Solved Paper 2022
Mathematics (TERM-II)
Class-XII

Time : 2 Hours **Max. Marks : 40**

General Instructions:

(i) *This question paper contains* ***three*** *Sections -* ***A, B*** *and* ***C****.*
(ii) ***Each*** *section is compulsory.*
(iii) ***Section-A*** *has 6 short answer type-I questions of* ***2*** *marks each.*
(iv) ***Section-B*** *has 4 short answer type-II questions of* ***3*** *marks each.*
(v) ***Section-C*** *has 4 long answer type questions of* ***4*** *marks each.*
(vi) *There is an internal choice in some questions.*
(vii) *Question 14 is a case study based question with two sub parts of* ***2*** *marks each.*

Series: ABCD/5/5, Delhi Set-I **65/5/1**

SECTION - A

Question numbers 1 to 6 carry 2 marks each.

1. Find $\int \frac{dx}{\sqrt{4x-x^2}}$ **2**

Ans. Let

$$I = \int \frac{dx}{\sqrt{4x-x^2}} = \int \frac{dx}{\sqrt{-(x^2-4x)}}$$

$$= \int \frac{dx}{\sqrt{-(x^2-4x+2^2-2^2)}}$$

$$= \int \frac{dx}{\sqrt{-(x-2)^2+2^2}} = \int \frac{dx}{\sqrt{2^2-(x-2)^2}}$$

$$= \sin^{-1}\left(\frac{x-2}{2}\right)+C$$

$$\left[\because \int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right)+C\right]$$

2. Find the general solution of the following differential equation: **2**

$$\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$$

Ans. Given differential equation is

$$\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$$

$$\Rightarrow \quad \frac{dy}{dx} = e^{-y}(e^x + x^2)$$

$$\Rightarrow \quad \frac{dy}{e^{-y}} = (e^x + x^2)\,dx$$

$$\Rightarrow \quad e^y dy = e^x dx + x^2 dx$$

On integrating both sides, we get

$$e^y = e^x + \frac{x^3}{3} + c$$

3. Let X be a random variable which assumes values x_1, x_2, x_3, x_4 such that $2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4)$. **2**

Find the probability distribution of X.

Ans. Given, $2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4)$

Let $2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4) = k$

$$\therefore \quad P(X = x_1) = \frac{k}{2} \quad \text{...(i)}$$

$$P(X = x_2) = \frac{k}{3} \quad \text{...(ii)}$$

$$P(X = x_3) = k \quad \text{...(iii)}$$

$$P(X = x_4) = \frac{k}{5} \quad \text{...(iv)}$$

On adding eqs. (i) – (iv), and equating sum of all probabilities is equal to 1, we get

$$\frac{k}{2}+\frac{k}{3}+k+\frac{k}{5} = 1$$

$$\Rightarrow \frac{15k+10k+30k+6k}{30} = 1 \Rightarrow 61k = 30 \Rightarrow k = \frac{30}{61}$$

The required probability distribution is:

$P(X = x_1)$	$P(X = x_2)$	$P(X = x_3)$	$P(X = x_4)$
$\frac{30}{61\times 2} = \frac{15}{61}$	$\frac{30}{61\times 3} = \frac{10}{61}$	$\frac{30}{61}$	$\frac{30}{61\times 5} = \frac{6}{61}$

4. If $\vec{a} = \hat{i}+\hat{j}+\hat{k}$, $\vec{a}.\vec{b} = 1$ and $\vec{a}\times\vec{b} = \hat{j}-\hat{k}$, then find $|\vec{b}|$. **2**

Ans. Given,

$$\vec{a} = \hat{i}+\hat{j}+\hat{k}$$

$$\vec{a}.\vec{b} = 1$$

and $\vec{a}\times\vec{b} = \hat{j}-\hat{k}$

Let $\vec{b} = b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$

Now, $\vec{a}.\vec{b} = 1$

$\Rightarrow (\hat{i}+\hat{j}+\hat{k})(b_1\hat{i}+b_2\hat{j}+b_3\hat{k}) = 1$

$\Rightarrow b_1 + b_2 + b_3 = 1$ ...(i)

and $\vec{a}\times\vec{b} = \hat{j}-\hat{k}$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ b_1 & b_2 & b_3 \end{vmatrix} = \hat{j}-\hat{k}$$

$\Rightarrow \hat{i}(b_3-b_2)-\hat{j}(b_3-b_1)+\hat{k}(b_2-b_1) = \hat{j}-\hat{k}$

On comparing both sides, we get

$-(b_3-b_1) = 1$ and $b_2-b_1 = -1$

$\Rightarrow b_3-b_1 = -1$ and $b_2-b_1 = -1$

$\Rightarrow b_3 = -1+b_1$ and $b_2 = -1+b_1$ ...(ii)

Now from eq. (i), we get

$b_1 + (-1+b_1) + (-1+b_1) = 1$

$\Rightarrow 3b_1 = 3$

$\Rightarrow b_1 = 1$

From eq. (ii), we get

$b_2 = 0$ and $b_3 = 0$

$\therefore \vec{b} = \hat{i}$

Therefore, $|\vec{b}| = 1$

5. If a line makes an angle α, β, γ with the coordinate axes, then find the value of cos2α + cos2β + cos2γ. **2**

Ans. We have, $\cos2\alpha + \cos2\beta + \cos2\gamma$

$= 2\cos^2\alpha - 1 + 2\cos^2\beta - 1 + 2\cos^2\gamma - 1$

$= 2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) - 3$

$= 2\times1-3$ $[\because \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1]$

$= 2-3$

$= -1$

6. (a) Events A and B are such that

$$\mathbf{P(A) = \frac{1}{2}, P(B) = \frac{7}{12} \text{ and } P(\bar{A}\cup\bar{B}) = \frac{1}{4}}$$

Find whether the events A and B are independent or not.

OR

(b) A box B_1 contains 1 white ball and 3 red balls. Another box B_2 contains 2 white balls and 3 red balls. If one ball is drawn at random from each of the boxes B_1 and B_2, then find the probability that the two balls drawn are of the same colour.

Ans. (a) Given $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$

and $P(\bar{A}\cup\bar{B}) = \frac{1}{4}$

For A and B are independent

$P(A\cap B) = P(A).P(B)$ ...(i)

Now, $P(\bar{A}\cup\bar{B}) = P(\overline{A\cap B})$

$\Rightarrow P(\bar{A}\cup\bar{B}) = 1 - P(A\cap B)$

$\Rightarrow P(A\cap B) = 1 - P(\bar{A}\cup\bar{B})$

$\Rightarrow P(A\cap B) = 1-\frac{1}{4} = \frac{3}{4}$ ...(ii)

Now, $P(A).P(B) = \frac{1}{2}\times\frac{7}{12} = \frac{7}{24}$ ...(iii)

Since from eqs. (ii) & (iii)

$P(A\cap B) \neq P(A).P(B)$

Therefore, events A and B are not independent.

OR

(b)

Box B_1	1 White Balls 3 Red Balls

Box B_2	2 White Balls 3 Red Balls

$\therefore$ P(Required) = P(Both are white) + P(Both are red)

$= \frac{1}{4}\times\frac{2}{5}+\frac{3}{4}\times\frac{3}{5}$

$= \frac{2}{20}+\frac{9}{20} = \frac{11}{20}$

SECTION - B

Question numbers 7 to 10 carry 3 marks each.

