ICSE Solved Paper 2022 Semester-2
Mathematics

Class-X

(Maximum Marks : 40)

(Time allowed : One hours and a half)

Attempt all questions from Section A and any three questions from Section B.
The marks intended for questions are given in brackets [ ]
Mathematical tables are provided.

SECTION-A **(10 marks)**

*(Attempt **all** questions.)*

1. Choose the correct answers to the questions from the given options. (Do not copy the question. Write the correct answer only.) [10]

(i) The probability of getting a number divisible by 3 in throwing a dice is:

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) $\frac{2}{3}$

(ii) The volume of a conical tent is 462 m^3 and the area of the base is 154 m^2. The height of the cone is:

(a) 15 m (b) 12 m

(c) 9 m (d) 24 m

(iii) The median class for the given distribution is:

Class Interval	0 - 10	10 - 20	20 - 30	30 - 40
Frequency	2	4	3	5

(a) 0-10 (b) 10-20

(c) 20-30 (d) 30-40

(iv) If two lines are perpendicular to one another then the relation between their slopes m_1 and m_2 is:

(a) $m_1 = m_2$ (b) $m_1 = \frac{1}{m_2}$

(c) $m_1 = -m_2$ (d) $m_1 \times m_2 = -1$

(v) A lighthouse is 80 m high. The angle of elevation of its top from a point 80 m away from its foot along the same horizontal line is:

(a) 60° (b) 45°

(c) 30° (d) 90°

(vi) The modal class of a given distribution always corresponds to the:

(a) interval with highest frequency

(b) interval with lowest frequency

(c) the first interval

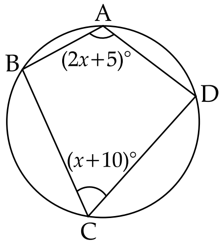
(d) the last interval

(vii) The coordinates of the point P(–3, 5) on reflecting on the X axis are:

(a) (3, 5) (b) (–3, –5)

(c) (3, –5) (d) (–3, 5)

(viii) ABCD is a cyclic quadrilateral. If ∠BAD = $(2x+5)°$ and ∠BCD = $(x + 10)°$ then x is equal to:



(a) 65° (b) 45°

(c) 55° (d) 5°

(ix) A(1, 4), B (4, 1) and C (x, 4) are the vertices of ΔABC. If the centroid of the triangle is G (4, 3) then x is equal to

(a) 2 (b) 1

(c) 7 (d) 4

(x) The radius of a roller 100 cm long is 14 cm. The curved surface area of the roller is:

(Take $\pi = \frac{22}{7}$)

(a) 13200 cm^2 (b) 15400 cm^2

(c) 4400 cm^2 (d) 8800 cm^2

Ans. (i) Option (b) is correct.

Explanation:

All possible out comes of a dice = {1, 2, 3, 4, 5, 6}

n (E) = 6

Favourable outcomes (divisible by 3)

= {3, 6}

n (F) = 2

Probability of getting a number divisible by 3

$$= \frac{\text{Favourable outcomes}}{\text{Total no. of outcomes}}$$

$$= \frac{n(F)}{n(E)} = \frac{2}{6} = \frac{1}{3}$$

(ii) Option (c) is correct.

Explanation:
Given, Volume of conical tent = 462 m^3
and Area of the base = 154 m^2
We know that

$$\text{Volume of a conical tent} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \text{Area of base} \times \text{height}$$

$$\Rightarrow \quad 462 = \frac{1}{3} \times 154 \times h$$

$$\Rightarrow \quad h = \frac{462 \times 3}{154} = 9 \text{ m}$$

(iii) Option (c) is correct.
Explanation:

C. I.	Frequency	*c. f.*
0 - 10	2	2
10 – 20	4	2 + 4 = 6
20 – 30	3 (*f*)	6 + 3 = 9
30 – 40	5	9 + 5 = 14
	14	

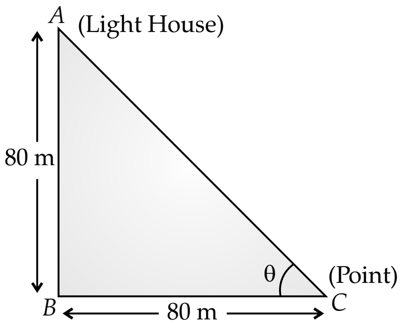
$$N = 14 \Rightarrow \frac{N}{2} = 7$$

We can see that cumulative frequency just greater than 7 is 9, which is lie in the class interval 20 – 30.
So, the median class is 20 – 30.

