ICSE Solved Paper 2019
Mathematics

Class-X

(Maximum Marks : 80)

(Time allowed : Two hours and a half)

*Attempt **all** questions from **Section A** and **any four** questions from **Section B.***

All working, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answer.

Omission of essential working will result in loss of marks.

The intended marks for questions or parts of question are given in brackets [ ].

Mathematical tables are provided.

SECTION-A (40 marks)

*Attempt **all** questions from this Section.*

1. (a) Solve the following inequation and write down the solution set: [3]

$11x - 4 < 15x + 4 \leq 13x + 14, x \in W$

Represent the solution on a real number line.

(b) A man invests ₹ 4500 in shares of a company which is paying 7.5% dividend. If ₹ 100 shares are available at a discount of 10%. [3]

Find:

(i) Number of shares he purchases.

(ii) His annual income.

(c) In a class of 40 students, marks obtained by the students in a class test (out of 10) are given below: [4]

Marks	1	2	3	4	5	6	7	8	9	10
Number of students	1	2	3	3	6	10	5	4	3	3

Calculate the following for the given distribution:

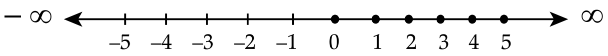
(i) Median

(ii) Mode

Ans. (a) $11x - 4 < 15x + 4 \leq 13x + 14, x \in W$

$11x - 4 < 15x + 4$	$15x + 4 \leq 13x + 14$
$11x - 15x < 4 + 4$	$15x - 13x \leq 14 - 4$
$-4x < 8$	$2x \leq 10$
$x > \frac{8}{-4}$	$x \leq \frac{10}{2}$
$x > -2$	$x \leq 5$
$-2 < x \leq 5$	

But $x \in W$, therefore required solution set is {0, 1, 2, 3, 4, 5}. The solution set on real number line is shown below.



(b) Investment = ₹ 4500,

Face value of shares = ₹ 100

Rate of dividend = 7.5%, Discount = 10%

(i) Market value of 1 share = 100 – 10% = 100 – 10 = ₹ 90

$$\text{Number of shares} = \frac{\text{Investment}}{\text{M.V. of 1 share}}$$

$$= \frac{4500}{90} = 50$$

Number of shares he purchases = 50

(ii) Annual Income = Rate of Dividend × Face value of 1 share × no. of shares

$$= \frac{7.5}{100} \times 100 \times 50$$

Annual Income = ₹ 375

(c)

Marks (x)	Number of students (f)	*c.f.*
1	1	1
2	2	3
3	3	6
4	3	9
5	6	15
6	10	25
7	5	30
8	4	34
9	3	37
10	3	40

As, n is even.

$\therefore$ Median $= \dfrac{\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th}}{2}$ term

$= \dfrac{\left(\frac{40}{2}\right)^{th} + \left(\frac{40}{2}+1\right)^{th}}{2}$ term

$= \dfrac{20^{th} + 21^{th}}{2}$ term

Median $= \dfrac{6+6}{2} = 6$ marks

Here, maximum frequency is 10 whose value is 6

$\therefore$ Mode = 6 marks

$\therefore$ Median = Mode = 6 marks

2. (a) Using the factor theorem, show that (x – 2) is a factor of $x^3 + x^2 - 4x - 4$. Hence, factorise the polynomial completely. [3]

(b) Prove that:

$(\text{cosec}\,\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta - \cot\theta) = 1$ [3]

(c) In an Arithmetic Progression (A.P.) the fourth and sixth terms are 8 and 14 respectively. Find the: [4]

(i) first term

(ii) common difference

(iii) sum of the first 20 terms.

Ans. (a) Let $f(x) = x^3 + x^2 - 4x - 4$

Put $x - 2 = 0$

$x = 2$

$\therefore$ $f(2) = (2)^3 + (2)^2 - 4(2) - 4$

$= 8 + 4 - 8 - 4$

Remainder $= f(2) = 0$

$\therefore$ $(x - 2)$ is a factor of $f(x) = x^3 + x^2 - 4x - 4$

Now, $f(x) = x^3 + x^2 - 4x - 4$

$= x^2(x+1) - 4(x+1)$

$= (x+1)(x^2 - 4)$

$= (x+1)\{(x)^2 - (2)^2\}$

$= (x + 1)(x + 2)(x - 2)$

Alternative Method:

As, $(x - 2)$ is a factor of $x^3 + x^2 - 4x - 4$.

