

# ICSE Solved Paper 2020 Mathematics

## Class-X

(Maximum Marks : 80)

(Time allowed : Two hours and a half)

Attempt all questions from Section A and any four questions from Section B.

All working, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answer.

Omission of essential working will result in loss of marks.

The intended marks for questions or parts of question are given in brackets [ ].

Mathematical tables are provided.

### SECTION-A

(40 marks)

Attempt all questions from this Section

1. (a) Solve the following Quadratic Equation : [3]

$$x^2 - 7x + 3 = 0$$

Give your answer correct to two decimal places.

(b) Given  $A = \begin{bmatrix} x & 3 \\ y & 3 \end{bmatrix}$

If  $A^2 = 3I$ , where  $I$  is the identity matrix of order 2, find  $x$  and  $y$ . [3]

- (c) Using ruler and compass construct a triangle  $ABC$  where  $AB = 3$  cm,  $BC = 4$  cm and  $\angle ABC = 90^\circ$ . Hence construct a circle circumscribing triangle  $ABC$ . Measure and write down the radius of the circle. [4]

- Ans. (a)  $x^2 - 7x + 3 = 0$   
Compare the equation by  $ax^2 + bx + c = 0$   
then,  $a = 1, b = -7$  and  $c = 3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{7 \pm \sqrt{49 - 4 \times 1 \times 3}}{2 \times 1}$$

$$= \frac{7 \pm \sqrt{37}}{2}$$

$$= \frac{7 \pm \sqrt{37}}{2} = \frac{7 \pm 6.08}{2}$$

$$\therefore x = \frac{7 + 6.08}{2} = \frac{13.08}{2}$$

$$= 6.54$$

and  $x = \frac{7 - 6.08}{2} = \frac{0.92}{2}$

$$= 0.46$$

Hence,  $x = 6.54$  (Approx.) and  $x = 0.46$  (Approx).

(b)  $A = \begin{bmatrix} x & 3 \\ y & 3 \end{bmatrix}$

$$A^2 = A \times A$$

$$= \begin{bmatrix} x & 3 \\ y & 3 \end{bmatrix} \begin{bmatrix} x & 3 \\ y & 3 \end{bmatrix}$$

$$= \begin{bmatrix} x^2 + 3y & 3x + 9 \\ xy + 3y & 3y + 9 \end{bmatrix}$$

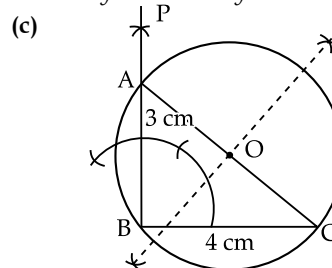
$$A^2 = 3I \text{ (Given)}$$

$$\begin{bmatrix} x^2 + 3y & 3x + 9 \\ xy + 3y & 3y + 9 \end{bmatrix} = 3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

Comparing the elements of matrices

$$3x + 9 = 0 \Rightarrow x = -3$$

$$3y + 9 = 3 \Rightarrow y = -2$$



Steps of construction

- (i) Draw  $BC = 4$  cm
- (ii) Make  $\angle PBC = 90^\circ$
- (iii) Taking centre  $B$  and radius  $3$  cm draw an arc which intersect  $BP$  at the point  $A$ .
- (iv) Join  $AC$ ,  $\triangle ABC$  is required triangle.  $\angle B = 90^\circ$   
 $\therefore AC$  is the diameter of circle.
- (v) Draw perpendicular bisector of  $AC$ .
- (vi) Taking centre  $O$  and radius equal to  $BO$  draw a circle which passes through vertex  $A, B$  and  $C$ .  
Radius of circumscribe circle =  $2.5$  cm.

2. (a) Use factor theorem to factorise  $6x^3 + 17x^2 + 4x - 12$  completely. [3]

- (b) Solve the following inequation and represent the solution set on the number line. [3]

$$\frac{3x}{5} + 2 < x + 4 \leq \frac{x}{2} + 5, x \in R$$

(c) Draw a Histogram for the given data, using a graph paper : [4]

Weekly Wages (in ₹)	No. of People
3000 - 4000	4
4000 - 5000	9
5000 - 6000	18
6000 - 7000	6
7000 - 8000	7
8000 - 9000	2
9000 - 10000	4

Estimate the mode from the graph.

Ans. (a)  $p(x) = 6x^3 + 17x^2 + 4x - 12$   
 $p(-2) = 6(-2)^3 + 17(-2)^2 + 4(-2) - 12$   
 $= -48 + 68 - 8 - 12$   
 $= 68 - 68 = 0$   
 $\therefore (x + 2)$  is a factor of given polynomial  $p(x)$

$$\begin{array}{r} x+2 \overline{) 6x^3 + 17x^2 + 4x - 12} \\ \underline{6x^3 + 12x^2} \phantom{+ 4x - 12} \\ 5x^2 + 4x - 12 \\ \underline{5x^2 + 10x} \phantom{- 12} \\ -6x - 12 \\ \underline{-6x - 12} \\ (+) (+) \\ 0 \end{array}$$

$$\begin{aligned} \therefore 6x^3 + 17x^2 + 4x - 12 &= (x + 2)(6x^2 + 5x - 6) \\ &= (x + 2)\{6x^2 + 9x - 4x - 6\} \\ &= (x + 2)\{3x(2x + 3) - 2(2x + 3)\} \\ &= (x + 2)(2x + 3)(3x - 2) \end{aligned}$$

(b)  $\frac{3x}{5} + 2 < x + 4 \leq \frac{x}{2} + 5, x \in R$

$$\frac{3x}{5} + 2 < x + 14 \qquad x + 4 \leq \frac{x}{2} + 5$$

$$\frac{3x}{5} - x < 14 - 2 \qquad x - \frac{x}{2} \leq 5 - 4$$

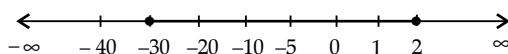
$$\frac{3x - 5x}{5} < 12 \qquad \frac{x}{2} \leq 1$$

