Solved Paper 2022
Mathematics (TERM I)
Class-XII

Time : 90 Minutes **Max. Marks : 40**

General Instructions :

(i) This question paper comprises of ***50*** *questions out of which* ***40*** *questions are to be attempted as per instructions. All questions carry equal marks.*

(ii) The question paper consists of three Sections – Section A, B and C.

(iii) ***Section – A*** *contains* ***20*** *questions. Attempt any* ***16*** *questions from Q. No.* ***1*** *to* ***20****.*

(iv) ***Section – B*** *also contains* ***20*** *questions. Attempt any* ***16*** *questions from Q. No.* ***21*** *to* ***40****.*

(v) ***Section – C*** *contains* ***10*** *questions including one Case Study. Attempt any* ***8*** *questions from Q. No.* ***41*** *to* ***50****.*

(vi) There is only one correct option for every Multiple Choice Question (MCQ). Marks will not be awarded for answering more than one option.

(vii) There is no negative marking.

Series: SS/J/2 **65/2/4**

SECTION - A

In this section, attempt any 16 questions out of Questions 1-20. Each question is of one mark.

1. Differential of log [log (log x^5)] w.r.t x is

(a) $\dfrac{5}{x\log(x^5)\log(\log x^5)}$

(b) $\dfrac{5}{x\log(\log x^5)}$

(c) $\dfrac{5x^4}{\log(x^5)\log(\log x^5)}$

(d) $\dfrac{5x^4}{\log x^5\log(\log x^5)}$

Sol. Option (a) is correct.

Explanation: Let $y = \log[\log(\log x^5)]$

$$\therefore \quad \frac{d}{dx} = \frac{1}{\log(\log x^5)}\frac{d}{dx}[\log(\log x^5)]$$

(By Chain Rule)

$$= \frac{1}{\log(\log x^5)}\cdot\frac{1}{\log x^5}\frac{d}{dx}\log x^5$$

$$= \frac{1}{\log(x^5)\log(\log x^5)}\frac{1}{x^5}\frac{d}{dx}(x^5)$$

$$= \frac{5}{x\log(x^5)\log(\log x^5)}$$

2. The number of all possible matrices of order 2 × 3 with each entry 1 or 2 is

(a) 16 **(b)** 6
(c) 64 **(d)** 24

Sol. Option (c) is correct.

Explanation: The order of the matrix = 2 × 3

The number of elements = 2 × 3 = 6

Each place can have either 1 or 2. So, each place can be filled in 2 ways.

Thus, the number of possible matrices = $2^6 = 64$

3. A function f : R → R is defined as $f(x) = x^3 + 1$. Then the function has

(a) no minimum value
(b) no maximum value
(c) both maximum and minimum values
(d) neither maximum value nor minimum value

Sol. Option (d) is correct.

Explanation: Given, $f(x) = x^3 + 1$

$\therefore \quad f'(x) = 3x^2$ and $f''(x) = 6x$

Put $\quad f'(x) = 0$

$\Rightarrow \quad 3x^2 = 0 \Rightarrow x = 0$

At $\quad x = 0, f''(x) = 0$

Thus, $f(x)$ has neither maximum value nor minimum value.

4. If $\sin y = x\cos(a + y)$, then $\dfrac{dx}{dy}$ is

(a) $\dfrac{\cos a}{\cos^2(a+y)}$ **(b)** $\dfrac{-\cos a}{\cos^2(a+y)}$

(c) $\dfrac{\cos a}{\sin^2 y}$ **(d)** $\dfrac{-\cos a}{\sin^2 y}$

Sol. Option (a) is correct.

Explanation: Given, $\sin y = x\cos(a + y)$

$$\Rightarrow \quad x = \frac{\sin y}{\cos(a+y)}$$

Differentiating with respect to y, we get

$$\frac{dx}{dy} = \frac{\cos(a+y)\frac{d}{dy}(\sin y)-\sin y\frac{d}{dy}\{\cos(a+y)\}}{\cos^2(a+y)}$$

$$\Rightarrow \frac{dx}{dy} = \frac{\cos(a+y)\cos y-\sin y[-\sin(a+y)]}{\cos^2(a+y)}$$

$$\Rightarrow \frac{dx}{dy} = \frac{\cos(a+y)\cos y+\sin y\sin(a+y)}{\cos^2(a+y)}$$

$$\Rightarrow \frac{dx}{dy} = \frac{\cos[(a+y)-y]}{\cos^2(a+y)}$$

$$\Rightarrow \frac{dx}{dy} = \frac{\cos a}{\cos^2(a+y)}$$

5. The points on the curve $\frac{x^2}{9}+\frac{y^2}{25} = 1$, where tangent is parallel to X-axis are

(a) $(\pm 5, 0)$ **(b)** $(0, \pm 5)$

(c) $(0, \pm 3)$ **(d)** $(\pm 3, 0)$

Sol. Option (b) is correct.

Explanation: The equation of the given curve:

$$\frac{x^2}{9}+\frac{y^2}{25} = 1 \quad ...(i)$$

On differentiating both sides w.r.t. x, we get

$$\frac{2x}{9}+\frac{2y}{25}\frac{dy}{dx} = 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{-25x}{9y}$$

Since, tangent is parallel to X-axis, then the slope of the tangent is zero.

$\therefore \quad \frac{-25}{9}\frac{x}{y} = 0$, which is possible if $x = 0$

Put $x = 0$ in eq (i), we get

$$\frac{y^2}{25} = 1 \Rightarrow y^2 = 25 \Rightarrow y = \pm 5$$

Hence, required points are $(0, \pm 5)$.

6. Three points P($2x$, $x + 3$), Q(0, x) and R($x + 3$, $x + 6$) are collinear, then x is equal to

(a) 0 **(b)** 2

(c) 3 **(d)** 1

Sol. Option (d) is correct.

