NDA / NA

National Defence Academy / Naval Academy

MATHEMATICS

I

QUESTION PAPER 2023

Time : 2 : 30 Hours **Total Marks : 300**

Instructions :

1. *This Test Booklet contains **120** items (questions). Each item comprises four responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose **ONLY ONE** response for each item.*
2. *You have to mark all your responses **ONLY** on the separate Answer Sheet provided. See directions in the Answer Sheet.*
3. ***All** items carry equal marks.*
4. ***Penalty for wrong answers :***
   THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.
   (i) *There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third** of the marks assigned to that question will be deducted as penalty.*
   (ii) *If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.*
   (iii) *If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.*

1. If ω is a non-real cube root of 1, then what is the value of $\left|\frac{1-\omega}{\omega+\omega^2}\right|$?

(a) $\sqrt{3}$ **(b)** $\sqrt{2}$

(c) 1 **(d)** $\frac{4}{\sqrt{3}}$

2. What is the number of 6-digit numbers that can be formed only by using 0, 1, 2, 3, 4 and 5 (each once); and divisible by 6 ?

(a) 96 **(b)** 120

(c) 192 **(d)** 312

3. What is the binary number equivalent to decimal number 1011 ?

(a) 1011 **(b)** 111011

(c) 11111001 **(d)** 111110011

4. Let A be a matrix of order 3×3 and $|A| = 4$. If $|2\,\text{adj}(3A)| = 2^{\alpha}3^{\beta}$ then what is the value of $(\alpha + \beta)$?

(a) 12 **(b)** 13

(c) 17 **(d)** 24

5. If α and β are the distinct roots of equation $x^2 - x + 1 = 0$, then what is the value of $\left|\frac{\alpha^{100}+\beta^{100}}{\alpha^{100}-\beta^{100}}\right|$?

(a) $\sqrt{3}$ **(b)** $\sqrt{2}$

(c) 1 **(d)** $\frac{1}{\sqrt{3}}$

6. Let A and B be symmetric matrices of same order, then which one of the following is correct regarding (AB – BA)?

1. Its diagonal entries are equal but nonzero
2. The sum of its non-diagonal entries is zero

Select the correct answer using the code given below:

(a) 1 only **(b)** 2 only

(c) Both 1 and 2 **(d)** Neither 1 nor 2

7. Consider the following statements in respect of square matrices A, B, C each of same order n :

1. $AB = AC \Rightarrow B = C$ if A is non-singular

2. If $BX = CX$ for every column matrix X having n rows then $B = C$

Which of the statements given above is/are correct ?

(a) 1 only **(b)** 2 only

(c) Both 1 and 2 **(d)** Neither 1 nor 2

8. The system of linear equations $x + 2y + z = 4$, $2x + 4y + 2z = 8$ and $3x + 6y + 3z = 10$ has

(a) a unique solution

(b) infinite many solutions

(c) no solution

(d) exactly three solutions

9. Let AX = B be a system of 3 linear equations with 3-unknowns. Let X_1 and X_2 be its two distinct solutions. If the combination $aX_1 + bX_2$ is a solution of AX = B; where a, b are real numbers, then which one of the following is correct ?

(a) $a = b$ **(b)** $a + b = 1$
(c) $a + b = 0$ **(d)** $a - b = 1$

10. What is the sum of the roots of the equation $\begin{vmatrix} 0 & x-a & x-b \\ 0 & 0 & x-c \\ x+b & x+c & 1 \end{vmatrix} = 0$?

(a) $a + b + c$ **(b)** $a - b + c$
(c) $a + b - c$ **(d)** $a - b - c$

11. If $2 - i\sqrt{3}$ where $i = \sqrt{-1}$ is a root of the equation $x^2 + ax + b = 0$, then what is the value of $(a + b)$?

(a) -11 **(b)** -3
(c) 0 **(d)** 3

12. If $z = \frac{1+i\sqrt{3}}{1-i\sqrt{3}}$ where $i = \sqrt{-1}$, then what is the argument of z ?

(a) $\frac{\pi}{3}$ **(b)** $\frac{2\pi}{3}$
(c) $\frac{4\pi}{3}$ **(d)** $\frac{5\pi}{6}$

13. If a, b, c are in AP, then what is $\begin{vmatrix} x+1 & x+2 & x+3 \\ x+2 & x+3 & x+4 \\ x+a & x+b & x+3 \end{vmatrix}$ equal to ?

(a) -1 **(b)** 0
(c) 1 **(d)** 2

14. $\log_x a$, a^x and $\log_b x$ are in GP, then what is x equal to ?

(a) $\log_a(\log_b a)$ **(b)** $\log_b(\log_a b)$
(c) $\frac{\log_a(\log_b a)}{2}$ **(d)** $\frac{\log_b(\log_a b)}{2}$

15. If $2^{\frac{1}{c}}$, $2^{\frac{b}{ac}}$, $2^{\frac{1}{a}}$ are in GP, then which one of the following is correct?

(a) a, b, c are in AP **(b)** a, b, c are in GP
(c) a, b, c are in HP **(d)** ab, be, ca are in AP

16. The first and the second terms of an AP are $\frac{5}{2}$ and $\frac{23}{12}$ respectively. If n^{th} term is the largest negative term, what is the value of n ?

