C.D.S.
Combined Defence Services

EXAM I 2023

Elementary Mathematics

Time Allowed : 2 Hours **Total Marks : 100**

Instructions

1. *This Test Booklet contains **100** items (questions). Each item comprises four responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose **ONLY ONE** response for each item.*
2. *You have to mark all your responses **ONLY** on the separate Answer Sheet provided. See directions in the Answer Sheet.*
3. ***All** items carry equal marks.*
4. ***Penalty for wrong answers :***
   THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.
   (i). *There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third** (0.33) of the marks assigned to that question will be deducted as penalty.*
   (ii). *If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.*
   (iii). *If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.*

1. What is the largest number which divides both $2^{35} - 1$ and $2^{91} - 1$?
(a) 34 **(b)** 90
(c) 127 **(d)** 129

2. What is the largest power of 10 that divides the product $29 \times 28 \times 27 \times ... 2 \times 1$?
(a) 4 **(b)** 5
(c) 6 **(d)** 7

3. What is the remainder when 65^{99} is divided by 11?
(a) 0 **(b)** 5
(c) 9 **(d)** 10

4. If the roots of the equation $x^2 - bx + c = 5$ differ by 5, then which one of the following is correct ?
(a) $b^2 = 4c + 5$ **(b)** $c^2 = 4b - 5$
(c) $b^2 + c^2 = 5$ **(d)** $b^2 - c^2 = 5$

5. In a party of 150 persons, 75 persons take tea, 60 persons take coffee and 50 persons take milk. 15 of them take both tea and coffee, but no one taking milk takes tea. If each person in the party takes at least one drink, then what is the number of persons taking milk only ?
(a) 50 **(b)** 40
(c) 30 **(d)** 20

6. A, B, C, D and E enter into a business. They invest money in the ratio 2 : 3 : 4 : 5 : 6. However, the time invested by them is in the ratio 6 : 5 : 4 : 3 : 2. If the profit distributed is directly proportional to time and money invested, then who receives the highest amount of profit ?
(a) C
(b) Both B and D
(c) Both C and D
(d) All get equal profit

7. Consider the following numbers :
1. 437
2. 797
3. 1073
How many of the above numbers are prime ?
(a) Only one **(b)** Only two
(c) All three **(d)** None

8. A can do a certain work at twice the speed of B. Further, B can do the same work at 1.5 times the speed of C. All of them together can finish the work in 12 days. In how many days can C alone finish the work ?
(a) 36 days **(b)** 45 days
(c) 60 days **(d)** 66 days

9. The sum of digits of a 2-digit number is 12. When the digits are reversed, the number becomes greater by eighteen. What is the difference between the digits in the number ?
(a) 1 **(b)** 2
(c) 3 **(d)** 4

10. The time taken by a train to cross a man travelling in another train is 10 seconds, when the other train is travelling in the opposite direction. However, it takes 20 seconds, if both the trains are travelling in the same direction. The length of the first train is 200 m and that of the second train is 150 m. What is the speed of the first train ?

(a) 60 km/hr **(b)** 56 km/hr
(c) 54 km/hr **(d)** 52 km/hr

11. If a, b, c, d, e and f satisfy $2a = 3b = 6c = 9d = 12e = 18f$, then what is the value of $(a + b)/(c + d + e + f)$?

(a) 4/7 **(b)** 2
(c) 5/2 **(d)** 9/2

12. If a, b, c are non-zero real numbers such that $a + b + c = 0$, then what are the roots of the quadratic equation $ax^2 + bx + c = 0$?

(a) $2, 1 + \left(\frac{c}{a}\right)$ **(b)** $1, \frac{a}{c}$

(c) $1, \frac{c}{a}$ **(d)** $2, \left(\frac{c}{a}\right) - 1$

13. Twelve percent of bananas bought by a fruit vendor got lost during transportation. On selling the remaining bananas, the vendor's overall profit turned out to be 4%. If the vendor had not lost any bananas and had sold them at the price of the remaining bananas, what would have been his profit percentage ?

(a) $8\frac{1}{9}\%$ **(b)** $9\frac{1}{11}\%$

(c) $18\frac{2}{11}\%$ **(d)** None of the above

14. If the positive square root of $(5 + 3\sqrt{2})(5 - 3\sqrt{2})$ is α, then what is the positive square root of $8 + 2\alpha$?

(a) $2 + \sqrt{3}$ **(b)** $3 - \sqrt{2}$
(c) $\sqrt{7} - 1$ **(d)** $\sqrt{7} + 1$

15. When every even power of every odd integer (greater than 1) is divided by 8, what is the remainder ?

(a) 3
(b) 2
(c) 1
(d) The remainder is not necessarily 1

16. Consider the following statements :

1. If n is a natural number, then the number $\frac{n(n^2+2)}{3}$ is also a natural number.
2. If m is an odd integer, then the number $\frac{m^4+4m^2+11}{16}$ is an integer.

Which of the statements given above is/are correct ?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

17. It is given that 5 does not divide $n - 1$, n and $n + 1$, where n is a positive integer. Which one of the following is correct ?

(a) 5 divides $(n^2 + 1)$
(b) 5 divides $(n^2 - 1)$
(c) 5 divides $(n + n)$
(d) 5 divides $(n^2 - n)$

18. What is the largest 5-digit number, which leaves remainder 7, when divided by 18 as well as by 11 ?

(a) 99981 **(b)** 99988
(c) 99997 **(d)** 99999

19. In a business dealing, A owes B ₹ 20,000 payable after 5 years, whereas B owes A ₹ 12,000 payable after 4 years. They want to settle it now at the rate of 5% simple interest. Who gives how much money in this settlement?

(a) Both are at par
(b) B gives ₹ 6,000 to A
(c) A gives ₹ 6,000 to B
(d) A gives ₹ 4,000 to B

20. Average marks in Mathematics of Section A comprising 30 students is 65 and that of Section B comprising 35 students is 70. What are the average marks (approximately) of both the sections if it was detected later that an entry of 47 marks was wrongly made as 74 ?

(a) 67.28 **(b)** 67.58
(c) 68.11 **(d)** 68.63

21. If α and P are the roots of the equation $x^2 - 7x + 1 = 0$, then what is the value of $\alpha^4 + \beta^4$?

(a) 2207 **(b)** 2247
(c) 2317 **(d)** 2337

22. Consider the following statements in respect of all factors of 360 :

1. The number of factors is 24.
2. The sum of all factors is 1170.

Which of the above statements is/are correct ?

(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

23. Consider a 6-digit number of the form XYXYXY. The number is divisible by :
(a) 3 and 7 only
(b) 7 and 13 only
(c) 3, 13 and 37 only
(d) 3, 7, 13 and 37

24. What is the HCF of $3^{29} - 9$ and $3^{38} - 9$?
(a) $3^1 - 1$ **(b)** $3^{11} - 1$
(c) $3^{11} - 3$ **(d)** $3^{11} - 9$

25. If $x = \sqrt{4\sqrt{4\sqrt{4\sqrt{4\ldots}}}}$, then what is the value of x ?
(a) 2 **(b)** 4
(c) 8 **(d)** 16

26. Let m and n be natural numbers. What is the minimum value of $(m + n)$ such that $33m + 22n$ is divisible by 121 ?
(a) 3 **(b)** 4
(c) 5 **(d)** 10

27. The product of two numbers is 2160 and their HCF is 12. If the sum of the squares of the two numbers is 4896, then what is the mean of the two numbers ?
(a) 24 **(b)** 36
(c) 48 **(d)** 96

28. The age of Q exceeds the age of P by 3 years. The age of R is twice the age of P and the age of Q is twice the age of S. Further, the age difference of R and S is 30 years. What is the sum of the ages of P and Q ?
(a) 35 years **(b)** 38 years
(c) 39 years **(d)** 45 years

29. If a, b and c are the sides of a triangle ABC, then $\sqrt{a} + \sqrt{b} - \sqrt{c}$ is always :
(a) Negative **(b)** Positive
(c) Non-negative **(d)** Non-positive

30. There are four bells which ring at an interval of 15 minutes, 25 minutes, 35 minutes and 45 minutes respectively. If all of them ring at 9 A.M., how many more times will they ring together in the next 72 hours ?
(a) 0 **(b)** 1
(c) 2 **(d)** 3

31. Let a, b, c and d be four positive integers such that $a + b + c + d = 200$. If $S = (-1)^a + (-1)^b + (-1)^c + (-1)^d$, then what is the number of possible values of S ?
(a) One **(b)** Two
(c) Three **(d)** Four

32. The number $97^{30} - 14^{30}$ is divisible by :
(a) 37 but not 83
(b) 83 but not 37
(c) Both 37 and 83
(d) Neither 37 nor 83

33. Consider the following statements :
1. $\log_{10} 50$ is a rational number.
2. $\log_{100} 10$ is an irrational number.
Which of the statements given above is/are correct ?
(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

34. If 17 women and 24 men can do a piece of work in 5 days and 12 women and 23 men can do it in 6 days, then which one of the following is correct ?
(a) Efficiency of 13 women = Efficiency of 18 men
(b) Efficiency of 11 women = Efficiency of 16 men
(c) Efficiency of 13 women = Efficiency of 17 men
(d) Efficiency of 11 women = Efficiency of 15 men

35. Three taps A, B and C together can fill a tank in 6 hours. Tap C alone can fill the tank in 12 hours. To fill the tank, when it is empty, all the three taps are started together. After working t hours, tap C is closed and the tank is filled in 8 more hours. What is t equal to ?
(a) 1 **(b)** 2
(c) 4 **(d)** 6

36. A, B and C can complete a work in x, $1.5x$ and $2x$ days respectively. If they complete the work together, in what ratio should they be paid ?
(a) 2:3:4 **(b)** 6:4:3
(c) 3:2:1 **(d)** 4:3:2

37. Consider the following statements :
1. $n^3 - n$ is divisible by 6.
2. $n^5 - n$ is divisible by 5.
3. $n^5 - 5n^3 + 4n$ is divisible by 120.
Which of the statements given above are correct ?
(a) 1 and 2 only **(b)** 2 and 3 only
(c) 1 and 3 only **(d)** 1, 2 and 3

38. What is the last digit of the sum $S = 9^{27} + 27^9$?
(a) 3 **(b)** 6
(c) 7 **(d)** 9

39. If $x = \frac{\sqrt{3}+1}{\sqrt{3}-1}$ and $y = \frac{\sqrt{3}-1}{\sqrt{3}+1}$, then what is the value of $x^3 - y^3$?

(a) 60 **(b)** $45\sqrt{3}$

(c) $30\sqrt{3}$ **(d)** 90

40. The speed of a boat in still water is 15 km/hr. If it can travel 42 km downstream and 28 km upstream in the same time, then what is the speed of the stream ?

(a) 2.5 km/hr **(b)** 3 km/hr

(c) 4.5 km/hr **(d)** 6 km/hr

41. What is the difference between simple interest and compound interest on ₹ 10,000 for two years at 20% per annum compounded half-yearly ?

(a) ₹ 842 **(b)** ₹ 756

(c) ₹ 641 **(d)** ₹ 542

42. Consider the following statements in respect of the polynomial $a(b-c)(x-b)(x-c) + b(c-a)(x-c)(x-a) + c(a-b)(x-a)(x-b)$:

1. The coefficient of x^2 is 0.

2. The coefficient of x is $(a-b)(b-c)(c-a)$.

Which of the statements given above is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

43. Consider the following statements in respect of the polynomial $1 - x - x^n + x^{n+1}$, where n is a natural number:

1. It is divisible by $1 - 2x + x^2$.

2. It is divisible by $1 - x^n$.

Which of the statements given above is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

44. For what values of m, is $mx^2 + mx + 8x + 9$ a perfect square ?

(a) 1, 4 **(b)** 4, 9

(c) 9, 16 **(d)** 4, 16

45. If $x = a + b + \frac{(a-b)^2}{4a+4b}$ and $y = \frac{a+b}{4} + \frac{ab}{a+b}$ then what is the value of $(x-a)^2 - (y-b)^2$?

