Solved Paper 2019
Mathematics
Class-XII

Time : 3 Hours **Max. Marks : 100**

General Instructions :

(i) *All questions are compulsory.*

(ii) *The question paper consists of **29 questions** divided into four sections A, B, C and D. Section A comprises of **4 questions** of **one mark** each, Section B comprises of **8 questions** of **two marks** each, Section C comprises of **11 questions** of **four marks** each and Section D comprises of **6 questions** of **six marks** each.*

(iii) *All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.*

(iv) *There is no overall choice. However, internal choice has been provided in 1 questions of section A, 3 questions of section B, 3 questions of section C and 3 questions of section D. You have to attempt only one of the alternatives in all such questions.*

(v) *Use of calculators is not permitted. You may ask for logarithmic tables, if required*

Delhi Set - III **Code No. 65/1/3**

SECTION - A

1. If $3A - B = \begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, then find the matrix A.

Ans. $3A = \begin{bmatrix} 9 & 3 \\ 3 & 6 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$ ½+ ½

[CBSE Marking Scheme 2019]

2. Write the order and the degree of the following differential equation:

$$x^3\left(\frac{d^2y}{dx^2}\right)^2 + x\left(\frac{dy}{dx}\right)^4 = 0$$

Ans. Order = 2, degree = 2 ½+ ½

[CBSE Marking Scheme 2019]

* **3. If $f(x) = x + 1$, find $\frac{d}{dx}$ $(fof)(x)$.**

4. If a line makes angles 90°, 135°, 45° with the X, Y and Z axes respectively, find its direction cosines.

OR

Find the vector equation of the line which passes through the point (3, 4, 5) and is parallel to the vector $2\hat{i} + 2\hat{j} + 3\hat{k}$.

Ans. d.c. 's = < cos 90°, cos 135°, cos 45°> ½

$$= < 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, >$$ ½

OR

$$\vec{r} = (3\hat{i} + 4\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + 2\hat{j} + 3\hat{k})$$ 1

[CBSE Marking Scheme 2019]

SECTION - B

5. Evaluate: $\int \sin x \log \cos x \; dxs$

Ans. $I = \int \sin x . \log(\cos x) dx$

$\cos x = t \Rightarrow I = -\int \log t . 1 dt$ 1

$= -\left[t . \log t - \int \frac{1}{t} . t dt\right]$ ½

$= t(1 - \log t) + c = \cos x\,(1 - \log(\cos x)) + c$ ½

[CBSE Marking Scheme, 2019]

6. Evaluate: $\int_{-\pi}^{\pi} (1 - x^2) \sin x \cos^2 x \; dx$.

OR

Evaluate: $\int_{-1}^{2} \frac{|x|}{x} dx$.

* *Out of Syllabus*

Ans. Let $f(x) = (1 - x^2).\sin x \cos^2 xs$

as $f(-x) = -f(x) \Rightarrow f$ is odd function.

$\therefore I = 0$

OR

$$I = \int_{-1}^{2} \frac{|x|}{x} dx = \int_{-1}^{0} -1 dx + \int_{0}^{2} 1 dx$$

$= -1 + 2 = 1$ 1

[CBSE Marking Scheme 2019]

***7. Examine whether the operation * defined on *R* by *a* * *b* = *ab* + 1 is (i) a binary or not. (ii) if a binary operation, is it associative or not ?**

8. Find a matrix *A* such that 2*A* – 3*B* + 5*C* = 0, where

$$B = \begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix} \text{ and } C = \begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}.$$

Ans.

$$2A - \begin{bmatrix} -6 & 6 & 0 \\ 9 & 3 & 12 \end{bmatrix} + \begin{bmatrix} 10 & 0 & -10 \\ 35 & 5 & 30 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} -8 & 3 & 5 \\ -13 & -1 & -9 \end{bmatrix}$$ **1**

[CBSE Marking Scheme, 2019]

9. A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event "number is even" and B be the event "number is marked red". Find whether the events A and B are independent or not.

Ans. $A = \{2, 4, 6\}, B = \{1, 2, 3\}, (A \cap B) = \{2\}$

Now, $P(A) = \frac{1}{2}, P(B) = \frac{1}{2}, P(A \cap B) = \frac{1}{6}$ **1**

As $P(A) \times P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{6} \neq P(A \cap B)$ ½

$\Rightarrow A$ and B are not independent. ½

[CBSE Marking Scheme, 2019]

10. Form the differential equation representing the family of curves $y = e^{2x}(a + bx)$, where '*a*' and '*b*' are arbitrary constants.

Ans. $y' = be^{2x} + 2y \Rightarrow b = \frac{y' - 2y}{e^{2x}}$

differentiating again

$$\frac{e^{2x}.(y'' - 2y') - (y' - 2y).2e^{2x}}{(e^{2x})^2} = 0$$ ½

$y'' - 4y' + 4y = 0$ or $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$ ½

[CBSE Marking Scheme, 2019]

* **11. A die is thrown 6 times. If "getting on odd number" is a "success", what is the probability of (i) 5 successes? (ii) atmost 5 successes ?**

OR

The random variable *X* has a probability distribution *P*(*X*) of the following form, where '*k*' is some number.

$$P(X = x) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$

Determine the value of '*k*'.

Here,

x	0	1	2
$P(x)$	k	$2k$	$3k$

Since, $P(0) + P(1) + P(2) = 1$ i.e., sum of all probabilities is 1.

$\therefore$ We have,

$k + 2k + 3k = 1$

$\Rightarrow \quad 6k = 1$

$\Rightarrow \quad k = \frac{1}{6}$

Hence, value of k is $= \frac{1}{6}$

12. If the sum of two unit vectors is a unit vector, prove that the magnitude of their difference is $\sqrt{3}$.

OR

If $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}, \vec{b} = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{c} = -3\hat{i} + \hat{j} + 2\hat{k}$, find $[\vec{a}\ \vec{b}\ \vec{c}]$.

Ans. Given $|\hat{a} + \hat{b}| = 1$

as $|\hat{a} + \hat{b}|^2 + |\hat{a} - \hat{b}|^2 = 2(|\hat{a}|^2 + |\hat{b}|^2)$ **1**

$\Rightarrow 1 + |\vec{a} - \vec{b}|^2 = 2(1 + 1)$

$\Rightarrow |\vec{a} - \vec{b}|^2 = 3 \Rightarrow |\vec{a} - \vec{b}| = \sqrt{3}$ **1**

OR

$$[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 2 & 3 & 1 \\ 1 & -2 & 1 \\ -3 & 1 & 2 \end{vmatrix}$$ **1**

$= -30$ **1**

[CBSE Marking Scheme, 2019]

* *Out of Syllabus*

SECTION - C

13. Using properties of determinants, prove the following:

$$\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3+b^3+c^3-3abc.$$

Ans. Refer the Q.33 in Outside Delhi Set-I

14. Solve: $\tan^{-1} 4x + \tan^{-1} 6x = \dfrac{\pi}{4}$.

Ans. $\tan^{-1}\left(\dfrac{4x+6x}{1-(4x)(6x)}\right) = \dfrac{\pi}{4}$ **1**

$\Rightarrow \dfrac{10x}{1-24x^2} = 1 \Rightarrow 24x^2+10x-1=0$ 1½

$\Rightarrow x = \dfrac{1}{12}$ or $-\dfrac{1}{2}$ **1**

as $x = -\dfrac{1}{2}$ does not satisfy the given equation,

so $x = \dfrac{1}{12}$ ½

[CBSE Marking Scheme, 2019]

15. Show that the relation R on R defined as $R = \{(a, b) : a \le b\}$, is reflexive, and transitive but not symmetric.

OR

Prove that the function $f : N \to N$, defined by $f(x) = x^2 + x + 1$ is one-one but not onto.

* **Find inverse of $f : N \to S$, where S is range of f.**

Ans. Clearly $a \le a \,\forall\, a \in R \Rightarrow (a,a) \in R \Rightarrow R$ is reflexive. **1**

For transitive :

Let $(a, b) \in R$ and $(b, c) \in R$, $a, b, c \in R$

$\Rightarrow a \le b$ and $b \le c \Rightarrow a \le c \Rightarrow (a,c) \in R$

$\Rightarrow R$ is transitive. **1½**

For non-symmetric:

Let $a = 1, b = 2$. As $1 \le 2 \Rightarrow (1,2) \in R$

but $2 \not\le 1 \Rightarrow (2,1) \notin R$

$\Rightarrow R$ is non-symmetric. **1½**

OR

For one-one. Let $x_1, x_2 \in N$.

$f(x_1) = f(x_2) \Rightarrow x_1^2 + x_1 + 1 = x_2^2 + x_2 + 1$

$\Rightarrow (x_1 - x_2)(x_1 + x_2 + 1) = 0$

$\Rightarrow x_1 = x_2$ as $x_1 + x_2 + 1 \ne 0$

$(\because x_1, x_2 \in N)$ **1½**

$\Rightarrow f$ is one-one.

