ISC Solved Paper 2022, Semester -1

Class-XII

Mathematics

(Maximum Marks : 40)

(Time allowed : One and a half hours)

Condidates are allowed an additional ***10 minutes*** *for* ***only*** *reading the paper.*

They must ***NOT*** *start writing during this time.*

The Question Paper consists of three sections A, B and C.

Candidates are required to attempt all questions from Section A and all questions either from Section B OR Section C.

Each question/subpart of a question carries two mark.

Select and write the correct option for each of the following question..

SECTION - A **[34 Marks]**

1. Let 'R' be a relation on N, set of all natural given by $R = \{(a, b) : a - b = 2\}$. Then :

(a) $(2, 4) \in R$ **(b)** $(10, 8) \in R$

(c) $(6, 8) \in R$ **(d)** $(8, 7) \in R$

Ans. Option (b) is correct.

The given relation is, $R = \{(a, b): a - b = 2\}$

As the relation is on set of all natural numbers therefore, 'a' should be greater than 'b'.

In the given options, in option (b) $10 > 8$ and $10 - 8 = 2$.

Thus, $(10, 8) \in R$.

2. If $A = \begin{vmatrix} zy & x & yz \\ xz & y & zx \\ yx & z & xy \end{vmatrix}$, then the value of A is equal to:

(a) 0 **(b)** xyz

(c) 1 **(d)** $\frac{1}{xyz}$

Ans. Option (a) is correct.

Given determinant $$A = \begin{vmatrix} zy & x & yz \\ xz & y & zx \\ yx & z & xy \end{vmatrix}$$

In A, C_1 and C_3 are identical.

According to the property of determinants, if two rows or columns are identical, then its value is zero.

Therefore, $A = 0$.

3. The function $f(x) = \frac{x^3}{3} - x$ is decreasing in the interval:

(a) $(-1, 1)$ **(b)** $(-\infty, -1)$

(c) $(1, \infty)$ **(d)** $(-\infty, -1) \cup (1, \infty)$

Ans. Option (a) is correct.

Given function

$$f(x) = \frac{x^3}{3} - x$$

Differentiating both sides w.r.t. x

$$f'(x) = \frac{3x^2}{3} - 1$$

or $f'(x) = x^2 - 1$

Put $f'(x) = 0$

or $x^2 - 1 = 0$

or $(x - 1)(x + 1) = 0$

or $x = -1, 1$

In the interval $(-1, 1)$ $f(x)$ is decreasing.

4. If $\cot^{-1}\frac{1}{5} + \tan^{-1}x = \frac{\pi}{2}$, than value of x is:

(a) $\frac{1}{5}$ **(b)** 1

(c) 0 **(d)** $\frac{-1}{5}$

Ans. Option (a) is correct.

$$\cot^{-1}\frac{1}{5} + \tan^{-1}x = \frac{\pi}{2} \quad \text{......(i)}$$

Also, $$\cot^{-1}x + \tan^{-1}x = \frac{\pi}{2} \quad \text{......(ii)}$$

From (i) & (ii), we get $x = \frac{1}{5}$

5. Let the two functions $f(x)$ and $g(x)$ be defined as $f(x) = x^2 - 1$ and $g(x) = \sqrt{x}$ then (fog) (6) is:

(a) 5 **(b)** 7
(c) 35 **(b)** –35

Ans. Option (a) is correct.

The given function are $f(x)=x^2-1$ and $g(x)=\sqrt{x}$

$$(fog)(x)=f(g(x))=f(\sqrt{x})=(\sqrt{x})^2-1=x-1$$

$$(fog)(6)=6-1=5$$

6. If $\begin{pmatrix}2&0&0\\0&3&0\\0&0&4\end{pmatrix}$, then the matrix A^2 is equal to:

(a) $A^2=\begin{pmatrix}4&0&0\\0&9&0\\0&0&16\end{pmatrix}$

(b) $A^2=\begin{pmatrix}0&0&4\\0&9&0\\16&0&0\end{pmatrix}$

(c) $A^2=\begin{pmatrix}4&0&16\\0&0&0\\0&9&0\end{pmatrix}$

(d) $A^2=\begin{pmatrix}0&0&0\\4&9&16\\0&0&0\end{pmatrix}$

Ans. Option (a) is correct.

$$A=\begin{pmatrix}2&0&0\\0&3&0\\0&0&4\end{pmatrix}$$

$$A^2=\begin{pmatrix}2&0&0\\0&3&0\\0&0&4\end{pmatrix}\begin{pmatrix}2&0&0\\0&3&0\\0&0&4\end{pmatrix}$$

$$=\begin{pmatrix}4+0+0&0+0+0&0+0+0\\0+0+0&0+9+0&0+0+0\\0+0+0&0+0+0&0+0+16\end{pmatrix}=\begin{pmatrix}4&0&0\\0&9&0\\0&0&16\end{pmatrix}$$

7. The value of $\lim_{x\to 0}\frac{1-\cos x}{x^2}$ is equal to:

(a) $\frac{1}{2}$ **(b)** 1

(c) –1 **(d)** $\frac{-1}{2}$

Ans. Option (a) is correct.

