ISC Solved Paper 2022 Semester -2

Class-XII

Mathematics

(Maximum Marks : 40)

(Time allowed : One and a half hours)

*Condidates are allowed an additional **10 minutes** for **only** reading the paper.*
*They must **NOT** start writing during this time.*

*The Question Paper consists of three **sections A, B and C.***
*Candidates are required to attempt all questions from **Section A** and all questions EITHER from **Section B OR Section C***
All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.
The intended marks for questions or parts of questions are given in brackets [ ].
Mathematical tables and graph papers are provided.

SECTION - A **[32 Marks]**

1. Choose the correct option to answer the following questions.

(i) $\int \frac{\sin 2x}{\cos x}\, dx$ **is equal to:** **[1]**

(a) $-2\cos x + c$ (b) $2\cos x + c$

(c) $\frac{-\cos x}{2} + c$ (d) $\frac{\cos x}{2} + c$

Ans. Option (a) is correct.

Explanation: Let

$$\begin{aligned} I &= \int \frac{\sin 2x}{\cos x} dx \\ &= \int \frac{2\sin x . \cos x}{\cos x} dx \\ &= 2\int \sin x\, dx \\ &= 2(-\cos x) + c \\ &= -2\cos x + c \end{aligned}$$

(ii) If A and B are two events such that $P(A) = \frac{4}{5}$ and $P(B/A) = \frac{7}{8}$ .Then $P(A \cap B)$ is equal to: **[1]**

(a) $\frac{7}{40}$ (b) $\frac{21}{40}$

(c) $\frac{32}{35}$ (d) $\frac{7}{10}$

Ans. Option (d) is correct.

Explanation: Given, $P(A) = \frac{4}{5}$, $P(B/A) = \frac{7}{8}$

$\because$ $P(B/A) = \frac{P(B \cap A)}{P(A)}$

$\therefore$
$$\begin{aligned} P(B \cap A) &= P(B/A).P(A) \\ &= \frac{7}{8} \times \frac{4}{5} \\ &= \frac{7}{10} \end{aligned}$$

Therefore, $P(A \cap B) = \frac{7}{10}$

(iii) $\int e^{\sin x}$ **Cos$x$ dx is equal to:** **[1]**

(a) $e^{\cos x} + c$ (b) $e^{\sin x} + c$

(c) $\frac{\sin^2 x}{2} + c$ (d) $e^{\sin^2 x} + c$

Ans. Option (b) is correct.

Explanation: Let, $I = \int e^{\sin x} \cos x\, dx$

Let, $\sin x = t$

$\Rightarrow$ $\cos x\, dx = dt$

$\therefore$
$$\begin{aligned} I &= \int e^t\, dt \\ &= e^t + c \\ &= e^{\sin x} + c \end{aligned}$$

(iv) The order and degree of the differential equation $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 3$ is: **[1]**

(a) order 3 and degree 1

(b) order 1 and degree 3

(c) order 2 and degree 1

(d) order 2 and degree 2

Ans. Option (a) is correct.

Explanation: Order is 3 because highest order derivative is $\frac{d^3y}{dx^3}$

Degree is 1 because power of highest order derivative is one.

(v) A bag contains 9 red, 7 white and 4 black balls. If two balls are drawn at random without replacement, the probability that both balls are red will be: **[1]**

(a) $\frac{11}{95}$ **(b)** $\frac{18}{95}$

(c) $\frac{18}{85}$ **(d)** $\frac{18}{23}$

Ans. Option (b) is correct.

Explanation:

No. of red balls = 9

No. of white balls = 7

No. of black balls = 4

Total no. of balls = 9 + 7 + 4 = 20

Required probability $= \frac{9}{20}\times\frac{8}{19}=\frac{18}{95}$

(vi) $\int a^{3x+2}dx$ **is equal to:** **[1]**

(a) $\left(\frac{a^{3x}}{3\log_e a}\right)+c$ **(b)** $a^2x+\left(\frac{a^{3x}}{3\log_e a}\right)+c$

(c) $a^2\left(\frac{a^{3x}}{3\log_e a}\right)+c$ **(d)** $a^2\left(\frac{a^{3x}}{\log_e a}\right)+c$

Ans. Option (c) is correct.