(iv) Option (d) is correct.
Explanation: If two lines, having slopes m_1 and m_2, perpendicular to one another, then $m_1 \times m_2 = -1$

(v) Option (b) is correct.
Explanation: Height of the light house = 80 m
Distance of the point from the foot of the light house = 80 m
Let the angle of elevation be θ, then tan θ

$$= \frac{\text{Perpendicular}}{\text{Base}}$$



$$\tan\theta = \frac{AB}{BC} = \frac{80}{80} = 1$$

$$\tan\theta = \tan 45°$$

$$(\because \tan 45° = 1)$$

$$\text{So, } \theta = 45°$$

(vi) Option (a) is correct.

(vii) Option (b) is correct.
Explanation: Given point (–3, 5) lies in II^{nd} quadrant. So, after reflection sign of Y-coordinate will invert and sign of X-coordinate remain same. Reflection of this point will be in III^{rd}-quadrant, i.e., (–3, –5).

(viii) Option (c) is correct.
Explanation: Given, $\angle BAD = (2x + 5)°$
and $\angle BCD = (x + 10)°$
So, $\angle BAD + \angle BCD = 180°$
(Sum of opposite angles of a cyclic quadrilateral are supplementary)

$$\Rightarrow \quad (2x + 5)° + (x + 10)° = 180°$$

$$(3x + 15)° = 180°$$

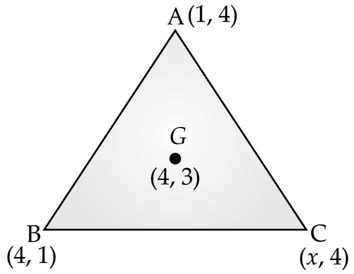
$$3x = 165°$$

$$x = \frac{165°}{3} = 55°$$

(ix) Option (c) is correct.
Explanation: Coordinates of centroid of a triangle is given by

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

So, coordinates of the centroid of triangle ABC is given by $\left(\frac{1+4+x}{3}, \frac{4+1+4}{3}\right)$ or $\left(\frac{5+x}{3}, 3\right)$



Also, coordinates of centroid of ΔABC (4, 3)

$$\text{So, } 4 = \frac{5+x}{3} \Rightarrow 12 = 5 + x \Rightarrow x = 7$$

(x) Option (d) is correct.
Explanation: We have,
radius of a roller = 14 cm
height of the roller = 100 cm
Now, C.S.A. of roller (Cylinder) = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 14 \times 100 = 8800 \text{ cm}^2$$

SECTION-B (30 marks)

(Attempt ***any three*** *questions from this Section.)*

2. (i) Prove that **[2]**

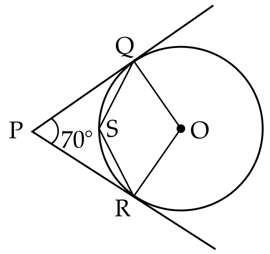
$$\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta} = 2\sec^2\theta$$

(ii) Find 'a' if A $(2a + 2, 3)$, B $(7, 4)$ and C $(2a + 5, 2)$ are collinear. **[2]**

(iii) Calculate the mean of the following frequency distribution **[3]**

Class Interval	5 - 15	15 - 25	25 - 35	35 - 45	45 - 55
Frequency	2	6	4	8	4

(iv) In the given figure O is the centre of the circle. PQ and PR are tangents and ∠QPR = 70° Calculate: **[3]**



(a) ∠QOR

(b) ∠QSR

Ans. (i) L.H.S.

$$= \frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}$$

$$= \frac{(1-\sin\theta)+(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$$

$$= \frac{2}{1-\sin^2\theta} = \frac{2}{\cos^2\theta} \quad (\because \sin^2\theta + \cos^2\theta = 1)$$

$$= 2\sec^2\theta = \text{R.H.S.} \left(\because \frac{1}{\cos\theta} = \sec\theta\right)$$

Hence Proved.