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}$$

$$=\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{x^2}\quad\left[\text{Using}, 1-\cos x=2\sin^2\frac{x}{2}\right]$$

$$=\lim_{x\to 0}2\left(\frac{\sin\frac{x}{2}}{x}\right)^2$$

$$=\lim_{x\to 0}2\left(\frac{\sin\frac{x}{2}}{\frac{2x}{2}}\right)^2\quad\left[\text{Using}, \lim_{x\to 0}\frac{\sin x}{x}=1\right]$$

$$=2\times\frac{1}{4}=\frac{1}{2}$$

8. The expression for cos ($\tan^{-1} x$) is equal to:

(a) $\frac{1}{\sqrt{1-x^2}}$ **(b)** $\frac{1}{\sqrt{1+x^2}}$

(c) $\frac{\sqrt{1-x^2}}{2}$ **(d)** $\sqrt{1-x^2}$

Ans. Option (b) is correct.

Let $\theta=\tan^{-1}x$

$\Rightarrow$ $x=\tan\theta$

Also, $\cos\theta=\frac{1}{\sec\theta}=\frac{1}{\sqrt{1+\tan^2\theta}}=\frac{1}{\sqrt{1+x^2}}$

$\therefore$ $\cos(\tan^{-1}x)=\frac{1}{\sqrt{1+x^2}}$

9. If $2\begin{pmatrix}a&9\\6&d\end{pmatrix}+3\begin{pmatrix}1&-1\\0&2\end{pmatrix}=3\begin{pmatrix}3&5\\4&6\end{pmatrix}$, then the values of a and d respectively are:

(a) 3, 6 **(b)** 3, 5
(c) 3, 9 **(d)** 3, 7

Ans. Option (a) is correct.

$$2\begin{pmatrix}a&9\\6&d\end{pmatrix}+3\begin{pmatrix}1&-1\\0&2\end{pmatrix}=3\begin{pmatrix}3&5\\4&6\end{pmatrix}$$

$$\begin{pmatrix}2a&18\\12&2d\end{pmatrix}+\begin{pmatrix}3&-3\\0&6\end{pmatrix}=\begin{pmatrix}9&15\\12&18\end{pmatrix}$$

$$\begin{pmatrix}2a+3&15\\12&2d+6\end{pmatrix}=\begin{pmatrix}9&15\\12&18\end{pmatrix}$$

On comparing the corresponding elements, we get

$2a+3=9$ and $2d+6=18$

or $2a=6$ and $2d=12$

or $a=3$ and $d=6$

10. Differentiation of log $(1+x^2)$ with respect to $\tan^{-1}x$ is:

(a) $\frac{1}{1+x^2}$ **(b)** $2x$

(c) $\frac{-1}{1+x^2}$ **(d)** $-x$

Ans. Option (b) is correct.

Let $u=\log(1+x^2)$ and $v=\tan^{-1}x$

$$\frac{du}{dx}=\frac{d}{dx}\left[\log(1+x^2)\right]=\frac{1}{1+x^2}(2x)=\frac{2x}{1+x^2}$$

$$\frac{dv}{dx}=\frac{d}{dx}\left(\tan^{-1}x\right)=\frac{1}{1+x^2}$$

$$\frac{du}{dv}=\frac{du}{dx}\times\frac{dx}{dv}=\frac{2x}{1+x^2}\times\frac{1+x^2}{1}=2x$$

11. The relation R = {(a, a), (b, b), (c, c)} on the set {a,b,c} is:

(a) symmetric only **(b)** reflexive only
(c) transitive only **(d)** an equivalence relation

Ans. Option (d) is correct.

The given relation R = {(a, a), (b, b), (c, c) is an identity relation.

We know that, Identity relation is always an equivalence relation, therefore the given relation is an equivalence relation.

12. If the function $f(x)=\begin{cases}3x-1, & x<2\\ k, & x=2\\ 2x+1, & x>2\end{cases}$ **is continuous at x= 2, then the value of 'k' is:**

(a) $k = 2$ **(b)** $k = 3$
(c) $k = 5$ **(d)** $k = 1$

Ans. Option (c) is correct.

Given the function $f(x)$ is continuous at x =2.

Then, $\lim_{x\to2^-} f(x)=\lim_{x\to2^+} f(x)=f(2)$

Now, $\lim_{x\to2^-}(3x-1)=3(2)-1=5$

and $\lim_{x\to2^+}(2x+1)=2(2)+1=5$

Hence, $k = 5$

13. If $x^2+y^3=42$, then $\frac{dy}{dx}$ is:

(a) $\frac{dy}{dx}=\frac{-3y^2}{2x}$ **(b)** $\frac{dy}{dx}=\frac{3y^2}{2x}$

(c) $\frac{dy}{dx}=\frac{2x}{3y^2}$ **(d)** $\frac{dy}{dx}=\frac{-2x}{3y^2}$

Ans. Option (d) is correct.