Explanation:

Let $I = \int a^{3x+2}\,dx$

Let $3x+2 = t$

$\Rightarrow$ $3dx = dt$

$\Rightarrow$ $dx = \frac{dt}{3}$

$\therefore$ $I = \int a^t \frac{dt}{3}$

$= \frac{1}{3}\int a^t dt$

$= \frac{1}{3}\frac{a^t}{\log_e a}+c$

$= \frac{1}{3}\frac{a^{3x+2}}{\log_e^a}+c = a^2\left(\frac{a^{3x}}{3\log_e a}\right)+C$

2. (a) Evaluate: $\int\frac{1}{\sin^2 x\cos^2 x}dx$ **[2]**

OR

(b) Evaluate: $\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 dx$

Ans. (a) Let $I = \int\frac{1}{\sin^2 x\cos^2 x}dx$

$= \int\frac{\sin^2 x+\cos^2 x}{\sin^2 x\cos^2 x}dx$

$= \int\frac{\sin^2 x}{\sin^2 x\cos^2 x}dx + \int\frac{\cos^2 x}{\sin^2 x\cos^2 x}dx$

$= \int\frac{1}{\cos^2 x}dx+\int\frac{1}{\sin^2 x}dx$

$= \int\sec^2 x\,dx + \int\text{cosec}^2 x\,dx$

$= \tan x - \cot x + c$

OR

(b) Let $I = \int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 dx$

$= \int\left\{\left(\sqrt{x}\right)^2+\left(\frac{1}{\sqrt{x}}\right)^2+2\sqrt{x}.\frac{1}{\sqrt{x}}\right\}dx$

$= \int\left(x+\frac{1}{x}+2\right)dx$

$= \int x\,dx+\int\frac{1}{x}dx+2\int dx$

$= \frac{x^2}{2}+\log x+2x+c$

3. (a) Solve: $\frac{dy}{dx}$ **= sin$x$ – x** **[2]**

OR

(b) Solve: $\frac{dy}{dx}$ **+ 2x =** e^{3x}

Ans. (a) Given, $\frac{dy}{dx} = \sin x - x$

$\Rightarrow$ $dy = (\sin x - x)\,dx$

$\Rightarrow$ $dy = \sin x\,dx - x dx$

On integrating both sides, we get

$\int dy = \int\sin x\,dx - \int x dx$

$\Rightarrow$ $y = -\cos x - \frac{x^2}{2} + c$

OR

(b) Given, $\frac{dy}{dx} + 2x = e^{3x}$

$$\Rightarrow \quad \frac{dy}{dx} = e^{3x} - 2x$$

$$\Rightarrow \quad dy = (e^{3x} - 2x)\, dx$$

$$\Rightarrow \quad dy = e^{3x}\, dx - 2x\, dx$$

On integrating both sides, we get

$$\int dy = \int e^{3x}\, dx - 2\int x\, dx$$

$$\Rightarrow \quad y = \frac{e^{3x}}{3} - 2\frac{x^2}{2} + C$$

$$\Rightarrow \quad y = \frac{1}{3}e^{3x} - x^2 + c$$

4. Evaluate: $\int_1^4 |x-2|\,dx$ **[2]**

Ans. Let $I = \int_1^4 |x-2|\,dx$

$$= \int_1^2 |x-2|\,dx + \int_2^4 |x-2|\,dx$$

$$= -\int_1^2 (x-2) + \int_2^4 (x-2)\,dx$$

$$= -\left[\frac{x^2}{2} - 2x\right]_1^2 + \left[\frac{x^2}{2} - 2x\right]_2^4$$

$$= -\left[\left(\frac{4}{2}-4\right)-\left(\frac{1}{2}-2\right)\right] + \left[\left(\frac{16}{2}-8\right)-\left(\frac{4}{2}-4\right)\right]$$

$$= -\left[\left(-2+\frac{3}{2}\right)\right] + [0+2]$$

$$= -\left(-\frac{1}{2}\right) + 2 = \frac{1}{2} + 2 = \frac{5}{2}$$

5. Two horses are considered for race. The probability of selection of first horse is $\frac{1}{5}$ and that of second is $\frac{2}{3}$. Find the probability that: **[4]**

(i) both will be selected.
(ii) only one of them will be selected.
(iii) none of them will be selected
(iv) at least one of them will be selected.