(a) a^2 **(b)** b^2

(c) ab **(d)** a^2b^2

46. Consider the following :

1. $\cos^4\theta - \sin^4\theta = \frac{2\tan\theta}{1-\tan^2\theta}, 0 < \theta < \frac{\pi}{2}$

2. $\operatorname{cosec}\theta + \cot\theta = \frac{1}{\operatorname{cosec}\theta - \cot\theta}, 0 < \theta < \frac{\pi}{2}$

3. $\cos^2\theta - \sin^2\theta = \frac{1-\tan^2\theta}{1-\tan^2\theta}, 0 < \theta < \frac{\pi}{2}$

Which of the above equations are identities ?

(a) 1 and 2 only **(b)** 2 and 3 only

(c) 1 and 3 only **(d)** 1, 2 and 3

47. If $\sin\theta = \frac{12}{13}$, then what is the value of $(\tan\theta + \sec\theta)^2 (\operatorname{cosec}\theta - \cot\theta)^{-2}$, $0 < \theta < \frac{\pi}{2}$

(a) $\frac{121}{4}$ **(b)** $\frac{169}{9}$

(c) $\frac{196}{9}$ **(d)** $\frac{225}{4}$

48. If $\tan^8\theta + \cot^8\theta = m$, then what is the value of $\tan\theta + \cot\theta$?

(a) $\sqrt{\sqrt{m+2}+2}$

(b) $\sqrt{\sqrt{\sqrt{m+4}+2}}$

(c) $\sqrt{\sqrt{\sqrt{m+2}+2}+2}$

(d) $\sqrt{\sqrt{\sqrt{m+4}+2}+2}$

49. What is the minimum value of $6 - 4\sin\theta$, $0 \le \theta \le \frac{\pi}{2}$?

(a) 1 **(b)** 2

(c) 4 **(d)** 6

50. What is the value of x that satisfies $4\cos^2 30° + 2x\sin 30° - \cot^2 30° - 6\tan 15°\tan 75° = 0$?

(a) 1 **(b)** 2

(c) 3 (d) 6

51. What is the value of $\frac{\cos^2 32° + \cos^2 58°}{\sec^2 50° - \cot^2 40°} + 4\tan 13°\tan 37°\tan 53°\tan 77°$?

(a) 2 **(b)** 3

(c) 4 **(d)** 5

52. What is the value of $(1 + \cot^2 \theta)(1 + \cos \theta)(1 - \cos \theta) - (1 + \tan^2 \theta)(1 + \sin \theta)(1 - \sin \theta)$?

(a) -1 **(b)** 0
(c) 1 **(d)** 2

53. If $2\cos^2 \theta + \sin \theta - 2 = 0$, $0 < \theta \leq \frac{\pi}{2}$, then what is the value of θ ?

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

54. A person on the top of a vertical tower observes a car moving at a uniform speed coming directly towards it. If it takes 6 minutes for the angle of depression to change from 30° to 45°, and further t minutes to reach the tower, which one of the following is correct ?

(a) $7.7 < t < 8$ **(b)** $8 < t < 8.3$
(c) $8.3 < t < 8.6$ **(d)** $8.6 < t < 8.9$

55. A woman is standing on the deck of a ship, which is h (in metres) above water level. She observes the angle of elevation of the top of a tower as 60° and the angle of depression of the base of the tower as 30°. What is the height of the tower ?

(a) $2h$ **(b)** $3h$
(c) $4h$ **(d)** $5h$

56. Let ABC be a right-angled triangle with sides 5 cm, 12 cm and 13 cm. If p is the length of the perpendicular drawn from vertex A on the hypotenuse BC, then what is the value of $13p$?

(a) 24 **(b)** 48
(c) 60 **(d)** 90

57. OABC is a rhombus whose three vertices lie on a circle with centre at O. If the area of the rhombus is $32\sqrt{3}$ square cm, then what is the radius of the circle ?

(a) 4 cm **(b)** 6 cm
(c) 8 cm **(d)** 16 cm

58. The surface area of a cube is increased by 25%. If p is the percentage increase in its length, then which one of the following is correct ?

(a) $16 < p < 18$ **(b)** $14 < p < 16$
(c) $12 < p < 14$ **(d)** $10 < p < 12$

59. A solid cube is cut into two cuboids of equal volume. What is the ratio of total surface area of the given cube to that of one of the cuboids ?

(a) 2 : 1 **(b)** 3 : 2
(c) 4 : 3 **(d)** 5 : 3

60. The length of a diagonal of a cuboid is 11 cm. The surface area is 240 square cm. What is the sum of its length, breadth and height ?

(a) 16 cm **(b)** 17 cm
(c) 18 cm **(d)** 19 cm

61. What is the area of the circle (approximately) inscribed in a triangle with side lengths 12 cm, 16 cm and 20 cm ?

(a) 48 square cm **(b)** 50 square cm
(c) 52 square cm **(d)** 54 square cm

62. Two times the total surface area of a solid right circular cylinder is three times its curved surface area. If h is the height and r is the radius of the base of the cylinder, then which one of the following is correct ?

(a) $h = r$ **(b)** $h = 2r$
(c) $2h = 3r$ **(d)** $3h = 4r$

63. A floor of a big hall has dimensions 30 m 60 cm and 23 m 40 cm. It is to be paved with square tiles of same size. What is the minimum number of tiles required ?

(a) 30 **(b)** 36
(c) 169 **(d)** 221

64. How long will a man take to walk around the boundary of a square field of area 25 hectares at the rate of 5 km/hr ?

(a) 36 minutes **(b)** 30 minutes
(c) 24 minutes **(d)** 18 minutes

65. Let x be the area of a square inscribed in a circle of radius r and y be the area of an equilateral triangle inscribed in the same circle. Which one of the following is correct ?

(a) $9x^2 = 16y^2$ **(b)** $27x^2 = 64y^2$
(c) $36x^2 = 49y^2$ **(d)** $16x^2 = 21y^2$

66. If the length of a rectangle is increased by $66\frac{2}{3}\%$, then by what percent should the width of the rectangle be decreased in order to maintain the same area ?

(a) 50% **(b)** 45%
(c) 40% **(d)** 35%

67. What is the maximum area that can be covered by three non-intersecting circles drawn inside a rectangle of sides 8 cm and 12 cm ?

(a) 16π square cm **(b)** 18π square cm
(c) 20π square cm **(d)** 24π square cm

68. ABCD is a square field with AB = x. A vertical pole OP of height $2x$ stands at the centre O of the square field. If $\angle APO = \theta$, then what is $\cot \theta$ equal to ?

(a) $\sqrt{2}$ **(b)** 2

(c) $2\sqrt{2}$ **(d)** $3\sqrt{2}$

69. A solid iron ball is melted and 64 smaller solid balls of equal size are made using the entire volume of iron. What is the ratio of the surface area of the larger ball to the sum of the surface areas of all the smaller balls ?

(a) 0.25 **(b)** 0.5

(c) 0.75 **(d)** 1

70. A triangle ABC has been divided into four smaller triangles P, Q, R, S whose perimeters are 16 cm, 12 cm, 4 cm and 12 cm respectively. P, R and S contain the vertices A, B and C respectively. What is the perimeter of the triangle ABC ?

(a) 18 cm **(b)** 20 cm

(c) 22 cm **(d)** 24 cm

*For the next **ten (10)** items that follow :*

Each item contains a Question followed by two Statements. Answer each item using the following instructions :

(a) Choose this option if the Question can be answered by one of the Statements alone but not by the other.

(b) Choose this option if the Question can be answered by either Statement alone.

(c) Choose this option if the Question can be answered by using both the Statements together, but cannot be answered by using either Statement alone.

(d) Choose this option if the Question cannot be answered even by using both Statements together.

71. A number 277XY5 (where X, Y are digits) is divisible by 25.

Question: What is the value of X ?

Statement I: The given number is divisible by 9.

Statement II: X > 5.

72. Question: What are the unique values of a, b and c if 2 is a root of the 2 equation $ax^2 + bx + c = 0$?

Statement I: Ratio of c to a is 1.

Statement II: Ratio of b to a is (– 5/2).

73. Question: Is $m > n$, where m, n are non-zero numbers ?

Statement I: $\frac{m}{n} > 1$.

Statement II: $m > 2n$.

74. Question: Can a circle be drawn through the points A, B and C ?

Statement I: AB = 5 cm, BC = 5 cm, CA = 6 cm.

Statement II: AB = 3 cm, BC = 4 cm, CA = 7 cm.

75. x and y are consecutive odd integers.

Question: Can the value of $(x + y)$ be determined uniquely ?

Statement I: $(x + y)^4 = 256$.

Statement II: $(x + y)^3 < 16$.

76. Question: Is $p^2 + q^2 + q$ odd, where p, q are positive integers ?

Statement I: $2p + q$ is odd.

Statement II: $q - 2p$ is odd.

77. Question: What is the area of the circle C ?

Statement I: An arc of length 7 cm subtends an angle 30° at the centre of C.

Statement II: A chord of length 10 cm subtends an angle 90° at the centre of C.

78. Question: Is triangle A right angled ?

Statement I: The length of the line segment joining the mid-points of two sides of A is half of the third side of A.

Statement II: The angles of A are in the ratio 1 : 2 : 3.

79. The lengths of two longer sides of the triangle are 25 cm and 24 cm.

Question: What is the length of the shortest side ?

Statement I: The angles of A are in the ratio 1 : 2 : 3.

Statement II: The length of the perpendicular drawn on the longest side of Δ from its opposite vertex is 6.72 cm.

80. A chord PQ of the circle C divides it into two segments such that 3 times the area of the major segment is 4 times the area of the minor segment.

Question: What is the radius of C ?

Statement I: Area of the minor segment is 66 square cm.

Statement II: Area of the major segment is 88 square cm.