The given equation is

$$x^2 + y^3 = 42$$

Differentiating both sides w.r.t. x, we get

$$\frac{d}{dx}\left(x^2\right)+\frac{d}{dx}\left(y^3\right)=\frac{d}{dx}(42)$$

$$\Rightarrow 2x+3y^2\frac{dy}{dx}=0$$

$$\Rightarrow \frac{dy}{dx}=\frac{-2x}{3y^2}$$

14. If the matrix $A=\begin{pmatrix}1 & x & -1\\ -1 & 3 & 2\\ 2 & 1 & 1\end{pmatrix}$ is singular, then the value of 'x' is:

(a) $x=\frac{8}{5}$ **(b)** $x=\frac{-8}{5}$

(c) $x=\frac{5}{8}$ **(d)** $x = 1$

Ans. Option (b) is correct.

When det(A) is zero , then A is a singular matrix.

Therefore, $|A| = 0$

$$\Rightarrow \begin{vmatrix}1 & x & -1\\ -1 & 3 & 2\\ 2 & 1 & 1\end{vmatrix}=0$$

$$\Rightarrow 1\begin{vmatrix}3 & 2\\ 1 & 1\end{vmatrix}-x\begin{vmatrix}-1 & 2\\ 2 & 1\end{vmatrix}+(-1)\begin{vmatrix}-1 & 3\\ 2 & 1\end{vmatrix}=0$$

$$\Rightarrow 1 + 5x + 7 = 0$$

$$\Rightarrow 5x = -8$$

$$\Rightarrow x = -\frac{8}{5}$$

15. The value of $\tan^{-1}1+\cos^{-1}\frac{1}{2}$ is:

(a) $\frac{5\pi}{12}$ **(b)** $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ **(d)** $\frac{7\pi}{12}$

Ans. Option (d) is correct.

$$\tan^{-1}1=\frac{\pi}{4}\text{ and }\cos^{-1}\frac{1}{2}=\frac{\pi}{3}$$

$$\Rightarrow \tan^{-1}1+\cos^{-1}\frac{1}{2}=\frac{\pi}{4}+\frac{\pi}{3}=\frac{7\pi}{12}$$

16. The slope of the tangent to the curve $\sqrt{x}+\sqrt{y}=a$ at $\left(\frac{a^2}{4},\frac{a^2}{4}\right)$ is:

(a) 1 **(b)** –1

(c) $\frac{a}{4}$ **(d)** $\frac{a}{2}$

Ans. Option (b) is correct.

The given equation is, $\sqrt{x}+\sqrt{y}=a$

Differentiating both sides w.r.t. x, we get

$$\frac{d}{dx}\left(\sqrt{x}\right)+\frac{d}{dx}\left(\sqrt{y}\right)=\frac{d}{dx}(a)$$

$$\Rightarrow \frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{y}}\frac{dy}{dx}=0$$

$$\Rightarrow \frac{dy}{dx}=-\sqrt{\frac{y}{x}}$$

At $\left(\frac{a^2}{4},\frac{a^2}{4}\right)$ $\frac{dy}{dx}=-1$

17. If $A=\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}$, then the matrix $A + A^T$ is:

(a) Symmetric matrix
(b) Skew-symmetric matrix

(c) Diagonal matrix
(d) Identity matrix

Ans. Option (a) is correct.

$$A=\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}$$

$\Rightarrow$ $$A^{T}=\begin{pmatrix}1 & 3\\ 2 & 4\end{pmatrix}$$

and $$A+A^{T}=\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}+\begin{pmatrix}1 & 3\\ 2 & 4\end{pmatrix}=\begin{pmatrix}2 & 5\\ 5 & 8\end{pmatrix} \quad ...(i)$$

Now, $$\left(A+A^{T}\right)^{T}=\begin{pmatrix}2 & 5\\ 5 & 8\end{pmatrix} \quad ...(ii)$$

From (i) and (ii),

$$A + A^T = (A+A^T)^T$$

Therefore, $A + A^T$ is a symmetric matrix.

18. If $x = a\cos\theta$, $y = a\sin\theta$, then $\dfrac{dy}{dx}$ at $\theta = \dfrac{\pi}{2}$ will be:

(a) $\dfrac{dy}{dx}=-1$ **(b)** $\dfrac{dy}{dx}=1$

(c) $\dfrac{dy}{dx}=0$ **(d)** $\dfrac{dy}{dx}=2$

Ans. Option (c) is correct.

