ISC Solved Paper 2020

Class-XII

Mathematics

(Maximum Marks : 80)

(Time allowed : Three hours)

*Condidates are allowed an additional **15 minutes** for **only** reading the paper.*

*They must **NOT** start writing during this time.*

The question Paper consists of three sections A, B and C.
*Candidates are required to attempt all questions from **Section A** and all questions*
EITHER** from **Section B** **OR** **Section C
***Section A :** Internal choice has been provided in three questions of four marks each and two question of six marks each.*
***Section B :** Internal choice has been provided in two questions of four marks each.*
***Section C :** Internal choice has been provided in two questions of four marks each.*
All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.
The intended marks for questions or parts of questions are given in brackets [ ].
Mathematical tables and graph papers are provided.

SECTION - A **[80 Marks]**

1. **[10 × 2]**

(i) Determine whether the binary operation * on R defined by $a * b = |a - b|$ is commutative. Also, find the value of (–3) * 2.

(ii) Prove that :

$$\tan^2(\sec^{-1} 2) + \cot^2(\text{cosec}^{-1} 3) = 11.$$

Ans. $\tan^2(\sec^{-1} 2) + \cot^2(\text{cosec}^{-1} 3) = 11$

L.H.S.

$= \tan^2(\sec^{-1} 2) + \cot^2(\text{cosec}^{-1} 3)$

$= \tan^2\left(\tan^{-1}\sqrt{3}\right) + \cot^2\left(\cot^{-1} 2\sqrt{2}\right)$

$$\begin{bmatrix} \sec^{-1} 2 = \tan^{-1}\sqrt{3} \\ \text{cosec}^{-1} 3 = \cot^{-1} 2\sqrt{2} \end{bmatrix}$$

$= \left[\tan\left(\tan^{-1}\sqrt{3}\right)\right]^2 + \left[\cot\left(\cot^{-1} 2\sqrt{2}\right)\right]^2$

$= \left(\sqrt{3}\right)^2 + \left(2\sqrt{2}\right)^2$

$= 3 + 8 = 11$

$=$ R.H.S. **Hence Proved**

(iii) Without expanding at any stage, find the value of the determinant :

Ans. $D = \begin{vmatrix} 20 & a & b+c \\ 20 & b & c+a \\ 20 & c & a+b \end{vmatrix}$

$= 20 \begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$

$C_3 \rightarrow C_3 + C_2$

$\Delta = 20 \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix}$

$\Delta = 20\,(a + b + c) \begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$

Here, $C_1 = C_3$ so, the determinant value will be zero

$\therefore \quad \Delta = 20(a + b + c)(0) = 0$

$\Delta = 0$

$\Delta = \begin{vmatrix} 20 & a & b+c \\ 20 & b & a+c \\ 20 & c & a+b \end{vmatrix}$

(iv) If $\begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}\begin{pmatrix} 1 & -3 \\ -2 & 4 \end{pmatrix} = \begin{pmatrix} -4 & 6 \\ -9 & x \end{pmatrix}$, find x.

Ans. $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$

$$\begin{bmatrix} 2-6 & -6+12 \\ 5-14 & -15+28 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$$

$$\begin{bmatrix} -4 & 6 \\ -9 & 13 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$$

$$\therefore \quad x = 13$$

(v) Find $\frac{dy}{dx}$ if $x^3 + y^3 = 3axy$.

Ans.

$$x^3 + y^3 = 3axy$$

$$\frac{d}{dx}(x^3) + \frac{d}{dx}(y^3) = \frac{d}{dx}(3axy)$$

$$3x^2 + 3y^2\frac{dy}{dx} = 3a\frac{d}{dx}(xy)$$

$$x^2 + y^2\frac{dy}{dx} = a\left\{x\frac{d}{dx}y + y\frac{d}{dx}x\right\}$$

$$x^2 + y^2\frac{dy}{dx} = a\left\{x\frac{dy}{dx} + y(1)\right\}$$

$$y^2\frac{dy}{dx} - ax\frac{dy}{dx} = ay - x^2$$

$$(y^2 - ax)\frac{dy}{dx} = ay - x^2$$

$$\therefore \quad \frac{dy}{dx} = \frac{ay - x^2}{y^2 - ax}$$

(vi) The edge of a variable cube is increasing at the rate of 10 cm/sec. How fast is the volume of the cube increasing when the edge is 5 cm long ?

Ans. Let the edge of cube be x cm

$$\therefore \quad \frac{dx}{dt} = 10 \text{ cm/sec (given)}$$

Volume of cube (V) = x^3

$$\frac{dV}{dt} = \frac{d}{dt}x^3 = 3x^2\frac{dx}{dt}$$

$$\left(\frac{dV}{dt}\right)_{x=5} = 3(5)^2(10)$$

$$= 750 \text{ cm}^3\text{/sec.}$$

$\therefore$ Volume of cube increasing by 750 cm^3/sec when edge of cube is 5 cm.

(vii) Evaluate : $\int_4^5 |x-5| dx$

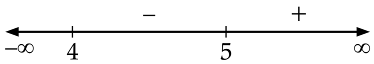
Ans.

$$\int_4^5 |x-5|\,dx = \int_4^5 -(x-5)dx$$

$$= -\left[\frac{x^2}{2} - 5x\right]_4^5$$

$$= -\left[\frac{25}{2} - 25 - \frac{16}{2} + 20\right]$$

$$= \frac{1}{2}$$



(viii) Form a differential equation of the family of the curves $y^2 = 4ax$.

Ans.

$$y^2 = 4ax \quad \text{...(i)}$$

$$\frac{d}{dx}y^2 = \frac{d}{dx}(4ax)$$

$$2y\frac{dy}{dx} = 4a(1)$$

$$\frac{dy}{dx} = \frac{4a}{2y} = \frac{2a}{y}$$

$$\frac{y}{2}\frac{dy}{dx} = a \quad \text{...(ii)}$$

From (i) and (ii)

$$y^2 = 4\frac{y}{2}\frac{dy}{dx}x$$

$$y = 2x\frac{dy}{dx}$$

(ix) A bag contains 5 white, 7 red and 4 black balls. If four balls are drawn one by one with replacement, what is the probability that none is white ?

Ans. Number of White balls = 5

Red balls = 7

Black balls = 4

Total balls = 5 + 7 + 4 = 16

Probability of the ball is not White

$$= \frac{11}{16}$$

Probability of the ball is not White when 4 balls are drawn one by one with replacement

$$= \left(\frac{11}{16}\right)^4$$

$$= \frac{14641}{65536}$$

$$= 0.2234$$

(x) Let A and B be two events such that

$$P(A) = \frac{1}{2},\ P(B) = p \text{ and } P(A \cup B) = \frac{3}{5}$$

find 'p' if A and B are independent events.