Explanation: As points are collinear

$\Rightarrow$ area of triangle formed by 3 points is zero.

$$\Rightarrow \frac{1}{2}\begin{vmatrix}(x_1-x_2) & (x_2-x_3)\\(y_1-y_2) & (y_2-y_3)\end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{2}\begin{vmatrix}(2x-0) & \{0-(x+3)\}\\(x+3-x) & \{x-(x+6)\}\end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix}2x & -(x+3)\\3 & -6\end{vmatrix} = 0$$

$$\Rightarrow -12x + 3(x + 3) = 0$$

$$\Rightarrow -12x + 3x + 9 = 0$$

$$\Rightarrow -9x = -9$$

$$\Rightarrow x = 1$$

7. The principal value of $\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is

(a) $\frac{\pi}{12}$ **(b)** π

(c) $\frac{\pi}{3}$ **(d)** $\frac{\pi}{6}$

Sol. Option (a) is correct.

Explanation: $\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(\frac{-1}{\sqrt{2}}\right)$

$$= \cos^{-1}\left(\cos\frac{\pi}{3}\right)-\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$= \frac{\pi}{3}-\sin^{-1}\left(\sin\frac{\pi}{4}\right)$$

$$= \frac{\pi}{3}-\frac{\pi}{4}=\frac{\pi}{12}$$

8. If $(x^2 + y^2)^2 = xy$, then $\frac{dy}{dx}$ is

(a) $\frac{y+4x(x^2+y^2)}{4y(x^2+y^2)-x}$ **(b)** $\frac{y-4x(x^2+y^2)}{x+4(x^2+y^2)}$

(c) $\frac{y-4x(x^2+y^2)}{4y(x^2+y^2)-x}$ **(d)** $\frac{4y(x^2+y^2)-x}{y-4x(x^2+y^2)}$

Sol. Option (c) is correct.

Explanation: Given, $(x^2 + y^2)^2 = xy$

$\Rightarrow \quad x^4 + 2x^2y^2 + y^4 - xy = 0$

Differentiating w.r.t. x, we get

$$4x^3+2\left[2xy^2+x^2.2y\frac{dy}{dx}\right]+4y^3\frac{dy}{dx}-\left[y+x\frac{dy}{dx}\right] = 0$$

$$\frac{dy}{dx}\left[4x^2y+4y^3-x\right]+\left[4x^3+4xy^2-y\right] = 0$$

$$\frac{dy}{dx} = \frac{-[4x^3+4xy^2-y]}{[4x^2y+4y^3-x]}$$

or $$\frac{dy}{dx} = \frac{y-4x(x^2+y^2)}{4y(x^2+y^2)-x}$$

9. If a matrix A is both symmetric and skew symmetric, then A is necessarily a

(a) Diagonal matrix

(b) Zero square matrix

(c) Square matrix

(d) Identity matrix

Sol. Option (b) is correct.

Explanation: If matrix A is symmetric

$$A^T = A$$

If matrix A is skew-symmetric

$$A^T = -A$$

Also, diagonal elements are zero.

Since, it is given that matrix A is both symmetric and skew-symmetric.

$\therefore \qquad A = A^T = -A$

Which is only possible if A is zero matrix.

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = A^T = -A$$

Thus, if a matrix A is both symmetric and skew symmetric, then A is necessarily a zero matrix.

10. Let set X = {1, 2, 3} and a relation R is defined in X as:

R = {(1, 3), (2, 2), (3, 2)}, then minimum ordered pairs which should be added in relation R to make it reflexive and symmetric are

(a) {(1, 1), (2, 3), (1, 2)}
(b) {(3, 3), (3, 1), (1, 2)}
(c) {(1, 1), (3, 3), (3, 1). (2, 3)}
(d) {(1, 1), (3, 3), (3, 1), (1, 2)}

Sol. Option (c) is correct.

Explanation:

(i) R is reflexive if it contains {(1, 1), (2, 2) and (3, 3)}.
Since, (2, 2) ∈ R. So, we need to add (1, 1) and (3, 3) to make R reflexive.

(ii) R is symmetric if it contains {(2, 2), (1, 3), (3, 1), (3, 2), (2, 3)}.
Since, {(2, 2), (1, 3), (3, 2)} ∈ R. So, we need to add (3, 1) and (2, 3).

Thus, minimum ordered pairs which should be added in relation R to make it reflexive and symmetric are {(1, 1), (3, 3), (3, 1), (2, 3)}.

11. A Linear Programming Problem is as follows:

Minimise $z = 2x + y$

subject to the constraints $x \geq 3, x \leq 9, y \geq 0$

$x - y \geq 0, x + y \leq 14$

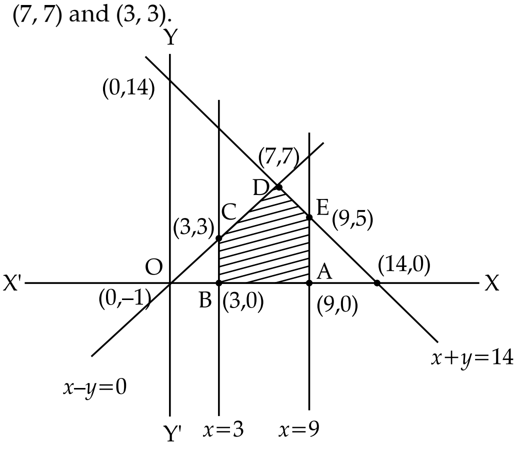
The feasible region has

(a) 5 corner points including (0, 0) and (9, 5)
(b) 5 corner points including (7, 7) and (3, 3)
(c) 5 corner points including (14, 0) and (9, 0)
(d) 5 corner points including (3, 6) and (9, 5)

Sol. Option (b) is correct.

Explanation: On plotting the constraints $x = 3$, $x = 9$, $x = y$ and $x + y = 14$, we get the following graph. From the graph given below it clear that feasible region is ABCDEA, including corner points A(9, 0), B(3, 0), C(3, 3), D(7, 7) and E(9, 5).

Thus feasible region has 5 corner points including (7, 7) and (3, 3).



12. The function $f(x) = \begin{cases} \dfrac{e^{3x} - e^{-5x}}{x}, & \text{if } x \neq 0 \\ k & \text{if } x = 0 \end{cases}$

is continuous at $x = 0$ for the value of k, as

(a) 3 **(b)** 5
(c) 2 **(d)** 8

Sol. Option (d) is correct.