	$x^2 + 3x + 2$
$x - 2$	$x^3 + x^2 - 4x - 4$
	$x^3 - 2x^2$
	$(-)\ (+)$
	$3x^2 - 4x - 4$
	$3x^2 - 6x$
	$(-)\ (+)$
	$2x - 4$
	$2x - 4$
	$(-)\ (+)$
	$\times$

$\therefore$ $x^3 + x^2 - 4x - 4 = (x-2)(x^2 + 3x + 2)$

$= (x-2)\{x^2 + 2x + x + 2\}$

$= (x-2)\{x(x+2) + 1(x+2)\}$

$= (x-2)(x+2)(x+1)$

$\therefore$ $x^3 + x^2 - 4x - 4 = (x-2)(x+1)(x+2)$

(b) L.H.S. $= (\text{cosec}\,\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta)$

$= \left(\dfrac{1}{\sin\theta} - \sin\theta\right)\left(\dfrac{1}{\cos\theta} - \cos\theta\right)\left(\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}\right)$

$= \left(\dfrac{1-\sin^2\theta}{\sin\theta}\right)\left(\dfrac{1-\cos^2\theta}{\cos\theta}\right)\left(\dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right)$

$= \left(\dfrac{\cos^2\theta}{\sin\theta}\right)\left(\dfrac{\sin^2\theta}{\cos\theta}\right)\left(\dfrac{1}{\sin\theta\cos\theta}\right)$

$= \dfrac{\cos^2\theta \cdot \sin^2\theta}{\cos^2\theta\sin^2\theta}$

= 1 = R.H.S. Hence proved.

(c) Let the first term of an A.P. be 'a' and common difference be 'd'.

Now $a_4 = 8$

$a + (4-1)d = 8$ $[a_n = a + (n-1)d]$

$a + 3d = 8$ ... (i)

and $a^6 = 14$

$a + (6 - 1) = 14$

$a + 5d = 14$ ... (ii)

By subtracting eq. (i) from eq. (ii)

$a + 5d = 14$

$a + 3d = 8$

(–) (–) (–)

2d-6

$d = \dfrac{6}{2} = 3$

From eq. (i)

a+3(3)=8

a= 8-9 = -1

$\therefore$ a = -1

$S_n = \dfrac{n}{2}\{2a + (n-1)d\}$

$S_{20} = \dfrac{20}{2}\{2 \times -1 + (20-1)3\}$

= 10 (-2 + 57)

= 10 × 55

S_{20} = 550

(i) Hence, First term of an A.P. = -1
(ii) Common difference of an A.P. =3
(iii) Sum of first 20 terms = 550

3. (a) Simplify **[3]**

$$\sin A\begin{bmatrix}\sin A & -\cos A\\ \cos A & \sin A\end{bmatrix}+\cos A\begin{bmatrix}\cos A & \sin A\\ -\sin A & \cos A\end{bmatrix}$$

(b) M and N are two points on the X axis and Y axis respectively. **[3]**
P(3, 2) divides the line segment MN in the ratio 2 : 3.

Find:
(i) the coordinates of M and N.
(ii) slope of the line MN.

(c) A solid metallic sphere of radius 6 cm is melted and made into a solid cylinder of height 32 cm. Find the: **[4]**
(i) radius of the cylinder.
(ii) curved surface area of the cylinder.
[Take $\pi = 3.1$**]**

Ans. (a)

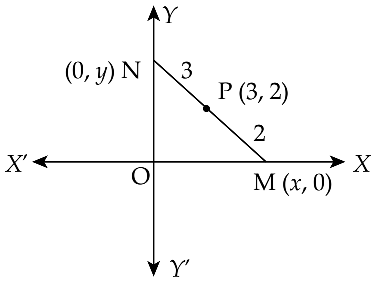
$$\sin A\begin{bmatrix}\sin A & -\cos A\\ \cos A & \sin A\end{bmatrix}+\cos A\begin{bmatrix}\cos A & \sin A\\ -\sin A & \cos A\end{bmatrix}$$

$$=\begin{bmatrix}\sin^2 A & -\sin A\cos A\\ \sin A\cos A & \sin^2 A\end{bmatrix}+\begin{bmatrix}\cos^2 A & \cos A\sin A\\ -\sin A\cos A & \cos^2 A\end{bmatrix}$$

$$=\begin{bmatrix}\sin^2 A+\cos^2 A & 0\\ 0 & \sin^2 A+\cos^2 A\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I_2$$

(b) Let the coordinate of the points M and N be $(x, 0)$ and $(0, y)$ respectively.
MP = 2 units and PN = 3 units (given)



(i) $P_x = \dfrac{m_1x_2 + m_2x_1}{m_1 + m_2}$

$$3 = \frac{2\times 0 + 3\times x}{2+3}$$

$$3\times 5 = 3x$$

$$\therefore\ x = 5$$

Hence, coordinate of point M is (5, 0).

$$P_y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

$$2 = \frac{2\times y + 3\times 0}{2+3}$$

$$2\times 5 = 2y$$

$$\therefore\ y = 5$$

Hence, coordinate of the point N is (0, 5).