7. Evaluate: $\int_0^{\pi/4}\frac{dx}{1+\tan x}$ **3**

Ans. Let $I = \int_0^{\pi/4}\frac{dx}{1+\tan x} = \int_0^{\pi/4}\frac{dx}{1+\frac{\sin x}{\cos x}}$

$= \int_0^{\pi/4}\frac{\cos x\,dx}{\cos x+\sin x} = \frac{1}{2}\int_0^{\pi/4}\frac{2\cos x}{\cos x+\sin x}dx$

$= \frac{1}{2}\int_0^{\pi/4}\frac{\cos x+\sin x+\cos x-\sin x}{\cos x+\sin x}dx$

$= \frac{1}{2}\left[\int_0^{\pi/4}\frac{\cos x+\sin x}{\cos x+\sin x}dx+\int_0^{\pi/4}\frac{\cos x-\sin x}{\cos x+\sin x}dx\right]$

$= \frac{1}{2}\left[\int_0^{\pi/4}1dx+\int_0^{\pi/4}\frac{\cos x-\sin x}{\cos x+\sin x}dx\right] = \frac{1}{2}(I_1+I_2)$

where, $I_1 = \int_0^{\pi/4}1dx = [x]_0^{\pi/4} = \frac{\pi}{4}$

and $I_2 = \int_0^{\pi/4}\frac{\cos x-\sin x}{\cos x+\sin x}dx$

Let $\cos x + \sin x = t$

$\Rightarrow (-\sin x + \cos x)dx = dt$

when $x = 0, t = 1$

and $x = \frac{\pi}{4}, t = \frac{2}{\sqrt{2}}$

$\therefore I_2 = \int_1^{2/\sqrt{2}}\frac{dt}{t} = [\log t]_1^{\frac{2}{\sqrt{2}}}$

$= \log\frac{2}{\sqrt{2}}-\log1 = \log\frac{2}{\sqrt{2}}-0$

$= \log2^{3/2} = \frac{3}{2}\log2$

$\therefore \quad I = \frac{1}{2}(I_1 + I_2)$

or $\quad I = \frac{1}{2}\left(\frac{\pi}{4} + \frac{3}{2}\log 2\right)$

8. (a) If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}+\vec{b}| = |\vec{b}|$, then prove that $(\vec{a}+2\vec{b})$ is perpendicular to $\vec{a}$. **3**

OR

(b) If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between them, then prove that $\sin\frac{\theta}{2} = \frac{1}{2}|\vec{a}-\vec{b}|$. **3**

Ans. (a) Given, $\quad |\vec{a}+\vec{b}| = |\vec{b}|$

On squaring both sides, we get

$$|\vec{a}+\vec{b}|^2 = |\vec{b}|^2$$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}| = |\vec{b}|^2$

$\Rightarrow \quad |\vec{a}|^2 + 2|\vec{a}||\vec{b}| = 0$

$\Rightarrow \quad |\vec{a}|.(|\vec{a}|+2|\vec{b}|) = 0$

$\Rightarrow \quad \vec{a}.(\vec{a}+2\vec{b}) = 0$

Since, dot product of $\vec{a}$ and $\vec{a}+2\vec{b}$ is zero, thus vectors are perpendicular. **Hence Proved**

OR

Given, $\quad |\vec{a}| = 1$ and $|\vec{b}| = 1$

Now, we take

$$|\vec{a}-\vec{b}|^2 = (\vec{a}-\vec{b}).(\vec{a}-\vec{b})$$

$$= |\vec{a}|^2 + |\vec{b}| - 2|\vec{a}|.|\vec{b}|$$

$$= 1 + 1 - 2|\vec{a}|.|\vec{b}|\cos\theta$$

$$= 2 - 2 \times 1 \times 1\cos\theta = 2(1-\cos\theta)$$

$$= 2\left[1-\left(1-2\sin^2\frac{\theta}{2}\right)\right]$$

$$= 2\left(2\sin^2\frac{\theta}{2}\right)$$

$$= 4\sin^2\frac{\theta}{2}$$

or, $\quad \sin^2\frac{\theta}{2} = \frac{|\vec{a}-\vec{b}|^2}{4}$

$\Rightarrow \quad \sin\frac{\theta}{2} = \frac{|\vec{a}-\vec{b}|}{2}$ **Hence proved**

*** 9. Find the equation of the plane passing through the line of intersection of the planes $\vec{r}.(\hat{i}+\hat{j}+\hat{k}) = 10$ and $\vec{r}.(2\hat{i}+3\hat{j}-\hat{k})+4 = 0$ and passing through (–2, 3, 1).** **3**

* *Out of Syllabus*

10. (a) Find: $\int e^x.\sin 2x dx$ **3**

OR

(b) Find: $\int \frac{2x}{(x^2+1)(x^2+2)}dx$ **3**

Ans. (a) Let $\quad I = \int e^x \sin 2x dx$

Applying integration by parts

$$I = \int e^x \sin 2x dx$$

I II

$$= e^x \int \sin 2x dx - \int\left[\frac{d}{dx}(e^x)\int \sin 2x dx\right]dx$$

$$= e^x\left(\frac{-\cos 2x}{2}\right) + \frac{1}{2}\int e^x \cos 2x dx$$

$$= \frac{1}{2}(-e^x \cos 2x) + \frac{1}{2}\left[e^x \int \cos 2x dx - \int\left(\frac{d}{dx}(e^x)\int \cos 2x dx\right)dx\right]$$

$$= \frac{1}{2}(-e^x\cos 2x) + \frac{1}{2}\left[\frac{e^x \sin 2x}{2} - \frac{1}{2}\int e^x \sin 2x dx\right]$$

$$I = \frac{1}{2}(-e^x\cos 2x) + \frac{1}{4}(e^x \sin 2x) - \frac{1}{4}\int e^x \sin 2x dx + K$$

$\therefore$ 4I = $-2e^x\cos 2x + e^x\sin 2x - I + K$

or 5I = $-2e^x\cos 2x + e^x\sin 2x + K$

$$I = \frac{1}{5}(e^x \sin 2x - 2e^x \cos 2x) + \frac{K}{5}$$

or $\quad I = \frac{1}{5}(e^x\sin 2x - 2e^x\cos 2x) + c \quad \left(c = \frac{K}{5}\right)$

OR

Let $\quad I = \int \frac{2x}{(x^2+1)(x^2+2)}dx$

By Partial Fractions

let $\quad \frac{1}{(x^2+1)(x^2+2)} = \frac{A}{x^2+1} + \frac{B}{x^2+2}$

$\Rightarrow \quad 1 = A(x^2+2) + B(x^2+1)$

$\Rightarrow \quad 1 = (A+B)x^2 + (2A+B)$

On comparing both sides, we get

$$A + B = 0 \text{ and } 2A + B = 1$$

On solving above equations, we get

$$A = 1 \text{ and } B = -1$$

$\therefore \quad I = \int\left(\frac{1}{x^2+1} - \frac{1}{x^2+2}\right)2x dx$

$$I = \int \frac{2x}{x^2+1}dx - \int \frac{2x}{x^2+2}dx$$

$$I = \log|x^2+1| - \log|x^2+2| + C$$

$$I = \log\left|\frac{x^2+1}{x^2+2}\right| + C$$

SECTION - C

Question numbers 11 to 14 carry 4 marks each.

11. Three persons A, B and C apply for a job a manager in a private company. Chances of their selection are in the ratio 1 : 2 : 4. The probability that A, B and C can introduce chances to increase the profits of a company are 0.8, 0.5 and 0.3 respectively. If increase in the profit does not take place, find the probability that it is due to the appointment of A. **4**

Ans. Let E_1 = Person A gets the job

E_2 = Person B gets the job

E_3 = Person C gets the job

A = No change takes place

The chances of selection of A, B and C are in the ratio 1 : 2 : 4

Hence, $P(E_1) = \frac{1}{7}, P(E_2) = \frac{2}{7}, P(E_3) = \frac{4}{7}$

Also, given $P\left(\frac{A}{E_1}\right) = 0.2 = \frac{2}{10}$, $P\left(\frac{A}{E_2}\right) = 0.5 = \frac{5}{10}$

and $P\left(\frac{A}{E_3}\right) = 0.7 = \frac{7}{10}$

Required probability is

$$P\left(\frac{E_1}{A}\right) = \frac{P\left(\frac{A}{E_1}\right).P(E_1)}{P\left(\frac{A}{E_1}\right).P(E_1)+P\left(\frac{A}{E_2}\right).P(E_2)+P\left(\frac{A}{E_3}\right).P(E_3)}$$

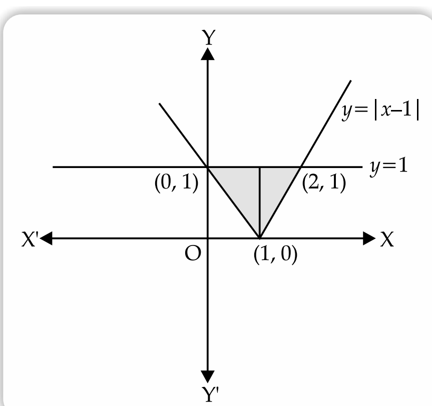
$$= \frac{\frac{2}{10}\times\frac{1}{7}}{\frac{2}{10}\times\frac{1}{7}+\frac{5}{10}\times\frac{2}{7}+\frac{7}{10}\times\frac{4}{7}}$$

$$= \frac{\frac{2}{70}}{\frac{2}{70}+\frac{10}{70}+\frac{28}{70}} = \frac{2}{40} = \frac{1}{20}$$

∴ If no change takes palace, the probability that it is due to appointment of person A is $\frac{1}{20}$.