(ii) Given A $(2a + 2, 3)$, B$(7, 4)$ and C $(2a + 5, 2)$ are collinear.

When three points are colliner, then

$$\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right] = 0$$

or $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

We have

$x_1 = 2a + 2, x_2 = 7, \quad x_3 = 2a + 5$

$y_1 = 3, y_2 = 4, \quad y_3 = 2$

$\Rightarrow \quad [(2a + 2)(4 - 2) + 7(2 - 3) + (2a + 5)(3 - 4)] = 0$

$4a + 4 - 7 - 2a - 5 = 0$

$2a - 8 = 0$

$$a = \frac{8}{2} = 4$$

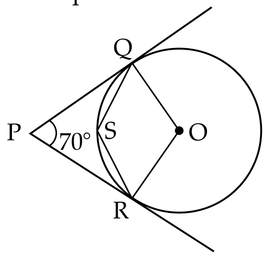
(iii)

C. I.	Class mark (x)	Frequency (f)	$f \times x$
5 – 15	10	2	20
15 – 25	20	6	120
25 – 35	30	4	120
35 – 45	40	8	320
45 – 55	50	4	200
Total		**24**	**780**

We know, Mean $(\bar{x}) = \frac{\sum fx}{\sum f}$

$$\bar{x} = \frac{780}{24} = 32.5$$

(iv) PQOR is a quadrilateral.



(a) $\angle QPR + \angle QOR = 180°$

(supplementary angles)

$70° + \angle QOR = 180°$

$\angle QOR = 110°$

(b) Reflex $\angle QOR = 360° - 110° = 250°$

$\angle QSR = \frac{1}{2}$ Reflex $\angle QOR$

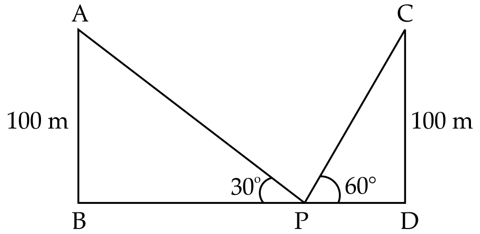
($\because$ The angle subtended by an arc at the centre is double the angle subtended by it on the remaining part of the circle.)

$\angle QSR = \frac{1}{2} \times 250° = 125°$

3. (i) A bag contains 5 white, 2 red and 3 black balls. A ball is drawn at random. What is the probability that the ball drawn is a red ball? **[2]**

(ii) A solid cone of radius 5 cm and height 9 cm is melted and made into small cylinders of radius of 0.5 cm and height 1.5 cm. Find the number of cylinders so formed. **[2]**

(iii) Two lamp posts AB and CD each of height 100 m are on either side of the road. P is a point on the road between the two lamp posts. The angles of elevation of the top of the lamp posts from the point P are 60°and 30°. Find the distances PB and PD. **[3]**



(iv) **Marks obtained by 100 students in an examination are given below. [3]**

Marks	0-10	10-20	20-30	30-40	40-50	50-60
No of students	5	15	20	28	20	12

Draw a histogram for the given data using a graph paper and find the mode.

Take 2 cm = 10 marks along one axis and 2 cm = 10 students along the other axis.

Ans. (i) Given, No. of white balls $= 5$

No. of red balls $= 2$

No. of black balls $= 3$

Total no. of balls $= 5 + 2 + 3$

$= 10$

Probability of getting a red ball

$$= \frac{\text{No. of favourable outcomes (red balls)}}{\text{Total no. of balls}}$$

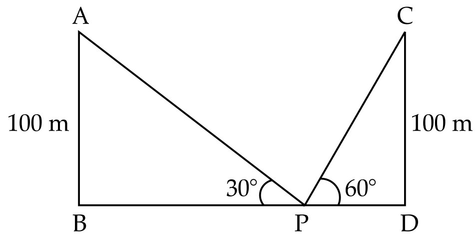
$$= \frac{2}{10} = \frac{1}{5}$$

(ii) Given,
For cone,
Radius (R) = 5 cm, Height (H) = 9 cm
For cylinder,
Radius (r) = 0.5 cm, Height (h) = 1.5 cm

$$\text{No. of cylinders} = \frac{\text{Volume of Cone}}{\text{Volume of cylinder}}$$

$$= \frac{\frac{1}{3}\pi R^2 H}{\pi r^2 h} = \frac{\frac{1}{3}\times(5)^2\times 9}{(0.5)^2\times 1.5} = \frac{25\times 3}{0.25\times 1.5} = 200$$