Ans. $P(A) = \frac{1}{2}$, $P(B) = p$ and $P(A \cup B) = \frac{3}{5}$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{3}{5} = \frac{1}{2} + p - P(A \cap B)$$

$$\frac{3}{5}-\frac{1}{2} = p - P(A)\,P(B)$$

Since, $[P(A \cap B) = P(A)P(B)$ as A and B are independent events$]$

$$\frac{6-5}{10} = p - \frac{1}{2}p$$

$$\frac{1}{10} = \frac{p}{2}$$

$$\therefore \quad p = \frac{1}{5}$$

***2. If the function $f : R \to R$ be defined as**

$$f(x) = \frac{3x+4}{5x-7}, \left(x \neq \frac{7}{5}\right) \text{ and}$$

$g : R \to R$ be defined as

$$g(x) = \frac{7x+4}{5x-3}, \left(x \neq \frac{3}{5}\right)$$ **[4]**

Show that $(gof)(x) = (fog)(x)$.

Hence Proved

3. (a) If $\cos^{-1}\frac{x}{2} + \cos^{-1}\frac{y}{3} = \theta$, then prove that

$$9x^2 - 12xy\cos\theta + 4y^2 = 36\sin^2\theta$$

OR

(b) Evaluate : $\cos(2\cos^{-1}x + \sin^{-1}x)$ at $x = \frac{1}{5}$ **[4]**

Ans. (a)

$$\cos^{-1}\frac{x}{2} + \cos^{-1}\frac{y}{3} = \theta$$

$$\cos^{-1}\left\{\frac{x}{2}.\frac{y}{3} - \sqrt{1-\frac{x^2}{4}}\sqrt{1-\frac{y^2}{9}}\right\} = \theta$$

$$\left[\cos^{-1}a + \cos^{-1}b = \cos^{-1}\left(ab - \sqrt{1-a^2}\sqrt{1-b^2}\right)\right]$$

$$\frac{xy}{6} - \sqrt{1-\frac{x^2}{4}}\sqrt{-\frac{y^2}{9}} = \cos\theta$$

$$\frac{xy}{6} - \cos\theta = \sqrt{1-\frac{x^2}{4}}\sqrt{1-\frac{y^2}{9}}$$

Squaring both sides

$$\frac{x^2y^2}{36} + \cos^2\theta - \frac{xy\cos\theta}{3} = \left(1-\frac{x^2}{4}\right)\left(1-\frac{y^2}{9}\right)$$

$$\frac{x^2y^2}{36} + \cos^2\theta - \frac{xy\cos\theta}{3} = 1 - \frac{x^2}{4} - \frac{y^2}{9} + \frac{x^2y^2}{36}$$

$$\frac{x^2}{4} + \frac{y^2}{9} - \frac{xy\cos\theta}{3} = 1 - \cos^2\theta$$

$$\frac{x^2}{4} + \frac{y^2}{9} - \frac{xy\cos\theta}{3} = \sin^2\theta$$

Multiply by 36

$$9x^2 + 4y^2 - 12xy\cos\theta = 36\sin^2\theta$$

$$9x^2 - 12xy\cos\theta + 4y^2 = 36\sin^2\theta$$

Hence Proved.

OR

(b) $\cos(2\cos^{-1}x + \sin^{-1}x)$ at $x = \frac{1}{5}$

$$= \cos(\cos^{-1}x + \sin^{-1}x + \cos^{-1}x)$$

$$= \cos\left(\frac{\pi}{2} + \cos^{-1}x\right)$$

$$= -\sin(\cos^{-1}x)$$

$$= -\sin\sin^{-1}\left(\sqrt{1-x^2}\right)$$

$$= -\left(\sqrt{1-x^2}\right)$$

$$= -\left(\sqrt{1-\frac{1}{25}}\right)$$

$$= -\sqrt{\frac{24}{25}} = -\frac{2\sqrt{6}}{5}$$

4. Using properties of determinants, show that

$$\begin{vmatrix} x & p & q \\ p & x & q \\ q & q & x \end{vmatrix} = (x-p)(x^2 + px - 2q^2)$$ **[4]**

Ans.

$$\text{L.H.S.} = \begin{vmatrix} x & p & q \\ p & x & q \\ q & q & x \end{vmatrix}$$

$C_1 \to C_1 - C_2$

$$= \begin{vmatrix} x-p & p & q \\ p-x & x & q \\ 0 & q & x \end{vmatrix}$$

***Out of Syllabus**

$$= (x-p)\begin{vmatrix} 1 & p & q \\ -1 & x & q \\ 0 & q & x \end{vmatrix}$$

$R_2 \to R_2 + R_2$

$$= (x-p)\begin{vmatrix} 1 & p & q \\ 0 & x+p & 2q \\ 0 & q & x \end{vmatrix}$$

Expand towards C_1

$$= (x-p)\begin{vmatrix} x+p & 2q \\ q & x \end{vmatrix}$$

$$= (x-p)(x^2 + px - 2q^2)$$

= R.H.S. **Hence Proved**

5. Verify Rolle's theorem for the function, $f(x) = -1 + \cos x$ in the interval $[0, 2\pi]$ [4]

Ans. $f(x) = -1 + \cos x \; [0, 2\pi]$

Since $\cos x$ is everywhere continuous and differential in the given interval

$\therefore$ $f(x) = -1 + \cos x$ is continuous in $[0, 2\pi]$

and $f(x) = -1 + \cos x$ is differentiable $(0, 2\pi)$

$$f(0) = -1 + \cos 0 = -1 + 1 = 0$$

$$f(2\pi) = -1 + \cos 2\pi = -1 + 1 = 0$$

$$f(0) = f(2\pi)$$

$$f'(x) = -\sin x$$

$$f'(c) = -\sin c$$

$$0 = -\sin c$$

$$\sin c = 0$$

$c = n\pi$ i.e, π when $n = 1$

$\therefore$ $0 < c < 2\pi$

Hence Rolles theorem is verified.