Explanation: Since, $f(x)$ is continuous at $x = 0$, then

LHL = RHL = $f(0)$ or LHL = RHL = k

Now, $\text{LHL} = \lim_{h \to 0} \dfrac{e^{3(0-h)} - e^{-5(0-h)}}{0-h}$

$= \lim_{h \to 0} \dfrac{e^{-3h} - e^{5h}}{-h}$

$= \lim_{h \to 0} \left(\dfrac{e^{-3h} - 1}{-h}\right) + \lim_{h \to 0} \left(\dfrac{e^{5h} - 1}{h}\right)$

$= 3 \lim_{h \to 0} \left(\dfrac{e^{-3h} - 1}{-3h}\right) + 5 \lim_{h \to 0} \left(\dfrac{e^{5h} - 1}{5h}\right)$

$= 3 \times 1 + 5 \times 1 = 8$

Thus, $k = 8$.

13. If C_{ij} denotes the cofactor of element P_{ij} of the matrix $P = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & 2 & 4 \end{bmatrix}$, **then the value of $C_{31}.C_{23}$ is**

(a) 5 **(b)** 24
(c) –24 **(d)** –5

Sol. Option (a) is correct.

Explanation:

Here, $C_{31} = (-1)^{3+1} \begin{vmatrix} -1 & 2 \\ 2 & -3 \end{vmatrix} = 3 - 4 = -1$

and $C_{23} = (-1)^{2+3} \begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix} = -(2 + 3) = -5$

Thus, $C_{31}.C_{23} = (-1)(-5) = 5$

14. The function $y = x^2 e^{-x}$ is decreasing in the interval

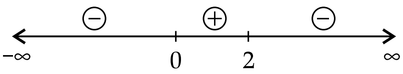
(a) (0, 2) **(b)** (2, ∞)
(c) (–∞, 0) **(d)** (–∞, 0) ∪ (2, ∞)

14. Option (d) is correct.

Explanation: We have,

$$f(x) = y = x^2 e^{-x}$$

$\therefore \dfrac{dy}{dx} = 2x\, e^{-x} + x^2(-1)e^{-x} = xe^{-x}(2 - x)$



Now, put $\dfrac{dy}{dx} = 0$

$\Rightarrow x = 0$ and $x = 2$

The points $x = 0$ and $x = 2$ divide the real line into three disjoint intervals i.e., (–∞, 0), (0, 2) and (2, ∞)

In intervals, $(-\infty, 0)$ and $(2, \infty)$, $f'(x) < 0$ as e^{-x} is always positive.

$\therefore f(x)$ or y is decreasing in $(-\infty, 0)$ and $(2, \infty)$.

15. If $R = \{(x, y); x, y \in Z, x^2 + y^2 \le 4\}$ is a relation in set Z, then domain of R is

(a) $\{0, 1, 2\}$
(b) $\{-2, -1, 0, 1, 2\}$
(c) $\{0, -1, -2\}$
(d) $\{-1, 0, 1\}$

Sol. Option (b) is correct.

Explanation: Given,

$$R = \{(x, y) : x, y \in Z, x^2 + y^2 \le 4\}$$

Since, $y = f(x)$

$$y = \sqrt{4 - x^2}$$

For, $x = 0, \pm 1, \pm 2, \Rightarrow y = \pm 2, \pm\sqrt{3}, 0$

Hence, the required domain will be 0 ±1, ±2

16. The system of linear equations

$5x + ky = 5,$

$3x + 3y = 5;$

will be consistent if

(a) $k \ne -3$ **(b)** $k = -5$
(c) $k = 5$ **(d)** $k \ne 5$

Sol. Option (d) is correct.

Explanation:

We have, $5x + ky - 5 = 0$

and $3x + 3y - 5 = 0$

For consistent system

$$\frac{5}{3} \ne \frac{k}{3}$$

$\Rightarrow$ $k \ne 5$

***17. The equation of the tangent to the curve $y(1 + x^2) = 2 - x$, where it crosses the X-axis is**

(a) $x - 5y = 2$ **(b)** $5x - y = 2$
(c) $x + 5y = 2$ **(d)** $5x + y = 2$

18. $\begin{bmatrix} 3c+6 & a-d \\ a+d & 2-3b \end{bmatrix} = \begin{bmatrix} 12 & 2 \\ -8 & -4 \end{bmatrix}$ are equal, then value of $ab - cd$ is

(a) 4 **(b)** 16
(c) –4 **(d)** –16

Sol. Option (a) is correct.

Explanation: Given, $\begin{bmatrix} 3c+6 & a-d \\ a+d & 2-3b \end{bmatrix} = \begin{bmatrix} 12 & 2 \\ -8 & -4 \end{bmatrix}$

$\therefore$ $3c + 6 = 12$ ...(i)

$a - d = 2$ ...(ii)

$a + d = -8$ ...(iii)

$2 - 3b = -4$ ...(iv)

From eq. (i), we get $c = 2$

On solving eqs. (ii) and (iii), we get $a = -3$ and $d = -5$ from eq. (iv), we get $b = 2$

Now, $ab - cd = (-3)2 - 2(-5)$

$\Rightarrow$ $ab - cd = -6 + 10 = 4$

19. The principal value of $\tan^{-1}\left(\tan\frac{9\pi}{8}\right)$ is

(a) $\frac{\pi}{8}$ **(b)** $\frac{3\pi}{8}$

(c) $-\frac{\pi}{8}$ **(d)** $-\frac{3\pi}{8}$

Sol. Option (a) is correct.