Multiply by 5  $\qquad x \leq 2$   
 $-2x < 12 \times 5$

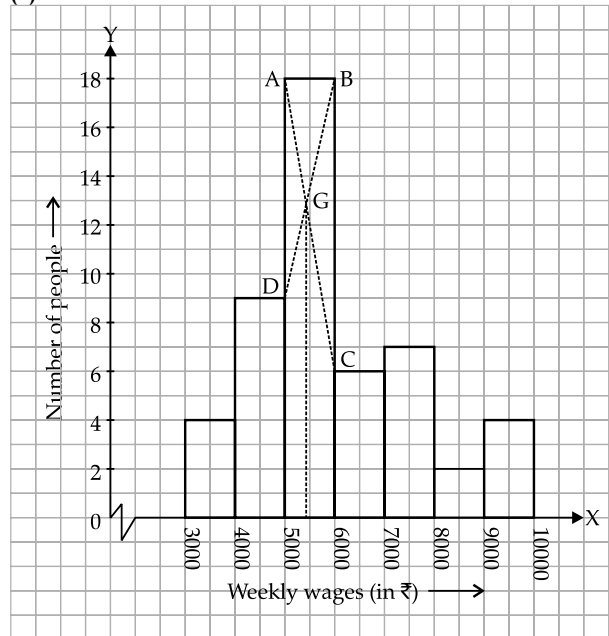
$$x > \frac{60}{-2}$$

$$x > -30$$

$$\therefore x > -30 < x \leq 2$$



(c)

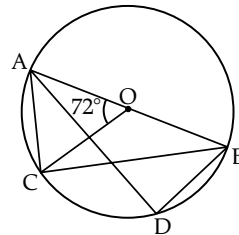


= Mode - 5400 (approx)

3. (a) In the figure given below, O is the centre of the circle and AB is a diameter. [3]

If  $AC = BD$  and  $\angle AOC = 72^\circ$ . Find:

- (i)  $\angle ABC$
- (ii)  $\angle BAD$
- (iii)  $\angle ABD$



(b) Prove that : [3]

$$\frac{\sin A}{1 + \cot A} = \frac{\cos A}{1 + \tan A} = \sin A - \cos A$$

(c) In what ratio is the line joining  $P(5, 3)$  and  $Q(-5, 3)$  divided by the  $y$ -axis? Also find the coordinates of the point of intersection. [4]

Ans. (a) AB is the diameter of circle

$$\therefore \angle ADB = 90^\circ \quad (\text{semicircle angle})$$

(i)  $\angle ABC = \frac{1}{2} \angle AOC$   
 (same arc angles at circumference and centres)

$$\therefore \angle ABC = \frac{1}{2} \times 72^\circ = 36^\circ$$

(ii)  $AC = BD$   
 $\therefore \angle ABC = \angle BAD$  (equal arcs make equal angle)

$$\therefore \angle BAD = 36^\circ$$

(iii) In  $\triangle ABD$

$$\angle ABD + \angle BAD + \angle ADB = 180^\circ$$

(sum of all angles of  $\Delta$ )

$$\begin{aligned} \angle ABD + 36^\circ + 90^\circ &= 180^\circ \\ \angle ABD &= 180^\circ - 126^\circ \\ \angle ABD &= 54^\circ \end{aligned}$$

(b) L.H.S. =  $\frac{\sin A}{1 + \cot A} - \frac{\cos A}{1 + \tan A}$

$$= \frac{\sin A}{1 + \frac{\cos A}{\sin A}} - \frac{\cos A}{1 + \frac{\sin A}{\cos A}}$$

$$= \frac{\sin^2 A}{\sin A + \cos A} - \frac{\cos^2 A}{\cos A + \sin A}$$

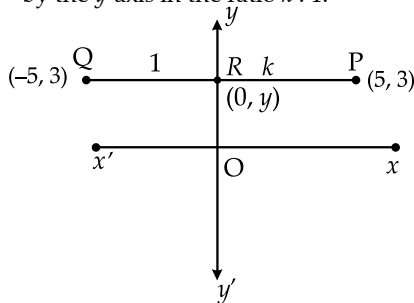
$$= \frac{\sin^2 A - \cos^2 A}{\sin A + \cos A}$$

$$= \frac{(\sin A + \cos A)(\sin A - \cos A)}{(\sin A + \cos A)}$$

$$= \sin A - \cos A$$

= R.H.S. **Hence proved.**

(c) Let the line joining the (5, 3) and (-5, 3) divided by the y-axis in the ratio k : 1.



Let the coordinate of the point of intersection R be (0, y).

$$R_x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

$$0 = \frac{1 \times 5 + k(-5)}{1 + k}$$

$$5 - 5k = 0 \Rightarrow k = 1$$

$$R_y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$$y = \frac{1 \times 3 + k \times 3}{1 + k}$$

$$y = \frac{3 + 3}{1 + 1} = 3 \quad [\text{Put, } k = 1]$$

Ratio = k : 1 = 1 : 1

Point of intersection of the line (0, 3).

4. (a) A solid spherical ball of radius 6 cm is melted and recast into 64 identical spherical marbles. Find the radius of each marble. [3]

(b) Each of the letters of the word 'AUTHORIZES' is written on identical circular discs and put in a bag. They are well shuffled. If a disc is drawn at random from the bag, what is the probability that the letter is :

(i) a vowel ?

(ii) one of the first 9 letters of the English alphabet which appears in the given word ?