(ii) Slope of line MN

$$(m) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-0}{0-5} = \frac{5}{-5}$$

$\therefore$ Slope of line MN = – 1

(c) Radius of sphere (R) = 6 cm
Height of cylinder (h) = 32 cm
Given, $\pi = 3.1$

(i) Volume of cylinder = Volume of sphere

$$\pi r^2h = \frac{4}{3}\pi R^3$$ [where, r is the radius of cylinder]

$$r^2h = \frac{4}{3}R^3$$

$$r^2\times 32 = \frac{4}{3}\times(6)^3$$

$$r^2 = \frac{4\times 6\times 6\times 6}{3\times 32}$$

$$r^2 = 9 \quad \therefore \quad r = 3$$

Radius of cylinder = 3 cm

(ii) Curved surface area of cylinder = $2\pi rh$
$= 2\times 3.1\times 3\times 32 = 18.6\times 32 = 595.2\ \text{cm}^2$
$\therefore$ C.S.A. of cylinder = 595.2 cm^2

4. (a) The following numbers, $k + 3, k + 2, 3k - 7$ **and** $2k - 3$ **are in proportion. Find** k**.** **[3]**

(b) Solve for x **the quadratic equation** $x^2 - 4x - 8 = 0$**.** **[3]**
Give your answer correct to three significant figures.

(c) Use ruler and compass only for answering this question. **[4]**
Draw a circle of radius 4 cm. Mark the centre as O. Mark a point P outside the circle at a distance of 7 cm from the centre. Construct two tangents to the circle from the external point P.
Measure and write down the length of any one tangent.

Ans. (a) $(k + 3)$, $(k + 2)$, $(3k - 7)$ and $(2k - 3)$ are in proportion.

$$\therefore\ \frac{k+3}{k+2} = \frac{3k-7}{2k-3}$$

$\Rightarrow (k + 3)(2k - 3) = (k + 2)(3k - 7)$
$\Rightarrow 2k^2+3k-9 = 3k^2-k-14$
$\Rightarrow k^2 - 4k - 5 = 0$

$\Rightarrow k^2 - 5k + k - 5 = 0$

$\Rightarrow k(k - 5) + 1(k - 5) = 0$

$\Rightarrow (k - 5)(k + 1) = 0$

$k - 5 = 0 \Rightarrow k = 5$

or $k + 1 = 0 \Rightarrow k = -1$

So, $k = 5$ and -1

(b) $x^2 - 4x - 8 = 0$

$x^2 - 4x + 4 - 4 - 8 = 0$

$(x^2 - 4x + 4) - 12 = 0$

$(x - 2)^2 = 12$

$x - 2 = \pm\sqrt{12}$ [Taking square root on both sides]

$x - 2 = \pm 2\sqrt{3}$

$x - 2 = \pm 2(1.732)$

$x = 2 + 3.464$ and $x = 2 - 3.464$

$x = 2 \pm 3.464$

$x = 5.464$ and $x = -1.464$

$\therefore\ x = 5.46$ and $x = -1.46$

(c) Steps of construction

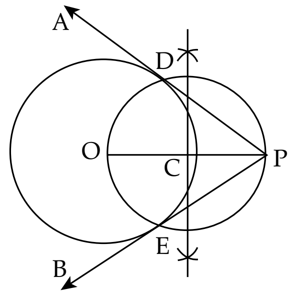
(i) Firstly, draw a circle of 4 cm having centre O.

(ii) Draw a line segment OP = 7 cm.

(iii) Draw a perpendicular bisector on OP which intersect at C.

(vi) With C as centre and OC as radius, draw a circle. This circle intersect the given circle at points D and E.

(v) Join PD and PE, which are the required tangents.



(vi) PD = 5.7 cm

SECTION-B (40 marks)

*Attempt any **four** questions from this Section*

5. (a) There are 25 discs numbered 1 to 25. They are put in a closed box and shaken thoroughly.A disc is drawn at random from the box. [3]

Find the probability that the number on the disc is:

(i) an odd number.

(ii) divisible by 2 and 3 both.

(iii) a number less than 16.

(b) Rekha opened a recurring deposit account for 20 months. The rate of interest is 9% per annum and Rekha receives ₹ 441 as interest at the time of maturity. [4]

Find the amount Rekha deposited each month.

(c) Use a graph sheet for this question. [4]

Take 1 cm = 1 unit along both x and y axis.

(i) Plot the following points:

A(0, 5), B(3, 0), C(1, 0) and D(1, –5)

(ii) Reflect the points B, C and D on the y axis and name them as B', C' and D' respectively.

(iii) Write down the coordinates of B', C' and D'.

(iv) Join the points A, B, C, D, D', C', B', A in order and give a name to the closed figure ABCDD'CB'A.