*Consider the following data for the next **two (02)** items that follow :*

Class	0 – 30	30 – 60	60 – 90	90 – 120
Frequency	4	5	7	4

81. What is the mode of the distribution ?
(a) 60 **(b)** 72
(c) 75 **(d)** 80

82. If the median (P) and mode (Q) satisfy the relation 7(Q – P) = 9R, then what is the value of R?
(a) 6 **(b)** 5
(c) 3 **(d)** 1

*Consider the following data for the next **two (02)** items that follow :*

Class	40 – 50	50 – 60	60 – 70	70 – 80
Frequency	4	3	1	2

83. What is the mean of the distribution ?
(a) 51 **(b)** 52
(c) 54 **(d)** 56

84. If M is the median, then what is the value of 3M?
(a) $53\frac{1}{3}$ **(b)** 60
(c) 160 **(d)** 180

85. The plinth of a house has an area of 200 square metres. It is rectangular in shape and its length and breadth are in the ratio 2 : 1. The owner of the house extends the terrace by 1 m on each side. What is the percentage of area that has increased in the terrace relative to the plinth ?
(a) 40% **(b)** 32%
(c) 20% **(d)** 15.5%

86. A square sheet of side length 44 cm is rolled along one of its sides to form a cylinder by making opposite edges just to touch each other. What is the volume of the cylinder ?
(Take $\pi = \frac{22}{3}$)
(a) 6776 cubic cm **(b)** 6248 cubic cm
(c) 5896 cubic cm **(d)** 5680 cubic cm

87. The volume of a cuboid is 3600 cubic cm. The areas of two adjacent faces are 225 square cm and 144 square cm. What is the area of the other adjacent face ?
(a) 400 square cm **(b)** 360 square cm
(c) 320 square cm **(d)** 300 square cm

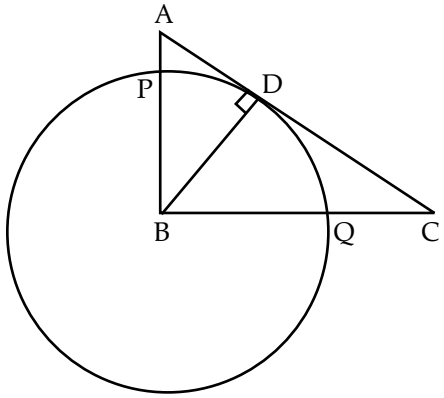
88. The perimeter and the area of a right-angled triangle are 36 cm and 54 square cm respectively. What is the length of the hypotenuse ?
(a) 12 cm **(b)** 14 cm
(c) 15 cm **(d)** 16 cm

89. Let
X={x | $x = 2 + 4k$, where $k = 0, 1, 2, 3, \ldots 24$}. Let S be a subset of X such that the sum of no two elements of S is 100. What is the maximum possible number of elements in S ?
(a) 10 **(b)** 11
(c) 12 **(d)** 13

90. The perimeter of a sector of a circle of radius 5.2 cm is 16.4 cm. What is the area of the sector?
(a) 15.6 square cm **(b)** 15 square cm
(c) 14.4 square cm **(d)** 14.1 square cm

*Consider the following for the next **three (03)** items that follow :*

In the triangle ABC, AB = 6 cm, BC = 8 cm and AC = 10 cm. The perpendicular dropped from B meets the side AC at D. A circle of radius BD (with centre B) cuts AB and BC at P and Q respectively as shown in the figure.



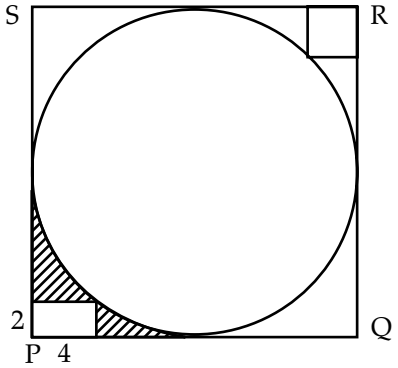
91. What is the length of QC ?
(a) 4.4 cm **(b)** 4.2 cm
(c) 3.6 cm **(d)** 3.2 cm

92. If $\angle ABD = \theta$, then what is $\sin\theta$ equal to ?
(a) 0.4 **(b)** 0.5
(c) 0.6 **(d)** 0.8

93. What is the radius of the circle ?
(a) 5 cm **(b)** 4.8 cm
(c) 4.4 cm **(d)** 4 cm

*Consider the following for the next **three (03)** items that follow :*

In the figure given below, a circle is inscribed in a square PQRS. A rectangle at the comer P that measures 4 cm × 2 cm and a square at the comer R.



94. What is the area of the circle ?

(a) 100π square cm
(b) 96π square cm
(c) 50π square cm
(d) 48π square cm

95. What is the area of the smaller square ?

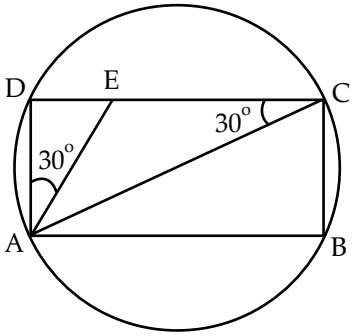
(a) $50\,(3-\sqrt{2}\,)$ square cm
(b) $25\,(3-2\sqrt{2}\,)$ square cm
(c) $25\,(3+2\sqrt{2}\,)$ square cm
(d) $50\,(3-2\sqrt{2}\,)$ square cm

96. What is the area of the shaded region ?

(a) (96 – 25π) square cm
(b) (92 – 25π) square cm
(c) (96 – 16π) square cm
(d) (92 – 16π) square cm

*Consider the following for the next **two (02)** items that follow :*

In the following figure, a rectangle ABCD is inscribed in a circle of radius r. Given ∠ DAE = 30° and ∠ ACD = 30°.



97. What is the ratio of the area of the circle to the area of the rectangle ?

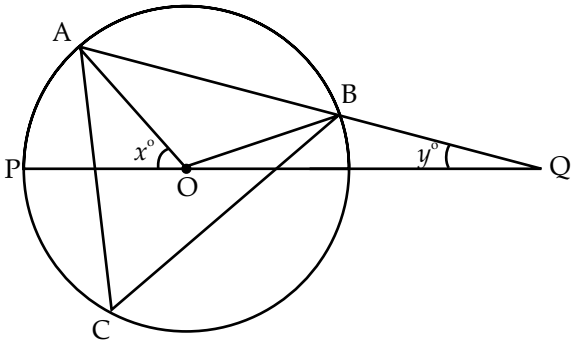
(a) $\frac{\pi}{\sqrt{2}}$ **(b)** $\frac{\pi}{\sqrt{3}}$
(c) $\frac{2\pi}{\sqrt{3}}$ **(d)** $\frac{3\pi}{\sqrt{2}}$

98. What is the area of Δ AEC ?

(a) $\frac{r^2}{\sqrt{3}}$ **(b)** $\frac{r^2}{2\sqrt{3}}$
(c) $\frac{r^2}{3\sqrt{3}}$ **(d)** $\frac{2r^2}{\sqrt{3}}$

*Consider the following for the next **two (02)** items that follow :*

In the following figure, a triangle ABC is inscribed in a circle with centre at O. Let ∠POA = $x°$ and ∠ OQB = $y°$. Further, OB = BQ



99. What is the relation between x and y ?

(a) $x = y$ **(b)** $2x = 3y$
(c) $x = 3y$ **(d)** $3x = 4y$

100. If $y = 15$, then what is ∠ACB equal to ?

(a) 30° **(b)** 40°
(c) 45° **(d)** 60°

Answer Key

Q.No.	Answer Key	Topic's Name	Chapter's Name
1	**c**	Divisibility	Number System
2	**c**	Divisibility	Number System
3	**d**	Remainder	Number System
4	**a**	Quadratic Equation	Equations
5	**c**	Venn Diagram	Set Theory
6	**a**	Ratio	Ratio and Proportion
7	**a**	Prime and Composite numbers	Number System
8	**d**	Time & Work	Time & Work
9	**b**	Numbers	Number System
10	**c**	Time & Distance	Time & Distance
11	**b**	Ratio	Ratio and Proportion
12	**c**	Quadratic Equation	Equations
13	**c**	Profit percentage	Profit loss
14	**d**	Numbers	Number System
15	**c**	Remainder	Number System
16	**c**	Numbers	Number System
17	**a**	Divisibility	Number System
18	**c**	Divisibility	Number System
19	**c**	SI & CI	SI & CI
20	**a**	Average	Statistics
21	**a**	Quadratic Equation	Equations
22	**c**	Factors	Number System
23	**d**	Divisibility	Number System
24	**d**	Divisibility	Number System
25	**b**	Numbers	Number System
26	**b**	Divisibility	Number system
27	**c**	HCF	HCF and LCM
28	**d**	Ages	Ratio-Proportion
29	**b**	Triangles	Mensuration
30	**c**	LCM	HCF and LCM
31	**c**	Integers	Number system
32	**c**	Divisibility Rule	Number system
33	**a**	Logarithm	Logarithm
34	**a**	Concept of Efficiency	Time and work
35	**b**	Pipe and Cistern	Time and work
36	**b**	Time and Work	Time and work
37	**d**	Divisibility	Number system
38	**b**	Surds and Indices	Number system
39	**c**	Algebraic Identities	Algebra
40	**b**	Boats and Stream	Speed, time and distance

41	**c**	CI And SI	Compound interest
42	**a**	Polynomial	Algebra
43	**c**	Polynomial	Algebra
44	**d**	Quadratic Equations	Algebra
45	**b**	Algebraic Identities	Algebra
46	**b**	Trigonometric Functions	Trigonometry
47	**d**	Trigonometric Ratios	Trigonometry
48	**c**	Trigonometric Functions	Trigonometry
49	**b**	Trigonometric Functions	Trigonometry
50	**d**	Trigonometric Functions	Trigonometry
51	**d**	Complementary angle	Trigonometry
52	**b**	Trigonometric Identity	Trigonometry
53	**a**	Trigonometric Identity	Trigonometry
54	**b**	Height and Distance	Trigonometry
55	**c**	Height and Distance	Trigonometry
56	**c**	Similarity	Trigonometry
57	**c**	Area of circle	Mensuration
58	**d**	Area of cube	Mensuration
59	**b**	Area of cube and cuboid	Mensuration
60	**d**	Area of cuboid	Mensuration
61	**b**	Area of triangle and circle	Mensuration
62	**b**	Cylinder	Mensuration
63	**d**	Area of floor	Mensuration
64	**c**	Area of square	Mensuration
65	**b**	Circle	Mensuration
66	**c**	Area of rectangle	Mensuration
67	**d**	Circle	Mensuration
68	**c**	Height and Distance	Mensuration
69	**a**	Sphere	Mensuration
70	**b**	Perimeter of triangle	Mensuration
71	**b**	Divisibility	Algebra
72	**b**	Quadratic equation	Algebra
73	**b**	Inequality	Algebra
74	**b**	Circumcircle	Mensuration
75	**c**	Integer	Algebra
76	**d**	Integer	Algebra
77	**b**	Area related to circle	Mensuration
78	**a**	Triangle	Geometry
79	**b**	Sine Rule	Properties of triangle
80	**b**	Area related to circle	Mensuration
81	**b**	Mode	Statistics
82	**a**	Median	Statistics

83	**d**	Mean	Statistics
84	**c**	Median	Statistics
85	**b**	Rectangle	Mensuration
86	**a**	Cylinder	Mensuration
87	**a**	Cuboid	Mensuration
88	**c**	Area of triangle	Mensuration
89	**d**	Sets	Algebra
90	**a**	Area related to circle	Mensuration
91	**d**	Similar Triangle	Geometry
92	**c**	Similar Triangle	Geometry
93	**b**	Area related to circle	Mensuration
94	**a**	Area related to circle	Mensuration
95	**d**	Area related to circle	Mensuration
96	**b**	Area related to circle	Mensuration
97	**b**	Area related to circle	Mensuration
98	**a**	Area related to circle	Mensuration
99	**c**	Circle	Geometry
100	**d**	Circle	Geometry

C.D.S.
Combined Defence Services

EXAM
I
2023

Mathematics

ANSWERS WITH EXPLANATION

1. Option (c) is correct.

$2^{35} - 1$
$= (2^7)^5 - 1$
$= (128)^5 - 1$
$= (128-1)(128^4 + 128^3 + 128^2 + 128 + 1)$
$= (127)$ (Integer)
$2^{91} - 1$
$= (2^7)^{13} - 1$
$= (128)^{13} - 1$
$= (128 - 1)(128^{12} + 128^{11} + \ldots + 128 + 1)$
$= (127)$ (Integer)
So, both numbers are divisible by 127.

