ICSE Solved Paper 2018
Mathematics

Class-X

(Maximum Marks : 80)

(Time allowed : Two hours and a half)

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*Attempt **all** questions from **Section A** and **any four** questions from **Section B**.*

All working, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answer.

Omission of essential working will result in loss of marks.

The intended marks for questions or parts of question are given in brackets [ ].

Mathematical tables are provided.

SECTION-A	(40 marks)

*Attempt **all** questions from this Section.*

1. (a) Find the value of 'x' and 'y' if : [3]

$$2\begin{bmatrix} x & 7 \\ 9 & y-5 \end{bmatrix} + \begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

(b) Sonia had a recurring deposit account in a bank and deposited ₹ 600 per month for 2½ years. If the rate of interest was 10% p.a., find the maturity value of this account. [3]

(c) Cards bearing numbers 2, 4, 6, 8, 10, 12, 14, 16, 18 and 20 are kept in a bag. A card is drawn at random from the bag. Find the probability of getting a card which is : [4]

(i) a prime number.

(ii) a number divisible by 4.

(iii) a number that is a multiple of 6.

(iv) an odd number.

Ans. (a) $$2\begin{bmatrix} x & 7 \\ 9 & y-5 \end{bmatrix} + \begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 2x & 14 \\ 18 & 2y-10 \end{bmatrix} + \begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

or $$\begin{bmatrix} 2x+6 & 7 \\ 22 & 2y-5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

By equating $2x + 6 = 10$ or $x = 2$

$2y - 5 = 15$ or $y = 10$

(b) $p = ₹ 600$

$n = 2½$ year $= 30$ months

$r = 10\%$ p.a.

$M.V. = ?$

$$M.V. = p \times n + \frac{p \times n\,(n+1) \times r}{2,400}$$

$$= 600 \times 30 + \frac{600 \times 30 \times 31 \times 10}{2,400}$$

$$= 18,000 + 2,325$$

$$= ₹ 20,325$$

(c) (i)   Prime number $= \{2\}$

No. of favourable cards $= 1$

Total number of cards $= 10$

Hence, probability of a getting a prime number card

$$= \frac{\text{Number of favourable cards}}{\text{Total no. of cards}} = \frac{1}{10}$$

(ii) Number divisible by 4 $= \{4, 8, 12, 16, 20\}$

No. of favourable cards $= 5$

Hence, probability of a getting card, where number divisible by 4.

$$= \frac{\text{Number of favourable cards}}{\text{Total no. of cards}} = \frac{5}{10} = \frac{1}{2}$$

(iii) Number which are multiple of 6 $= \{6, 12, 18\}$

No. of favourable cards $= 3$

Hence, probability of getting card, which is multiple of 6.

$$= \frac{\text{Number of favourable cards}}{\text{Total No. of cards}} = \frac{3}{10}$$

(iv) Odd number $= 0$ (No. odd card)

No. of favourable cards $= 0$

Hence, probability of getting an odd number card

$$= \frac{\text{Number of favourable cards}}{\text{Total No. of cards}} = \frac{0}{10} = 0$$

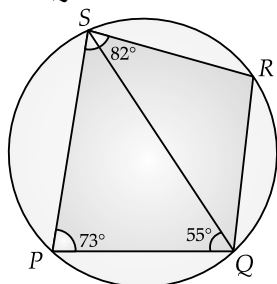**2. (a)** The circumference of the base of a cylindrical vessel is 132 cm and its height is 25 cm. Find the **[3]**
(i) radius of the cylinder
(ii) volume of cylinder. (use $\pi = \dfrac{22}{7}$)

(b) If $(k - 3)$, $(2k + 1)$ and $(4k + 3)$ are three consecutive terms of an *A.P.*, find the value of *k*. **[3]**

(c) *PQRS* is a cyclic quadrilateral. Given $\angle QPS = 73°$, $\angle PQS = 55°$ and $\angle PSR = 82°$, calculate : **[4]**
(i) $\angle QRS$
(ii) $\angle RQS$
(iii) $\angle PRQ$

Ans. (a) Given

circumference = 132 cm
height = 25 cm
(i) circumference = $2\pi r$
$132 = 2\pi r$
$132 = \dfrac{2 \times 22}{7} \times r$
$r = \dfrac{132 \times 7}{2 \times 22} = 21$ cm

Hence, radius of cylinder = 21 cm.
(ii) volume of cylinder = $\pi r^2 h$
$= \dfrac{22}{7} \times 21 \times 21 \times 25$
$= 34,650$ cm^3

(b) For three consecutive terms of an A.P., the common difference should be same, i.e,.
$(2k + 1) - (k - 3) = (4k + 3) - (2k + 1)$
$2k + 1 - k + 3 = 4k + 3 - 2k - 1$
$k + 4 = 2k + 2$
$k = 2$

