Solved Paper 2020
Mathematics
Class-XII

Time : 3 Hours **Max. Marks : 80**

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper comprises **four** sections – **A, B, C** and **D**.*

*This question paper carries **36** questions. **All** questions are compulsory.*

(ii) ***Section A** – Question no. **1** to **20** comprises of **20** questions of **one** mark each.*

(iii) ***Section B** – Question no. **21** to **26** comprises of **6** questions of **two** marks each.*

(iv) ***Section C** – Question no. **27** to **32** comprises of **6** questions of **four** marks each.*

(v) ***Section D** – Question no. **33** to **36** comprises of **4** questions of **six** marks each.*

(vi) *There is no overall choice in the question paper. However, an internal choice has been provided in **3** questions of one mark, **2** questions of two marks, **2** questions of four marks and **2** questions of six marks. Only one of the choices in such questions have to be attempted.*

(vii) *In addition to this, separate instructions are given with each section and question, wherever necessary.*

(viii) *Use of calculators is not permitted.*

Delhi Set-I 65/5/1

SECTION - A

Question numbers 1 to 10 are multiple choice questions of 1 mark each. Select the correct option :

1. If A is a square matrix of order 3, such that $A(adj\ A) = 10\ I$, then $|adj\ A|$ is equal to

(a) 1 **(b)** 10
(c) 100 **(d)** 101

Ans. Option (c) is correct.

Explanation:

Consider the equation

$$A\ (adj\ A) = |A|\ I$$

Here, $A\ (adj\ A) = 10\ I$

Then, $|A| = 10$

Since, $|adj\ A| = |A|^{n-1}$

Where n is order of matrix

Here, $= |A|^{3-1} = 10^2$

$= 100$

2. If A is a 3×3 matrix such that $|A| = 8$, then $|3A|$ equals

(a) 8 **(b)** 24
(c) 72 **(d)** 216

Ans. Option (d) is correct.

Explanation:

Here $|A| = 8$

Then $|3A| = 3^3|A| = 27 \times 8 = 216$

3. If $y = Ae^{5x} + Be^{-5x}$, then $\dfrac{d^2y}{dx^2}$ is equal to

(a) $25\ y$ **(b)** $5\ y$
(c) $-25\ y$ **(d)** $15\ y$

Ans. Option (a) is correct.

Explanation:

$$y = Ae^{5x} + Be^{-5x}$$

$$\Rightarrow \quad \frac{dy}{dx} = 5Ae^{5x} - 5Be^{-5x}$$

$$\Rightarrow \quad \frac{d^2y}{d^2x} = 25Ae^{5x} + 25Be^{-5x}$$

$$= 25\ y$$

4. $\int x^2 e^{x^3}\ dx$ equals

(a) $\frac{1}{3}e^{x^3} + C$ **(b)** $\frac{1}{3}e^{x^4} + C$
(c) $\frac{1}{2}e^{x^3} + C$ **(d)** $\frac{1}{2}e^{x^2} + C$

Ans. Option (a) is correct.

Explanation:

Given $\int x^2 e^{x^3} dx$

Put $x^3 = y$

$\Rightarrow \quad 3x^2 dx = dy$

Then $\int x^2 e^{x^3} dx = \frac{1}{3}\int e^y dy$

$$= \frac{1}{3}e^y + C$$

$$= \frac{1}{3}e^{x^3} + C$$

5. If $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along three mutually perpendicular directions, then

(a) $\hat{i} \cdot \hat{j} = 1$ **(b)** $\hat{i} \times \hat{j} = 1$

(c) $\hat{i} \cdot \hat{k} = 0$ **(d)** $\hat{i} \times \hat{k} = 0$

Ans. Option (c) is correct.

Explanation:

$$\hat{i}.\hat{k} = |\hat{i}||\hat{k}|\cos\frac{\pi}{2} = 1 \times 1 \times 0 = 0$$

6. *ABCD* is a rhombus whose diagonals intersect at *E*. Then $\vec{EA} + \vec{EB} + \vec{EC} + \vec{ED}$ equals

(a) $\vec{0}$ **(b)** $\vec{AD}$

(c) $2\vec{BC}$ **(d)** $2\vec{AD}$

Ans. Option (a) is correct.

Explanation:

$$\overrightarrow{EA} + \overrightarrow{EB} + \overrightarrow{EC} + \overrightarrow{ED} = \overrightarrow{EA} + \overrightarrow{EB} - \overrightarrow{EA} - \overrightarrow{EB} = \vec{0}$$

7. The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{4-z}{k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{-2}$ are mutually perpendicular if the value of *k* is

(a) $-\frac{2}{3}$ **(b)** $\frac{2}{3}$

(c) -2 **(d)** 2

Ans. Option (a) is correct.

Explanation:

If $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$

And $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{-2}$

are perpendicular then

$1 \times k + 1 \times 2 + (-k) \times (-2) = 0$

$\Rightarrow \quad k + 2 + 2k = 0$

$\Rightarrow \quad 2 + 3k = 0$

$\Rightarrow \quad k = -\frac{2}{3}$

8. The graph of the inequality $2x + 3y > 6$ is

(a) half plane that contains the origin.

(b) half plane that neither contains the origin nor the points of the line $2x + 3y = 6$.

(c) whole *XOY*-plane excluding the points on the line $2x + 3y = 6$.

(d) entire *XOY* plane.

Ans. Option (b) is correct.

Explanation: Since, the equation $2x + 3y > 6$, hence half plane that neither contains the origin nor the points of the line $2x + 3y = 6$.

9. A card is picked at random from a pack of 52 playing cards. Given that the picked card is a queen, the probability of this card to be a card of spade is

(a) $\frac{1}{3}$ **(b)** $\frac{4}{13}$

(c) $\frac{1}{4}$ **(d)** $\frac{1}{2}$

Ans. Option (c) is correct.

Explanation:

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

where, A : The card is spade

B : The picked card is queen

$$\Rightarrow \quad P(A/B) = \frac{\frac{1}{52}}{\frac{4}{52}} = \frac{1}{4}$$

10. A die is thrown once. Let *A* be the event that the number obtained is greater than 3. Let *B* be the event that the number obtained is less than 5. Then $P(A \cup B)$ is

(a) $\frac{2}{5}$ **(b)** $\frac{3}{5}$

(c) 0 **(d)** 1

Ans. Option (d) is correct.

Explanation :

$A = \{4, 5, 6\}$ and $B = \{1, 2, 3, 4\}$

Here $A \cap B = \{4\}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{3}{6} + \frac{4}{6} - \frac{1}{6} = 1$

Fill in the blanks in Questions from 11 to 15.

11. A relation in a set *A* is called .... relation, if each element of *A* is related to itself.

Ans. "Identity"

Explanation: Since, in Identity relation; every element of a set always related to itself.

12. If $A + B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $A - 2B = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$, then A = ...

Ans. $2(A + B) + (A - 2B) = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$

$$3A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}$$

13. The least value of the function $f(x) = ax + \frac{b}{x} (a > 0, b > 0, x > 0)$ is ....

Ans. Here $$f(x) = ax + \frac{b}{x}$$

For the least value

$$f'(x) = a - \frac{b}{x^2} = 0$$

$$\Rightarrow \quad a = \frac{b}{x^2}$$

$$\Rightarrow \quad x = \sqrt{\frac{b}{a}}$$

Also $$f''(x) = +\frac{2b}{x^3}$$

Then $$f''\left(\sqrt{\frac{b}{a}}\right) = \frac{2b}{\left(\frac{b}{a}\right)^{\frac{3}{2}}} > 0$$

Least value of the function is

$$f\left(\sqrt{\frac{b}{a}}\right) = a\sqrt{\left(\frac{b}{a}\right)} + \frac{b}{\sqrt{\left(\frac{b}{a}\right)}}$$

$$= \sqrt{ab} + \sqrt{ab}$$

$$= 2\sqrt{ab}$$

$$\therefore \quad f\left(\sqrt{\frac{b}{a}}\right) = 2\sqrt{ab}$$

14. The integrating factor of the differential equation $x\frac{dy}{dx} + 2y = x^2$ is ....

OR

The degree of the differential equation $1 + \left(\frac{dy}{dx}\right)^2 = x$ is ....

Ans. Given that

$$x\frac{dy}{dx} + 2y = x^2$$

$$\frac{dy}{dx} + \frac{2}{x}y = x$$

This differential eqn. is of the form

$$\frac{dy}{dx} + Py = Q$$

$$P = \frac{2}{x} \text{ and } Q = x$$

$$\text{I.F.} = e^{\int \frac{2}{x}dx}$$

$$= e^{\log x^2} = x^2$$

OR

Degree= 2

15. The vector equation of a line which passes through the point (3, 4, – 7) and (1, – 1, 6) is ....

OR

The line of shortest distance between two skew lines is .... to both the lines.

Ans. Required vector equation

$$\vec{r} = 3\hat{i} + 4j - 7k + \lambda[\hat{i} - \hat{j} + 6\hat{k} - 3\hat{i} - 4\hat{j} + 7\hat{k}]$$

$$\therefore \quad \vec{r} = 3\hat{i} + 4\hat{j} - 7\hat{k} + \lambda(-2\hat{i} - 5\hat{j} + 13\hat{k})$$

OR

"Perpendicular"

Question numbers 16 to 20 are very short answer type questions.

16. Find the value of $\sin^{-1}\left[\sin\left(-\frac{17\pi}{8}\right)\right]$.

Ans.
$$\sin^{-1}\left[\sin\left(\frac{-17\pi}{8}\right)\right] = -\sin^{-1}\left[\sin\left(\frac{17\pi}{8}\right)\right]$$

$$= -\sin^{-1}\left[\sin\left(2\pi + \frac{\pi}{8}\right)\right]$$

$$\sin^{-1}\left[\sin\left(\frac{-17\pi}{8}\right)\right] = -\sin^{-1}\sin\frac{\pi}{8}$$

$$= -\frac{\pi}{8}$$

17. For $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ write A^{-1}.

Ans.
$$A^{-1} = \frac{1}{(-3+4)}\begin{bmatrix} -1 & 4 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 4 \\ -1 & 3 \end{bmatrix}$$

18. If the function f defined as

$$f(x) = \begin{cases} \frac{x^2 - 9}{x - 3}, & x \neq 3 \\ k, & x = 3 \end{cases}$$

is continuous at $x = 3$, find the value of k.

Ans. Here f is continuous at $x = 3$, then $\lim_{x \to 3} f(x) = f(3)$

$$\Rightarrow \quad \lim_{x \to 3} \frac{(x-3)(x+3)}{(x-3)} = k$$

$$\Rightarrow \quad 3 + 3 = k$$

$$\Rightarrow \quad k = 6$$

19. If $f(x) = x^4 - 10$, then find the approximate value of $f(2.1)$.

OR

* **Find the slope of the tangent to the curve $y = 2\sin^2(3x)$ at $x = \frac{\pi}{6}$.**

Ans. Since, $f(x + h) = f(x) + h f'(x)$,

where $h \to 0$

here $x = 2, h = 0{\cdot}1$

$$\therefore \quad f(2{\cdot}1) = f(2) + (0{\cdot}1)f'(2) \quad \text{...(i)}$$

Also $$f'(x) = 4x^3$$

$$f'(2) = 4 \times 2^3 = 32$$

* *Out of Syllabus*

From (i)

$$f(2{\cdot}1) = 16 - 10 + (0{\cdot}1) \times 32$$
$$f(2{\cdot}1) = 6 + 3{\cdot}2 = 9{\cdot}2$$

20. Find the value of $\int_1^4 |x-5|\,dx$.

Ans.
$$\int_1^4 |x-5|\,dx = \int_1^4 -(x-5)dx$$
$$= -\frac{1}{2}[(x-5)^2]_1^4$$
$$\int_1^4 |x-5|\,dx = -\frac{1}{2}[1-16] = \frac{15}{2}$$

SECTION - B

Question numbers 21 to 26 carry 2 marks each.

***21. If $f(x)=\dfrac{4x+3}{6x-4}$, $x \neq \dfrac{2}{3}$, then show that $(fof)(x) = x$, for all x $\dfrac{2}{3}$. Also, write inverse of f.**

OR

Check if the relation R in the set R of real numbers defined as $R = \{(a, b) : a < b\}$ is (i) symmetric, (ii) transitive.

Ans.OR

(i) It is not symmetric because if $a < b$ then $b < a$ is not true

(ii) Here, if $a < b$ and $b < c$ then $a < c$ is also true for all a, b, $c \in$ Real numbers. Therefore R is transitive.

22. Find $\int \dfrac{x}{x^2+3x+2}dx$

Ans.
$$\int \frac{xdx}{x^2+3x+2} = \frac{1}{2}\int\frac{2x+3}{x^2+3x+2} - \frac{3}{2}\int\frac{dx}{x^2+3x+2}$$
$$= \frac{1}{2}\log|x^2+3x+2| - \frac{3}{2}\int\frac{dx}{\left(x+\frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2}$$
$$= \frac{1}{2}\log|x^2+3x+2| - \frac{3}{2}\times\frac{1}{2\times\frac{1}{2}}\log\left|\frac{x+\frac{3}{2}-\frac{1}{2}}{x+\frac{3}{2}+\frac{1}{2}}\right| + C$$
$$\Rightarrow \int\frac{x\,dx}{x^2+3x+2} = \frac{1}{2}\log|x^2+3x+2| - \frac{3}{2}\log\left|\frac{x+1}{x+2}\right| + C$$

23. If $x = a\cos\theta$; $y = b\sin\theta$, then find $\dfrac{d^2y}{dx^2}$.

OR

Find the differential of $\sin^2 x$ w.r.t. $e^{\cos x}$.

