2| + \tan^{-1} t + C$$

1½

$$= -\log(1 - \sin x) + \frac{1}{2} \log(1 + \sin^2 x) +$$

$$\tan^{-1}(\sin x) + C \frac{1}{2}$$

[CBSE Marking Scheme, 2018]

19. Find the particular solution of the differential equation $e^x \tan y dx + (2 - e^x) \sec^2 y dy = 0$, given that $y = \frac{\pi}{4}$ when $x = 0$.

OR

Find the particular solution of the differential equation $\frac{dy}{dx} + 2y \tan x = \sin x$, given that $y =$

$$0 \text{ when } x = \frac{\pi}{3}.$$

Ans. Separating the variables, we get:

$$\int \frac{\sec^2 y}{\tan y} dy = \int \frac{e^x}{e^x - 2} dx \quad 1\frac{1}{2}$$

$$\Rightarrow \log |\tan y| = \log |e^x - 2| + \log C \quad 1\frac{1}{2}$$

$$\Rightarrow \tan y = C(e^x - 2), \text{ for } x = 0, y = \pi/4,$$

$$C = -1 \quad \frac{1}{2}$$

$$\therefore \text{ Particular solution is: } \tan y = 2 - e^x. \quad \frac{1}{2}$$

OR

$$\text{Integrating factor} = e^{\int 2 \tan x dx} = \sec^2 x \quad 1$$

$$\therefore \text{ Solution is: } y \cdot \sec^2 x = \int \sin x \cdot \sec^2 x dx \quad 1$$

$$= \int \sec x \cdot \tan x dx$$

$$y \cdot \sec^2 x = \sec x + C, \text{ for } x = \frac{\pi}{3}, y = 0,$$

$$\therefore C = -2 \quad 1 + \frac{1}{2}$$

$$\therefore \text{ Particular solution is: } y \cdot \sec^2 x = \sec x - 2$$

$$\text{or } y = \cos x - 2 \cos^2 x \quad \frac{1}{2}$$

[CBSE Marking Scheme, 2018]

20. Let $\vec{a} = 4\hat{i} + 5\hat{j} - \hat{k}$, $\vec{b} = \hat{i} - 4\hat{j} + 5\hat{k}$ and

$$\vec{c} = 3\hat{i} + \hat{j} - \hat{k}. \text{ Find a vector } \vec{d} \text{ which is}$$

perpendicular to both \vec{c} and \vec{b} and $\vec{d} \cdot \vec{a} =$

21.

$$\text{Ans. } \vec{d} = \lambda(\vec{c} \times \vec{b}) = \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -1 \\ 1 & -4 & 5 \end{vmatrix} \quad 1$$

$$\therefore \vec{d} = \lambda \hat{i} - 16\lambda \hat{j} - 13\lambda \hat{k} \quad 1$$

$$\vec{d} \cdot \vec{a} = 21 \Rightarrow 4\lambda - 80\lambda + 13\lambda = 21$$

$$\Rightarrow \lambda = -\frac{1}{3} \quad 1$$

$$\therefore \vec{d} = -\frac{1}{3}\hat{i} + \frac{16}{3}\hat{j} + \frac{13}{3}\hat{k} \quad 1$$

[CBSE Marking Scheme, 2018]

21. Find the shortest distance between the lines

$$\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k}) \text{ and}$$

$$\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \alpha(2\hat{i} + 4\hat{j} - 5\hat{k}).$$

$$\text{Ans. Here } \vec{a}_1 = 4\hat{i} - \hat{j}, \vec{a}_2 = \hat{i} - \hat{j} + 2\hat{k} \quad 1$$

$$\vec{a}_2 - \vec{a}_1 = -3\hat{i} + 2\hat{k} \quad 1$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix} = 2\hat{i} - \hat{j} \quad 1$$

$$\text{Shortest distance} = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$$

$$= \frac{|-6|}{\sqrt{5}} = \frac{6\sqrt{5}}{5} \quad 1$$

[CBSE Marking Scheme, 2018]

22. Suppose a girl throws a die. If she gets 1 or 2, she tosses a coin three times and notes the number of tails. If she gets 3, 4, 5 or 6, she tosses a coin once and notes whether a 'head' or 'tail' is obtained. If she obtained exactly one 'tail', what is the probability that she threw 3, 4, 5 or 6 with the die?