Ans. Let

Event selection of first horse be A.
Event selection of second horse be B.

Given, $P(A) = \frac{1}{5}$ and $P(B) = \frac{2}{3}$

(i) Probability (both will be selected)

$$= P(A)\,.\,P(B)$$

$$= \frac{1}{5} \times \frac{2}{3} = \frac{2}{15}$$

(ii) Probability (only one of them will be selected)

$$= P(A)\,.\,P(\overline{B}) + P(\overline{A})\,.\,P(B)$$

$$= \frac{1}{5} \times \left(1-\frac{2}{3}\right) + \left(1-\frac{1}{5}\right) \times \frac{2}{3}$$

$$= \frac{1}{5} \times \frac{1}{3} + \frac{4}{5} \times \frac{2}{3}$$

$$= \frac{1}{15} + \frac{8}{15}$$

$$= \frac{9}{15} = \frac{3}{5}$$

(iii) Probability (none of them will be selected)= $P(\overline{A})\,.\,P(\overline{B})$

$$= \left(1-\frac{1}{5}\right)\left(1-\frac{2}{3}\right)$$

$$= \frac{4}{5} \times \frac{1}{3}$$

$$= \frac{4}{15}$$

(iv) Probability (at least one of them will be selected)= 1 – Probability (none of them will be selected)

$$= 1 - \frac{4}{15}$$

$$= \frac{11}{15}$$

6. (a) Evaluate: $\int \frac{dx}{x\left[(\log x)^2 - 6\log x + 5\right]}$ **[4]**

OR

(b) Evaluate: $\int x \tan^{-1} x\, dx$

Ans. (a) Let, $I = \int \frac{dx}{x\left[(\log x)^2 - 6\log x + 5\right]}$

Let $\log x = t \Rightarrow \frac{1}{x}dx = dt$

$$\therefore \quad I = \int \frac{dt}{t^2 - 6t + 5}$$

$$= \int \frac{dt}{(t-5)(t-1)}$$

Let, $\frac{1}{(t-5)(t-1)} = \frac{A}{t-5} + \frac{B}{t-1}$

$$\Rightarrow \quad 1 = A(t-1) + B(t-5)$$

$$\Rightarrow \quad 1 = (A+B)t + (-A-5B)$$

On comparing coefficients both sides, we get

$$A + B = 0$$

$$\Rightarrow \quad A = -B \quad \text{....(i)}$$

$$\text{and} \quad -A - 5B = 1 \quad \text{....(ii)}$$

from eqs (i) & (ii), we get

$$-(-B) - 5B = 1$$

$$-4B = 1$$

$$B = -\frac{1}{4}$$

$$\text{and } A = \frac{1}{4}$$

$$\therefore \quad \frac{1}{(t-5)(t-1)} = \frac{1}{4(t-5)} - \frac{1}{4(t-1)}$$

$$= \frac{1}{4}\left[\frac{1}{t-5} - \frac{1}{t-1}\right]$$

$$\text{Thus} \quad I = \frac{1}{4}\int\left[\frac{1}{(t-5)} - \frac{1}{(t-1)}\right]dt$$

$$= \frac{1}{4}\left[\int\frac{1}{(t-5)}dt - \int\frac{1}{(t-1)}dt\right]$$

$$= \frac{1}{4}\left[\log(t-5) - \log(t-1)\right] + C$$

$$= \frac{1}{4}\log\left(\frac{t-5}{t-1}\right) + C$$

$$= \frac{1}{4}\log\left(\frac{\log x - 5}{\log x - 1}\right) + C$$

OR

(b) Let, $I = \int \underset{\text{II}}{x} \ \underset{\text{I}}{\tan^{-1} x}\, dx$

$$= \tan^{-1} x \int x\,dx - \int\left\{\frac{d}{dx}\left(\tan^{-1} x\right)\int x\,dx\right\}dx$$