Given, $x = a\cos\theta$ & $y = a\sin\theta$

$\therefore$ $$\frac{dx}{d\theta}=\frac{d}{d\theta}(a\cos\theta)=-a\sin\theta$$

and $$\frac{dy}{d\theta}=\frac{d}{d\theta}(a\sin\theta)=a\cos\theta$$

Now, $$\frac{dy}{dx}=\frac{dy}{d\theta}\times\frac{d\theta}{dx}=\frac{a\cos\theta}{-a\sin\theta}=-\cot\theta$$

At $\theta=\dfrac{\pi}{2}$, $$\frac{dy}{dx}=\cot\frac{\pi}{2}=0$$

19. If $y = \tan^{-1} x$, then:

(a) $\left(1+x^2\right)\dfrac{d^2y}{dx^2}+2x\dfrac{dy}{dx}=0$

(b) $\sqrt{\left(1-x^2\right)}\dfrac{d^2y}{dx^2}+2x\dfrac{dy}{dx}=0$

(c) $\left(1-x^2\right)\dfrac{d^2y}{dx^2}+2x\dfrac{dy}{dx}=0$

(d) $\sqrt{\left(1+x^2\right)}\dfrac{d^2y}{dx^2}+2x\dfrac{dy}{dx}=0$

Ans. Option (a) is correct.

Given, $y = \tan^{-1}x$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx}=\frac{d}{dx}\left(\tan^{-1}x\right)=\frac{1}{1+x^2}$$

or $$\left(1+x^2\right)\frac{dy}{dx}=1$$

Again differentiating w.r.t. x, we get

$$\left(1+x^2\right)\frac{d^2y}{dx^2}+\frac{dy}{dx}(2x)=1$$

20. If $A=\begin{pmatrix}-1 & 1\\ 2 & 3\end{pmatrix}$, then $A(\text{adj } A)$ is equal to:

(a) $\begin{pmatrix}4 & 0\\ 10 & 24\end{pmatrix}$ **(b)** $\begin{pmatrix}-5 & 0\\ 0 & -5\end{pmatrix}$

(c) $\begin{pmatrix}4 & 5\\ 5 & 4\end{pmatrix}$ **(d)** $\begin{pmatrix}5 & 0\\ 0 & 5\end{pmatrix}$

Ans. Option (b) is correct.

Given, $$A=\begin{pmatrix}-1 & 1\\ 2 & 3\end{pmatrix}$$

Minors of A are:

$M_{11} = 3$, $M_{12} = 2$, $M_{21} = 1$, $M_{22} = -1$

Cofactors of A are:

$C_{11} = 3$, $C_{12} = -2$, $C_{21} = -1$, $C_{22} = -1$

Therefore,

$$Adj(A)=\begin{pmatrix}3 & -2\\ -1 & -1\end{pmatrix}^{T}=\begin{pmatrix}3 & -1\\ -2 & -1\end{pmatrix}$$

Now,

$$A(adj\ A)=\begin{pmatrix}-1 & 1\\ 2 & 3\end{pmatrix}\begin{pmatrix}3 & -1\\ -2 & -1\end{pmatrix}\begin{pmatrix}-3-2 & 1-1\\ 6-6 & -2-3\end{pmatrix}$$

$$=\begin{pmatrix}-5 & 0\\ 0 & -5\end{pmatrix}$$

21. Consider the two functions $f : R \to R$ given by $f(x) = x - 2$ and $g : R \to R$ given by $g(x) = x^2$

(i) The function $f(x)$ is

(a) One to one but not onto
(b) Onto but not one to one
(c) Neither one to one nor onto
(d) Bijective

(ii) The value of $f(1) + g(1)$ is:

(a) $\dfrac{1}{2}$ **(b)** 0

(c) 1 **(d)** $\dfrac{-1}{2}$

(iii) The expression for $(gof)(x)$ is:

(a) $x - 2$ **(b)** $(x - 2)^2$

(c) $x^2 - 2$ **(d)** $x^2 - 3$

(iv) If $(gof)(x) = 0$, then the value of x will be:

(a) $x = \pm 2$ **(b)** $x = 2$

(c) $x = \pm\sqrt{2}$ **(d)** $x = 3$

Ans. (i) Option (d) is correct.

Let $x_1, x_2 \in R$

Now, $f(x_1) = f(x_2)$

$\Rightarrow$ $x_1 - 2 = x_2 - 2$

$\Rightarrow$ $x_1 = x_2$

$\Rightarrow$ $f(x)$ is one-one.

Let $y \in R$

Let $y = f(x_0)$

Then, $x_0 - 2 = y$

$\Rightarrow x_0 = y + 2$

Now, $y \in R$

$\Rightarrow y + 2 \in R$

$\Rightarrow x_0 \in R$

$f(x_0) = x_0 + 2 = y$

Therefore, for each $y \in R$, there exists $x_0 \in R$ such that $f(x_0) = y$

So, $f(x)$ is onto.

Thus, $f(x)$ is one-one and onto or bijective.

(ii) Option (b) is correct.

$f(1) = 1 - 2 = -1$ and $g(1) = (1)^2 = 1$

Now, $f(1) + g(1) = -1 + 1 = 0$

(iii) Option (b) is correct.