For not onto.

for $y = 1 \in N$, there is no $x \in N$ for which

$f(x) = 1$ **1½**

[CBSE Marking Scheme 2019]

* **16. Find the equation of tangent to the curve $y = \sqrt{3x-2s}$ which is parallel to the line $4x - 2y + 5 = 0$. Also, write the equation of normal to the curve at the point of contact.**

17. If $\log(x^2 + y^2) = 2\tan^{-1}\left(\dfrac{y}{x}\right)$, show that $\dfrac{dy}{dx} = \dfrac{x+y}{x-y}.c$

OR

If $x^y - y^x = a^b$, find $\dfrac{dy}{dx}$.

Ans. $\log(x^2+y^2) = 2\tan^{-1}\left(\dfrac{y}{x}\right)$

differentiating both sides w.r.t. x,

$$\frac{1}{x^2+y^2}\left(2x+2y\frac{dy}{dx}\right) = 2\cdot\frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\left(\frac{x\cdot\frac{dy}{dx}-y\cdot 1}{x^2}\right) \quad \textbf{2}$$

$$\Rightarrow \frac{2}{x^2+y^2}\left(x+y\frac{dy}{dx}\right) = \frac{2x^2}{x^2+y^2}\cdot\frac{1}{x^2}\cdot\left(x\frac{dy}{dx}-y\right) \quad \textbf{1}$$

$$\Rightarrow (x+y) = (x-y)\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{x+y}{x-y} \quad \textbf{1}$$

OR

Let $u = x^y$, $v = y^x$. Then $u - v = a^b$

$\Rightarrow \dfrac{du}{dx} - \dfrac{dv}{dx} = 0$ **...(1) 1**

Now, $\log u = y.\log x$

$$\Rightarrow \frac{1}{u}\cdot\frac{du}{dx} = y\cdot\frac{1}{x} + \log x\cdot\frac{dy}{dx} \Rightarrow \frac{du}{dx} = x^y\left(\frac{y}{x}+\log x\cdot\frac{dy}{dx}\right)$$

...(2) 1

Again, $\log v = x \cdot \log y$

$$\Rightarrow \frac{1}{v}\cdot\frac{dv}{dx} = x\cdot\frac{1}{y}\cdot\frac{dy}{dx} + \log y \cdot 1 \Rightarrow \frac{dv}{dx} = y^x\left(\frac{x}{y}\frac{dy}{dx}+\log y\right)$$

...(3) 1

From (1), (2) and (3)

$$x^y\left(\frac{y}{x}+\log x\frac{dy}{dx}\right) - y^x\left(\frac{x}{y}\frac{dy}{dx}+\log y\right) = 0 \quad ½$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^x\cdot\log y - x^{y-1}\cdot y}{x^y\cdot\log x - y^{x-1}\cdot x} \quad ½$$

[CBSE Marking, Scheme, 2019]

* *Out of Syllabus*

18. If $y = (\sin^{-1}x)^2$, prove that

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$$

Ans. $y = (\sin^{-1} x)^2$

$$\Rightarrow y' = 2\cdot\sin^{-1}x\cdot\frac{1}{\sqrt{1-x^2}} \qquad 1$$

$$\Rightarrow \sqrt{1-x^2}\cdot y' = 2\sin^{-1}x$$

$$\Rightarrow \sqrt{1-x^2}\cdot y'' + y'\cdot\frac{1}{2\sqrt{1-x^2}}(-2x) = \frac{2}{\sqrt{1-x^2}} \qquad 2$$

$$\Rightarrow (1-x^2)\cdot y'' - xy' = 2 \text{ or } (1-x^2)\cdot\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0. \qquad 1$$

[CBSE Marking, Scheme 2019]

19. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$, hence evaluate $\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx$.

Ans. Please refer the Q.29 in Outside Delhi Paper 2022.

20. Find: $\int\frac{\cos x}{(1+\sin x)(2+\sin x)}dx$

Ans. $I = \int\frac{\cos x}{(1+\sin x)(2+\sin x)}dx$. Put $\sin x = t$ ½

$$= \int\frac{dt}{(1+t)(2+t)} = \int\left(\frac{1}{1+t} - \frac{1}{2+t}\right)dt \qquad 2$$

$$= \log\left|\frac{1+t}{2+t}\right| + c = \log\left|\frac{1+\sin x}{2+\sin x}\right| + c \qquad 1½$$

[CBSE Marking Scheme, 2019]

21. Solve the differential equation : $\frac{dy}{dx} - \frac{2x}{1+x^2}y = x^2 + 2$

OR

Solve the differential equation :

$$(x+1)\frac{dy}{dx} = 2e^{-y} - 1;\ y(0) = 0.$$

Ans. I.F. $= e^{-\int\frac{2x}{1+x^2}dx} = \frac{1}{1+x^2}$ 1

Solution is given by,

$$y\cdot\left(\frac{1}{1+x^2}\right) = \int\frac{x^2+2}{1+x^2}dx \qquad 1$$

$$y\cdot\frac{1}{1+x^2} = \int\left(1+\frac{1}{1+x^2}\right)dx = x + \tan^{-1}x + c \qquad 1$$

or $y = (1 + x^2)(x + \tan^{-1} x + c)$ 1

OR

Given equation can be written as

$$\int\frac{dy}{2e^{-y}-1} = \int\frac{dx}{x+1}$$

$$\Rightarrow \int\frac{e^y}{2-e^y}dy = \int\frac{dx}{x+1} \qquad 1$$

$\Rightarrow -\log|2 - e^y| + \log c = \log|x + 1|$ 1½

$\Rightarrow (2 - e^y)(x + 1) = c$

When $x = 0, y = 0 \Rightarrow c = 1$ 1

$\therefore$ Solution is $(2 - e^y)(x + 1) = 1$ ½

[CBSE Marking Scheme 2019]

22. If $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}+5\hat{j}$, $3\hat{i}+2\hat{j}-3\hat{k}$ and $\hat{i}-6\hat{j}-\hat{k}$ respectively are the position vectors of points *A, B, C* and *D*, then find the angle between the straight lines *AB* and *CD*. Find whether $\vec{AB}$ and $\vec{CD}$ are collinear or not.

Ans. $\overrightarrow{AB} = \hat{i} + 4\hat{j} - \hat{k}$ 1

$\overrightarrow{CD} = -2\hat{i} - 8\hat{j} + 2\hat{k}$ 1

Let required angle be θ.

$$\text{Then } \cos\theta = \left|\frac{\overrightarrow{AB}\cdot\overrightarrow{CD}}{|\overrightarrow{AB}||\overrightarrow{CD}|}\right| = \left|\frac{-2-32-2}{\sqrt{18}\sqrt{72}}\right| = -1 \qquad 1$$

$\Rightarrow \theta = 180°$ or π ½

Since $\theta = \pi$ so $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear. ½

[CBSE Marking Scheme 2019]

23. Find the value of λ, so that the lines $\frac{1-x}{3} = \frac{7y-14}{\lambda} = \frac{z-3}{2}$ and $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles. Also, find whether the lines are intersecting or not.

Ans. Given lines are:

$$\frac{x-1}{-3} = \frac{y-2}{\left(\frac{\lambda}{7}\right)} = \frac{z-3}{2} \text{ and } \frac{x-1}{\left(\frac{-3\lambda}{7}\right)} = \frac{y-5}{1} = \frac{z-6}{-5} \qquad 1$$

As lines are perpendicular,

$$(-3)\left(\frac{-3\lambda}{7}\right) + \left(\frac{\lambda}{7}\right)(1) + 2(-5) = 0 \Rightarrow \lambda = 7 \qquad 1$$

So, lines are

$$\frac{x-1}{-3} = \frac{y-2}{1} = \frac{z-3}{2} \text{ and } \frac{x-1}{-3} = \frac{y-5}{1} = \frac{z-6}{-5} \qquad ½$$

Consider

$$\Delta = \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 0 & 3 & 3 \\ -3 & 1 & 2 \\ -3 & 1 & -5 \end{vmatrix} = -63 \qquad 1$$

as $\Delta \neq 0 \Rightarrow$ lines are not intersecting. ½

[CBSE Marking Scheme 2019]

SECTION - D

24. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m^3. If building of tank ₹70 per square metre for the base and ₹45 per square metre for the sides, what is the cost of least expensive tank ?

Ans. $V = 2xy \Rightarrow 2xy = 8$ (given)

$\Rightarrow y = \frac{4}{x}$

Now, cost, $C = 70xy + 45 \times 2 \times (2x + 2y)$ **1**

$= 280 + 180x + \frac{720}{x}$ **1**

$\frac{dC}{dx} = 180 - \frac{720}{x^2}$ **1**

$\frac{dC}{dx} = 0 \Rightarrow x = 2\text{m}$ **½**

$\frac{d^2C}{dx^2} = \frac{1440}{x^3} = 180 > 0$ at $x = 2$ **1**

$\Rightarrow$ C is minimum at $x = 2$m **½**

Minimum cost $= 280 + 180(2) + \frac{720}{2} =$ ₹ 1,000 **1**

[CBSE Marking Scheme, 2019]

25. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$, find A^{-1}. Hence, solve the system of equations $x + y + z = 6$, $x + 2z = 7$, $3x + y + z = 12$.