6. If $y = e^{m\sin^{-1}x}$, prove that

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = m^2y$$

[4]

Ans.

$$y = e^{m\sin^{-1}x}$$

$$\frac{d}{dx}y = \frac{d}{dx}(e^{m\sin^{-1}x})$$

$$\frac{dy}{dx} = e^{m\sin^{-1}x}\frac{d}{dx}(m\sin^{-1}x)$$

$$\frac{dy}{dx} = e^{m\sin^{-1}x}\frac{m}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = \frac{my}{\sqrt{1-x^2}}$$

Squaring both sides

$$\left(\frac{dy}{dx}\right)^2 = \frac{m^2y^2}{1-x^2}$$

$$(1-x^2)\left(\frac{dy}{dx}\right)^2 = m^2y^2$$

Differentiate *w.r.t.* x

$$(1-x^2)\frac{d}{dx}\left(\frac{dy}{dx}\right)^2 + \left(\frac{dy}{dx}\right)^2\frac{d}{dx}(1-x^2) = \frac{d}{dx}m^2y^2$$

$$(1-x^2)\frac{2dy}{dx}\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2(0-2x) = m^2 2y\frac{dy}{dx}$$

$$\frac{2dy}{dx}\left\{(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx}\right\} = 2m^2y\frac{dy}{dx}$$

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = m^2y$$ **Hence Proved.**

7. (a) The equation of tangent at (2, 3) on the curve $y^2 = px^3 + q$ is $y = 4x - 7$. Find the values of 'p' and 'q'. [4]

OR

(b) $\lim_{x\to 0}\frac{xe^x - \log(1+x)}{x^2}$ [4]

Ans. (a)

$$y^2 = px^3 + q$$

$$\frac{d}{dx}(y^2) = \frac{d}{dx}px^3 + \frac{d}{dx}q$$

$$2y\frac{dy}{dx} = 3px^2 + 0$$

$$\left(\frac{dy}{dx}\right) = \frac{3px^2}{2y}$$

Slope of the line, $\left(\frac{dy}{dx}\right)_{(2,3)} = \frac{3p(2)^2}{2\times 3} = 2p$

Since the slope of given tangent line is 4.

Hence, $2p = 4$

or, $p = 2$

Since, the equation of the curve $y^2 = px^3 + q$ is also passing through (2, 3)

$\therefore$ $(3)^2 = p(2)^3 + q$

$9 = p(8) + q$

$9 = 16 + q$

$\therefore$ $q = -7$

Hence, $p = 2, q = -7$.

OR

7. (b) $\lim_{x\to 0} \frac{xe^x - \log(1+x)}{x^2}$

$$\lim_{x\to 0} \frac{\frac{d}{dx}\left[xe^x - \log(1+x)\right]}{\frac{d}{dx}x^2}$$

$$\lim_{x\to 0} \frac{xe^x + e^x - \frac{1}{1+x}(1+0)}{2x}$$

$$\lim_{x\to 0} \frac{xe^x + e^x - \frac{1}{1+x}}{2x}$$

$$\lim_{x\to 0} \frac{\frac{d}{dx}\left(xe^x + e^x - \frac{1}{1+x}\right)}{\frac{d}{dx}(2x)}$$

$$\lim_{x\to 0} \frac{xe^x + e^x + e^x \frac{1}{(1+x)^2}}{2}$$

$$\lim_{x\to 0} \frac{xe^x + 2e^x + \frac{1}{(1+x)^2}}{2}$$

$$= \frac{0\times e^0 + 2\times e^0 + \frac{1}{(1+0)^2}}{2} = \frac{0+2+1}{2}$$

$$\lim_{x\to 0} \frac{xe^x - \log(1+x)}{x^2} = \frac{3}{2}$$

8. (a) Evaluate : $\int \frac{dx}{\sqrt{5x-4x^2}}$ **[4]**

OR

(b) Evaluate : $\int \sin^3 x \cos^4 x\, dx$

Ans. (a)

$$\int \frac{dx}{\sqrt{5x-4x^2}} = \frac{1}{2}\int \frac{dx}{\sqrt{\frac{5x}{4} - x^2}}$$

$$= \frac{1}{2}\int \frac{dx}{\sqrt{\frac{25}{64} - \left(x^2 - \frac{5x}{4} + \frac{25}{64}\right)}}$$

$$= \frac{1}{2}\int \frac{dx}{\sqrt{\left(\frac{5}{8}\right)^2 - \left(x - \frac{5}{8}\right)^2}}$$

$$= \frac{1}{2}\sin^{-1}\left|\frac{x-\frac{5}{8}}{\frac{5}{8}}\right| + c$$

$$= \frac{1}{2}\sin^{-1}\left|\frac{8x-5}{5}\right| + c$$

OR

(b) $I = \int \sin^3 x \cos^4 x\, dx$

Let $\cos x = t$

$-\sin x\, dx = dt$

$$dx = \frac{-dt}{\sin x}$$

$$\therefore \quad I = \int \sin^3 x\, t^4 \left(\frac{-dt}{\sin x}\right)$$

$$I = \int \sin^2 x\, t^4 (-dt)$$

$$= -\int \left(1 - \cos^2 t\right) t^4\, dt$$

$$= -\int (1-t^2)t^4 dt$$

$$= -\int (t^4 - t^6) dt$$

$$I = \frac{-t^5}{5} + \frac{t^7}{7} + c$$

$$= \frac{-\cos^5 x}{5} + \frac{\cos^7 x}{7} + c$$

9. Solve the differential equation **[4]**

$$(1 + x^2)\frac{dy}{dx} = 4x^2 - 2xy$$

Ans.

$$(1+x^2)\frac{dy}{dx} = 4x^2 - 2xy$$

$$\frac{dy}{dx} = \frac{4x^2}{1+x^2} - \frac{2xy}{1+x^2}$$

$$\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{4x^2}{1+x^2}$$

Compare $\frac{dy}{dx} + Py = Q(x)$

$\therefore \quad P = \frac{2x}{1+x^2}$ and $Q(x) = \frac{4x^2}{1+x^2}$

$$I.F. = e^{\int P dx} = e^{\int \frac{2x}{1+x^2}dx}$$

$$= e^{\log|1+x^2|} = (1 + x^2)$$

Solution of $\frac{dy}{dx} + Py = Q(x)$ is

$$y \times I.F. = \int I.F. \times Q(x) dx$$

$$(1 + x^2)y = \int (1+x^2)\frac{4x^2}{1+x^2}dx$$

$$(1 + x^2)y = \int 4x^2 dx$$

$$(1 + x^2)y = \frac{4x^3}{3} + c$$

10. Three persons *A*, *B* and *C* shoot to hit a target. Their probabilities of hitting the target are $\frac{5}{6}$, $\frac{4}{5}$ and $\frac{3}{4}$ respectively. Find the probability that :

(i) Exactly two persons hit the target.

(ii) At least one person hits the target.

