

# ICSE Solved Paper 2023 Mathematics

## Class-X

(Maximum Marks : 80)

(Time allowed : Two hours and a half)

Attempts all questions from Section A and any four questions from Section B.

All working, including rough work, must be clearly shown, and must be done on the same sheet as the rest of the answer.

Omission of essential working will result in loss of marks.

The intended marks for questions or parts of questions are given in brackets. []

Mathematical tables and graph papers are provided.

### SECTION-A

(40 marks)

(Attempt all questions from this Section)

1. Choose the correct answer to the questions from the given options. [15]

(Do not copy the questions, write the correct answer only.)

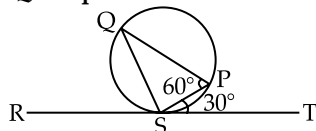
- (i) If  $\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -8 \end{bmatrix}$ , the value of  $x$  and  $y$  respectively are:

- (a) 1, -2                      (b) -2, 1  
(c) 1, 2                        (d) -2, -1

- (ii) If  $x - 2$  is a factor of  $x^3 - kx - 12$ , then the value of  $k$  is:

- (a) 3                              (b) 2  
(c) -2                            (d) -3

- (iii) In the given diagram RT is a tangent touching the circle at S. If  $\angle PST = 30^\circ$  and  $\angle SPQ = 60^\circ$  then  $\angle PSQ$  is equal to:



- (a)  $40^\circ$                         (b)  $30^\circ$   
(c)  $60^\circ$                         (d)  $90^\circ$

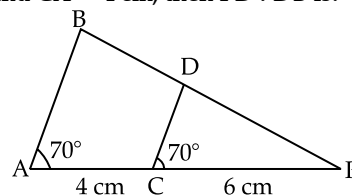
- (iv) A letter is chosen at random from all the letters of the English alphabets. The probability that the letter chosen is a vowel, is:

- (a)  $\frac{4}{26}$                             (b)  $\frac{5}{26}$   
(c)  $\frac{21}{26}$                             (d)  $\frac{5}{24}$

- (v) If 3 is a root of the quadratic equation  $x^2 - px + 3 = 0$  then  $p$  is equal to:

- (a) 4                                (b) 3  
(c) 5                                (d) 2

- (vi) In the given figure  $\angle BAP = \angle DCP = 70^\circ$ ,  $PC = 6$  cm and  $CA = 4$  cm, then  $PD : DB$  is:



- (a) 5 : 3                        (b) 3 : 5  
(c) 3 : 2                        (d) 2 : 3

- (vii) The printed price of an article is ₹3080. If the rate of GST is 10% then the GST charged is:

- (a) ₹154                        (b) ₹308  
(c) ₹30.80                      (d) ₹15.40

- (viii)  $(1 + \sin A)(1 - \sin A)$  is equal to:

- (a)  $\operatorname{cosec}^2 A$                 (b)  $\sin^2 A$   
(c)  $\sec^2 A$                       (d)  $\cos^2 A$

- (ix) The coordinates of the vertices of  $\Delta ABC$  are respectively  $(-4, -2)$ ,  $(6, 2)$  and  $(4, 6)$ . The centroid  $G$  of  $\Delta ABC$  is:

- (a)  $(2, 2)$                         (b)  $(2, 3)$   
(c)  $(3, 3)$                         (d)  $(0, -1)$

- (x) The  $n^{\text{th}}$  term an Arithmetic Progression (A.P) is  $2n + 5$ . The  $10^{\text{th}}$  term is:

- (a) 7                                (b) 15  
(c) 25                                (d) 45

- (xi) The mean proportional between 4 and 9 is:

- (a) 4                                (b) 6  
(c) 9                                (d) 36

- (xii) Which of the following cannot be determined graphically for a grouped frequency distribution?

- (a) Median                        (b) Mode  
(c) Quartiles                      (d) Mean

- (xiii) Volume of a cylinder of height 3 cm is  $48\pi \text{ cm}^3$ .  
Radius of the cylinder is:  
(a) 48 cm (b) 16 cm  
(c) 4 cm (d) 24 cm
- (xiv) Naveen deposits ₹800 every month in a recurring deposit account for 6 months. If he receives ₹4884 at the time of maturity, then the interest he earns is:  
(a) ₹84 (b) ₹42  
(c) ₹24 (d) ₹284
- (xv) The solution set for the inequation  $2x + 4 \leq 14$ ,  $x \in W$  is:  
(a) {1, 2, 3, 4, 5} (b) {0, 1, 2, 3, 4, 5}  
(c) {1, 2, 3, 4} (d) {0, 1, 2, 3, 4}

Ans.

(A) (i) Option (a) is correct.

Explanation:  $\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -8 \end{bmatrix}$

$$\begin{bmatrix} 2x \\ 4y \end{bmatrix} = \begin{bmatrix} 2 \\ -8 \end{bmatrix}$$

$$2x = 2, x = 1$$

$$4y = -8, y = -2$$

(ii) Option (c) is correct.