OR

*** Find the inverse of the following matrix using elementary operations.**

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

Ans. $|A| = 4 \neq 0 \Rightarrow A^{-1}$ exists. **1**

$$\text{Adj } A = \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$ **2**

$$\therefore A^{-1} = \frac{1}{|A|} \cdot adj\ A = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$ **½**

Given system of equations can be written as AX = B where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$

$\therefore$ $X = A^{-1} \cdot B$ **1**

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$ **1**

$\Rightarrow x = 3, y = 1, z = 2$ **½**

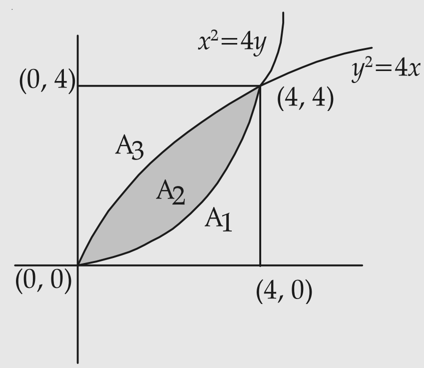
[CBSE Marking Scheme, 2019]

26. Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by sides $x = 0$, $x = 4$, $y = 4$ and $y = 0$ into three equal parts.

OR

Using integration, find the area of the triangle whose vertices are (2, 3), (3, 5) and (4, 4).

Ans.



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Point of intersection are (0, 0) and (4, 4) **1**

here, $A_1 = \int_0^4 \frac{x^2}{4} dx = \frac{16}{3}$ **...(1)** **1**

$$A_2 = \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right) dx = \frac{16}{3}$$ **...(2)** **1**

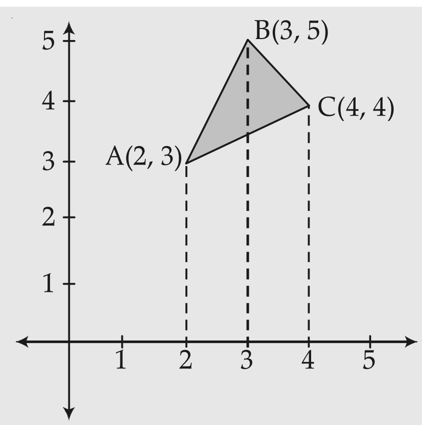
$$\left[\frac{4}{3}x^{3/2} - \frac{x^3}{2}\right]_0^4 = \frac{16}{3}$$

$$A_3 = \int_0^4 \frac{y^2}{4} dy = \frac{16}{3}$$ **...(3)** **1**

From (1), (2) and (3), $A_1 = A_2 = A_3$ **1**

[CBSE Marking Scheme, 2019]

OR



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* *Out of Syllabus*

$$\left.\begin{array}{l}\text{Equation of AB}: y = 2x - 1\\ \text{Equation of BC}: y = -x + 8\\ \text{Equation of AC}: y = \frac{1}{2}(x+4)\end{array}\right\}$$ **1 ½**

Required Area

$$= \int_2^3 (2x-1)dx + \int_3^4 (-x+8)dx - \int_2^4 \left(\frac{x+4}{2}\right)dx$$ **1 ½**

$$= \left[x^2 - x\right]_2^3 + \left[\frac{-x^2}{2} + 8x\right]_3^4 - \frac{1}{2}\left[\frac{x^2}{2} + 4x\right]_2^4$$ **1 ½**

$$= 4 + \frac{9}{2} - 7 = \frac{3}{2}$$ ½

[CBSE Marking Scheme, 2019]

27. A manufacturer has employed 5 skilled men and 10 semi-skilled men and makes two models *A* and *B* of an article. The making of one item of model *A* requires 2 hours work by a skilled man and 2 hours work by a semi-skilled man. One item of model *B* requires 1 hour by a skilled man 3 hours by a semi-skilled man. No man is expected to work more than 8 hours per day. The manufacturer's profit on an item of model *A* is ₹15 and on an item of model *B* is ₹10. How many of items of each model should be made per day in order to maximize daily profit ? Formulate the above LPP and solve it graphically and find the maximum profit.

Ans. Let number of items produced of model A be x and that of model B be y.

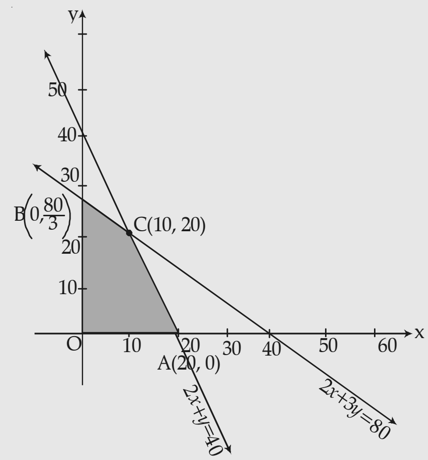
LPP is:

Maximize, profit $z = 15x + 10y$ **1**

subject to

$$\left.\begin{array}{ll}2x + y \le 5(8) & \text{i.e.,}\ 2x + y \le 40\\ 2x + 3y \le 10(8) & \text{i.e.,}\ 2x + 3y \le 80\\ x \ge 0, y \ge 0 & \end{array}\right\}$$ **2**

Correct Figure **2**



Corner point	z = 15x + 10y	
A(20, 0)	300	
$B\left(0, \frac{80}{3}\right)$	$\frac{800}{3} = 266.6$	
C(10, 20)	350 ← maximum	½

Maximum profit = ₹ 350

when $x = 10$, $y = 20$. ½

If a student has interpreted the language of the question in a different way, then the LPP will be of the type :

Maximise profit $z = 15x + 10y$

Subject to $2x + y \le 8$

$2x + 3y \le 8$

$x \ge 0, y \ge 0$

This is be accepted and marks may be given accordingly. **1**

[CBSE Marking Scheme, 2019]

* **28. Find the vector and Cartesian equations of the plane passing through the points (2, 2, – 1), (3, 4, 2) and (7, 0, 6). Also find the vector equation of a plane passing through (4, 3, 1) and parallel to the plane obtained above.**

OR

* **Find the vector equation of the plane that contains the lines $\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and the point (–1, 3, – 4). Also, find length of the perpendicular drawn from the point (2, 1, 4) to the plane thus obtained.**

* **29. Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean and variance of the number of kings.**

Outside Delhi Set - I — Code No. 65/2/1

SECTION - A

1. If *A* is a square matrix satisfying *A*' *A* = *I*, write the value of $|A|$.

Ans. $|A'|\ |A| = |I| \Rightarrow |A|^2 = 1$ ½

$\therefore$ $|A| = 1$ or $|A| = -1$ ½

[CBSE Marking Scheme, 2019]

* *Out of Syllabus*

2. If $y = x|x|$, find $\frac{dy}{dx}$ for $x < 0$.

Ans. For $x < 0, y = x\,|x| = -x^2$ ½

$\therefore \frac{dy}{dx} = -2x$ ½

[CBSE Marking Scheme, 2019]

3. Find the order and degree (if defined) of the differential equation.

$$\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 = 2x^2 \log\left(\frac{d^2y}{dx^2}\right)$$

Ans. Order = 2, Degree not defined ½+ ½

[CBSE Marking Scheme, 2019]

4. Find the direction cosines of a line which makes equal angles with the coordinate axes.

OR

A line passes through the point with position vector $2\hat{i} - \hat{j} + 4\hat{k}$ and is in the direction of the vector $\hat{i} + \hat{j} - 2\hat{k}$. Find the equation of the line in cartesian form.

Ans. D. Rs are 1, 1, 1

$\therefore$ Direction cosines of the line are:

$$\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$$ 1

OR

Equation of the line is :

$$\frac{x-2}{1} = \frac{y+1}{1} = \frac{z-4}{-2}$$ 1

[CBSE Marking Scheme, 2019]

SECTION - B

*** 5. Examine whether the operation * defined on R, the set of all real numbers, by $a * b = \sqrt{a^2 + b^2}$ is a binary operation or not, and if it is a binary operation, find whether it is associative or not.**

6. If $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$, show that $(A - 2I)(A - 3I) = 0$.

Ans. $(A-2I)(A-3I) = \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$ 1

[CBSE Marking Scheme, 2019]

7. Find: $\int \sqrt{3 - 2x - x^2}\, dx$

Ans. $\int \sqrt{3-2x-x^2}\, dx = \int \sqrt{2^2 - (x+1)^2}\, dx$ 1

$= \frac{x+1}{2}\sqrt{3-2x-x^2} + 2\sin^{-1}\left(\frac{x+1}{2}\right) + c$ 1

[CBSE Marking Scheme, 2019]

8. Find: $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx$

OR

Find: $\int \frac{x-3}{(x-1)^3} e^x dx$

Ans.

$\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx = \int \left(\sec x \cdot \tan x + \operatorname{cosec} x . \cot x\right) dx$

1

$= \sec x - \operatorname{cosec} x + c$ 1

OR

$\int \frac{x-3}{(x-1)^3} e^3\, dx = \int e^x \left\{(x-1)^{-2} - 2(x-1)^{-3}\right\} dx$ 1

$= e^x (x-1)^{-2} + c$

or

$\frac{e^x}{(x-1)^2} + c$ 1

[CBSE Marking Scheme, 2019]

9. Find the differential equation of the family of curves $y = Ae^{2x} + Be^{-2x}$, where A and B are arbitrary constants.