12. Find the area bounded by the curve $y = |x-1|$ and $y = 1$, using integration. **4**

Ans.



We have, $y = (x-1)$

$y = x-1$, if $x-1 \geq 0$

$y = -x+1$, if $x-1 < 0$

Required Area = Area of shaded region

$$A = \int_0^2 y\,dx = \int_0^1 (1-x)dx + \int_1^2 (x-1)dx$$

$$= \left[x-\frac{x^2}{2}\right]_0^1 + \left[\frac{x^2}{2}-x\right]_1^2$$

$$= \left(1-\frac{1}{2}\right)-\left(0-\frac{0}{2}\right)+\left(\frac{4}{2}-2\right)-\left(\frac{1}{2}-1\right)$$

$$= \frac{1}{2}+\frac{1}{2}$$

= 1 sq. unit

13. (a) Solve the following differential equation:

$(y - \sin^2 x)dx + \tan x\, dy = 0$ **4**

OR

(b) Find the general solution of the differential equation: $(x^3 + y^3)dy = x^2 y dx$ **4**

Ans. **(a)** Given differential equation is

$(y - \sin^2 x)dx + \tan x dy = 0$

$(y - \sin^2 x)dx = -\tan x dy$

$$\frac{dy}{dx} = \frac{y-\sin^2 x}{-\tan x} \quad \frac{dy}{dx} = \frac{\sin^2 x - y}{\tan x}$$

$$\frac{dy}{dx} = \frac{\sin^2 x}{\tan x} - \frac{y}{\tan x}$$

$$\frac{dy}{dx} = \sin x\cos x - y\cot x$$

$$\frac{dy}{dx} + y\cot x = \sin x\cos x$$

which is a linear differential equation of the form

$$\frac{dy}{dx} + Py = Q$$

where $P = \cot x$

$Q = \sin x \cos x$

Here, $I.f. = e^{\int P dx} = e^{\int \cot x dx}$

$= e^{\log|\sin x|} = \sin x$

∴ Solution is given by

$$y.I.f. = \int Q.I.f.\, dx + C_1$$

$$y.\sin x = \int (\sin x\cos x\sin x)dx + C_1$$

$$y.\sin x = \int \sin^2 x\cos x dx + C_1$$

$$y.\sin x = I + C_1 \quad \text{...(i)}$$

where $I = \int \sin^2 x \cos dx$

let $\sin x = t$

⇒ $\cos x dx = dt$

$$\therefore \quad I = \int t^2 dt = \frac{t^3}{3} + C_2$$

or $$I = \frac{\sin^3 x}{3} + C_2$$

from eq. (i), we have

$$y.\sin x = \frac{\sin^3 x}{3} + C_2 + C_1$$

or $$y.\sin x = \frac{\sin^3 x}{3} + C$$

(where $C = C_1 + C_2$)

OR

Given differential equation is

$$(x^3 + y^3)dy = x^2ydx$$

$$\therefore \quad \frac{dx}{dy} = \frac{x^3+y^3}{x^2y} \quad \text{...(i)}$$

Put $x = vy$

$$\Rightarrow \quad \frac{dx}{dy} = v + y\frac{dv}{dy}$$

from eq. (i), we have

$$v + y\frac{dv}{dy} = \frac{(vy)^3+y^3}{(vy)^2y}, \quad v + y\frac{dv}{dy} = \frac{v^3y^3+y^3}{v^2y^3}$$

$$v + y\frac{dv}{dy} = \frac{v^3+1}{v^2}, \quad y\frac{dv}{dy} = \frac{v^3+1}{v^2} - v$$

$$y\frac{dv}{dy} = \frac{1}{v^2}, \quad v^2dv = \frac{dy}{y}$$

(variable separable method)

Integrating both sides, we get

$$\int v^2dv = \int\frac{dy}{y}$$

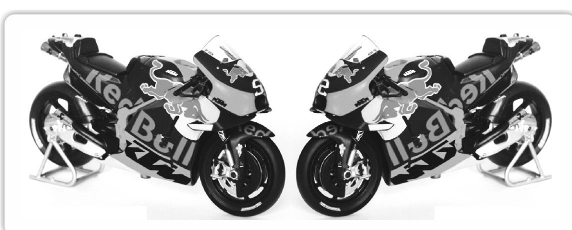
$$\frac{v^3}{3} = \log y + C$$

Putting $v = \frac{x}{y}$, we get

$$\frac{x^3}{3y^3} = \log y + c$$

Case Study Based Question

14. Two motorcycles A and B are running at the speed more than the allowed speed on the roads represented by the lines $\vec{r} = \lambda(\hat{i}+2\hat{j}-\hat{k})$ and $\vec{r} = (3\hat{i}+3\hat{j}) + \mu(2\hat{i}+\hat{j}+\hat{k})$ respectively. 2 × 2 = 4



Based on the above information, answer the following questions:

(a) Find the shortest distance between the given lines. 2

(b) Find the point at which the motorcycles may collide. 2

Ans. **(a)** Given, lines are:

$\vec{r} = \lambda(\hat{i}+2\hat{j}-\hat{k})$ and $\vec{r} = (3\hat{i}+3\hat{j}) + \mu(2\hat{i}+\hat{j}+\hat{k})$

We know that, shortest distance between the lines $\vec{r_1} = \vec{a} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_1}$ is

$$d = \frac{|(\vec{a_2}-\vec{a_1}).(\vec{b_1}\times\vec{b_2})|}{|\vec{b_1}\times\vec{b_2}|}$$

Here, $\vec{a_1} = 0, \ \vec{a_2} = (3\hat{i}+3\hat{j})$

$$\vec{b_1} = \hat{i}+2\hat{j}-\hat{k}$$

and $$\vec{b_2} = 2\hat{i}+\hat{j}+\hat{k}$$

$$\therefore \quad \vec{a_2}-\vec{a_1} = (3\hat{i}+3\hat{j}) - 0 = 3\hat{i}+3\hat{j}$$

$$\vec{b_1}\times\vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}(2+1) - \hat{j}(1+2) + \hat{k}(1-4)$$

$$= 3\hat{i} - 3\hat{j} - 3\hat{k}$$

and $$|\vec{b_1}\times\vec{b_2}| = \sqrt{3^2+(-3)^2+(-3)^2}$$

$$= \sqrt{9+9+9} = 3\sqrt{3}$$

Also, $$(\vec{a_2}-\vec{a_1}).(\vec{b_1}\times\vec{b_2}) = (3\hat{i}+3\hat{j}).(3\hat{i}-3\hat{j}-3\hat{k})$$

$$= 9 - 9 = 0$$

$$d = \frac{0}{3\sqrt{3}} = 0$$

Thus, distance between lines is 0.

(b) We have, $$\vec{r} = \lambda(\hat{i}+2\hat{j}-\hat{k}) \quad \text{...(i)}$$

and $$\vec{r} = 3\hat{i}+3\hat{j}+\mu(2\hat{i}+\hat{j}+\hat{k})$$

or $$\vec{r} = (3+2\mu)\hat{i} + (3+\mu)\hat{j} + \mu\hat{k} \quad \text{...(ii)}$$

Now, from eq. (i) & eq. (ii), we get

$$\lambda(\hat{i}+2\hat{j}-\hat{k}) = (3+2\mu)\hat{i} + (3+\mu)\hat{j} + \mu\hat{k}$$

On comparing both sides, we get

$$3 + 2\mu = \lambda, \ 3 + \mu = 2\lambda \text{ and } \mu = -\lambda$$

On solving for values of λ and μ, we get

$$\lambda = 1 \text{ and } \mu = -1$$

from eq. (i), we get $\vec{r} = \hat{i}+2j-\hat{k}$

$$x\hat{i}+y\hat{j}+z\hat{k} = \hat{i}+2\hat{j}-\hat{k}$$

So, required point is (1, 2, –1).

Series: ABCD/5/5, Delhi Set-II 65/5/2

Note: Except these, all other Questions are from Set-I.

SECTION - A

1. Find the vector equation of a line passing trough a point with position vector $2\hat{i}-\hat{j}+\hat{k}$ and parallel to the line joining the points $-\hat{i}+4\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+2\hat{k}$. **2**

Ans. Let A, B and C be the points with position vectors $2\hat{i}-\hat{j}+\hat{k}, -\hat{i}+4\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+2\hat{k}$, respectively.

We have to find the equation of a line passing through the point A and parallel to vector BC.