Ans. Here $x = a\cos\theta$, $y = b\sin\theta$

$$\Rightarrow \frac{dx}{d\theta} = -a\sin\theta,\ \frac{dy}{d\theta} = b\cos\theta$$
$$\Rightarrow \frac{dy}{dx} = \frac{b\cos\theta}{-a\sin\theta} = -\frac{b}{a}\cot\theta$$
$$\Rightarrow \frac{d^2y}{dx^2} = \frac{b}{a}\text{cosec}^2\theta\times\frac{d\theta}{dx}$$
$$= \frac{b}{a}\text{cosec}^2\theta\times\frac{(-1)}{a\sin\theta}$$
$$\Rightarrow \frac{d^2y}{dx^2} = -\frac{b}{a^2}\text{cosec}^3\theta$$

OR

Here, suppose $u = \sin^2 x$, $v = e^{\cos x}$

Then, we need to differentiate u w.r. to v

i.e.
$$\frac{du}{dv} = \frac{du}{dx}\cdot\frac{dx}{dv}$$
$$= \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{\frac{d(\sin^2 x)}{dx}}{\frac{d(e^{\cos x})}{dx}}$$
$$\Rightarrow \frac{du}{dv} = \frac{2\sin x\cos x}{e^{\cos x}.(-\sin x)} = -\frac{2\cos x}{e^{\cos x}}$$

24. Evaluate : $\int_1^2\left[\dfrac{1}{x}-\dfrac{1}{2x^2}\right]e^{2x}dx$

Ans. $\int_1^2\left[\dfrac{1}{x}-\dfrac{1}{2x^2}\right]e^{2x}dx$

Put $2x = y \Rightarrow 2dx = dy \Rightarrow dx = \dfrac{dy}{2}$

For $x = 1$, $y = 2$ and for $x = 2$, $y = 4$

$$= \frac{1}{2}\int_2^4\left(\frac{2}{y}-\frac{2}{y^2}\right)e^y dy$$
$$= \int_2^4\left(\frac{1}{y}-\frac{1}{y^2}\right)e^y dy$$
$$= \left[\frac{1}{y}e^y\right]_2^4 = \frac{1}{4}e^4 - \frac{1}{2}e^2$$
$$= \frac{e^2}{2}\left[\frac{e^2}{2}-1\right]$$

25. Find the value of $\int_0^1 x(1-x)^n\,dx$.

Ans. Suppose

$$\text{I} = \int_0^1 x(1-x)^n\,dx$$
$$\Rightarrow \text{I} = \int_0^1 (1-x)[1-(1-x)]^n\,dx \qquad \text{[By King Rule]}$$
$$\Rightarrow \text{I} = \int_0^1 (1-x)x^n dx$$
$$\Rightarrow \text{I} = \int_0^1 (x^n - x^{n+1})dx = \left[\frac{x^{n+1}}{n+1}-\frac{x^{n+2}}{n+2}\right]_0^1$$
$$\Rightarrow \text{I} = \left[\frac{1}{n+1}-\frac{1}{n+2}\right]-[0-0] = \frac{1}{(n+1)(n+2)}$$

* *Out of Syllabus*

26. Given two independent events A and B such that $P(A) = 0.3$ and $P(B) = 0.6$, find $P(A' \cap B')$.

Ans.
$$P(A' \cap B') = P(A').P(B')$$
$$= [1 - P(A)]\,[1 - P(B)]$$
$$= [1 - 0.3]\,[1 - 0.6]$$
$$= (0.7).(0.4)$$
$$\Rightarrow \quad P(A' \cap B') = 0.28$$

SECTION - C

Question numbers 27 to 32 carry 4 marks each.

27. Solve for x : $\sin^{-1}(1 - x) - 2\sin^{-1}(x) = \dfrac{\pi}{2}$.

Ans. Given that,

$$\sin^{-1}(1 - x) - 2\sin^{-1}(x) = \frac{\pi}{2} \qquad \text{....(i)}$$

$$\Rightarrow \quad \sin^{-1}(1 - x) = \frac{\pi}{2} + 2\sin^{-1}(x)$$

$$\Rightarrow \quad 1 - x = \sin\left(\frac{\pi}{2} + 2\sin^{-1}(x)\right)$$

$$\Rightarrow \quad 1 - x = \cos(2\sin^{-1}(x))$$

$$\left[\text{Because } \sin\left(\frac{\pi}{2} + \theta\right) = \cos\theta\right]$$

$$\Rightarrow \quad 1 - x = \cos\{\cos^{-1}(1 - 2x^2)\}$$

$$[\text{Because } 2\sin^{-1}x = \cos^{-1}(1 - 2x^2)]$$

$$\Rightarrow \quad 1 - x = 1 - 2x^2$$
$$\Rightarrow \quad x = 2x^2$$
$$\Rightarrow \quad 2x^2 - x = 0$$
$$\Rightarrow \quad x(2x - 1) = 0$$
$$\Rightarrow \quad x = 0,\ x = 1/2$$

for $\quad x = 1/2$

$$\sin^{-1}(1 - x) - 2\sin^{-1}(x) = \sin^{-1}\left(1 - \frac{1}{2}\right) - 2\sin^{-1}(1/2)$$

$$= \sin^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(\frac{1}{2}\right)$$

$$= -\sin^{-1}\left(\frac{1}{2}\right)$$

$$= -\frac{\pi}{6} \neq \text{R.H.S}$$

Clearly $x = \dfrac{1}{2}$ is not a solution.

Therefore, $x = 0$ is the required solution.

28. If $y = (\log x)^x + x^{\log x}$, then find $\dfrac{dy}{dx}$.

Ans. Let us Suppose $\quad y = u + v$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \qquad \text{...(i)}$$

where $\quad u = [\log(x)]^x$

and $\quad v = x^{\log x} \qquad$ ...(ii)

First take $\quad u = [\log(x)]^x$

Taking log of both the sides

$$\log u = x\log\log x \qquad \text{...(iii)}$$

Now differentiating the above w.r. to x

$$\Rightarrow \quad \frac{1}{u}.\frac{du}{dx} = x.\frac{1}{\log x}.\frac{1}{x} + \log\log x.$$

$$\Rightarrow \quad \frac{du}{dx} = u\left(\frac{1}{\log x} + \log\log x\right)$$

$$\Rightarrow \quad \frac{du}{dx} = (\log(x))^x\left(\frac{1}{\log x} + \log\log x\right) \qquad \text{...(iv)}$$

Now take $\quad v = x^{\log x}$

$$\log v = \log x.\log x$$

[Taking log on both sides]

$$= (\log x)^2$$

Differentiating w.r.t. x

$$\Rightarrow \quad \frac{1}{v^a}.\frac{dv}{dx} = 2\log x.\frac{1}{x}$$

$$\Rightarrow \quad \frac{dv}{dx} = v\left(2\log x.\frac{1}{x}\right)$$

$$\Rightarrow \quad \frac{dv}{dx} = x^{\log x}.\frac{2\log x}{x}$$

$$\Rightarrow \quad \frac{dv}{dx} = 2x^{\log x - 1}.\log x \qquad \text{...(v)}$$

From equation (i), (iv) and (v)

$$\frac{dy}{dx} = (\log x)^x\left(\frac{1}{\log x} + \log\log x\right) + 2x^{\log x - 1}.\log x$$

29. Solve the differential equation:

$$x\sin\left(\frac{y}{x}\right)\frac{dy}{dx} + x - y\sin\left(\frac{y}{x}\right) = 0$$

Given that $x = 1$ when $y = \dfrac{\pi}{2}$.

Ans. We have,

$$x\sin\left(\frac{y}{x}\right)\frac{dy}{dx} + x - y\sin\frac{y}{x} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y\sin\left(\frac{y}{x}\right) - x}{x\sin\left(\frac{y}{x}\right)} \qquad \text{...(i)}$$

Above differential equation is a homogeneous equation

Put $\quad y = vx$

Then, $\quad \dfrac{dy}{dx} = v + x\dfrac{dv}{dx} \qquad$ ...(ii)

From (i) and (ii)

$$\Rightarrow \quad v + x\frac{dv}{dx} = \frac{vx.\sin\left(\frac{vx}{x}\right) - x}{x\sin\left(\frac{vx}{x}\right)}$$

$$\Rightarrow \quad v + x\frac{dv}{dx} = \frac{x(v\sin v - 1)}{x\sin v}$$

$$\Rightarrow \quad v + x\frac{dv}{dx} = \frac{v\sin v - 1}{\sin v}$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{v\sin v - 1}{\sin v} - v$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{v\sin v - 1 - v\sin v}{\sin v}$$

$$\Rightarrow \quad x\frac{dv}{dx} = -\frac{1}{\sin v}$$

$$\Rightarrow \quad \sin v\, dv = -\frac{1}{x}dx \quad [\text{Here } x \neq 0]$$

Now, integrating both sides

$$\Rightarrow \quad \int \sin v\, dv = -\int \frac{1}{x}dx$$

$$\Rightarrow \quad -\cos v = -\log|x| + C$$

Put, $v = \dfrac{y}{x}$

$$\Rightarrow \quad -\cos\left(\frac{y}{x}\right) = -\log|x| + C \quad \text{...(iii)}$$

Also, given that $x = 1$, when $y = \dfrac{\pi}{2}$

Put $x = 1$ and $y = \dfrac{\pi}{2}$ in (iii)

$$\Rightarrow \quad -\cos\left(\frac{\pi}{2}\right) = -\log 1 + C$$

$$C = 0$$

$$\Rightarrow \quad -\cos\left(\frac{y}{x}\right) + \log|x| = 0$$

Therefore $\log|x| = \cos\left(\dfrac{y}{x}\right)$ is the required solution.

30. If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ represent two adjacent sides of a parallelogram, find unit vectors parallel to the diagonals of the parallelogram.

OR

Using vectors, find the area of the triangle *ABC* with vertices *A*(1, 2, 3), *B*(2, –1, 4) and *C*(4, 5, –1).

Ans. Given that, $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ **and** $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ are two adjacent sides of a parallelogram.

Let us suppose $\vec{d_1}$ and $\vec{d_2}$ are two diagonals of parallelogram.

Then,
$$\vec{d_1} = \vec{a} + \vec{b}$$
$$= \hat{i} + 2\hat{j} + 3\hat{k} + 2\hat{i} + 4\hat{j} - 5\hat{k}$$
$$= 3\hat{i} + 6\hat{j} - 2\hat{k}$$

and
$$\vec{d_2} = \vec{b} - \vec{a}$$
$$= 2\hat{i} + 4\hat{j} - 5\hat{k} - \hat{i} - 2\hat{j} - 3\hat{k}$$
$$= \hat{i} + 2\hat{j} - 8\hat{k}$$

Now, unit vector parallel to $\vec{d_1}$ is

$$\hat{d}_1 = \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{9 + 36 + 4}}$$
$$= \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{49}}$$
$$\hat{d}_1 = \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{7}$$

And unit vector parallel to $\vec{d_2}$ is

$$\vec{d_2} = \frac{\hat{i} + 2\hat{j} - 8\hat{k}}{\sqrt{1 + 4 + 64}}$$
$$= \frac{\hat{i} + 2\hat{j} - 8\hat{k}}{\sqrt{69}}$$

OR

Given that,
$$\overrightarrow{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$$
$$\overrightarrow{OB} = 2\hat{i} - \hat{j} + 4\hat{k}$$
and
$$\overrightarrow{OC} = 4\hat{i} + 5\hat{j} - \hat{k}$$

Now,
$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$
$$= 2\hat{i} - \hat{j} + 4\hat{k} - \hat{i} - 2\hat{j} - 3\hat{k}$$
$$= \hat{i} - 3\hat{j} + \hat{k}$$

and
$$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$$
$$= 4\hat{i} + 5\hat{j} - \hat{k} - \hat{i} - 2\hat{j} - 3\hat{k}$$
$$= 3\hat{i} + 3\hat{j} - 4\hat{k}$$

We know the area of the given triangle

$$= \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$$

Now,
$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 3 & 3 & -4 \end{vmatrix}$$

$$= \hat{i}(12 - 3) + \hat{j}(3 + 4) + \hat{k}(3 + 9)$$
$$= 9\hat{i} + 7\hat{j} + 12\hat{k}$$

Therefore,
$$|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(9)^2 + (7)^2 + (12)^2}$$
$$= \sqrt{81 + 49 + 144}$$
$$= \sqrt{274}$$

Hence, Required area $= \dfrac{1}{2}\sqrt{274}$ unit^2

31. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type *A* requires 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type *B* require

8 minutes each for cutting and 8 minutes each for assembling. Given that total time for cutting is 3 hours 20 minutes and for assembling 4 hours. The profit for type *A* souvenir is ₹ 100 each and for type *B* souvenir, profit is ₹ 120 each. How many souvenirs of each type should the company manufacture in order to maximize the profit ? Formulate the problem as an LPP and solve it graphically.

Ans. Suppose number of souvenirs of type *A* and type *B* are x and y respectively.

Then, the LPP is as follows :

Maximize $Z = 100x + 120y$

Subject to $5x + 8y \le 200$

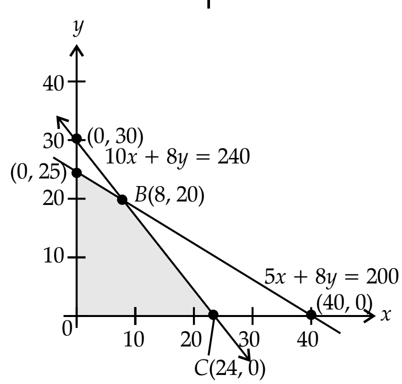
$10x + 8y \le 240$

and $x, y \ge 0$

To solve the LPP graphically first we convert inequalities into equations and draw the corresponding lines.

Then,

Corner points	Value of Z in ₹
A (0, 25)	3000
B (8, 20)	3200 maximum
C (24, 0)	2400



Clearly, maximum profit is obtained when 8 souvenirs of type *A* and 20 souvenirs of type *B* is manufactured.

Then Maximum profit $= 100 \times 8 + 120 \times 20$

$= ₹3200$

32. Three rotten apples are mixed with seven fresh apples. Find the probability distribution of the number of rotten apples, if three apples are drawn one by one with replacement. Find the mean of the number of rotten apples.

OR

In a shop *X*, 30 tins of ghee of type *A* and 40 tins of ghee of type *B* which look alike, are kept for sale. While in shop *Y*, similar 50 tins of ghee of type *A* and 60 tins of ghee of type *B* are there. One tin of ghee is purchased from one of the randomly selected shop and is found to be of type *B*. Find the probability that it is purchased from shop *Y*.

Ans. Given that rotten apples are 3 and fresh apples are 7.

Total apples = 10

Suppose X : no of rotten apples then X can take the values 0, 1, 2 and 3.

Now suppose E : getting a rotten apple

$$\therefore \quad P(E) = \frac{3}{10} \text{ and } P(E') = \frac{7}{10}$$

Then
$$P(X=0) = P(E)P(E')P(E') = \frac{343}{1000}$$

$$P(X=1) = 3P(E)P(E')P(E') = \frac{441}{1000}$$

$$P(X=2) = 3P(E)P(E)P(E') = \frac{189}{1000}$$

$$P(X=3) = P(E)P(E)P(E) = \frac{27}{1000}$$

Now,
$$\text{Mean} = \Sigma XP(X)$$

$$= 0\times\frac{343}{1000}+1\times\frac{441}{1000}+2\times\frac{189}{1000}+3\times\frac{27}{1000}$$

$$\text{Mean} = \frac{900}{1000} = \frac{9}{10}$$

OR

Suppose A : getting type B ghee

E_1 : Ghee purchased from *X*

E_2 : Ghee purchased from *Y*

$$\therefore \quad P(E_1) = \frac{1}{2} \text{ and } P(E_2) = \frac{1}{2}$$

Now
$$P\left(\frac{A}{E_1}\right) = \frac{40}{70} = \frac{4}{7}$$

$$P\left(\frac{A}{E_2}\right) = \frac{60}{110} = \frac{6}{11}$$

From Bayes' Theorem

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)}$$

$$P\left(\frac{E_2}{A}\right) = \frac{\frac{1}{2}\times\frac{6}{11}}{\frac{1}{2}\times\frac{4}{7}+\frac{1}{2}\times\frac{6}{11}}$$

$$= \frac{\frac{6}{11}}{\frac{4}{7}+\frac{6}{11}}$$

$$= \frac{42}{86}$$

Therefore
$$P\left(\frac{E_2}{A}\right) = \frac{21}{43}$$

SECTION - D

Question numbers 33 to 36 carry 6 marks each.

33. Find the vector and cartesian equations of the line which is perpendicular to the lines with equations

$$\frac{x+2}{1}=\frac{y-3}{2}=\frac{z+1}{4} \text{ and } \frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$$

and passes through the point (1, 1, 1). Also find the angle between the given lines.