Ans. Differentiating $y = Ae^{2x} + Be^{-2x}$, we get

$\frac{dy}{dx} = 2Ae^{2x} - 2Be^{-2x}$, differentiate again to get, **1**

$\frac{d^2y}{dx^2} = 4Ae^{2x} + 4Be^{-2x} = 4y$ or $\frac{d^2y}{dx^2} - 4y = 0$ 1

[CBSE Marking Scheme, 2019]

10. If $|\vec{a}| = 2, |\vec{b}| = 7$ and $\vec{a} \times \vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$, find the angle between $\vec{a}$ and $\vec{b}$.

OR

Find the volume of a cuboid whose edges are given by $-3\hat{i} + 7\hat{j} + 5\hat{k}, -5\hat{i} + 7\hat{j} - 3\hat{k}$ and $7\hat{i} - 5\hat{j} - 3\hat{k}$.

Ans. Let θ be the angle between $\vec{a}$ and $\vec{b}$, then

$\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|} = \frac{7}{2.7} = \frac{1}{2}$ 1½

* *Out of Syllabus*

$$\Rightarrow \theta = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}$$ ½

OR

$$(-3\hat{i}+7\hat{j}+5\hat{k})\cdot\left\{\left(-5\hat{i}+7\hat{j}-3\hat{k}\right)\times\left(7\hat{i}-5\hat{j}-3\hat{k}\right)\right\}$$

$$= \begin{vmatrix} -3 & 7 & 5 \\ -5 & 7 & -3 \\ 7 & -5 & -3 \end{vmatrix}$$ 1

$$= -264$$ ½

$\therefore$ Volume of cuboid = 264 cubic units ½

[CBSE Marking Scheme, 2019]

11. If P(not A) = 0.7, $P(B)$ = 0.7 and $P(B/A)$ = 0.5, then find $P(A/B)$.

Ans. $P(\overline{A}) = 0.7 \Rightarrow 1 - \text{P(A)} = 0.7 \Rightarrow P(A) = 0.3$ ½

$P(A \cap B) = P(A)\cdot P(B\,|\,A) = 0.3 \times 0.5 = 0.15$ ½

$$P(A\,|\,B) = \frac{P(A\cap B)}{P(B)} = \frac{0.15}{0.7} = \frac{15}{70}\, or\, \frac{3}{14}$$ 1

[CBSE Marking Scheme, 2019]

*** 12. A coin is tossed 5 times. What is the probability of getting (i) 3 heads, (ii) atmost 3 heads ?**

OR

*** Find the probability distribution of X, the number of heads in a simultaneous toss of two coins.**

SECTION - C

13. Check whether the relation R defined on the set A = {1, 2, 3, 4, 5, 6} as R= {(a, b) : $b = a + 1$} is reflexive, symmetric or transitive.

OR

*** Let $f : N \to Y$ be a function defined as $f(x) = 4x + 3$, where $Y = \{y \in N ; y = 4x + 3$, for some $x \in N\}$. Show that f is invertible. Find its inverse.**

Ans. R = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)}

For $1 \in A$, $(1, 1) \notin R \Rightarrow R$ is not reflexive 1

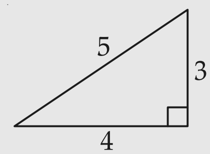
For $1, 2 \in A$, $(1, 2) \in R$ but $(2, 1) \notin R \Rightarrow R$ is not symmetric 1½

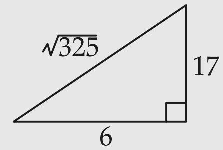
For $1, 2, 3 \in A$, $(1, 2), (2, 3) \in R$ but $(1, 3) \notin R \Rightarrow R$ is not transitive 1½

14. Find the value of $\sin\left(\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3}\right)$.

Ans. $$\sin\left[\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right]$$ 1

$$= \sin\left[\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right]$$





$$= \sin\left[\tan^{-1}\left(\frac{3/4 + 2/3}{1 - 3/4\cdot 2/3}\right)\right] = \sin\left[\tan^{-1}\left(\frac{17}{6}\right)\right]$$ 1½

$$= \sin\left[\sin^{-1}\left(\frac{17}{\sqrt{325}}\right)\right] = \frac{17}{\sqrt{325}} = \frac{17}{5\sqrt{13}}$$ 1½

[CBSE Marking Scheme, 2019]

15. Using properties of determinants, show that

$$\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab + bc + ca)$$

Ans. $$\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix}$$

$$= \begin{vmatrix} a+b+c & -a+b & -a+c \\ a+b+c & 3b & -b+c \\ a+b+c & -c+b & 3c \end{vmatrix} \begin{pmatrix} \textit{By applying} \\ C_1 \to C_1 + C_2 + C_3 \end{pmatrix}$$ 1

$$= \begin{vmatrix} a+b+c & -a+b & -a+c \\ 0 & 2b+a & a-b \\ 0 & a-c & 2c+a \end{vmatrix} \begin{cases} \textit{By applying} \\ R_2 \to R_2 - R_1; \\ R_3 \to R_3 - R_1 \end{cases}$$ 2

$= (a + b + c)\,\{4bc + 2ab + 2ac + a^2 - (a^2 - ac - ba + bc)\}$

$= 3(a + b + c)\,(ab + bc + ca)$ 1

[CBSE Marking Scheme, 2019]

16. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ and $x \neq y$, prove that $\frac{dy}{dx} = -\frac{1}{(x+1)^2}$.

OR

If $(\cos x)^y = (\sin y)^x$ find $\frac{dy}{dx}$.

Ans. $x\sqrt{1+y} + y\sqrt{1+x} = 0 \Rightarrow x\sqrt{1+y} = -y\sqrt{1+x}$ ½

Squaring to get: $x^2(1 + y) = y^2(1 + x)$ ½

Simplifying to get : $(x - y)(x + y + xy) = 0$ 1

* *Out of Syllabus*

As, $x \neq y \therefore \; y = -\frac{x}{1+x}$ **1**

Differentiating w.r.t x, we get

$$\frac{dy}{dx} = \frac{-1(1+x)-(-x)\cdot 1}{(1+x)^2} = \frac{-1}{(1+x)^2}$$ **1**

OR

$(\cos x)^y = (\sin y)^x \Rightarrow y \cdot \log(\cos x) = x \cdot \log(\sin y)$ **1**

Differentiating w.r.t 'x'

$$\Rightarrow \frac{dy}{dx}\cdot \log\cos x + y(-\tan x)$$

$$= \log(\sin y) + x\cdot\cot y\cdot\frac{dy}{dx}$$ **2**

$$\Rightarrow \frac{dy}{dx} = \frac{y\cdot\tan x + \log(\sin y)}{\log(\cos x) - x\cot y}$$ **1**

[CBSE Marking Scheme, 2019]

17. If $(x-a)^2 + (y-b)^2 = c^2$, for some $c > 0$, Prove that $\dfrac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$ is a constant independent of a and b.

Ans. $(x-a)^2 + (y-b)^2 = c^2, c > 0$

$$2(x-a) + 2(y-b)\cdot\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x-a}{y-b}$$ **1½**

Differentiating again with respect to x, we get :

$$\frac{d^2y}{dx^2} = -\frac{(y-b)-(x-a)\cdot\frac{dy}{dx}}{(y-b)^2}$$

$$= \frac{-c^2}{(y-b)^3}\left(\text{By substitution } '\frac{dy}{dx}'\right)$$ **1**

$$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\left[1+\frac{(x-a)^2}{(y-b)^2}\right]^{6/2}}{-\frac{c^2}{(y-b)^3}} = \frac{\frac{c^3}{(y-b)^3}}{-\frac{c^2}{(y-b)^3}} = -c$$ **1**

Which is a constant independent of 'a' and 'b'. **½**

[CBSE Marking Scheme, 2019]

*** 18. Find the equation of the normal to the curve $x^2 = 4y$ which passes through the point (–1, 4).**

19. Find: $\displaystyle\int \frac{x^2+x+1}{(x+2)(x^2+1)}dx$

Ans. $$\int \frac{x^2+x+1}{(x-2)(x^2+1)}dx$$

$$= \frac{3}{5}\int\frac{1}{x+2}dx + \frac{1}{5}\int\frac{2x}{x^2+1}dx + \frac{1}{5}\int\frac{1}{x^2+1}dx$$ **2**

$$= \frac{3}{5}\log|x+2| + \frac{1}{5}\log|x^2+1| + \frac{1}{5}\tan^{-1}x + c$$ **2**

[CBSE Marking Scheme, 2019]

20. Prove that $\displaystyle\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and hence evaluate $\displaystyle\int_0^{\pi/2}\frac{x}{\sin x + \cos x}dx$

Ans. $$\int_0^a f(x)dx = -\int_a^0 f(a-t)dx$$

Put $x = a - t, dx = -dt$

Upper limit $= t = a - x = a - a = 0$

Lower limit $= t = a - x = a - 0 = a$

$$= \int_0^a f(a-t)dt = \int_0^a f(a-x)dx$$ **1**

Let $$I = \int_0^{\pi/2}\frac{x}{\sin x + \cos x}dx$$ **...(i)**

$$I = \int_0^{\pi/2}\frac{\pi/2 - x}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}dx$$

$$\therefore I = \int_0^{\pi/2}\frac{\pi/2 - x}{\cos x + \sin x}dx$$ **...(ii) 1**