(c) Given $\angle QPS = 73°$, $\angle PQS = 55°$, $\angle PSR = 82°$

(i) Since, sum of opposite pairs of angles in cyclic quadrilaterals is 180°.
Hence,
$\angle QRS + \angle QPS = 180°$
$\angle QRS + 73° = 180°$
$\angle QRS = 180° - 73°$
$= 107°$
(ii) $\angle PQR + \angle PSR = 180°$
(PQRS is a cyclic quadrilateral)
$\angle PQR + 82° = 180°$
$\angle PQR = 180° - 82$
$= 98°$
$\because$ $\angle PQS + \angle RQS = \angle PQR$
$55° + \angle RQS = 98°$
$\angle RQS = 98° - 55°$
$= 43°$

(iii) Join *P* to *R*
In ΔPSQ,
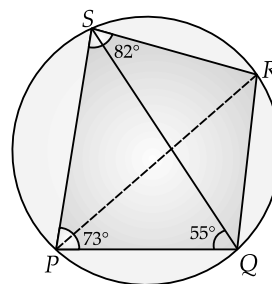$\angle SPQ + \angle PQS + \angle PSQ = 180°$
$73° + 55° + \angle PSQ = 180°$
$\angle PSQ = 180° - 128°$
$= 52°$
Now,
$\angle PRQ = \angle PSQ (\because$ Angle in same segment)
$\therefore$ $\angle PRQ = 52°$

3. (a) If $(x + 2)$ and $(x + 3)$ are factors of $x^3 + ax + b$, find the values of '*a*' and '*b*'. **[3]**

(b) Prove that $\sqrt{\sec^2 \theta + \text{cosec}^2 \theta}$ = tan θ + cot θ **[3]**

(c) Using a graph paper draw a histogram for the given distribution showing the number of runs scored by 50 batsmen. Estimate the mode of the data : **[4]**

Runs scored	3000 - 4000	4000-5000	5000-6000	6000-7000	7000-8000	8000-9000	9000-10000
No. of batsmen	4	18	9	6	7	2	4

Ans. (a) $x^3 + ax + b$
$(x + 2)$ is a factor of given polynomial then
$x = -2$ will satisfy the polynomial

$(-2)^3 + a(-2) + b = 0$
$-8 - 2a + b = 0$
$b - 2a = 8$ ...(i)

$(x + 3)$ is also a factor of given polynomial then $x = -3$ will satisfy the polynomial

$$(-3)^3 + a(-3) + b = 0$$

$$-27 - 3a + b = 0$$

$$b - 3a = 27 \qquad ...(ii)$$

On solving (i) and (ii), we get

$$a = -19, b = -30$$

(b) L.H.S. $= \sqrt{\sec^2\theta + \csc^2\theta}$

$$= \sqrt{1 + \tan^2\theta + 1 + \cot^2\theta}$$

[Since, $1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \csc^2\theta$]
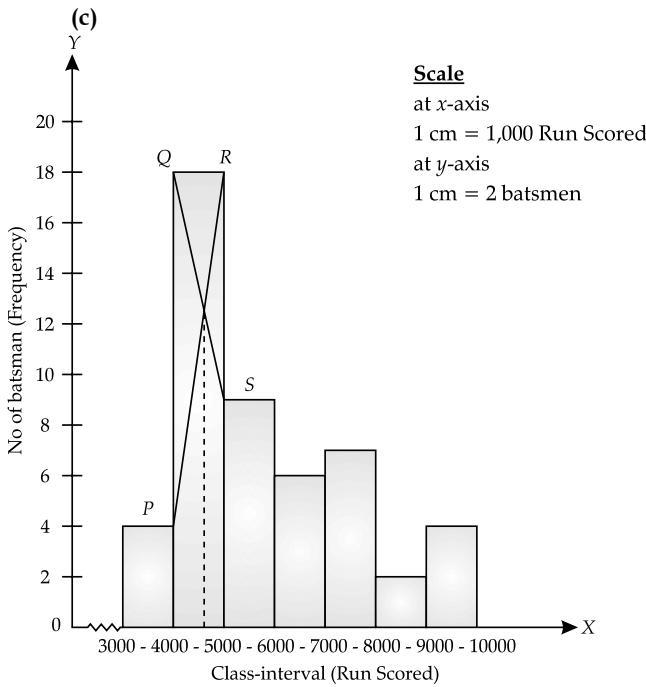
$$= \sqrt{\tan^2\theta + \cot^2\theta + 2}$$

$$= \sqrt{\tan^2\theta + \cot^2\theta + 2\tan\theta.\cot\theta}$$

$$[\because \tan\theta.\cot\theta = 1]$$

$$= \sqrt{(\tan\theta + \cot\theta)^2}$$

$$= \tan\theta + \cot\theta = \text{R.H.S.}$$

(c)

Scale
at x-axis
1 cm = 1,000 Run Scored
at y-axis
1 cm = 2 batsmen

Mode = 4600

4. (a) Solve the following inequation, write down the solution set and represent it on the real number line : **[3]**