(iii) Given,
Heights of the lamp posts AB = CD = 100 m
and $\angle APB = 30°$;
$\angle CPD = 60°$



Now, in ΔAPR

$$\tan P = \frac{\text{Perpendicular(AB)}}{\text{Base}}$$

$$\tan 30° = \frac{AB}{BP} = \frac{100}{BP}$$

$$\frac{1}{\sqrt{3}} = \frac{100}{BP} \Rightarrow BP = 100\sqrt{3}\text{ m}$$

$$BP = 100 \times 1.732 = 173.2\text{ m}$$

Correction–In this question paper $\angle APB = 40°$ is given, which should be 30° to solve it.

Now, In ΔPCD

$$\tan P = \frac{CD}{PD} = \frac{100}{PD}$$

$$\tan 60° = \frac{100}{PD}$$

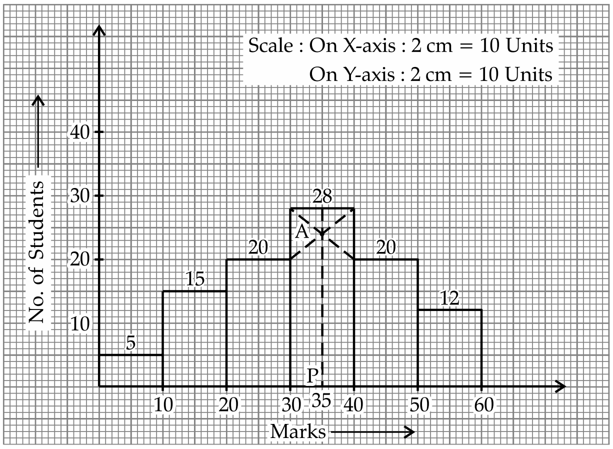
$$\sqrt{3} = \frac{100}{PD}$$

$$\Rightarrow \quad PD = \frac{100}{\sqrt{3}}\text{ m}$$

$$= \frac{100\sqrt{3}}{3} = \frac{173.2}{3}$$

$$PD = 57.73\text{ m}$$

(iv) Mode = 35 Marks



To find mode,

(i) First identify the rectangle with highest frequency (Modal class); here 30–40.

(ii) Join the top corners of the modal rectangle with immediate next corners of the adjacent rectangles.

(iii) Let the point where the joining lines cut each other (here A). Draw a perpendicular from A to X-axis. The point 'P', where the perpendicular meet the X-axis will give the mode.

4. (i) **Find a point P which divides internally the line segment joining the points A (–3, 9) and B (1, –3) in the ratio 1 : 3. [2]**

(ii) **A letter of the word 'SECONDARY' is selected at random. What is the probability that the letter selected is not a vowel? [2]**

(iii) **Use a graph paper for this question. Take 2 cm - 1 unit along both the axes. [3]**

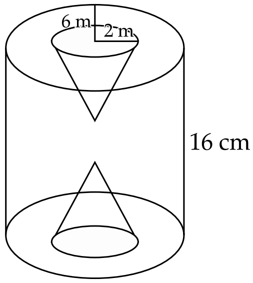
(a) **Plot the points A (0, 4), B (2, 2), C(5, 2) and D (4, 0), E(0, 0) is the origin.**

(b) **Reflect B, C, D on the Y-axis and name them as B′, C′ and D′ respectively.**

(c) Join the points ABCDD′C′B′ and A in order and give a geometrical name to the closed figure.