Adding, (i) and (ii), we get

$$2I = \frac{\pi}{2}\int_0^{\pi/2}\frac{1}{\cos x + \sin x}dx = \frac{\pi}{2\sqrt{2}}\int_0^{\pi/2}\frac{1}{\cos\left(x-\frac{\pi}{4}\right)}dx$$

$$= \frac{\pi}{2\sqrt{2}}\int_0^{\pi/2}\sec\left(x-\frac{\pi}{4}\right)dx$$ **½**

$$\Rightarrow 2I = \frac{\pi}{2\sqrt{2}}\left\{\log\left|\sec\left(x-\frac{\pi}{4}\right)+\tan\left(x-\frac{\pi}{4}\right)\right|\right\}_0^{\frac{\pi}{2}}$$ **½**

$$= \frac{\pi}{2\sqrt{2}}\{\log(\sqrt{2}+1) - \log(\sqrt{2}-1)\}$$ **½**

* *Out of Syllabus*

$$\Rightarrow I = \frac{\pi}{4\sqrt{2}}\{\log(\sqrt{2}+1) \div (\sqrt{2}-1)\} \text{ or}$$

$$\frac{\pi}{4\sqrt{2}}\log\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$$

$$\frac{\pi}{2\sqrt{2}}\log(\sqrt{2}+1)$$ ½

[CBSE Marking Scheme, 2019]

21. Solve the differential equation:

$$x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$$

OR

Solve the differential equation:

$$\frac{dy}{dx} = -\left[\frac{x + y\cos x}{1+\sin x}\right]$$

Ans. The given differential equation can be written as:

Put $\frac{dy}{dx} = \frac{y}{x} - \tan\left(\frac{y}{x}\right)$ **1**

$\frac{y}{x} = v$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$, *to get* ½

$$v + x\frac{dv}{dx} = v - \tan v$$

$$\Rightarrow x\frac{dv}{dx} = -\tan v$$

$$\Rightarrow \cot v\, dv = -\frac{1}{x}dx,$$ **1**

Integrating both sides we get,

$$\left.\begin{array}{l}\log|\sin v| = -\log x + \log c \\ \Rightarrow \log|\sin v = \log\left|\frac{c}{x}\right|\end{array}\right\}$$ **1**

∴ Solution of differential equation is

$$\sin\left(\frac{y}{x}\right) = \frac{c}{x} \text{ or } x\cdot\sin\left(\frac{y}{x}\right) = c$$ ½

OR

$$\frac{dy}{dx} + \frac{\cos x}{1+\sin x}.y = \frac{-x}{1+\sin x}$$

$$I.F. = \int_e \frac{\cos x}{1+\sin x}dx = e^{\log(1+\sin x)} = 1+\sin x$$

∴ Solution of the given differential equation is :

$$y(1+\sin x) = \int\frac{-x}{1+\sin x}\times(1+\sin x)dx + c$$

$$\Rightarrow y(1+\sin x) = -\frac{x^2}{2} + c \text{ or } y = \frac{-x^2}{2(1+\sin x)} + \frac{c}{1+\sin x}$$

1

[CBSE Marking Scheme, 2019]

22. The scalar product of the vector $\vec{a} = \hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of the vectors $\vec{b} = 2\hat{i}+4\hat{j}-5\hat{k}$ and $\vec{c} = \lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to 1. Find the value of λ and hence find the unit vector along $\vec{b}+\vec{c}$.

Ans. $\vec{a}.\frac{(\vec{b}+\vec{c})}{|(\vec{b}+\vec{c})|} = 1$

$$\Rightarrow \frac{(\hat{i}+\hat{j}+\hat{k})\cdot\{(2+\lambda)\ \hat{i}+6\hat{j}-2\hat{k}\}}{\left|\sqrt{(2+\lambda)^2+36+41}\right|} = 1$$ **1½**

$$\Rightarrow \lambda + 6 = \sqrt{(2+\lambda)^2 + 40}$$ **2**

Squaring to get

$\lambda^2 + 12\lambda + 36 = \lambda^2 + 4\lambda + 44 \Rightarrow \lambda = 1$ **1½**

∴ Unit vector along $(\vec{b}+\vec{c})$ is $\frac{3}{7}\hat{i}+\frac{6}{7}\hat{j}-\frac{2}{7}\hat{k}$ **1**

[CBSE Marking Scheme, 2019]

23. If the lines $\frac{x-1}{-3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$ and $\frac{x-1}{3\lambda} = \frac{y-1}{2} = \frac{z-6}{-5}$ are perpendicular, find the value of λ. Hence find whether the lines are intersecting or not.

Ans. Lines are perpendicular

$\therefore\ -3(3\lambda)+2\lambda(2)+2(-5) = 0 \Rightarrow \lambda = -2$

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 1-1 & 1-2 & 6-3 \\ -3 & 2(-2) & 2 \\ 3(-2) & 2 & -5 \end{vmatrix}$$

$$= -63 \neq 0$$

Lines are not intersecting **2**

[CBSE Marking Scheme, 2019]

SECTION - D

24. If $A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \\ 5 & 1 & 1 \end{bmatrix}$, find A^{-1}.

Hence solve the system of equations

$x + 3y + 4z = 8$

$2x + y + 2z = 5$

and $5x + y + z = 7$

OR

* **Find the inverse of the following matrix, using elementary transformations:**

$$\mathbf{A} = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

* *Out of Syllabus*

Ans. $|A| = 11;\ Adj\ (A) = \begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}$ **1+2**

$\therefore A^{-1} = \frac{1}{|A|}.Adj\ A = \frac{1}{11}\begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}$ ½

Taking; $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix};\quad B = \begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix}$

The system of equations in matrix form is

$A \cdot X = B \ \therefore X = A^{-1} \cdot B$ **1**

$\therefore$ **Solution is:**

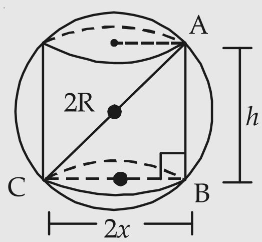
$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}\begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ **1**

$\therefore\ x = 1, y = 1, z = 1$ ½

[CBSE Marking Scheme, 2019]

25. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

Ans. **1**



In rt. ΔABC; $4x^2 + h^2 = 4R^2, x^2 = \frac{4R^2 - h^2}{4}$ **1**

V(Volume of cylinder)

$= \pi x^2 h = \frac{\pi}{4}\left(4R^2h - h^3\right)$ **1**

$V'(h) = \frac{\pi}{4}\left(4R^2 - 3h^2\right);\ V''(h) = \frac{\pi}{4}(-6h)$ ½+ ½

$V'(h) = 0 \Rightarrow h = \frac{2R}{\sqrt{3}}$ **1**

$V''\left(\frac{2R}{\sqrt{3}}\right) = \frac{-6\pi}{4}\left(\frac{2R}{\sqrt{3}}\right) < 0 \Rightarrow Volume\text{'V'}is\max.$ ½

for $h = \frac{2R}{\sqrt{3}}$

Max. Volume: $V = \frac{4}{3\sqrt{3}}\pi R^3$

[CBSE Marking Scheme, 2019]

26. Using method of integration, find the area of the triangle whose vertices are (1, 0), (2, 2) and (3, 1).

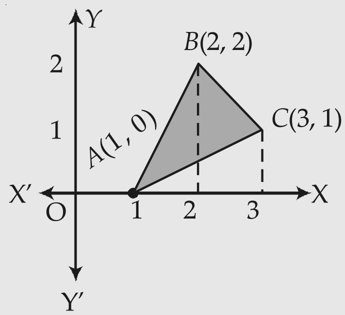
OR

Using method of integration, find the area of the region enclosed between two circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.

Ans. Equation of AB : $y = 2(x-1)$

Equation of BC : $y = 4 - x$

Equation of AC : $y = \frac{1}{2}(x-1)$ 1½



1 ½

$\text{ar }(\Delta ABC) = 2\int_1^2 (x-1)dx + \int_2^3 (4-x)dx - \frac{1}{2}\int_1^3 (x-1)dx$

$= \left[(x-1)^2\right]_1^2 - \left[\frac{1}{2}(4-x)^2\right]_2^3 - \left[\frac{1}{4}(x-1)^2\right]_1^3$

$= 1 + \frac{3}{2} - 1 = \frac{3}{2}$

1

[CBSE Marking Scheme, 2019]

OR

Getting the point of intersection as x = 1 **1**

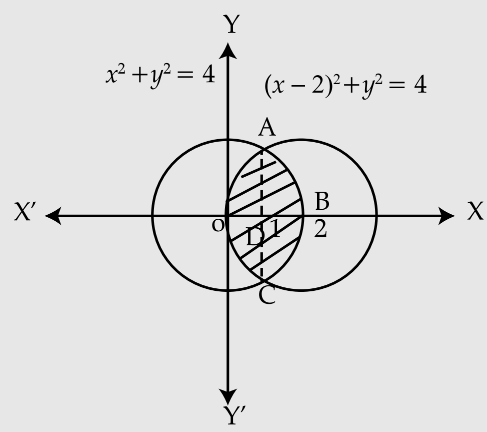
Area $(OABCO) = 4\ ar\ (ABD)$

$= 4\int_1^2 \sqrt{2^2 - x^2}$

$= 4\left\{\frac{x\sqrt{4-x^2}}{2} + 2\sin^{-1}\left(\frac{x}{2}\right)\right\}_1^2$

$= \left(\frac{8\pi}{3} - 2\sqrt{3}\right)$

2



2

[CBSE Marking Scheme, 2019]

*** 27. Find the vector and cartesian equations of the plane passing through the points having position vectors $\hat{i}+\hat{j}-2\hat{k}$, $2\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$. Write the equation of a plane passing through a point (2, 3, 7) and parallel to the plane obtained above. Hence, find the distance between the two parallel planes.**

OR

*** Find the equation of the line passing (2, –1, 2) and (5, 3, 4) and of the plane passing through (2, 0, 3), (1, 1, 5) and (3, 2, 4). Also, find their point of intersection.**

28. There are three coins. One is a two-headed coin, another is a biased coin that comes up heads 75% of the time and the third is an unbiased coin. One of the three coins is chosen at random and tossed. If it shows heads, what is the probability that it is the two-headed coin ?