Explanation:  $(x - 2)$  is a factor of  $f(x)$

$$\therefore \text{Remainder} = f(2) = 0$$

$$f(x) = x^3 - kx - 12$$

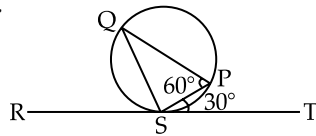
$$f(2) = (2)^3 - k(2) - 12$$

$$0 = 8 - 2k - 12$$

$$k = -2$$

(iii) Option (d) is correct.

Explanation:



$$\angle PST = \angle PQS = 30^\circ$$

(alternate segment angle)

In  $\Delta PSQ$ ,

$$\angle PSQ + \angle PQS + \angle SPQ = 180^\circ$$

(Sum of all angles of a triangle)

$$\angle PSQ + 30^\circ + 60^\circ = 180^\circ$$

$$\angle PSQ = 90^\circ$$

(iv) Option (b) is correct.

Explanation: Total no. of letters in

English alphabets,  $n(S) = 26$

Total no. of vowels in

English alphabets,  $n(E) = 05$

$$\therefore \text{Required probability } P(E) = \frac{5}{26}$$

(v) Option (a) is correct.

Explanation:  $f(x) = x^2 - px + 3 = 0$

3 is a root of  $f(x)$

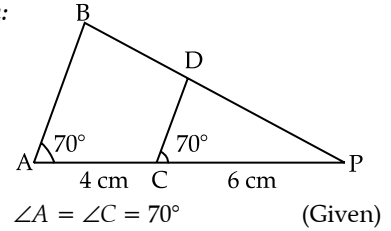
$$\therefore f(3) = 0$$

$$(3)^2 - 3p + 3 = 0$$

$$\therefore p = 4$$

(vi) Option (c) is correct.

Explanation:



$\angle A = \angle C = 70^\circ$  (Given)

These are corresponding angles

$\therefore DC \parallel BA$

$$\therefore \frac{PD}{DB} = \frac{PC}{AC}$$

(Basic proportional Theorem)

$$\frac{PD}{DB} = \frac{6}{4} = \frac{3}{2}$$

(vii) Option (b) is correct.

Explanation: G.S.T. = %Rate  $\times$  printed price

$$= \frac{10}{100} \times 3080$$

$$= ₹308$$

Therefore, the GST charged on the article is (rupees symbol) 308.

(viii) Option (d) is correct.

Explanation:  $(1 + \sin A)(1 - \sin A)$

$$= (1)^2 - (\sin A)^2$$

$$= 1 - \sin^2 A$$

$$= \sin^2 A + \cos^2 A - \sin^2 A$$

$$= \cos^2 A$$

(ix) Option (a) is correct.

$$\text{Explanation: } G\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

$$= G\left(\frac{-4 + 6 + 4}{3}, \frac{-2 + 2 + 6}{3}\right)$$

$$= G(2, 2)$$

(x) Option (c) is correct.

Explanation:  $t_n = 2n + 5$

$$t_{10} = 2 \times 10 + 5 = 25$$

(xi) Option (b) is correct.

Explanation: Mean proportional =  $\sqrt{4 \times 9} = 6$

(xii) Option (d) is correct.

Explanation: Mean is a numerical measure that requires the individual data values, which cannot be determined graphically for a grouped frequency distribution.

(xiii) Option (c) is correct.

Explanation: Vol. of cylinder =  $48\pi \text{ cm}^3$

$$\pi r^2 h = 48\pi$$

$$r^2(3) = 48$$

$$r = 4 \text{ cm}$$

(xiv) Option (a) is correct.

Explanation: Amount = Principle + Interest

$$4884 = 800 \times 6 + \text{Interest}$$

$$\therefore \text{Interest} = 4884 - 4800 = ₹84$$

(xv) Option (b) is correct.

Explanation:  $2x + 4 \leq 14$   
 $2x \leq 14 - 4$   
 $x \leq 5$

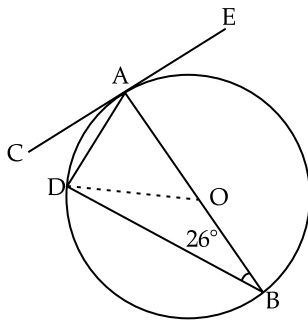
$\therefore$  Solution set  $\{0, 1, 2, 3, 4, 5\}$

2.(i) Find the value of 'a' if  $x - a$  is a factor of the polynomial  $3x^3 + x^2 - ax - 81$ . [4]

(ii) Salman deposits ₹1000 every month in a recurring deposit account for 2 years. If he receives ₹26000 on maturity, find: [4]

- (a) the total interest Salman earns.
- (b) the rate of interest.