Ans. (a) Total number of discs, $n(S) = 25$

(i) An odd number on disc

$= \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25\}$

$n(E) = 13$

Probability of getting odd number on disc

$$= \frac{\text{Odd number on disc}}{\text{Total number of disc}}$$

$$= \frac{n(E)}{n(S)} = \frac{13}{25}$$

(ii) Number on disc, which is divisible by 2 and 3 both = {6, 12, 18, 24}

$n(E) = 4$

Probability of a number divisible by 2 and 3 both $P(E) = \frac{n(E)}{n(S)}$

$$P(E) = \frac{4}{25}$$

(iii) Number of disc having a number less than 16, $n(E) = 15$

$$P(E) = \frac{n(E)}{n(S)} = \frac{15}{25} = \frac{3}{5}$$

(b) Interest on recurring deposit account

$$= \frac{P.n(n+1) \times r}{2400}$$

where P = Amount deposit in each month

n = number of months = 20 months

r = Rate of Interest = 9%

$$I = \frac{P.n(n+1)r}{2400}$$

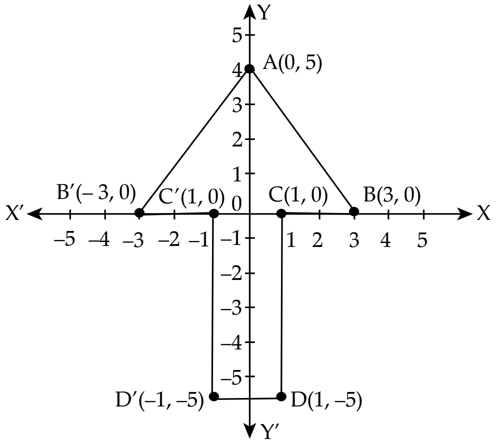
$$441 = \frac{P(20)(20+1) \times 9}{2400}$$

$$P = \frac{441 \times 2400}{20 \times 21 \times 9}$$

P = ₹ 280

Hence, Rekha deposited each month of amount ₹ 280.

(c) (i) Plot the given points on graph paper.



(ii) The reflection of the points B, C and D on y-axis are B', C' and D′ are shown in the figure.

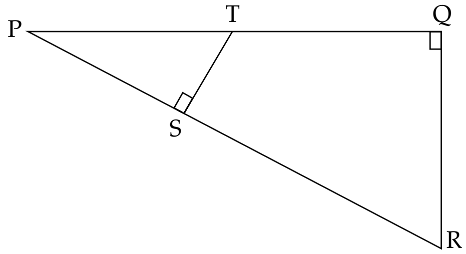
(iii) $B'(-3,0)$, $C'(-1,0)$ and $D'(-1,-5)$

(iv) Arrow

6. (a) In the given figure, ∠PQR = ∠PST = 90°, PQ = 5 cm and PS = 2 cm. [3]

(i) Prove that ΔPQR ~ ΔPST.

(ii) Find Area of ΔPQR : Area of quadrilateral SRQT.



(b) The first and last term of a Geometrical Progression (G.P.) are 3 and 96 respectively. If the common ratio is 2, find: [3]

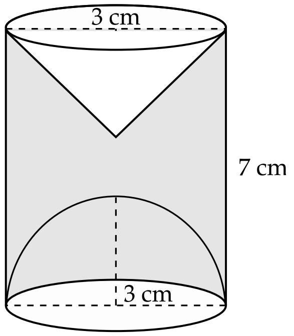
(i) 'n' the number of terms of the G.P.

(ii) Sum of the n terms.

(c) A hemispherical and a conical hole is scooped out of a solid wooden cylinder. Find the volume of the remaining solid where the measurements are as follows: [4]

The height of the solid cylinder is 7 cm, radius of each of hemisphere, cone and cylinder is 3 cm. Height of cone is 3 cm.

Give your answer correct to the nearest whole number. $\left|\text{Take } \pi = \frac{22}{7}\right|$.



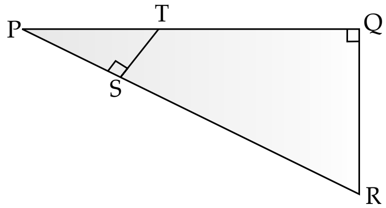
Ans.

(a) (i) In ΔPQR and ΔPST,

∠RPQ = ∠TPS (common)

∠PQR = ∠PST = 90° (given)

∴ ΔPQR ~ ΔPST (AA similarity test)



(ii) $\frac{\text{ar } \Delta PQR}{\text{ar}\Delta \text{ PST}} = \frac{PQ^2}{PS^2}$

$\frac{ar\ \Delta \text{ PQR}}{ar\ \Delta \text{ PST}} = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$

arΔPQR = 25k and ar ΔPST = 4k (where $k \neq 0$)

Area of quad. SRQT = arΔPQR – arΔPST

= 25k – 4k = 21k

arΔPQR : ar quad. SRQT = 25k : 21k

= 25 : 21

(b) First term of G.P. (a) = 3

last term of G.P. $(a_n) = 96$

and common ratio r = 2

(i) Let the number in G.P. be n, then

$a_n = 96$

$ar^{n-1} = 96$

$3(2)^{n-1} = 96$

$2^{n-1} = \frac{96}{3} = 32$

$2^{n-1} = 2^5$

Comparing the powers, we get

$n - 1 = 5$

$n = 6$

Number of terms in G.P. = 6

(ii) $S_n = \dfrac{a(r^n - 1)}{r - 1}$

$= \dfrac{3(2^6 - 1)}{2 - 1}$

$= 3(64 - 1)$

$= 3 \times 63 \ = 189$

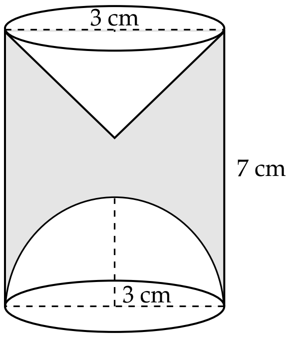
Sum of n terms of G.P. = 189

(c) Given, height of cylinder (H) = 7 cm

Radius of hemisphere (r) = 3 cm

Height of cone(h) = 3 cm

$\pi = \dfrac{22}{7}$



Volume of remaining figure = (Volume of cylinder – Volume of cone – Volume of hemisphere)