(iii) one of the last 9 letters of the English alphabet which appears in the given word ? [3]

(c) Mr. Bedi visits the market and buys the following articles :

Medicines costing ₹ 950, GST @ 5%

A Pair of shoes costing ₹ 3000, GST @ 18%

A Laptop bag costing ₹ 1000 with a discount of 30% GST @ 18%

(i) Calculate the total amount of GST paid.

(ii) The total bill amount including GST paid by Mr. Bedi. [4]

Ans. (a) Volume of solid spherical ball =  $\frac{4}{3} \pi r^3$

$$= \frac{4}{3} \pi (6)^3$$

$$= \frac{4}{3} \pi \times 216$$

$$= \frac{216 \times 4}{3} \pi \text{ cm}^3$$

Let the radius of spherical marble be R cm.

Then 64 × volume of spherical marbles = volume of spherical ball

$$64 \times \frac{4}{3} \pi R^3 = \frac{4 \times 216}{3} \pi \text{ cm}^3$$

$$R^3 = \frac{216}{64}$$

$$R = \sqrt[3]{\frac{216}{64}} = \frac{6}{4} = \frac{3}{2} \text{ cm}$$

$$= 1.5 \text{ cm}$$

∴ Radius of spherical marble is 1.5 cm.

(b) 'AUTHORIZES'

Total number of disc n(S) = 10

(i) Number of discs a letter of vowel = 5

(A, U, O, I, E) ∴ n(E) = 5

Probability of a disc is vowel

$$= \frac{n(E)}{n(S)} = \frac{5}{10} = \frac{1}{2}$$

(ii) One of the first 9 letter in english alphabet

$$= 4 (A, E, H, I)$$

∴ n(E) = 4

Probability of a disc is drawn is written one of first 9 letters of english alphabet

$$= \frac{n(E)}{n(S)} = \frac{4}{10}$$

$$= \frac{2}{5}$$

(iii) One of the last 9 (R, S, T, U, Z) letters of english alphabet

$$n(E) = 5$$

Probability of a disc is drawn is written one of the last 9 letters of english alphabet

$$= \frac{n(E)}{n(S)} = \frac{5}{10}$$

$$= \frac{1}{2}$$

(c) Medicine costing = ₹ 950

$$\text{GST @ 5\%} = \frac{5}{100} \times 950 = ₹ 47.50$$

A pair of shoes costing = ₹ 3000

$$\text{GST @ 18\%} = \frac{18}{100} \times 3000 = ₹ 540$$

Laptop bag costing ₹ 1000 with a discount of 30%

$$\therefore \text{Net cost} = 1000 - \frac{30}{100} \times 1000$$

$$= ₹ 700$$

$$\text{GST @ 18\%} = \frac{18}{100} \times 700 = ₹ 126$$

(i) Total amount of GST = ₹ 47.50 + 540 + 126 = ₹ 713.50

(ii) Total bill amount including GST = ₹ 950 + 3000 + 700 + 713.50 = ₹ 4650 + 713.50 = ₹ 5363.50

**SECTION-B**

**(40 marks)**

Attempt any four questions from this Section

5. (a) A company with 500 shares of nominal value ₹ 120 declares an annual dividend of 15%. Calculate :

(i) the total amount of dividend paid by the company.

(ii) annual income of Mr. Sharma who holds 80 shares of the company. [3]  
If the return percent of Mr. Sharma from his shares is 10%. Find the market value of each share.

(b) The mean of the following data is 16. Calculate the value of  $f$ . [3]

Marks	5	10	15	20	25
No. of Students	3	7	$f$	9	6

(c) The 4<sup>th</sup>, 6<sup>th</sup> and the last term of a geometric progression are 10, 40 and 640 respectively. If the common ratio is positive, find the first term, common ratio and the number of terms of the series. [4]

Ans. (a) Number of shares = 500  
Nominal value = ₹ 120  
Annual dividend = 15%

(i) Total amount of dividend paid by the company = Rate of dividend × Nominal value of share × Number of share

$$= \frac{15}{100} \times 120 \times 500$$

$$= 15 \times 600 = ₹ 9000$$

∴ Total amount paid by the company = ₹ 9000

(ii) Mr sharma's Annual Income = Rate of dividend × Nominal value of share × Number of share

$$= \frac{15}{100} \times 120 \times 80$$

$$= 15 \times 96 = ₹ 1440$$

∴ Mr sharma's Annual Income = ₹ 1440

$$\text{Return of Investment} = \frac{\text{Income}}{\text{Investment}} \times 100$$

$$10 = \frac{1440}{\text{Investment}} \times 100$$

$$\therefore \text{Investment} = \frac{144000}{10} = ₹ 14400$$

$$\text{Market value of share} = \frac{\text{Investment}}{\text{No. of share}}$$

$$= \frac{14400}{80}$$

∴ Market value of share = ₹ 180

(b)

Marks	No. of Students	
(x)	(f)	$f \times x$
5	3	15
10	7	70
15	$f$	$15f$
20	9	180
25	6	150
	$25 + f$	$415 + 15f$

Mean = 16 (Given)

$$\frac{\sum fx}{\sum f} = 16$$

$$\frac{415 + 15f}{25 + f} = 16$$

$$415 + 15f = 400 + 16f$$

$$16f - 15f = 415 - 400$$

$$\therefore f = 15$$

(c) Let the first term of G.P. be  $a$ , common ratio be  $r$  and number of terms be  $n$ .