(iv) A solid wooden cylinder is of radius 6 cm and height 16 cm. Two cones each of radius 2 cm and height 6 cm are drilled out of the cylinder. Find the volume of the remaining solid. [3]

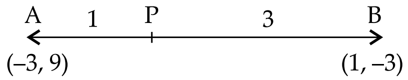
(Take $\pi = \frac{22}{7}$)



Ans. (i) Given points are

$$A(x_1, y_1) = (-3, 9)$$

and $B(x_2, y_2) = (1, -3)$



Let P(x, y) divides the line segment AB in the ratio 1:3. Then by using section formula,

$$= \left(\frac{1\times1+3\times-3}{1+3}, \frac{1\times-3+3\times9}{1+3}\right)$$

$$P(x, y) = \left(\frac{-8}{4}, \frac{24}{4}\right)$$

$$P(x, y) = (-2, 6)$$

(ii) Given word is 'SECONDARY'

No. of vowels = 3

No. of consonants = 6

Total no. of letters = 9

Probability of not selecting a vowel

= Probability of selecting a consonant

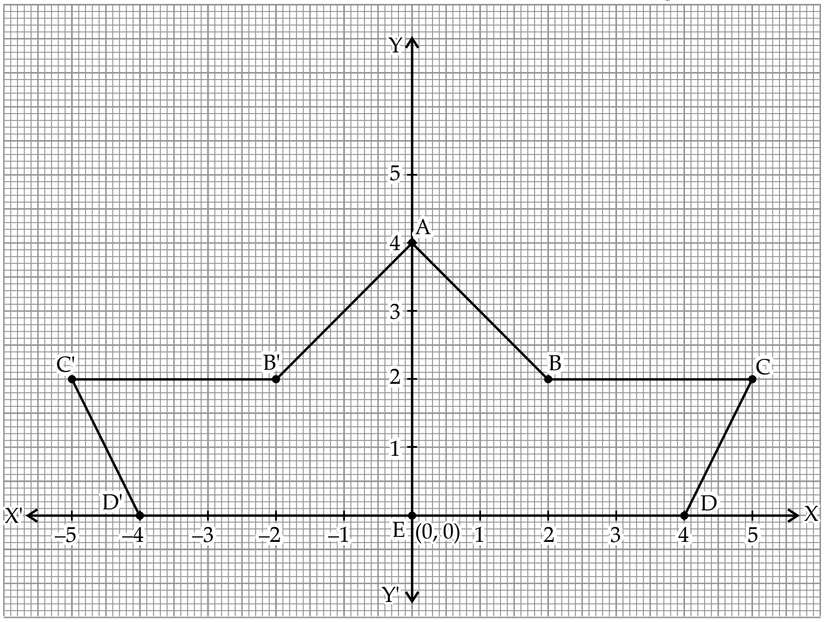
$$P\text{ (not a vowel)} = \frac{6}{9} = \frac{2}{3}$$

(iii) (a) See graph

(b) Reflected points are :

B' (–2, 2), C'(–5, 2) and D' (–4, 0)

(c) On joining the points ABCDD'C'B'A, the shape obtain is not proper geometrical shape, but collection of a quadrilateral and triangle.



(iv) Given:

For Cylinder,

Radius (R) = 6 cm

Height (H) = 16 cm

We know, Volume $= \pi R^2 H$

$$= \frac{22}{7} \times (6)^2 \times (16)$$

$$= \frac{12672}{7}\text{cm}^3$$

For cone,

Radius (r) = 2 cm

Height (h) = 6 cm

We know, Volume $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times (2)^2 \times 6$$

$$= \frac{176}{7}\text{cm}^3$$

Remaining Volume = Volume of cylinder – 2 × Volume of a cone

$$= \frac{12672}{7} - 2 \times \frac{176}{7}$$

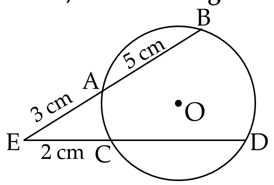
$$= \frac{12672}{7} - \frac{352}{7}$$

$$= \frac{12672 - 352}{7}$$

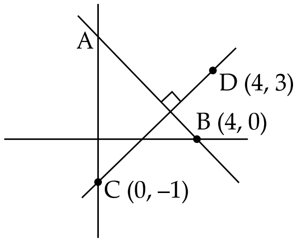
$$= \frac{12320}{7}$$

$= 1760 \text{ cm}^3$

5. (i) Two chords AB and CD of a circle intersect externally at E. If EC = 2 cm, EA = 3 cm and AB = 5 cm, find the length of CD. **[2]**



(ii) Line AB is perpendicular to CD coordinates of B, C and D respectively (4, 0), (0, –1) and (4, 3). **[2]**