[Using integration by parts]

$$= \tan^{-1} x \left(\frac{x^2}{2}\right) - \int\left(\frac{1}{1+x^2}\right)\left(\frac{x^2}{2}\right)dx$$

$$= \frac{x^2}{2}\tan^{-1} x - \frac{1}{2}\int\left(\frac{x^2+1}{1+x^2} - \frac{1}{1+x^2}\right)dx$$

$$= \frac{x^2}{2}\tan^{-1} x - \frac{1}{2}\int\left(1 - \frac{1}{1+x^2}\right)dx$$

$$= \frac{x^2}{2}\tan^{-1} x - \frac{1}{2}\left(x - \tan^{-1} x\right) + c$$

$$= \frac{x^2}{2}\tan^{-1} x - \frac{x}{2} + \frac{1}{2}\tan^{-1} x + c$$

7. An insurance company insured 1000 scooter drivers, 2000 car drivers and 4000 truck drivers. The probability of accidents by scooter, car and truck drivers are 0·02, 0.05 and 0.03 respectively. If one of the insured persons meets with an accident, find the probability that he is a truck driver. [6]

Ans. Let S : Scooter driver met with accident

C : Car driver met with accident

T : Truck driver met with accident

A : The driver is insured

By Bayes' theorem

Required Probability= $P(T/A)$

$$= \frac{P(T)\cdot P\left(A/T\right)}{P(T).P\left(A/T\right) + P(S).P\left(A/S\right) + P(C).P\left(A/C\right)} \quad ....(i)$$

Given, $P(S)= 0.02$, $P(C) = 0.05$ and $P(T) = 0.03$

$P(A/S)$= Probability that a scooter driver is insured

$$= \frac{1000}{1000+2000+4000} = \frac{1000}{7000} = \frac{1}{7}$$

$P(A/C)$= Probability that a car driver is insured

$$= \frac{2000}{1000+2000+4000} = \frac{2}{7}$$

$P(A/T)$= Probability that a truck driver is insured

$$= \frac{4000}{1000+2000+4000} = \frac{4}{7}$$

Now, from eq (i)

$$P(T/A) = \frac{0.03\times\frac{4}{7}}{0.03\times\frac{4}{7} + 0.02\times\frac{1}{7} + 0.05\times\frac{2}{7}}$$

$$= \frac{0.12}{0.12+0.02+0.10} = \frac{0.12}{0.24} = \frac{1}{2}$$

8. (a) Write a particular solution of the differential equation, [6]

$$\frac{dy}{dx} = \frac{y^2}{xy - x^2}, \text{ when } x = 1 \text{ and } y = 1$$

OR

(b) Write a particular solution of the differential equation,

$$\left(1+x^2\right)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2} \text{ when } y = 0, x = 0$$

Ans. (a) Given different equation is

$$\frac{dy}{dx} = \frac{y^2}{xy - x^2}$$

The given differential equation is homogeneous in degree 2.

Put $\quad y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\therefore \quad v + x\frac{dv}{dx} = \frac{(vx)^2}{x(vx) - x^2}$$

$$\Rightarrow \quad v + x\frac{dv}{dx} = \frac{v^2x^2}{x^2v - x^2}$$

$$\Rightarrow \quad v + x\frac{dv}{dx} = \frac{v^2}{v-1}$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{v^2}{v-1} - v$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{v^2 - v^2 + v}{v-1}$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{v}{v-1}$$

$$\Rightarrow \quad \left(\frac{v-1}{v}\right)dv = \frac{1}{x}dx$$

on integating both sides, we get

$$\int dv - \int \frac{1}{v}dv = \int \frac{1}{x}dx$$

$$\Rightarrow \quad v - \log v = \log x + \log c$$

$$\Rightarrow \quad v - \log v = \log x + \log c$$

$$\Rightarrow \quad \frac{y}{x} = \log\left(\frac{y}{x}\right) + \log x + \log c$$