$$a_4 = 10, a_6 = 40 \text{ and } a_n = 640 \text{ (given)}$$

$$a_4 = 10$$

$$ar^3 = 10 \quad \dots(i)$$

Again,

$$a_6 = 40$$

$$ar^5 = 40 \quad \dots(ii)$$

From eqs. (i) & (ii)

$$\frac{ar^5}{ar^3} = \frac{40}{10}$$

$$\Rightarrow r^2 = 4$$

$$\therefore r = \pm 2$$

$$\therefore r = 2 \quad (r > 0)$$

From eq. (i)

$$ar^3 = 10$$

$$a(2)^3 = 10$$

$$a = \frac{10}{8} = \frac{5}{4}$$

Again,

$$a_n = 640$$

$$ar^{n-1} = 640$$

$$\frac{5}{4}(2)^{n-1} = 640$$

$$2^{n-1} = \frac{640 \times 4}{5}$$

$$2^{n-1} = 128 \times 4 = 2^9$$

Comparing the power

$$n - 1 = 9 \Rightarrow n = 10$$

$\therefore$  First term of G.P. =  $\frac{5}{4}$ , common ratio = 2

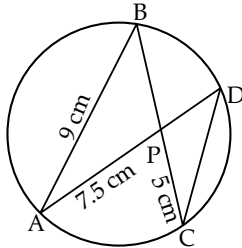
and number of terms = 10

6. (a) If  $A = \begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} -4 & 2 \\ 1 & 0 \end{bmatrix}$  [3]

Find  $A^2 - 2AB + B^2$ .

(b) In the given figure  $AB = 9$  cm,  $PA = 7.5$  cm and  $PC = 5$  cm. [3]

Chords  $AD$  and  $BC$  intersect at  $P$ .



(i) Prove that  $\triangle PAB \sim \triangle PCD$ .

(ii) Find the length of  $CD$ .

(iii) Find area of  $\triangle PAB$  : area of  $\triangle PCD$ .

(c) From the top of a cliff, the angle of depression of the top and bottom of a tower are observed to be  $45^\circ$  and  $60^\circ$  respectively. If the height of the tower is 20 m. [4]

Find :

(i) the height of the cliff.

(ii) the distance between the cliff and the tower.

Ans.(a)  $A = \begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} -4 & 2 \\ 1 & 0 \end{bmatrix}$

$$A^2 = A \times A = \begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9+0 & 0+0 \\ 15+5 & 0+1 \end{bmatrix}$$

$$\therefore A^2 = \begin{bmatrix} 9 & 0 \\ 20 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} -4 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -12+0 & 6+0 \\ -20+1 & 10+0 \end{bmatrix}$$

$$AB = \begin{bmatrix} -12 & 6 \\ -19 & 10 \end{bmatrix}$$

$$B^2 = B \times B = \begin{bmatrix} -4 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -4 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 16+2 & -8+0 \\ -4+0 & 2+0 \end{bmatrix}$$

$$\therefore B^2 = \begin{bmatrix} 18 & -8 \\ -4 & 2 \end{bmatrix}$$

$$\therefore A^2 - 2AB + B^2$$

$$\begin{bmatrix} 9 & 0 \\ 20 & 1 \end{bmatrix} - 2 \begin{bmatrix} -12 & 6 \\ -19 & 10 \end{bmatrix} + \begin{bmatrix} 18 & -8 \\ -4 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 9 & 0 \\ 20 & 1 \end{bmatrix} + \begin{bmatrix} 24 & -12 \\ 38 & -20 \end{bmatrix} + \begin{bmatrix} 18 & -8 \\ -4 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 9+24+18 & 0-12-8 \\ 20+38-4 & 1-20+2 \end{bmatrix} = \begin{bmatrix} 51 & -20 \\ 54 & -17 \end{bmatrix}$$

(b) (i)  $\triangle PAB$  and  $\triangle PCD$

$$\angle APB = \angle CPD$$

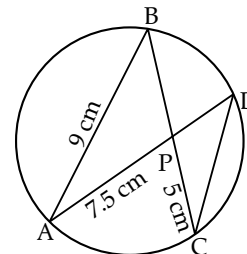
(Vertically opposite angle)

$$\angle ABP = \angle PDC$$

(same arc  $\widehat{AC}$  angles)

$$\therefore \triangle PAB \sim \triangle PCD$$

(AA similarly test)



(ii)  $\frac{PA}{PC} = \frac{AB}{CD} = \frac{PB}{PD}$

(Property of similar triangle)

$$\frac{PA}{PC} = \frac{AB}{CD}$$

$$\frac{7.5}{5} = \frac{9}{CD}$$

$$\therefore CD = \frac{9 \times 5}{7.5} = \frac{9 \times 5 \times 10}{75}$$

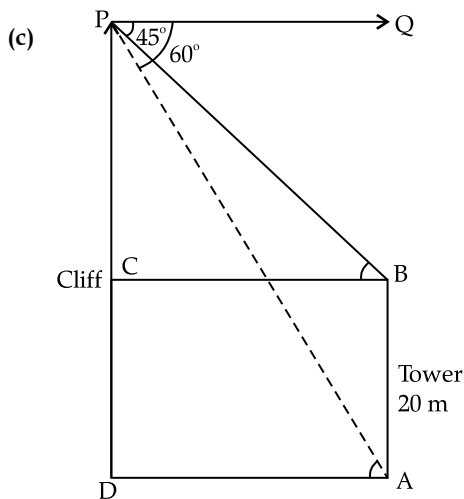
$$\therefore CD = 6 \text{ cm}$$

(iii)  $\frac{\text{ar}\triangle PAB}{\text{ar}\triangle PCD} = \frac{AP^2}{CP^2} = \left(\frac{AP}{CP}\right)^2$

$$= \left(\frac{7.5}{5}\right)^2$$

$$= \frac{9}{4}$$

$$\therefore \text{ar}\triangle PAB : \text{ar}\triangle PCD = 9 : 4$$



OP is the cliff and AB is the tower.