Ans. Probability of hitting the target

$P(A) = 5/6, P(B) = 4/5, P(C) = 3/4$ (Given)

(i) Exactly two persons hit the target

$$P(A) \times P(B) \times P(\bar{C}) + P(A) \times P(\bar{B}) \times P(C) + P(\bar{A}) \times P(B) \times P(C)$$

$$= \frac{5}{6}\times\frac{4}{5}\times\left(1-\frac{3}{4}\right)+\frac{5}{6}\times\left(1-\frac{4}{5}\right)\left(\frac{3}{4}\right) + \left(1-\frac{5}{6}\right)\left(\frac{4}{5}\right)\left(\frac{3}{4}\right)$$

$$= \frac{5}{6}\times\frac{4}{5}\times\frac{1}{4}+\frac{5}{6}\times\frac{1}{5}\times\frac{3}{4}+\frac{1}{6}\times\frac{4}{5}\times\frac{3}{4}$$

$$= \frac{20}{120}+\frac{15}{120}+\frac{12}{120}$$

$$= \frac{47}{120} = 0.3917 \text{ (Approx.)}$$

∴ Probability of exactly two persons hit the target $= \frac{47}{120}$

(ii) At least one person hits the target

$$= 1 - P(\bar{A})\, P(\bar{B})\, P(\bar{C})$$

$$= 1-\left(1-\frac{5}{6}\right)\left(1-\frac{4}{5}\right)\left(1-\frac{3}{4}\right)$$

$$= 1-\left(\frac{1}{6}\right)\left(\frac{1}{5}\right)\left(\frac{1}{4}\right)$$

$$= 1-\frac{1}{120} = \frac{119}{120}$$

∴ Probability of at least one person hits the target

$$= \frac{119}{120}$$

$$= 0.9917 \text{ approx.}$$

11. Solve the following system of linear equations using matrices :

$x - 2y = 10, 2x - y - z = 8, -2y + z = 7$ [6]

Ans. $x - 2y = 10, 2x - y - z = 8$ and $-2y + z = 7$

Equations can be rearrange in matrix form

$$\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$AX = B \quad ...(i)$$

$$\left[\text{where } A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}\right]$$

Now, $A^{-1}AX = A^{-1}B$

$IX = A^{-1}B$

$X = A^{-1}B$ ...(ii)

$$A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{vmatrix}$$

$$= 1\begin{vmatrix} -1 & -1 \\ -2 & 1 \end{vmatrix} + 2\begin{vmatrix} 2 & -1 \\ 0 & 1 \end{vmatrix} + 0\begin{vmatrix} 2 & -1 \\ 0 & -2 \end{vmatrix}$$

$$= 1(-1-2) + 2(2+0) + 0$$

$$= -3 + 4 = 1$$

$|A| = 1$ A^{-1} exist.

$$C_{11} = (-1)^{1+1}\begin{vmatrix} -1 & -1 \\ -2 & 1 \end{vmatrix} = -1-2 = -3$$

$$C_{12} = (-1)^{1+2}\begin{vmatrix} 2 & -1 \\ 0 & 1 \end{vmatrix} = -(2+0) = -2$$

$$C_{13} = (-1)^{1+3}\begin{vmatrix} 2 & -1 \\ 0 & -2 \end{vmatrix} = -4+0 = -4$$

$$C_{21} = (-1)^{2+1}\begin{vmatrix} -2 & 0 \\ -2 & 1 \end{vmatrix} = -(-2+0) = 2$$

$$C_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1(1-0) = 1$$

$$C_{23} = (-1)^{2+3}\begin{vmatrix} 1 & -2 \\ 0 & -2 \end{vmatrix} = -(-2+0) = 2$$

$$C_{31} = (-1)^{3+1}\begin{vmatrix} -2 & 0 \\ -1 & -1 \end{vmatrix} = 2+0 = 2$$

$$C_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 0 \\ 2 & -1 \end{vmatrix} = -(-1-0) = 1$$

$$C_{33} = (-1)^{3+3}\begin{vmatrix} 1 & -2 \\ 2 & -1 \end{vmatrix} = (-1+4) = 3$$

$$A_{adj} = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}^t = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{A_{adj}}{|A|} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}$$

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -30+16+14 \\ -20+8+7 \\ -40+16+21 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$$

$\therefore$ $x = 0, y = -5$ and $z = -3$

12. (a) Show that the radius of a closed right circular cylinder of given surface area and maximum volume is equal to half of its height. [6]

OR

(b) Prove that the area of right-angled triangle of given hypotenuse is maximum when the triangle is isosceles. [6]

Ans. (a) Given : Total surface area of cylinder

$$= 2\pi r^2 + 2\pi rh$$

$$A = 2\pi r^2 + 2\pi rh$$

$$\therefore \quad h = \frac{A - 2\pi r^2}{2\pi r} \quad \text{...(i)}$$

[Where r is the radius and h is the height of the cylinder]

Volume of cylinder (V)

$$= \pi r^2 h$$

$$V = \pi r^2 \frac{(A - 2\pi r^2)}{2\pi r} \quad \text{from (i)}$$

$$V = \frac{r}{2}(A - 2\pi r^2)$$

$$V = \frac{Ar}{2} - \pi r^3$$

Now, $$\frac{dV}{dr} = \frac{d}{dr}\frac{Ar}{2} - \frac{d}{dr}(\pi r^3)$$

$$\frac{dV}{dr} = \frac{A}{2} - 3\pi r^2$$

For maximum/minimum;

$$\frac{dV}{dr} = 0$$

$$\therefore \quad \frac{A}{2} - 3\pi r^2 = 0$$

$$\frac{A}{2} = 3\pi r^2$$

$$A = 6\pi r^2$$

So, $$2\pi r^2 + 2\pi rh = 6\pi r^2$$

$$2\pi rh = 4\pi r^2$$

$$h = 2r$$

$$r = \frac{h}{2} \quad \text{...(iii)}$$

Again, $$\frac{dV}{dr} = \frac{A}{2} - 3\pi r^2$$

and, $$\frac{d^2V}{dr^2} = 0 - 6\pi r$$

for any value of r i.e, $r > 0$

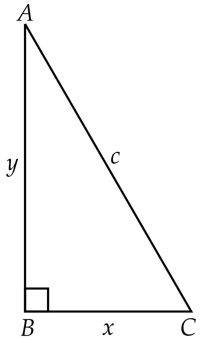
$$\frac{d^2V}{dr^2} < 0$$

Hence, volume is maximum when $r = \frac{h}{2}$

Hence Proved.

OR

(b) Given ΔABC is right angle triangle and hypotenuse is c unit.



To Prove : Area is maximum when triangle is an isosceles triangle.