(iii) In the given figure O, is the centre of the circle. CE is a tangent to the circle at A. If  $\angle ABD = 26^\circ$ , then find: [4]



- (a)  $\angle BDA$
- (b)  $\angle BAD$
- (c)  $\angle CAD$
- (d)  $\angle ODB$

Ans. (i)  $p(x) = 3x^3 + x^2 - ax - 81$   
 $(x - a)$  is a factor of  $p(x)$   
 $p(a) = 0$   
 $3(a)^3 + (a)^2 - a(a) - 81 = 0$   
 $3a^3 + a^2 - a^2 - 81 = 0$   
 $a = 3$

(ii)  $P = ₹1000$ ,  $n = 2$ , years = 24 months,  $r =$  rate of interest

$$I = P \times \frac{n(n+1)}{2} \times \frac{r}{100} \times \frac{1}{12}$$

$$I = 1000 \times \frac{24(24+1)}{2} \times \frac{r}{1200}$$

$$I = 250r \quad \dots(i)$$

$$A = P + I$$

$$26000 = 1000 \times 24 + I$$

$$2000 = I \quad \dots(ii)$$

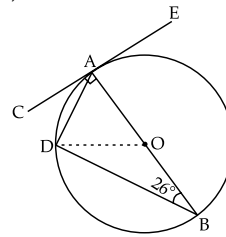
(a) Salman earns interest as ₹2000

From (i) and (ii)

$$250r = 2000$$

$$r = 8$$

(iii) (a)  $\therefore$  (b) rate of interest is 8%



$\angle BDA = 90^\circ$  (Angle in a semicircle)

- (b) In  $\triangle BAD$   
 $\angle BAD + \angle ABD + \angle ADB = 180^\circ$   
 (Sum of all angles of  $\Delta$ )  
 $\angle BAD + 26^\circ + 90^\circ = 180^\circ$   
 $\therefore \angle BAD = 64^\circ$
- (c)  $\angle CAD = \angle ABD$   
 $\angle CAD = 26^\circ$   
 (alternate segment angle)
- (d)  $OD = OB$  (radius of circle)  
 $\therefore \angle ODB = \angle OBD$   
 (Angles opposite to equal sides of a triangle are equal)  
 $\therefore \angle ODB = 26^\circ$

3.(i) Solve the following quadratic equation: [4]  
 $x^2 + 4x - 8 = 0$

Give your answer correct to one decimal place.  
 (Use mathematical tables if necessary.)

(ii) Prove the following identity: [4]

$$(\sin^2\theta - 1)(\tan^2\theta + 1) + 1 = 0$$

(iii) Use graph sheet to answer this question. Take 2 cm = 1 unit along both the axes. [5]

- (a) Plot A, B, C where A(0, 4), B(1, 1) and C(4, 0)
- (b) Reflect A and B on the x-axis and name them as E and D respectively.
- (c) Reflect B through the origin and name it F. Write down the coordinates of F.
- (d) Reflect B and C on the y-axis and name them as H and G respectively.
- (e) Join points A, B, C, D, E, F, G, H and A in order and name the closed figure formed.

Ans.

(i)  $x^2 + 4x - 8 = 0$

Compare the equation  $ax^2 + bx + c = 0$

$$\therefore a = 1, b = 4 \text{ and } c = -8$$

$$D = b^2 - 4ac = (4)^2 - 4(1)(-8)$$

$$= 16 + 32 = 48$$

$\therefore$  Roots are real and distinct

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

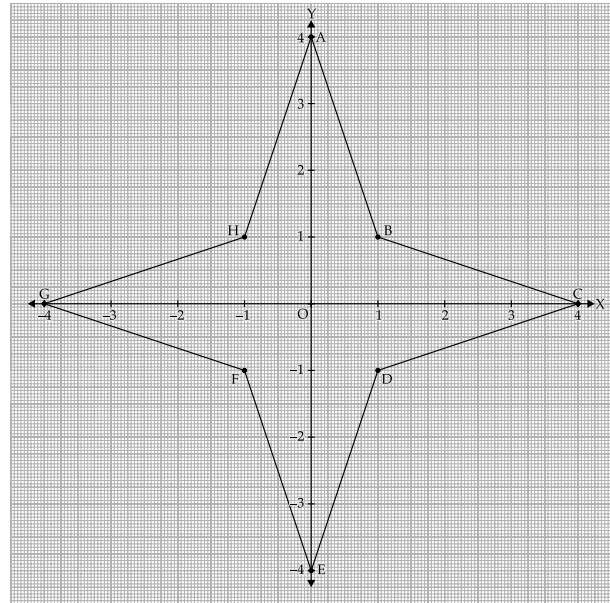
$$= \frac{-4 \pm \sqrt{48}}{2 \times 1} = \frac{-4 \pm 4\sqrt{3}}{2}$$

$$\begin{aligned}
 x &= -2 \pm 2\sqrt{3} \\
 x &= -2 \pm 2 \times 1.732 \\
 x &= -2 \pm 3.464 \\
 x &= -2 + 3.464 \\
 x &= -2 - 3.464 \\
 x &= 1.464 \text{ and } x = -5.464 \\
 x &= 1.5 \text{ and } x = -5.5
 \end{aligned}$$

and

(ii) L.H.S. =  $(\sin^2\theta - 1)(\tan^2\theta + 1) + 1$   
 $= (\sin^2\theta - \sin^2\theta - \cos^2\theta)(\sec^2\theta) + 1$   
 $1 + \tan^2\theta = \sec^2\theta$   
 $= (-\cos^2\theta)\left(\frac{1}{\cos^2\theta}\right) + 1$   
 $= -1 + 1$   
 $= 0 = \text{R.H.S.}$  **Hence Proved.**