In  $\triangle PBC$ ,  $\tan B = \frac{CP}{BC}$   
 $\tan 45^\circ = \frac{CP}{BC}$   
 $1 = \frac{CP}{BC}$   
 $\therefore CP = BC$  ... (i)

In  $\triangle OAP$ ,  $\tan A = \frac{OP}{OA}$   
 $(\because \angle QPA = \angle PAO = 60^\circ)$   
 $\tan 60^\circ = \frac{OC + CP}{OA}$

$\sqrt{3} = \frac{AB + CP}{OA}$   
 $(\because OC = AB = 20 \text{ m})$

$\sqrt{3} = \frac{20 + CP}{BC}$  ( $OA = BC$ )

$\sqrt{3} = \frac{20 + CP}{CP}$  ( $BC = CP$ )

$\sqrt{3} CP = 20 + CP$

$(\sqrt{3} - 1)CP = 20$

$CP = \frac{20}{\sqrt{3} - 1} \text{ m}$

$CP = \frac{20}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$

$CP = \frac{20(\sqrt{3} + 1)}{3 - 1}$   
 $= 10(\sqrt{3} + 1) \text{ m}$

$CP = 10(1.732 + 1) \text{ m}$   
 $CP = 27.32 \text{ m}$

- (i) height of the cliff  
 $OP = OC + CP$

$= 20 + 27.32$

$\therefore$  Height of the cliff = 47.32 m

- (ii) Distance between the cliff and the tower

$OA = BC = CP$

$OA = 27.32 \text{ m}$

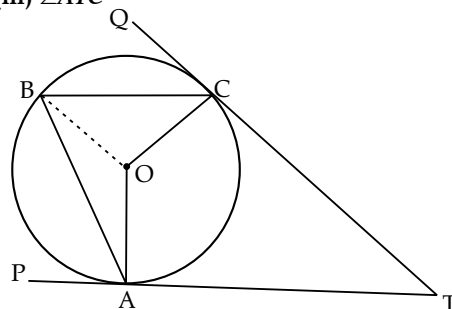
7. (a) Find the value of 'p' if the lines,  $5x - 3y + 2 = 0$  and  $6x - py + 7 = 0$  are perpendicular to each other. Hence, find the equation of a line passing through  $(-2, -1)$  and parallel to  $6x - py + 7 = 0$ . [3]

- (b) Using properties of proportion find  $x : y$ , given : [3]

$\frac{x^2 + 2x}{2x + 4} = \frac{y^2 + 3y}{3y + 9}$

- (c) In the given figure TP and TQ are two tangents to the circle with centre O, touching at A and C respectively. If  $\angle BCQ = 55^\circ$  and  $\angle BAP = 60^\circ$ , find : [4]

- (i)  $\angle OBA$  and  $\angle OBC$   
 (ii)  $\angle AOC$   
 (iii)  $\angle ATC$



Ans. (a) Slope of line  $5x - 3y + 2 = 0$  is  
 $= -\frac{\text{coefficient of } x}{\text{coefficient of } y}$   
 $= \frac{-5}{-3} = \frac{5}{3}$

Slope of line  $6x - py + 7 = 0$  is  $= \frac{-6}{-p} = \frac{6}{p}$

Lines are perpendicular each other

$\therefore$  Product of slopes = -1

$\frac{5}{3} \times \frac{6}{p} = -1$

$3p = -30 \Rightarrow p = -10$

$\therefore p = -10$

Given line  $6x - py + 7 = 0$  is  $6x + 10y + 7 = 0$

Equation of the line, parallel to  $6x + 10y + 7 = 0$  is  $6x + 10y + k = 0$

Since, line passes through  $(-2, -1)$ ,

$\therefore 6(-2) + 10(-1) + k = 0$

$-12 - 10 + k = 0 \Rightarrow k = 22$

Equation of the line parallel to  $6x + 10y + 7$  is

$6x + 10y + 22 = 0$  or  $3x + 5y + 11 = 0$

(b) 
$$\frac{x^2 + 2x}{2x + 4} = \frac{y^2 + 3y}{3y + 9}$$

(using compodendo dividendo  $\frac{a}{b} = \frac{c}{d}$ )

$$\Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$$

$$\frac{x^2 + 2x + 2x + 4}{x^2 + 2x - 2x - 4} = \frac{y^2 + 3y + 3y + 9}{y^2 + 3y - 3y - 9}$$

$$\frac{x^2 + 4x + 4}{x^2 - 4} = \frac{y^2 + 6y + 9}{y^2 - 9}$$

$$\frac{(x+2)^2}{(x+2)(x-2)} = \frac{(y+3)^2}{(y+3)(y-3)}$$

$$\frac{x+2}{x-2} = \frac{y+3}{y-3}$$

(using compodendo - dividendo)

$$\frac{x+2+x-2}{x+2-x+2} = \frac{y+3+y-3}{y+3-y+3}$$

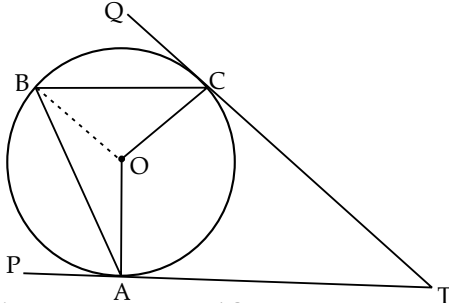
$$\frac{2x}{4} = \frac{2y}{6}$$

$$\frac{x}{2} = \frac{y}{3}$$

$$\frac{x}{y} = \frac{2}{3}$$

$\therefore x : y = 2 : 3$

(c) Given,  $\angle BCO = 55^\circ$  and  $\angle BAP = 60^\circ$



(i)  $\therefore \angle PAO = 90^\circ$   
(As PAT is the tangent)

$$\therefore \angle OAB = 90^\circ - 60^\circ = 30^\circ$$

In  $\triangle OAB$ ,  $OA = OB$  (radii of circle)

$$\angle OBA = \angle OAB = 30^\circ$$

Similarly  $\therefore \angle QCO = 90^\circ$

(As QCT is the tangent)

$$\therefore \angle OCB = 90^\circ - 55^\circ = 35^\circ$$

In  $\triangle OCB$ ,  $OC = OB$  (radii of circle)

$$\angle OBC = \angle OCB = 35^\circ$$

(ii) Now,  $\angle ABC = \angle OBA + \angle OBC$   
 $= 30^\circ + 35^\circ = 65^\circ$