Proof : In ΔABC, $\angle B = 90°$

$$\therefore \quad c^2 = x^2 + y^2 \quad \text{...(i)}$$

$$\text{Area of } \Delta ABC = \frac{1}{2} \text{ base} \times \text{height}$$

$$A = \frac{1}{2}xy$$

$$A^2 = \frac{1}{4}x^2y^2 \quad (\text{Let } A^2 = Z)$$

$$Z = \frac{1}{4}x^2(c^2 - x^2)$$

$$Z = \frac{c^2x^2}{4} - \frac{x^4}{4} \quad \text{...(ii)}$$

$$\frac{dZ}{dx} = \frac{d}{dx}\frac{c^2x^2}{4} - \frac{d}{dx}\frac{x^4}{4}$$

$$\frac{dZ}{dx} = \frac{2c^2x}{4} - \frac{4x^3}{4}$$

$$\frac{dZ}{dx} = \frac{c^2x}{2} - x^3$$

For maximum/minimum,

$$\frac{dZ}{dx} = 0$$

$$\frac{c^2x}{2} - x^3 = 0$$

$$x\left(\frac{c^2}{2} - x^2\right) = 0$$

either $x = 0$ (Not possible $x > 0$)

So, $\frac{c^2}{2} - x^2 = 0$

$$\frac{c^2}{2} = x^2$$

$$c^2 = 2x^2$$

Now, from equation (i) $x^2 + y^2 = 2x^2$

$$y^2 = x^2$$

$$y = x$$

Again, $\frac{dZ}{dx} = \frac{c^2x}{2} - x^3$

Again, $\frac{d^2Z}{dx^2} = \frac{c^2}{2} - 3x^2$

$$\frac{d^2Z}{dx^2} = \frac{x^2+y^2}{2} - 3x^2$$

$$\frac{d^2Z}{dx^2} = \frac{2x^2}{2} - 3x^2 = -2x^2$$

$$\frac{d^2Z}{dx^2} < 0$$

∴ Area is maximum when $x = y$.

∴ ΔABC is an isosceles right angle triangle.

Hence Proved.

13. (a) Evaluate :

$$\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}dx$$

[6]

$$\mathbf{I} = \int \tan^{-1}\sqrt{\frac{1-x}{1+x}}dx$$

Ans. (a) $I = \int \tan^{-1}\sqrt{\frac{1-x}{1+x}}dx$

Let $x = \cos 2\theta$

$dx = -2\sin 2\theta\, d\theta$

$$I = \int \tan^{-1}\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}(-2\sin 2\theta)d\theta$$

$$I = \int \tan^{-1}\sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}}(-2\sin 2\theta)d\theta$$

$$I = \int \tan^{-1}\tan\theta(-2\sin 2\theta)d\theta$$

$$I = --2\int \theta \sin 2\theta d\theta$$

$$= -2\left[\theta\int \sin 2\theta d\theta - \int\left[\frac{d}{d\theta}\theta\int \sin 2\theta d\theta\right]d\theta\right]$$

$$= -2\left[-\theta\frac{\cos 2\theta}{2} - \int \frac{\{\cos 2\theta\}}{2}d\theta\right]$$

$$= \theta\cos 2\theta - \int \cos 2\theta d\theta$$

$$= \theta\cos 2\theta - \frac{\sin 2\theta}{2} + C$$

$$= \theta\cos 2\theta - \frac{1}{2}\sqrt{1-\cos^2 2\theta} + C$$

$$= \frac{1}{2}x\cos^{-1}x - \frac{1}{2}\sqrt{1-x^2} + C$$

OR

(b) Evaluate :

$$\int \frac{2x+7}{x^2-x-2}dx$$

Ans. (b) $\int \frac{2x+7}{x^2-x-2}dx = \int \frac{2x+7}{(x-2)(x+1)}dx$

$$\frac{2x+7}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1}$$

$$2x+7 = A(x+1)+B(x-2)$$

Put, $x = -1$

$-2+7 = B$ **(-1-2)**

$$\Rightarrow B = -\frac{5}{3}$$

Put, $x = 2$

$$4+7 = A(1+2)$$

$$\Rightarrow A = \frac{11}{3}$$

$$\int \frac{2x+7}{(x-2)(x+1)}dx$$

$$= \frac{11}{3}\int \frac{dx}{x-2} - \frac{5}{3}\int \frac{dx}{x+1}$$

$$= \frac{11}{3}\log|x-2| - \frac{5}{3}\log|x+1| + C$$

$$= \frac{1}{3}\left[\log\left|(x-2)^{11}\right| - \log\left|(x+1)^5\right|\right] + C$$

$$\int \frac{2x+7}{x^2-x-2}dx$$

$$= \frac{1}{3}\log\left|\frac{(x-2)^{11}}{(x+1)^5}\right| + C$$

14. The probability that a bulb produced in a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs :

(i) None will fuse after 150 days of use.

(ii) Not more than one will fuse after 150 days of use.

(iii) More than one will fuse after 150 days of use.

(iv) At least, one will fuse after 150 days of use. [6]

Ans. $n = 5, p = 0.05$

$\therefore \quad d = 1 - p = 0.95$

(i) None will fuse after 150 days of use

i.e, $r = 0$

$P(r) = {}^nC_r\, p^r\, d^{n-r}$

$P(0) = {}^5C_0\,(0.05)^0(0.95)^{5-0}$

$= (0.95)^5$

$P(0) = (0.95)^5$

Probability of bulb is none is fused

$= 0.77378$ (approx.)

(ii) Not more than one will fuse after 150 days of use

$r = 0, 1$

$P(0) + P(1) = (0.95)^5 + {}^5C_1(0.05)^1(0.95)^4$

$= (0.95)^5 + 5 \times 0.05(0.95)^4$

$= (0.95)^4(0.95 + 0.25)$

$= (0.95)^4(1.20)$

$= 0.97740$ (approx.)

Probability of bulb is not more than one will fuse after 150 days of use

$= 0.97740$

(iii) More then one will fuse after 150 days of use

$= 1\{P(0) + P(1)\}$

$= 1 - 0.97740$

$= 0.0226$

Probability of bulb more than one will fuse after 150 days of use

$= 0.0226$

(iv) At least one will fuse after 150 days of use

$= 1 - P(0)$

$= 1 - 0.77378$

$= 0.2262$ (approx.)

$\therefore$ Probability of bulb at least one will be fuse after 150 days of use

$= 0.2262$ (approx.)