Find

(a) Slope of CD

(b) Equation of AB

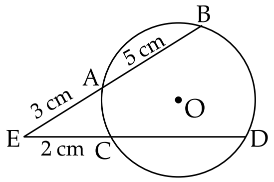
(iii) Prove that: **[3]**

$$\frac{(1+\sin\theta)^2+(1-\sin\theta)^2}{2\cos^2\theta} = \sec^2\theta + \tan^2\theta$$

(iv) The mean of the following distribution is 50. Find the unknown frequency. **[3]**

Class Interval	Frequency
0 - 20	6
20 - 40	f
40 - 60	8
60 - 80	12
80 - 100	8

Ans. (i) From the given figure,



We have EA = 3 cm, AB = 5 cm

EC = 2 cm

When two chords intersect externally, then (from figure)

$$EA \times EB = EC \times ED$$

or $EA \times (EA + AB) = EC \times (EC + CD)$

$\Rightarrow 3 \times (3 + 5) = 2\,(2 + CD)$

$24 = 4 + 2CD$

$\Rightarrow 2CD = 20$

$CD = 10 \text{ cm}$

(ii) (a) Slope of line CD is given by

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-3}{0-4} = 1$$

(b) Slope of line AB, Which is perpendicular to CD given by

$$m_1 = \frac{1}{-m_2} = -\frac{1}{1} = -1$$

Now, equation of a line (AB), when slope and a coordinates is given

$$y - y_1 = m_1 (x - x_1)$$

where $y_1 = 0, m_1 = -1, x_1 = 4$

So, $y - 0 = -1\,(x - 4)$

$y = -x + 4$

$x + y = 4$

(iii) L.H.S.

$$= \frac{(1+\sin\theta)^2 + (1-\sin\theta)^2}{2\cos^2\theta}$$

$$= \frac{1^2 + \sin^2\theta + 2\sin\theta + 1^2 + \sin^2\theta - 2\sin\theta}{2\cos^2\theta}$$

$$= \frac{2 + 2\sin^2\theta}{2\cos^2\theta} = \frac{1+\sin^2\theta}{\cos^2\theta}$$

$$= \frac{1}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta}$$

$= \sec^2\theta + \tan^2\theta$ = R.H.S. **Hence Proved.**

(iv)

C. I.	Class mark (x)	Frequency (f)	$f \times x$
0 – 20	10	6	60
20 – 40	30	f	$30f$
40 – 60	50	8	400
60 – 80	70	12	840
80 – 100	90	8	720
Total		$34 + f$	$2020 + 30f$

We know, Mean $(\bar{x}) = \frac{\sum fx}{\sum f}$

$$50 = \frac{2020 + 30f}{34 + f} \quad \text{(given)}$$

$1700 + 50f = 2020 + 30f$

$50f - 30f = 2020 - 1700$

$20f = 320$

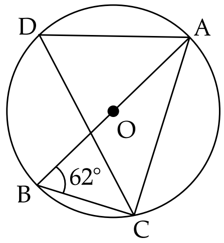
$f = 16$

6. (i) Prove that: **[2]**

$$1+\frac{\tan^2\theta}{1+\sec\theta}=\sec\theta$$

(ii) In the given figure A, B, C and D are points on the circle with centre O. Given $\angle ABC = 62°$. **[2]**

Find:



(a) $\angle ADC$ (b) $\angle CAB$

(iii) Find the equation of a line parallel to the line $2x + y - 7 = 0$ and passing through the intersection of the lines $x + y - 4 = 0$ and $2x - y = 8$. **[3]**

(iv) Marks obtained by 40 students in an examination are given below. **[3]**

Marks	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70
No of students	3	8	14	9	4	2

Using graph paper draw an ogive and estimate the median marks. Take 2 cm = 10 marks along one axis and 2 cm = 5 students along the other axis.

Ans. (i) L.H.S.

$$= 1 + \frac{\tan^2\theta}{1+\sec\theta}$$

$$= 1 + \frac{(\sec^2\theta-1)}{1+\sec\theta} \qquad (\because \sec^2\theta - \tan^2\theta = 1)$$

$$= 1+ \frac{(\sec\theta+1)(\sec\theta-1)}{(1+\sec\theta)}$$

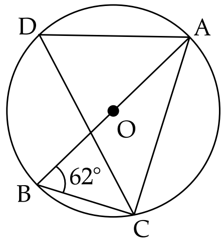
$= 1 + \sec\theta - 1$

$= \sec\theta$ = R.H.S.