(iii)



- (c)  $F(-1, -1)$   
 (e) Star

**SECTION-B**

**(40 marks)**

(Attempt any four questions from this Section)

4.(i) If  $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ ,  $C = \begin{bmatrix} 4 & 1 \\ 1 & 5 \end{bmatrix}$  and  $I =$

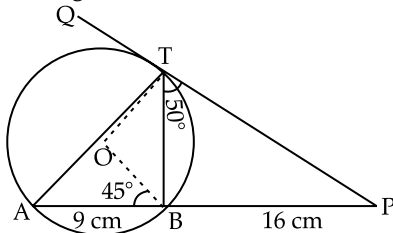
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Find  $A(B + C) - 14I$  [3]

(ii) ABC is a triangle whose vertices are  $A(1, -1)$ ,  $B(0, 4)$  and  $C(-6, 4)$ . D is the midpoint of BC. Find the: [3]

- (a) coordinates of D.  
 (b) equation of the median AD.

(iii) In the given figure, O is the centre of the circle. PQ is a tangent to the circle at T. Chord AB produced meets the tangent at P. [4]



$AB = 9 \text{ cm}$ ,  $BP = 16 \text{ cm}$ ,  $\angle PTB = 50^\circ$

$\angle OBA = 45^\circ$

Find:

- (a) length of PT  
 (b)  $\angle BAT$   
 (c)  $\angle BOT$   
 (d)  $\angle ABT$

Ans.

(i)  $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ ,  $C = \begin{bmatrix} 4 & 1 \\ 1 & 5 \end{bmatrix}$

$$A(B + C) - 14I$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \left\{ \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ 1 & 5 \end{bmatrix} \right\} - 14 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

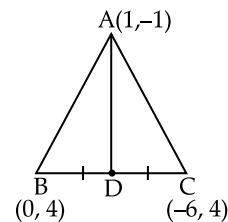
$$= \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 5 & 3 \\ 3 & 9 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 5+9 & 3+27 \\ 10+12 & 6+36 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 30 \\ 22 & 42 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 30 \\ 22 & 28 \end{bmatrix}$$

(ii)



(a)

$$D = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left( \frac{0 - 6}{2}, \frac{4 + 4}{2} \right)$$

therefore coordinates of D is (-3, 4)

$$\text{Slope of median AD} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 + 1}{-3 - 1} = \frac{-5}{4}$$

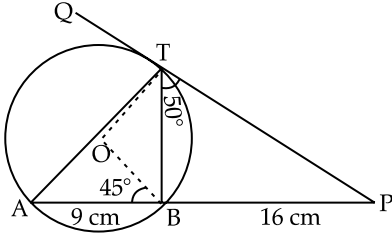
Equation of the AD

$$y - y_1 = m(x - x_1)$$

$$y + 1 = \frac{-5}{4}(x - 1)$$

$$\Rightarrow 5x + 4y - 1 = 0$$

(iii) (a)



$$PT^2 = PA \times PB$$

$$= (16 + 9) \times 16$$

$$PT = \sqrt{25 \times 16}$$

$$\therefore PT = 5 \times 4 = 20 \text{ cm}$$

(b)  $\angle BAT = \angle PTB = 50^\circ$   
(alternate segment angle)

(c)  $\angle BOT = 2\angle BAT$   
(Degree measure theorem)

$$\angle BOT = 2 \times 50 = 100^\circ$$

(d) In  $\triangle OBT$

$$\angle OBT + \angle OTB + \angle BOT = 180^\circ$$

(Sum of all angles of  $\triangle$ )

$$\angle OBT + \angle OBT + 100^\circ = 180^\circ$$

$$\angle OBT = \angle OTB$$

(angles opposite to equal sides of a triangle are equal)

$$2\angle OBT = 180^\circ - 100^\circ = 80^\circ$$

$$\angle OBT = \frac{80^\circ}{2} = 40^\circ$$

$$\angle ABT = \angle ABO + \angle OBT$$

$$= 45^\circ + 40^\circ$$

$$\therefore \angle ABT = 85^\circ$$

5.(i) Mrs. Arora bought the following articles from a departmental store: [3]

S.No.	Item	Price	Rate of GST	Discount
1.	Hair oil	₹1200	18%	₹100
2.	Cashew nuts	₹600	12%	-

Find the:

(a) Total GST paid.

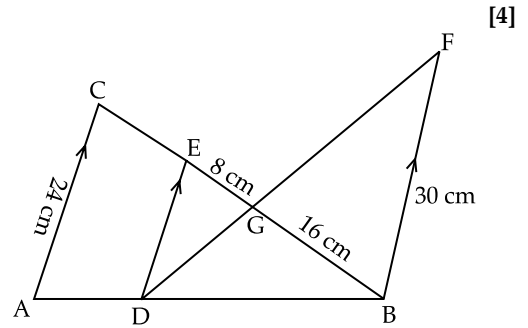
(b) Total bill amount including GST.