2. Option (c) is correct.

Let $P = 1 \times 2 \times 3 \times \ldots \times 28 \times 29$
$\Rightarrow P = 29!$
Also, $10 = 2 \times 5$
Exponent of 2 in P

$$= \left[\frac{29}{2}\right] + \left[\frac{29}{2^2}\right] + \left[\frac{29}{2^3}\right] + \left[\frac{29}{2^4}\right] + \left[\frac{29}{2^5}\right]$$

where [.] = GIF
$= 14 + 7 + 3 + 1 + 0 = 25$
Exponent of 5 in P

$$= \left[\frac{29}{5}\right] + \left[\frac{29}{5^2}\right] + \left[\frac{29}{5^3}\right]$$

$= 5 + 1 + 0 = 6$
So, largest power of 10 that divides P is 6.

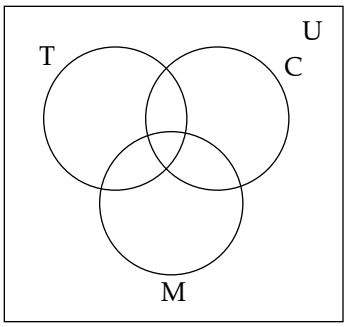
3. Option (d) is correct.

Let $P = 65^{99}$
$\Rightarrow P = (66 - 1)^{99}$
$\Rightarrow P = {}^{99}C_0 (66)^{99} - {}^{99}C_1 (66)^{98} + {}^{99}C_2 (66)^{97} - {}^{99}C_3 (66)^{96} + \ldots - {}^{99}C_{99}$
$\Rightarrow P =$ (Multiple of 11) $- 1$
$\Rightarrow P =$ (Multiple of 11) $+ 11 - 1$
$\Rightarrow P =$ (Multiple of 11) $+ 10$
So, remainder will be 10, when 65^{99} is divided by 11.

4. Option (a) is correct.

Let the roots of equation $x^2 - bx + c = 5$ be α & β.
Given, $|\alpha - \beta| = 5$
$\Rightarrow (\alpha - \beta)^2 = 25$
$\Rightarrow (\alpha^2 + \beta^2) - 2\alpha\beta = 25$
$\Rightarrow (\alpha^2 + \beta^2 + 2\alpha\beta) - 4\alpha\beta = 25$
$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = 25$ ($\because$ Sum of roots $= b$ and Product of roots $= c - 5$)
$\Rightarrow (b)^2 - 4(c - 5) = 25$
$\Rightarrow b^2 - 4c + 20 = 25$
$\Rightarrow b^2 = 4c + 5$

5. Option (c) is correct.



T – Tea
C – Coffee
M – Milk
$n(U) = 150$
$n(T) = 75$
$n(C) = 60$
$n(M) = 50$
$n(T \cap C) = 15$, $n(T \cap M) = 0$, $n(T \cap M \cap C) = 0$
Also, $n(U) = n(T) + n(M) + n(C) - n(M \cap T) - n(C \cap T) - n(C \cap M) + n(T \cap M \cap C)$
$\Rightarrow 150 = 75 + 60 + 50 - 0 - 15 - n(C \cap M) + 0$
$\Rightarrow n(C \cap M) = 20$
So, people having milk only
$= n(M) - [n(M \cap T) + n(C \cap M) - n(C \cap M \cap T)$
$= 50 - [0 + 20 - 0] = 30$

6. Option (a) is correct.

Let A, B, C, D, E are investing ₹ 200, 300, 400, 500 & 600 respectively for months as 6, 5, 4, 3 & 2 respectively.

A's Capital	1200
B's Capital	1500
C's Capital	1600
D's Capital	1500
E's Capital	1200

$\Rightarrow$ Profit of C will be highest because C's Capital is highest.

7. Option (a) is correct.

Let A = 437

$\Rightarrow$ A = 19 × 23

So, A is not prime:

B = 797

$\Rightarrow$ B = 797 × 1

C = 1073

$\Rightarrow$ C = 29 × 37

So, C is not prime.

8. Option (d) is correct.

Let speed of C be x.

So, speed of B will be $3x/2$.

And, speed of A will be $2(3x/2) = 3x$

So, work with effeciency of (A+B+C) i.e $11x/2$ is done in 12 days.

Now, C who is having efficiency /speed as "x" will complete the work in $= 12 \times \frac{11}{2} = 66$ days

9. Option (b) is correct.

Let the number be xy.

Sum of digits = $x + y = 12$ (1)

Reversed digits, yx

Now, $(10y + x) - (10x + y) = 18$

$\Rightarrow 9(y - x) = 18$

$\Rightarrow y - x = 2$

$\Rightarrow y = x + 2$ (2)

From eq. (1) ,

$x + x + 2 = 12$

$\Rightarrow x = 5$

So, $y = 7$

Difference of digits = 7 – 5 = 2

10. Option (c) is correrct.

Let speed of trains be v_1 and v_2 respectively.

When trains are travelling in opposite direction,

$$10 = \frac{200}{v_1 + v_2}$$

$\Rightarrow v_1 + v_2 = 20$ (1)

When trains are travelling in same direction,

$$20 = \frac{200}{v_1 - v_2}$$

$\Rightarrow v_1 - v_2 = 10$ (2)

By eq. (1) + (2),

$2v_1 = 30$

$v_1 = 15$ m/sec

$v_1 = (15 \times 18/5)$ km/hr = 54 km/hr

11. Option (b) is correct.

Given, $2a = 3b = 6c = 9d = 12e = 18f = k$(let)

Now, $\frac{(a+b)}{(c+d+e+f)}$

$$= \frac{\frac{k}{2}+\frac{k}{3}}{\frac{k}{6}+\frac{k}{9}+\frac{k}{12}+\frac{k}{18}} = \frac{\left(\frac{3+2}{6}\right)}{\left(\frac{6+4+3+2}{36}\right)}$$

$$= \left(\frac{5}{6}\right)\left(\frac{36}{15}\right) = 2$$

12. Option (c) is correct.

Given equation : $ax^2 + bx + c = 0$

Put $x = 1$, $a + b + c = 0$

So, $x = 1$ is a root of $ax^2 + bx + c = 0$

Product of roots $= \frac{c}{a}$

$\Rightarrow$ (1) (α) $= \frac{c}{a}$, where α be the other root

$\Rightarrow \alpha = \frac{c}{a}$

So, roots are 1 & $\frac{c}{a}$.

13. Option (c) is correct.

Solution:

Let total number of banana = 100

and cost price of one banana = ₹ 1.

On selling 88% of banana, vendor's overall profit turned out to be 4%.

$\therefore$ 88 × (SP of one banana) = 104

$\Rightarrow$ SP of one banana $= \frac{104}{88}$

If, vendor had not just any banana, then selling price of all banana $= \frac{104}{88} \times 100$

So, profit percentage $= \frac{SP - CP}{CP} \times 100\%$

$$= \frac{\frac{104}{88} \times 100 - 100}{100} \times 100\%$$

$$= \left(\frac{104}{88} - 1\right) \times 100\%$$

$$= \frac{200}{11}\% = 18\frac{2}{11}\%$$

14. Option (d) is correct.

Let $p = \left(5 + 3\sqrt{2}\right)\left(5 - 3\sqrt{2}\right)$

$\Rightarrow p = (5)^2 - (3\sqrt{2})^2$

$\Rightarrow p = 25 - 18$

$\Rightarrow p = 7$

So, $\alpha = \sqrt{7}$

Now, $\sqrt{8 + 2\alpha} = \sqrt{8 + 2\sqrt{7}}$

$$= \sqrt{\left(\sqrt{7}\right)^2 + (1)^2 + 2(1)\left(\sqrt{7}\right)}$$

$= \sqrt{7} + 1$

15. Option (c) is correct.

Here, we are dividing by 8, so whenever an odd number is divided, the remainder is 1, 3, 5, 7.

i. $(8k + 1)^{2n} \rightarrow$ Remainder = 1

ii. $(8k + 3)^{2n}$ = (Multiple of 8) + 3^{2n}

$3^{2n} = 9^n = (8+1)^n$ = (Multiple of 8) + 1

So, remainder = 1

iii. $(8k + 5)^{2n}$ = (Multiple of 8) + 5^{2n}

$5^{2n} = (25)^n = (24 + 1)^n$ = (Multiple of 8) + 1

So, remainder = 1

iv. $(8k + 7)^{2n} = (8k - 1)^{2n}$ = (Multiple of 8) + 1

So, remainder = 1

16. Option (c) is correct.

Statement 1: If n is natural number.

Then, let n = 1, 2, 3, ... etc.

Case1: When $n = 1$, then $\frac{n(n^2+2)}{3} = \frac{1(1^2+2)}{3} = 1$

Case 2: When n = 2, $\frac{2(2^2+2)}{3} = \frac{2\times 6}{3} = 4$

Case 3: When n = 3, $\frac{3(3^2+2)}{3} = \frac{3\times 11}{3} = 11$

Means in each case, the result is a natural number. So, statement (1) is true.

Statement 2: If m is odd integer. Let m = 1

Then $\frac{m^4 + 4m^2 + 11}{16} = \frac{1^2 + 4\times 1^2 + 11}{16} = 1$

For m = 3,

$$\frac{m^4 + 4m^2 + 11}{16} = \frac{3^4 + 4\times 3^2 + 11}{16}$$

$$= \frac{81 + 36 + 11}{16} = 8$$

So, statement (2) is also true.

Hence, option (c) is correct.

17. Option (a) is correct.

(i) Let n = 2

$n + 1 = 3$, $n - 1 = 1$

$n^2 + 1 = 5$, $n^2 - 1 = 3$, $n^2 + n = 6$, $n^2 - n = 2$

(ii) Let n = 3

$n + 1 = 4$, $n - 1 = 2$

$n^2 + 1 = 10$, $n^2 - 1 = 8$,

$n^2 + n = 12$, $n^2 - n = 6$

So, we can say that $(n^2 + 1)$ is divisible by 5.

18. Option (c) is correct.

99981 = 5554(18) + 9

99988 = 5554(18) + 16

99997 = 5555(18) + 7

= 9090(11) + 7

So, required number = 99997

19. Option (c) is correct.

Present worth of A

$$= \frac{100\times \text{Amount}}{100 + (\text{Rate}\times\text{Time})} = \frac{100\times 20000}{100 + (5\times 5)}$$

= ₹ 16,000

Present worth of B

$$= \frac{100\times \text{Amount}}{100 + (\text{Rate}\times\text{Time})} = \frac{100\times 12000}{100 + (5\times 4)}$$

= ₹ 10,000

So, A gives ₹ 6,000 to B.

20. Option (a) is correct.

Average marks of section A students = 65

Total marks of section A students

= 30 × 65 = 1950

Average marks of section B students = 70

Total marks of section B students

= 70 × 35 = 2450

Total students = 30 + 35 = 65

Total marks = 1950 + 2450 = 4400

Corrected marks = 4400 + 47 – 74 = 4373

Average marks = 4373/65 = 67.28

21. Option (a) is correct.

The roots $x^2 - 7x + 1 = 0$, are α & β.