$= \pi r^2 H - \left(\dfrac{1}{3}\pi r^2 h + \dfrac{2}{3}\pi r^3\right)$

$= \pi r^2 H - \dfrac{1}{3}\pi r^2 h - \dfrac{2}{3}\pi r^3$

$= \pi r^2 \left(H - \dfrac{1}{3}h - \dfrac{2}{3}r\right)$

$= \dfrac{22}{7} \times 3 \times 3\left(7 - \dfrac{3}{3} - \dfrac{2}{3} \times 3\right)$

$= \dfrac{22 \times 3 \times 3}{7}$ (7 – 1 – 2)

$= \dfrac{198}{7} \times 4$

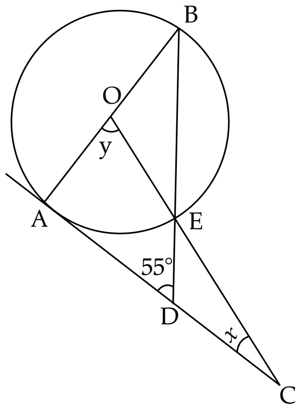
$= \dfrac{792}{7} \text{cm}^3$

$\therefore$ Volume of remaining solid = $113\dfrac{1}{7} \text{cm}^3$

= 113 cm^3

7. (a) In the given figure AC is a tangent to the circle with centre O. [3]

If ∠ADB = 55°, find x and y. Give reasons for your answers.



(b) The model of a building is constructed with the scale factor 1 : 30. [3]

(i) If the height of the model is 80 cm, find the actual height of the building in meters.

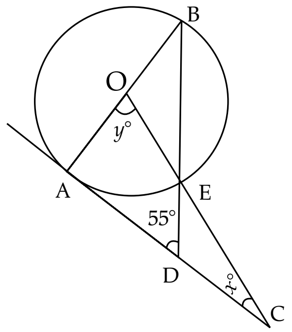
(ii) If the actual volume of a tank at the top of the building is 27 m^3, find the volume of the tank on the top of the model.

(c) Given $\begin{vmatrix} 4 & 2 \\ -1 & 1 \end{vmatrix}$**, where M is a matrix and I is unit matrix of order 2×2. [4]**

(i) State the order of matrix M.

(ii) Find the matrix M.

Ans. (a) In ΔABD,



∠A + ∠D + ∠B = 180° (sum of all angles of Δ)

90° + 55° +∠B = 180°

(AC is the tangent of circle)

∠B = 180° – 145°

$\therefore$ ∠B = 35°

∠AOE = 2∠ABE

(same arc make angles at center and circumference)

$y = 2 \times 35°$

$\therefore$ y = 70° = ∠AOE

In Δ AOC,

∠A + ∠O + ∠C = 180°

90° + 70° + x° = 180°

x = 180 – 160°

x = 20°

(b) (i) $\frac{\text{Height of model}}{\text{Actual height of the building}} = \frac{1}{30}$

$\frac{80 \text{ cm}}{\text{Actual height of the building}} = \frac{1}{30}$

$$\text{Actual height of the building} = 80 \times 30 \text{ cm}$$
$$= 2400 \text{ cm} = 24 \text{ m}$$

(ii)

$$\frac{\text{Volume of the tank on the top of model}}{\text{Actual volume of the tank at the top of building}} = \left(\frac{1}{30}\right)^3$$

$$\frac{\text{Volume of the tank on the top of model}}{27\text{m}^3} = \frac{1}{27000}$$

$$\text{Volume of the tank on the top of model} = \frac{27}{27000}$$
$$= 0.001 \text{ m}^3$$

(c) Given, $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} \text{M} = 6\text{I}$

(i) $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ is the order of 2×2

and I is also the order of 2×2

∴ M will be order of 2×2.

(ii) Let $\text{M} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = 6\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4a + 2c & 4b + 2d \\ -a + c & -b + d \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$$

$4a + 2c = 6$ ...(i)

$4b + 2d = 0$ ...(ii)

$-a + c = 0$ ...(iii)

$-b + d = 6$ ...(iv)

From (i) and (iii)

$a = c = 1$

From (ii) and (iv)

$b = -2$ and $d = 4$

∴ $\text{M} = \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$

8. (a) The sum of the first three terms of an Arithmetic Progression (A.P.) is 42 and the product of the first and third term is 52. Find the first term and the common difference. [3]

(b) The vertices of a ΔABC are A(3, 8), B(–l, 2) and C(6, –6), Find: [3]

(i) Slope of BC.