Ans.

Let E_1 = Event that two-headed coin is chosen **1**

E_2 = Event that biased coin is chosen

E_3 = Event that unbiased coin is chosen

A = Event that coin tossed shows head

Then, $P(E_1) = P(E_2) = P(E_3) = 1/3$ **1**

$P(A|E_1) = 1, P(A|E_2) = \frac{75}{100} = \frac{3}{4}, P(A|E_3) = \frac{1}{2}$ **1½**

$$P(E_1|A) = \frac{P(E_1).P(A|E_1)}{P(E_1).P(A|E_1) + P(E_2).P(A|E_2) + P(E_3).P(A|E_3)}$$

$$= \frac{\frac{1}{3}.1}{\frac{1}{3}.1 + \frac{1}{3}.\frac{3}{4} + \frac{1}{3}.\frac{1}{2}} = \frac{4}{9}$$

2½

[CBSE Marking Scheme, 2019]

29. *A* company produces two types of goods, *A* and *B*, that require gold and silver. Each unit of type *A* requires 3 *g* of silver and 1 *g* of gold while that of type *B* requires 1 *g* of silver and 2 *g* of gold. The company can use at the most 9 *g* of silver and 8 *g* of gold. If each unit of type *A* brings a profit of ₹40 and that of type *B* ₹50, find the number of units of each type that the company should produce to maximize profit. Formulate the above *LPP* and solve it graphically and also find the maximum profit.

Ans. Let the company produce:

Goods $A = x$ units

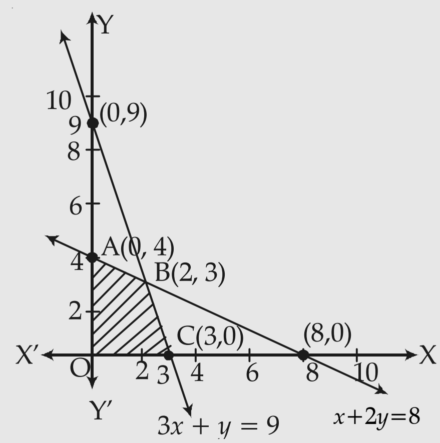
Goods $B = y$ units

then, the linear programming problem is:

Maximize profit: $z = 40x + 50y$ (₹) ½

Subject to constraints:

$$\left.\begin{array}{l} 3x + y \le 9 \\ x + 2y \le 8 \\ x, y \ge 0 \end{array}\right\}$$ **2½**



Correct graph: **2**

Corner point	Value of z (₹) = 40x + 50y
$A(0, 4)$	200
$B(2, 3)$	230 (Max)
$C(3, 0)$	120

½

∴ Maximum profit = ₹ 230 at:
Goods A produced = 2 units, Goods B produced = 3 units ½

[CBSE Marking Scheme, 2019]

Outside Delhi Sets - II 65/2/2

***Note :** Except these, all other question are from outside Delhi Set-I*

SECTION - A

1. Find $|AB|$, if $A = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$.

Ans. $|A| = |B| = 0$ ½

$\Rightarrow |AB| = 0$ **[CBSE Marking Scheme, 2019]** ½

2. Differentiate $e^{\sqrt{3x}}$, with respect to x.

Ans. $\frac{d}{dx}\left(e^{\sqrt{3x}}\right) = \frac{\sqrt{3}}{2\sqrt{x}}e^{\sqrt{3x}}$ **1**

[CBSE Marking Scheme, 2019]

* *Out of Syllabus*

SECTION - B

6. If $A = \begin{bmatrix} p & 2 \\ 2 & p \end{bmatrix}$ and $|A^3| = 125$, then find the values of p.

Ans. $|A| = p^2 - 4$ ½

$|A^3| = 125 \Rightarrow |A|^3 = 125 \Rightarrow |A| = 5$ **1**

$\therefore$ $p^2 - 4 = 5 \Rightarrow p = \pm 3$ ½

[CBSE Marking Scheme, 2019]

12. Find the general solution of the differential equation $\dfrac{dy}{dx} = e^{x+y}$.

Ans. Given differential equation can be written as:

$\dfrac{dy}{dx} = e^x \cdot e^y \Rightarrow e^{-y}\,dy = e^x\,dx$ **1**

Integrating both sides, we get

$-e^{-y} = e^x + c$ **1**

[CBSE Marking Scheme, 2019]

SECTION - C

21. If $(a + bx)\, e^{y/x} = x$, then prove that

$$x^3 \frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$$

Ans. $e^{y/x} = \dfrac{x}{a+bx}$, taking log, on both sides, we get

$\dfrac{y}{x} = \log x - \log(a+bx)$ **1**

Differentiating with respect to 'x'

$$\frac{x \cdot y' - y}{x^2} = \frac{1}{x} - \frac{b}{a+bx} = \frac{a}{(a+bx)x}$$

$$\Rightarrow x \cdot y' - y = \frac{ax}{a+bx} \qquad ...(1)$$ **1½**

Differentiating with respect to 'x'

$$\Rightarrow x \cdot y'' + y' - y' = \frac{(a+bx)\cdot a - ax \cdot b}{(a+bx)^2}$$

$$= \left(\frac{a}{a+bx}\right)^2$$ **1**

$$\Rightarrow x \cdot y'' = \left(\frac{a}{a+bx}\right)^2 \Rightarrow x^3 \cdot y'' = \left(\frac{ax}{a+bx}\right)^2$$

$$\Rightarrow x^3 \frac{d^2y}{dx^2} = \left\{x \cdot \frac{dy}{dx} - y\right\}^2 \quad \text{(Using (i))}$$ ½

[CBSE Marking Scheme, 2019]

22. The volume of a cube is increasing at the rate of 8 cm^3/s. How fast is the surface area increasing when the length of its edge is 12 cm ?

Ans. Let Edge = x cm, then

V(Volume of cube) $= x^3$, S(Surface area) $= 6x^2$

$\dfrac{dV}{dt} = 8\,\text{cm}^3/\text{s} \Rightarrow 3x^2\dfrac{dx}{dt} = 8 \Rightarrow \dfrac{dx}{dt} = \dfrac{8}{3x^2}$

$\dfrac{dS}{dt} = 12x\dfrac{dx}{dt} = 12x.\dfrac{8}{3x^2} = \dfrac{32}{x}$

$\therefore \left.\dfrac{ds}{dt}\right]_{x=12} = \dfrac{32}{12} = \dfrac{8}{3}\text{cm}^2/\text{s}$ **2**

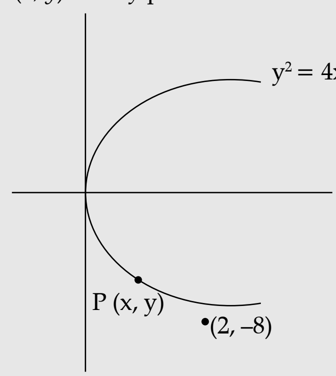
[CBSE Marking Scheme, 2019]

*** 23. Find the cartesian and vector equations of the plane passing through the points $A(2, 5, -3)$, $B(-2, -3, 5)$ and $C(5, 3, -3)$.**

SECTION - D

24. Find the point on the curve $y^2 = 4x$, which is nearest to the point $(2, -8)$.

Ans. Let $P(x, y)$ be any point on the curve $y^2 = 4x$



$$z = AP = \sqrt{(x-2)^2 + (y+8)^2}$$ **1**

let $s = z^2 = \left(\dfrac{y^2}{4} - 2\right)^2 + (y+8)^2$ **1**

$$\frac{ds}{dy} = 2\left(\frac{y^2}{4} - 2\right)\left(\frac{y}{2}\right) + 2(y+8) = \frac{y^3}{4} + 16$$ **1**

$$\frac{d^2s}{dy^2} = \frac{3y^2}{4}$$ ½

Let $\dfrac{ds}{dy} = 0 \Rightarrow y^3 = -64 \Rightarrow y = -4$ **1**

$$\left.\frac{d^2s}{dy^2}\right]_{y=-4} = \frac{3(16)}{4} > 0$$ ½

$\therefore$ s or z is minimum at $y = -4$; $x = \dfrac{y^2}{4} = 4$

$\therefore$ The nearest point is P(4, –4) **1**

[CBSE Marking Scheme, 2019]

* *Out of Syllabus*

* **25.** **Find $\int_1^3 (x^2 + 2 + e^{2x})\,dx$ as the limit of sums.**

OR

Using integration, find the area of the triangular region whose sides have the equations $y = 2x + 1$, $y = 3x + 1$ and $x = 4$.