$gof(x) = g(f(x)) = g(x-2) = (x-2)^2$

(iv) Option (b) is correct.

$gof(x) = 0$

$\Rightarrow (x-2)^2 = 0$

$\Rightarrow x - 2 = 0$

$\Rightarrow x = 2$

22. A school wants to award its students for their achievement in Sports, Music and Literature with a total cash prize of ₹ 6000.

Three times the prize money for Literature added to the prize money given for Sports is equal to ₹11000.

The prize money given for Sports and Literature together is equal to two times of the prize money given for Music.

If x, y and z represent the prize money given for Sports, Music and Literature respectively, then:

(i) The set of linear equations representing the above information will be:

(a) $x + y + z = 6000$, $x + 3z = 11000$ and $x - 2y + z = 0$

(b) $x + y + z = 6000$, $x + 3z = 11000$ and $x + y + 2z = 0$

(c) $x + y + z = 6000$, $3x + z = 11000$ and $x + y - 2z = 0$

(d) $x + y + z = 6000$, $x + 3z = 11000$ and $2x + 2y - z = 0$

(ii) Consider $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{pmatrix}$

Then the value of $|A|$ is:

(a) 3 **(b)** 6

(c) 4 **(d)** –1

(iii) The adj $(A) = \begin{pmatrix} 6 & p & 3 \\ q & 0 & r \\ -2 & 3 & -1 \end{pmatrix}$, **the values of p, q and r respectively will be:**

(a) 3, 2, –2 **(b)** 3, –2, –2

(c) –3, 2, –2 **(d)** 3, 2, 2

(iv) Using $|A|$ and adj A, calculate the prize money for Sports(x).

(a) $x = 1500$ **(b)** $x = 500$

(c) $x = 1000$ **(d)** $x = 2000$

Ans. (i) Option (a) is correct.

Given,

Cash prize for achievement in sports is ₹x.

Cash prize for achievement in music is ₹y.

Cash prize for achievement in literature is ₹z.

Three times the prize money for literature is added to the money given for sports is equal to ₹11000.

$\Rightarrow x + 3z = 11000$

The prize money given for sports and literature s equal to two times of prize money for music.

$\Rightarrow x + z = 2y$ or $x - 2y + z = 0$

Total cash prize for all the three subjects is ₹6000

$x + y + z = 6000$

(ii) Option (b) is correct

$$|A| = 1\begin{vmatrix} 0 & 3 \\ -2 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 1 & 0 \\ 1 & -2 \end{vmatrix}$$

$= 1[0+6] - 1[1-3] + 1[-2-0] = 6 + 2 - 2$

$= 6$

(iii) Option (c) is correct.

The above system of equations can be written in matrix form as $AX = B$.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix}$$

Here, $|A| = 6$ [From above part (ii)]

Which means A is non-singular. Hence, it is invertible.

Now, cofactors of A are:

$$A_{11} = (-1)^{1+1}\begin{vmatrix} 0 & 3 \\ -2 & 1 \end{vmatrix} = 0 + 6 = 6$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} = -(1-3) = 2$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 1 & 0 \\ 1 & -2 \end{vmatrix} = -2 - 0 = -2$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} 1 & 1 \\ -2 & 1 \end{vmatrix} = -(1+2) = -3$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 1 - 1 = 0$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix}1 & 1\\1 & -2\end{vmatrix} = -(-2-1) = 3$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix}1 & 1\\0 & 3\end{vmatrix} = 3-0 = 3$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix}1 & 1\\1 & 3\end{vmatrix} = -(3-1) = -2$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix}1 & 1\\1 & 0\end{vmatrix} = 0-1 = -1$$

$$\therefore \quad adj(A) = \begin{bmatrix}6 & 2 & -2\\-3 & 0 & 3\\3 & -2 & -1\end{bmatrix}^T$$

Therefore, $$\text{adj}(A) = \begin{bmatrix}6 & -3 & 3\\2 & 0 & -2\\-2 & 3 & -1\end{bmatrix}$$

On comparing, we get

$p = -3, q = 2$ and $r = -2$

(iv) Option (b) is correct.

$$X = A^{-1} B$$

or $$X = \frac{adj(A)}{|A|}.B$$

$$\Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix} = \frac{1}{6}\begin{bmatrix}6 & -3 & 3\\2 & 0 & -2\\-2 & 3 & -1\end{bmatrix}\begin{bmatrix}6000\\11000\\0\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix} = \frac{1}{6}\begin{bmatrix}36000-33000+0\\12000+0-0\\-12000+33000-0\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}\frac{3000}{6}\\\frac{12000}{6}\\\frac{21000}{6}\end{bmatrix} = \begin{bmatrix}500\\2000\\3500\end{bmatrix}$$

Therefore, the prize money for sports (x) is ₹ 500.