$$\Rightarrow \quad \frac{y}{x} = \log (cy)$$

When $x = 1,\ y = 1$

$$\therefore \quad \frac{1}{1} = \log c.\ 1$$

or $\log c = 1$

Therefore particular solution of given differetial equation is

$$\frac{y}{x} = \log y + \log c$$

$$\Rightarrow \quad \frac{y}{x} = \log y + 1$$

$$\Rightarrow \quad \log y = \frac{y}{x} - 1$$

$$\Rightarrow \quad y = e^{\left(y/x - 1\right)}$$

OR

(b) Given differential equation is

$$\left(1+x^2\right)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$$

$$\Rightarrow \quad \frac{dy}{dx} + \left(\frac{2x}{1+x^2}\right)y = \frac{1}{(1+x^2)^2}$$

[On dividing both sides of $(1+ x^2)$]

Which is a linear different equation of the from $\frac{dy}{dx} + Py = Q$

Here, P= $\frac{2x}{1+x^2}$ and Q = $\frac{1}{\left(1+x^2\right)^2}$

$$\therefore \quad I.F. = e^{\int P dx} = e^{\int \frac{2x}{1+x^2}dx} = e^{\log\left(1+x^2\right)} = (1+x^2)$$

Now, Solution is:

$$y\,(I.F.) = \int Q.\ I.F.\ dx + c$$

$$\Rightarrow \quad y\,(1 + x^2) = \int \frac{1}{\left(1+x^2\right)^2}\left(1+x^2\right)dx + c$$

$$\Rightarrow \quad y\,(1 + x^2) = \int \frac{1}{1+x^2}dx + c$$

$$\Rightarrow \quad y\,(1 + x^2) = \tan^{-1}x + c$$

When $x = 0,\ y = 0$

$$\therefore \quad 0 = \tan^{-1}(0) + c$$

$$\Rightarrow \quad c = 0$$

Particular solution is:

$$y\,(1+x^2) = \tan^{-1}x \text{ or } y\ \frac{\tan^{-1}x}{\left(1+x^2\right)}$$

SECTION - B **[8 Marks]**

9. Choose the correct option to answer the following questions. **[2]**

(i) If the intercept form of the equation of the plane $2x - 3y + 4z = 12$ is $x/a + y/b + z/c = 1$ then the values of a, b, c are respectively.

(a) $a = 6, b = -4, c = 3$

(b) $a = -6, b = -4, c = 3$

(c) $a = 6, b = 4, c = 3$

(d) $a = 6, b = 4, c = -3$

Ans. Option (a) is correct.

Explanation: Given equation of plane $2x - 3y + 4z = 12$

Rewriting the equation of plane as:

$$\frac{2x}{12} - \frac{3y}{12} + \frac{4z}{12} = \frac{12}{12}$$

$$\Rightarrow \quad \frac{x}{6} - \frac{y}{4} + \frac{z}{3} = 1$$

Comparing the above equation with $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1,$ we get $a = 6,\ b = -4,\ c = 3$

(ii) The distance of the plane, whose equation is given by $3x - 4y + 12z = 3$, from the origin will be:

(a) $\frac{3}{13}$ units **(b)** $\frac{-2}{13}$ units

(c) -3 units **(d)** $\frac{13}{19}$ units

Ans. Option (a) is correct.

Explanation: Distance from the origin (0, 0, 0) to the plane $3x - 4y + 12z - 3 = 0$ is given by

$$d = \frac{|3(0)-4(0)+12(0)-3|}{\sqrt{(3)^2+(-4)^2+(12)^2}}$$

$$\Rightarrow \quad = \frac{3}{\sqrt{9+16+144}} = \frac{3}{\sqrt{169}} = \frac{3}{13} \text{ units}$$

10. Find the equation of the plane passing through the points (–2, 6, 6), (1. –1, 0) and (1,2, –1) **[2]**

Ans. Equation of the plane passing through (–2, 6, 6), (1,–1, 0) and (1, 2, –1) is given by

$$\begin{vmatrix} x-(-2) & y-6 & z-6 \\ 1-(-2) & -1-6 & 0-6 \\ 1-(-2) & 2-6 & -1-6 \end{vmatrix} = 0$$

$$\Rightarrow \quad \begin{vmatrix} x+2 & y-6 & z-6 \\ 3 & -7 & -6 \\ 3 & -4 & -7 \end{vmatrix} = 0$$