Explanation: $\tan^{-1}\left(\tan\frac{9\pi}{8}\right) = \tan^{-1}\left(\tan\left(\pi + \frac{\pi}{8}\right)\right)$

$= \tan^{-1}\left(\tan\frac{\pi}{8}\right) = \frac{\pi}{8}$ $\left[\because \frac{\pi}{8} \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)\right]$

20. For two matrices $P = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $Q^T = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ P – Q is

(a) $\begin{bmatrix} 2 & 3 \\ -3 & 0 \\ 0 & -3 \end{bmatrix}$ **(b)** $\begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & 3 \\ -0 & -3 \\ -1 & -2 \end{bmatrix}$ **(d)** $\begin{bmatrix} 2 & 3 \\ 0 & -3 \\ 0 & -3 \end{bmatrix}$

Sol. Option (b) is correct.

Explanation:

Here, $Q = (Q^T)^T = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$

Now, $P - Q = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

SECTION - B

In this Section attempt any 16 questions out of the Questions 21-40. Each question is of one mark.

21. The function $f(x) = 2x^3 - 15x^2 + 36x + 6$ is increasing in the interval

(a) $(-\infty, 2) \cup (3, \infty)$ **(b)** $(-\infty, 2)$
(c) $(-\infty, 2] \cup [3, \infty)$ **(d)** $[3, \infty)$

Sol. Option (c) is correct.

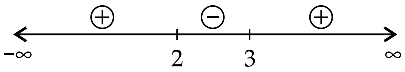
Explanation: Given, $f(x) = 2x^3 - 15x^2 + 36x + 6$

$\therefore$ $f'(x) = 6x^2 - 30x + 36$

It $f'(x) \ge 0$, then $f(x)$ is increasing.

So, $6x^2 - 30x + 36 \ge 0$

* *Out of Syllabus*



or, $x^2 - 5x + 6 \geq 0$

or, $(x-3)(x-2) \geq 0$

$\therefore$ $x \in (-\infty, 2] \cup [3, \infty)$

22. If $x = 2\cos\theta - \cos 2\theta$ and $y = 2\sin\theta - \sin 2\theta$, then $\dfrac{dy}{dx}$ is

(a) $\dfrac{\cos\theta + \cos 2\theta}{\sin\theta - \sin 2\theta}$ **(b)** $\dfrac{\cos\theta - \cos 2\theta}{\sin 2\theta - \sin\theta}$

(c) $\dfrac{\cos\theta - \cos 2\theta}{\sin\theta - \sin 2\theta}$ **(d)** $\dfrac{\cos 2\theta - \cos\theta}{\sin 2\theta + \sin\theta}$

Sol. Option (b) is correct.

Explanation: Given, $x = 2\cos\theta - \cos 2\theta$

and $y = 2\sin\theta - \sin 2\theta$

Therefore, $\dfrac{dx}{d\theta} = -2\sin\theta + 2\sin 2\theta$

and $\dfrac{dy}{d\theta} = 2\cos\theta - 2\cos 2\theta$

$\therefore$ $\dfrac{dy}{dx} = \dfrac{2\cos\theta - 2\cos 2\theta}{-2\sin\theta + 2\sin 2\theta}$

or $\dfrac{dy}{dx} = \dfrac{\cos\theta - \cos 2\theta}{\sin 2\theta - \sin\theta}$

23. What is the domain of the function $\cos^{-1}(2x-3)$?

(a) $[-1, 1]$ **(b)** $(1, 2)$

(c) $(-1, 1)$ **(d)** $[1, 2]$

Sol. Option (d) is correct.

Explanation: Let, $f(x) = \cos^{-1}(2x-3)$

$\because$ $-1 \leq 2x - 3 \leq 1$

$\Rightarrow$ $2 \leq 2x \leq 4$

$\Rightarrow$ $1 \leq x \leq 2$

$\therefore$ $x \in [1, 2]$ or domain of x is $[1, 2]$.

24. A matrix $A = [a_{ij}]_{3\times 3}$ is defined by

$$a_{ij} = \begin{cases} 2i+3j &, \ i<j \\ 5 &, \ i=j \\ 3i-2j &, \ i>j \end{cases}$$

The number of elements in A which are more than 5, is:

(a) 3 **(b)** 4

(c) 5 **(d)** 6

Sol. Option (b) is correct.

Explanation: Here, $A = \begin{bmatrix} 5 & 8 & 11 \\ 4 & 5 & 13 \\ 7 & 5 & 5 \end{bmatrix}$

Thus, number of elements more than 5, is 4.

25. If a function f defined by

$$f(x) = \begin{cases} \dfrac{k\cos x}{\pi - 2x}, & \text{if } x \neq \dfrac{\pi}{2} \\ 3 & \text{if } x = \dfrac{\pi}{2} \end{cases}$$

is continuous at $x = \dfrac{\pi}{2}$, then the value of k is

(a) 2 **(b)** 3

(c) 6 **(d)** –6

Sol. Option (c) is correct.

Explanation: Since, $f(x)$ is continuous at $x = \dfrac{\pi}{2}$

Therefore, $\lim\limits_{x\to\frac{\pi}{2}} f(x) = f\left(\dfrac{\pi}{2}\right)$

$\Rightarrow$ $\lim\limits_{x\to\frac{\pi}{2}} \dfrac{k\cos x}{\pi - 2x} = 3$

$\Rightarrow$ $k \lim\limits_{x\to\frac{\pi}{2}} \dfrac{\sin\left(\frac{\pi}{2} - x\right)}{2\left(\frac{\pi}{2} - x\right)} = 3$

$\Rightarrow$ $\dfrac{k}{2} \lim\limits_{x\to\frac{\pi}{2}} \dfrac{\sin\left(\frac{\pi}{2} - x\right)}{\left(\frac{\pi}{2} - x\right)} = 3$

$\Rightarrow$ $\dfrac{k}{2} \times 1 = 3 \Rightarrow k = 6$

26. For the matrix $X = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, $(X^2 - X)$ is

(a) 2I **(b)** 3I

(c) I **(d)** 5I

Sol. Option (a) is correct.

Explanation:

Here $X^2 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$\Rightarrow$ $X^2 = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

$\Rightarrow$ $X^2 - X = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$= 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= 2I$

27. Let $X = \{x^2 : x \in N\}$ and the function $f : N \to X$ is defined by $f(x) = x^2, x \in N$. Then this function is

(a) injective only **(b)** not bijective

(c) surjective only **(d)** bijective

Sol. Option (a) is correct.