Again,  $\angle AOC = 2\angle ABC = 2 \times 65^\circ = 130^\circ$

( $\therefore$  Angle subtended at the center of the circle by an arc is twice the angle subtended at the circle)

(iii)  $\angle AOC + \angle ATC = 180^\circ$   
 $130^\circ + \angle ATC = 180^\circ$   
 $\therefore \angle ATC = 180^\circ - 130^\circ = 50^\circ$   
 $\therefore \angle ATC = 50^\circ$

8. (a) What must be added to the polynomial  $2x^3 - 3x^2 - 8x$ , so that it leaves a remainder 10 when divided by  $2x + 1$ ? [3]

(b) Mr. Sona has a recurring deposit account and deposits ₹ 750 per month for 2 years. If he gets ₹ 19125 at the time of maturity, find the rate of interest. [3]

(c) Use graph paper for this question.

Take 1 cm = 1 unit on both  $x$  and  $y$  axes. [4]

(i) Plot the following points on your graph sheets.

$A(-4, 0)$ ,  $B(-3, 2)$ ,  $C(0, 4)$ ,  $D(4, 1)$  and  $E(7, 3)$

(ii) Reflect the points  $B$ ,  $C$ ,  $D$  and  $E$  on the  $x$ -axis and name them as  $B'$ ,  $C'$ ,  $D'$  and  $E'$  respectively.

(iii) Join the points  $A$ ,  $B$ ,  $C$ ,  $D$ ,  $E$ ,  $E'$ ,  $D'$ ,  $C'$ ,  $B'$  and  $A$  in order.

(iv) Name the closed figure formed.

Ans. (a)  $p(x) = 2x^3 - 3x^2 - 8x$   
 $q(x) = 2x + 1$   
Remainder = 10

Let  $k$  be added to get remainder 10 when divided by  $(2x + 1)$ .

$$\therefore p(x) = 2x^3 - 3x^2 - 8x + k$$

Remainder = 10

$$p\left(-\frac{1}{2}\right) = 10$$

$$2\left(-\frac{1}{2}\right)^3 - 3\left(-\frac{1}{2}\right)^2 - 8\left(-\frac{1}{2}\right) + k = 10$$

$$-2 \times \frac{1}{8} - 3 \times \frac{1}{4} + 4 + k = 10$$

$$-1 + 4 + k = 10$$

$$k = 7$$

Hence, 7 be added to get remainder 10 when divided by  $2x + 1$  to given polynomial.

(b) Maturity amount (M.A.) = ₹ 19125

Monthly deposit (P) = ₹ 750

Time ( $n$ ) = 2 years = 24 months

$$\text{M.A.} = P \times n + P \times \frac{r}{100} \times \frac{n(n+1)}{2} \times \frac{1}{12}$$

$$750 \times 24 + \frac{750 \times r}{2400} (24)(24+1) = 19125$$

$$18000 + \frac{750}{4} r = 19125$$

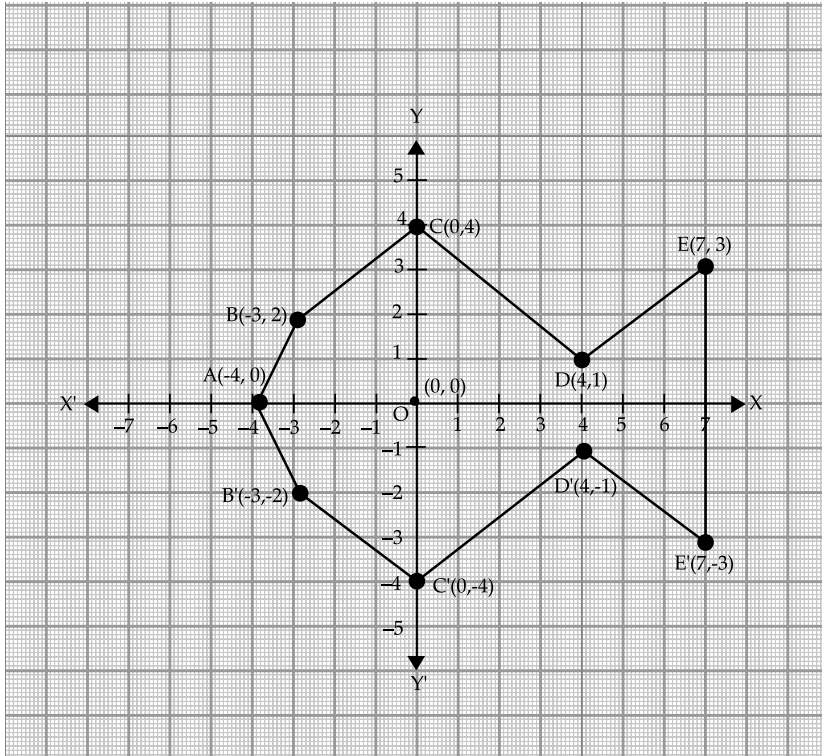
$$\frac{750r}{4} = 19125 - 18000$$

$$r = \frac{1125 \times 4}{750} = \frac{4500}{750}$$

$$r = 6$$

$\therefore$  Rate of interest = 6%

(c) (i)



(ii) B'(-3, -2), C'(0, -4), D'(4, -1) and E'(7, -3) respectively are the reflected points through the x-axis.