Now,

$\overrightarrow{BC}$ = position vector of C – position vector of $\vec{B}$

$= (\hat{i}+2\hat{j}+2\hat{k})-(-\hat{i}+4\hat{j}+\hat{k})$

$= 2\hat{i}-2\hat{j}+\hat{k}$

We know that, the equation of a line passing through a position vector $\vec{a}$ and parallel to vector $\vec{b}$ is

$$\vec{r} = \vec{a}+\lambda\vec{b}$$

$$\therefore \quad \vec{r} = (2\hat{i}-\hat{j}+\hat{k})+\lambda(2\hat{i}-2\hat{j}+\hat{k})$$

is the required equation of line in vector from.

[Here, $\overrightarrow{BC}=\vec{b}$]

SECTION - B

9. (a) Let $\vec{a}=\hat{i}+\hat{j}, \vec{b}=\hat{i}-\hat{j}$ and $\vec{c}=\hat{i}+\hat{j}+\hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{a}.\hat{n}=0$ and $\vec{b}.\hat{n}=0$, then find $|\vec{c}.\hat{n}|$. **3**

OR

(b) If $\vec{a}$ and $\vec{b}$ are unit vectors inclined at an angle 30° to each other, then find the area of the parallelogram with $(\vec{a}+3\vec{b})$ and $(3\vec{a}+\vec{b})$ as adjacent sides. **3**

Ans. Given, $\vec{a} = \hat{i}+\hat{j}$, $\vec{b} = \hat{i}-\hat{j}$ and $\vec{c} = \hat{i}+\hat{j}+\hat{k}$

Also, given $\vec{a}.\hat{n} = 0$ and $\vec{b}.\hat{n} = 0$

Here, $\hat{n} = \dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$

Here, $\vec{a}\times\vec{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 0\\ 1 & -1 & 0\end{vmatrix}$

$= \hat{i}(0-0)-\hat{j}(0-0)+\hat{k}(-1-1) = -2\hat{k}$

$\therefore \quad \hat{n} = \dfrac{-2\hat{k}}{\sqrt{(-2)^2}} = -\hat{k}$

Therefore, $|\vec{c}.\hat{n}| = |(\hat{i}+\hat{j}+\hat{k}).(-\hat{k})| = |-1| = 1$

OR

We know, Area of parallelogram with adjacents sides $\vec{p}$ and $\vec{q}$ is given by

$$A = |\vec{p}\times\vec{q}|$$

Here, Area $= |(\vec{a}+3\vec{b})\times(3\vec{a}+\vec{b})|$

$= |3(\vec{a}\times\vec{a})+(\vec{a}\times\vec{b})+9(\vec{b}\times\vec{a})+3(\vec{b}\times\vec{b})|$

$= |3\times 0+(\vec{a}\times\vec{b})-9(\vec{a}\times\vec{b})+3\times 0|$

$[\because \vec{a}\times\vec{a}=0=\vec{b}\times\vec{b}$ and $\vec{b}\times\vec{a}=-\vec{a}\times\vec{b}]$

$= |-8(\vec{a}\times\vec{b})| = 8|\vec{a}\times\vec{b}|$

$= 8|\vec{a}|.|\vec{b}|\sin\theta$

= 8.1. 1. sin 30° [Given, $|\vec{a}| = 1 = |\vec{b}|$ and θ = 30°]

$= 8.\dfrac{1}{2}$

= 4 sq. units

10. Evaluate: $\displaystyle\int_0^{\pi/2}\frac{1}{1+(\tan x)^{2/3}}dx$

Ans. Let $I = \displaystyle\int_0^{\pi/2}\frac{1}{1+(\tan x)^{2/3}}dx$ ...(i)

$$I = \int_0^{\pi/2}\frac{1}{1+\left[\tan\left(\frac{\pi}{2}-x\right)\right]^{2/3}}dx$$

[Using property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$]

$$I = \int_0^{\pi/2}\frac{1}{1+(\cot x)^{2/3}}dx$$

$$I = \int_0^{\pi/2}\frac{(\tan x)^{2/3}}{(\tan x)^{2/3}+1}dx$$

$$I = \int_0^{\pi/2}\frac{(\tan x)^{2/3}+1-1}{(\tan x)^{2/3}+1}dx$$

$$I = \int_0^{\pi/2}\frac{1+(\tan x)^{2/3}}{1+(\tan x)^{2/3}}dx-\int_0^{\pi/2}\frac{1}{1+(\tan x)^{2/3}}dx$$

$I = \displaystyle\int_0^{\pi/2}1.dx - I$ [From eq.(i)]

$2I = \displaystyle\int_0^{\pi/2}1.dx \quad 2I = [x]_0^{\pi/2} \quad 2I = \frac{\pi}{2} \quad I = \frac{\pi}{4}$

SECTION - C

13. In a factory, machine A produces 30% of total output, machine B produces 25% and the machine C produces the remaining output. The defective items produced by machines A, B and C are 1%, 1.2%, 2% respectively. An item is picked at random from a day's output and found to be defective. Find the probability that it was produced by machine B? 4

Ans. Let E_1 = choosing machine A

E_2 = choosing machine B

E_3 = choosing machine C

A = Producing a defective output

Given, $P(E_1) = 30\% = \frac{30}{100} = 0.3$

$P(E_2) = 25\% = \frac{25}{100} = 0.25$

$P(E_3) = [100 - (30 + 25)]\% = 45\%$

$= \frac{45}{100} = 0.45$

and $P\left(\frac{A}{E_1}\right)$

= P(Producing defective output from machine A)

$= 1\% = \frac{1}{100} = 0.01$

$P\left(\frac{A}{E_2}\right)$

= P(Producing defective output from machine B)

$= 1.2\% = \frac{1.2}{100} = 0.012$

$P\left(\frac{A}{E_3}\right)$

= P(Producing defective output from machine C)

$= 2\% = \frac{2}{100} = 0.02$

Required probability = $P\left(\frac{E_2}{A}\right)$

= P(The found defective item is produced by machine B)

Using Bayes' theorem,

$$P\left(\frac{E_2}{A}\right)$$

$$= \frac{P(E_2).P\left(\frac{A}{E_2}\right)}{P(E_1).P\left(\frac{A}{E_1}\right) + P(E_2).P\left(\frac{A}{E_2}\right) + P(E_3).P\left(\frac{A}{E_3}\right)}$$

$$= \frac{0.25 \times 0.012}{(0.3 \times 0.01) + (0.25 \times 0.012) + (0.45 \times 0.02)}$$

$$= \frac{300}{300 + 300 + 900} = \frac{300}{1500} = \frac{1}{5}$$

Thus, required probability is $\frac{1}{5}$.

Series: ABCD/5/5, Delhi Set-III 65/5/3

Note: Except these, all other Questions are from Set-I, II

SECTION - A

1. The Cartesian equation of a line AB is: $\frac{2x-1}{12} = \frac{y+2}{2} = \frac{z-3}{3}$ 2

Find the direction cosines of a line parallel to line AB.

Ans. We have, $\frac{2x-1}{12} = \frac{y+2}{2} = \frac{z-3}{3}$

The equation of line AB can be rewritten as

$$\frac{x - \frac{1}{2}}{6} = \frac{y-(-2)}{2} = \frac{z-3}{3}$$

Thus, direction ratios of the line parallel to AB are proportional to 6, 2, 3.

Hence, the direction cosines of the line parallel to AB are

$$\frac{6}{\sqrt{6^2+2^2+3^2}}, \frac{2}{\sqrt{6^2+2^2+3^2}}, \frac{3}{\sqrt{6^2+2^2+3^2}}$$

or $\frac{6}{\sqrt{49}}, \frac{2}{\sqrt{49}}, \frac{3}{\sqrt{49}}$

or $\frac{6}{7}, \frac{2}{7}, \frac{3}{7}$

SECTION - B

9. Evaluate: $\int_1^3 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{4-x}} dx$. 3

Ans. Let $I = \int_1^3 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{4-x}}$ ...(i)

Using property $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$, we get

$I = \int_1^3 \frac{\sqrt{4-x}}{\sqrt{4-x}+\sqrt{x}} dx$ ...(ii)

On adding eqs. (i) and (ii), we get

$$2I = \int_1^3 \frac{\sqrt{x}+\sqrt{4-x}}{\sqrt{x}+\sqrt{4-x}} dx = \int_1^3 1dx$$

$= [x]_1^3$

$= 3 - 1 = 2$

$\therefore \quad I = 1$

***10. Find the distance of the point (2,3,4) measured along the line $\frac{x-4}{3} = \frac{y+5}{6} = \frac{z+1}{2}$ from the plane $3x + 2y + 2z + 5 = 0$.**

SECTION - C

13. There are two boxes, namely box-I and box-II. Box-I contains 3 red and 6 black balls. Box-II contains 5 red and 5 black balls. One of the two boxes, is selected at random and a ball is drawn at random. The ball drawn is found to be red. Find the probability that this red ball comes out from box-II.