$-2 + 10x \le 13x + 10 < 24 + 10x, x \in \mathbf{Z}$

(b) If the straight lines $3x - 5y = 7$ and $4x + ay + 9 = 0$ are perpendicular to one another, find value of a. **[3]**

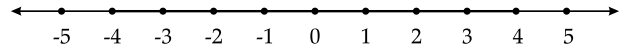**(c)** Solve $x^2 + 7x = 7$ and give your answer correct to two decimal places. **[4]**

Ans. (a) $$-2 + 10x \le 13x + 10 < 24 + 10x$$

or $$-2 \le 3x + 10 < 24$$

or $$-12 \le 3x < 14$$

$$-4 \le x < \frac{14}{3}$$

or

$$-4 \le x < 4\frac{2}{3}$$

$\therefore$ The solution set is $x \in [-4, 4]$ and $x \in \mathbf{Z}$

(b) Eqn. (i) $$3x - 5y = 7$$

$$y = \frac{3}{5}x - \frac{7}{5}$$

$\therefore$ $$m_1 = \frac{3}{5}$$

Eqn. (ii) $$4x + ay + 9 = 0$$

$$y = \frac{-4}{a}x - \frac{9}{a}$$

$\therefore$ $$m_2 = \frac{-4}{a}$$

Both lines are perpendicular to each other.

$\therefore$ $$m_1 \times m_2 = -1$$

or $$\frac{3}{5} \times \left(\frac{-4}{a}\right) = -1$$

or $$\frac{-12}{5a} = -1$$

$$a = \frac{12}{5}$$

(c) $$x^2 + 7x = 7$$

$$x^2 + 7x - 7 = 0$$

Compare the equation to

$$ax^2 + bx + c = 0$$

$$a = 1, b = 7, c = -7$$

$$D = b^2 - 4ac$$

$$= (7)^2 - 4 \times 1 \times (-7)$$

$$= 49 + 28$$

$$= 77$$

Roots are real, different and irrational number,

or $$x = \frac{-b \pm \sqrt{D}}{2a}$$

or $$x = \frac{-7 \pm \sqrt{77}}{2}$$

$$x = \frac{-7 \pm 8.7749}{2}$$

Taking (+) sign

$$x = \frac{-7 + 8.7749}{2}$$

$$= \frac{1.7749}{2} = 0.89$$

Taking (–) sign

$$x = \frac{-7 - 8.7749}{2}$$

$$= -\frac{15.7749}{2}$$

$$= -7.89$$

SECTION-B	**40 Marks**

*Attempt any **four** questions from this Section*

5. **(a)** The 4th term of a G.P. is 16 and the 7th term is 128. Find the first term and common ratio of the series. **[3]**

(b) A man invests ₹ 22,500 in ₹ 50 shares available at 10% discount. If the dividend paid by the company is 12%, calculate : **[3]**

(i) The number of shares purchased.

(ii) The annual dividend received.

(iii) The rate of return he gets on his investment. Give your answer correct to the nearest whole number.

(c) Use graph paper for this question (Take 2 cm = 1 unit along both *x* and *y* axis). **[4]**

ABCD is a quadrilateral whose vertices are *A*(2, 2), *B*(2, – 2), *C* (0 – 1) and *D*(0, 1)

(i) Reflect quadrilateral *ABCD* on the *y*-axis and name it as *A'B'CD*.

(ii) Write down the coordinates of *A'* and *B'*.

(iii) Name two points which are invariant under the above reflection.

(iv) Name the polygon *A'B'CD*.

Ans. (a) Given, $T_4 = 16$ and $T_7 = 128$

Let first term of G.P. be a and common ratio be *r*.

so $\qquad T_4 = ar^3 = 16 \qquad$ ....(i)

$\qquad T_7 = ar^6 = 128 \qquad$ ...(ii)

Dividing (ii) by (i), we get

$$\frac{ar^6}{ar^3} = \frac{128}{16}$$

$$r^3 = 8 = 2^3$$

$$r = 2$$

From (i)

$$a.(2)^3 = 16$$

$$a \times 8 = 16$$

$$a = 2$$

(b) Given,

Total investment = ₹ 22,500, Face value = ₹ 50

Market value = ₹ $\left(50 - \frac{10}{100} \times 50\right)$ = ₹ 45

(∵ 10% discount)