Ans. Let us suppose the direction ratios of the required line L is a, b, c and it is perpendicular to the given lines

Then, $a + 2b + 4c = 0$ ...(i)

$2a + 3b + 4c = 0$ ...(ii)

Solving (i) and (ii) by cross multiplication

$$\frac{a}{8-12} = \frac{b}{8-4} = \frac{c}{3-4}$$

$$\frac{a}{-4} = \frac{b}{4} = \frac{c}{-1}$$

$\therefore$ Direction ratios of line L are –4, 4, –1.

Then required vector and cartesian equations of the line L are respectively

$$\vec{r} = i+\hat{j}+\hat{k}+\lambda(-4i+4\hat{j}-\hat{k})$$

and $$\frac{x-1}{-4} = \frac{y-1}{4} = \frac{z-1}{-1}$$

Now, Suppose θ is the angle between given lines,

So $$\cos\theta = \frac{|1\times2+2\times3+4\times4|}{\sqrt{1+4+16}\sqrt{4+9+16}}$$

$$= \frac{24}{\sqrt{21}\sqrt{29}}$$

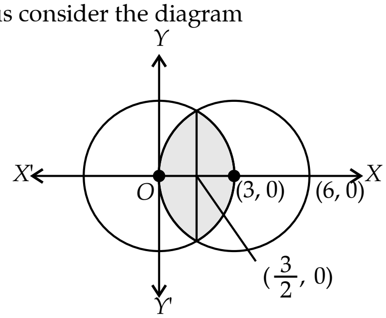
$\therefore$ $$\theta = \cos^{-1}\left(\frac{24}{\sqrt{609}}\right)$$

34. Using integration find the area of the region bounded between the two circles $x^2 + y^2 = 9$ and $(x - 3)^2 + y^2 = 9$.

OR

Evaluate the following integral as the limit of sums $\int_1^4 (x^2 - x)dx$.

Ans. Let us consider the diagram



Here $x^2 + y^2 = 9$ ...(i)

$(x - 3)^2 + y^2 = 9$ ...(ii)

are two circles with centres (0, 0) and (3, 0) respectively.

After solving (i) and (ii),

$$9 - 6x = 0$$

$\Rightarrow$ $$x = \frac{3}{2}$$

Then required area =

$$2\int_0^{3/2}\sqrt{9-(x-3)^2}\,dx+2\int_{3/2}^{3}\sqrt{9-x^2}\,dx$$

$$= 2\left[\frac{(x-3)}{2}\sqrt{9-(x-3)^2}+\frac{9}{2}\sin^{-1}\frac{(x-3)}{3}\right]_0^{3/2}$$

$$+2\left[\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\sin^{-1}\frac{x}{3}\right]_{3/2}^{3}$$

$$\text{Area} = 2\left\{\left[\frac{-3}{4}\sqrt{9-\frac{9}{4}}+\frac{9}{2}\sin^{-1}\left(\frac{-1}{2}\right)\right]-\left[0+\frac{9}{2}\sin^{-1}(-1)\right]\right.$$
$$\left.+\left[0+\frac{9}{2}\sin^{-1}1\right]-\left[\frac{3}{4}\sqrt{9-\frac{9}{4}}+\frac{9}{2}\sin^{-1}\frac{1}{2}\right]\right\}$$

$$\Rightarrow \text{Area} = 2\left\{-\frac{9\sqrt{3}}{8}-\frac{9}{2}\times\frac{\pi}{6}+\frac{9}{2}\times\frac{\pi}{2}+\frac{9}{2}\times\frac{\pi}{2}-\frac{9\sqrt{3}}{8}-\frac{9}{2}\times\frac{\pi}{6}\right\}$$

$$\Rightarrow \text{Area} = 2\left\{-2\times\frac{9\sqrt{3}}{8}-9\times\frac{\pi}{6}+\frac{9}{2}\times\pi\right\}$$

$$\Rightarrow \text{Area} = 2\left\{3\pi-\frac{9\sqrt{3}}{4}\right\} \text{ sq. units}$$

35. Find the minimum value of $(ax + by)$, where $xy = c^2$.

Ans. Given that $xy = c^2$

$$y = \frac{c^2}{x} \quad ...(i)$$

Now, suppose $S = ax + by$

$\Rightarrow$ $$S = ax+b\times\frac{c^2}{x} \quad \text{[From (i)]}$$

$\Rightarrow$ $$\frac{dS}{dx} = a-\frac{bc^2}{x^2}$$

For local points of maxima or minima

$\Rightarrow$ $$\frac{dS}{dx} = 0$$

$\Rightarrow$ $$a-\frac{bc^2}{x^2} = 0$$

$\Rightarrow$ $$x = \pm c\sqrt{\frac{b}{a}}$$

Also, $$\frac{d^2S}{dx^2} = \frac{2bc^2}{x^3}$$

$$\left.\frac{d^2S}{dx^2}\right]_{\text{at } x=c\sqrt{\frac{b}{a}}} = \frac{2bc^2}{c^3\left(\frac{b}{a}\right)^{3/2}} > 0$$

$\therefore$ $S = ax + by$ is minimum at $x = c\sqrt{\frac{b}{a}}$

$\Rightarrow$ Minimum value of $S = a \times c\sqrt{\frac{b}{a}} + b \times \frac{c^2}{c\sqrt{\frac{b}{a}}}$

$= c\sqrt{ab} + c\sqrt{ab} = 2c\sqrt{ab}$.

$\therefore$ Minimum value of $ax + by$, where $xy = c^2$ is $2c\sqrt{ab}$.

36. If a, b, c are p^{th}, q^{th} and r^{th} terms respectively of a G.P., then prove that

$$\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$$

OR

If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then find A^{-1}.

Using A^{-1}, solve the following system of equations:

$2x - 3y + 5z = 11$

$3x + 2y - 4z = -5$

$x + y - 2z = -3$

Ans. Suppose A is the first term and R is the common ratio of the G.P.

Then, $a = AR^{p-1}$ ...(i)

$b = AR^{q-1}$ ...(ii)

$c = AR^{r-1}$ ...(iii)

From (i), (ii) and (iii)

$\log a = \log A + (p-1)\log R$

$\log b = \log A + (q-1)\log R$

$\log c = \log A + (r-1)\log R$

$$\therefore \quad \begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = \begin{vmatrix} \log A + (p-1)\log R & p & 1 \\ \log A + (q-1)\log R & q & 1 \\ \log A + (r-1)\log R & r & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \log A + (p-1)\log R & p-1 & 1 \\ \log A + (q-1)\log R & q-1 & 1 \\ \log A + (r-1)\log R & r-1 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & p-1 & 1 \\ 0 & q-1 & 1 \\ 0 & r-1 & 1 \end{vmatrix}$$

[Applying $C_1 \to C_1 - \log A\, C_3 - \log R C_2$]

$$\Rightarrow \quad \begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$$ **Proved.**

OR

Given $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$

$$|A| = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix}$$

$= 2(-4+4) + 3(-6+4) + 5(3-2)$

$= 3(-2) + 5(1)$

$= -1 \neq 0$

$\therefore A^{-1}$ exists

Suppose A_{ij} is the cofactor of element a_{ij} of A

Now, $A_{11} = 0, A_{12} = -(-2) = 2, A_{13} = 1,$

$A_{21} = -1, A_{22} = -9, A_{23} = -5,$

$A_{31} = 2, A_{32} = 23,$

$A_{33} = 13$

$$\therefore \quad \text{Adj } A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{Adj}A$$

$$= \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

Also, we need to solve the following system of equations

$2x - 3y + 5z = 11$

$3x + 2y - 4z = -5$

$x + y - 2z = -3$

Here, $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ $B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$$\therefore \quad X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \times 11 + 1 \times (-5) + (-2) \times (-3) \\ (-2) \times 11 + 9 \times (-5) + (-23) \times (-3) \\ (-1) \times 11 + 5 \times (-5) + (-13) \times (-3) \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

From equality of matrices,

$x = 1, y = 2, z = 3$

Delhi Set-II 65/5/2

Note: *Except these, all other questions are from Delhi Set-I*

SECTION - A

Question numbers 1 to 10 are multiple choice questions of 1 mark each. Select the correct option :

1. If $[x \ \ 1]\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = 0$, then x equals

(a) 0 **(b)** –2

(c) –1 **(d)** 2

Ans. Option (d) is correct.

Explanation:

$$\begin{bmatrix} x & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x-2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

$$\Rightarrow \quad x-2 = 0 \qquad \text{[By def. of equality]}$$

$$\Rightarrow \quad x = 2$$

2. $\int 4^x 3^x \, dx$ **equals**

(a) $\dfrac{12^x}{\log 12}+C$ (b) $\dfrac{4^x}{\log 4}+C$

(c) $\left(\dfrac{4^x.3^x}{\log 4.\log 3}\right)+C$ (d) $\dfrac{3^x}{\log 3}+C$

Ans. Option (a) is correct.

Explanation:

$$\int 4^x 3^x \, dx = \int 12^x \, dx = \frac{12^x}{\log 12}+C$$

3. A number is chosen randomly from numbers 1 to 60. The probability that the chosen number is a multiple of 2 or 5 is

(a) $\dfrac{2}{5}$ (b) $\dfrac{3}{5}$

(c) $\dfrac{7}{10}$ (d) $\dfrac{9}{10}$

Ans. Option (b) is correct.

Explanation:

Let us suppose A : Number is multiple of 2.

Let us suppose B : Number is multiple of 5.

Then $n(A) = 30$

$n(B) = 12$

$n(A \cap B) = 6$

$$\Rightarrow \quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\Rightarrow \quad P(A \text{ or } B) = \frac{30}{60}+\frac{12}{60}-\frac{6}{60} = \frac{36}{60}$$

$$\Rightarrow \quad P(A \text{ or } B) = \frac{3}{5}$$

Fill in the blanks in Questions from 11 to 15.

11. A relation R on a set A is called .... If $(a_1, a_2) \in R$ and $(a_2, a_3) \in R$ implies that $(a_1, a_3) \in R$, for $a_1, a_2, a_3 \in A$.

Ans. Transitive

Question numbers 16 to 20 are very short answer type questions.

16. Evaluate: sin $\left[\dfrac{\pi}{3} - \sin^{-1}\left(-\dfrac{1}{2}\right)\right]$.

Ans. $$\sin\left[\frac{\pi}{3}-\left(-\frac{\pi}{6}\right)\right] = \sin\frac{\pi}{2} = 1$$

* *Out of Syllabus*

***17. Using differential, find the approximate value of $\sqrt{36.6}$ upto 2 decimal places.**

OR

Find the slope of tangent to the curve $y = 2\cos^2(3x)$ at $x = \dfrac{\pi}{6}$.

SECTION - B

Question numbers 21 to 26 carry 2 marks each.

21. Find $\displaystyle\int \frac{x+1}{x(1-2x)}\, dx$.

Ans. $$\int \frac{x+1}{x(1-2x)}dx = \int \frac{x}{x(1-2x)}dx + \int \frac{1}{x(1-2x)}dx$$

$$\Rightarrow \quad \int \frac{x+1}{x(1-2x)}dx = \int \frac{1}{(1-2x)}dx + \int \frac{1}{x}dx + \int \frac{2}{(1-2x)}dx$$

$$\Rightarrow \quad \int \frac{x+1}{x(1-2x)}dx = -\frac{1}{2}\log|1-2x| + \log|x| -2\times\frac{1}{2}\log|1-2x| + C$$

$$\Rightarrow \quad \int \frac{x+1}{x(1-2x)}dx = -\frac{3}{2}\log|1-2x| + \log|x| + C$$

22. Evaluate $\displaystyle\int \frac{x\sin^{-1}(x^2)}{\sqrt{1-x^4}}\, dx$.

Ans. Given that $\displaystyle\int \frac{x\sin^{-1}(x^2)}{\sqrt{1-x^4}}dx$

Put $\sin^{-1}(x^2) = y$

$$\Rightarrow \quad \frac{x}{\sqrt{1-x^4}}dx = \frac{dy}{2}$$

$$\Rightarrow \quad \frac{1}{2}\int y\, dy = \frac{1}{4}\times y^2 + C$$

$$\therefore \quad \int \frac{x\sin^{-1}(x^2)}{\sqrt{1-x^4}}dx = \frac{1}{4}(\sin^{-1}(x^2))^2 + C$$

SECTION - C

Question numbers 27 to 32 carry 4 marks each.

27. Prove that tan $\left[2\tan^{-1}\left(\dfrac{1}{2}\right) - \cot^{-1}3\right] = \dfrac{9}{13}$.

Ans. $$\text{L.H.S.} = \tan\left[2\tan^{-1}\left(\frac{1}{2}\right)-\cot^{-1}3\right]$$

$$\Rightarrow \quad \text{L.H.S.} = \tan\left[\tan^{-1}\frac{2\times\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2} - \tan^{-1}\frac{1}{3}\right]$$

[Because $2\tan^{-1}x = \tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$ if $-1 < x < 1$

and $\cot^{-1}x = \tan^{-1}\frac{1}{x}, x > 0]$

$$\Rightarrow \quad \text{L.H.S.} = \tan\left[\tan^{-1}\frac{4}{3} - \tan^{-1}\frac{1}{3}\right]$$

$$\Rightarrow \quad \text{L.H.S.} = \tan\left[\tan^{-1}\frac{\left(\frac{4}{3}-\frac{1}{3}\right)}{1+\frac{4}{3}\times\frac{1}{3}}\right]$$

$$\Rightarrow \quad \text{L.H.S.} = \tan\left[\tan^{-1}\frac{1}{13/9}\right]$$

$$\Rightarrow \quad \text{L.H.S.} = \frac{9}{13} = \text{R.H.S.}$$

$$\therefore \tan\left[2\tan^{-1}\left(\frac{1}{2}\right) - \cot^{-1}3\right] = \frac{9}{13}$$

28. If $y = (\cos x)^x + \tan^{-1}\sqrt{x}$, find $\frac{dy}{dx}$

Ans. Given that $\quad y = (\cos x)^x + \tan^{-1}\sqrt{x}$

[Because $e^{\log x} = x$]

$$y = e^{x\log(\cos x)} + \tan^{-1}\sqrt{x}$$

$$\Rightarrow \frac{dy}{dx} = e^{x\log(\cos x)}\left\{x\left(\frac{-\sin x}{\cos x}\right) + \log\cos x \times 1\right\} + \frac{1}{1+x}\times\frac{1}{2\sqrt{x}}$$

Then, $\frac{dy}{dx} = (\cos x)^x\,\{\log\cos x - x\tan x\} + \frac{1}{(1+x)}\times\frac{1}{2\sqrt{x}}$

SECTION - D

Question numbers 33 to 36 carry 6 marks each.

36. Find the point on the curve $y^2 = 4x$ which is nearest to the point (2, 1).

Ans. Suppose the required point on the curve is $K(p, q)$ and the given point is $A(2, 1)$.