$$\text{Ans. } \left. \begin{array}{l} E_1 : \text{ She gets 1 or 2 on die.} \\ E_2 : \text{ She gets 3, 4, 5 or 6 on die.} \\ A : \text{ She obtained exactly 1 tail} \end{array} \right\} \quad 1$$

$$P(E_1) = \frac{1}{3}, P(E_2) = \frac{2}{3}$$

$$P(A/E_1) = \frac{3}{8}, P(A/E_2) = \frac{1}{2}$$

1

$$P(E_2/A) = \frac{P(E_2) \cdot P(A/E_2)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)} \quad 1$$

$$= \frac{\frac{2}{3} \times \frac{1}{2}}{\frac{1}{3} \times \frac{3}{8} + \frac{2}{3} \times \frac{1}{2}} = \frac{8}{11} \quad 1$$

[CBSE Marking Scheme, 2018]

- * 23. Two numbers are selected at random (without replacement) from the first five positive integers. Let X denote the larger of the two numbers obtained. Find the mean and variance of X .

SECTION - D

24. Let $A = \{x \in \mathbb{Z} : 0 < x < 12\}$. Show that $R = \{(a, b) : a, b \in A, \{ |a - b| \text{ is divisible by } 4\} \}$ is an equivalence relation. Find the set of all elements related to 1. Also write the equivalence class.

OR

- * Show that the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \frac{x}{x^2 + 1}$, $\forall x \in \mathbb{R}$ is neither one-one nor onto. Also, if $g : \mathbb{R} \rightarrow \mathbb{R}$ is defined as $g(x) = 2x - 1$, find $f \circ g(x)$.

Ans. Reflexive: $|a - a| = 0$, which is divisible by 4, $\forall a \in A$ 1

$\therefore (a, a) \in R, \forall a \in A \therefore R$ is reflexive

Symmetric: let $(a, b) \in R$

$\Rightarrow |a - b|$ is divisible by 4

$\Rightarrow |b - a|$ is divisible by 4 ($\because |a - b| = |b - a|$) 1

$\Rightarrow (b, a) \in R \therefore R$ is symmetric

Transitive: let $(a, b), (b, c) \in R$

$\Rightarrow |a - b|$ & $|b - c|$ are divisible by 4

$\Rightarrow a - b = \pm 4m, b - c = \pm 4n, n \in \mathbb{Z}$

Adding we get, $a - c = 4(\pm m \pm n)$ 2

$\Rightarrow (a - c)$ is divisible by 4

$\Rightarrow |a - c|$ is divisible by 4 $\therefore (a, c) \in R$

$\Rightarrow R$ is transitive

Hence R is an equivalence relation in A 1

set of elements related to 1 is $\{1, 5, 9\}$

and $[2] = \{2, 6, 10\}$. 1

[CBSE Marking Scheme, 2018]

25. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1} . Use it to solve the

system of equations

$$2x - 3y + 5z = 11$$

$$3x + 2y - 4z = -5$$

$$x + y - 2z = -3.$$

OR

- * Using elementary row transformations, find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}.$$

Ans. $|A| : -1 \neq 0 \therefore A^{-1}$ exists 1

Co-factors of A are :

$$\left. \begin{array}{lll} A_{11} = 0; & A_{12} = 2; & A_{13} = 1 & \text{1 mark for} \\ A_{21} = -1; & A_{22} = -9; & A_{23} = -5 & \text{4 correct} \\ A_{31} = 2; & A_{32} = 23; & A_{33} = 13 & \text{cofactors} \end{array} \right\}$$

2

$$\text{adj}(A) = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{|A|} \cdot \text{adj}(A)$$

$$= \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \quad \frac{1}{2}$$

For, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$, the system of

equation is $A \cdot X = B$ 1/2

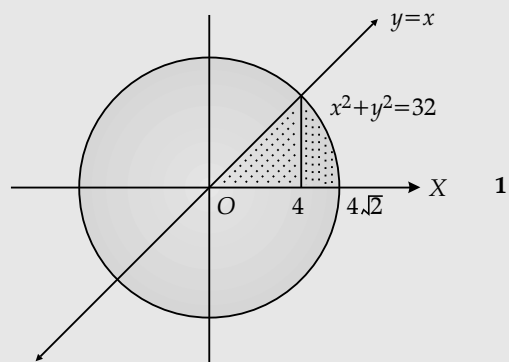
$$\therefore X = A^{-1} \cdot B = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad 1$$

$\therefore x = 1, y = 2, z = 3$ 1

[CBSE Marking Scheme, 2018]

26. Using integration, find the area of the region in the first quadrant enclosed by the X -axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.