Then, $\alpha + \beta = 7$,

$\alpha\beta = 1$

Now, $(\alpha + \beta)^2 = (7)^2$

$\Rightarrow \alpha^2 + \beta^2 + 2\alpha\beta = 49$

$\Rightarrow \alpha^2 + \beta^2 = 49 + 2(-1)$

$\Rightarrow \alpha^2 + \beta^2 = 47$

Again, $(\alpha^2 + \beta^2)^2 = (47)^2$

$\Rightarrow \alpha^4 + \beta^4 + 2\alpha^2\beta^2 = 2209$

$\Rightarrow \alpha^4 + \beta^4 = 2209 - 2(1)^2$

$\Rightarrow \alpha^4 + \beta^4 = 2207$

22. Option (c) is correct.

$360 = 36 \times 10$

$= 2^2 \times 3^2 \times 2^1 \times 5^1$

$= 2^3 \times 3^2 \times 5^1$

Number of factors $= (3 + 1)(2 + 1)(1 + 1) = 24$

Sum of all factors

$= (2^0 + 2^1 + 2^2 + 2^3)(3^0 + 3^1 + 3^2)(5^0 + 5^1)$

$= (1 + 2 + 4 + 8)(1 + 3 + 9)(1 + 5)$

$= 15 \times 13 \times 6 = 1170$

23. Option (d) is correct.

Let P = X

$\Rightarrow$ P = X$(10^5 + 10^3 + 10^1) + y(10^4 + 10^2 + 10^0)$

$\Rightarrow$ P = 101010X + 10101Y

$\Rightarrow$ P = 10101(10X + Y)

$\Rightarrow$ P = (10101)(10X + Y)

10101 = (37)(273)

= (13)(777)

= (7)(1443)

= (3)(3367)

So, the given number is divisible by all numbers 3, 7, 13 and 37.

24. Option (d) is correct.

Solution:

$3^{29} - 9 = 3^2 [3^{27} - 1]$

$= 3^2 [(3^9)^3 - 1^3]$

As we know, $a^n - b^n$ is divisible by $a - b$.

So, $3^{29} - 9$ is divisible by $3^2 [3^9 - 1]$

Now, $3^{38} - 9 = 3^2 [3^{36} - 1]$

$= 3^2 [(3^9)^4 - 1^4]$

So, $3^{38} - 9$ is divisible by $3^2[3^9 - 1]$

$\therefore$ HCF of $3^{29} - 9$ and $3^{38} - 9$ is $3^2[3^9 - 1]$

$= 3^{11} - 9$

25. Option (b) is correct.

$x = \sqrt{4\sqrt{4\sqrt{4.....}}}$

$\Rightarrow x = \sqrt{4x}$

$\Rightarrow x^2 = 4x$

$\Rightarrow x^2 - 4x = 0$

$\Rightarrow x(x - 4) = 0$

$\Rightarrow x = 0$ (not possible), 4

So, $x = 4$

26. Option (b) is correct.

$$\frac{33m + 22n}{121} = \frac{11(3m + 2n)}{11 \times 11} = \frac{3m + 2n}{11}$$

So, if $3m + 2n$ will divisible by 11 then $33m + 22n$ will also divisible by 121.

Using hit and trial method,

Verifying option (a) i.e $3 = 2 + 1$

Let $m = 2$, $n = 1$

$3 \times 2 + 2 \times 1 = 8$, which is not divisible by 11

Now take $m = 1$, $n = 2$

$3 \times 1 + 2 \times 2 = 7$, which is also not divisible by 11

Verifying option (b) i.e $4 = 3 + 1$

Let $m = 3$, $n = 1$

$3 \times 3 + 2 \times 1 = 9 + 2 = 11$, which is divisible by 11 and minimum value among other options.

Hence, $m + n = 3 + 1 = 4$

27. Option (c) is correct.

Explanation: Let one number is a and other number be b.

According to question

$a \times b = 2160$ (1)

Their HCF is 12. So, $a = 12x$ (2)

$b = 12y$ (3)

Putting value of a and b in (1),

$12x \times 12y = 2160$

$xy = 15$, i.e., $5 \times 3 = 15$

Using value of x, y in (2) and (3)

$a = 12 \times 5 = 60$

$b = 12 \times 3 = 36$

Hence, mean of these two numbers $\frac{(60 + 36)}{2} = 48$.

28. Option (d) is correct.

Assuming ages of Q, P, R and S is Q, P, R and S respectively.

According to question

Q – P = 3, R = 2P and Q = 2S

R – S = 30

So, 2P – S = 30 (1)

2S – P = 3 (2)

Multiplying (2) by 2

$(2S - P = 3) \times 2 \Rightarrow 4S - 2P = 6$ or $-2P + 4S = 6$ (3)

Solving (1) and (3),

$2P - S = 30$

$-2P + 4S = 6$

$3S = 36$

$S = 12$

So, $Q = 2 \times 12 = 24$

and $24 - P = 3$

So, $P = 21$

Hence, P + Q = 21 + 24 = 45 years

29. Option (b) is correct.

We know that sum of two sides of triangle is always greater than third side

i.e. $a + b > c$

So, $\sqrt{a} + \sqrt{b} > \sqrt{c}$

Hence, $\sqrt{a} + \sqrt{b} - \sqrt{c} > 0$

30. Option (c) is correct.

LCM of 15, 25, 35, 45 = 1575

So, four bells will ring together after every 1575 minutes.

72 hours = 72 × 60 = 4320 minutes

Number of times they will ring together

= 4320 ÷ 1575 = 2.7429

So, bell will ring 2 times in 72 hours.

31. Option (c) is correct.

Given, $a + b + c + d = 200$

Here 200 is a even number we know that if we add 4 even numbers or 4 odd numbers or 2 even and 2 odd numbers then sum is positive. So here is only three possibility of S.

32. Option (c) is correct.

$x^n - y^n$ is divisible by both $(x - y)$ and $(x + y)$

If n is an even number.

So, $97^{30} - 14^{30}$ is divisible by $(97 - 14) = 83$

and $(97 + 14) = 111$, which is a multiple of 37

Hence, $97^{30} - 14^{30}$ is divisible by 83 and 37 both.

33. Option (a) is correct.

1. $\log_{10}50 = \log50/\log10 = \log5 + \log10 = \log5$, which is rational.

   So, the statement is correct

2. $\log_{100}10 = \log10/2\log10 = 1/2$ which is rational.

   So, given statement is incorrect

   Hence only 1 is correct.

34. Option (a) is correct.

Assuming efficiency of 1 woman = w and efficiency of 1 men = m

According to question

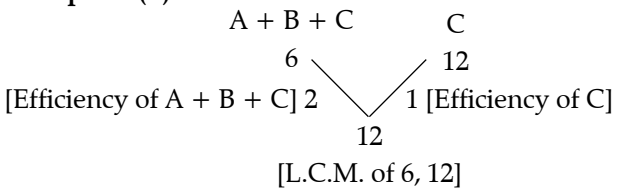
(17w + 24m) × 5 = (12w + 23m) × 6

$\Rightarrow$ 85w + 120m = 72w + 138m

$\Rightarrow$ 13w = 18m

Hence, efficiency of 13 women = efficiency of 18 men

35. Option (b) is correct.



So efficiency of A + B = 2 – 1 = 1

Now according to question

$(A + B + C)t + (A + B)8 = 12$

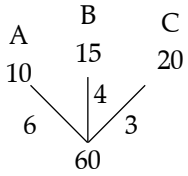
$\Rightarrow 2t + 8 = 12$

$\Rightarrow 2t = 4$

Hence, $t = 2$

36. Option (b) is correct.

Assuming $x = 10$



Then, efficiency of A : B : C is 6 : 4 : 3. So, they will be paid in the same ratio i.e 6 : 4 : 3.

37. Option (d) is correct.

Assuming value of $n = 2$

1. $2^3 - 2 = 6$ (which is divisible by 6)
2. $2^5 - 2 = 30$ (which is divisible by 5)
3. $2^5 - 5 \times 2^3 + 4 \times 2$

   $= 32 - 40 + 8$

   $= 0$ which is divisble by 120

Hence, all three statements are correct.

38. Option (b) is correct.

We know that if power of 9 is even then its units digit will be 1 and if power of 9 is odd then unit digit will be 9.

Unit digit of 27^9 = Unit digit of 71 (as Cyclicity of powers of 7 is 4).

So, in $9^{27} + 27^9$

unit digit of 9^{27} is 9 and unit digit of 27^9 is 7. So, 9 + 7 = 16

Hence, unit digit is 6.

39. Option (c) is correct.

$$x = \frac{\left(\sqrt{3}+1\right)\times\left(\sqrt{3}+1\right)}{\left(\sqrt{3}-1\right)\times\left(\sqrt{3}+1\right)} = \frac{\left(\sqrt{3}+1\right)^2}{3-1} = \frac{4+2\sqrt{3}}{2}$$

$$= 2+\sqrt{3}$$

$$y = \frac{\left(\sqrt{3}-1\right)\times\left(\sqrt{3}-1\right)}{\left(\sqrt{3}+1\right)\times\left(\sqrt{3}-1\right)} = \frac{\left(\sqrt{3}-1\right)^2}{3-1}$$

$$= \frac{4-2\sqrt{3}}{2} = 2-\sqrt{3}$$

$x = 2+\sqrt{3}$

$y = 2-\sqrt{3}$

$\Rightarrow x - y = 2\sqrt{3}$ and $xy = 4 - 1 = 3$

Using, $x^3 - y^3 = (x - y)(x^2 + y^2 + xy)$

$= 2\sqrt{3}\,[(x - y)^2 + 3xy]$

$= 2\sqrt{3}\,[(2\sqrt{3})^2 + 3]$

$= 30\sqrt{3}$

40. Option (b) is correct.

Assuming speed of boat is still water = a

Speed of current = b

Downstream speed = $a + b$
Upstream speed = $a - b$
According to question

$$\frac{42}{(a+b)} = \frac{28}{(a-b)}$$

$\Rightarrow 3a - 3b = 2a + 2b$
$\Rightarrow a = 5b$
Putting value of $a = 15$

$$b = \frac{15}{5} = 3 \text{ km/hr}$$

41. Option (c) is correct.
For half yearly, rate of interest

$$= \frac{R}{2} = \frac{20}{2} = 10\%$$

Time period = $2T = 2 \times 2 = 4$

$$\text{Compound interest} = P\left(1+\frac{R}{100}\right)^T - P$$

$$= 10000\left(1+\frac{10}{100}\right)^4 - 10000$$

$$= \left(10000 \times \frac{11}{10} \times \frac{11}{10} \times \frac{11}{10} \times \frac{11}{10}\right) - 10000$$

= ₹ 4,641

$$\text{Simple interest} = \frac{10000 \times 20 \times 2}{100}$$

$$\left[\text{Using, } SI = \frac{PRT}{100}\right]$$

= ₹ 4,000
Hence, required difference = 4641 – 4000
= ₹ 641

42. Option (a) is correct.
Assuming $a = 3, b = 2$ and $c = 1$

$$a(b-c)(x-b)(x-c) + b(c-a)(x-c)(x-a) + c(a-b)(x-a)(x-b)$$
$$= 3(2-1)(x-2)(x-1) + 2(1-3)(x-1)(x-3) + 1(3-2)(x-3)(x-2)$$
$$= 3(x-2)(x-1) + (-4)(x-1)(x-3) + (x-3)(x-2)$$
$$= (x-1)[3(x-2) + (-4)(x-3)] + x^2 - 2x - 3x + 6$$
$$= (x-1)[3x - 6 - 4x + 12] + x^2 - 2x - 3x + 6$$
$$= (x-1)(6-x) + x^2 - 5x + 6$$
$$= -x^2 + 7x - 6 + x^2 - 5x + 6$$
$$= 2x$$

Coeffcient of $x^2 = 0$
and coefficient of $x = 2 = -(a-b)(b-c)(c-a)$
Hence, only statement (1) is true

43. Option (c) is correct.
Assuming value of $n = 2$

$$1 - x - x^2 + x^{2+1}$$
$$= 1 - x - x^2(1-x)$$
$$= (1-x)(1-x^2)$$
$$= (1-x)(1-x)(1+x)$$

1. $1 - 2x + x^2 = (x-1)^2$
Here, $(1-x)^2(1+x)$ is divisible by $(x-1)^2$
So, statement 1 is correct.
2. $1 - x^n = 1 - x^2 = (1-x)(1+x)$
Yes, it can also divide $(1-x)(1-x)(1+x)$
Hence, both statements are true.