Explanation: Let $x_1, x_2 \in N$

$f(x_1) = f(x_2)$

$\Rightarrow$ $x_1^2 = x_2^2$

$\Rightarrow$ $x_1^2 - x_2^2 = 0$

$\Rightarrow \quad (x_1 + x_2)(x_1 - x_2) = 0$

$\Rightarrow \quad x_1 = x_2$

$\{x_1 + x_1 \neq 0 \text{ as } x_1, x_2 \in N\}$

Hence, $f(x)$ is injective.

Also, the elements like 2 and 3 have no pre-image in N. Thus, $f(x)$ is not surjective.

28. The corner points of the feasible region for a Linear Programming problem are P(0, 5), Q(1, 5), R(4, 2) and S(12, 0). The minimum value of the objective function Z= 2*x* + 5*y* is at the point

(a) P (b) Q

(c) R (d) S

Sol. Option (c) is correct.

Explanation:

Corner Points	Value of $Z = 2x + 5y$
P(0, 5)	Z = 2(0) + 5(5) = 25
Q(1, 5)	Z = 2(1) + 5(5) = 27
R(4, 2)	Z = 2(4) + 5(2) = 18 Minimum
S(12, 0)	Z = 2(12) + 5(0) = 24

Thus, minimum value of Z occurs at R(4, 2).

***29. The equation of the normal to the curve $ay^2 = x^3$ at the point (am^2, am^3) is**

(a) $2y - 3mx + am^3 = 0$

(b) $2x + 3my\, 3am^4 - am^2 = 0$

(c) $2x + 3my + 3am^4 - 2am = 0$

(d) $2x + 3my - 3am^4 - 2am^2 = 0$

30. If A is a square matrix of order 3 and |A| = –5, then |adj A| is

(a) 125 (b) – 25

(c) 25 (d) ±25

Sol. Option (c) is correct.

Explanation: We know that,

$$|\text{adj } A| = |A|^{n-1}$$

where n is the order of the matrix

$$\therefore \quad |\text{adj } A| = (5)^{3-1} = 5^2 = 25$$

31. The simplest form of $\tan^{-1}\left[\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]$ is

(a) $\dfrac{\pi}{4} - \dfrac{x}{2}$ (b) $\dfrac{\pi}{4} + \dfrac{x}{2}$

(c) $\dfrac{\pi}{4} - \dfrac{1}{2}\cos^{-1}x$ (d) $\dfrac{\pi}{4} + \dfrac{1}{2}\cos^{-1}x$

Sol. Option (c) is correct.

Explanation: We have,

$$\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$$

Put $x = \cos 2\theta$, so that $\theta = \dfrac{1}{2}\cos^{-1}x$

$$\tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\right)$$

$$= \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$$

$$= \tan^{-1}(1) - \tan^{-1}(\tan\theta)$$

$$\left[\because \tan^{-1}\left(\frac{x-y}{1+xy}\right) = \tan^{-1}x - \tan^{-1}y\right]$$

$$= \tan^{-1}\left(\tan\frac{\pi}{4}\right) - \theta$$

$$= \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$$

32. If for the matrix A = $\begin{bmatrix} \alpha & -2 \\ -2 & \alpha \end{bmatrix}$, $|A^3| = 125$, then the value of α is

(a) ±3 (b) –3

(c) ±1 (d) 1

Sol. Option (a) is correct.

Explanation: Given, $A = \begin{bmatrix} \alpha & -2 \\ -2 & \alpha \end{bmatrix}$

$\Rightarrow \quad |A| = \alpha^2 - 4 \quad$ ...(i)

Also, given $\quad |A^3| = 125$

$\Rightarrow \quad |A|^3 = 125$

$\Rightarrow \quad |A| = 5$

$\Rightarrow \quad \alpha^2 - 4 = 5 \quad$ [from eq. (i)]

$\Rightarrow \quad \alpha^2 = 9$

$\Rightarrow \quad \alpha = \pm 3$

33. If $y = \sin(m\sin^{-1}x)$, then which one of the following equations is true ?

(a) $(1-x^2)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + m^2y = 0$

(b) $(1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + m^2y = 0$

(c) $(1+x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - m^2y = 0$

(d) $(1+x^2)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} - m^2x = 0$

Sol. Option (b) is correct.

Explanation: Given, $y = \sin(m(\sin^{-1}x))$ ...(i)

* *Out of Syllabus*

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \cos(m\sin^{-1}x)\times\frac{m}{\sqrt{1-x^2}}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{m\cos(m\sin^{-1}x)}{\sqrt{1-x^2}} \quad ...(ii)$$

$$\Rightarrow \quad y' = \frac{m\cos(m\sin^{-1}x)}{\sqrt{1-x^2}} \quad ...(ii)$$

$$\Rightarrow \quad \left(\sqrt{1-x^2}\right)y' = m\cos(m\sin^{-1}x)$$

Differentiating again w.r.t. 'x', we get

$$y''\left(\sqrt{1-x^2}\right)+y'\frac{(-2x)}{2\sqrt{1-x^2}}$$

$$= -m^2\sin(m\sin^{-1}x)\frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \quad y''(1-x^2)-xy' = -m^2y$$

$$\Rightarrow y''(1-x^2)-xy'+m^2y = 0$$

or, $$(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+m^2y=0$$

34. The principal value of $[\tan^{-1}\sqrt{3}-\cot^{-1}(-\sqrt{3})]$ is

(a) π (b) $-\frac{\pi}{2}$

(c) 0 (d) $2\sqrt{3}$

Sol. Option (b) is correct.

Explanation: We have,

$$\tan^{-1}\sqrt{3}-\cot^{-1}\left(-\sqrt{3}\right)$$

$$= \frac{\pi}{3}-\left(\pi-\frac{\pi}{6}\right)$$

$$= \tan^{-1}\left(\tan\frac{\pi}{3}\right)-\pi+\cot^{-1}\cot\frac{\pi}{6}$$

$$= \frac{\pi}{3}-\frac{5\pi}{6}$$

$$= -\frac{\pi}{2}$$

35. The maximum value of $\left(\frac{1}{x}\right)^x$ is

(a) $e^{1/e}$ (b) e

(c) $\left(\frac{1}{e}\right)^{1/e}$ (d) e^e

Sol. Option (a) is correct.