Ans.



1

Point of intersection, $x = 4$ 1
 Area of shaded region 1
 $= \int_0^4 x dx + \int_4^{4\sqrt{2}} \sqrt{32-x^2} dx$ 1
 $= \left[\frac{x^2}{2} \right]_0^4 + \left[\frac{x}{2} \sqrt{32-x^2} + 16 \sin^{-1} \frac{x}{4\sqrt{2}} \right]_4^{4\sqrt{2}}$ 1
 $= 8 + 16 \frac{\pi}{2} - 8 - 4\pi = 4\pi$ sq. units. 1
 [CBSE Marking Scheme, 2018]

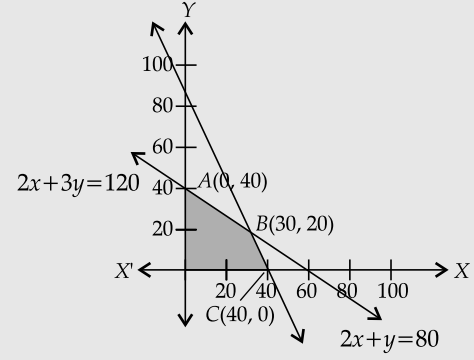
27. Evaluate: $\int_0^{\pi/4} \frac{\sin x + \cos x}{16 + 9 \sin 2x} dx$
 OR
 * Evaluate: $\int_1^3 (x^2 + 3x + e^x) dx$, as the limit of the sum.

Ans. Put $\sin x - \cos x = t$, $(\cos x + \sin x) dx = dt$, $1 - \sin 2x = t^2$ 1
 when $x = 0, t = -1$ }
 and $x = \pi/4, t = 0$ } 1/2
 $\therefore I = \int_0^{\pi/4} \frac{\sin x + \cos x}{16 + 9 \sin 2x} dx$
 $= \int_{-1}^0 \frac{1}{16 + 9(1-t^2)} dt$ 2
 $= \int_{-1}^0 \frac{1}{25 - 9t^2} dt$ 2
 $\Rightarrow I = \left[\frac{1}{30} \log \left| \frac{5+3t}{5-3t} \right| \right]_{-1}^0$ 1 1/2
 $= \frac{1}{30} \left[0 - \log \frac{1}{4} \right]$
 $= -\frac{1}{30} \log \frac{1}{4}$ or $\frac{1}{15} \log 2$ 1
 [CBSE Marking Scheme, 2018]

- * 28. Find the distance of the point $(-1, -5, -10)$ from the point of intersection of the line

$\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane
 $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$.

29. A factory manufactures two types of screws A and B, each type requiring the use of two machines, an automatic and a hand-operated. It takes 4 minutes on the automatic and 6 minutes on the hand-operated machines to manufacture a packet of screws 'A' while it takes 6 minutes on the automatic and 3 minutes on the hand-operated machine to manufacture a packet of screws 'B'. Each machine is available for at most 4 hours on any day. The manufacturer can sell a packet of screws 'A' at a profit of 70 paise and screws 'B' at a profit of ₹ 1. Assuming that he can sell all the screws he manufactures, how many packets of each type should the factory owner produce in a day in order to maximize his profit? Formulate the above LPP and solve it graphically and find the maximum profit.

Ans.  1
 Let number of packets of type A = x
 and number of packets of type B = y
 \therefore L.P.P. is: Maximize, $Z = 0.7x + y$ 1
 Subject to constraints:
 $4x + 6y \leq 240$ or $2x + 3y \leq 120$ } 2
 $6x + 3y \leq 240$ or $2x + y \leq 80$ }
 $x \geq 0, y \geq 0$
 Correct graph 2
 $Z(40, 0) = 28, Z(30, 20) = 41$ (Max.)
 $Z(0, 0) = 0, Z(0, 40) = 40$
 \therefore Max. profit is ₹ 41 at $x = 30, y = 20$. 1
 [CBSE Marking Scheme, 2018]