Dividend = 12%

(i) No. of shares = $\dfrac{\text{Investment}}{\text{Market value of share}}$

$$= \frac{22500}{45} = 500$$

(ii) Annual dividend = No. of shares × Nominal value of share × dividend rate

$$= 500 \times 50 \times \frac{12}{100}$$

$$= ₹ \, 3,000$$

(iii) Return percentage

$$= \frac{\text{Total dividend}}{\text{Total investment}} \times 100$$

$$= \frac{3,000}{22,500} \times 100$$

$$= 13.33\% = 13\%$$

(c) (i)
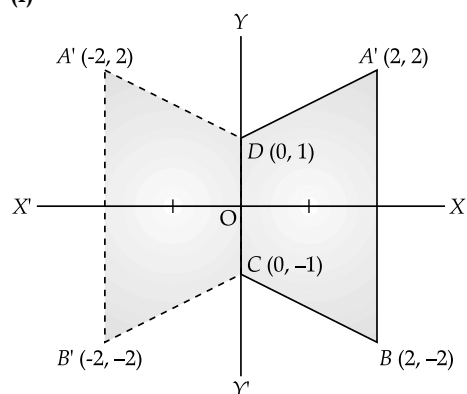
(ii) Co-ordinates of $A' \rightarrow (-2, 2)$

Co-ordinates of $B' \rightarrow (-2, -2)$

(iii) Two invariant points are $C(0, -1)$ and $D(0, 1)$

(iv) *A'B'CD* is a trapezium quadrilateral

6. **(a)** Using properties of proportion, solve for *x*. Given that *x* is positive : **[3]**

$$\frac{2x + \sqrt{4x^2 - 1}}{2x - \sqrt{4x^2 - 1}} = 4$$

(b) If $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$, and $C = \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$, find $AC + B^2 - 10C$. **[3]**

(c) Prove that $(1 + \cot \theta - \csc \theta)(1 + \tan \theta + \sec\theta) = 2$ **[4]**

Ans. (a) $\dfrac{2x + \sqrt{4x^2 - 1}}{2x - \sqrt{4x^2 - 1}} = 4$

Applying componendo & dividendo

$$\frac{2x + \sqrt{4x^2 - 1} + 2x - \sqrt{4x^2 - 1}}{2x + \sqrt{4x^2 - 1} - 2x + \sqrt{4x^2 - 1}} = \frac{4 + 1}{4 - 1}$$

$$\frac{4x}{2\sqrt{4x^2 - 1}} = \frac{5}{3}$$

$$12x = 10\sqrt{4x^2 - 1}$$

Squaring both sides,

$$144x^2 = 100(4x^2 - 1)$$
$$36x^2 = 25(4x^2 - 1)$$
$$36x^2 = 100x^2 - 25$$
$$100x^2 - 36x^2 = 25$$
$$64x^2 = 25$$
$$x^2 = \frac{25}{64}$$
$$x = \pm\frac{5}{8}$$

$\because$ x is positive, hence $x = \dfrac{5}{8}$

(b) $AC = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 2\times1+3\times-1 & 2\times0+3\times4 \\ 5\times1+7\times-1 & 5\times0+7\times4 \end{bmatrix}$$

$$= \begin{bmatrix} 2-3 & 0+12 \\ 5-7 & 0+28 \end{bmatrix} = \begin{bmatrix} -1 & 12 \\ -2 & 28 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 0\times0+4\times-1 & 0\times4+4\times7 \\ -1\times0+7\times-1 & -1\times4+7\times7 \end{bmatrix}$$

$$= \begin{bmatrix} 0-4 & 0+28 \\ 0-7 & -4+49 \end{bmatrix} = \begin{bmatrix} -4 & 28 \\ -7 & 45 \end{bmatrix}$$

$$10C = 10\begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ -10 & 40 \end{bmatrix}$$

$\therefore$ $AC + B^2 - 10C = \begin{bmatrix} -1 & 12 \\ -2 & 28 \end{bmatrix} + \begin{bmatrix} -4 & 28 \\ -7 & 45 \end{bmatrix} - \begin{bmatrix} 10 & 0 \\ -10 & 40 \end{bmatrix}$

$$= \begin{bmatrix} -1-4-10 & 12+28-0 \\ -2-7+10 & 28+45-40 \end{bmatrix}$$

$$= \begin{bmatrix} -15 & 40 \\ 1 & 33 \end{bmatrix}$$

(c) L.H.S. $= (1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta)$

$$= \left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)\left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)$$

$$= \left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)\left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)$$

$$= \frac{(\sin\theta + \cos\theta - 1)(\sin\theta + \cos\theta + 1)}{\sin\theta\cos\theta}$$

$$= \frac{(\sin\theta + \cos\theta)^2 - (1)^2}{\sin\theta\cos\theta}$$

$$[\because a^2 - b^2 = (a-b)(a+b)]$$

$$= \frac{(\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta) - 1}{\sin\theta.\cos\theta}$$

$$= \frac{(1 + 2\sin\theta\cos\theta) - 1}{\sin\theta.\cos\theta}$$

$$[\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2 = \text{R.H.S. Hence proved}$$

7. (a) Find the value of k for which the following equation has equal roots. [3]
$$x^2 + 4kx + (k^2 - k + 2) = 0$$

(b) One map drawn to a scale of 1 : 50,000, a rectangular plot of land $ABCD$ has the following dimensions. $AB = 6$ cm ; $BC = 8$ cm and all angles are right angles. Find : [3]
(i) the actual length of the diagonal distance AC of the plot in km.
(ii) the actual area of the plot in sq km.