$\therefore \quad q^2 = 4p \quad$ ...(i)

Then $\quad AK = \sqrt{(p-2)^2 + (q-1)^2}$

$$\Rightarrow \quad S = \sqrt{\left(\frac{q^2}{4}-2\right)^2 + (q-1)^2}$$

[Let $S = AK$ from (i) $p = \frac{q^2}{4}$]

Then $\quad S^2 = \left(\frac{q^2}{4}-2\right)^2 + (q-1)^2$

To find nearest point let us suppose $S^2 = T$

Then $\quad T = \left(\frac{q^2}{4}-2\right)^2 + (q-1)^2$

$$\Rightarrow \quad T' = 2\left(\frac{q^2}{4}-2\right)\times\frac{2q}{4} + 2(q-1)$$

For critical points $\quad T' = 0$

$$\Rightarrow 2\left(\frac{q^2}{4}-2\right)\times\frac{2q}{4} + 2(q-1) = 0$$

$$\Rightarrow \left(\frac{q^3}{4} - 2q\right) + (2q-2) = 0$$

$\Rightarrow \quad q^3 = 8$

$\Rightarrow \quad q = 2 \quad$ ...(ii)

To find the maxima or minima

$$T'' = \frac{3q^2}{4} - 2 + 2 = \frac{3q^2}{4}$$

$$\Rightarrow \quad T'' \text{ at } 2 = \frac{3\times2\times2}{4} = 3 > 0$$

Therefore, T is least

From (i) $\quad q^2 = 4p$

$\Rightarrow \quad 2^2 = 4p$

$\Rightarrow \quad p = 1$

Therefore, the required point is $K(1, 2)$.

Delhi Set-III 65/5/3

Note: *Except these, all other questions are from Delhi Set-I & II*

SECTION - A

Question numbers 1 to 10 are multiple choice questions of 1 mark each. Select the correct option:

1. If A is skew symmetric matrix of order 3, then the value of $|A|$ is

(a) 3 **(b)** 0

(c) 9 **(d)** 27

Ans. Option (b) is correct.

Explanation:

Determinant value of skew symmetric matrix is always '0'.

6. If $y = \log_e\left(\frac{x^2}{e^2}\right)$, then $\frac{d^2y}{dx^2}$ equals

(a) $-\frac{1}{x}$ **(b)** $-\frac{1}{x^2}$

(c) $\frac{2}{x^2}$ **(d)** $-\frac{2}{x^2}$

Ans. Option (d) is correct.

Explanation:

Given $\quad y = \log_e\left(\frac{x^2}{e^2}\right)$

$\Rightarrow \quad y = 2\log_e x - \log_e e^2$

$\Rightarrow \quad y = 2\log_e x - 2$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{2}{x}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{-2}{x^2}$$

* 9. **The distance of the origin (0, 0, 0) from the plane $-2x + 6y - 3z = -7$ is**

(a) 1 unit (b) $\sqrt{2}$ units

(c) $2\sqrt{2}$ units (d) 3 units

Fill in the blanks in Questions from 11 to 15.

11. **If A and B are square matrices each of order 3 and $|A| = 5$, $|B| = 3$, then the vlaue of $|3\ AB|$ is ....**

Ans.
$$|3AB| = 3^3\,|A|\,|B| = 27 \times 5 \times 3$$
$$|3AB| = 405$$

Question numbers 16 to 20 are very short answer type questions.

16. **Find the cofactors of all the elements of $\begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix}$.**

Ans. Given that $A = \begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix}$

Then $A_{11} = 3, A_{12} = -4, A_{21} = 2, A_{22} = 1$

17. **Let $f(x) = x|x|$, for all $x \in R$ check its differentiability at $x = 0$.**

Ans. Here
$$f(x) = \begin{cases} x^2 & if\ x \ge 0 \\ -x^2 & if\ x < 0 \end{cases}$$
$$R.f'(0) = \lim_{x\to 0^+} \frac{f(x)-f(0)}{x-0}$$
$$= \lim_{x\to 0^+} \frac{x^2-0}{x-0} = 0$$
$$L.f'(0) = \lim_{x\to 0^-} \frac{f(x)-f(0)}{x-0}$$
$$= \lim_{x\to 0^-} \frac{-x^2-0}{x-0} = 0$$
$$R.f'(0) = L.f'(0)$$

Therefore $f(x)$ is differentiable at $x = 0$.

SECTION - B

Question numbers 21 to 26 carry 2 marks each.

21. **Find $\int \frac{x+1}{(x+2)(x+3)}dx$.**

Ans.
$$\int \frac{x+1}{(x+2)(x+3)}dx = \int \frac{x+2-1}{(x+2)(x+3)}dx$$
$$\Rightarrow \int \frac{x+1}{(x+2)(x+3)}dx = \int \frac{1}{(x+3)}dx - \int\left\{\frac{1}{(x+2)} - \frac{1}{(x+3)}\right\}dx$$
$$= \log|x+3) - \log|x+2| + \log|x+3| + C$$
$$\Rightarrow \int \frac{x+1}{(x+2)(x+3)}dx$$
$$= 2\log|x+3| - \log|x+2| + C$$

26. **Find the value of $\int_0^1 \tan^{-1}\left(\frac{1-2x}{1+x-x^2}\right)dx$.**

Ans.
$$\int_0^1 \tan^{-1}\left(\frac{1-2x}{1+x-x^2}\right)dx$$
$$= \int_0^1 \tan^{-1}\left(\frac{(1-x)-x}{1+x(1-x)}\right)dx$$
$$= \int_0^1 \left[\tan^{-1}(1-x) - \tan^{-1}x\right]dx$$
$$= \int_0^1 \tan^{-1}(1-(1-x))dx - \int_0^1 \tan^{-1}x\,dx$$

[Because $\int_0^a f(x)dx = \int_0^a f(a-x)dx$]

$$= \int_0^1 \tan^{-1}x\,dx - \int_0^1 \tan^{-1}x\,dx = 0$$
$$\therefore \int_0^1 \tan^{-1}\left(\frac{1-2x}{1+x-x^2}\right)dx = 0$$

SECTION - C

Question numbers 27 to 32 carry 4 marks each.

27. **Solve the equation x: $\sin^{-1}\left(\frac{5}{x}\right) + \sin^{-1}\left(\frac{12}{x}\right) = \frac{\pi}{2} (x \ne 0)$**

Ans.
$$\sin^{-1}\left(\frac{5}{x}\right) + \sin^{-1}\left(\frac{12}{x}\right) = \frac{\pi}{2}$$
$$\Rightarrow \sin^{-1}\left(\frac{5}{x}\right) = \frac{\pi}{2} - \sin^{-1}\left(\frac{12}{x}\right)$$
$$\Rightarrow \sin^{-1}\left(\frac{5}{x}\right) = \cos^{-1}\left(\frac{12}{x}\right)$$

[$\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$]

$$\Rightarrow \sin^{-1}\left(\frac{5}{x}\right) = \cos^{-1}\left(\frac{12}{x}\right) = \theta$$
$$\Rightarrow \sin\theta = \frac{5}{x}, \cos\theta = \frac{12}{x}$$

Since,
$$\sin^2\theta + \cos^2\theta = 1$$
$$\Rightarrow \left(\frac{5}{x}\right)^2 + \left(\frac{12}{x}\right)^2 = 1$$
$$\Rightarrow \frac{25}{x^2} + \frac{144}{x^2} = 1$$
$$\Rightarrow \frac{169}{x^2} = 1$$
$$\Rightarrow x = \pm 13$$

Since $x = -13$ does not satisfy the given equation so $x = 13$ is the answer

* *Out of Syllabus*

28. Find the general solution of the differential equation

$ye^{x/y}\,dx = (xe^{x/y} + y^2)\,dy,\ y \neq 0$

Ans. Given $\quad ye^{x/y}dx = (xe^{x/y} + y^2)\,dy,\ y \neq 0$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{ye^{x/y}}{xe^{x/y}+y^2}$$

$$\frac{dx}{dy} = \frac{xe^{x/y}+y^2}{ye^{x/y}}$$

Put $\quad x = vy$

$$\frac{dx}{dy} = v + y\frac{dv}{dy}$$

$$\Rightarrow \quad v + y\frac{dv}{dy} = \frac{vye^v + y^2}{ye^v}$$

$$\Rightarrow \quad y\frac{dv}{dy} = \frac{vye^v + y^2}{ye^v} - v$$

$$\Rightarrow \quad y\frac{dv}{dy} = \frac{vye^v + y^2 - vye^v}{ye^v}$$

$$\Rightarrow \quad y\frac{dv}{dy} = \frac{y^2}{ye^v}$$

$$\Rightarrow \quad \frac{dv}{dy} = \frac{1}{e^v}$$

$$\Rightarrow \quad \int e^v dv = \int dy + C$$

$$\Rightarrow \quad e^v = y + C$$

$\Rightarrow \quad e^{x/y} = y + C$ is the required solution

SECTION - D

Question numbers numbers 33 to 36 carry 6 marks each.

***33. Find the distance of the point P(3, 4, 4) from the point, where the line joining the points A(3, – 4, – 5) and B(2, – 3, 1) intersects the plane $2x + y + z = 7$.**

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SECTION - A

Question numbers 1 to 10 are multiple choice questions of 1 mark each. Select the correct option:

1. The value of $\sin^{-1}\left(\cos\frac{3\pi}{5}\right)$ is

(a) $\frac{\pi}{10}$ (b) $\frac{3\pi}{5}$

(c) $-\frac{\pi}{10}$ (d) $\frac{-3\pi}{5}$

Ans. Option (c) is correct.

Explanation:

$$= \sin^{-1}\left[\cos\left(\frac{3\pi}{5}\right)\right]$$

$$= \sin^{-1}\left[\cos\left(\frac{\pi}{2}+\frac{\pi}{10}\right)\right]$$

$$= \sin^{-1}\left(-\sin\frac{\pi}{10}\right)$$

$$\left[\because \cos\left(\frac{\pi}{2}+x\right) = -\sin x\right]$$

$$= -\sin^{-1}\left(\sin\frac{\pi}{10}\right)$$

$$[\because \sin^{-1}(-x) = -\sin^{-1}x]$$

$$= -\frac{\pi}{10}$$

$$\left[\because \sin^{-1}(\sin x) = x, x \in \left(\frac{-\pi}{2},\frac{\pi}{2}\right)\right]$$

2. If $A = [2\ \ -3\ \ 4]$, $B = \begin{bmatrix}3\\2\\2\end{bmatrix}$, $X = [1\ \ 2\ \ 3]$ and $Y = \begin{bmatrix}2\\3\\4\end{bmatrix}$, then $AB + XY$ equals

(a) [28] (b) [24]

(c) 28 (d) 24

Ans. Option (a) is correct.

Explanation:

Given, $\quad A = \begin{bmatrix}2 & -3 & 4\end{bmatrix}$,

$$B = \begin{bmatrix}3\\2\\2\end{bmatrix},$$

$$X = \begin{bmatrix}1 & 2 & 3\end{bmatrix},$$

$$Y = \begin{bmatrix}2\\3\\4\end{bmatrix}$$

$$AB + XY = \begin{bmatrix}2 & -3 & 4\end{bmatrix}\begin{bmatrix}3\\2\\2\end{bmatrix} + \begin{bmatrix}1 & 2 & 3\end{bmatrix}\begin{bmatrix}2\\3\\4\end{bmatrix}$$

$$= [6 - 6 + 8] + [2 + 6 + 12]$$

$$= [8] + [20] = [28]$$

3. If $\begin{vmatrix}2 & 3 & 2\\ x & x & x\\ 4 & 9 & 1\end{vmatrix} + 3 = 0$, then the value of x is

(a) 3 (b) 0

(c) – 1 (d) 1

* *Out of Syllabus*

Ans. Option (c) is correct.

Explanation:

$$\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$$

On expanding along R_1

$$2(x - 9x) - 3(x - 4x) + 2(9x - 4x) + 3 = 0$$
$$2(-8x) - 3(-3x) + 2(5x) + 3 = 0$$
$$-16x + 9x + 10x + 3 = 0$$
$$3x + 3 = 0$$
$$3x = -3$$
$$x = -\frac{3}{3}$$
$$x = -1$$

4. $\int_0^{\pi/8} \tan^2(2x)$ **is equal to**

(a) $\frac{4-\pi}{8}$ (b) $\frac{4+\pi}{8}$

(c) $\frac{4-\pi}{4}$ (d) $\frac{4-\pi}{2}$

Ans. Option (a) is correct.

Explanation:

$$\int_0^{\frac{\pi}{8}} \tan^2(2x)dx = \int_0^{\frac{\pi}{8}} [\sec^2(2x) - 1]dx$$
$$= \int_0^{\frac{\pi}{8}} \sec^2(2x)dx - \int_0^{\frac{\pi}{8}} 1dx$$
$$= \left[\frac{\tan 2x}{2}\right]_0^{\frac{\pi}{8}} - [x]_0^{\frac{\pi}{8}}$$
$$= \frac{1}{2}\left[\tan 2\left(\frac{\pi}{8}\right) - \tan 2(0)\right] - \left[\left(\frac{\pi}{8}\right) - (0)\right]$$
$$= \frac{1}{2}\left[\tan\left(\frac{\pi}{4}\right) - \tan(0)\right] - \left[\frac{\pi}{8}\right]$$
$$= \frac{1}{2}[1-0] - \frac{\pi}{8} = \frac{1}{2} - \frac{\pi}{8}$$
$$= \frac{4-\pi}{8}$$

5. **If** $\vec{a}\cdot\vec{b} = \frac{1}{2}|\vec{a}||\vec{b}|$, **then the angle between** $\vec{a}$ **and** $\vec{b}$ **is**

(a) 0° (b) 30°

(c) 60° (d) 90°

Ans. Option (c) is correct.

Explanation:

Given, $\vec{a}.\vec{b} = \frac{1}{2}|\vec{a}||\vec{b}|$

$$\theta = \cos^{-1}\left[\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}\right]$$
$$\theta = \cos^{-1}\left[\frac{\frac{1}{2}|\vec{a}||\vec{b}|}{|\vec{a}||\vec{b}|}\right]$$
$$\theta = \cos^{-1}\left[\frac{1}{2}\right] = \cos^{-1}[\cos 60°]$$
$$\theta = 60°$$

6. **The two lines $x = ay + b, z = cy + d$; and $x = a'y + b', z = c'y + d'$ are perpendicular to each other, if**

(a) $\frac{a}{a'} + \frac{c}{c'} = 1$ (b) $\frac{a}{a'} + \frac{c}{c'} = -1$

(c) $aa' + cc' = 1$ (d) $aa' + cc' = -1$

Ans. Option (d) is correct.

Explanation :

Line 1 : $\frac{x-b}{a} = \frac{y}{1} = \frac{z-d}{c}$ ...(i)

Line 2 : $\frac{x-b'}{a'} = \frac{y}{1} = \frac{z-d'}{c'}$ ...(ii)

Given both line perpendicular to each other

Hence,

$$aa' + (1)(1) + cc' = 0$$
$$aa' + cc' = -1$$

* 7. **The two planes $x - 2y + 4z = 10$ and $18x + 17y + kz = 50$ are perpendicular, if k is equal to**

(a) – 4 (b) 4

(c) 2 (d) – 2

8. **In an LPP, if the objective function $z = ax + by$ has the same maximum value on two corner points of the feasible region, then the number of points at which z_{max} occurs is**

(a) 0 (b) 2

(c) finite (d) infinite

Ans. Option (d) is correct.