44. Option (d) is correct.
$mx^2 + x(m+8) + 9$
For perfect square, D = 0
$\Rightarrow (m+8)^2 - 4m \times 9 = 0$
$\Rightarrow m^2 + 64 + 16m - 36m = 0$
$\Rightarrow m^2 - 20m + 64 = 0$
$\Rightarrow (m-4)(m-16) = 0$
$\Rightarrow m = 4, m = 16$

45. Option (b) is correct.
Given,

$$x = a + b + \frac{(a-b)^2}{4a+4b}$$

$$y = \frac{a+b}{4} + \frac{ab}{a+b}$$

Putting $a = 2, b = 1$

$$x = 2 + 1 + \frac{(2-1)}{4\times2+4\times1} = 3 + \frac{1}{12} = \frac{37}{12}$$

$$y = \frac{2+1}{4} + \frac{2\times1}{2+1} = \frac{3}{4} + \frac{2}{3} = \frac{17}{12}$$

So, $(x-a)^2 - (y-b)^2$

$$= \left(\frac{37}{12} - 2\right)^2 - \left(\frac{17}{12} - 1\right)^2$$

$$= \frac{13}{12} \times \frac{13}{12} - \frac{5}{12} \times \frac{5}{12}$$

$$= \frac{1}{12\times12}[13\times13 - 5\times5]$$

$$= \frac{144}{144} = 1 = b^2$$

46. Option (b) is correct.
1. $\cos^4\theta - \sin^4\theta = 2\tan\theta/(1 - \tan2\theta)$
$\Rightarrow (\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)$
$= 2\tan\theta/(1 - \tan2\theta)$
$\Rightarrow (\cos^2\theta - \sin^2\theta) = 2\tan\theta/(1 - \tan2\theta)$

$$\Rightarrow 1 \times \cos2\theta \neq \frac{2\tan\theta}{1-\tan^2\theta}$$

2. $\text{cosec}\theta + \cot\theta = \dfrac{1}{\text{cosec}\theta - \cot\theta}$, given

$$\frac{1}{\operatorname{cosec}\theta-\cot\theta}\times\frac{\operatorname{cosec}\theta+\cot\theta}{\operatorname{cosec}\theta+\cot\theta}$$

$$=\frac{\operatorname{cosec}\theta+\cot\theta}{\operatorname{cosec}^2\theta-\cot^2\theta}=\operatorname{cosec}\theta+\cot\theta$$

So, statement 2 is correct.

3. $\cos^2\theta-\sin^2\theta=\cos 2\theta$

$$\frac{1-\tan^2\theta}{1-\tan^2\theta}=\cos 2\theta$$

$$\text{So, } \cos^2\theta-\sin^2\theta=\frac{1-\tan^2\theta}{1+\tan^2\theta}$$

So, statement 3 is correct.

47. Option (d) is correct.

$$\sin\theta=\frac{P}{H}=\frac{12}{13}$$

We know that 12, 13 and 5 forms a triplet of Pythagoras. So, base = 5

Now, $(\tan\theta+\sec\theta)^2(\operatorname{cosec}\theta-\cot\theta)^{-2}$

$$=\left(\frac{P}{B}+\frac{H}{B}\right)^2\left(\frac{H}{P}-\frac{B}{P}\right)^{-2}$$

$$=\left(\frac{12}{5}+\frac{13}{5}\right)^2\left(\frac{13}{12}-\frac{5}{12}\right)^{-2}$$

$$=25\times\left(\frac{2}{3}\right)^{-2}=25\times\frac{9}{4}=\frac{225}{4}$$

48. Option (c) is correct.

$\tan^2\theta+\cot^2\theta=m$

We have, $(a+b)^2=a^2+b^2+2ab$

$a^2+b^2=(a+b)^2-2ab$ (1)

but here power is 8, so we keep to use (1) those time which give three times faster and addition of 2, three time. So, correct option is

$$\sqrt{\sqrt{\sqrt{m+2}+2}+2}$$

49. Option (b) is correct.

Maximum value of $\sin\theta = 1$

So, minimum value of $6-4\sin\theta$

$= 6-4\times1=2$

50. Option (d) is correct.

$4\cos^2 30°+2x\sin 30°-\cot^2 30°-6\tan 15°\tan 75°=0$

$$\Rightarrow 4\left(\frac{\sqrt{3}}{2}\right)^2+2x\times\frac{1}{2}-\left(\sqrt{3}\right)^2-6\ \tan 15\,\frac{1}{\tan 15}=0$$

$$\Rightarrow 4\times\frac{3}{4}+x-3-6=0$$

$$\Rightarrow 3+x-9=0$$

$$\Rightarrow x-6=0$$

Hence, $x = 6$

51. Option (d) is correct.

$$\frac{\cos^2 32°+\cos^2 58°}{\sec^2 50°-\cot^2 40°}+4\tan 13°\cdot\tan 37°\cdot\tan 53°\cdot\tan 77°$$

$$=\frac{\sin^2(90°-32°)+\cos^2 58°}{\operatorname{cosec}^2(90°-50°)-\cot^2 40°}+4\cot(90°-13°)$$

$$\tan(90°-37°)\cdot\tan 53°\cdot\tan 77°$$

$$=\frac{\sin^2 58°+\cos^2 58°}{\operatorname{cosec}^2 40°-\cot^2 40°}+4\cot 77°\cdot\cot 53°\cdot\tan 53°\cdot\tan 77°$$

$= 1 + 4 = 5$

52. Option (b) is correct.

$(1+\cot^2\theta)(1+\cos\theta)(1-\cos\theta)$

$$-(1+\tan^2\theta)(1+\sin\theta)\times(1-\sin\theta)$$

$=\operatorname{cosec}^2\theta(1-\cos^2\theta)-\sec^2\theta(1-\sin^2\theta)$

$$=\frac{1}{\sin^2\theta}\times\sin^2\theta-\frac{1}{\cos^2\theta}\times\cos^2\theta$$

$= 1-1=0$

53. Option (a) is correct.

$2\cos^2\theta+\sin\theta-2=0$

$\Rightarrow 2(1-\sin^2\theta)+\sin\theta-2=0$

$\Rightarrow 2-2\sin^2\theta+\sin\theta-2=0$

$\Rightarrow \sin\theta-2\sin^2\theta=0$

$\Rightarrow \sin(1-2\sin\theta)=0$

$\Rightarrow \sin\theta\neq 0$ ($\theta\in(0,\pi/2]$)

$1-2\sin\theta=0\Rightarrow\sin\theta=1/2$

So, $\theta=\pi/6$

Shortcut method

Put $x = \pi/6, \pi/4, \pi/3$ and $\pi/6$ in given equation.

$2\cos^2(\pi/6)=\sin\pi/6-2=0$

$$2\times\frac{3}{4}+\frac{1}{2}-2=0$$

Satisfy the given equation.

54. Option (b) is correct.

Let speed of can be x m/min.

CD = $6x$ m and BD = tx m

In ΔABD,

$\tan 45° = AB/BD$

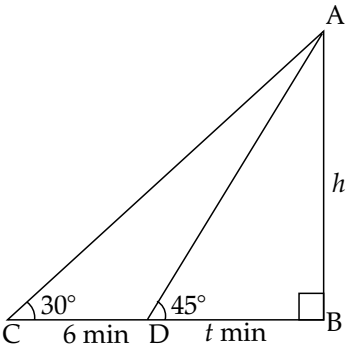
$$1=\frac{h}{tx}$$

$\Rightarrow h = tx$ (1)

In ΔABC,

$\tan 30° = AB/BC$

$$\Rightarrow \frac{1}{\sqrt{3}}=\frac{h}{6x+tx}$$



$\Rightarrow 6x+tx=\sqrt{3}\,tx$

$\Rightarrow 6+t=\sqrt{3}\,t$

$\Rightarrow 6=t(\sqrt{3}-1)$

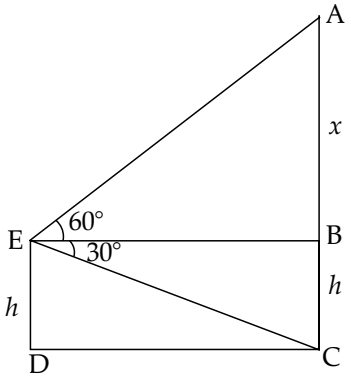
$$t = \frac{6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{6(\sqrt{3}+1)}{3-1}$$

$$= 3(\sqrt{3}+1) = 3(1\cdot732+1) = 8\cdot196$$

$\therefore 8 < t < 8.3$

55. Option (c) is correct.

In ΔBCE,



$$\tan 30° = \frac{h}{BE}$$

$$\Rightarrow BE = \sqrt{3}h$$

In ΔABE

$$\tan 60° = \frac{x}{BE}$$

$$\Rightarrow \sqrt{3} = \frac{x}{\sqrt{3}h}$$

$$\Rightarrow x = 3h$$

Height of tower $= x + h = 3h + h = 4h$

56. Option (c) is correct.

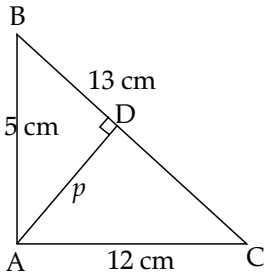
In ΔABC and ΔABD

$\angle BAC = \angle ADB = 90°$

$\angle B = \angle B$ (common)

$\angle BAC = \angle A$

$\therefore \Delta ABC = \Delta DBA$



$$\Rightarrow \frac{BC}{BA} = \frac{AC}{DA}$$

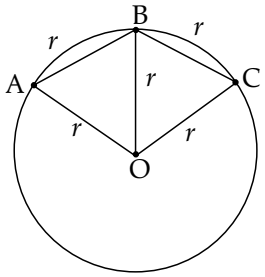
$$\Rightarrow \frac{13}{5} = \frac{12}{DA}$$

$$\Rightarrow AD = p = \frac{5\times12}{13} = \frac{60}{13}$$

$$\therefore 13p = 13 \times \frac{60}{13} = 60$$

57. Option (c) is correct.

Since, ΔOAB and ΔOBC are equilateral triangles with side r.



$$\therefore \text{Area of rhombus} = 2 \times \frac{\sqrt{3}}{4} r^2 = 32\sqrt{3}$$