$\Rightarrow$ $(x + 2)(49 - 24) - (y - 6)(-21 + 18) + (z - 6)(-12 + 21) = 0$

$\Rightarrow$ $25(x + 2) + 3(y - 6) + 9(z - 6) = 0$

$\Rightarrow$ $25x + 3y + 9z - 22 = 0$

$\Rightarrow$ $25x + 3y + 9z = 22$

Thus, required equation of plane is $25x + 3y + 9z = 22$.

11. Find the area of the region bounded by the curves $y = x^2 + 2$, $y = x$, $x = 0$ and $x = 3$. **[4]**

Ans. Let $y = x^2 + 2 = f(x)$ and $y = g(x) = x$

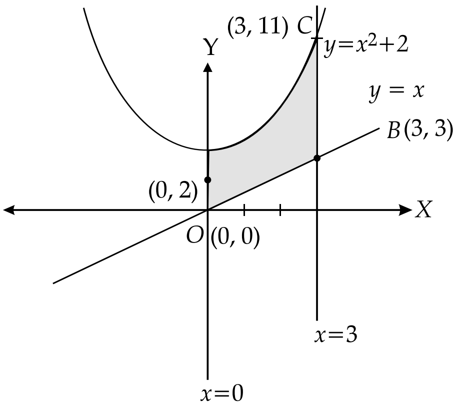
Here, coordinates of points O, A, B and C are:

$A = (0, 2)$

$B = (3, 3)$

$C = (3, 11)$

$O = (0, 0)$



$$\therefore \quad \text{Required Area} = \int_0^3 [f(x) - g(x)]dx$$

$$= \int_0^3 (x^2 + 2 - x)dx$$

$$= \int_0^3 x^2 dx + 2\int_0^3 dx - \int_0^3 x dx$$

$$= \left[\frac{x^3}{3}\right]_0^3 + 2[x]_0^3 - \left[\frac{x^2}{2}\right]_0^3$$

$$= \frac{27}{3} + 6 - \frac{9}{2}$$

$$= 9 + 6 - \frac{9}{2}$$

$$= 15 - \frac{9}{2}$$

$$= \frac{21}{2} \text{ square units}$$

SECTION - C [18 Marks]

12. Choose the correct option to answer the following questions. **[2]**

(i) If the two regression coefficients b_{xy} and b_{yx} are –0.8 and –0.2 respectively, then the correlation coefficient (r) will be,

(a) 0.16 **(b)** – 0.16

(c) 0.4 **(d)** – 0.4

Ans. Option (c) is correct

Explanation: Given, $b_{xy} = -0.8$ and $b_{yx} = -0.2$

$$r = \sqrt{b_{xy}.b_{yx}}$$

$$= \sqrt{(-0.8)(-0.2)}$$

$$= \sqrt{0.16}$$

$[b_{xy} < 0, b_{yx} < 0, \text{ then } r > 0]$

$$= -0.4$$

(ii) The line of regression of y on x is, $4x - 5y + 33 = 0$ and the line of regression of x on y is, $20x - 9y - 107 = 0$, then the value of x when $y = 7$ is,

(a) 8.5 **(b)** – 8.5

(c) 0.5 **(d)** – 0.5

Ans. Option (a) is correct.

Explanation: Given, regression line of x on y is

$$20x - 9y - 107 = 0$$

$$\Rightarrow \quad 20x = 9y + 107$$

$$x = \frac{9}{20}y + \frac{107}{20}$$

when $y = 7$

$$x = \frac{9}{20}(7) + \frac{107}{20}$$

$$= \frac{63}{20} + \frac{107}{20}$$

$$= \frac{170}{20} = \frac{17}{2} = 8.5$$

13. The mean and standard deviation of the two variables x and y are given as $\bar{x} = 6$, $\bar{y} = 8$, $\sigma_x = 4$, $\sigma_y = 12$. The correlation coefficient is given as $r = \frac{2}{3}$.