(ii) Equation of a line perpendicular to BC and passing through A.

(c) Using ruler and a compass only construct a semi-circle with diameter BC = 7 cm. Locate a point A on the circumference of the semicircle such that A is equidistant from B and C. Complete the cyclic quadrilateral ABCD, such that D is equidistant from AB and BC. Measure ∠ADC and write it down. [4]

Ans. (a) Let the first three terms be $(a - d)$, a and $(a + d)$.

Sum of the first three terms = 42

$$a - d + a + a + d = 42$$
$$3a = 42$$
$$\therefore \quad a = \frac{42}{3} = 14$$

Product of the first and third term = 52

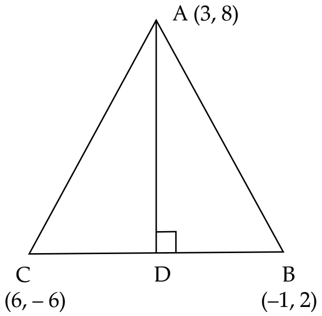
$$(a - d)(a + d) = 52$$
$$a^2 - d^2 = 52$$
$$(14)^2 - d^2 = 52$$
$$d^2 = 196 - 52 = 144$$
$$\therefore \quad d = \pm\sqrt{144} = \pm 12$$

∴ First term = 2

and common difference = ± 12

(b) (i) Slope of BC $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6 - 2}{6 + 1}$

Slope of BC $= \frac{-8}{7}$



(ii) Slope of BC × Slope of AD = – 1 (BC ⊥ AD)

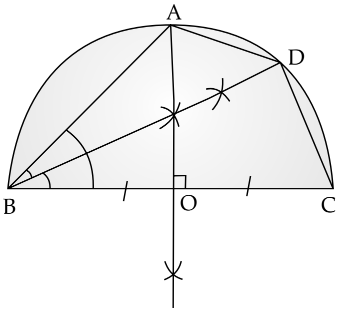
$\frac{-8}{7} \times$ Slope of AD $(m) = -1$

$m = \frac{7}{8}$

Equation of line perpendicular to BD i.e,. AD is

$$\therefore \quad y - y_1 = m(x - x_1)$$
$$y - 8 = \frac{7}{8}(x - 3)$$
$$8y - 64 = 7x - 21$$
$$7x - 8y + 43 = 0$$

(c) Firstly, draw a semicircle of radius 3.5 cm having centre O.



Justification :

A is the equidistant from the point B and C.

$\therefore$ A is lying on perpendicular bisector of BC. D is the equidistant from the side AB and BC.

In ΔAOB,

$AO = OB$ and $\angle AOB = 90°$

$\therefore$ $\angle ABO = 45°$

In cyclic quadrilateral ABCD,

$\angle ABC + \angle ADC = 180°$ (Opposite angles of cyclic quadrilateral)

$45° + \angle ADC = 180°$

$\angle ADC = 180° - 45°$

$\therefore$ $\angle ADC = 135°$

9. (a) The data on the number of patients attending a hospital in a month are given below. Find the average (mean) number of patients attending the hospital in a month by using the shortcut method. [3]

Take the assumed mean as 45. Give your answer correct to 2 decimal places.

Number of patients	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Number of days	5	2	7	9	2	5

(b) Using properties of proportion solve for *x*, given [3]

$$\frac{\sqrt{5x}+\sqrt{2x-6}}{\sqrt{5x}-\sqrt{2x-6}}=4$$

(c) Sachin invests ₹ 8500 in 10%, ₹ 100 shares at ₹ 170. He sells the shares when the price of each share rises by ₹ 30. He invests the proceeds in 12%, ₹ 100 shares at ₹ 125. Find: [4]

(i) the sale proceeds.

(ii) the number of ₹ 125 shares he buys.

(iii) the change in his annual income.

Ans. (a) Here class width $h = 10$,

Number of patients	Number of days (f)	x	$d = \frac{x - A}{h}$	fd
10 – 20	5	15	– 3	– 15
20 – 30	2	25	– 2	– 4
30 – 40	7	35	– 1	– 7
40 – 50	9	$\boxed{45 = A}$	0	0
50 – 60	2	55	1	2
60 – 70	5	65	2	10
	$\Sigma f = 30$			$\Sigma fd = -14$

$$\bar{x} = A + \frac{\Sigma fd}{\Sigma f} \times h$$

$$= 45 + \frac{(-14)}{30} \times 10$$

$= 45 - 4.666$

$= 40.334$

$\bar{x} = 40.33$

(b) $$\frac{\sqrt{5x}+\sqrt{2x-6}}{\sqrt{5x}+\sqrt{2x-6}} = 4$$

By componendo-dividendo

$$\frac{\sqrt{5x}+\sqrt{2x-6}+\sqrt{5x}-\sqrt{2x-6}}{\sqrt{5x}+\sqrt{2x-6}-\sqrt{5x}+\sqrt{2x-6}} = \frac{4+1}{4-1}$$