Ans. Let E_1 = Selecting Box-I

E_2 = Selecting Box-II

A = getting a red ball from the selected box

Here, $P(E_1) = \frac{1}{2}$, $P(E_2) = \frac{1}{2}$

$$P\left(\frac{A}{E_1}\right) = \frac{3}{9} = \frac{1}{3}$$

$$P\left(\frac{A}{E_2}\right) = \frac{5}{10} = \frac{1}{2}$$

Required probability $= P\left(\frac{E_2}{A}\right)$

= P(Red ball comes out from Box-II) Using Bayes' theorem,

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1).P\left(\frac{A}{E_1}\right)+P(E_2).P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{2}\times\frac{1}{2}}{\frac{1}{2}\times\frac{1}{3}+\frac{1}{2}\times\frac{1}{2}}$$

$$= \frac{\frac{1}{4}}{\frac{1}{6}+\frac{1}{4}}$$

$$= \frac{\frac{1}{4}}{\frac{10}{24}} = \frac{1}{4}\times\frac{24}{10} = \frac{3}{5}$$

Thus, probability that the red ball comes out form Box-II is $\frac{3}{5}$.

Series: ABCD/4/3, Outside Delhi Set-I 65/3/1

SECTION - A

Question Nos. 1 to 6 carry 2 marks each.

1. Find: $\int \frac{dx}{x^2 - 6x + 13}$ **2**

Ans. Given integral is

$$I = \int \frac{dx}{x^2 - 6x + 13} = \int \frac{dx}{(x-3)^2 + 13 - 9}$$

$$= \int \frac{dx}{(x-3)^2 + 4} = \int \frac{dx}{(x-3)^2 + 2^2}$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{x-3}{2}\right) + C$$

$$\left[\text{Using } \int \frac{1}{x^2 + a^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C\right]$$

2. Find the general solution of the differential equation : $e^{dy/dx} = x^2$. **2**

Ans. Given differential equation is

$$e^{dy/dx} = x^2$$

Taking log both sides, we get

$$\frac{dy}{dx}\log e = 2\log x \qquad [\because \log_e = 1]$$

$$\Rightarrow \quad \frac{dy}{dx} = 2\log x$$

$$\Rightarrow \quad dy = 2\log x\, dx$$

On integrating both sides, we get

$$\int dy = 2\int \log x\, dx$$

$$\Rightarrow \quad y = 2\int 1.\log x\, dx$$

$$\Rightarrow \quad y = 2\left[\log x \int 1 dx - \int \frac{d}{dx}(\log x)\left(\int 1.dx\right)dx\right]$$

[Using integration by parts]

$$\Rightarrow \quad y = 2\left[\log x(x) - \int \frac{1}{x}(x)dx\right]$$

$$\Rightarrow \quad y = 2[x\log x - x] + C$$

$$\Rightarrow \quad y = 2x(\log x - 1) + C$$

3. Write the projection of the vector $(\vec{b}+\vec{c})$ on the vector $\vec{a}$, where $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. **2**

Ans. Given vectors,

$$\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k} \quad \vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$$

* *Out of Syllabus*

$$\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$$

$$\vec{b} + \vec{c} = \left(\hat{i} + 2\hat{j} - 2\hat{k}\right) + \left(2\hat{i} - \hat{j} + 4\hat{k}\right)$$

or, $$\vec{b} + \vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$$

Projection of $\left(\vec{b} + \vec{c}\right)$ on $\vec{a}$ $= \dfrac{\left(\vec{b} + \vec{c}\right).\vec{a}}{|\vec{a}|}$

$$= \frac{\left(3\hat{i} + \hat{j} + 2\hat{k}\right).\left(2\hat{i} - 2\hat{j} + \hat{k}\right)}{\sqrt{(2)^2 + (2)^2 + (1)^2}}$$

$$= \frac{6 - 2 + 2}{3}$$

$$= 2$$

* **4. If the distance of the point (1, 1, 1) from the plane $x - y + z + \lambda = 0$ is $\frac{5}{\sqrt{3}}$, find the value(s) of λ.** **2**

5. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of spade cards. **2**

Ans. Let X denote the number of spades in a sample of 2 drawn cards from a well shuffle pack of 52 cards.

Then, X can take the values 0, 1, 2.

Now, P(X = 0) = P(no spade)

$$= \frac{^{39}C_2}{^{52}C_2} = \frac{741}{1326}$$

$$= \frac{19}{34}$$

P(X = 1) = P(one spade card)

$$= \frac{^{13}C_1 \times {^{39}C_1}}{^{52}C_2} = \frac{507}{1326} = \frac{13}{34}$$

P(X = 2) = P(both cards are spade)

$$= \frac{^{13}C_2}{^{52}C_2} = \frac{78}{1326} = \frac{1}{17}$$

Thus, the probability distribution of X is given by

X	P(X)
0	$\frac{19}{34}$
1	$\frac{13}{34}$
2	$\frac{1}{17}$

* *Out of Syllabus*

6. A pair of dice is thrown and the sum of the numbers appearing on the dice is observed to be 7. Find the probability that the number 5 has appeared on atleast one die. **2**

OR

The probability that A hits the target is $\frac{1}{3}$ and the probability that B hits it, is $\frac{2}{5}$. If both try to hit the target independently, find the probability that the target is hit. **2**

Ans. Let E = event that 5 has appeared on atleast one die

$\therefore$ E = {(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 5), (4, 5), (3, 5), (2, 5), (1, 5)}

Let F = event that sum of no. on die is 7.

$\therefore$ F = {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}

$E \cap F$ = {(2, 5), (5, 2)}

$\therefore n(E \cap F) = 2$

Now, $P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)} = \frac{n(E \cap F)}{n(F)} = \frac{2}{6} = \frac{1}{3}$

OR

$$P(A) = P(\text{A hits target}) = \frac{1}{3}$$

$$P(B) = P(\text{B hits target}) = \frac{2}{5}$$

Now, $P(A \cup B)$ = P(target will be hit)

$= P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A).P(B)$

[$\because$ A and B are independent]

$$= \frac{1}{3} + \frac{2}{5} - \frac{1}{3} \times \frac{2}{5}$$

$$= \frac{5 + 6 - 2}{15}$$

$$= \frac{9}{15} = \frac{3}{5}$$

SECTION - B

Question Nos. **7** to **10** carry **3** marks each.

7. Evaluate: $\int_0^{2\pi} \frac{dx}{1 + e^{\sin x}}$ **3**

Ans. Let
$$I = \int_0^{2\pi} \frac{dx}{1 + e^{\sin x}}$$

$$= \int_0^{\pi} \left\{\frac{1}{1 + e^{\sin x}} + \frac{1}{1 + e^{\sin(2\pi - x)}}\right\} dx$$

$$\left[\because \int_0^{2a} f(x)dx = \int_0^{a} \{f(x) + f(2a - x)\} dx\right]$$

$$= \int_0^{\pi}\left\{\frac{1}{1+e^{\sin x}}+\frac{1}{1+e^{-\sin x}}\right\}dx$$

$$= \int_0^{\pi}\left\{\frac{1}{1+e^{\sin x}}+\frac{e^{\sin x}}{1+e^{\sin x}}\right\}dx$$

$$= \int_0^{\pi}\frac{1+e^{\sin x}}{1+e^{\sin x}}dx$$

$$= \int_0^{\pi}1.dx = [x]_0^{\pi} = \pi$$

8. Find the particular solution of the differential equation $x\frac{dy}{dx}-y=x^2.e^x$, given $y(1)=0$. **3**

OR

Find the general solution of the differential equation $x\frac{dy}{dx}=y(\log y-\log x+1)$.