Find the regression line of x on y. [2]

Ans. Given,

Variable	x	y
Mean	$\bar{x} = 6$	$\bar{y} = 8$
Standard (σ) deviation	$\sigma_x = 4$	$\sigma_y = 12$

Correlation Coefficient, $r = \frac{2}{3}$

Now, $$b_{xy} = r\frac{\sigma_{\hat{x}}}{\sigma_{\hat{y}}} = \frac{2}{3}\cdot\frac{4}{12} = \frac{2}{9}$$

Regression line x on y is given by

$$(x - \bar{x}) = b_{xy}(y - \bar{y})$$

$$\Rightarrow \quad x - 6 = \frac{2}{9}(y - 8)$$

$$\Rightarrow \quad 9x - 54 = 2y - 16$$

$$\Rightarrow x = \frac{2y}{9} + \frac{38}{9}$$

14. A manufacturer has two machines X and Y that may run at the most 360 minutes in a day to produce two types of toys A and B.

To produce each Toy A, machines X and Y need to run at the most 12 minutes and 6 minutes respectively.

To produce each Toy B, machines X and Y need to run at the most 6 minutes and 9 minutes respectively.

By selling the toys A and B, the manufacturer makes the profits of ₹30/- and ₹20/ respectively.

Formulate a Linear Programming Problem and find the number of toys A and B that should be manufactured in a day to get maximum profit. [4]

Ans. Let x and y toys of type A and type B, respectively be manufactured in a day.

According to the question, we construct the following table

Types of Toys	Machine X	Machine Y
A	**12**	**6**
B	6	9

The given problem can be formulated as follows

$$\text{Max } Z = 30x + 20y$$

Subject to constraints:

$$12x + 6y \leq 360$$
$$6x + 9y \leq 360$$
$$x, y \geq 0$$

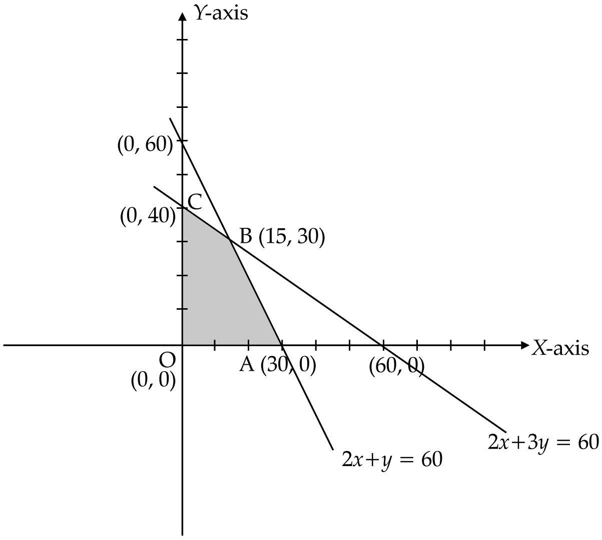
Now, $12x + 6y = 360 \Rightarrow 2x + y = 60$

x	0	30
y	60	0

and $6x + 9y = 360 \Rightarrow 2x + 3y = 120$

x	0	60
y	40	0

Also, intersection point of $2x + y = 60$ and $2x + 3y = 120$ is (15, 30)



The feasible region determined by the constraints is shown in above graph.

The corner points of the feasible region are O (0, 0), A (30, 0), B (15, 30) and C (0, 40).

The value of Z at these corner points are as follows

Corner points	$Z = 30x + 20y$
O (0, 0)	$Z = 0$
A (30, 0)	$Z = 30(30) + 20(0) = 900$
B (15, 30)	$Z = 30(15) + 20(30) = 450 + 600 = 1050$ (Max.)
C (0, 40)	$Z = 30(0) + 20(40) = 800$

The maximum value of Z is 1050 at (15, 30)

Thus, the manufacturer should manufacture 15 toys of type A and 30 toys if type B to maximize profit i.e., ₹1050