(ii) (a) Given, $\angle ABC = 62°$

$\angle ABC = \angle ADC = 62°$

($\because$ Angles in the same segment are equal)



(b) In ΔABC,

$\angle ABC = 62°$

$\angle ACB = 90°$

($\because$ Angles in a semicircle is a right angle)

Now, $\angle ABC + \angle ACB + \angle CAB = 180°$

(Angle sum property)

$62° + 90° + \angle CAB = 180°$

$\angle CAB = 180° - 152°$

$\angle CAB = 28°$

(iii) Given,

$x + y - 4 = 0$ or $x + y = 4$

and $2x - y = 8$

On solving above two equations we get the intersection points as (4, 0).

Now, equation of line parallel to $2x + y - 7 = 0$ is given by

$2x + y = \lambda$

which is passing through (4, 0)

So, $2 \times 4 + 0 = \lambda$

$\lambda = 8$

So,the required equation of the line is $2x + y = 8$

or, $2x + y - 8 = 0$

(iv)

Marks	**No. of students**	**Marks less than**	**Cumulative frequency**
10 – 20	3	20	3
20 – 30	8	30	11
30 – 40	14	40	25
40 – 50	9	50	34
50 – 60	4	60	38
60 – 70	2	70	40

Other than the given class intervals, we assume a class interval 0 – 10 prior the first class with zero frequency.

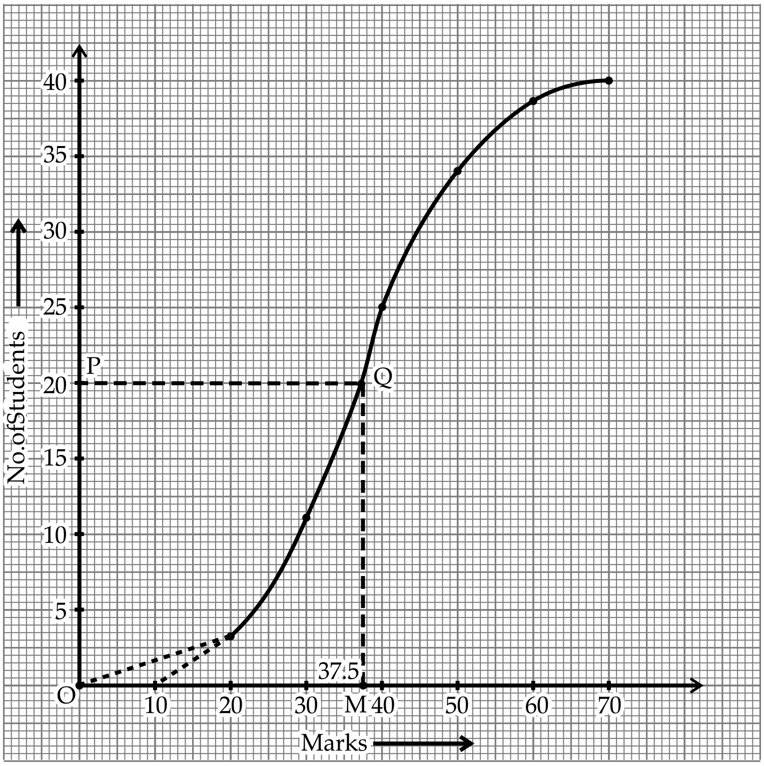
Now, plot the points (10, 0), (20, 3), (30, 11), (40, 25), (50, 34), (60, 38), (70, 40) on the graph paper.

In order to obtain ogive, we draw a smooth curve passing through these points.

In order to find the median, we first locate the point $\frac{N}{2}=\frac{40}{2}$ = 20 on Y-axis. Let the point P from this draw a line parallel to X-axis cutting the curve at Q. From Q draw a line parallel to Y-axis meeting the X-axis at point M. The coordinate of M is 37.5. Hence, the median is 37.5.

Now, plot the points (10, 0), (20, 3), (30, 11), (40, 25), (50, 34), (60, 38), (70, 40) on the graph paper.

In order to obtain ogive, we draw a smooth curve passing through these, points.



❑❑❑