(c) $A(2, 5)$, $B(-1, 2)$ and $C(5, 8)$ are the vertices of a triangle ABC, 'M' is a point on AB such that $AM : MB = 1 : 2$. Find the co-ordinates of 'M'. Hence find the equation of the line passing through the points C and M. [4]

Ans. (a) $x^2 + 4kx + (k^2 - k + 2) = 0$

Compare the equation to
$$ax^2 + bx + c = 0$$
$a = 1, b = 4k, c = k^2 - k + 2$
Since, roots of the equation are equal
$$b^2 - 4ac = 0$$
$$(4k)^2 - 4 \times 1 \times (k^2 - k + 2) = 0$$
$$16k^2 - 4k^2 + 4k - 8 = 0$$
$$12k^2 + 4k - 8 = 0$$
$$3k^2 + k - 2 = 0$$
$$3k^2 + 3k - 2k - 2 = 0$$
$$3k(k + 1) - 2(k + 1) = 0$$
$$(k + 1)(3k - 2) = 0$$
$$k = -1, k = 2/3$$

(b) (i) Since, $AB = 6$ cm
$$BC = 8 \text{ cm}$$
By Pythagorean theorem
$$\text{Diagonal } AC = \sqrt{AB^2 + BC^2}$$
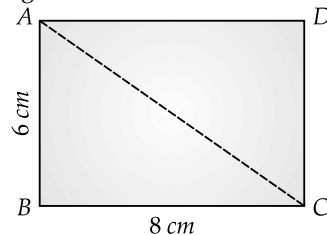$$= \sqrt{(6)^2 + (8)^2}$$
$$= \sqrt{100} = 10 \text{ cm}$$

Since, it is given that
$$1 \text{ cm} = 50,000 \text{ cm} \qquad (1 : 50000)$$
$\therefore$ $10 \text{ cm} = 5,00,000 \text{ cm}$
Diagonal $AC = 5$ km

(ii) $\text{Area} = AB \times BC$
$$= 6 \times 8$$

$$= 48 \text{ cm}^2$$
$$= 48 \times (50,000 \times 50,000)$$
$$= 1,20,00,00,00,000 \text{ cm}^2$$
$$= 12 \text{ sq. km}$$
$$[1 \text{ km}^2 = 10,00,00,00,000 \text{ cm}^2]$$

(c) Let $M(x_1, y_1)$ be the point which divides the line segment AB in the ratio 1 : 2.

Hence, Coordinate of M are

$$M(x_1, y_1) = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

$$M(x_1, y_1) = \left(\frac{1 \times (-1) + 2 \times 2}{1+2}, \frac{1 \times 2 + 2 \times 5}{1+2} \right)$$

$$= \left(\frac{-1+4}{3}, \frac{2+10}{3} \right)$$

$$= (1, 4)$$

Coordinates of M are (1, 4).

Now equation of line passing through $C(5, 8)$ and $M(1, 4)$ is

$$y - 8 = \frac{4-8}{1-5}(x - 5)$$

$$y - 8 = \frac{-4}{-4}(x - 5)$$

$$y - 8 = x - 5$$
$$y = x + 3$$

8. (a) ₹ 7500 were divided equally among a certain number of children. Had there been 20 less children, each would have received ₹ 100 more. Find the original number of children. **[3]**

(b) If the mean of the following distribution is 24, find the value of 'a'. **[3]**

Marks	0-10	10-20	20-30	30-40	40-50
Number of students	7	a	8	10	5

(c) Using ruler and compass only, construct a $\triangle ABC$ such that $BC = 5$ cm and $AB = 6.5$ cm and $\angle ABC = 120°$. **[4]**

(i) Construct a circum-circle of $\triangle ABC$.

(ii) Construct a cyclic quadrilateral $ABCD$, such that D is equidistant from AB and BC.

Ans. (a) Let the number of children be x and ₹ y is given to each children then

$$x.y = 7,500 \qquad ...(i)$$
and $(x - 20)(y + 100) = 7,500$
$$xy + 100x - 20y - 2,000 = 7,500$$
$$7,500 + 100x - 20y - 2,000 = 7,500$$
$$[\because xy = 7,500]$$
$$100x - 20y - 2,000 = 0$$
$$100x - 20 \times \frac{7,500}{x} - 2,000 = 0$$

[From (i) $y = \frac{7,500}{x}$]

$$100\, x^2 - 1,50,000 - 2,000\, x = 0$$
$$x^2 - 20x - 1,500 = 0$$
$$x^2 - 50x + 30x - 1,500 = 0$$
$$x(x - 50) + 30(x - 50) = 0$$
$$(x - 50)(x + 30) = 0$$
$$x = 50, -30$$

The number of children can not be – ve.
$$x = 50$$
$\therefore$ Number of children = 50

(b)

Marks	Mid x_i	f_i	$f_i x_i$
0-10	5	7	35
10-20	15	a	15a
20-30	25	8	200
30-40	35	10	350
40-50	45	5	225
		30 + a	810 + 15a

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$24 = \frac{810 + 15a}{30 + a}$$

$$720 + 24a = 810 + 15a$$
$$24a - 15a = 810 - 720$$
$$9a = 90$$
$$a = 10$$

(c) (a) Steps of construction :

(i) Draw a line segment $BC = 5$ cm.