Explanation: Let $y = \left(\frac{1}{x}\right)^x$

Then, $$\log y = x\log\left(\frac{1}{x}\right) = -x\log x \quad ...(i)$$

Differentiating both sides w.r.t. x

$$\therefore \quad \frac{1}{y}\frac{dy}{dx} = -\left[x.\frac{1}{x}+\log x\right]$$

$$= -(1+\log x) \quad ...(ii)$$

On differentiating again eq. (ii), we get

$$\frac{1}{y}\frac{d^2y}{dx^2}-\frac{1}{y^2}\left(\frac{dy}{dx}\right)^2 = \frac{-1}{x} \quad ...(iii)$$

From eq. (ii), we get

$$\frac{dy}{dx} = -y(1+\log x)$$

$$= -\left(\frac{1}{x}\right)^x(1+\log x)$$

For maximum or minimum values of y, put $\frac{dy}{dx} = 0$

Therefore, $\left(\frac{1}{x}\right)^x(1+\log x) = 0$

However, $\left(\frac{1}{x}\right)^x \neq 0$ for any value of x. Therefore

$$1+\log x = 0$$

$$\Rightarrow \quad \log x = -1 \Rightarrow x = e^{-1} \Rightarrow x = \frac{1}{e}$$

When $x = \frac{1}{e}$, from eq. (iii)

$$\frac{1}{y}\frac{d^2y}{dx^2}-0 = -e$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -e(e)^{1/e} < 0$$

Hence, y is maximum when $x=\frac{1}{e}$ and maximum value of $y = e^{1/e}$.

36. Let matrix X = $[x_{ij}]$ is given by X = $\begin{bmatrix}1 & -1 & 2\\ 3 & 4 & -5\\ 2 & -1 & 3\end{bmatrix}$.

Then the matrix Y = $[m_{ij}]$, where m_{ij} = Minor of x_{ij}, is

(a) $\begin{bmatrix}7 & -5 & -3\\ 19 & 1 & -11\\ -11 & 1 & 7\end{bmatrix}$ (b) $\begin{bmatrix}7 & -19 & 11\\ 5 & -1 & -1\\ 3 & 11 & 7\end{bmatrix}$

(c) $\begin{bmatrix}7 & 19 & -11\\ -3 & 11 & 7\\ -2 & -1 & -1\end{bmatrix}$ (d) $\begin{bmatrix}7 & 19 & -11\\ -1 & -1 & 1\\ -3 & -11 & 7\end{bmatrix}$

Sol. Option (d) is correct.

Explanation: $$m_{11} = \begin{vmatrix}4 & -5\\ -1 & 3\end{vmatrix} = 12-5 = 7$$

$$m_{12} = \begin{vmatrix}3 & -5\\ 2 & 3\end{vmatrix} = 9+10 = 19$$

$$m_{13} = \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} = -3-8 = -11$$

$$m_{21} = \begin{vmatrix} -1 & 2 \\ -1 & 3 \end{vmatrix} = -3+2 = -1$$

$$m_{22} = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 3-4 = -1$$

$$m_{23} = \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = -1+2 = 1$$

$$m_{31} = \begin{vmatrix} -1 & 2 \\ 4 & -5 \end{vmatrix} = 5-8 = -3$$

$$m_{32} = \begin{vmatrix} 1 & 2 \\ 3 & -5 \end{vmatrix} = -5-6 = -11$$

$$m_{33} = \begin{vmatrix} 1 & -1 \\ 3 & 4 \end{vmatrix} = 4+3 = 7$$

$$\therefore \quad Y = \begin{bmatrix} 7 & 19 & -11 \\ -1 & -1 & 1 \\ -3 & -11 & 7 \end{bmatrix}$$

37. A function $f : R \to R$ defined by $f(x) = 2 + x^2$ is

(a) not one-one

(b) one-one

(c) not onto

(d) neither one-one nor onto

Sol. Option (d) is correct.

Explanation: Given, $f(x) = 2 + x^2$

For one-one, $f(x_1) = f(x_2)$

$\Rightarrow \quad 2 + x_1^2 = 2 + x_2^2$

$\Rightarrow \quad x_1^2 = x_2^2$

$\Rightarrow \quad x_1 = \pm x_2$

$\Rightarrow \quad x_1 = x_2$

or $\quad x_1 = -x_2$

Thus, $f(x)$ is not one-one.

For onto

Let $\quad f(x) = y$ such that $y \in R$

$\therefore \quad x^2 = y - 2$

$\Rightarrow \quad x = \pm\sqrt{y-2}$

Put $y = -3$, we get

$$x = \pm\sqrt{-3-2} = \pm\sqrt{-5}$$

Which is not possible as root of negative is not a real number.

Hence, x is not real.

So, $f(x)$ is not onto.

38. A Linear Programming Problem is as follows:

Maximise / Minimise objective function Z = $2x - y + 5$

Subject to the constraints

$$3x + 4y \le 60$$
$$x + 3y \le 30$$
$$x \ge 0, y \ge 0$$

If the corner points of the feasible region are A(0, 10), B(12, 6), C(20, 0) and O(0,0), then which of the following is true ?

(a) Maximum value of Z is 40

(b) Minimum value of Z is – 5

(c) Difference of maximum and minimum values of Z is 35

(d) At two corner points, value of Z are equal

Sol. Option (b) is correct.

Explanation:

Corner Points	Value of $Z = 2x - y + 5$
A(0, 10)	Z = 2(0) – 10 + 5 = –5 (Minimum)
B(12, 6)	Z = 2(12) – 6 + 5 = 23
C(20, 0)	Z = 2(20) – 0 + 5 = 45 (Maximum)
O(0, 0)	Z = 0(0) – 0 + 5 = 5

So the minimum value of Z is – 5.