(ii) Solve the following inequation. Write down the solution set and represent it on the real number line.

$$-5(x - 9) \geq 17 - 9x > x + 2, x \in \mathbb{R} \quad [3]$$

(iii) In the given figure,  $AC \parallel DE \parallel BF$ .

If  $AC = 24 \text{ cm}$ ,  $EG = 8 \text{ cm}$ ,  $GB = 16 \text{ cm}$ ,  $BF = 30 \text{ cm}$ .



- (a) Prove  $\triangle GED \sim \triangle GBF$   
(b) Find DE  
(c)  $DB : AB$

Ans.

(i) Mrs Arora paid for

1. Hair oil  $\text{₹}(1200 - 100) = \text{₹}1100$

$$\text{G.S.T} = \frac{18}{100} \times 1100 = \text{₹}198$$

2. Cashew nuts

$$\text{G.S.T} = 600 \times \frac{12}{100} = \text{₹}72$$

(a) Total G.S.T paid =  $\text{₹}198 + 72 = \text{₹}270$

(b) Total Bill Amount including G.S.T.  
=  $\text{₹}1100 + 600 + 270$   
=  $\text{₹}1970$

(ii)

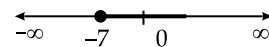
$$-5(x - 9) \geq 17 - 9x > x + 2$$

$$-5(x - 9) \geq 17 - 9x \text{ and } 17 - 9x > x + 2$$

$$-5x + 9x \geq 17 - 45$$

$$4x \geq -28$$

$$x \geq -7$$



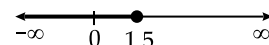
$$17 - 9x > x + 2$$

$$-9x - x > 2 - 17$$

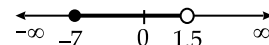
$$-10x > -15$$

$$x < \frac{-15}{-10}$$

$$x < 1.5$$

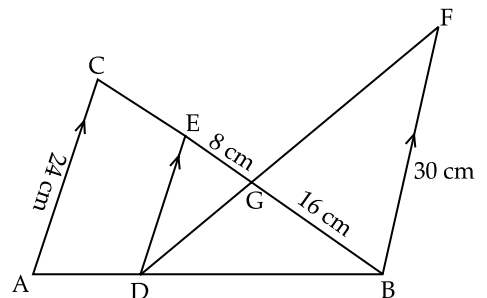


Solution set



$$-7 \leq x < 1.5$$

(iii) (a)



In  $\triangle GED$  and  $\triangle GBF$

$$\angle EGD = \angle BGF$$

(Vertically opposite angles)

$\angle DEG = \angle FBG$   
 (Alternative angles  $DE \parallel BF$ )  
 $\therefore \triangle GED \sim \triangle GBF$   
 (AA Similarity Criterion)  
**Hence Proved.**

(b)  $\triangle GED \sim \triangle GBF$  (Proved)  
 $\frac{GE}{GB} = \frac{ED}{BF} = \frac{GD}{GF}$  (by BPT)  
 $\frac{ED}{BF} = \frac{GE}{GB}$   
 $\frac{ED}{30} = \frac{8}{16}$

$$ED = \frac{8}{16} \times 30 = 15 \text{ cm}$$

$$DE = 15 \text{ cm}$$

(c) In  $\triangle ABC$ ,  $DE \parallel AC$   
 $\angle BDE = \angle BAC$   
 and  $\angle BED = \angle BCA$   
 by AA Similarity criterion, triangle  $BDE \sim$   
 triangle  $BAC$ .

$$\frac{DB}{AB} = \frac{BE}{BC} = \frac{DE}{AC} \quad (\text{by BPT})$$

$$\frac{DB}{AB} = \frac{DE}{AC}$$

$$\frac{DB}{AB} = \frac{15}{24} = \frac{5}{8}$$

$$\therefore DB : AB = 5 : 8$$

6.(i) The following distribution gives the daily wages of 60 workers of a factory. [3]

Daily income ₹	Number of worker (f)
200-300	6
300-400	10
400-500	14
500-600	16
600-700	10
700-800	4

Use graph paper to answer this question.

Take 2 cm = ₹100 along one axis and 2 cm = 2 workers along the other axis. Draw a histogram and hence find the mode of the given distribution.

(ii) The 5<sup>th</sup> term and the 9<sup>th</sup> term of an Arithmetic Progression are 4 and -12 respectively. [3]

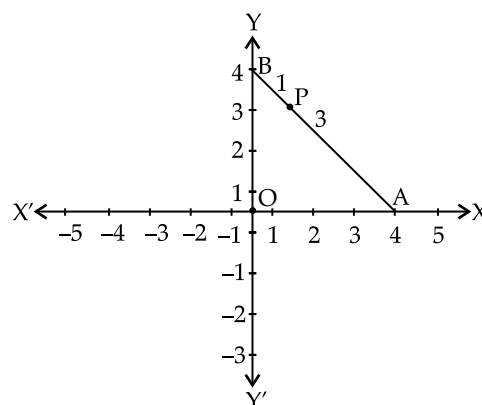
Find:

(a) the first term

(b) common difference

(c) sum of 16 terms of the AP.