$$\frac{2\sqrt{5x}}{2\sqrt{2x-6}} = \frac{5}{3}$$

$$\frac{\sqrt{5x}}{\sqrt{2x-6}} = \frac{5}{3}$$ [squaring both sides]

$$\frac{5x}{2x-6} = \frac{25}{9}$$

$50x - 150 = 45x$

$5x = 150$

$$x = \frac{150}{5}$$

$\therefore$ $x = 30$

(c) Total investment = ₹ 8500

Market value of each share = ₹ 170

Number of shares purchased = $\frac{8500}{170}$ = 50

Dividend received = ₹ $\left(\frac{10}{100}\right)$ × 50 × 100 = ₹500

Now, market value of each share = ₹ (170 + 30) = ₹ 200

Amount received on selling = ₹ (50 × 200) = ₹10000

Market value of new shares = ₹ 125 each

Number of shares purchased = $\frac{10000}{125}$ = 80

Dividend received = $\left(\frac{12}{100}\right)$ × 80 ×100 = ₹960

Change in income = ₹ (960 – 500) = ₹ 460

Selling price of these shares = Market value of one share × Number of shares

= 200 × 50

= ₹ 10,000

Number of shares purchased = $\frac{10000}{125}$

= 80

Annual Income of these shares = $\frac{12}{100}$ × 100 × 80

= ₹ 960

The change in his Annual Income = ₹ (960 – 500) = ₹ 460

(i) Sale proceeds = ₹ 10000

(ii) Number of ₹ 125 shares = 80

(iii) Change in his annual Income = ₹ 460

10. (a) Use graph paper for this question. **[6]**

The marks obtained by 120 students in an English test are given below:

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100
No. of students	5	9	16	22	26	18	11	6	4	3

Draw the ogive and hence, estimate:

(i) the median marks.

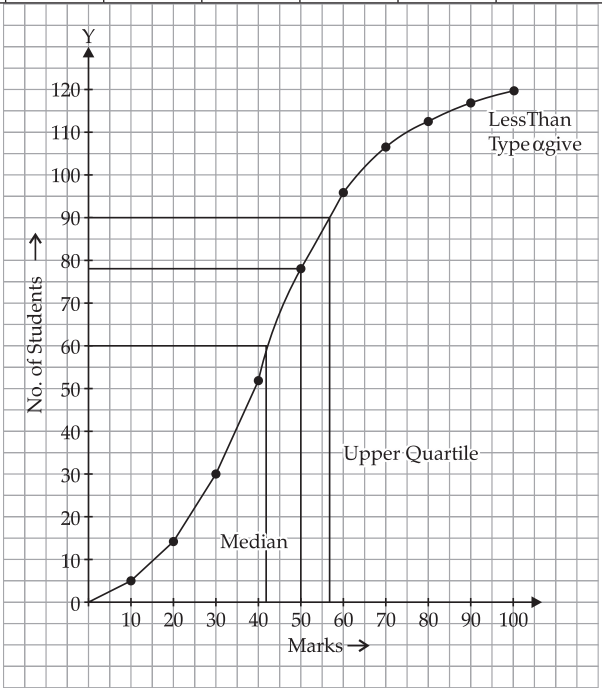
(ii) the number of students who did not pass the test if the pass percentage was 50.

(iii) the upper quartile marks.

(b) A man observes the angle of elevation of the top of the tower to be 45°. He walks towards it in a horizontal line through its base. On covering 20 m the angle of elevation changes to 60°. Find the height of the tower correct to 2 significant figures. **[4]**

Ans. (a)

Marks	No. of students	Less than	*c.f.* Less Than Type
0-10	5	10	5
10-20	9	20	14
20-30	16	30	30
30-40	22	40	52
40-50	26	50	78
50-60	18	60	96
60-70	11	70	107
70-80	6	80	113
80-90	4	90	117
90-100	3	100	120
	120		



Here, N = 120

$\frac{N}{2} = \frac{120}{2} = 60$

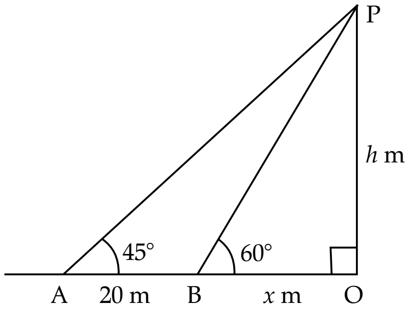
(i) Median = 42 marks

(ii) The number of student who did not pass the test if the pass percentage was 50 is 78.

(iii) The upper quartile marks is 57.

(b) Let the height of tower be *h* m.

Angle of elevation of the top of tower is 45° and 60° from the points A and B respectively.



$\therefore$ AB = 20 m

Let OB be x m,

and OP is the tower having height = h m.