39. If $x = -4$ is a root of $\begin{vmatrix} x & 2 & 3 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix} = 0$, then the sum of the other two roots is

(a) 4 **(b)** –3

(c) 2 **(d)** 5

Sol. Option (a) is correct.

Explanation: Given, $\begin{vmatrix} x & 2 & 3 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix} = 0$

$\Rightarrow \quad x(x^2 - 2) - 2(x - 3) + 3(2 - 3x) = 0$

$\Rightarrow \quad x^3 - 13x + 12 = 0$

Since $(x + 4)$ is one root of above cubic equation.

Sum roots $= 0$

$\therefore \quad$ Sum of two roots + (– 4) = 0

Sum of two roots = 4

40. The absolute maximum value of the function $f(x) = 4x - \frac{1}{2}x^2$ in the interval $\left[-2, \frac{9}{2}\right]$ is

(a) 8 **(b)** 9

(c) 6 **(d)** 10

Sol. Option (a) is correct.

Explanation:

Given, $\quad f(x) = 4x - \frac{1}{2}x^2$

$\therefore \quad f'(x) = 4 - \frac{1}{2}(2x) = 4 - x$

put $\quad f'(x) = 0$

$\Rightarrow \quad 4 - x = 0$

$\Rightarrow \quad x = 4$

Then, we evaluate the f at critical point $x = 4$ and at the end points of the interval $\left[-2, \frac{9}{2}\right]$.

$$f(4) = 16 - \frac{1}{2}(16) = 16 - 8 = 8$$

$$f(-2) = -8 - \frac{1}{2}(4)$$
$$= -8 - 2 = -10$$

$$f\left(\frac{9}{2}\right) = 4\left(\frac{9}{2}\right) - \frac{1}{2}\left(\frac{9}{2}\right)^2$$
$$= 18 - \frac{81}{8} = 7.875$$

Thus, the absolute maximum value of f on $\left[-2, \frac{9}{2}\right]$ is 8 occurring at $x = 4$.

SECTION - C

Attempt any 8 questions out of the Questions 41-50. Each question is of one mark.

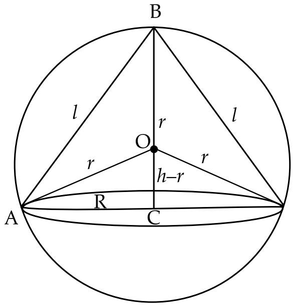
41. In a sphere of radius r, a right circular cone of height h having maximum curved surface area is inscribed. The expression for the square of curved surface of cone is

(a) $2\pi^2 rh(2rh + h^2)$ **(b)** $\pi^2 hr(2rh + h^2)$

(c) $2\pi^2 r(2rh^2 - h^3)$ **(d)** $2\pi^2 r^2(2rh - h^2)$

Sol. Option (c) is correct.

Explanation:



Here, CSA of cone $= \pi Rl$

Radius of sphere $= r$

height of cone $= h$

In ΔAOC,

$$AO^2 = AC^2 + OC^2$$
$$\Rightarrow \quad r^2 = R^2 + (h - r)^2$$
$$\Rightarrow \quad R^2 = 2hr - h^2$$
$$\therefore \quad \text{Radius of cone, } R = \sqrt{2hr - h^2} \quad ...(i)$$

In ΔABC,

$$AB^2 = AC^2 + BC^2$$
$$\Rightarrow \quad l^2 = R^2 + h^2$$
$$\Rightarrow \quad l^2 = 2hr - h^2 + h^2$$
$$\therefore \quad \text{slant height} = \sqrt{2hr} \quad ...(ii)$$

$$\text{CSA of cone} = \pi Rl$$
$$= \pi\sqrt{2hr - h^2}\sqrt{2hr}$$
$$(\text{CSA of cone})^2 = \pi^2(2hr - h^2)(2hr)$$
$$= 2\pi^2 hr(2hr - h^2)$$
$$= 2\pi^2 r(2rh^2 - h^3)$$

42. The corner points of the feasible region determined by a set of constraints (linear inequalities) are P(0, 5), Q(3, 5), R(5, 0) and S(4, 1) and the objective function is Z = ax + 2by where a, b > 0. The condition on a and b such that the maximum Z occurs at Q and S is

(a) $a - 5b = 0$ **(b)** $a - 3b = 0$

(c) $a - 2b = 0$ **(d)** $a - 8b = 0$

Sol. Option (d) is correct.

Explanation: Given, Max. $Z = ax + 2by$

Max. value of Z on Q(3, 5) = Max. value of Z on $S(4, 1)$

$$\Rightarrow \quad 3a + 10b = 4a + 2b$$
$$\Rightarrow \quad a - 8b = 0$$

43. If curves $y^2 = 4x$ and $xy = c$ cut at right angles, then the value of c is

(a) $4\sqrt{2}$ **(b)** 8

(c) $2\sqrt{2}$ **(d)** $-4\sqrt{2}$

Sol. Option (a) is correct.

Explanation: Given curves, $y^2 = 4x$ and $xy = c$ cuts orthogonally.

Let they intersect at (x_1, y_1).