Ans. Given differential equation is

$$x\frac{dy}{dx}-y = x^2.e^x$$

$\Rightarrow$ $\frac{dy}{dx}-\frac{y}{x} = xe^x$, which is of the form

$$\frac{dy}{dx}+Py = Q$$

Here, $P = -\frac{1}{x}$ and $Q = xe^x$

$$\text{I.F.} = e^{\int pdx} = e^{\int\frac{-1}{x}dx} = e^{-\log x}$$

$$= e^{\log\frac{1}{x}} = \frac{1}{x}\log_e e = \frac{1}{x}$$

The solution is given by

$$y.\text{I.F.} = \int Q\times\text{I.F.}dx + C$$

$$y.\frac{1}{x} = \int xe^x\times\frac{1}{x}dx + C$$

$$\frac{y}{x} = \int e^x dx + C \quad \frac{y}{x} = e^x + C$$

$$\frac{y}{x} = e^x \qquad \text{...(i)}$$

Given $y = 0$ when $x = 1$

from eq (i), we get

$$0 = 1.e^1 + C.1$$

$\Rightarrow$ $C = -e$

OR

$$y = xe^x - ex$$

Given differential equation is

$$x\frac{dy}{dx} = y(\log y - \log x + 1)$$

$\Rightarrow$ $\frac{dy}{dx} = \frac{y}{x}\left(\log\frac{y}{x}+1\right)$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow$ $v + x\frac{dv}{dx} = v(\log v + 1)$

$\Rightarrow$ $\frac{dv}{v\log v} = \frac{dx}{x}$

On integrating both sides, we get

$$\int\frac{dv}{v\log v} = \int\frac{dx}{x}$$

$\Rightarrow$ $\log(\log v) = \log x + \log C$

$\Rightarrow$ $\log(\log v) = \log Cx$

$\Rightarrow$ $\log(y/x) = Cx$

9. The two adjacent sides of a parallelogram are represented by vectors $2\hat{i}-4\hat{j}+5\hat{k}$ and $\hat{i}-2\hat{j}-3\hat{k}$. Find the unit vector parallel to one of its diagonals, Also, find the area of the parallelogram. **3**

OR

If $\vec{a}=2\hat{i}+2\hat{j}+3\hat{k}, \vec{b}=-\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}$ are such that the vector $(\vec{a}+\lambda\vec{b})$ is perpendicular to vector $\vec{c}$, then find the value of λ. **3**

Ans. Given two adjacent sides of a parallelogram are

$$\vec{a} = 2\hat{i}-4\hat{j}+5\hat{k}$$

$$\vec{b} = \hat{i}-2\hat{j}-3\hat{k}$$

Let $\vec{c}$ be the diagonal of given parallelogram.

$$\vec{c} = \vec{a}+\vec{b}$$

$$= (2\hat{i}-4\hat{j}+5\hat{k})+(\hat{i}-2\hat{j}-3\hat{k})$$

$$= 3\hat{i}-6\hat{j}+2\hat{k}$$

$\therefore$ $|\vec{c}| = \left|\sqrt{(3)^2+(-6)^2+(2)^2}\right| = 7$

Unit vector in direction of $\vec{c} = \frac{\vec{c}}{|\vec{c}|} = \frac{3\hat{i}-6\hat{j}+2\hat{k}}{7}$

Now, Area of parallelogram $= |\vec{a}\times\vec{b}|$

$\therefore$ $\vec{a}\times\vec{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -4 & 5\\ 1 & -2 & -3\end{vmatrix}$

$$= (12+10)\hat{i}-(-6-5)\hat{j}+(-4+4)\hat{k}$$

$$= 22\hat{i}+11\hat{j}$$

Therefore, Area of parallelogram $= |\vec{a}\times\vec{b}|$

$= \left|\sqrt{(22)^2 + (11)^2}\right|$

$= \left|\sqrt{484 + 121}\right| = 11\sqrt{5}$ sq. units

OR

Given vectors are $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$

Now, $\vec{a} + \lambda\vec{b} = \left(2\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(-\hat{i} + 2\hat{j} + \hat{k}\right)$

$= (2-\lambda)\hat{i} + (2+2\lambda)\hat{j} + (3+\lambda)\hat{k}$

If $\vec{a} + \lambda\vec{b}$ is perpendicular $\vec{c}$, then

$$\left(\vec{a} + \lambda\vec{b}\right).\vec{c} = 0$$

$\Rightarrow (2-\lambda)\hat{i} + (2+2\lambda)\hat{j} + (3+\lambda)\hat{k}.\left(3\hat{i} + \hat{j}\right) = 0$

$\Rightarrow (2-\lambda).3 + (2+2\lambda).1 + (3+\lambda).0 = 0$

$\Rightarrow 6 - 3\lambda + 2 + 2\lambda = 0$

$\Rightarrow -\lambda + 8 = 0$

$\Rightarrow \lambda = 8$

10. Show that the lines: **3**

$\frac{1-x}{2} = \frac{y-3}{4} = \frac{z}{-1}$ and $\frac{x-4}{3} = \frac{2y-2}{-4} = z-1$ are coplanar.

Ans. Given lines are:

$\frac{1-x}{2} = \frac{y-3}{4} = \frac{z}{-1}$ and $\frac{x-4}{3} = \frac{2y-2}{-4} = z-1$

or $\frac{x-1}{-2} = \frac{y-3}{4} = \frac{z-0}{-1}$ and $\frac{x-4}{3} = \frac{y-1}{-2} = \frac{z-1}{1}$

These lines will be coplanar if

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

$$\begin{vmatrix} 4-1 & 1-3 & 1-0 \\ -2 & 4 & -1 \\ 3 & -2 & 1 \end{vmatrix} = \begin{vmatrix} 3 & -2 & 1 \\ -2 & 4 & -1 \\ 3 & -2 & 1 \end{vmatrix} = 0$$

[Since, $R_1 = R_3$]

Thus, given lines are coplanar,

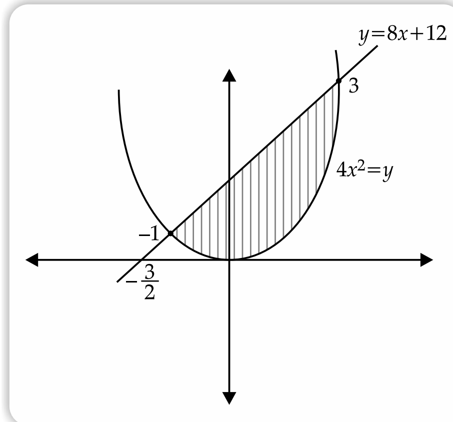
SECTION - C

Question Nos. 11 to 14 carry 4 marks each.

11. Find the area of the region bounded by curve $4x^2 = y$ and the line $y = 8x + 12$, using integration. **4**

Ans. Given curve is $4x^2 = y$ and line is $y = 8x + 12$

On solving both equation, we get



$4x^2 = 8x + 12$

$\Rightarrow x^2 = 2x + 3$

$\Rightarrow x^2 - 2x - 3 = 0$

$\Rightarrow x = 3, -1$

Required area $= \int_{-1}^{3}\left\{(8x+12) - 4x^2\right\}dx$

$= 4\int_{-1}^{3}\left(2x + 3 - x^2\right)dx$

$= 4\left[x^2 + 3x - \frac{x^3}{3}\right]_{-1}^{3}$

$= 4\left[(9+9-9) - \left(1 - 3 + \frac{1}{3}\right)\right]$

$= 4\left(9 + 2 - \frac{1}{3}\right)$

$= 4\left(11 - \frac{1}{3}\right)$

$= 4 \times \frac{32}{3} = \frac{128}{3}$ sq. units

12. Find: $\int \frac{x^2}{(x^2+1)(3x^2+4)}dx$ **4**

OR

Evaluate: $\int_{-2}^{1}\sqrt{5 - 4x - x^2}\,dx$

Ans. $\frac{x^2}{(x^2+1)(3x^2+4)}$

Put $t = x^2$

$$\frac{t}{(t+1)(3t+4)} = \frac{A}{t+1} + \frac{B}{3t+4}$$

$t = A(3t+4) + B(t+1)$

$t = (3A+B)t + (4A+B)$

On comparing both sides, we get

$3A + B = 1$ and $4A + B = 0$

$\therefore \quad I = \int \frac{-1}{x^2+1}dx + \int \frac{+4}{3x^2+4}dx$

$= -\int \frac{1}{x^2+1}dx + 4\int \frac{1}{3x^2+4}dx$

$= -\int \frac{1}{x^2+1^2}dx + 4\int \frac{1}{\left(\sqrt{3}x\right)^2+2^2}dx$

$= -1\tan^{-1}x + \frac{4}{2\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}x}{2}\right)+C$

$= -\tan^{-1}x + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}x}{2}\right)+C$

OR

Let $\quad I = \int_{-2}^{1}\sqrt{5-4x-x^2}\,dx = \int_{-2}^{1}\sqrt{-(x^2+4x-5)}\,dx$