9. **From the set {1, 2, 3, 4, 5}, two numbers a and b ($a \neq b$) are chosen at random. The probability that $\frac{a}{b}$ is an integer is:**

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$

(c) $\frac{1}{2}$ (d) $\frac{3}{5}$

Ans. Option (b) is correct.

Explanation:

Sample Space

$$= \left\{\frac{1}{2},\frac{1}{3},\frac{1}{4},\frac{1}{5},\frac{2}{1},\frac{2}{3},\frac{2}{4},\frac{2}{5},\frac{3}{1},\frac{3}{2},\frac{3}{4},\frac{3}{5},\frac{4}{1},\frac{4}{2},\frac{4}{3},\frac{4}{5},\frac{5}{1},\frac{5}{2},\frac{5}{3},\frac{5}{4}\right\}$$

* *Out of Syllabus*

Cases in which $\frac{a}{b}$ is an integer $= \left[\frac{2}{1}, \frac{3}{1}, \frac{4}{1}, \frac{4}{2}, \frac{5}{1}\right]$

The probability that $\left(\frac{a}{b}\right)$ is an integer $= \frac{5}{20} = \frac{1}{4}$

10. A bag contains 3 white, 4 black and 2 red balls. If 2 balls are drawn at random (without replacement), then the probability that both the balls are white is

(a) $\frac{1}{18}$ **(b)** $\frac{1}{36}$

(c) $\frac{1}{12}$ **(d)** $\frac{1}{24}$

Ans. Option (c) is correct.

Explanation :

The probability that both the balls are white

$$= \frac{3}{9} \times \frac{2}{8} = \frac{1}{12}$$

In Q. Nos. 11 to 15, fill in the blanks with correct word/sentence :

11. If $f : R \to R$ be given by $f(x) = (3 - x^3)^{1/3}$, then $fof\,(x)$ = ...

Ans.
$$\begin{aligned} f(x) &= (3 - x^3)^{1/3} \\ fof\,(x) &= f[f(x)] \\ fof\,(x) &= f[3 - x^3]^{1/3} \\ &= [3 - ((3 - x^3)^{1/3})^3]^{1/3} \\ &= [3 - (3 - x^3)]^{1/3} \\ &= [3 - 3 + x^3]^{1/3} \\ &= (x^3)^{1/3} = x \end{aligned}$$

12. If $\begin{bmatrix} x+y & 7 \\ 9 & x-y \end{bmatrix} = \begin{bmatrix} 2 & 7 \\ 9 & 4 \end{bmatrix}$, then $x \cdot y$ = ...

Ans. $\begin{bmatrix} x+y & 7 \\ 9 & x-y \end{bmatrix} = \begin{bmatrix} 2 & 7 \\ 9 & 4 \end{bmatrix}$

Here $x + y = 2$ ...(i)

and $x - y = 4$ ...(ii)

On solving eqn. (i) and eqn. (ii)

$x = 3$

and $y = -1$

Now, $x.y = (3).(-1)$

$= -3$

13. The number of points of discontinuity of f defined by $f(x) = |x| - |x + 1|$ is ....

Ans. $f(x) = |x| - |x + 1|$

$$f(x) = \begin{cases} 1, x < -1 \\ -2x - 1, -1 \le x < 0 \\ -1, x > 0 \end{cases}$$

Here, at $x = 0, -1$ $f(x)$ is continuous.

Hence, there is no point of discontinuity.

***14. The slope of the tangent to the curve $y = x^3 - x$ at the point (2, 6) is ....**

OR

The rate of change of the area of a circle with respect to its radius r, when $r = 3$ cm, is ....

* *Out of Syllabus*

Ans. **OR**

Area of circle with radius $r = \pi r^2$

$$A = \pi r^2$$

Differentiate w.r.t. r

$$\frac{dA}{dr} = \pi(2r)$$

$$\frac{dA}{dr} = 2\pi r$$

When, $r = 3$ cm

$$\left.\frac{dA}{dr}\right\}_{r=3} = 2\pi(3)$$

$$= 6\pi \text{ cm}^2$$

15. If $\vec{a}$ is a non-zero vector, then $(\vec{a} \cdot \hat{i}).\hat{i} + (\vec{a} \cdot \hat{j}).\hat{j} + (\vec{a} \cdot \hat{k}).\hat{k}$ ....

OR

The projection of the vector $\hat{i} - \hat{j}$ on the vector $\hat{i} + \hat{j}$ is ....

Ans. $(\vec{a}.\hat{i})\hat{i} + (\vec{a}.\hat{j})\hat{j} + (\vec{a}.\hat{k})\hat{k}$

$= a_1\hat{i} + a_2\hat{j} + a_3\hat{k} \quad [\vec{a}.\hat{i} = a_1, \vec{a}.\hat{j} = a_2, \vec{a}.\hat{k} = a_3] = \vec{a}$

OR

Let, $\vec{a} = \hat{i} - \hat{j}, b = \hat{i} + \hat{j}$

The projection of the vector $\vec{a}$ on the vector $\vec{b}$

$$= \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

$$= \left(\frac{(\hat{i} - \hat{j}).(\hat{i} + \hat{j})}{\sqrt{(1)^2 + (1)^2 + (0)^2}}\right)$$

$$= \frac{0}{1} = 0$$

Q. 16 to 20 are very short answer questions.

16. Find *adj A*, if $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$.

Ans. Given, $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$

$C_{11} = 3, C_{12} = -4$

$C_{21} = 1, C_{22} = 2$

$$\text{adj } A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T = \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}^T$$

$$\text{adj } A = \begin{bmatrix} 3 & 1 \\ -4 & 2 \end{bmatrix}$$

17. Find $\int \frac{2^{x+1}-5^{x-1}}{10^x}dx$

Ans.
$$\int \frac{2^{x+1}-5^{x-1}}{10^x}dx = \int \frac{2^{x+1}-5^{x-1}}{2^x.5^x}dx$$
$$= \int\left(\frac{2^{x+1}}{2^x.5^x}-\frac{5^{x-1}}{2^x.5^x}\right)dx$$
$$= \int \frac{2}{5^x}dx - \int \frac{1}{5\cdot 2^x}dx$$
$$= 2\int 5^{-x}dx - \frac{1}{5}\int 2^{-x}dx$$
$$= 2\times\frac{-5^{-x}}{\log 5} - \frac{1}{5}\times\frac{-2^{-x}}{\log 2}$$
$$= \frac{1}{5.2^x\log 2} - \frac{2}{5^x \log 5}$$

18. Evaluate $\int_0^{2\pi}|\sin x|dx$

Ans. $\int_0^{2\pi}|\sin x|dx$

Let
$$I = \int_0^{2\pi}|\sin x|dx$$
$$= \int_0^{\pi}|\sin x|dx + \int_{\pi}^{2\pi}|\sin x|dx$$
$$= \int_0^{\pi}\sin x dx - \int_{\pi}^{2\pi}\sin x dx$$
$$= [-\cos x]_0^{\pi} - [-\cos x]_{\pi}^{2\pi}$$
$= [-\cos\pi + \cos 0] - [-\cos 2\pi + \cos\pi]$

$= [1 + 1] - [-1 - 1] = 2 + 2 = 4$

19. If $\int_0^a \frac{dx}{1+4x^2} = \frac{\pi}{8}$, **then find the value of** a.

OR

Find $\int \frac{dx}{\sqrt{x}+x}$

Ans.
$$\int_0^a \frac{dx}{1+4x^2} = \frac{\pi}{8}$$
$$\int_0^a \frac{dx}{1+(2x)^2} = \frac{\pi}{8}$$
$$\left[\int \frac{1}{1+x^2}dx = \tan^{-1}x + C\right]$$
$$\frac{1}{2}\left[\tan^{-1}(2x)\right]_0^a = \frac{\pi}{8}$$
$$\tan^{-1}2a - \tan^{-1}0 = \frac{\pi}{8}\times 2$$
$$\tan^{-1}2a - 0 = \frac{\pi}{4}$$
$$2a = \tan\frac{\pi}{4}$$
$$2a = 1$$
$$a = \frac{1}{2}$$

OR

$$\int \frac{dx}{\sqrt{x}+x}dx = \int \frac{1}{\sqrt{x}(1+\sqrt{x})}dx$$
Let, $1+\sqrt{x} = t$
$$0 + \frac{1}{2\sqrt{x}}dx = dt$$
$$\frac{1}{\sqrt{x}}dx = 2dt$$
$$= 2\int \frac{1}{t}dt$$
$= 2\log t + C$

$= 2\log(1 + \sqrt{x}) + C$

20. Show that the function $y = ax + 2a^2$ **is a solution of the differential equation** $2\left(\frac{dy}{dx}\right)^2 + x\left(\frac{dy}{dx}\right) - y = 0.$

Ans. $y = ax + 2a^2$

Differentiate w.r.t. x
$$\frac{dy}{dx} = a(1) + 0$$
$$\frac{dy}{dx} = a$$
$$2\left(\frac{dy}{dx}\right)^2 + x\left(\frac{dy}{dx}\right) - y = 2(a)^2 + x(a) - y$$
$= 2a^2 + ax - (ax + 2a^2)$ $[y = ax + 2a^2]$

$= 2a^2 + ax - ax - 2a^2$

$= 0$
$$2\left(\frac{dy}{dx}\right)^2 + x\left(\frac{dy}{dx}\right) - y = 0$$
Hence, $y = ax + 2a^2$ is a solution of the differential equation $2\left(\frac{dy}{dx}\right)^2 + x\left(\frac{dy}{dx}\right) - y = 0$

SECTION - B

Q. 21 to 26 carry 2 marks each.

21. Check if the relation R **on the set** $A = \{1, 2, 3, 4, 5, 6\}$ **defined as** $R = \{(x, y): y$ **is divisible by** $x\}$ **is (i) symmetric (ii) transitive.**

OR

Prove that:
$$\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right) = \frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

Ans. $A = \{1, 2, 3, 4, 5, 6\}$

$R = \{(x, y) : y \text{ is divisible by } x\}$

(i) Symmetric

Let $(x, y) \in R$ y is divisible by x

$\therefore$ x is not necessarily divisible by y

$$(y, x) \notin R$$

e.g., $(1, 2) \in R$

2 is divisible by 1

but 1 is not divisible by 2

$$(2, 1) \notin R$$

Hence, Given Relation is not symmetric

(ii) Transitive

Let $(x, y) \in R$

y is divisible by x ...(i)

and $(y, z) \in R$

z is divisible by y ...(ii)

From eq(i) and eq(ii)

z is divisible by x

$\therefore$ $(x, z) \in R$

e.g., $(1, 2) \in R$

2 is divisible by 1 ...(i)

$(2, 4) \in R$

4 is divisible by 2 ...(ii)

From eq(i) and eq(ii)

4 is divisible by 1

$(1, 4) \in R$

Hence, Given Relation is transitive.

OR

To prove:

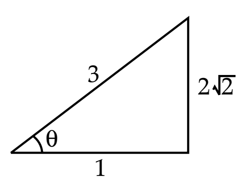
$$\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right) = \frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

$$\text{L.H.S.} = \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3}$$

$$= \frac{9}{4}\left[\frac{\pi}{2} - \sin^{-1}\frac{1}{3}\right]$$

[Using, $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2} \Rightarrow \cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x$]

$$= \frac{9}{4}\cos^{-1}\left(\frac{1}{3}\right) \quad \text{...(i)}$$



let $\cos^{-1}\frac{1}{3} = \theta$

$$\cos\theta = \frac{1}{3}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta + \left(\frac{1}{3}\right)^2 = 1$$

$$\sin^2\theta = 1 - \frac{1}{9}$$

$$\sin^2\theta = \frac{8}{9}$$

$$\sin\theta = \frac{2\sqrt{2}}{3}$$

$$\theta = \sin^{-1}\frac{2\sqrt{2}}{3}$$

$$\cos^{-1}\left(\frac{1}{3}\right) = \sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

Now, from equation (i) $= \frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

= R.H.S. **Hence proved.**

22. Find the value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$, if $x = \cos\theta - \cos 2\theta$, $y = \sin\theta - \sin 2\theta$.

Ans. Given, $x = \cos\theta - 2\cos 2\theta$

$y = \sin\theta - \sin 2\theta$

Differentiate $x = \cos\theta - 2\cos 2\theta$ w.r.t θ

$$\frac{dx}{d\theta} = -\sin\theta + 2\sin 2\theta \quad \text{...(i)}$$

Differentiate $y = \sin\theta - \sin 2\theta$ w.r.t θ

$$\frac{dy}{d\theta} = \cos\theta - 2\cos 2\theta \quad \text{...(ii)}$$

On dividing eq(ii) by eq(i),

$$\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\cos\theta - 2\cos 2\theta}{-\sin\theta + 2\sin 2\theta}$$

$$\frac{dy}{dx} = \frac{\cos\theta - 2\cos 2\theta}{2\sin 2\theta - \sin\theta}$$

$$\left.\frac{dy}{dx}\right]_{\theta=\frac{\pi}{3}} = \frac{\cos\left(\frac{\pi}{3}\right) - 2\cos 2\left(\frac{\pi}{3}\right)}{2\sin 2\left(\frac{\pi}{3}\right) - \sin\left(\frac{\pi}{3}\right)}$$

$$= \frac{\left(\frac{1}{2}\right) - 2\left(\frac{-1}{2}\right)}{2\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)}$$

$$= \frac{\frac{1}{2} + 1}{\frac{\sqrt{3}}{2}} = \frac{\frac{3}{2}}{\frac{\sqrt{3}}{2}} = \sqrt{3}$$

23. Show that the function f defined by $f(x) = (x-1)e^x + 1$ is an increasing function for all $x > 0$.

Ans.
$$f(x) = (x-1)e^x + 1, x > 0$$
$$f'(x) = (x-1)e^x + e^x(1-0) + 0$$
$$= xe^x - e^x + e^x$$
$$= xe^x$$

For all $x > 0$
$$xe^x > 0$$
$$f'(x) > 0$$

Hence, function $f(x) = (x-1)e^x + 1$ is an increasing function for all $x > 0$.

24. Find $|\vec{a}|$ and $|\vec{b}|$, if $|\vec{a}| = 2|\vec{b}|$ and $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = 12$.

OR

Find the unit vector perpendicular to each of the vectors $\vec{a} = 4\hat{i}+3\hat{j}+\hat{k}$ and $\vec{b} = 2\hat{i}-\hat{j}+2\hat{k}$.