23. A person has manufactured a water tank in the shape of a closed right circular cylinder. The volume of the cylinder is $\frac{539}{2}$ cubic units. If the height and radius of the cylinder be h and r, then:

(i) The height h in terms of radius r and the given volume will be:

(a) $h = \frac{539}{\pi r^2}$ **(b)** $h = \frac{539}{2\pi r^2}$

(c) $h = \frac{539}{2\pi r}$ **(d)** $h = \frac{539}{\pi r}$

(ii) Let the total surface area of the closed cylindrical tank be S, given by $S = \frac{539}{r} + 2\pi r^2$

If the total surface area of the tank is minimum, then the value of r will be:

(a) $r = 7$ cm **(b)** $r = 14$ cm

(c) $r = 49$ cm **(d)** $r = \frac{7}{2}$cm

(iii) The height of the tank h is equal to:

(a) $h = 7$ cm **(b)** $h = 14$ cm

(c) $h = 28$ cm **(d)** $h = 2$ cm

(iv) The minimum total surface area of the tank S will be:

(a) 231 sq. cm **(b)** 321 sq. cm

(c) 230 sq. cm **(d)** 221 sq. cm

Ans. (i) Option (b) is correct.

It is mentioned in the question that radius $= r$ units & height $= h$ units

Also, volume of cylinder $= \frac{539}{2}$ cubic units

$$\Rightarrow \quad \pi r^2 h = \frac{539}{2}$$

$$\Rightarrow \quad h = \frac{539}{2\pi r^2}$$

(ii) Option (d) is correct.

Total surface area,

$$S = 2\pi rh + 2\pi r^2$$

$$= 2\pi r\frac{539}{2\pi r^2} + 2\pi r^2 \text{ [From above part (i)]}$$

$$= \frac{539}{r} + 2\pi r^2$$

Now,

$$\frac{ds}{dr} = -\frac{539}{r^2} + 4\pi r \text{ and } \frac{d^2s}{dr^2} = \frac{1078}{r^3} + 4\pi$$

Put $\frac{ds}{dr} = 0 \Rightarrow -\frac{539}{r^2} + 4\pi r = 0$

$$\Rightarrow \quad r^3 = \frac{539\times 7}{4\times 22}$$

$$\Rightarrow \quad r^3 = \frac{7\times 7\times 7}{2\times 2\times 2}$$

$$\Rightarrow \quad r = \frac{7}{2}$$

Here, at $r = \frac{7}{2}, \frac{d^2s}{dr^2} > 0.$ Hence, minimum.

(iii) Option (a) is correct.

As, $$h = \frac{539}{2\pi r^2} = \frac{539}{2\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}} = 7 \text{ cm}$$

(iv) Option (a) is correct.

Surface area is minimum at $r = \frac{7}{2}$ cm

Therefore,

$$S=\frac{539}{r}+2\pi r^2=\frac{539}{\frac{7}{2}}+2\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}$$

$=154 + 77 = 231$ sq. cm

SECTION - B [16 Marks]

(Answer all Questions)

24. If $\vec{a}=2\hat{i}+4\hat{j}-\hat{k}$ and $b=3\hat{i}-2\hat{j}-\lambda\hat{k}$ such that they are perpendicular to each other, then the value of λ will be:

(a) 2 **(b)** –2
(c) 3 **(d)** –3

Ans. Option (a) is correct.

When two vectors $\vec{a}$ and $\vec{b}$ are perpendicular, then $\vec{a}.\vec{b}=0$

Therefore, $(2\vec{i}+4\vec{j}-\vec{k}).(3\vec{i}-2\vec{j}-\gamma\vec{k})=0$

$\Rightarrow$ $6-8+\gamma=0$

$\Rightarrow$ $\gamma=2$

25. The equation of the line passing through the points (0, 1, 2) and (1, 3, 5) is:

(a) $\frac{x-1}{0}=\frac{y-2}{1}=\frac{z-3}{2}$

(b) $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$

(c) $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$

(d) $\frac{x-1}{1}=\frac{y-3}{3}=\frac{z-5}{5}$

Ans. Option (c) is correct.

The equation of a line passing through two points (x_1, y_1, z_1) (x_2, y_2, z_2) is given by,

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

Therefore, The line is passing through (0, 1, 2) & (1, 3, 5) is

$$\frac{x-0}{1-0}=\frac{y-1}{3-1}=\frac{z-2}{5-2}$$

$$\Rightarrow \quad \frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$$

26. The direction cosines of a line parallel to $\frac{x-1}{2}=\frac{y+3}{3}=\frac{z-6}{-6}$ are:

(a) $\left(\frac{-2}{7},\frac{-3}{7},\frac{-6}{7}\right)$ **(b)** $\left(\frac{2}{7},\frac{-3}{7},\frac{6}{7}\right)$

(c) $\left(\frac{2}{7},\frac{3}{7},\frac{6}{7}\right)$ **(d)** $\left(\frac{2}{7},\frac{3}{7},\frac{-6}{7}\right)$

Ans. Option (d) is correct.