(ii) Construct $\angle ABC = 120°$.

(iii) Cut $BA = 6.5$ cm

(iv) Join A to C.

(v) Construct perpendicular bisectors of AB and BC, intersecting at O. Join AO.

(vi) Taking O as centre, and OA as radius draw a circle, passing through A, B and C.

(b) (i) Draw the bisector of $\angle ABC$ such that it touches the circle at point D.

(ii) Join A to D and C to D.

(iii) $ABCD$ is required cyclic quadrilateral.
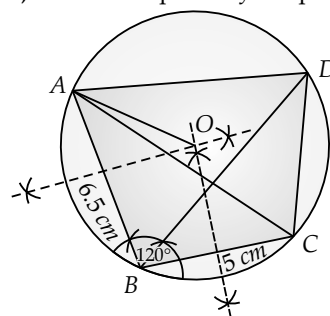
9. (a) Priyanka has a recurring deposit account of ₹ 1,000 per month at 10% per annum. If she gets ₹ 5,550 as interest at the time of maturity, find the total time for which the account was held. **[3]**
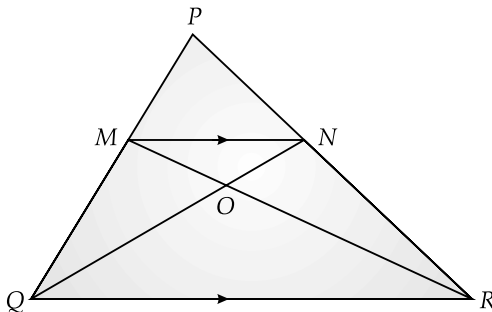
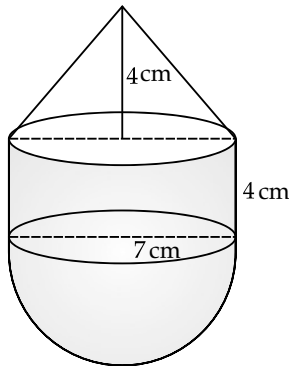**(b)** In $\triangle PQR$, MN is parallel to QR and

$$\frac{PM}{MQ} = \frac{2}{3}$$

(i) Find $\dfrac{MN}{QR}$.

(ii) Prove that $\triangle OMN$ and $\triangle ORQ$ are similar.

(iii) Find, area of $\triangle OMN$: Area of $\triangle ORQ$.

(c) The following figure represents a solid consisting of a right circular cylinder with a hemisphere at one end and a cone at the other. Their common radius is 7 cm. The height of the cylinder and cone are each of 4 cm. Find volume of the solid.

Ans. (a) $P = ₹\,1,000$, $R = 10\%$, p.a., S.I.$= ₹\,5,550$

$$n = ?$$

$$\text{SI} = \frac{Pn(n+1)}{2} \times \frac{1}{12} \times \frac{R}{100}$$

$$5,550 = \frac{1000 \times n\,(n+1) \times 10}{2 \times 12 \times 100}$$

$$n(n+1) = 1332$$
$$n^2 + n - 1332 = 0$$
$$n^2 + 37n - 36n - 1332 = 0$$
$$n(n+37) - 36\,(n+37) = 0$$
$$(n+37)(n-36) = 0$$
$$n = 36, -37$$

The value of time cannot be – ve. Hence

$n = 36$ months

or $n = 3$ years

(b) (i) Given $MN \parallel QR$, then

or $\therefore$ $\dfrac{PM}{PQ} = \dfrac{MN}{QR}$ (By Thales theorem)

or $\dfrac{PM}{PM+MQ} = \dfrac{MN}{QR}$

or $\dfrac{2}{2+3} = \dfrac{MN}{QR}$

$\Rightarrow \dfrac{MN}{QR} = \dfrac{2}{5}$

(ii) $MN \parallel QR$ (Given)

$\angle OMN = \angle ORQ$	(Alternate angle)
$\angle ONM = \angle OQR$	(Alternate angle)
$\angle MON = \angle ROQ$	(Vert. Opp. angles)

$\therefore \quad \triangle OMN \sim \triangle ORQ$ (by AAA similarity)

(iii) $\because \triangle OMN$ and $\triangle ORQ$ are similar triangle.