Ans.
$$|\vec{a}| = ?$$
$$|\vec{b}| = ?$$

Given, $|\vec{a}| = 2|\vec{b}|$

and $(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = 12$

$\Rightarrow$ $\vec{a}.\vec{a} - \vec{a}.\vec{b} + \vec{b}.\vec{a} - \vec{b}.\vec{b} = 12$

$\Rightarrow$ $\vec{a}.\vec{a} - \vec{b}.\vec{b} = 12$

$\Rightarrow$ $|\vec{a}|^2 - |\vec{b}|^2 = 12$

$\Rightarrow$ $(2|\vec{b}|)^2 - (|\vec{b}|)^2 = 12$

$\Rightarrow$ $4|\vec{b}|^2 - |\vec{b}|^2 = 12$

$\Rightarrow$ $3|\vec{b}|^2 = 12$

$\Rightarrow$ $|\vec{b}|^2 = 4$

$\Rightarrow$ $|\vec{b}| = 2$

$\Rightarrow$ $|\vec{a}| = 2|\vec{b}| = 2(2) = 4$

Hence, $|\vec{a}| = 4$ and $|\vec{b}| = 2$

OR

Given, $\vec{a} = 4\hat{i}+3\hat{j}+\hat{k}$

and $\vec{b} = 2\hat{i}-\hat{j}+2\hat{k}$

Hence,
$$\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ 4 & 3 & 1 \\ 2 & -1 & 2 \end{vmatrix}$$
$$= \hat{i}(6+1) - \hat{j}(8-2) + \hat{k}(-4-6)$$
$$= 7\hat{i} - 6\hat{j} - 10\hat{k}$$

Unit vector perpendicular to each at the vector $\vec{a}$ and $\vec{b}$
$$= \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$$
$$= \frac{7\hat{i}-6\hat{j}-10\hat{k}}{\sqrt{(7)^2+(-6)^2+(-10)^2}}$$
$$= \frac{7}{\sqrt{185}}\hat{i} - \frac{6}{\sqrt{185}}\hat{j} - \frac{10}{\sqrt{185}}\hat{k}$$

***25. Find the equation of the plane with intercept 3 on the *y*-axis and parallel to *xz* – plane.**

26. Find $[P(B/A) + P(A/B)]$, if $P(A) = \frac{3}{10}$, $P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{5}$.

Ans. Find $P\left(\frac{B}{A}\right) + P\left(\frac{A}{B}\right)$

Given, $P(A) = \frac{3}{10}$

$P(B) = \frac{2}{5}$

and $P(A \cup B) = \frac{3}{5}$

Since,
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$\frac{3}{5} = \frac{3}{10} + \frac{2}{5} - P(A \cap B)$$
$$P(A \cap B) = \frac{7}{10} - \frac{3}{5}$$
$$P(A \cap B) = \frac{1}{10}$$
$$P\left(\frac{B}{A}\right) + P\left(\frac{A}{B}\right) = \frac{P(B \cap A)}{P(A)} + \frac{P(A \cap B)}{P(B)}$$
$$= \frac{\frac{1}{10}}{\frac{3}{10}} + \frac{\frac{1}{10}}{\frac{2}{5}} = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$$

SECTION - C

Q. 27 to 32 carry 4 marks each.

27. Prove that the relation *R* on *Z*, defined by *R* = {(*x*, *y*): (*x* – *y*) is divisible by 5} is an equivalence relation.

Ans. The given relation is $R = \{(x, y) : x, y \in Z$ and $x - y$ is divisible by $5\}$.

To prove R is an equivalence relation, we have to prove R is reflexive, symmetric and transitive.

Reflexive As for any $x \in Z$, we have $x - x = 0$, which is divisible by 5.

$\Rightarrow (x - x)$ is divisible by $5 \Rightarrow (x, x) \in R, \forall x \in Z$

Therefore, R is reflexive.

* *Out of Syllabus*

Symmetric Let $(x, y) \in$ R, where $x, y \in Z$

$\Rightarrow (x-y)$ is divisible by 5 [by definition of R]

$\Rightarrow x - y = 5A$ for some $A \in Z \Rightarrow y - x = 5(-A)$

$\Rightarrow (y-x)$ is also divisible by 5 $\Rightarrow (y, x) \in R$

Therefore, R is symmetric.

Transitive Let $(x, y) \in R$, where $x, y \in Z$

$\Rightarrow (x-y)$ is divisible by 5

$\Rightarrow x - y = 5A$ for some $A \in Z$

Again, let $(y, z) \in R$, where $y, z \in Z$

$\Rightarrow (y-z)$ is divisible by 5

$\Rightarrow y - z = 5B$ for some $B \in Z$

Now, $(x-y) + (y-z) = 5A + 5B$

$\Rightarrow \quad x - z = 5(A+B)$

$\Rightarrow (x-z)$ is divisible by 5 for some

$(A+B) \in Z \Rightarrow (x, z) \in R$

Therefore, R is transitive.

Thus, R is reflexive, symmetric and transitive.

Hence, it is an equivalence relation.

28. If $y = \sin^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right)$, then show that $\frac{dy}{dx} = \frac{-1}{2\sqrt{1-x^2}}$.

OR

Verify the Rolle's Theorem for the function cos x in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Ans. To Prove: $\frac{dy}{dx} = \frac{-1}{2\sqrt{1-x^2}}$

Given: $y = \sin^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right)$

Put $x = \cos 2\theta$

$$y = \sin^{-1}\left[\frac{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}{2}\right]$$

$$y = \sin^{-1}\left[\frac{\sqrt{1+2\cos^2\theta-1}+\sqrt{1-(1-2\sin^2\theta)}}{2}\right]$$

$[\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta]$

$$y = \sin^{-1}\left[\frac{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}{2}\right]$$

$$y = \sin^{-1}\left[\frac{1}{\sqrt{2}}\cos\theta+\frac{1}{\sqrt{2}}\sin\theta\right]$$

$$y = \sin^{-1}\left[\sin\frac{\pi}{4}\cos\theta+\cos\frac{\pi}{4}\sin\theta\right]$$

[Using $\sin A \cos B + \cos A \sin B = \sin(A+B)$]

$$y = \sin^{-1}\left[\sin\left(\frac{\pi}{4}+\theta\right)\right]$$

$$y = \frac{\pi}{4}+\theta$$

Put $\theta = \frac{1}{2}\cos^{-1}x$ $[x = \cos 2\theta]$

$$y = \frac{\pi}{4}+\frac{1}{2}\cos^{-1}x$$

Differentiate above equation w.r.t x :

$$\frac{dy}{dx} = 0+\frac{1}{2}\times\frac{-1}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = \frac{-1}{2\sqrt{1-x^2}}$$

Hence Proved.

OR

$f(x) = e^x \cos x$ in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$f\left(-\frac{\pi}{2}\right) = e^{-\frac{\pi}{2}}\cos\left(-\frac{\pi}{2}\right)$$

$$= +e^{-\frac{\pi}{2}}\cos\left(\frac{\pi}{2}\right) \qquad [\cos(-\theta) = \cos\theta]$$

$$= +e^{-\frac{\pi}{2}}\times 0 \qquad \left[\cos\frac{\pi}{2} = 0\right]$$

$f(x) = e^x = 0$

$$f\left(\frac{\pi}{2}\right) = e^{\frac{\pi}{2}}\cos\frac{\pi}{2}$$

$$= e^{\frac{\pi}{2}}\times 0$$

$$= 0$$

Since Rolle's theorem holds true, $f\left(-\frac{\pi}{2}\right) = f\left(\frac{\pi}{2}\right)$

Hence, there exists $c \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ such that

$$f'(c) = 0$$

$$-e^c \sin c + \cos c \cdot e^c = 0$$

$$\sin c \cdot e^c = e^c \cos c$$

$$\tan c = 1$$

$$\tan c = \tan\frac{\pi}{4}$$

$$c = \frac{\pi}{4}$$

$$c = \frac{\pi}{4} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

Hence, Rolle's theorem verified.

29. Evaluate: $\displaystyle\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx.$

Ans. $$I = \int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx \quad ....(i)$$

On Applying Property

$$\int_0^{\pi} f(x)dx = \int_0^{\pi} f(\pi - x)dx$$

$$I = \int_0^{\pi} \frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)}dx$$

or, $$I = \int_0^{\pi} \frac{(\pi-x)\sin x}{1+\cos^2 x}dx \quad ....(ii)$$

Adding eqn. (i) and eqn. (ii), we get

$$2I = \int_0^{\pi} \frac{\pi\sin x}{1+\cos^2 x}dx$$

$$I = \frac{\pi}{2}\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}dx$$

Let $\cos x = t$

$-\sin x\, dx = dt$

$\sin x\, dx = -dt$

When $x = 0$

$t = \cos 0 = 1$

When $x = \pi$

$t = \cos \pi = -1$

$$I = -\frac{\pi}{2}\int_1^{-1} \frac{1}{1+t^2}dt$$

Using Property

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

$$I = \frac{\pi}{2}\int_{-1}^{1} \frac{1}{1+t^2}dt$$

$$I = \frac{\pi}{2}\left[\tan^{-1} t\right]_{-1}^{1}$$

$$I = \frac{\pi}{2}\left[\tan^{-1}(1) - \tan^{-1}(-1)\right]$$

$$= \frac{\pi}{2}\left[\frac{\pi}{4} + \frac{3\pi}{4}\right]$$

$$= \frac{\pi}{2}\left[+\frac{2\pi}{4}\right]$$

$$= \frac{\pi^2}{4}$$

30. For the differential equation given below, find a particular solution satisfying the given condition

$$(x+1)\frac{dy}{dx} = 2e^{-y} + 1;\ y = 0 \text{ when } x = 0.$$

Ans. $$(x+1)\frac{dy}{dx} = 2e^{-y} + 1;\ y = 0 \text{ when } x = 0$$

$$\frac{dy}{dx} = \frac{2e^{-y}+1}{x+1}$$

or, $$\frac{1}{2e^{-y}+1}dy = \frac{1}{x+1}dx$$

or, $$\frac{e^y}{2+e^y}dy = \frac{1}{x+1}dx$$

Integrating both sides,

$$\int \frac{e^y}{2+e^y}dy = \int \frac{1}{x+1}dx$$

or, $\log|2 + e^y| = \log|x + 1| + \log C$

$\log|2 + e^y| = \log|(x + 1)C|$

$2 + e^y = C(x + 1) \quad$ ...(i)

When $x = 0, y = 0$

$2 + e^0 = C(0 + 1)$

$2 + 1 = C$

$C = 3$

Put $C = 3$ in eqn. (i),

$2 + e^y = 3(x + 1)$

$e^y = 3x + 3 - 2$

$e^y = 3x + 1$

$y = e^{(3x + 1)}$

This is the required Particular solution.

31. A manufacturer has three machines I, II and III installed in his factory. Machine I and II are capable of being operated for atmost 12 hours whereas machine III must be operated for atleast 5 hours a day. He produces only two items M and N each requiring the use of all the three machines.

The number of hours required for producing 1 unit of M and N on three machines are given in the following table :

Items	Number of hours required on machines		
	I	II	III
M	1	2	1
N	2	1	1.25

He makes a profit of ₹ 600 and ₹ 400 on one unit of items M and N respectively. How many units of each item should he produce so as to maximize his profit assuming that he can sell all the items that he produced. What will be the maximum profit ?

Ans. Let manufacturer produces x units of Product M and y units of product N.

	Product M (x)	Product N (y)	Time
Machine I	1	2	12
Machine II	2	1	12
Machine III	1	1.25	5

Subject to constraints

$$x + 2y \leq 12$$
$$2x + y \leq 12$$
$$x + 1.25y \geq 5$$

or, $4x + 5y \geq 20, x \geq 0, y \geq 0$

Maximize $Z = 600x + 400y$

$L_1 : \quad x + 2y = 12$

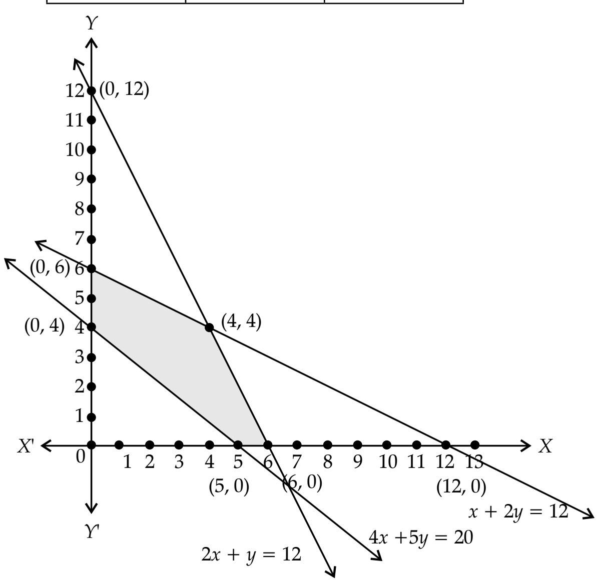
x	0	12	4
y	6	0	4

$L_2 : \quad 2x + y = 12$

x	0	6	4
y	12	0	4

$L_3 : \quad 4x + 5y = 20$

x	0	5
y	4	0



Vertices of feasible region are (0, 4), (5, 0), (6, 0), (4, 4) and (0, 6)

(x, y)	$Z = 600x + 400y$
(0, 4)	Z = 1600
(5, 0)	Z = 3000
(6, 0)	Z = 3600
(4, 4)	Z = 4000 (Max.)
(0, 6)	Z = 2400

Maximum Value of $Z = 4000$ at $x = 4$ and $y = 4$

32. A coin is biased so that the head is three times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails. Hence find the mean of the number of tails.

OR

Suppose that 5 men out of 100 and 25 women out of 1000 are good orators. Assuming that there are equal number of men and women, find the probability of choosing a good orator.

Ans. $P(\text{Head}) = \frac{3}{4}$, $P(\text{Tail}) = \frac{1}{4}$

Let X be the number of tails in two tosses :

X	0	1	2
	$\frac{3}{4}\times\frac{3}{4}$	$2\times\frac{3}{4}\times\frac{1}{4}$	$\frac{1}{4}\times\frac{1}{4}$
$P(X)$	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

$$\text{Mean} = \Sigma XP(X)$$
$$= 0\times\frac{9}{16}+1\times\frac{6}{16}+2\times\frac{1}{16}$$
$$= 0+\frac{6}{16}+\frac{2}{16}$$
$$= \frac{8}{16}$$
$$= \frac{1}{2}$$

OR

$$\text{P(Men good orator)} = \frac{5}{100}$$
$$\text{P(Women good orator)} = \frac{25}{1000}$$

P(of choosing good orator) = ?

Let E_1 and E_2 denote the events that the person is a good orator.