$\Rightarrow r^2 = 64$ or $r = 8$ cm

58. Option (d) is correct.

Let length of side be x unit

$\therefore$ Surface area of cube $= 6x^2$

New length of side $= x\left(1 + \frac{p}{100}\right)$

New surface area $= 6x^2\left(1 + \frac{25}{100}\right)$

$$\Rightarrow \quad 6x^2\left(1+\frac{p}{100}\right)^2 = 6x^2\left(1+\frac{25}{100}\right)$$

$$\Rightarrow \quad \left(1+\frac{p}{100}\right)^2 = \frac{125}{100}$$

$$\Rightarrow \quad 1+\frac{p}{100} = \frac{5\sqrt{5}}{10}$$

$$\Rightarrow \quad \frac{p}{100} = \frac{5\sqrt{5}-10}{10}$$

$$\Rightarrow \quad p = 50\sqrt{5} - 100 = 11.80$$

$\therefore 10 < p < 12$

59. Option (b) is correct.

Let side of cube be x unit

Then length of cuboids be $x/2$ unit

Breadth $= x$ unit and height $= x$ unit

$$\text{So, } \frac{\text{surface area of cube}}{\text{surface area of cuboids}} = \frac{6x^2}{2\left(\frac{x^2}{2} + x^2 + \frac{x^2}{2}\right)}$$

$$= \frac{6x^2}{x^2+2x^2+x^2} = \frac{6x^2}{4x^2} = \frac{3}{2} = 3:2$$

60. Option (d) is correct.

Diagonal of cuboid $= \sqrt{l^2+b^2+h^2} = 11$

$\Rightarrow l^2 + b^2 + h^2 = 121$ (i)

Surface area $= 2(lb + bh + hl) = 240$

$\Rightarrow lb + bh + hl = 120$ (ii)

We know that

$(l + b + h)^2 = l^2 + b^2 + h^2 + 2(lb + bh + hl)$

$= 121 + 2(120)$

$= 121 + 240 = 361$

$$\Rightarrow \quad l+b+h = \sqrt{361} = 19 \text{ cm}$$

61. Option (b) is correct.

$$S = \frac{a+b+c}{2} = \frac{12+16+20}{2} = \frac{48}{2} = 24 \text{ cm}$$

Δ = area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{24(24-12)(24-16)(24-20)}$

$= \sqrt{24\times12\times8\times4} = \sqrt{12\times2\times12\times4\times2\times4}$

$= 12 \times 2 \times 4 = 96 \text{ cm}^2$

Inradius (r) $= \frac{\Delta}{S} = \frac{96}{24} = 4$ *cm*

Area of circle $= \pi r^2 = 3.14 \times (4)^2$

$= 3.14 \times 16 = 50.24 \approx 50 \text{ cm}^2$

62. Option (b) is correct.

According to question,

$2 \times$ T.S.A. of cylinder $= 3 \times$ C.S.A. of cylinder

$2 \times 2\pi r\,(r + h) = 3 \times 2\pi rh$

$2r + 2h = 3h$

$\Rightarrow \quad 2r = h$

63. Option (d) is correct.

l = Length of floor = 30 m 60 cm = 3060 cm

b = Breadth of floor = 23 m 40 cm = 2340 cm

HCF of (3060, 2340) = side of square tiles

= 180 cm

$$\text{Number of tiles} = \frac{\text{Area of floor}}{\text{Area of each tile}}$$

$$= \frac{3060 \times 2340}{180 \times 180} = 17 \times 13$$

$$= 221$$

64. Option (c) is correct.

Area of square $= a^2 = 25$ hectares

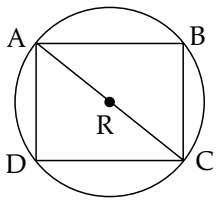
$\Rightarrow a^2 = 2{,}50{,}000 \text{ m}^2 \Rightarrow a = 500$ m

Length of boundary $= 4a = 2000$ m $= 2$ km

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{2}{5}\text{h} = \left(\frac{2}{5} \times 60\right) \text{min}.$$

= 24 minutes

65. Option (b) is correct.



Area of square $= x$

$\Rightarrow \quad (\text{Side})^2 = x$

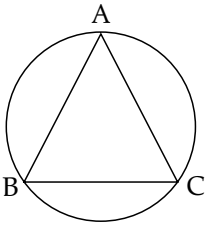
$\Rightarrow \quad \text{Side} = \sqrt{x}$

In right angle triangle ADC,

$$(2R)^2 = (\sqrt{x})^2 + (\sqrt{x})^2$$

$$\Rightarrow \quad 2R^2 = x \Rightarrow R^2 = \frac{x}{2} \qquad \text{(i)}$$

Area of equivaleteral triangle $= y$



$$\therefore \quad y = \frac{\sqrt{3}}{4}(\text{Side})^2$$

$$\Rightarrow \quad (\text{Side})^2 = \frac{4y}{\sqrt{3}} \qquad \text{(ii)}$$

We know that

Radius of circumcircle

$$= \frac{\text{Side of equilateral triangle}}{\sqrt{3}}$$

$$\text{R} = \frac{\text{side}}{\sqrt{3}} \Rightarrow \text{side} = \sqrt{3}\text{R} \qquad \text{[from (ii)]}$$

$$\Rightarrow 3\text{R}^2 = \frac{4y}{\sqrt{3}} \Rightarrow \text{R}^2 = \frac{4y}{3\sqrt{3}}$$

$$\Rightarrow \frac{x}{2} = \frac{4y}{3\sqrt{3}} \qquad \text{[from (i)]}$$

$$\Rightarrow 3\sqrt{3}x = 8y$$

Squaring both sides

$$27x^2 = 64y^2$$

66. Option (c) is correct.

$$\text{New length} = l\left(1 + \frac{200}{300}\right) = \frac{5l}{3}$$

Let new width $= b'$

According to question,

$$\frac{5l}{3} \times b' = lb$$

$$\Rightarrow \quad b' = \frac{3b}{5}$$

$\therefore$ Percentage decreased in width

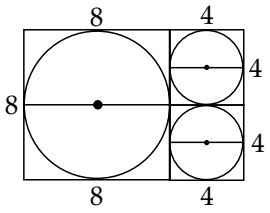
$$= \frac{b - \frac{3b}{5}}{b} \times 100 = \frac{2}{5} \times 100 = 40\%$$

67. Option (d) is correct.

$r_1 = 8/2 = 4$ cm

$r_2 = 4/2 = 2$ cm

$r_3 = 4/2 = 2$ cm



Area covered by these three circles

$= \pi {r_1}^2 + \pi {r_2}^2 + \pi {r_3}^2$

$= \pi(4)^2 + \pi(2)^2 + \pi(2)^2$

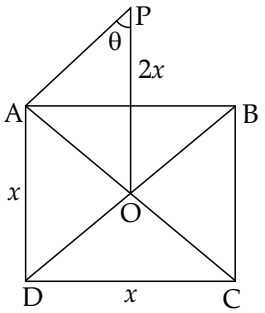
$= 16\pi + 4\pi + 4\pi = 24\pi \text{ cm}^2$

68. Option (c) is correct.

Given that side of square be x

$\therefore \quad \text{AC} = \sqrt{2}x$

$$\Rightarrow \quad \text{OA} = \frac{1}{2}\text{AC} = \frac{\sqrt{2}x}{2}$$



In ΔAOP,

$$\cot\theta = \frac{OP}{OA} = \frac{2x}{\frac{\sqrt{2}x}{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

69. Option (a) is correct.

Let radius of larger and smaller balls be r_1 and r_2 respectively.

$$\text{Number of balls} = \frac{\text{Volume of larger ball}}{\text{Volume of smaller ball}}$$

$$\Rightarrow \qquad 64 = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{r_1^3}{r_2^3}$$

$$\Rightarrow \qquad \frac{r_1}{r_2} = 4$$

$$\text{So, } \frac{\text{Surface are of larger ball}}{\text{Sum of the surface area of smaller balls}}$$

$$= \frac{4\pi r_1^2}{64 \times 4\pi r_2^2} = \frac{1}{64} \times \left(\frac{r_1}{r_2}\right)^2$$

$$= \frac{1}{64} \times 16 = \frac{1}{4} = 0.25$$

70. Option (b) is correct.

Perimeter of P:

$AF + FE + AE = 16$ (1)

Perimeter of Q:

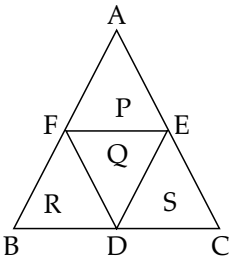
$FE + FD + ED = 12$ (ii)

Perimeter of R:

$BF + FD + BD = 4$ (iii)

Perimeter of S:

$CD + ED + EC = 12$ (iv)



Adding equation (i), (iii) and (iv), we get

$\Rightarrow$ $AB + BC + AC + EF + FD + ED$
$= 16 + 4 + 12$

$\Rightarrow$ $AB + BC + AC + 12 = 32$ [Using (ii)]

$\Rightarrow$ $AB + BC + AC = 32 - 12 = 20$

So, perimeter of $\Delta ABC = 20$ cm

71. Option (a) is correct.

A number 277XY5 is divisible by 25.

If Y5 is divisible by 25.

$\therefore$ Y = 2

Statement 1, since given number D.

Divisible by 9, so sum of digit is divisible by 9.

$2 + 7 + 7 + X + 2 + 5 = 23 + X$ is divisible by 9.

So, possible value of X is 4 which is not satisfy the statement II.

Hence, cannot be answered by using both statement together.

72. Option (b) is correct.

Since 2 is root of the given equation

$\therefore$ $4a + 2b + c = 0$ (i)

Statement I: $c/a = 1$ (ii)

Statement II: $b/a = -5/2$ (iii)

Hence, unique value of a, b, c can be obtained by using both statement I and statement II.

73. Option (b) is correct.

Statement I: $\frac{m}{n} > 1$ (i)

Statement II: $m > 2n \Rightarrow \frac{m}{n} > 2$ (ii)

From (i) or (ii) we get,

$$m > n$$

Hence, the question can be answered by either statement alone.

74. Option (b) is correct.

We can draw a circum circle if all sides of triangle are given.

75. Option (c) is correct.

Let consecutive odd integer x and y are k and $k + 2$ respectively.

From statement-I:

$(x + y)^4 = 256$

$\Rightarrow$ $x + y = 4$

$\therefore$ $(x, y) = (1, 3); (-1, -3)$

From statement-II:

$(x + y)^3 < 16$

$(x + y) < (16)^{1/3}$

$(x + y) < 2(2)^{1/3}$

76. Option (d) is correct.

Let, $p = 2$ and $q = 5$, then

Statement I: $2p + q = 4 + 5 = 9$ is odd

Statement II: $q - 2p = 5 - 4 = 1$ is odd

But

$p^2 + q^2 + q = 4 + 25 + 5 = 34$ is even

So, question cannot be answered even by using both statement together.

77. Option (b) is correct.

Statement I:

$l = 7$ cm

$\theta = 30° = \pi/6$

We have, $l = r\theta$

$$7 = r \times \frac{\pi}{6} \Rightarrow r = 7 \times \frac{6}{\pi}$$

$\Rightarrow \quad r = \dfrac{7\times6\times7}{22}$

Area of circle $= \dfrac{22}{7}\times\dfrac{7\times7\times6}{22}\times\dfrac{7\times6\times7}{22}$

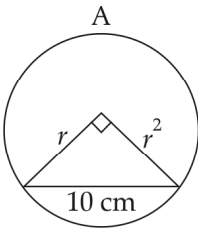
$= \dfrac{7\times6\times7\times6\times7}{22}$

Statement II:

$r^2 + r^2 = 100$

$\Rightarrow \quad r = 5\sqrt{2}$

Area of circle $= \dfrac{22}{7}\times 25 \times 2$



So, question can be answered by either statement alone.

78. Option (a) is correct.

Statement I is true for all the triangle. So, it may be right triangle.

Statement II

$x + 2x + 3x = 180°$

$\Rightarrow 6x = 180° \Rightarrow x = 30°$

So, $3x = 3 \times 30° = 90°$

So, triangle is right triangle.