(iii) On joining the points A, B, C, D, E, E', D', C', B' and A in order, we get a closed figure.

(iv) Closed figure formed is nine sided polygon or nonagon, polygon fish or kite.

9. (a) 40 Students enter for a game of shot-put competition. The distance thrown (in metres) is recorded below :

Distance in m	12 – 13	13 – 14	14 – 15	15 – 16	16 – 17	17 – 18	18 – 19
Number of Students	3	9	12	9	4	2	1

Use a graph paper to draw an ogive for the above distribution.

Use a scale of 2 cm = 1 m on one axis and 2 cm = 5 students on the other axis.

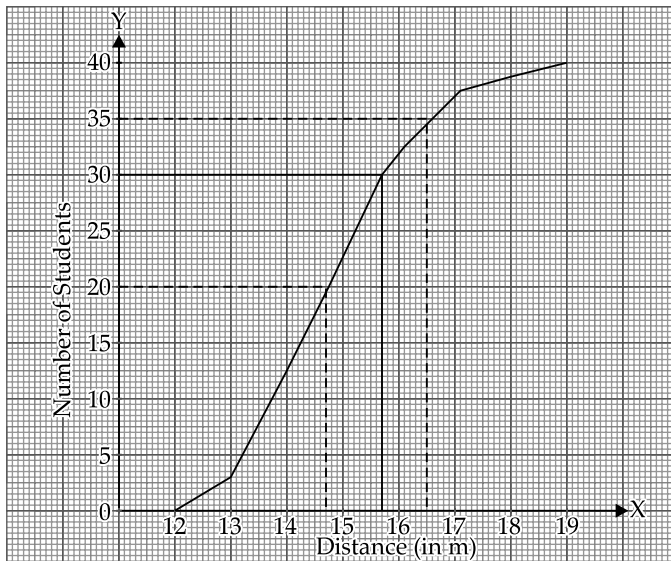
Hence, using your graph find :

- (i) the median.
- (ii) upper quartile.

(iii) number of students who cover a distance which is above  $16\frac{1}{2}$  m. [6]

(b) If  $x = \frac{\sqrt{2a+1} + \sqrt{2a-1}}{\sqrt{2a+1} - \sqrt{2a-1}}$ , prove that  $x^2 - 4ax + 1 = 0$ . [4]

Ans. (a)





Distance in (m)	Number of students	Less than	Cumulative frequency
12 -13	3	13	3
13 -14	9	14	12
14 - 15	12	15	24
15 - 16	9	16	33
16 - 17	4	17	37
17 - 18	2	18	39
18 - 19	1	19	40

- (i) Median =  $14.6 \pm 0.2$
- (ii) Upper quartile =  $15.75 \pm 0.2$
- (iii) Number of students above  $16\frac{1}{2} m = 40 - 35 = 5$

(b)  $x = \frac{\sqrt{2a+1} + \sqrt{2a-1}}{\sqrt{2a+1} - \sqrt{2a-1}}$

$$\frac{x+1}{x-1} = \frac{\sqrt{2a+1} + \sqrt{2a-1} + \sqrt{2a+1} - \sqrt{2a-1}}{\sqrt{2a+1} + \sqrt{2a-1} - \sqrt{2a+1} + \sqrt{2a-1}}$$

(Apply compodendo-dividendo  $\frac{A}{B} = \frac{C}{D}$ )  
 $\Rightarrow \frac{A+B}{A-B} = \frac{C+D}{C-D}$

$$\frac{x+1}{x-1} = \frac{2\sqrt{2a+1}}{2\sqrt{2a-1}} = \frac{\sqrt{2a+1}}{\sqrt{2a-1}}$$

Squaring both sides

$$\frac{(x+1)^2}{(x-1)^2} = \left(\frac{\sqrt{2a+1}}{\sqrt{2a-1}}\right)^2$$

$$\frac{x^2 + 2x + 1}{x^2 - 2x + 1} = \frac{2a+1}{2a-1}$$

Apply compodendo - dividendo

$$\frac{x^2 + 2x + 1 + x^2 - 2x + 1}{x^2 + 2x + 1 - x^2 + 2x - 1} = \frac{2a+1 + 2a-1}{2a+1 - 2a+1}$$

$$\frac{2(x^2 + 1)}{4x} = \frac{4a}{2}$$

$$\frac{x^2 + 1}{2x} = \frac{2a}{1}$$

$$x^2 + 1 = 4ax$$

$$x^2 - 4ax + 1 = 0 \quad \text{Hence proved.}$$

10. (a) If the 6<sup>th</sup> term of an A.P. is equal to four times its first term and the sum of first six terms is 75, find the first term and the common difference. [3]

(b) The difference of two natural numbers is 7 and their product is 450. [3]  
 Find the numbers.

(c) Use ruler and compass for this question. Construct a circle of radius 4.5 cm. Draw a chord  $AB = 6$  cm.

(i) Find the locus of points equidistant from  $A$  and  $B$ .

Mark the point where it meets the circle as  $D$ .

(ii) Join  $AD$  and find the locus of points which are equidistant from  $AD$  and  $AB$ . Mark the point where it meets the circle as  $C$ .

(iii) Join  $BC$  and  $CD$ , Measure and write down the length of side  $CD$  of the quadrilateral  $ABCD$ .