(iii) A and B are two points on the x-axis and y-axis respectively. [4]

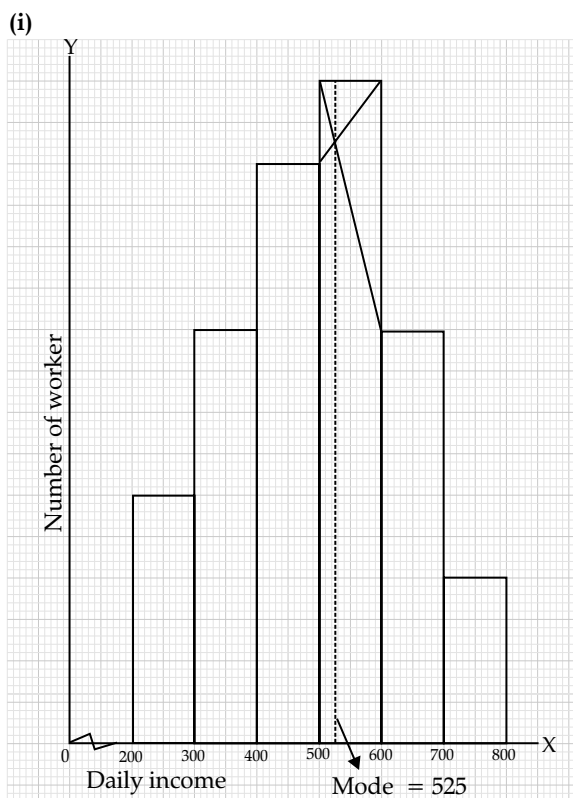


(a) Write down the coordinates of A and B.

(b) P is a point on AB such that  $AP : PB = 3 : 1$ . Using section formula find the coordinates of point P.

(c) Find the equation of a line passing through P and perpendicular to AB.

Ans.



(ii) Given,  $t_5 = 4$  and  $t_9 = -12$

Let the first term be  $a$  and common difference be  $d$  respectively

$$a_n = a + (n - 1)d$$

$$t_5 = 4$$

$$a + (5 - 1)d = 4$$

$$a + 4d = 4 \quad \dots(i)$$

$$\text{Similarly, } a + 8d = -12 \quad \dots(ii)$$

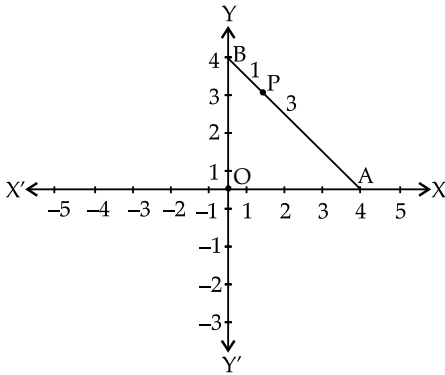
from (i) and (ii)

$d = -4$  and  $a = 20$

- (a) first term is 20
- (b) Common difference =  $-4$
- (c)  $S_n = \frac{n}{2}[2a + (n + 1)d]$   
 $S_{16} = \frac{16}{2}[2 \times 20 + (16 - 1)(-4)]$   
 $= 8[40 - 60]$   
 $= -160$

$\therefore$  Sum of 16 terms is  $-160$

(iii) (a)



Coordinates of A(4, 0)  
 Coordinates of B(0, 4)

(b)  $P = \left( \frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2} \right)$   
 $= \left( \frac{3 \times 0 + 1 \times 4}{3 + 1}, \frac{3 \times 4 + 1 \times 0}{3 + 1} \right)$   
 $P = (1, 3)$

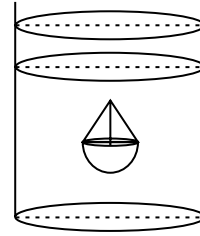
(c) Slope of line AB =  $\frac{4 - 0}{0 - 4} = -1$

$\therefore$  Slope of perpendicular line AB = 1

$y - y_1 = m(x - x_1)$   
 $y - 3 = 1(x - 1)$   
 $y - 3 = x - 1$   
 $x - y + 2 = 0$

- 7.(i) A bag contains 25 cards, numbered through 1 to 25. A card is drawn at random. What is the probability that the number on the card drawn is: [3]
  - (a) multiple of 5
  - (b) a perfect square
  - (c) a prime number?
- (ii) A man covers a distance of 100 km, travelling with a uniform speed of  $x$  km/hr, had the speed been 5 km/hr more it would have taken 1 hour less. Find  $x$  the original speed. [3]
- (iii) A solid in the shape of a hemisphere of radius 7 cm, surmounted by a cone of height 4 cm. The solid

is immersed completely in a cylindrical container filled with water to a certain height. If the radius of the cylinder is 14 cm, find the rise in the water level. [4]



Ans.