Equation of the line given in the question is, $\frac{x-1}{2}=\frac{y+3}{3}=\frac{z+6}{-6}$

Direction ratios of this line is <2, 3, –6>

Direction ratios of the line parallel to the given line are also same, <2, 3, –6>

Let the direction cosines of the second line are <l,m , n>

$$l=\frac{2}{\sqrt{2^2+3^2+(-6)^2}}=\frac{2}{\sqrt{4+9+36}}=\frac{2}{\sqrt{49}}=\frac{2}{7}$$

Similarly, $m=\frac{3}{7}$ and $n=\frac{-6}{7}$

27. The angle between the pairs of lines $\vec{r}=3\hat{i}+2\hat{j}-4\hat{k}+\lambda(\hat{i}+2\hat{j}+2\hat{k})$ and $\vec{r}=5\hat{i}-2\hat{k}+\mu(3\hat{i}+2\hat{j}+3\hat{k})$ is:

(a) $\theta=\sin^{-1}\frac{19}{21}$ **(b)** $\theta=\cos^{-1}\frac{22}{21}$

(c) $\theta=\cos^{-1}\frac{19}{20}$ **(d)** $\theta=\cos^{-1}\frac{19}{21}$

Ans. Option (d) is correct.

The angle between two line $\vec{r}_1=\vec{a}_1+\gamma\vec{b}_1$ and $\vec{r}_2=\vec{a}_2+\mu\vec{b}_2$ is

$$\cos\theta=\frac{\vec{b}_1,\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}$$

Therefore,

$$\cos\theta=\frac{(\vec{i}+2\vec{j}+2\vec{k})(3\vec{i}+2\vec{j}+6\vec{k})}{\sqrt{1^2+2^2+2^2}\sqrt{3^2+2^2+6^2}}$$

$$=\frac{3+4+12}{\sqrt{9}\times\sqrt{49}}=\frac{19}{21}$$

$$\theta=\cos^{-1}\frac{19}{21}$$

28. Consider the two vectors $\vec{a}=3\hat{i}+2\hat{j}+4\hat{k}$ and $\hat{b}=\hat{i}-3\hat{j}+\hat{k}$

(i) The vector perpendicular to both $\vec{a}$ and $\vec{b}$ will be:

(a) $14\hat{i}+\hat{j}-12\hat{k}$ **(b)** $14\hat{i}-\hat{j}+11\hat{k}$

(c) $14\hat{i}+\hat{j}-11\hat{k}$ **(d)** $14\hat{i}+\hat{j}+11\hat{k}$

(ii) The unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ **are:**

(a) $\frac{14\hat{i}+\hat{j}-12\hat{k}}{\sqrt{308}}$ **(b)** $\frac{14\hat{i}-\hat{j}+11\hat{k}}{\sqrt{318}}$

(c) $\frac{14\hat{i}+\hat{j}-11\hat{k}}{\sqrt{318}}$ **(d)** $\frac{14\hat{i}+\hat{j}+11\hat{k}}{\sqrt{318}}$

(iii) The value of $|2\vec{a}+\vec{b}|$ **will be:**

(a) $\sqrt{130}$ **(b)** $\sqrt{131}$

(c) $\sqrt{141}$ **(d)** $\sqrt{140}$

(iv) The area of the parallelogram formed by $\vec{a}$ *and* $\vec{b}$ **as its diagonals will be**

(a) $\frac{1}{2}\sqrt{318}$ **(b)** $2\sqrt{318}$

(c) $\frac{1}{2}\sqrt{308}$ **(d)** $2\sqrt{308}$

Ans. (i) Option (c) is correct.

The vector which is perpendicular to both $\vec{a}$ and $\vec{b}$ is $\vec{a}\times\vec{b}$.

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 2 & 4\\ 1 & -3 & 1\end{vmatrix}=\hat{i}(2+12)-\hat{j}(3-4)+\hat{k}(-9-2)$$

$$\vec{a}\times\vec{b}=14\hat{i}+\hat{j}-11\hat{k}$$

(ii) Option (c) is correct.

Let unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ is $\hat{C}$

$$\hat{C}=\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}=\frac{14\hat{i}+\hat{j}-11\hat{k}}{\sqrt{14^2+1^2+11^2}}=\frac{14\hat{i}+\hat{j}-11\hat{k}}{\sqrt{318}}$$

(iii) Option (b) is correct.

$$|2\vec{a}+\vec{b}|=\left|2\left(3\hat{i}+2\hat{j}+4\hat{k}\right)+\left(\hat{i}-3\hat{j}+\hat{k}\right)\right|$$

$$=\left|6\hat{i}+4\hat{j}+8\hat{k}+\hat{i}-3\hat{j}+\hat{k}\right|$$

$$=\left|7\hat{i}+\hat{j}+9\hat{k}\right|=\sqrt{7^2+1^2+9^2}$$

$$=\sqrt{49+1+81}=\sqrt{131}$$

(iv) Option (a) is correct.