In ΔAOP, $\angle$O = 90°

$$\tan A = \frac{OP}{AO}$$

$$\tan 45° = \frac{h}{x+20}$$

$$1 = \frac{h}{x+20}$$

$$x + 20 = h \quad ...(i)$$

In ΔBOP, $\angle$O = 90°

$$\tan B = \frac{OP}{OB}$$

$$\tan 60° = \frac{h}{x}$$

$$\sqrt{3} = \frac{h}{x}$$

$$h = \sqrt{3}x \quad ...(ii)$$

from eq. (i) and (ii)

$$h = \sqrt{3}(h-20) \Rightarrow h(\sqrt{3}-1) = 20\sqrt{3}$$

$$h = \frac{20\sqrt{3}}{\sqrt{3}-1}$$

$$h = \frac{20\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$h = \frac{20\sqrt{3}(\sqrt{3}+1)}{3-1}$$

$$h = \frac{20(3+\sqrt{3})}{2}$$

$$h = 10(3+\sqrt{3})$$

$$h = 10(3+1.73)$$

$$h = 47.3 \text{ m}$$

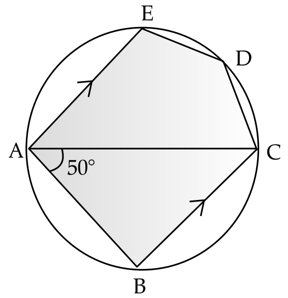
$\therefore$ Height of the tower = 47 m

11. (a) Using the Remainder Theorem, find the remainders obtained when x^3 + (Rx+8)x +R is divided by x + 1 and x – 2. [3]

Hence, find k if the sum of the two remainders is 1.

(b) The product of two consecutive natural numbers which are multiples of 3 is equal to 810. Find the two numbers. [3]

(c) In the given figure, ABCDE is a pentagon inscribed in a circle such that AC is a diameter and side BC||AE. If $\angle$BAC = 50°, find giving reasons: [4]



(i) $\angle$ACB

(ii) $\angle$EDC

(iii) $\angle$BEC

Hence, prove that BE is also a diameter.

Ans. (a) Let R_1 and R_2 are remainders when given polynomial is divided by (x + 1) and (x – 2) respectively.

Let $P(x) = x3 + (kx+8)\,x+8$

Remainder

$R_1 = P(-1) = (-1)^3 + (K(-1)+8)(-1)+k$

$$R_1 = P(-1) = -1 + k - 8 + k = 2k - 9$$

$R_1 = 2k - 9$

Remainder

$R_2 = P(2) = (2)^3 + \{k(2)+8\}(2) + k$

$R_2 = P(2) = 8 + 4k + 16 + k$

$R_2 = 5k + 24$

$R_1 + R_2 = 1$ (Given)

$2k - 9 + 5k + 24 = 1$

$7k = -14$

$k = -2 \quad \therefore \; k = -2$

(b) Let the two consecutive natural numbers that are multiple of 3 be $3x$ and $3x + 3$.

Given : Product of two consecutive numbers = 810

$$3x(3x+3) = 810$$

$$9x(x+1) = 810$$

$$x(x+1) = \frac{810}{9}$$

$$x^2 + x - 90 = 0$$

$$x^2 + 10x - 9x - 90 = 0$$

$$x(x+10) - 9(x+10) = 0$$

$$(x+10)(x-9) = 0$$

If, $x + 10 = 0 \Rightarrow x = -10$

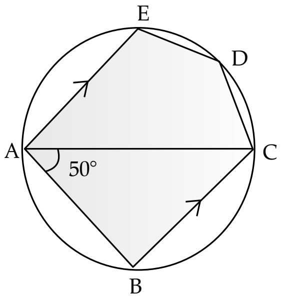
It is not natural number

If, $x - 9 = 0 \Rightarrow x = 9$

$\therefore$ Numbers are $3x = 3 \times 9 = 27$

and $3x + 3 = 27 + 3 = 30$

(c) (i) AC is a diameter of circle.



$\therefore$ $\angle ABC = 90°$

In ΔABC,

$\angle BAC + \angle ABC + \angle ACB = 180°$ (sum of angles of Δ)

$50° + 90° + \angle ACB = 180°$

$\therefore$ $\angle ACB = 180° - 140°$

$\therefore$ $\angle ACB = 40°$

(ii) $BC \| AE$

$\therefore$ $\angle EAC = \angle ACB$ (Alternative angles)

$\angle EAC = 40°$

Since, ACDE is a cyclic quadrilateral.

$\therefore$ $\angle EAC + \angle EDC = 180°$

(Sum of opposite angles of cyclic quadrilateral is 180°)

$\therefore$ $\angle EDC = 180° - 40°$

$\angle EDC = 140°$

(iii) $\angle BEC = \angle BAC$ (same arc (BC))

$\therefore$ $\angle BEC = 50°$

$\angle BAE = \angle BAC + \angle CAE = 50° + 40° = 90°$

$\therefore$ BE is a diameter, as the angle subtended by a diameter on any point of circle is 90°.

❑❑❑