(i)  $S = \{1, 2, 3, 4, \dots, 25\}$

$\therefore n(S) = 25$

(a) multiple of 5

$E = \{5, 10, 15, 20, 25\}$

$n(E) = 5$

Probability of multiple of 5 =  $\frac{n(E)}{n(S)} = \frac{5}{25}$   
 $= \frac{1}{5}$

(b) Perfect square = (1, 4, 9, 16, 25)

Required probability =  $\frac{5}{25} = \frac{1}{5}$

(c) a prime number {2, 3, 5, 7, 11, 13, 17, 19, 23}

Required probability =  $\frac{9}{25}$

(ii) Let the original speed be  $x$  km/h

Time =  $\frac{\text{Distance}}{\text{Speed}}$

$t_1 = \frac{100}{x}$  h

$t_2 = \frac{100}{x + 5}$  h

$t_1 - t_2 = 1$  (Given)

$\frac{100}{x} - \frac{100}{x + 5} = 1$

$\frac{100x + 500 - 100x}{x(x + 5)} = 1$

$500 = x^2 + 5x$

$x^2 + 5x - 500 = 0$

$(x + 25)(x - 20) = 0$

If  $x + 25 = 0 \Rightarrow x = -25$

it is not possible because speed can not be negative

if  $x - 20 = 0$

$x = 20$

$\therefore$  original speed be 20 km/h

(iii) Volume of solid = Vol. of hemisphere + Vol. of Cone.

$$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h = \frac{\pi r^2}{3}(2r + h)$$

$$= \frac{\pi \times 7 \times 7}{3}(2 \times 7 + 4) = \frac{49\pi}{3}(18)$$

$$= 294\pi \text{ cm}^3$$

Let the level of water rise in cylinder be  $x$  cm

$\therefore$  Rise in Vol. of water =  $294\pi$

$$\pi R^2 x = 294\pi$$

$$(14)^2 x = 294$$

$$x = \frac{294}{14 \times 14} = \frac{21}{14}$$

$$x = \frac{3}{2} \text{ cm} = 1.5 \text{ cm}$$

$\therefore$  level of water rise in cylinder = 1.5 cm

8. (i) The following table gives the marks scored by a set of students in an examination. Calculate the mean of the distribution by using the short cut method. [3]

Marks	Number of Students (f)
0-10	3
10-20	8
20-30	14
30-40	9
40-50	4
50-60	2

(ii) What number must be added to each of the numbers 4, 6, 8, 11 in order to get the four numbers in proportion? [3]

(iii) Using ruler and compass construct a triangle ABC in which  $AB = 6$  cm,  $\angle BAC = 120^\circ$  and  $AC = 5$  cm. Construct a circle passing through A, B and C. Measure and write down the radius of the circle. [4]

Ans. (i)

Marks	Number of Students (f)	$x$	$d = x - A$	$fd$
0-10	3	5	-20	-60
10-20	8	15	-10	-80
20-30	14	$25 = A$	0	0
30-40	9	35	10	90
40-50	4	45	20	80
50-60	2	55	30	60
	40			90

$$\bar{x} = A + \frac{\sum fd}{\sum f}$$

$$= 25 + \frac{90}{40}$$

$$= 25 + 2.25$$

$$\text{Mean} = 27.25$$

(ii) Let  $x$  be added to get the four numbers in proportion

$$\therefore (4 + x) : (6 + x) :: (8 + x) : (11 + x)$$

$$(4 + x)(11 + x) = (6 + x)(8 + x)$$

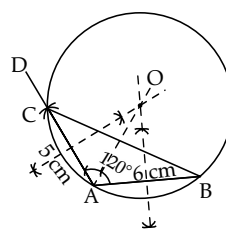
$$x^2 + 15x + 44 = x^2 + 14x + 48$$

$$15x - 14x = 48 - 44$$

$$x = 4$$

$\therefore$  4 should be added to get the four numbers in proportion

(iii)



Step of construction:

(a) Draw  $AB = 6$  cm

(b) Make  $\angle BAD = 120^\circ$

(c) Taking centre A and radius of AC 5 cm draw an arc which intersect at the point C

(d) join BC. triangle ABC is required triangle

(e) Draw perpendicular bisectors of side AB and AC respectively which intersect at the point O

(f) Taking O as centre and OA as radius draw a circle which passes through the points A, B and C respectively.

Now, measured the radius  $OA = 5.5$  cm

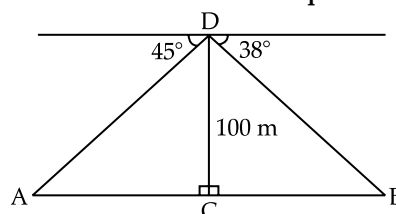
9. (i) Using Componendo and Dividendo solve for  $x$ . [3]

$$\frac{\sqrt{2x+2} + \sqrt{2x-1}}{\sqrt{2x+2} - \sqrt{2x-1}} = 3$$

(ii) Which term of the Arithmetic Progression (A.P) 15, 30, 45, 60 ... is 300? Hence find the sum of the terms of the Arithmetic Progression (A.P) [3]

(iii) From the top of a tower 100 m high a man observes the angles of depression of two ships A and B, on opposite sides of the tower as  $45^\circ$  and  $38^\circ$  respectively. If the foot of the tower and the ships are in the same horizontal line find the distance between the two ships A and B to the nearest metre.