E_1 = Man orator, $P(E_1) = \frac{1}{2}$

E_2 = Woman orator, $P(E_2) = \frac{1}{2}$

A = Selecting good orator

$$P(A/E_1) = \frac{5}{100},\ P(A/E_2) = \frac{25}{1000}$$

Required Probability

$$P(A) = P(E_1)\times P(A/E_1)+P(E_2)\times P(A/E_2)$$
$$= \frac{1}{2}\times\frac{5}{100}+\frac{1}{2}\times\frac{25}{1000}$$
$$= \frac{3}{80}$$

SECTION - D

Q. 33 to 36 carry 6 marks each.

33. Using properties of determinants prove that :

$$\begin{vmatrix} a-b & b+c & a \\ b-c & c+a & b \\ c-a & a+b & c \end{vmatrix} = a^3+b^3+c^3-3abc.$$

OR

If $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$**, then show that** $A^3 - 4A^2 - 3A$ + **11** $I = 0$**, Hence find** A^{-1}**.**

Ans. To Prove

$$\begin{vmatrix} a-b & b+c & a \\ b-c & c+a & b \\ c-a & a+b & c \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$$

From L.H.S. $\begin{vmatrix} a-b & b+c & a \\ b-c & c+a & b \\ c-a & a+b & c \end{vmatrix}$

$R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} 0 & 2(a+b+c) & a+b+c \\ b-c & c+a & b \\ c-a & a+b & c \end{vmatrix}$$

Taking $(a + b + c)$ common from R_1

$$= (a+b+c)\begin{vmatrix} 0 & 2 & 1 \\ b-c & c+a & b \\ c-a & a+b & c \end{vmatrix}$$

$C_2 \to C_2 - 2C_3$

$$= (a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ b-c & c+a-2b & b \\ c-a & a+b-2c & c \end{vmatrix}$$

Expand along R_1

$= (a + b + c)[0 - 0 + 1\{(b - c)(a + b - 2c) - (c - a)(c + a - 2b)\}]$

$= (a + b + c)[(ab + b^2 - 2bc - ac - bc + 2c^2) - (c^2 + ac - 2bc - ac - a^2 + 2ab)]$

$= (a + b + c)[ab + b^2 - 3bc - ac + 2c^2 - c^2 + 2bc + a^2 - 2ab]$

$= (a + b + c)[a^2 + b^2 + c^2 - ab - bc - ac]$

$= a^3 + b^3 + c^3 - 3abc$ [Using identity]

= R.H.S. **Hence Proved.**

OR

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$$

$$A^2 = A.A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 9 & 7 & 5 \\ 1 & 4 & 1 \\ 8 & 9 & 9 \end{bmatrix}$$

$$A^3 = A^2.A = \begin{bmatrix} 9 & 7 & 5 \\ 1 & 4 & 1 \\ 8 & 9 & 9 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 28 & 37 & 26 \\ 10 & 5 & 1 \\ 35 & 42 & 34 \end{bmatrix}$$

$$\therefore A^3 - 4A^2 - 3A + 11I = \begin{bmatrix} 28 & 37 & 26 \\ 10 & 5 & 1 \\ 35 & 42 & 34 \end{bmatrix} - 4\begin{bmatrix} 9 & 7 & 5 \\ 1 & 4 & 1 \\ 8 & 9 & 9 \end{bmatrix} - 3\begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix} + 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 28 & 37 & 26 \\ 10 & 5 & 1 \\ 35 & 42 & 34 \end{bmatrix} - \begin{bmatrix} 36 & 28 & 20 \\ 4 & 16 & 4 \\ 32 & 36 & 36 \end{bmatrix} - \begin{bmatrix} 3 & 9 & 6 \\ 6 & 0 & -3 \\ 3 & 6 & 9 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 28-36-3+11 & 37-28-9+0 & 26-20-6+0 \\ 10-4-6+0 & 5-16-0+11 & 1-4+3+0 \\ 35-32-3+0 & 42-36-6+0 & 34-36-9+11 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

$$A^3 - 4A^2 - 3A + 11I = 0$$

$$A^3 \cdot A^{-1} - 4A^2 \cdot A^{-1} - 3A \cdot A^{-1} + 11IA^{-1} = 0.A^{-1}$$

$$A^2I - 4AI - 3I + 11A^{-1} = 0$$

$$A^2 - 4A - 3I + 11A^{-1} = 0$$

$$11A^{-1} = 3I + 4A - A^2$$

$$= 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + 4\begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix} - \begin{bmatrix} 9 & 7 & 5 \\ 1 & 4 & 1 \\ 8 & 9 & 9 \end{bmatrix}$$

$$11A^{-1} = \begin{bmatrix} 3+4-9 & 0+12-7 & 0+8-5 \\ 0+8-1 & 3+0-4 & 0-4-1 \\ 0+4-8 & 0+8-9 & 3+12-9 \end{bmatrix}$$

$$11A^{-1} = \begin{bmatrix} -2 & 5 & 3 \\ 7 & -1 & -5 \\ -4 & -1 & 6 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -\frac{2}{11} & \frac{5}{11} & \frac{3}{11} \\ \frac{7}{11} & -\frac{1}{11} & -\frac{5}{11} \\ -\frac{4}{11} & -\frac{1}{11} & \frac{6}{11} \end{bmatrix}$$

34. Find the intervals on which the function $f(x) = (x-1)^3 (x-2)^2$ is (a) strictly increasing (b) strictly decreasing.

OR

Find the dimensions of the rectangle of perimeter 36 cm which will sweep out a volume as large as possible, when revolved about one of its side. Also, find the maximum volume.

Ans. Given, $f(x) = (x-1)^3 (x-2)^2$

On differentiating both sides w.r.t. x, we get

$$f'(x) = (x-1)^3 . \frac{d}{dx}(x-2)^2$$

$$+(x-2)^2 . \frac{d}{dx}(x-1)^3 \quad \left[\because \frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}\right]$$

$$f'(x) = (x-1)^3 . 2(x-2) + (x-2)^2 . 3(x-1)^2$$

$$= (x-1)^2 (x-2) [2(x-1) + 3(x-2)]$$

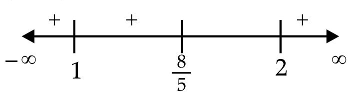
$$= (x-1)^2 (x-2) (2x-2+3x-6)$$

or, $$f'(x) = (x-1)^2 (x-2) (5x-8)$$

Now, put $f'(x) = 0$

or, $(x-1)^2 (x-2)(5x-8) = 0$

Either $(x-1)^2 = 0$ or $x-2 = 0$ or $5x-8 = 0$



$\therefore$ $$x = 1, \frac{8}{5}, 2$$

Now, we find intervals and check in which interval $f(x)$ is strictly increasing and strictly decreasing.

Interval	$f'(x) = (x-1)^2 (x-2)(5x-8)$	Sign of $f'(x)$
$x < 1$	(+)(–)(–)	+ve
$1 < x < \frac{8}{5}$	(+)(–)(–)	+ve
$\frac{8}{5} < x < 2$	(+)(–)(+)	–ve
$x > 2$	(+)(+)(+)	+ve

We know that, a function $f(x)$ is said to be an strictly increasing function, if $f'(x) > 0$ and strictly decreasing if $f'(x) < 0$. So, the given function $f(x)$ is increasing on the intervals and $(-\infty, 1)$ $\left(1, \frac{8}{5}\right)$ and $(2, \infty)$ and decreasing on $\left(\frac{8}{5}, 2\right)$.

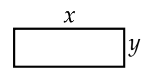
Since, $f(x)$ is a polynomial function, so it is continuous at $x = 1, \frac{8}{5}, 2$. Hence, $f(x)$ is

(a) increasing on intervals $\left(-\infty, \frac{8}{5}\right) \cup [2, \infty)$

(b) decreasing on interval $\left(\frac{8}{5}, 2\right)$

OR

Let the length and breadth of rectangle be x and y respectively.



Given, $$P = 36$$

$$2(x+y) = 36$$

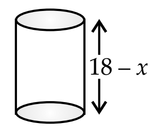
$$x + y = 18$$

$$y = 18 - x$$

Let rectangle revolved around side x and form cylinder with radius r and height $(18 - x)$

$$2\pi r = x$$

$$r = \frac{x}{2\pi}$$



Volume, $$V = \pi r^2 h$$

$$= \pi\left(\frac{x}{2\pi}\right)^2 y$$

$$= \pi \frac{x^2}{4\pi^2}(18-x)$$

$$= \frac{1}{4\pi}x^2(18-x)$$

$$V = \frac{1}{4\pi}(18x^2 - x^3)$$

Differentiate w.r.t. x

$$\frac{dV}{dx} = \frac{1}{4\pi}(36x - 3x^2)$$

$$\frac{dV}{dx} = 0$$

For maxima and minima,

$$\frac{1}{4\pi}(36x - 3x^2) = 0$$

$$x \neq 0, x = 12$$

$$\frac{d^2V}{dx^2} = \frac{1}{4\pi}(36 - 6x)$$

$$\left.\frac{d^2V}{dx^2}\right]_{x=12} = \frac{1}{4\pi}[36 - 6(12)]$$

$$= \frac{1}{4\pi}(-36) < 0$$

Hence, Volume is maximum when $x = 12$

$$y = 18 - x$$
$$= 18 - 12$$
$$= 6$$

Required length and breadth of rectangle are 12 and 6 respectively.

Volume $$V = \pi\left(\frac{12}{2\pi}\right)^2(6)$$

$$= \frac{36}{22}\times 6\times 7$$

$$= \frac{756}{11}$$

$$= 68.72 \text{ unit}^3 \text{ (Approx)}$$

35. Find the area of the region lying in the first quadrant and enclosed by the X-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.

Ans. We have $y = 0$, $y = x$ and the circle $x^2 + y^2 = 32$ in the first quadrant.

Solving $y = x$ with the circle

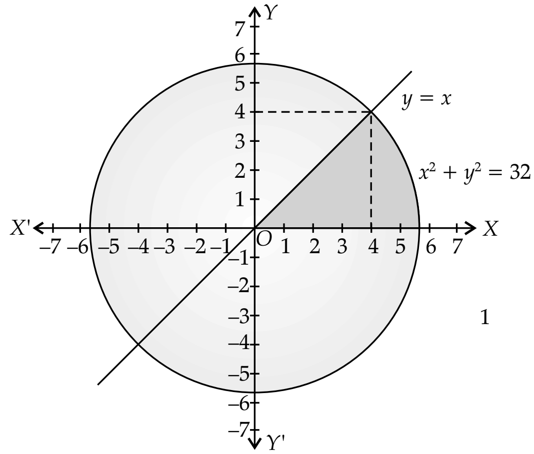
$$x^2 + x^2 = 32$$
$$x^2 = 16$$
$$x = 4 \quad \text{(In the first quadrant)}$$

When $x = 4$,

$y = 4$ for the point of intersection of the circle with the X-axis.

Put $$y = 0$$
$$x^2 + 0 = 32$$
$$x = \pm 4\sqrt{2}$$

So, the circle intersects the X-axis at $(\pm 4\sqrt{2}, 0)$.



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From the above figure, area of the shaded region,

$$A = \int_0^4 x\,dx + \int_4^{4\sqrt{2}} \sqrt{(4\sqrt{2})^2 - x^2}\,dx$$

$$= \left[\frac{x^2}{2}\right]_0^4 + \left[\frac{x}{2}\sqrt{(4\sqrt{2})^2 - x^2} + \frac{(4\sqrt{2})^2}{2}\sin^{-1}\frac{x}{4\sqrt{2}}\right]_4^{4\sqrt{2}}$$

$$= \left[\frac{16}{2}\right] + \left[\begin{array}{l} 0 + 16\sin^{-1}1 - \frac{4}{2}\sqrt{(4\sqrt{2})^2 - 16^2} \\ -16\sin^{-1}\frac{4}{4\sqrt{2}} \end{array}\right]$$

$$= 8 + \left[16\ \pi/2 - 2\sqrt{16} - 16\ \frac{\pi}{4}\right]$$

$$= 8 + [8\pi - 8 - 4\pi]$$
$$= 4\pi \text{ sq. units}$$

*** 36. Show that the lines $\vec{r} = \vec{a} + \lambda\vec{b}$ and $\vec{r} = \vec{b} + \mu\vec{a}$ are coplanar and the plane containing them is given by $\vec{r}\cdot(\vec{a}\times\vec{b}) = 0$.**

Outside Delhi Set-II 65/4/2

Note : Except these, all other questions from outside Delhi Set-I

SECTION - A

Question numbers 1 to 10 are multiple choice questions of 1 mark each. Select the correct option:

8. Let $A = \begin{bmatrix} 200 & 50 \\ 10 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 50 & 40 \\ 2 & 3 \end{bmatrix}$, then $|AB|$ is equal to

(a) 460 **(b)** 2000

(c) 3000 **(d)** – 7000

Ans. Option (d) is correct.

Explanation:

$$A = \begin{bmatrix} 200 & 50 \\ 10 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 50 & 40 \\ 2 & 3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 200 & 50 \\ 10 & 2 \end{bmatrix}\begin{bmatrix} 50 & 40 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 10000 + 100 & 8000 + 150 \\ 500 + 4 & 400 + 6 \end{bmatrix}$$

$$AB = \begin{bmatrix} 10100 & 8150 \\ 504 & 406 \end{bmatrix}$$

$$|AB| = (10100)(406) - (504)(8150)$$
$$= 4100600 - 4107600 = -7000$$

* *Out of Syllabus*

9. Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$. If $\vec{b}$ is a vector such that $\vec{a}\cdot\vec{b} = |\vec{b}|^2$ and $|\vec{a} - \vec{b}| = \sqrt{7}$ then $|\vec{b}|$ equals

(a) 7 **(b)** 14

(c) $\sqrt{7}$ **(d)** 21

Ans. Option (c) is correct.

Explanation:

$$|\vec{a} - \vec{b}| = \sqrt{7}$$

On squaring both sides,

$$|\vec{a} - \vec{b}|^2 = (\sqrt{7})^2$$

$$|\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\cdot\vec{b} = 7$$

Given,

$$\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$$

$$|\vec{a}| = \sqrt{(1)^2 + (-2)^2 + (3)^2}$$

$$|\vec{a}| = \sqrt{14}$$

$$|\vec{a}|^2 = 14$$

$$14 + |\vec{b}|^2 - 2\vec{a}.\vec{b} = 7$$

$$14 + |\vec{b}|^2 - 2|\vec{b}|^2 = 7 \quad [\text{Given, } \vec{a}.\vec{b} = |\vec{b}|^2]$$

$$14 - |\vec{b}|^2 = 7$$

$$|\vec{b}|^2 = 7$$

$$|\vec{b}| = \sqrt{7}$$

10. Three dice are thrown simultaneously. The probability of obtaining a total score of 5 is

(a) $\frac{5}{216}$ **(b)** $\frac{1}{6}$

(c) $\frac{1}{36}$ **(d)** $\frac{1}{49}$

Ans. Option (c) is correct.

Explanation:

Total no. of cases when three dice are thrown

$$= 6 \times 6 \times 6$$

$$n(S) = 216$$

Number of cases where getting sum of 5

$$= \{(1, 1, 3), (1, 3, 1), (1, 2, 2), (2, 1, 2), (2, 2, 1), (3, 1, 1)\}$$

$$n(E) = 6$$

Probability of obtaining a total score of 5

$$P(E) = \frac{n(E)}{n(S)}$$

$$= \frac{6}{216}$$

$$= \frac{1}{36}$$

In Q. Nos. 11 to 15, fill in the blanks with correct word / sentence:

15. **If $f(x) = 2|x| + 3|\sin x| + 6$, then the right hand derivative of $f(x)$ at $x = 0$ is ....**

Ans.

$$f(x) = 2|x| + 3|\sin x| + 6$$

$$f(0) = 2|0| + 3|\sin 0| + 6$$

$$= 0 + 3(0) + 6$$

$$= 0 + 0 + 6 = 6$$

$$\text{R.H.D.} = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h\to 0} \frac{[2|0+h| + 3|\sin(0+h)| + 6] - [6]}{h}$$

$$= \lim_{h\to 0} \frac{2h + 3\sin h + 6 - 6}{h}$$

$$= \lim_{h\to 0} \frac{2h}{h} + \lim_{h\to 0} \frac{3\sin h}{h}$$

$$= 2 + 3\lim_{h\to 0} \frac{\sin h}{h}$$

$$\left[\lim_{x\to 0} \frac{\sin x}{x} = 1\right]$$

$$= 2 + 3(1)$$

$$= 2 + 3 = 5$$

Question numbers 16 to 20 are very short answer type questions.