$= \int_{-2}^{1}\sqrt{-(x^2+4x+2^2-2^2-5)}\,dx$

$= \int_{-2}^{1}\sqrt{-\left\{(x+2)^2-9\right\}}\,dx$

$= \int_{-2}^{1}\sqrt{3^2-(x+2)^2}\,dx$

$= \left[\frac{x+2}{2}\sqrt{3^2-(x+2)^2}+\frac{3^2}{2}\sin^{-1}\left(\frac{x+2}{3}\right)\right]_{-2}^{1}$

$= 0+\frac{9}{2}.\frac{\pi}{2}-(0+0) = \frac{9\pi}{4}$

***13. Find the distance of the point (1, –2, 0) from the point of the line $\vec{r} = 4\hat{i}+2\hat{j}+7\hat{k}+\lambda\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$ and the plane $\vec{r}.\left(\hat{i}-\hat{j}+\hat{k}\right)=10.$** **4**

Case Study Based Problem:

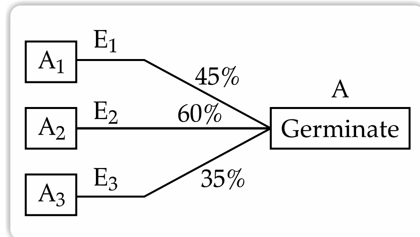
14. A shopkeeper sells three types of flower seeds A1, A2, A3. They are sold is the form of a mixture, where the proportions of these seeds are 4 : 4 : 2 respectively. The germination rates of the three types of seeds are 45%, 60% and 35% respectively.4

Based on the above information :

(a) Calculate the probability that a randomly chosen seed will germinate: **2**

(b) Calculate the probability that the seed is of type A2, given that a randomly chosen seed germinates. **2**

Ans. (a)



Here, $P(E_1) = \frac{4}{10}$, $P(E_2) = \frac{4}{10}$, $P(E_3) = \frac{2}{10}$

$P\left(\frac{A}{E_1}\right) = \frac{45}{100},\ P\left(\frac{A}{E_2}\right) = \frac{60}{100},\ P\left(\frac{A}{E_3}\right) = \frac{35}{100}$

$\therefore P(A) = P(E_1).P\left(\frac{A}{E_1}\right)+P(E_2).P\left(\frac{A}{E_2}\right)+P(E_3).P\left(\frac{A}{E_3}\right)$

$= \frac{4}{10}\times\frac{45}{100}+\frac{4}{10}\times\frac{60}{100}+\frac{2}{10}\times\frac{35}{100}$

$= \frac{180}{1000}+\frac{240}{1000}+\frac{70}{100}$

$= \frac{490}{1000}$

(b) Required probability $= P\left(\frac{E_2}{A}\right)$

$= \frac{P(E_2).P\left(\frac{A}{E_2}\right)}{P(A)}$

$= \frac{\frac{4}{10}\times\frac{60}{100}}{\frac{490}{1000}}$

$= \frac{240}{490} = \frac{24}{49}$

* *Out of Syllabus*

Series: ABCD/4/3, Outside Delhi Set-II 65/3/2

Note: Except these, all other Questions are from Set-I.

SECTION - A

2. Find the general solution of the differential equation: $\log\left(\frac{dy}{dx}\right) = ax + by$. **2**

Ans. Given differential equation is

$$\log\left(\frac{dy}{dx}\right) = ax + by$$

$$\Rightarrow \quad \frac{dy}{dx} = e^{ax + by}$$

$$\Rightarrow \quad \frac{dy}{dx} = e^{ax}.e^{by}$$

$$\Rightarrow \quad \frac{dy}{e^{by}} = e^{ax}\,dx$$

$$\Rightarrow \quad e^{-by}\,dy = e^{ax}\,dx$$

On integrating both sides, we get

$$\int e^{-by}dy = \int e^{ax}dx$$

$$\frac{e^{-by}}{-b} = \frac{e^{ax}}{a} + C$$

$$\Rightarrow \quad \frac{e^{ax}}{a} + \frac{e^{-by}}{b} + C = 0$$

SECTION - B

7. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c}$ and $\vec{a}\times\vec{b} = \vec{a}\times\vec{c}$, $\vec{a} \neq 0$, then show that $\vec{b} = \vec{c}$. **3**

OR

If $|\vec{a}| = 3, |\vec{b}| = 5, |\vec{c}| = 4$ and $\vec{a}+\vec{b}+\vec{c} = \vec{0}$, then find the value of $(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})$. **3**

Ans. Given, $\vec{a}.\vec{b} = \vec{a}.\vec{c}$

$$\Rightarrow \quad \vec{a}.\vec{b} - \vec{a}.\vec{c} = 0$$

$$\Rightarrow \quad \vec{a}.(\vec{b} - \vec{c}) = 0$$

$\Rightarrow$ But $\vec{a} \neq \vec{c}$

So, $\vec{b} - \vec{c} = 0$

or, $\vec{b} = \vec{c}$

Also, given $\vec{a}\times\vec{b} = \vec{a}\times\vec{c}$

$$\Rightarrow \quad \vec{a}\times\vec{b} - \vec{a}\times\vec{c} = 0$$

$$\Rightarrow \quad \vec{a}\times(\vec{b} - \vec{c}) = 0$$

$\Rightarrow$ Since, $\vec{a} \neq \vec{0}$

So, $\vec{b} - \vec{c} = 0$

Hence, vector $\vec{b} = \vec{c}$.

OR

Given, $|\vec{a}| = 3, |\vec{b}| = 5, |\vec{c}| = 4$

and $\vec{a}+\vec{b}+\vec{c} = \vec{0}$

$$\therefore \quad |\vec{a}+\vec{b}+\vec{c}| = |\vec{0}|$$

$$\Rightarrow \quad |\vec{a}+\vec{b}+\vec{c}|^2 = 0$$

$$\Rightarrow (\vec{a}+\vec{b}+\vec{c}).(\vec{a}+\vec{b}+\vec{c}) = 0$$

$$\Rightarrow \vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{a}.\vec{c}+\vec{b}.\vec{a}+\vec{b}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}+\vec{c}.\vec{b}+\vec{c}.\vec{c} = 0$$

$$\Rightarrow |\vec{a}|^2+\vec{a}.\vec{b}+\vec{c}.\vec{a}+\vec{a}.\vec{b}+|\vec{b}|^2+\vec{b}.\vec{c}+\vec{c}.\vec{a}+\vec{b}.\vec{c}+|\vec{c}|^2 = 0$$

$$\left[\because \vec{a}.\vec{b} = \vec{b}.\vec{a},\ \vec{a}.\vec{c} = \vec{c}.\vec{a},\ \vec{b}.\vec{c} = \vec{c}.\vec{b}\right]$$

$$\Rightarrow \quad |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}) = 0$$

$$\Rightarrow (3)^2+(5)^2+(4)^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}) = 0$$

$$\Rightarrow \quad 9+25+16+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}) = 0$$

$$\Rightarrow \quad 50+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}) = 0$$

$$\Rightarrow \quad \vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a} = -25$$

8. Evaluate: $\int_{-1}^{2} |x^3 - x|\,dx$ **3**

Ans. Let

$$I = \int_{-1}^{2}|x^3 - x|dx$$

$$= \int_{-1}^{2}|x(x^2-1)|dx$$

$$= \int_{-1}^{2}|x(x-1)(x+1)|dx$$

Here, $x^3 - x = 0$, when $x = 0, 1, -1$

Value of x	Value of $(x^3 - x)$
$-1 < x < 0$	+ve
$0 < x < 1$	–ve
$1 < x < 2$	+ve

$$\therefore |x^3 - x| = \begin{cases} x^3 - x & \text{if } -1 < x < 0 \text{ and } 1 < x < 2 \\ -x^3 + x & \text{if } 0 < x < 1 \end{cases}$$

$$I = \int_{-1}^{0}(x^3 - x)dx + \int_{0}^{1}(-x^3 + x)dx + \int_{1}^{2}(x^3 - x)dx$$

$$= \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{-x^4}{4} + \frac{x^2}{2}\right]_{0}^{1} + \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{1}^{2}$$

$$= \frac{1}{4} + \frac{1}{4} + 2 + \frac{1}{4}$$

$$= 2 + \frac{3}{4} = \frac{11}{4}$$

SECTION - C

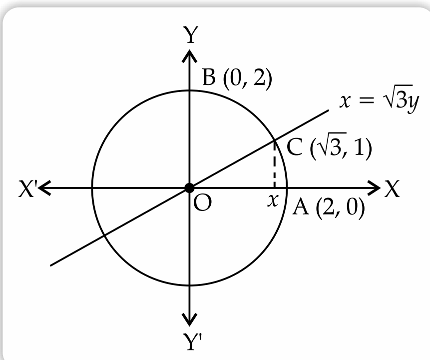
12. Using integration, find the area of the region bounded by the curves $x^2 + y^2 = 4$, $x = \sqrt{3}y$ and X-axis lying in the first quadrant. **4**