[4]

Ans. (a) Let the first term of an A.P. be  $a$  and common difference be  $d$  respectively

$$a_6 = 4 \times a \text{ (given)}$$

$$a_6 = 4a \quad \dots(i)$$

$$a + 5d = 4a \quad \{a_n = a + (n-1)d\}$$

$$3a = 5d$$

$$a = \frac{5}{3}d \quad \dots(ii)$$

$$S_6 = 75 \text{ (given)}$$

$$\frac{n}{2}[a + a_6] = 75$$

$$\frac{6}{2}(a + a_6) = 75$$

$$a + a_6 = \frac{75}{3}$$

$$a + 4a = 25$$

$$a = \frac{25}{5} = 5$$

$$\therefore a = 5$$

From equation (i)

$$a = \frac{5}{3}d$$

$$5 = \frac{5}{3}d$$

$$\therefore d = \frac{5 \times 3}{5} \times 3$$

$$\therefore d = 3$$

First term of an A.P. = 5 and common difference = 3

(b) Let the numbers be  $x$  and  $y$ .

According the given condition

$$x - y = 7 \Rightarrow y = x - 7 \quad [x > y] \quad \dots(i)$$

$$xy = 450 \quad \dots(ii)$$

From (i) & (ii)

$$x(x - 7) = 450$$

$$x^2 - 7x - 450 = 0$$

$$x^2 - 25x + 18x - 450 = 0$$

$$x(x - 25) + 18(x - 25) = 0$$

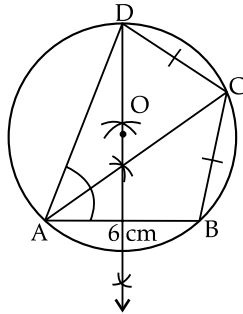
$$(x - 25)(x + 18) = 0$$

If  $x + 18 = 0 \Rightarrow x = -18$  it is not a natural number.

$$\text{If } x - 25 = 0 \Rightarrow x = 25, y = 25 - 7 = 18$$

Thus, numbers 25 and 18.

(c)



$$BC = CD = 5.1 \text{ cm (Approx.)}$$

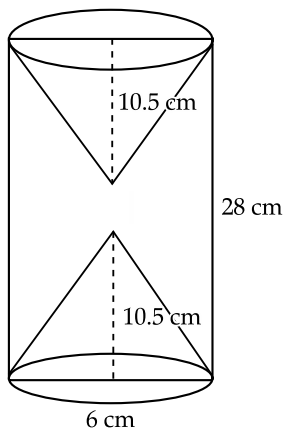
11. (a) A model of a high rise building is made to a scale of 1 : 50. [3]

(i) If the height of the model is 0.8 m, find the height of the actual building.

(ii) If the floor area of a flat in the building is  $20 \text{ m}^2$ , find the floor area of that in the model.

(b) From a solid wooden cylinder of height 28 cm and diameter 6 cm, two conical cavities are hollowed out. The diameters of the cones are also of 6 cm and height 10.5 cm.

Taking  $\pi = \frac{22}{7}$  find the volume of the remaining solid.



[3]

(c) Prove the identity

$$\left( \frac{1 - \tan \theta}{1 - \cot \theta} \right)^2 = \tan^2 \theta \quad [4]$$

Ans. (a) Scale = 1 : 50

Height of the model 0.8 m

$$\therefore \text{Height of the building} = 0.8 \times 50 \text{ m} = 40 \text{ m}$$

$$\text{Floor area of building} = 20 \text{ m}^2$$

$$\text{Floor area of the model} = \frac{20}{50 \times 50} \text{ m}^2$$

$$= \frac{1}{125} \text{ m}^2$$

$$= 0.008 \text{ m}^2$$

$\therefore$  Floor area of the model

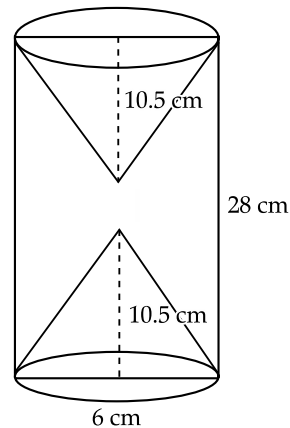
$$= 0.008 \times 10000 \text{ cm}^2$$

$$= 80 \text{ cm}^2$$

(b) Dimension of cylinder

$$\text{Height } (h) = 28 \text{ cm}$$

$$\text{Diameter } (2r) = 6 \text{ cm}$$



Dimension of conical cavities

$$\text{Height } (H) = 10.5 \text{ cm}$$

$$\text{diameter } (2r) = 6 \text{ cm}$$

Volume of remaining solid = Volume of cylinder - 2  $\times$  Volume of cavity

$$= \pi r^2 h - 2 \times \frac{1}{3} \pi r^2 H$$

$$= \pi r^2 \left( h - \frac{2}{3} H \right)$$

$$= \frac{22}{7} \times 3 \times 3 \left( 28 - \frac{2}{3} \times 10.5 \right)$$

$$= \frac{22}{7} \times 3 \times 3 (28 - 7)$$

$$= \frac{22}{7} \times 3 \times 3 \times 21$$

$$= 22 \times 3 \times 3 \times 3 = 594 \text{ cm}^3$$

Hence, volume of remaining solid =  $594 \text{ cm}^3$

$$(c) \quad \left( \frac{1 - \tan \theta}{1 - \cot \theta} \right)^2 = \tan^2 \theta$$

$$\text{L.H.S.} = \left( \frac{1 - \tan \theta}{1 - \cot \theta} \right)^2$$

$$= \left( \frac{1 - \tan \theta}{1 - \frac{1}{\tan \theta}} \right)^2$$

$$= \left( \frac{1 - \tan \theta}{\tan \theta - 1} \right)^2$$

$$= \left\{ \frac{\tan \theta (1 - \tan \theta)}{-(1 - \tan \theta)} \right\}^2$$

$$= \left\{ \frac{\tan \theta}{-1} \right\}^2 = \tan^2 \theta$$

= R.H.S. Hence proved.

□□□