SECTION - B [20 Marks]

15. (a) Write a vector of magnitude of 18 units in the direction of the vector $\hat{i} - 2\hat{j} - 2\hat{k}$. [3 × 2]

Ans. $\vec{a} = \hat{i} - 2\hat{j} - 2\hat{k}$

$|\vec{a}| = \left|\sqrt{1^2 + (-2)^2 + (-2)^2}\right|$

$= \left|\sqrt{1+4+4}\right| = 3$

So, its Unit Vector, $\vec{a} = \dfrac{\vec{a}}{|\vec{a}|} = \dfrac{\hat{i} - 2\hat{j} - 2\hat{k}}{3}$

$18\vec{a} = 18 \times \dfrac{\hat{i} - 2\hat{j} - 2\hat{k}}{3}$

$= 6(\hat{i} - 2\hat{j} - 2\hat{k})$

Required vector $= 6\hat{i} - 12\hat{j} - 12\hat{k}$

(b) Find the angle between the two lines :

$$\frac{x+1}{2} = \frac{y-2}{5} = \frac{z+3}{4}$$

and $$\frac{x-1}{5} = \frac{y+2}{2} = \frac{z-1}{-5}$$

Ans. Angle between two lines

$$\cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right|$$

Where $a_1 = 2, b_1 = 5, c_1 = 4$

and $a_2 = 5, b_2 = 2, c_2 = -5$

$$\cos\theta = \left|\frac{2\times5 + 5\times2 + 4(-5)}{\sqrt{2^2 + 5^2 + 4^2}\sqrt{5^2 + 2^2 + (-5)^2}}\right|$$

$$\cos\theta = \left|\frac{10 + 10 - 20}{\sqrt{4+25+16}\sqrt{25+4+25}}\right|$$

$\cos\theta = 0$

$\theta = \dfrac{\pi}{2}$

Lines are perpendicular each other

(c) Find the equation of the plane passing through the point (2, –3, 1) and perpendicular to the line joining the points (4, 5, 0) and (1,2,4).

Ans. Plane passing through the point (2, –3, 1) and perpendicular to the line joining the point (4, 5, 0) and (1, –2, 4) direction ratio of the line is

$\{(1 - 4), (-2 - 5), (4 - 0)\}$

$= (-3, -7, 4)$

direction ratio of the plane

= direction ratio of the line

$= (-3, -7, 4)$

Equation of the plane

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$
$$-3(x - 2) - 7(y + 3) + 4(z - 1) = 0$$
$$-3x + 6 - 7y - 21 + 4z - 4 = 0$$
$$-3x - 7y + 4z - 19 = 0$$
$$3x + 7y - 4z + 19 = 0$$

Required equation of the plane is

$$3x + 7y - 4z + 19 = 0$$

16. (a) Prove that $\vec{a}.\left[\left(\vec{b}+\vec{c}\right)\times\left(\vec{a}+3\vec{b}+4\vec{c}\right)\right] = \left[\vec{a}\ \vec{b}\ \vec{c}\right]$ **[4]**

OR

(b) Using vectors, find the area of the triangle whose vertices are :

$A(3, -1, 2)$, $B(1, -1, -3)$ and $C(4, -3, 1)$

Ans. (a) $\vec{a}[(\vec{b}+\vec{c})\times(\vec{a}+3\vec{b}+4\vec{c})] = [\vec{a}\ \vec{b}\ \vec{c}]$

$$\text{L.H.S.} = \vec{a}[(\vec{b}+\vec{c})\times(\vec{a}+3\vec{b}+4\vec{c})]$$
$$= \vec{a}[\vec{b}\times\vec{a}+3\vec{b}\times\vec{b}+4\vec{c}\times\vec{b}+\vec{c}\times\vec{a} +3\vec{b}\times\vec{c}+4\vec{c}\times\vec{c}]$$
$$= \vec{a}[\vec{b}\times\vec{a}+4\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+3\vec{c}\times\vec{b}]$$
$$= \vec{a}[\vec{b}\times\vec{a}+4\vec{b}\times\vec{c}+\vec{c}\times\vec{a}-3\vec{b}\times\vec{c}]$$
$$= \vec{a}[\vec{b}\times\vec{a}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}]$$
$$= \vec{a}.\vec{b}\times\vec{a}+\vec{a}.\vec{b}\times\vec{c}+\vec{a}.\vec{c}\times\vec{a}$$
$$= 0+\vec{a}.\vec{b}\times\vec{c}+0$$
$$= \vec{a}.\vec{b}\times\vec{c} = \left[\vec{a}.\vec{b}\times\vec{c}\right]$$
$$= \text{R.H.S.}$$

Hence Proved.

OR

(b) $A(3, -1, 2)$; $B(1, -1, -3)$ and $C(4, -3, 1)$

$\vec{a} = 3\hat{i}-\hat{j}+2\hat{k}$; $\vec{b} = \hat{i}-\hat{j}-3\hat{k}$; $\vec{c} = 4\hat{i}-3\hat{j}+\hat{k}$

$$\vec{AB} = \vec{b}-\vec{a}$$
$$= \hat{i}-\hat{j}-3\hat{k}-3\hat{i}+\hat{j}-2\hat{k}$$
$$\vec{AB} = -2\hat{i}-5\hat{k}$$
$$\vec{AC} = \vec{c}-\vec{a}$$
$$= 4\hat{i}-3\hat{j}+\hat{k}-3\hat{i}+\hat{j}-2\hat{k}$$
$$= \hat{i}-2\hat{j}-\hat{k}$$
$$\text{ar }\Delta ABC = \frac{1}{2}\vec{AB}\times\vec{AC}$$
$$= \frac{1}{2}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -2 & 0 & -5\\ 1 & -2 & -1\end{vmatrix}$$
$$= \frac{1}{2}\left|\hat{i}\begin{vmatrix}0 & -5\\ -2 & 1\end{vmatrix}-\hat{j}\begin{vmatrix}-2 & -5\\ 1 & -1\end{vmatrix} +\hat{k}\begin{vmatrix}-2 & 0\\ 1 & -2\end{vmatrix}\right|$$
$$= \frac{1}{2}\left|-10\hat{i}-7\hat{j}+4\hat{k}\right|$$
$$\text{ar }\Delta ABC = \frac{1}{2}\left|\sqrt{(-10)^2+(-7)^2+(4)^2}\right|$$
$$= \frac{1}{2}\left|\sqrt{100+49+16}\right|$$
$$\text{ar }\Delta ABC = \frac{1}{2}\sqrt{165}\text{ unit}^2.$$

17. (a) Find the image of the point $(3, -2, 1)$ in the plane $3x - y + 4z = 2$. **[4]**

OR

(b) Determine the equation of the line passing through the point $(-1, 3, -2)$ and perpendicular to the lines :

$$\frac{x}{1}=\frac{y}{2}=\frac{z}{3} \quad \text{and} \quad \frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$$

Ans. (a) Equation of the plane

$$3x - y + 4z = 2$$

$\therefore$ direction ratio of perpendicular line to the plane is $(3, -1, 4)$

Equation of the line is passing through $(3, -2, 1)$ and direction ratios are $(3, -1, 4)$ is

$$\frac{x-3}{3} = \frac{y+2}{-1} = \frac{z-1}{4}$$

General point on the line is $(3\lambda + 3, -\lambda - 2, 4\lambda + 1)$

Foot of perpendicular lies in the plane

$$\therefore\ 3(3\lambda + 3) - (-\lambda - 2) + 4(4\lambda + 1) = 2$$
$$9\lambda + 9 + \lambda + 2 + 16\lambda + 4 = 2$$
$$26\lambda = 2 - 15$$
$$\lambda = \frac{-13}{26} = \frac{-1}{2}$$