Ans. Given equation of circle

$$x^2 + y^2 = 4$$

or $\quad x^2 + y^2 = (2)^2$

$\therefore \quad$ radius = 2



So, point A is (2, 0) and point B is (0, 2)

Let line $x = \sqrt{3}y$ intersect the circle at point C

On solving $x^2 + y^2 = 4$ and $x = \sqrt{3}y$, we get

$$\left(\sqrt{3}y\right)^2 + y^2 = 4$$

$\Rightarrow 3y^2 + y^2 = 4 \Rightarrow 4y^2 = 4 \Rightarrow y^2 = 1 \Rightarrow y = \pm 1$

for $y = 1, x = \sqrt{3}$ and $y = -1, x = -\sqrt{3}$

Since point C is in 1st quadrant

$\therefore$ C = $\left(\sqrt{3}, 1\right)$

$$\therefore \text{ Required Area} = \int_0^{\sqrt{3}} y_{\text{line}}\, dx + \int_{\sqrt{3}}^{2} y_{\text{circle}}\, dx$$

$$= \int_0^{\sqrt{3}} \frac{x}{\sqrt{3}} dx + \int_{\sqrt{3}}^{2} \sqrt{4 - x^2}\, dx$$

$$= \frac{1}{\sqrt{3}} \int_0^{\sqrt{3}} x\, dx + \int_{\sqrt{3}}^{2} \sqrt{4 - x^2}\, dx$$

$$= \frac{1}{\sqrt{3}} \left[\frac{x^2}{2}\right]_0^{\sqrt{3}} + \left[\frac{1}{2} x\sqrt{4 - x^2} + \frac{(2)^2}{2} \sin^{-1}\frac{x}{2}\right]_{\sqrt{3}}^{2}$$

$$= \frac{1}{2\sqrt{3}} \left\{\left(\sqrt{3}\right)^2 - 0\right\} + \left[\begin{array}{l}\left\{\frac{1}{2}(2)\sqrt{4 - 2^2} + 2\sin^{-1}\left(\frac{2}{2}\right)\right\} \\ -\left\{\frac{1}{2}\left(\sqrt{3}\right)\sqrt{4 - \left(\sqrt{3}\right)^2} - 2\sin^{-1}\frac{\sqrt{3}}{2}\right\}\end{array}\right]$$

$$= \frac{\sqrt{3}}{2} + 2\sin^{-1}(1) - \frac{\sqrt{3}}{2} - 2\sin^{-1}\frac{\sqrt{3}}{2}$$

$$= 2\frac{\pi}{2} - 2\frac{\pi}{3}$$

$$= \pi - \frac{2\pi}{3} = \frac{\pi}{3} \text{ sq. units}$$

Series: ABCD/4/3, Outside Delhi Set-III — 65/3/3

Note: Except these all other Questions are from Set-I.

SECTION - A

3. Find the general solution of the differential equation: $\frac{dy}{dx} = \frac{3e^{2x} + 3e^{4x}}{e^x + e^{-x}}$ **2**

Ans. Given differential equation is

$$\frac{dy}{dx} = \frac{3e^{2x} + 3e^{4x}}{e^x + e^{-x}}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{3e^{2x}(1 + e^{2x})}{e^x + \frac{1}{e^x}}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{3e^{2x}(1 + e^{2x})}{(e^{2x} + 1)} \times e^x$$

$$\Rightarrow \quad \frac{dy}{dx} = 3e^{3x}$$

$$\Rightarrow \quad dy = 3e^{3x} dx$$

Integrating both sides, we get

$$\int dy = 3\int e^{3x} dx$$

$$\Rightarrow \quad y = 3\frac{e^{3x}}{3} + C \Rightarrow y = e^{3x} + C$$

SECTION - B

7. Find the shortest distance between the following lines: **3**

$\vec{r} = 3\hat{i} + 5\hat{j} + 7\hat{k} + \lambda\left(\hat{i} - 2\hat{j} + \hat{k}\right)$ and

$\vec{r} = \left(-\hat{i} - \hat{j} - \hat{k}\right) + \mu\left(7\hat{i} - 6\hat{j} + \hat{k}\right)$.

Ans. Given lines are:

$$\vec{r} = 3\hat{i} + 5\hat{j} + 7\hat{k} + \lambda\left(\hat{i} - 2\hat{j} + \hat{k}\right)$$

and $\quad \vec{r} = \left(-\hat{i} - \hat{j} - \hat{k}\right) + \mu\left(7\hat{i} - 6\hat{j} + \hat{k}\right)$

Let the given lines be $\vec{r} = \vec{a_1} + \lambda\vec{b_2}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$

Shortest distance between two lines

$$d = \left|\frac{(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$$

$$\therefore\ \vec{a}_2 - \vec{a}_1 = (-\hat{i} - \hat{j} - \hat{k}) - (3\hat{i} + 5\hat{j} + 7\hat{k}) = -4\hat{i} - 6\hat{j} - 8\hat{k}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ 7 & -6 & 1 \end{vmatrix}$$

$$= \hat{i}(-2+6) - \hat{j}(1-7) + \hat{k}(-6+14)$$

$$= 4\hat{i} + 6\hat{j} + 8\hat{k}$$

$$\therefore |\vec{b}_1 \times \vec{b}_2| = \left|\sqrt{4^2 + 6^2 + 8^2}\right|$$

$$= \left|\sqrt{16 + 36 + 64}\right|$$

$$= \sqrt{116} = 2\sqrt{29}$$

$$\text{Therefore, } d = \left|\frac{(-4\hat{i} - 6\hat{j} - 8\hat{k}).(4\hat{i} + 6\hat{j} + 8\hat{k})}{\sqrt{116}}\right|$$

$$= \left|\frac{-16 - 36 - 64}{\sqrt{116}}\right| = \left|\frac{-116}{\sqrt{116}}\right| = \sqrt{116} = 2\sqrt{29} \text{ units}$$

10. Evaluate: $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (\sin|x| + \cos|x|)dx$ **3**

Ans. We have, $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (\sin|x| + \cos|x|)dx$

Let $f(x) = \sin|x| + \cos|x|$

Then, $f(x) = f(-x)$

Since, $f(x)$ is an even function

So, $I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (\sin|x| + \cos|x|)dx$

$$= 2\int_0^{\frac{\pi}{2}} (\sin x + \cos x)dx$$

$$= 2[-\cos x + \sin x]_0^{\frac{\pi}{2}}$$

$$= 2\left[-\cos\frac{\pi}{2} + \sin\frac{\pi}{2} + \cos 0 - \sin 0\right]$$

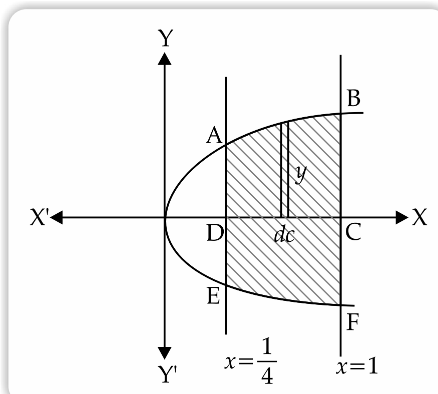
$$= 2[0 + 1 + 1 - 0]$$

$$= 2(2) = 4$$

SECTION - C

13. Find the area of the region enclosed by the curves $y^2 = x$, $x = \frac{1}{4}$, $y \geq 0$ and $x = 1$, using integration. 4

Ans. The area of the region bounded by the curve, $y^2 = x$, the lines $x = \frac{1}{4}$ and $x = 1$ and $y = 0$ (i.e., X-axis) is the a ABCD



Thus, area of ABEF = 2 area of ABCD

$$\text{Required area} = \int_{\frac{1}{4}}^{1} y\,dx = \int_{\frac{1}{4}}^{1} \sqrt{x}\,dx$$

$$= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_{\frac{1}{4}}^{1} = \frac{2}{3}\left[(1)^{\frac{3}{2}} - \left(\frac{1}{4}\right)^{\frac{3}{2}}\right]$$

$$= \frac{2}{3}\left[1 - \frac{1}{8}\right] = \frac{2}{3}\left[\frac{7}{8}\right] = \frac{7}{12} \text{ units}$$

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