We know that ratio of the areas of two similar triangle is equal to ratio of the squares of the corres-ponding sides.

$$\therefore \frac{ar\,(\triangle OMN)}{ar\,(\triangle ORQ)} = \frac{MN^2}{QR^2}$$

$$= \left(\frac{MN}{QR}\right)^2 = \left(\frac{2}{5}\right)^2$$

$$= \frac{4}{25}$$

Hence, $ar\,(\triangle OMN) : ar\,(\triangle ORQ) = 4:25$

(c) Common radius $(r) = 7$ cm

Height of cylinder = Height of cone

$$= h = 4 \text{ cm}$$

Volume of solid = volume of hemisphere + volume of cylinder + volume of cone

$$= \frac{2}{3}\pi r^3 + \pi r^2 h + \frac{1}{3}\pi r^2 h$$

$$= \frac{2}{3}\pi \times 7 \times 7 \times 7 + \pi \times 7 \times 7 \times 4 + \frac{1}{3} \times \pi \times 7 \times 7 \times 4$$

$$= \frac{686\pi}{3} + 196\,\pi + \frac{196\pi}{3}$$

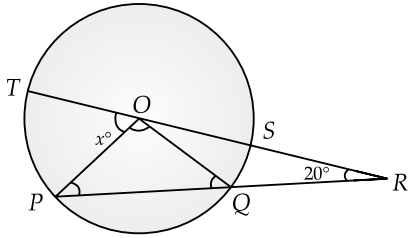$$= \frac{686\pi + 588\pi + 196\pi}{3}$$

$$= \frac{1,470\pi}{3} = 490\pi \text{ cm}^3$$

$$= \frac{490 \times 22}{7} = 1540 \text{ cm}^3$$

10. (a) Use Remainder theorem to factorize the following polynomial : **[3]**

$$2x^3 + 3x^2 - 9x - 10$$

(b) In the figure given below 'O' is the centre of the circle. If $QR = OP$ and $\angle ORP = 20°$. Find the value of 'x' giving reasons. **[3]**

(c) **The angle of elevation from a point P of the top of a tower QR, 50 m high is 60° and that of the tower PT from a point Q is 30°. Find the height of the tower PT, correct to the nearest metre.** **[4]**
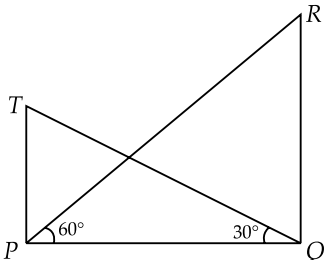

Ans. (a) Let $f(x) = 2x^3 + 3x^2 - 9x - 10$

The factors of the constant terms are $\pm 1, \pm 2, \pm 5$ and ± 10.

We have

$f(-1) = 2(-1)^3 + 3(-1)^2 - 9(-1) - 10$

$= -2 + 3 + 9 - 10 = 0$

So, $(x + 1)$ is a factor of $f(x)$.

We now divide $f(x) = 2x^3 + 3x^2 - 9x - 10$ by $(x + 1)$ to get the other factor of $f(x)$.

$$
\begin{array}{r}
2x^2 + x - 10 \\
x + 1 \overline{\smash{\big)}\ 2x^3 + 3x^2 - 9x - 10} \\
\underline{2x^3 + 2x^2} \\
\ \ x^2 - 9x - 10 \\
\underline{\ \ x^2 + x} \\
\ -10x - 10 \\
\underline{\ -10x - 10} \\
\ 0
\end{array}
$$

$\therefore$ $2x^3 + 3x^2 - 9x - 10 = (x + 1)(2x^2 + x - 10)$

$= (x + 1)[2x^2 + 5x - 4x - 10]$

$= (x + 1)[x(2x + 5) - 2(2x + 5)]$

$= (x + 1)(x - 2)(2x + 5)$

(b) Given $QR = OP$ ...(i)

and $\angle ORP = 20°$

$\because$ $\angle ORQ = \angle ORP = 20°$

$\because$ $OP = OQ$

(Radius of the circle)

$\therefore$ $QR = OQ$ (From (i))

In ΔOQR,

$\because$ $OQ = QR$

$\therefore$ $\angle QOR = \angle ORQ$

$= 20°$
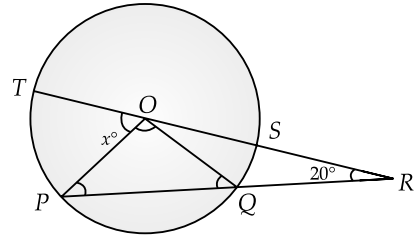
Now $\angle OQR + \angle ORQ + \angle QOR = 180°$

(By angle sums property)

$\angle OQR + 20° + 20° = 180°$

$\angle OQR = 180° - 40°$

$= 140°$

Now $\angle OQR + \angle OQP = 180°$ (By linear pair)