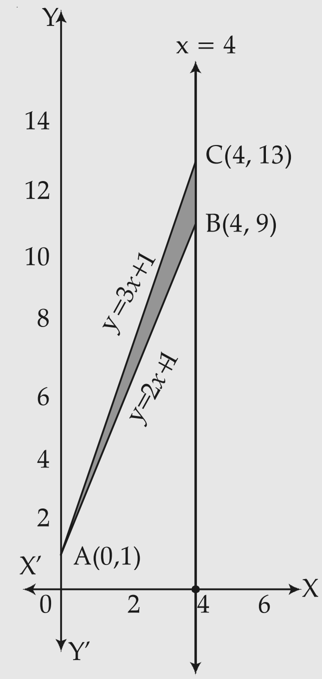
OR

Points of intersection of given lines are $A(0, 1)$, $B(4, 9)$, $C(4, 13)$ **1½**

$\therefore$ Req. Area $= \int_0^4 (3x+1)\,dx - \int_0^4 (2x+1)\,dx$ **2**

$$= \int_0^4 x\,dx = \frac{1}{2}x^2\Big]_0^4 = 8$$ **1½**

Figure **1**



[CBSE Marking Scheme, 2019]

Delhi Set-I 65/1/1

SECTION - A

1. **If A and B are square matrices of the same order 3, such that |A| = 2 and AB = 2I, write the value of |B|.**

Ans. $AB = 2I \Rightarrow |AB| = |2I| \Rightarrow |A| \cdot |B| = 2^3|I|$ ½

$\Rightarrow 2 \times |B| = 8 \Rightarrow |B| = 4$ ½

[CBSE Marking Scheme, 2019]

3. **Find the order and the degree of the differential equation $x^2 \dfrac{d^2y}{dx^2} = \left\{1 + \left(\dfrac{dy}{dx}\right)^2\right\}^4$.**

Ans. Order = 2, degree = 1 ½ + ½

[CBSE Marking Scheme, 2019]

SECTION - B

7. **Find: $\displaystyle\int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}}\,dx$.**

Ans. Put $\tan x = t \Rightarrow \sec^2 x\,dx = dt$ ½

$$\int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}}\,dx = \int \frac{dt}{\sqrt{t^2 + 4}}$$

$$= \log|t + \sqrt{t^2 + 4}| + c$$ ½

$$= \log|\tan x + \sqrt{\tan^2 x + 4}| + c$$ ½

[CBSE Marking Scheme, 2019]

8. **Find: $\displaystyle\int \sqrt{1 - \sin 2x}\,dx, \frac{\pi}{4} < x < \frac{\pi}{2}$**

OR

Find: $\int \sin^{-1}(2x)\,dx$.

Ans. Let $I = \sqrt{1 - \sin 2x\,dx}$

$$= \int(\sin x - \cos x)\,dx$$ **1**

as $\sin x > \cos x$ when $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.

$$= -\cos x - \sin x + c$$ **1**

[CBSE Marking Scheme, 2019]

OR

$$I = \int \sin^{-1}(2x).1dx$$

$$= x.\sin^{-1}(2x) - \int \frac{2x}{\sqrt{1-4x^2}}\,dx$$ **1**

$$= x.\sin^{-1}(2x) + \frac{1}{4}\int \frac{-8x}{\sqrt{1-4x^2}}\,dx$$

$$= x.\sin^{-1}(2x) + \frac{1}{2}\sqrt{1-4x^2} + c$$ **1**

[CBSE Marking Scheme, 2019]

* *Out of Syllabus*

SECTION - C

15. Using properties of determinants, prove that

$$\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3.$$

Ans. $$\text{LHS} = \begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+1 & 1 \\ 3 & 3 & 1 \end{vmatrix}$$

$R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$

$$= \begin{vmatrix} a^2-1 & a-1 & 0 \\ 2(a-1) & a-1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$ **2**

$$= (a-1)^2\begin{vmatrix} a+1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$ **1**

Expanding along C_3,

$=(a-1)^2.(a-1) = (a-1)^3$ = RHS. **1**

[CBSE Marking Scheme, 2019]

19. Find : $\int \frac{3x+5}{x^2+3x-18}dx.$

Ans. $$I=\int\frac{3x+5}{x^2+3x-18}dx = \frac{3}{2}\int\frac{2x+3}{x^2+3x-18}dx + \frac{1}{2}\int\frac{1}{x^2+3x-18}dx$$ **1**

$$= \frac{3}{2}\int\frac{2x+3}{x^2+3x-18}dx + \frac{1}{2}\int\frac{1}{\left(x+\frac{3}{2}\right)^2-\left(\frac{9}{2}\right)^2}dx$$ **1**

$$= \frac{3}{2}\log|x^2+3x-18| + \frac{1}{18}\log\left|\frac{x-3}{x+6}\right| + C$$ **1+1**

[CBSE Marking Scheme, 2019]

21. Solve the differential equation :

$x\,dy - y\,dx = \sqrt{x^2+y^2}\,dx$**, given that** $y = 0$ **when** $x = 1$**.**

OR

Solve the differential equation :

$(1+x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$**, subject to the initial condition** $y(0) = 0$**.**

Ans. Writing $\frac{dy}{dx} = \frac{y+\sqrt{x^2+y^2}}{x} = \frac{y}{x} + \sqrt{1+\left(\frac{y}{x}\right)^2}$ ½

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ ½

Differential equation becomes

$$v + x\frac{dv}{dx} = v + \sqrt{1+v^2}$$

$$\Rightarrow \int\frac{dv}{\sqrt{1+v^2}} = \int\frac{dx}{x}$$ ½

$$\Rightarrow \log\left|v+\sqrt{1+v^2}\right| = \log|x| + \log c$$ ½

$$\Rightarrow v+\sqrt{1+v^2} = cx \Rightarrow y + \sqrt{x^2+y^2} = cx^2$$ **1**

when $x = 1, y = 0 \Rightarrow c = 1$ ½

$\therefore y + \sqrt{x^2+y^2} = x^2$ ½

[CBSE Marking Scheme, 2019]

OR

Given equation is $\frac{dy}{dx} + \frac{2x}{1+x^2}.y = \frac{4x^2}{1+x^2}$ ½

$$I.F. = e^{\int\frac{2x}{1+x^2}dx} = 1+x^2$$ **1**

Solution is given by,

$$y\cdot(1+x^2) = \int\frac{4x^2}{1+x^2}.(1+x^2)dx = \int 4x^2\,dx$$ **1**

$$\Rightarrow y\cdot(1+x^2) = \frac{4x^3}{3} + c$$ ½

when $x = 0, y = 0 \Rightarrow c = 0$ ½

$$y\cdot(1+x^2) = \frac{4x^3}{3} \text{ or } y = \frac{4x^3}{3(1+x^2)}$$ ½

[CBSE Marking Scheme, 2019]

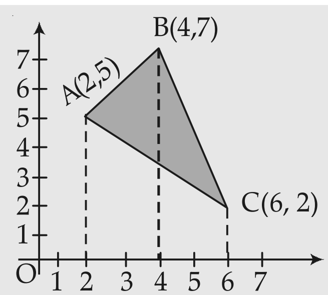
SECTION - D

26. Using integration, find the area of triangle ABC**, whose vertices are** $A(2, 5)$**,** $B(4, 7)$ **and** $C(6, 2)$**.**

OR

Find the area of the region lying above X**-axis and included between the circle** $x^2 + y^2 = 8x$ **and inside of the parabola** $y^2 = 4x$**.**

Ans.



1

Equation of AB : $y = x+3$

Equation of BC : $y = \frac{-5x}{2}+17$

Equation of AC : $y = \frac{-3x}{4}+\frac{13}{2}$ **1½**

$$\text{Required Area} = \int_2^4 (x+3)\,dx + \int_4^6 \left(\frac{-5x}{2}+17\right)dx - \int_2^6 \left(\frac{-3x}{4}+\frac{13}{2}\right)dx \quad 1\tfrac{1}{2}$$

$$= \left[\frac{(x+3)^2}{2}\right]_2^4 + \left[\frac{-5x^2}{4}+17x\right]_4^6 - \left[\frac{-3x^2}{8}+\frac{13x}{2}\right]_2^6$$

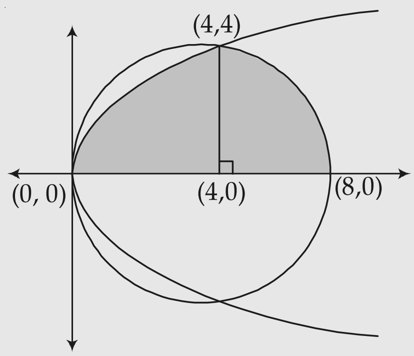
1½

$= 12 + 09 - 14$

$= 7$ units ½

[CBSE Marking Scheme, 2019]

OR



Correct Figure 1

Given circle $x^2 - 8x + y^2 = 0$

or $(x-4)^2 + y^2 = 4^2$

Point of intersection (0, 0) and (4, 4) 1

$$\text{Required Area} = \int_0^4 2\sqrt{x}\,dx + \int_4^8 \sqrt{4^2-(x-4)^2}\,dx \quad 1\tfrac{1}{2}$$

$$= \left[\frac{4}{3}x^{3/2}\right]_0^4 + \left[\frac{x-4}{2}\sqrt{16-(x-4)^2} + \frac{16}{2}\sin^{-1}\left(\frac{x-4}{4}\right)\right]_4^8 \quad 1\tfrac{1}{2}$$

$$= \left(4\pi + \frac{32}{3}\right)$$

1

[CBSE Marking Scheme, 2019]

28. A manufacturer has three machine operators *A, B* and *C*. The first operator *A* produces 1% of defective items, whereas the other two operators *B* and *C* produces 5% and 7% defective items respectively. *A* is on the job for 50% of the time, *B* on the job 30% of the time and *C* on the job for 20% of the time. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by *A* ?