(Use Mathematical Tables for this question.) [4]





Ans.

$$(i) \frac{\sqrt{2x+2} + \sqrt{2x-1}}{\sqrt{2x+2} - \sqrt{2x-1}} = 3$$

Applying componendo and dividendo

$$\frac{\sqrt{2x+2} + \sqrt{2x-1} + \sqrt{2x+2} - \sqrt{2x-1}}{\sqrt{2x+2} + \sqrt{2x-1} - \sqrt{2x+2} + \sqrt{2x-1}} = \frac{3+1}{3-1}$$

$$\frac{2\sqrt{2x+2}}{2\sqrt{2x-1}} = \frac{4}{2} = 2$$

$$\frac{\sqrt{2x+2}}{\sqrt{2x-1}} = 2$$

squaring both sides

$$\frac{2x+2}{2x-1} = 4$$

$$2x+2 = 8x-4$$

$$2x-8x = -4-2$$

$$-6x = -6$$

$$x = 1$$

(ii) 15, 30, 45, 60, ..... 300

$$a = 15, d = 30 - 15 = 15$$

$$a_n = 300$$

$$a + (n-1)d = 300$$

$$15 + (n-1)(15) = 300$$

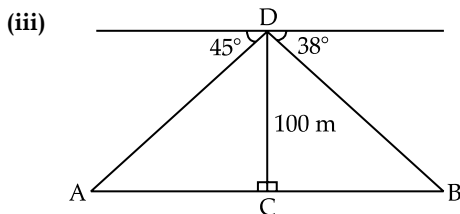
$$1 + (n-1) = 20$$

$$n = 20$$

20<sup>th</sup> term is 300

$$S_n = \frac{n}{2}[a + a_n] = \frac{20}{2}[15 + 300]$$

$$S_{20} = 3150$$



In  $\triangle ACD$

$$\tan A = \frac{CD}{AC}$$

$$\tan 45^\circ = \frac{100}{AC}$$

$$1 = \frac{100}{AC}$$

$$\therefore AC = 100 \text{ m}$$

In  $\triangle BCD$

$$\tan B = \frac{CD}{BC}$$

$$\tan 38^\circ = \frac{100}{BC}$$

$$0.7813 = \frac{100}{BC}$$

$$\therefore BC = \frac{100}{0.7813} = 127.99 \text{ m}$$

Distance between two ships =  $AC + BC$

$$= 100 + 127.99$$

$$227.99 \text{ m} = 228 \text{ m}$$

10.(i) Factorize completely using factor theorem: [4]

$$2x^3 - x^2 - 13x - 6$$

(ii) Use graph paper to answer this question. [6]

During a medical checkup of 60 students in a school, weights were recorded as follows:

Weight (in kg)	Number of Students
28-30	2
30-32	4
32-34	10
34-36	13
36-38	15
38-40	9
40-42	5
42-44	2

Taking 2 cm = 2 kg along one axis and 2 cm = 10 students along the other axis draw an ogive. Use your graph to find the:

(a) median

(b) upper Quartile

(c) number of students whose weight is above 37 kg

Ans.(i)  $2x^3 - x^2 - 13x - 6$

Put  $x = -2$

$$= 2(-2)^3 - (-2)^2 - 13(-2) - 6$$

$$= -16 - 4 + 26 - 6$$

$$= 0$$

$\therefore (x + 2)$  is a factor of polynomial

$$x + 2 \overline{) 2x^3 - x^2 - 13x - 6} \quad (2x^2 - 5x - 3)$$

$$2x^3 + 4x^2$$

$$\underline{(-) \quad (-)}$$

$$-5x^2 - 13x - 6$$

$$-5x^2 - 10x$$

$$\underline{(+) \quad (+)}$$

$$-3x - 6$$

$$-3x - 6$$

$$\underline{(+) \quad (+)}$$

$$\times$$

$$2x^3 - x^2 - 13x - 6 = (x + 2)(2x^2 - 5x - 3)$$

$$= (x + 2)\{2x^2 - 6x + x - 3\}$$

$$= (x + 2)\{2x(x - 3) + 1(x - 3)\}$$

$$= (x + 2)(x - 3)(2x + 1)$$

$$\therefore 2x^3 - x^2 - 13x - 6 = (x - 3)(2x + 1)(x + 2)$$

(ii)

Weight (in kg)	Number of Students ( <i>f</i> )	Cumulative frequency ( <i>cf</i> )
28-30	2	2
30-32	4	6
32-34	10	16
34-36	13	29
36-38	15	44
38-40	9	53
40-42	5	58
42-44	2	60

Weight in kgs. (less than)	Number of Students
28	0
30	2
32	6
34	16
36	29
38	44
40	53
42	58
44	60

(a) median = 36.2 kg

(b) upper quartile = 38.2 kg

(c) Number of students where weight is above 37 kg = 60 – 37 = 23 students

(ii)