$140° + \angle OQP = 180°$

$\angle OQP = 180° - 140° = 40°$

In ΔPOQ,

$\because$ $OP = OQ$

$\therefore$ $\angle OPR = \angle OQP = 40°$

Now $\angle POQ + \angle OPR + \angle OQP = 180°$

(By angle sum property)

$\angle POQ + 40° + 40° = 180°$

$\angle POQ = 180° - 80° = 100°$

Now

$\angle ROQ + \angle POQ + x = 180°$ (By linear pair)

$20° + 100° + x = 180°$

$x = 180° - 120°$

$x = 60°$

(c) Let the height of the tower PT be h and distance $PQ = x$ m

In ΔPRQ,

$$\tan 60° = \frac{RQ}{PQ}$$
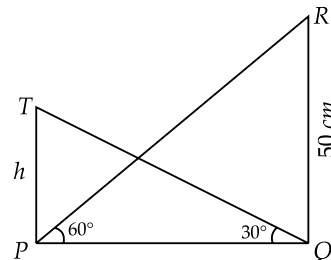
$$\sqrt{3} = \frac{50}{x}$$

or $$x = \frac{50}{\sqrt{3}}$$

In right ΔPTQ,

$$\tan 30° = \frac{PT}{PQ} = \frac{h}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{50/\sqrt{3}}$$

$$3h = 50$$

$$h = \frac{50}{3} = 16.66 = 17 \text{ m}$$

11. (a) The 4th term of an A.P. is 22 and 15th term is 66. Find the first term and the common difference. Hence find the sum of the series to 8 terms. **[4]**

(b) Use Graph paper for this question. **[6]**

A survey regarding height (in cm) of 60 boys belonging to Class 10 of a school was conducted. The following data was recorded :

Height in cm	135-140	140-145	145-150	150-155	155-160	160-165	165-170
No. of boys	4	8	20	14	7	6	1

Taking 2 cm = height of 10 cm along one axis and 2 cm = 10 boys along the other axis draw an ogive of the above distribution. Use the graph to estimate the following :

(i) the median

(ii) lower quartile

(iii) if above 158 cm is considered as the tall boys of the class. Find the number of boys in the class who are tall.

Ans. (a) Let first term be a and common difference be d.

$$T_4 = a + 3d = 22, \qquad ...(i)$$
$$T_{15} = a + 14d = 66 \qquad ...(ii)$$

Subtracting (ii) from (i)

$$a + 14d - a - 3d = 66 - 22$$
$$11d = 44$$
$$d = 4$$

From (i)

$$a + 3 \times 4 = 22$$
$$a = 22 - 12$$
$$= 10$$

Thus,
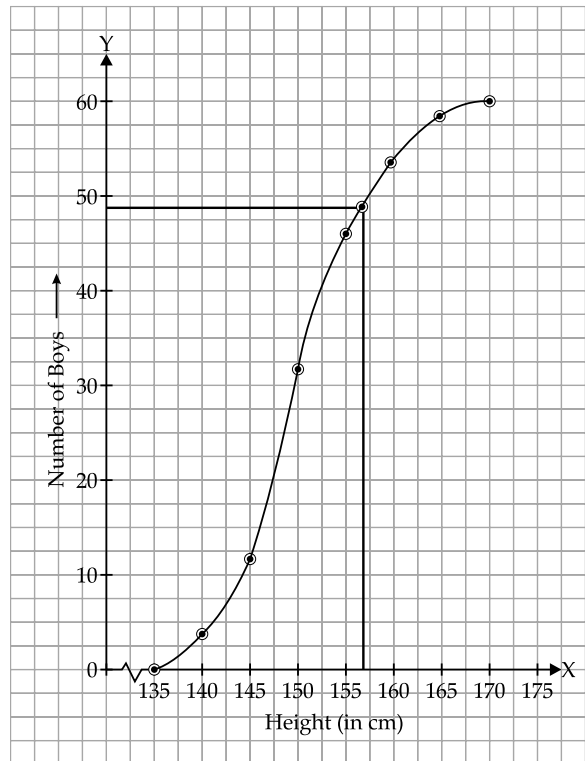
$$S_8 = \frac{8}{2} \ [2 \times 10 + (8 - 1) \ 4]$$
$$= 4 \ [20 + 28]$$
$$= 4 \times 48 = 192$$

(b)

Height (in cm)	Frequency (f)	Cumulative Frequency ($c. f.$)
135-140	4	4
140-145	8	12
145-150	20	32
150-155	14	46
155-160	7	53
160-165	6	59
165-170	1	60
	N = 60	

Here, $n = 60$

(i) Median = $\frac{60}{2}$ th term = 30th term = 149.5

(ii) Lower quartile = $\frac{60}{4}$ th term = 15th term = 145.5

(iii) Through mark of 158 on x-axis, draw a vertical line which meets the graph at a point on the ogive. Then through that point, draw a horizontal line which meets the y-axis at the mark of 48. Therefore, number of boys in the class who are tall = 60 – 48 = 12.

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