Hence point of intersection of the line and plane is

$$\left(\frac{-3}{2}+3, \frac{1}{2}-2, -2+1\right) = \left(\frac{3}{2}, \frac{-3}{2}, -1\right)$$

Let the image of the point $(3, -2, 1)$ be (x, y, z)

$$\therefore\ \frac{x+3}{2} = \frac{3}{2} \Rightarrow x = 0$$
$$\frac{y-2}{2} = \frac{-3}{2} \Rightarrow y = -1$$

$$\frac{z-1}{2} = -1 \quad z = -1$$

Hence, coordinate of image (0, – 1, – 1).

OR

(b) Equation of the line passes through (– 1, 3, – 2) is :

$$\frac{x+1}{a} = \frac{y-3}{b} = \frac{z+2}{c}$$

it is perpendicular to

$$\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$$

$\therefore \quad a + 2b + 3c = 0 \qquad ...(i)$

it is also perpendicular to

$$\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$$

$\therefore \quad -3a + 2b + 5c = 0 \qquad ...(ii)$

$a + 2b + 3c = 0 \qquad ...(iii)$

$$\frac{a}{6-10} = \frac{b}{5+9} = \frac{c}{-6-2}$$

$$\frac{a}{-4} = \frac{b}{14} = \frac{c}{-8}$$

$$\frac{a}{2} = \frac{b}{-7} = \frac{c}{4}$$

Hence, required equation of the line is :

$$\frac{x+1}{2} = \frac{y-3}{-7} = \frac{z+2}{4}$$

18. Draw a rough sketch of the curves $y^2 = x$ and $y^2 = 4 - 3x$ and find the area enclosed between them. [6]

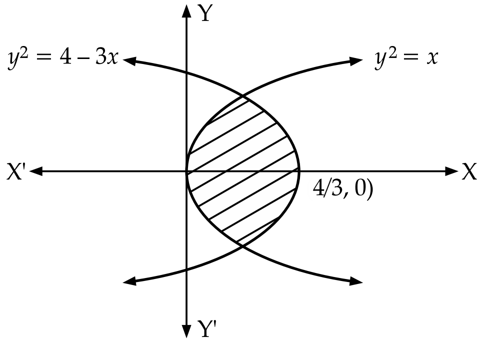
Ans. $y^2 = x \qquad ...(i)$

$y^2 = 4 - 3x \qquad ...(ii)$

from (i) & (ii)

$x = 4 - 3x$

$x = 1$



$\therefore$ Point of intersection (1, 1) and (1, – 1)

Required area

$$= 2\left[\int_0^1 \sqrt{x}dx + \int_1^{4/3} \sqrt{4-3x}dx\right]$$

$$= 2\left[\frac{2}{3}\left(x^{3/2}\right)_0^1 - \frac{2}{3x3}\left\{(4-3x)^{3/2}\right\}_1^{4/3}\right]$$

$$= 2\left[\frac{2}{3}(1-0) - \frac{2}{9}(0-1)\right] = 2\left[\frac{2}{3} + \frac{2}{9}\right]$$

Required area

$$= \frac{16}{9} \text{ unit}^2$$

SECTION - C [20 Marks]

19. (a) The selling price of a commodity is fixed at ₹60 and its cost function is :

***C(x)* = 35*x* + 250**

(i) Determine the profit function.

(ii) Find the break even points. [3 × 2]

Ans. Let the number of articles be x

Total sell price = ₹60x

Total cost price = $35x + 250$

(i) Profit function $P(x)$

$= S.P. - C.P.$

$= 60x - 35x - 250$

$= 25x - 250$

Hence, profit function is $25x - 250$.

(ii) Break-even point i.e, $P(x) = 0$

$25x - 250 = 0 \Rightarrow x = 10$ units

(b) The revenue function is given by $R(x) = 100x - x^2 - x^3$. Find

(i) The demand function.

(ii) Marginal revenue function.

Ans. (i) $R(x) = 100x - x^2 - x^3$

Demand function

$$= \frac{R(x)}{x} = 100 - x - x^2$$

(ii) Marginal revenue function (*M.R.*)

$$= \frac{dR(x)}{dx}$$

$$= \frac{d}{dx}(100x) - \frac{d}{dx}x^2 - \frac{d}{dx}x^3$$

Marginal revenue function

$= 100 - 2x - 3x^2$

(c) For the lines of regression $4x - 2y = 4$ and $2x - 3y + 6 = 0$, find the mean of 'x' and the mean of 'y'.

Ans. $4x - 2y - 4 = 0$

$2x - 3y + 6 = 0$

$$\frac{x}{-12-12} = \frac{y}{-8-24} = \frac{1}{-12+4}$$

$$\frac{x}{-24} = \frac{y}{-32} = \frac{1}{-8}$$

$$x = \frac{-24}{-8} = 3$$

$$y = \frac{-32}{-8} = 4$$

$\therefore \quad \bar{x} = 3$ and $\bar{y} = 4$

20. (a) The correlation coefficient between x and y is 0.6. If the variance of x is 225, the variance of y is 400, mean of x is 10 and mean of y is 20, find

(i) the equations of two regression lines.

(ii) the expected value of y when $x = 2$. [4]

Or

(b) Find the regression coefficients b_{yx}, b_{xy} and correlation coefficient 'r' for the following data : (2, 8), (6, 8), (4, 5), (7, 6), (5, 2) [4]

Ans. **(a)** Given : Correlation coeffiecient $= 0.6$

Variance of $x = 225$

Variance of $y = 400$

Mean of $x = 10$

Mean of $y = 20$

$$\sigma_x = \sqrt{\text{Variance}} = \sqrt{225} = \pm 15$$

$$\sigma_y = \sqrt{\text{Variance}} = \sqrt{400} = \pm 20$$

(i) $$b_{yx} = r\frac{\sigma_y}{\sigma_x} = 0.6 \times \frac{20}{15} = 0.8$$

$$b_{xy} = r\frac{\sigma_x}{\sigma_y} = 0.6 \times \frac{15}{20} = 0.45$$

Regression line y on x

$$y - \bar{y} = by_x(x - \bar{x})$$
$$y - 20 = 0.8(x - 10)$$
$$y - 20 = 0.8x - 8$$
$$y = 0.8x + 12 \quad ...(1)$$