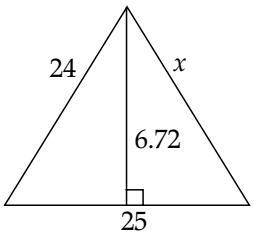
Hence, question can be answered by one of the statement alone but not by the other.

79. Option (b) is correct.

Statement-I:

$x + 2x + 3x = 180° \Rightarrow 6x = 180°$

$\Rightarrow x = 30°$



So, angles are 30°, 60°, 90°

$\dfrac{a}{\sin 30°} = \dfrac{24}{\sin 60°} = \dfrac{25}{\sin 90°}$

$a = \dfrac{25}{\sin 90°}\times \sin 30° = 25\times\dfrac{1}{2}$

Shortest side = 12.5 cm

Statement-II: We can find the value of x by Pythagoras theorem which is different from Statement I.

So, question can be answered by either statement alone.

80. Option (b) is correct.

Given that,

3X area of major segment

= 4 × area of minor segment

Area of major segment

= 4/3 × area of minor segment

Area of minor segment

= 3/4 × area of major segment

Area of minor segment + Major segment

$= \pi r^2$

Area of major segment = $4/7\ \pi r^2$

So, we can find the radius of circle if either area of minor or major segment given.

81. Option (b) is correct.

Class	f_i	C.F
0-30	4	4
30-60	5	9
60-90	7	16
90-120	4	20

Mode $= l + \dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\times h$

$= 60 + \dfrac{7-5}{14-5-4}\times 30$

$= 60 + 2/5 \times 30$

$= 60 + 12 = 72$

82. Option (a) is correct.

Mode (Q) = 72

Median (P) $= l + \dfrac{\frac{N}{2} - C.F}{f}\times h$

$= 60 + \dfrac{10-9}{7}\times 30$

$= 60 + \dfrac{30}{7} = 30\left(2 + \dfrac{1}{7}\right)$

$= 30\times\dfrac{15}{7} = \dfrac{450}{7}$

Now, 7(Q–P) = 9R

$\Rightarrow 7\left(72 - \dfrac{450}{7}\right) = 9R$

$\Rightarrow 7\left(\dfrac{504-450}{7}\right) = 9R$

54 = 9R

R = 6

83. Option (d) is correct.

Class	Frequency (f_i)	Class mark (x_i)	$f_i x_i$
40–50	4	45	180
50–60	3	55	165
60–70	1	65	65
70–80	2	75	150
Total	10		560

Mean $= \dfrac{\sum x_i f_i}{\sum f_i} = \dfrac{560}{10} = 56$

84. Option (c) is correct.

Class	Frequency (f_i)	C.F	
40-50	4	4	
50-60	3	7	→ Median class
60-70	1	8	
70-80	2	10	

Here, $\dfrac{N}{2} = \dfrac{10}{2} = 5$

$$\text{Median (M)} = l + \frac{\frac{N}{2} - \text{C.F}}{f} \times h$$

$$= 50 + \frac{5-4}{3} \times 10$$

$$= 50 + \frac{10}{3} = \frac{160}{3}$$

So, $3M = 3 \times \dfrac{160}{3} = 160$

85. Option (b) is correct.

Let length be $2x$ and breadth be x

Area $= 2x \times x = 200$

$2x^2 = 200 \Rightarrow x = 10$ m

Length = 20 m

Breadth = 10 m

New length = 20 + 2 = 22 m

New breadth = 10 + 2 = 12 m

New area = $22 \times 12 = 264$ m^2

Increased area percentage

$= \dfrac{264 - 200}{200} \times 100 = 32\%$

86. Option (a) is correct.

Circumference = 44 cm

$2\pi r = 44 \Rightarrow r = 44 \times 7/2 \times 22$

$r = 7$ cm

Height (h) = 44 cm

Volume $= \pi r^2 h$

$= \dfrac{22}{7} \times 7 \times 7 \times 44$

$= 6776$ cm^3

87. Option (a) is correct.

Given that,

$lbh = 3600$ cm^3 (i)

$lb = x$ cm^2 (ii)

$bh = 225$ cm^2 (iii)

$lh = 144$ cm^2 (iv)

Multiply (ii), (iii) and (iv), we get

$l^2b^2h^2 = 225 \times 144 \times x$

$\Rightarrow \quad x = \dfrac{3600 \times 3600}{225 \times 144} = 400$ cm^2

88. Option (c) is correct.

Given that,

$1/2 \times p \times b = 54 \Rightarrow pb = 108$

$p + b + h = 36$

$p + b = 36 - h$

Squaring both sides

$(p+b)^2 = (36 - h)^2$

$\Rightarrow p^2 + b^2 + 2pb = (36 - h)^2$

$\Rightarrow h^2 + 2 \times 108 = (36 - h)^2$

$\Rightarrow (36 - h)^2 - h^2 = 216$

$\Rightarrow (36 - h + h)(36 - h - h) = 216$

$\Rightarrow 36(36 - 2h) = 216$

$\Rightarrow 36 - 2h = 6 \Rightarrow h = 15$ cm

89. Option (d) is correct.

$x = \{2, 6, 10, 14, ..., 98\}$

Sum of two numbers is 100 = (2, 98), (6, 94), (10, 90), (14, 86), (18, 82), (22, 78), (26, 74)(30, 70) (34,66)(38,62), (42, 58), (46, 54), (50, 50)

S = {2, 6, 10, 14, 18, 22, 26, 30, 34, 38, 42, 46, 50} and {50, 54, 58, 62, 66, 70, 74, 78, 82, 86, 90, 94, 98}

Required numbers = 13

90. Option (a) is correct.

Perimeter of a sector of a circle

$= 2r$ + Length of arc

$16.4 = 2(5.2)$ + Length of arc

Length of arc $= 16.4 - 10.4 = 6$

$$\Rightarrow \frac{\theta}{360^\circ} \times 2\pi r = 6$$

$$\Rightarrow \theta = \frac{6 \times 360}{2\pi \times 5.2} = \frac{6 \times 360 \times 7}{2 \times 22 \times 5.2}$$

So, area of sector $= \dfrac{\theta}{360^\circ} \times \pi r^2$

$= \dfrac{1}{360^\circ} \times \dfrac{6 \times 360 \times 7}{2 \times 22 \times 5.2} \times \dfrac{22}{7} \times (5.2)^2 = 15.6$ cm^2

91. Option (d) is correct.

$10^2 = 6^2 + 8^2$

$\therefore \angle ABC = 90^\circ$

$\therefore \Delta ABC \sim \Delta BDC$

$$\Rightarrow \frac{AB}{BD} = \frac{AC}{BC}$$

$$\Rightarrow \frac{6}{BD} = \frac{10}{8}$$

$$\Rightarrow BD = \frac{6 \times 8}{10} = 4.8 \text{ cm} = BQ$$

So, CQ = BC – BQ = 8 – 4.8 = 3.2 cm

92. Option (c) is correct.

$\Delta ABC \sim \Delta ADB$

$\therefore \frac{AB}{AD} = \frac{AC}{AB} \Rightarrow \frac{6}{AD} = \frac{10}{6}$

$\Rightarrow AD = \frac{36}{10} = 3.6$ cm

Given that $\angle ABD = \theta$

$\sin\theta = \frac{AD}{AB} = \frac{3.6}{6} = 0.6$

93. Option (b) is correct.

$\Delta ABC \sim \Delta BDC$

$\therefore \frac{AB}{BD} = \frac{AC}{BC}$

$\frac{6}{BD} = \frac{10}{8}$

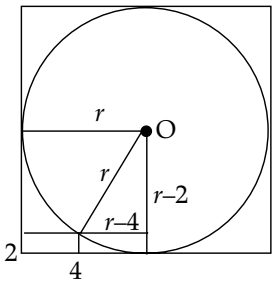
$\Rightarrow BD = \frac{6\times 8}{10} = 4.8$ cm

Radius = 4.8 cm

94. Option (a) is correct

$r^2 = (r-4)^2 + (r-2)^2$

$r^2 = r^2 - 8r + 16 + r^2 - 4r + 4$



$r^2 - 12r + 20 = 0$

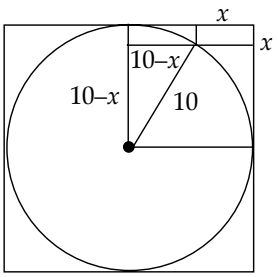
$r = 10$ ($r = 2$ is not possible)

$\therefore$ Area of circle $= \pi r^2 = \pi(10)^2 = 100\pi$ cm^2

95. Option (d) is correct.

$(10-x)^2 + (10-x)^2 = 10^2$

$\Rightarrow 2(10-x)^2 = 10^2$



$\Rightarrow \sqrt{2}\,(10-x) = 10$

$\Rightarrow 10 - x = 10/\sqrt{2} = 5\sqrt{2}$

$\Rightarrow x = 10 - 5\sqrt{2} = 5(2-\sqrt{2})$

Area of smaller square

$= [5(2-\sqrt{2}\,]^2 = 25\,[4 + 2 - 4\sqrt{2}\,]$

$= 50\,[3 - 2\sqrt{2}\,]$ cm^2

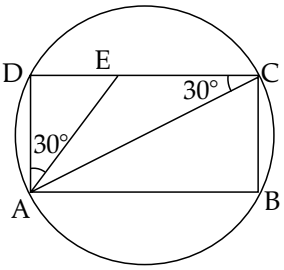
96. Option (b) is correct.

Area of the shaded region

$= (10)^2 - \frac{1}{4}\pi(10)^2 - 2\times 4$

$= 100 - 25\pi - 8 = (92 - 25\pi)$ cm^2

97. Option (b) is correct.



$AC = 2r$

In ΔACD

$\sin 30° = AD/AC$

$\Rightarrow AD = 1/2\ AC$

$= 1/2 \times 2r = r$

$\cos 30° = CD/AC$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{CD}{AC}$

$\Rightarrow CD = \frac{\sqrt{3}}{2}\times AC = \frac{\sqrt{3}}{2}\times 2r = \sqrt{3}r$

So, $\frac{\text{Area of circle}}{\text{Area of rectangle}} = \frac{\pi r^2}{(r)\times(\sqrt{3}vr)} = \frac{\pi}{\sqrt{3}}$

98. Option (a) is correct.

In ADE,

$\tan 30° = \frac{DE}{AD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{DE}{r} \Rightarrow DE = \frac{r}{\sqrt{3}}$

Area of ΔAEC = Area of ΔADC – Area of ΔADE

$= \frac{1}{2}\times r \times \sqrt{3}r - \frac{1}{2}\times r \times \frac{r}{\sqrt{3}}$

$= \frac{r^2}{2}\left(\sqrt{3} - \frac{1}{3}\right) = \frac{r^2}{2}\times\frac{2}{\sqrt{3}} = \frac{r^2}{\sqrt{3}}$

99. Option (c) is correct.

OB = BQ

$\therefore \angle BOQ = \angle BQO = y°$

$\Rightarrow \angle ABO = y° + y = 2y$ (exterior angle)

$\Rightarrow \angle OAB = \angle ABO = 2y$ (OA = OB)

In ΔAOB,

$\angle AOB = 180° - 4y°$

By angle sum property of a triangle,

$\angle POA + \angle AOB + \angle BOQ = 180°$

$\Rightarrow x° + 180° - 4y° + y° = 180°$

$\Rightarrow x° - 3y° = 0$

$\Rightarrow x° = 3y°$

100. Option (d) is correct.

$\therefore \angle AOB = 180° - 4y°$

$= 180° - 4 \times 15 = 120°$

$\angle ACB = 1/2\ \angle AOB$

$= 1/2 \times 120° = 60°$