Ans. Let $\left.\begin{array}{l} E_1 : \text{item is produced by } A \\ E_2 : \text{item is produced by } B \\ E_3 : \text{item is produced by } C \end{array}\right\}$ 1

$$P(E_1) = \frac{50}{100}, P(E_2) = \frac{30}{100}, P(E_3) = \frac{20}{100} \quad 1$$

$$P(A/E_1) = \frac{1}{100},\ P(A/E_2) = \frac{5}{100},$$

$$P(A/E_3) = \frac{7}{100} \quad 1$$

$$P(E_1|A) = \frac{\frac{50}{100}\times\frac{1}{100}}{\frac{50}{100}\times\frac{1}{100}+\frac{30}{100}\times\frac{5}{100}+\frac{20}{100}\times\frac{7}{100}} \quad 2$$

$$= \frac{5}{34} \quad 1$$

[CBSE Marking Scheme, 2019]

Delhi Set-II 65/1/2

Note : *Except these, all other questions from Delhi Set-I*

SECTION - A

* **2. If $f(x) = x + 7$ and $g(x) = x - 7$, $x \in R$, then find $\frac{d}{dx}(fog)(x)$.**

3. Find the value of $x - y$, if

$$2\begin{bmatrix}1 & 3\\0 & x\end{bmatrix} + \begin{bmatrix}y & 0\\1 & 2\end{bmatrix} = \begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$$

Ans. $\begin{bmatrix}2+y & 6\\1 & 2x+2\end{bmatrix} = \begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$ ½

$\Rightarrow x = 3, y = 3 \qquad \therefore x - y = 0$ ½

[CBSE Marking Scheme, 2019]

* *Out of Syllabus*

SECTION - B

6. If $A = \begin{bmatrix}2 & 0 & 1\\2 & 1 & 3\\1 & -1 & 0\end{bmatrix}$, then find $(A^2 - 5A)$.

Ans. $A^2 = \begin{pmatrix}5 & -1 & 2\\9 & -2 & 5\\0 & -1 & -2\end{pmatrix}$ 1

$$A^2 - 5A = \begin{pmatrix}5 & -1 & 2\\9 & -2 & 5\\0 & -1 & -2\end{pmatrix} - \begin{pmatrix}10 & 0 & 5\\10 & 5 & 15\\5 & -5 & 0\end{pmatrix}$$

$$=\begin{pmatrix}-5 & -1 & -3\\ -1 & -7 & -10\\ -5 & 4 & -2\end{pmatrix}$$ **1**

[CBSE Marking Scheme, 2019]

12. Find: $\int \frac{\tan^2 x \sec^2 x}{1-\tan^6 x}dx.$

Ans. $I=\int \frac{\tan^2 x.\sec^2 x}{1-(\tan^3 x)^2}dx$

$\text{Put} \tan^3 x = t \Rightarrow \text{I} = \frac{1}{3}\int \frac{dt}{1-t^2}$ **1**

$=\frac{1}{6}\log\left|\frac{1+t}{1-t}\right|+c=\frac{1}{6}\log\left|\frac{1+\tan^3 x}{1-\tan^3 x}\right|+c$ **½ + ½**

[CBSE Marking Scheme, 2019]

SECTION - C

13. Solve for x: $\tan^{-1}(2x)+\tan^{-1}(3x)=\frac{\pi}{4}$.

Ans. $\tan^{-1}\left(\frac{2x+3x}{1-(2x)(3x)}\right)=\frac{\pi}{4}$ **1**

$\Rightarrow \frac{5x}{1-6x^2}=\tan\frac{\pi}{4}=1 \Rightarrow 6x^2+5x-1=0$ **1½**

$\Rightarrow x=-1 \text{ or } x=\frac{1}{6}$ **1**

as $x = -1$ does not satisfy the given equation.

$\therefore \quad x=\frac{1}{6}$ **½**

[CBSE Marking Scheme, 2019

18. Using properties of determinants, prove the following:

$$\begin{vmatrix}a+b+c & -c & -b\\ -c & a+b+c & -a\\ -b & -a & a+b+c\end{vmatrix}$$

= 2(a + b) (b + c) (c + a).

Ans. LHS = $\begin{vmatrix}a+b+c & -c & -b\\ -c & a+b+c & -a\\ -b & -a & a+b+c\end{vmatrix}$

$C_2 \to C_2+C_1,\ C_3 \to C_3+C_1$

$$=\begin{vmatrix}a+b+c & a+b & a+c\\ -c & a+b & -(a+c)\\ -b & -(a+b) & (a+c)\end{vmatrix}$$ **1½**

Taking common (a+b) from c_2 and (a+c) from c_3.

$$=(a+b)(a+c)\begin{vmatrix}a+b+c & 1 & 1\\ -c & 1 & -1\\ -b & -1 & 1\end{vmatrix}$$ **½**

$C_3 \to C_3+C_2$

$$=(a+b)(a+c)\begin{vmatrix}a+b+c & 1 & 2\\ -c & 1 & 0\\ -b & -1 & 0\end{vmatrix}$$ **1**

Expanding along C_3

= 2 (a+b) (b+c) (c+a) = RHS. **1**

[CBSE Marking Scheme, 2019]

19. If $x=\cos t+\log\tan\left(\frac{t}{2}\right)$, $y=\sin t$, then find the values of $\frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$ at $t=\frac{\pi}{4}$.

Ans. $\frac{dx}{dt}=-\sin t+\frac{1}{\tan\frac{t}{2}}\times\left(\sec^2\frac{t}{2}\times\frac{1}{2}\right)=\frac{\cos^2 t}{\sin t}$

$\frac{dy}{dt}=\cos t$

$\frac{d^2y}{dt^2}=-\sin t \Rightarrow \left.\frac{d^2y}{dt^2}\right]_{t=\frac{\pi}{4}}=-\frac{1}{\sqrt{2}}$

$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\tan t$

$\frac{d^2y}{dx^2}=\sec^2 t.\frac{dt}{dx}=\sec^4 t.\sin t$

$\Rightarrow \left.\frac{d^2y}{dx^2}\right]_{t=\frac{\pi}{4}}=2\sqrt{2}$

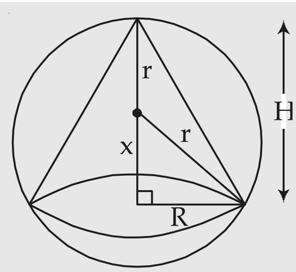
1

[CBSE Marking Scheme, 2019]

SECTION - D

24. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$. Also find the maximum volume of cone.

Ans.



Correct Figure

$r^2=x^2+R^2$ **1**

$V=\frac{1}{3}\pi R^2 H$

$$= \frac{1}{3}\pi(r^2 - x^2)(r + x)$$

$$= \frac{1}{3}\pi(r + x)^2(r - x)$$ **1**

$$\frac{dV}{dx} = \frac{1}{3}\pi\left[(r + x)^2(-1) + (r - x).2(r + x)\right]$$

$$= \frac{1}{3}\pi(r + x)(r - 3x)$$ **1**

$$\frac{dV}{dx} = 0 \Rightarrow x = -r \text{ or } x = \frac{r}{3}$$ ½

(Rejected)

$$\frac{d^2V}{dx^2} = \frac{1}{3}\pi[(r + x)(-3) + (r - 3x)]$$

$$= -\frac{\pi}{3}(2r + 6x) < 0$$ **1**

$\Rightarrow$ V is maximum when $x = \frac{r}{3}\cdot$

$$H = r + x = r + \frac{r}{3} = \frac{4r}{3}$$ ½

Maximum volume $V = \frac{1}{3}\pi\left(r + \frac{r}{3}\right)^2\left(r - \frac{r}{3}\right)$

$$= \frac{32}{81}\pi r^3$$ **1**

[CBSE Marking Scheme, 2019]

25. If A = $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then find A^{-1}. Hence solve the following system of equations:

$2x - 3y + 5z = 11, 3x + 2y - 4z = -5,$
$x + y - 2z = -3.$

OR

* **Obtain the inverse of the following matrix using elementary operations:**

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

Ans. $|A| = -1 \neq 0 \Rightarrow A^{-1}$ exists. **1**

$$\text{Adj } A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$ **2**

$$\therefore A^{-1} = \frac{1}{|A|}\cdot adj\ A = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$ ½

Given system of equations can be written as

AX = B where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

Now, $X = A^{-1}B$ **1**

$$= \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$ **1**

$\Rightarrow x = 1, y = 2, z = 3$ ½

[CBSE Marking Scheme, 2019]

❑❑

* *Out of Syllabus*