Now, $y^2 = 4x$

$$\therefore \quad 2y\frac{dy}{dx} = 4$$
$$\Rightarrow \quad \frac{dy}{dx} = \frac{2}{y}$$
$$\Rightarrow \quad \left.\frac{dy}{dx}\right|_{(x_1, y_1)} = \frac{2}{y_1} \quad ...(i)$$

and $xy = c$

$$\therefore \quad x\frac{dy}{dx} + y = 0$$
$$\therefore \quad \frac{dy}{dx} = \frac{-y}{x}$$
$$\Rightarrow \quad \left.\frac{dy}{dx}\right|_{(x_1, y_1)} = -\frac{y_1}{x_1} \quad ...(ii)$$

From eqs. (i) and (ii)

$$\frac{2}{y_1} \times \left(\frac{-y_1}{x_1}\right) = -1 \quad [\because m_1 m_2 = -1]$$
$$\Rightarrow \quad x_1 = 2$$

Put $x_1 = 2$ in $y_1^2 = 4x_1$, we get

$$y_1^2 = 4(2) = 8$$
$$\Rightarrow \quad y_1 = 2\sqrt{2}$$

Now, put value of x_1 and y_1 in $x_1 y_1 = c$, we get

$$c = 2\left(2\sqrt{2}\right) = 4\sqrt{2}$$

44. The inverse of the matrix X = $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is

(a) $24\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix}$ **(b)** $\frac{1}{24}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $\frac{1}{24}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ **(d)** $\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix}$

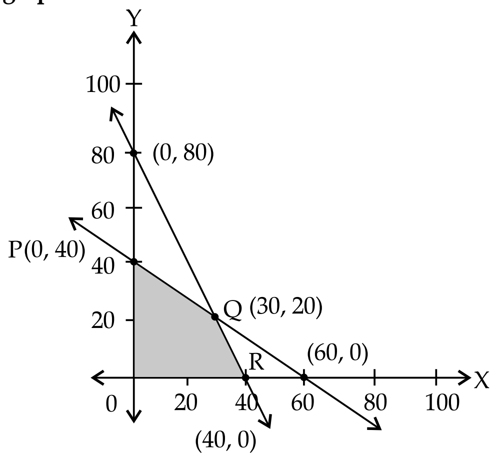
Sol. Option (d) is correct.

Explanation: The inverse of a diagonal matrix is obtained by replacing each element in the diagonal with its reciprocal.

Since, $$X = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

Therefore $$X^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$$

45. For an L.P.P. the objective function is Z = 4*x* + 3*y*, and the feasible region determined by a set of constraints (linear inequations) is shown in the graph.



Which one of the following statements is true ?

(a) Maximum value of Z is at R.

(b) Maximum value of Z is at Q.

(c) Value of Z at R is less than the value at P.

(d) Value of Z at Q is less than the value at R.

Sol. Option (b) is correct.

Explanation:

Corner Points of Feasible Region	Value of $z = (z = 4x + 3y)$
O(0, 0)	Z = 4(0) + 3(0) = 0
P(0, 40)	Z = 4(0) + 3(40) = 120
Q(30, 20)	Z = 4(30) + 3(20) = 180 (Maximum)
R(40, 0)	Z = 4(40) + 3(0) = 160

Thus, Maximum value of z is at Q, which is 180.

Case Study

In a residential society comprising of 100 houses, there were 60 children between the ages of 10-15 years. They were inspired by their teachers to start composting to ensure that biodegradable waste is recycled. For this purpose, instead of each child doing it for only his/her house, children convinced the Residents welfare association to do it as a society initiative. For this they identified a square area in the local park. Local authorities charged amount of ₹ 50 per square metre for space so that there is no misuse of the space and Resident welfare association takes it seriously. Association hired a labourer for digging out 250 m^3 and he charged ₹ 400 X $(\text{depth})^2$. Association will like to have minimum cost.

46. Let side of square plot is *x* m and its depth is *h* metres, then cost *C* for the pit is

(a) $\frac{50}{h} + 400h^2$ **(b)** $\frac{12500}{h} + 400h^2$

(c) $\frac{250}{h} + h^2$ **(d)** $\frac{250}{h} + 400h^2$

Sol. Option (b) is correct.

Explanation: $$C = \frac{250 \times 50}{h} + 400 \times h^2$$

$$\Rightarrow \quad C = \frac{12500}{h} + 400h^2$$

47. Value of *h* (in m) for which $\frac{dC}{dh} = 0$ is

(a) 1.5 **(b)** 2

(c) 2.5 **(d)** 3

Sol. Option (c) is correct.

Explanation: Since,

$$C = \frac{12500}{h} + 400h^2$$

$$\therefore \quad \frac{dC}{dh} = \frac{-12500}{h^2} + 800h$$

Put $$\frac{dC}{dh} = 0$$

$$\therefore \quad \frac{-12500}{h^2} + 800h = 0$$

$$\Rightarrow \quad 800h^3 = 12500$$

$$\Rightarrow \quad h^3 = \frac{125}{8}$$

$$\Rightarrow \quad h = \frac{5}{2} = 2.5 \text{ m}$$

48. $\frac{d^2C}{dh^2}$ is given by

(a) $\frac{25000}{h^3} + 800$ **(b)** $\frac{500}{h^3} + 800$

(c) $\frac{100}{h^3} + 800$ **(d)** $\frac{500}{h^3} + 2$

Sol. Option (a) is correct.

Explanation: Since,

$$\therefore \quad \frac{dC}{dh} = \frac{-12500}{h^2} + 800h$$

$$\therefore \quad \frac{d^2C}{dh^2} = \frac{-(-2) \times 12500}{h^3} + 800$$

$$\Rightarrow \quad \frac{d^2C}{dh^2} = \frac{25000}{h^3} + 800$$

49. Value of x (in m) for minimum cost is

(a) 5 **(b)** $10\sqrt{\frac{5}{3}}$

(c) $5\sqrt{5}$ **(d)** 10

Sol. Option (d) is correct.

Explanation: For minimum cost, put $\frac{dC}{dh} = 0$, we get

$$h = 2.5 \text{ m}$$

At $h = 2.5, \frac{d^2C}{dh^2} > 0$

(Hence, minimum)

Value of x at minimum cost

$$x = \sqrt{\frac{250}{h}}$$

$$= \sqrt{\frac{250}{2.5}} = 10 \text{ m}$$

50. Total minimum cost of digging the pit (in ₹) is

(a) 4,100 **(b)** 7,500

(c) 7,850 **(d)** 3,220

Sol. Option (b) is correct.

Explanation: Total minimum cost,

$$C = \frac{12500}{h} + 400h^2 \quad \text{(At 2.5)}$$

$$\Rightarrow \quad C = \frac{12500}{2.5} + 400(2.5)^2$$

$$\Rightarrow \quad C = 5000 + 2500$$

$$\Rightarrow \quad C = ₹\ 7500$$

❑❑