Here,

Area of parallelogram

$$=\frac{1}{2}|\vec{a}\times\vec{b}|=\frac{1}{2}\sqrt{318}\text{ sq. units}$$

[Given $\vec{a}$ and $\vec{b}$ are diagonals]

SECTION - C [16 Marks]

(Answer all Questions)

29. The cost function of a firm is given by $C(x)=1500+25x+\frac{x^2}{10}$**. Then the marginal cost of the firm** $MC(x)$ **will be:**

(a) $1500+\frac{x}{5}$ **(b)** $\frac{-1500}{x^2}+\frac{1}{10}$

(c) $25-\frac{x}{5}$ **(d)** $25+\frac{x}{5}$

Ans. Option (d) is correct.

Given, $C(x) = 1500+25x+\frac{x^2}{10}$

Therefore, $MC(x)=\frac{d}{dx}\left(1500+25x+\frac{x^2}{10}\right)$

$$=25+\frac{2x}{10}=25+\frac{x}{5}$$

30. The revenue of a monopolist is given by $R(x)$ **=** $120x^2+300-x$**. Then, the average revenue function** $AR(x)$ **at** $x = 10$ **will be:**

(a) 1229 **(b)** 1500

(c) 1210 **(d)** 12310

30. Option (a) is correct.

Given, $R(x) = 120x^2 + 300 - x$

Therefore,

$$AR(x)=\frac{R(x)}{x}=\frac{120x^2+300-x}{x}$$

At $x = 10$,

$$AR(10)=\frac{120(10)^2+300-10}{10}$$

$$=\frac{12000+300-10}{10}$$

$$=\frac{12290}{10}=1229$$

31. A company sells its product at the rate of ₹10 per unit. The variable costs are estimated to be 25% of the total revenue received. If the fixed costs for the product are ₹4500, then the cost function will be:

(a) $\frac{15}{2}-4500x$ **(b)** $\frac{15}{x}-4500$

(c) $\frac{5x}{2}+4500$ **(d)** $\frac{25x}{2}-4500$

Ans. Option (c) is correct.

Let x be the number of units sold.

Price of 1 unit ₹10.

Total revenue = ₹$10x$

Cost function, $C(x) = 4500 + 25\%$ of ₹$10x$

$$= 4500 + \frac{25}{100} \times 10x = 4500 + \frac{5}{2}x$$

32. Let the total cost function be $C(x) = 5x + 350$ and the total revenue function be $R(x) = 50x - x^2$ for a company.

Then, the break-even points will be:

(a) –35 and 10 **(b)** 35 and 10

(c) 35 and –10 **(d)** –35 and –10

Ans. Option (b) is correct.

For break-even values, $C(x) = R(x)$

Therefore, $5x + 350 = 50x - x^2$

or $x^2 - 45x + 350 = 0$

Which is a quadratic equation in x, using quadratic formula

$$D = b^2 - 4ac \text{ and } x = \frac{-b \pm \sqrt{D}}{2a}$$

Here, $D = (-45)^2 - 4(1)(350) = 2025 - 1400 = 625$

Therefore,

$$x = \frac{-(-45) \pm \sqrt{625}}{2(1)} = \frac{45+25}{2}, \frac{45-25}{2}$$

or,

$$x = \frac{70}{2}, \frac{20}{2} = 35, 10$$

33. The demand function of a firm producing x units is given by $p = 200 - 5x$

(i) The revenue function at $x = 20$ will be:

(a) 4000 **(b)** 2000

(c) 100 **(d)** –100

(ii) The marginal revenue MR(x) will be:

(a) $200 - 10x^2$ **(b)** $200 - 5x$

(c) $200 - 10x$ **(d)** $-5x^2$

(iii) The value of x, for which revenue increases, will be:

(a) $x < 20$ **(b)** $x > 20$

(c) $x = 20$ **(d)** $x = 200$

(iv) The slope of the marginal revenue will be:

(a) –45 **(b)** 45

(c) 10 **(d)** –10

Ans. (i) Option (b) is correct.

Demand function $p = 200 - 5x$

Revenue function, $R(x) = px = 200x - 5x^2$

At $x = 20$, $R(x) = 200(20) - 5(20)^2$

$= 4000 - 2000$

$= 2000$

(ii) Option (c) is correct.

Here, $R(x) = 200x - 5x^2$

Therefore, $MR(x) = \frac{d}{dx}(200x - 5x^2)$

$= 200 - 10x$

(iii) Option (a) is correct.

$$MR > 0$$

$\Rightarrow$ $200 - 10x > 0$

$\Rightarrow$ $-10x > -200$

$\Rightarrow$ $x < \frac{200}{10}$

$\Rightarrow$ $x < 20$

(iv) Option (d) is correct.

Slope of Marginal revenue,

$$\frac{d}{dx}(MR) = \frac{d}{dx}(200 - 10x) = -10$$

■■