19. **Find $\int \sin^5\left(\frac{x}{2}\right).\cos\left(\frac{x}{2}\right)dx$**

Ans. $\int \sin^5\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)dx$

let $\quad \sin\frac{x}{2} = t$

$$\cos\frac{x}{2} \times \frac{1}{2}dx = dt$$

$$\cos\left(\frac{x}{2}\right)dx = 2dt$$

$$I = 2\int t^5 dt$$

$$I = \frac{2t^6}{6} + C$$

$$I = \frac{\sin^6\frac{x}{2}}{3} + C$$

20. **If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, then find A^3.**

Ans.

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$A^2 = A\cdot A$$

$$= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0+0 \\ 1+1 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$A^3 = A^2.A$$

$$= \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$$

SECTION - B

Q. 21 to 26 carry 2 marks each.

25. Find the derivative of $x^{\log x}$ w.r.t. $\log x$.

Ans. let $u = x^{\log x}$, $v = \log x$ and $u = x^{\log x}$

taking log on both sides

$$\log u = \log x^{\log x}$$

$$\log u = \log x \log x$$

$$\log u = (\log x)^2$$

Differentiate w.r.t. x

$$\frac{1}{u}\frac{du}{dx} = 2\log x \times \frac{1}{x}$$

$$\frac{du}{dx} = u \cdot \frac{2\log x}{x}$$

$$\frac{du}{dx} = \frac{x^{\log x}.2\log x}{x} \quad ...(i)$$

$$v = \log x$$

Differentiate w.r.t. to x

$$\frac{dv}{dx} = \frac{1}{x} \quad ...(ii)$$

On dividing eq(i) by eqn. (ii)

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{x^{\log x}.\frac{2\log x}{x}}{\frac{1}{x}}$$

$$\frac{du}{dv} = 2\log x.x^{\log x}$$

The derivative of $x^{\log x}$ w.r.t. to $\log x$ is $2\log x.x^{\log x}$.

SECTION - C

Q. 27 to 32 carry 4 marks each.

31. Show that the function $f : R \to R$ defined by $f(x) = \frac{x}{x^2+1}$, $\forall\, x \in R$ is neither one-one nor onto.

Ans. Given, $f : R \to R;\ f(x) = \frac{x}{1+x^2}$

To show that f is neither one-one nor onto

(i) f is one-one : Let $x_1, x_2 \in R$ (domain)

and $f(x_1) = f(x_2)$

$$\Rightarrow \frac{x_1}{1+x_1^2} = \frac{x_2}{1+x_2^2}$$

$$\Rightarrow x_1(1+x_2^2) = x_2(1+x_1^2)$$

$$\Rightarrow x_1 + x_1 \cdot x_2^2 - x_2 - x_2 x_1^2 = 0$$

$$\Rightarrow (x_1 - x_2)(1 - x_1 \cdot x_2) = 0$$

Taking $x_1 = 4$, $x_2 = \frac{1}{4} \in R$

$$f(x_1) = f(4) = \frac{4}{17}$$

$$f(x_2) = f\left(\frac{1}{4}\right) = \frac{4}{17}$$

$\therefore f$ not is one-one.

(ii) f is onto: Let $y \in R$ (co-domain)

$$f(x) = y$$

$$\Rightarrow \frac{x}{1+x^2} = y \Rightarrow y \cdot (1 + x^2) = x$$

$$\Rightarrow yx^2 + y - x = 0$$

$$\Rightarrow x = \frac{1 \pm \sqrt{1-4y^2}}{2y}$$

since, $x \in R$,

$$\therefore 1 - 4y^2 \geq 0$$

$$\Rightarrow -\frac{1}{2} \leq y \leq \frac{1}{2}$$

So Range$(f) \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

Range $(f) \neq$ R (Co-domain)

$\therefore f$ is not onto.

Hence f is neither one-one nor onto.

32. Evaluate: $\int_{-1}^{2} |x^3 - x|\, dx$

Ans. $\int_{-1}^{2} |x^3 - x|\, dx = \int_{-1}^{0} (x^3 - x)dx - \int_{0}^{1} (x^3 - x)dx + \int_{1}^{2} (x^3 - x)dx$

$$= \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} - \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{0}^{1} + \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{1}^{2}$$

$$= \left[(0-0) - \left(\frac{1}{4} - \frac{1}{2}\right)\right] - \left[\left(\frac{1}{4} - \frac{1}{2}\right) - (0-0)\right] + \left[(4-2) - \left(\frac{1}{4} - \frac{1}{2}\right)\right]$$

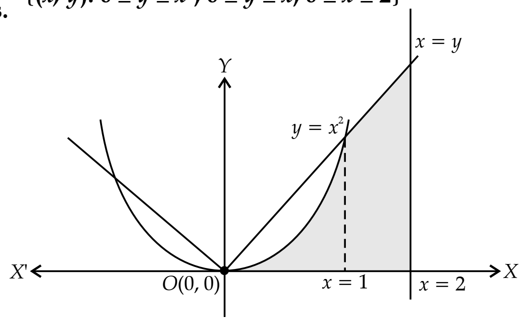
$$= \frac{1}{4} + \frac{1}{4} + 2 + \frac{1}{4} = \frac{11}{4}$$

SECTION - D

Q. 33 to 36 carry 6 marks each.

36. Using integration find the area of the region :
$\{(x, y): 0 \le y \le x^2, 0 \le y \le x, 0 \le x \le 2\}$

Ans.



$$\text{Area of the shaded region} = \int_0^1 x^2 dx + \int_1^2 x dx$$

$$= \left[\frac{x^3}{3}\right]_0^1 + \left[\frac{x^2}{2}\right]_1^2$$

$$= \left[\frac{1}{3} - \frac{0}{3}\right] + \left[2 - \frac{1}{2}\right]$$

$$= \frac{1}{3} + \frac{3}{2}$$

$$= \frac{2+9}{6}$$

$$= \frac{11}{6} \text{ sq. unit}$$

Outside Delhi Set-III — 65/4/3

***Note :** Except these, all other questions from outside Delhi Set-I & II*

SECTION - A

Question numbers 1 to 10 are multiple choice questions of 1 mark each. Select the correct option:

8. The value of $\tan\left[\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right]$ is

(a) $\frac{3+\sqrt{5}}{2}$ **(b)** $\frac{3-\sqrt{5}}{2}$

(c) $\frac{-3+\sqrt{5}}{2}$ **(d)** $\frac{-3-\sqrt{5}}{2}$

Ans. Option (b) is correct.

Explanation :

$$x = \tan\left[\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right]$$

Let $\cos^{-1}\frac{\sqrt{5}}{3} = \theta$

$$\cos\theta = \frac{\sqrt{5}}{3}$$

$$x = \frac{\sqrt{1-\frac{\sqrt{5}}{3}}}{\sqrt{1+\frac{\sqrt{5}}{3}}}$$

$$= \frac{\sqrt{3-\sqrt{5}}}{\sqrt{3+\sqrt{5}}}$$

$$= \frac{\sqrt{3-\sqrt{5}}}{\sqrt{3+\sqrt{5}}} \times \frac{\sqrt{3-\sqrt{5}}}{\sqrt{3-\sqrt{5}}}$$

$$= \frac{3-\sqrt{5}}{\sqrt{(3)^2-(\sqrt{5})^2}}$$

$$= \frac{3-\sqrt{5}}{\sqrt{9-5}}$$

$$= \frac{3-\sqrt{5}}{2}$$

9. If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then det ($adj\ A$) equals

(a) a^{27} **(b)** a^9

(c) a^6 **(d)** a^2

Ans. Option (c) is correct.

Explanation:

$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$$

$$\text{Det}(A) = a(a \times a - 0 \times 0) - 0 + 0$$
$$= a^3$$
$$\text{Det}(\text{adj } A) = (a^3)^2$$
$$= a^6$$

*** 10. The line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ is parallel to the plane**

(a) $2x + 3y + 4z = 0$ **(b)** $3x + 4y - 5z = 7$

(c) $2x + y - 2z = 0$ **(d)** $x - y + z = 2$

In Q. Nos. 11 to 15, fill in the blanks with correct word / sentence :

15. If $f(x) = x|x|$, then $f'(x) =$ ....

Ans.
$$f(x) = x|x|$$

$$f(x) = \begin{cases} x^2, & x \ge 0 \\ -x^2, & x < 0 \end{cases}$$

* *Out of Syllabus*

$$f'(x) = \begin{bmatrix} 2x, & x \geq 0 \\ -2x, & x < 0 \end{bmatrix}$$

Hence, $f'(x)$ is differential everywhere

$$f'(x) = 2|x|$$

Q. Nos. 16 to 20 are very short answer type questions.

17. Find *adj A*, if $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$

Ans.
$$A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$$

$$C_{11} = 3, C_{12} = -4$$
$$C_{21} = 1, C_{22} = 2$$

$$Adj\,A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$$

$$= \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}^T$$

$$Adj\,A = \begin{bmatrix} 3 & 1 \\ -4 & 2 \end{bmatrix}$$

19. Find $\int \frac{1}{x(1+x^2)} dx$

Ans. $\int \frac{1}{x(1+x^2)} dx$

let $1 + x^2 = t$

$$2xdx = dt$$

$$xdx = \frac{dt}{2}$$

$$= \frac{1}{2} \int \frac{1}{(t-1)t} dt$$

$$\frac{1}{(t-1)(t)} = \frac{A}{t} + \frac{B}{t-1} \quad \text{...(i)}$$

$$1 = A(t-1) + Bt$$

Put $t = 0$

$$A = -1$$

Put $t = 1$

$$B = 1$$

Put $A = -1$ and $B = 1$ ...(ii)

$$\frac{1}{(t-1)t} = \frac{-1}{t} + \frac{1}{t-1}$$

$$\frac{1}{2} \int \frac{1}{(t-1)t} dt = \frac{1}{2} \left[-\int \frac{1}{t} dt + \int \frac{1}{t-1} dt \right]$$

$$= \frac{1}{2} \left[-\log t + \log|t-1| \right] + C$$

$$= \frac{1}{2} \left[\log \frac{t-1}{t} \right] + C$$

Put $t = x^2 + 1$

$$= \frac{1}{2} \left[\log \frac{x^2+1-1}{x^2+1} \right] + C$$

$$= \frac{1}{2} \log \frac{x^2}{1+x^2} + C$$

*** 20. If [*x*] denotes the greatest integer function, then find** $\int_0^{3/2} [x^2] dx$

SECTION - B

Q. Nos. 21 to 26 carry 2 marks each.

25. Show that the function $f(x) = \frac{x}{3} + \frac{3}{x}$ **decreases in the intervals (– 3, 0) ∪ (0, 3).**

Ans.
$$f(x) = \frac{x}{3} + \frac{3}{x}$$

$$f'(x) = \frac{1}{3} - \frac{3}{x^2}$$

$$f'(x) = 0$$

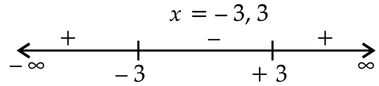
$$\frac{1}{3} - \frac{3}{x^2} = 0$$

$$\frac{x^2 - 9}{3x^2} = 0$$

$x^2 - 9 = 0$, x not equal sig 0

$$(x+3)(x-3) = 0$$

$$x = -3, 3$$



Here $x \neq 0$

the function decreases in the interval

(–3, 0) ∪ (0, 3)

26. Three distinct numbers are chosen randomly from the first 50 natural numbers. Find the probability that all the three numbers are divisible by both 2 and 3.

Ans.
$$S = \{1, 2, 3........ 50\}$$
$$n(S) = 50$$
E = [no's divisible by 2 and 3]

E = [6, 12, 18, 24, 30, 36, 42, 48]

$$n(E) = 8$$

Probability that all the three numbers are divisible by both 2 and 3.

$$P(E) = \frac{^8C_3}{^{50}C_3}$$

$$= \frac{\frac{\lfloor 8}{\lfloor 3 \lfloor 5}}{\frac{\lfloor 50}{\lfloor 3 \lfloor 47}}$$

* *Out of Syllabus*

$$= \frac{8 \times 7 \times 6}{50 \times 49 \times 48}$$

$$= \frac{1}{350}$$

SECTION - C

Q. Nos. 27 to 32 carry 4 marks each.

31. Evaluate $\int_0^1 \sqrt{3-2x-x^2}\,dx$

Ans.

$$\int_0^1 \sqrt{3-2x-x^2}\,dx = \int_0^1 \sqrt{-(x^2+2x-3)}\,dx$$

$$= \int_0^1 \sqrt{-(x^2+2x+1-4)}\,dx$$

$$= \int_0^1 \sqrt{(2)^2-(x+1)^2}\,dx$$

let $x + 1 = t$

$dx = dt$

when $x = 0, t = 1$

$x = 1, t = 2$

$$= \int_1^2 \sqrt{(2)^2-t^2}\,dt$$

[using $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$]

$$= \left[\frac{t}{2}\sqrt{(2)^2-t^2} + \frac{(2)^2}{2}\sin^{-1}\frac{t}{2}\right]_1^2$$

$$= \left[\frac{2}{2}\sqrt{4-4} + 2\sin^{-1}\frac{2}{2}\right] - \left[\frac{1}{2}\sqrt{4-1} + 2\sin^{-1}\frac{1}{2}\right]$$

$$= 0 + 2\left(\frac{\pi}{2}\right) - \left[\frac{\sqrt{3}}{2} + 2\left(\frac{\pi}{6}\right)\right]$$

$$= \pi - \frac{\sqrt{3}}{2} - \frac{\pi}{3}$$

$$= \frac{2\pi}{3} - \frac{\sqrt{3}}{2}$$

$$= \frac{4\pi - 3\sqrt{3}}{6}$$

32. Find the general solution of the differential equation $\frac{dy}{dx} + \frac{1}{x} = \frac{e^y}{x}$.

Ans.

$$\frac{dy}{dx} + \frac{1}{x} = \frac{e^y}{x}$$

$$\frac{dy}{dx} = \frac{e^y - 1}{x}$$

$$\frac{1}{e^y - 1}dy = \frac{1}{x}dx$$

Integrating both side

$$\int \frac{1}{e^y - 1}dy = \int \frac{1}{x}dx$$

$$\int \frac{1}{e^y(1-e^{-y})}dy = \int \frac{1}{x}dx$$

$$\int \frac{e^{-y}}{1-e^{-y}}dy = \int \frac{1}{x}dx$$

let $1 - e^{-y} = t$

$0 - e^{-y}(-1)\,dy = dt$

$e^{-y}\,dy = dt$

$$\int \frac{1}{t}dt = \int \frac{1}{x}dx$$

$\log t = \log x + \log C$

$\log(1 - e^{-y}) = \log xC$

$1 - e^{-y} = xC$

$1 - xC = e^{-y}$

$$1 - xC = \frac{1}{e^y}$$

$$e^y = \frac{1}{1 - xC}$$

SECTION - D

Q. Nos. 33 to 36 carry 6 marks each.

*** 36. Find the image of the point (– 1, 3, 4) in the plane $x - 2y = 0$.**

❑❑

* *Out of Syllabus*