Regression line x on y

$$x - \bar{x} = b_{yz}(y - \bar{y})$$
$$x - 10 = 0.45(y - 20)$$
$$x = 0.45y - 9 + 10$$
$$x = 0.45y + 1 \quad ...(2)$$

(ii) When $x = 2$

$$y = 0.8 \times 2 + 12 = 13.6$$

OR

(b)

x	y	x^2	y^2	xy
2	8	04	64	16
6	8	36	64	48
4	5	16	25	20
7	6	49	36	42
5	2	25	04	10
24	29	130	193	136

$n = 5$

$$by_x = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{n}}{\Sigma x^2 - \frac{(\Sigma x)^2}{n}} = \frac{136 - \frac{24 \times 29}{5}}{130 - \frac{(24)^2}{5}} = \frac{680 - 696}{5(130 - 115.2)}$$

$$\therefore \quad b_{yx} = \frac{-16}{5(14.8)} = \frac{-16}{74} = \frac{-8}{37} = -0.2162 \text{ (approx.)}$$

$$b_{xy} = \frac{\Sigma xy - \frac{1}{n}\Sigma x \Sigma y}{\Sigma y^2 - \frac{1}{n}(\Sigma y)^2} = \frac{136 - \frac{1}{5}24 \times 29}{193 - \frac{1}{5}(29)^2} = \frac{680 - 696}{5(193 - 168.2)} = \frac{-16}{5 \times 24.8} = \frac{-16}{124} = \frac{-4}{31}$$

$$\therefore \quad b_{xy} = -0.1290 \text{ (approx.)}$$

$$r = \pm\sqrt{b_{xy} \times b_{yx}} = -\sqrt{\frac{8}{37} \times \frac{4}{31}} = -\sqrt{\frac{32}{1147}} = -\sqrt{0.02790}$$

$$r = -0.1670 \text{ (approx.)}$$

21. (a) The marginal cost of the production of the commodity is $30 + 2x$, it is known that fixed costs are ₹200, find

(i) The total cost.

(ii) The cost of increasing output from 100 to 200 units. [4]

OR

(b) The total cost function of a firm is given by

$$C(x) = \frac{1}{3}x^3 - 5x^2 + 30x - 15,$$

where the selling price per unit is given as ₹6. Find for what value of x will the profit be maximum.

Ans. (a) (i) Marginal Cost ($M.C.$) $= 30 + 2x$

$$\text{Total Cost} = \int M.C.dx$$

$$= \int (30+2x)dx$$

$$= 30x + 2\frac{x^2}{2} + C$$

Total Cost $= 30x + x^2 + 200$

(C is the fixed cost)

Total Cost $= x^2 + 30x + 200$

(ii) Cost of increasing output from 100 to 200 is $\int_{100}^{200} M.C.dx$

$$= \int_{100}^{200} (30+2x)dx$$

$$= [30x + x^2]_{100}^{200}$$

$= 30(200 - 100) + 200^2 - 100^2$

$= 3000 + (200 - 100)(200 + 100)$

$= 3000 + 100 \times 300$

$=$ ₹33000

OR

(b) $C(x) = \frac{1}{3}x^3 - 5x^2 + 30x - 15$

$R(x) =$ ₹$6x$

(where x is the number of articles)

$P(x) = R(x) - C(x)$

$= 6x - \frac{1}{3}x^3 + 5x^2 - 30x + 15$

[where $P(x)$ is profit function]

$P(x) = -\frac{1}{3}x^3 + 5x^2 - 24x + 15$

$$\frac{dP(x)}{dx} = -\frac{d}{dx}\frac{1}{3}x^3 + 5\frac{d}{dx}x^2 - 24\frac{d}{dx}x + \frac{d}{dx}15$$

$$\frac{dP(x)}{dx} = -x^2 + 10x - 24 + 0$$

$$\frac{dP(x)}{dx} = -x^2 + 10x - 24$$

For maximum or minimum $\frac{dP(x)}{dx} = 0$

$-x^2 + 10x - 24 = 0$

$x^2 - 10x + 24 = 0$

$(x - 6)(x - 4) = 0$

$x = 6$ or $x = 4$

$$\frac{d^2P(x)}{dx^2} = -2x + 10$$

When $x = 6$ $\frac{d^2P}{dx^2} = -2$

$$\left(\frac{d^2P}{dx^2}\right)_{x=6} < 0$$

$\therefore$ Profit is maximum at $x = 6$

Hence profit is maximum at 6 units.

22. A company uses three machines to manufacture two types of shirts, half sleeves and full sleeves. The number of hours required per week on machine M_1, M_2 and M_3 for one shirt of each type is given in the following table :

	M_1	M_2	M_3
Half sleeves	1	2	8/5
Full sleeves	2	1	8/5

None of the machines can be in operation for more than 40 hours per week. The profit on each half sleeve shirt is ₹1 and the profit on each full sleeve shirt is ₹1.50. How many of each type of shirts should be made per week to maximize the company's profit ? **[6]**

Ans. Let number of half-sleeves shirts be x and full-sleeves shirts be y

Objective function $Z = 1x + 1.5y$

Constraints $x + 2y \leq 40$

$2x + y \leq 40$

$\frac{8}{5}x + \frac{8}{5}y \leq 40$ or $x + y \leq 25$

$x + 2y = 40$

x	0	40	10
y	20	0	15

$2x + y = 40$

x	0	20	10
y	40	0	20

$x + y = 25$

x	0	25	10
y	25	0	15

Maximum profit at (0, 0)

$Z = x + 1.5y$

$= 0 + 1.5 \times 0 =$ ₹0

Maximum profit at (0, 20)

$Z = 0 + 1.5 \times 20$

$= 0 + 30 =$ ₹30

Maximum profit at (10, 15)

$Z = 10 + 1.5 \times 15$

$= 10 + 22.5 =$ ₹32.5

Maximum profit at (15, 10)

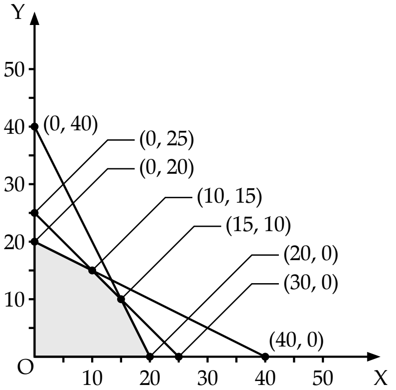
$Z = 15 + 1.5 \times 10$

$= 15 + 15 =$ ₹30

Maximum profit at (20, 0)

$Z = 20 + 1.5 \times 0$

$= 20 + 0 = 0$



Hence, Z is maximum at (10, 15)

$Z = 32.5.$