(a) 5
(b) 6
(c) 7
(d) n cannot be determined

17. For how many integral values of k, the equation $x^2 - 4x + k = 0$, where k is an integer has real roots and both of them lie in the interval (0, 5) ?

(a) 3 **(b)** 4
(c) 5 **(d)** 6

18. In an AP, the first term is x and the sum of the first n terms is zero. What is the sum of next m terms ?

(a) $\frac{mx(m+n)}{n-1}$ **(b)** $\frac{mx(m+n)}{1-n}$
(c) $\frac{nx(m+n)}{m-1}$ **(d)** $\frac{nx(m+n)}{1-m}$

19. Consider the following statements :
1. (25)! + 1 is divisible by 26
2. (6)! + 1 is divisible by 7
Which of the above statements is/are correct ?

(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

20. If z is a complex number such that $\frac{z-1}{z+1}$ is purely imaginary, then what is $|z|$ equal to ?

(a) $\frac{1}{2}$ **(b)** $\frac{2}{3}$
(c) 1 **(d)** 2

21. How many real numbers satisfy the equation $|x - 4| + |x - 7| = 15$?

(a) Only one **(b)** Only two
(c) Only three **(d)** Infinitely many

22. A mapping $f : A \to B$ defined as $f(x) = \frac{2x+3}{3x+5}, x \in A$. If f is to be onto, then what are A and B equal to ?

(a) $A = R \setminus \{-\frac{5}{3}\}$ and $B = R \setminus \{-\frac{2}{3}\}$

(b) $A = R$ and $B = R \setminus \{-\frac{5}{3}\}$

(c) $A = R \setminus \{-\frac{3}{2}\}$ and $B = R \setminus \{0\}$

(d) $A = R \setminus \{-\frac{5}{3}\}$ and $B = R \setminus \{\frac{2}{3}\}$

23. α and β are distinct real roots of the quadratic equation $x^2 + ax + b = 0$. Which of the following statements is/are sufficient to find α ?

1. $\alpha + \beta = 0, \alpha^2 + \beta^2 = 2$
2. $\alpha\beta^2 = -1, a = 0$

Select the correct answer using the code given below:

(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

24. If the sixth term in the binomial expansion of $\left(x^{-\frac{8}{3}} + x^2 log_{10} x\right)^8$ is 5600, then what is the value of x ?

(a) 6 **(b)** 8
(c) 9 **(d)** 10

25. How many terms are there in the expansion of $(3x - y)^4(x + 3y)^4$?

(a) 9 **(b)** 12
(c) 15 **(d)** 17

26. p, q, r and s are in AP such that $p + s = 8$ and $qr = 15$. What is the difference between largest and smallest numbers ?

(a) 6 **(b)** 5
(c) 4 **(d)** 3

27. Consider the following statements for a fixed natural number n :

1. $C(n, r)$ is greatest if $n = 2r$
2. $C(n, r)$ is greatest if $n = 2r - 1$ and $n = 2r + 1$

Which of the statements given above is/are correct ?

(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

28. m parallel lines cut n parallel lines giving rise to 60 parallelograms. What is the value of $(m + n)$?

(a) 6 **(b)** 7
(c) 8 **(d)** 9

29. Let x be the number of permutations of the word 'PERMUTATIONS' and y be the number of permutations of the word 'COMBINATIONS'. Which one of the following is correct ?

(a) $x = y$ **(b)** $y = 2x$
(c) $x = 4y$ **(d)** $y = 4x$

30. 5-digit numbers are formed using the digits 0, 1, 2, 4, 5 without repetition. What is the percentage of numbers which are greater than 50,000 ?

(a) 20% **(b)** 25%
(c) $\frac{100}{3}\%$ **(d)** $\frac{110}{3}\%$

*Consider the following for the next **two (02)** items that follow :*

Let $\sin\beta$ be the GM of $\sin\alpha$ and $\cos\alpha$; $\tan\gamma$ be the AM of $\sin\alpha$ and $\cos\alpha$.

31. What is $\cos 2\beta$ equal to ?

(a) $(\cos\alpha - \sin\alpha)^2$ **(b)** $(\cos\alpha + \sin\alpha)^2$
(c) $(\cos\alpha - \sin\alpha)^3$ **(d)** $\frac{(\cos\alpha - \sin\alpha)^2}{2}$

32. What is the value of $\sec 2\gamma$?

(a) $\frac{3 - \sin 2\alpha}{5 + 2\sin 2\alpha}$ **(b)** $\frac{5 + \sin 2\alpha}{3 - \sin 2\alpha}$
(c) $\frac{3 - 2\sin 2\alpha}{4 + \sin 2\alpha}$ **(d)** $\frac{3 - \sin 2\alpha}{4 + 3\sin 2\alpha}$

*Consider the following for the next **two (02)** items that follow :*

A flagstaff 20 m long standing on a pillar 10 m high subtends an angle $\tan^{-1}(0.5)$ at a point P on the ground. Let θ be the angle subtended by the pillar at this point P

33. If x is the distance of P from bottom of the pillar, then consider the following statements:

1. x can take two values which are in the ratio 1 : 3
2. x can be equal to height of the flagstaff

Which of the statements given above is/are correct ?

(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

34. What is a possible value of $\tan\theta$?

(a) $\frac{3}{4}$ **(b)** $\frac{2}{3}$
(c) $\frac{1}{3}$ **(d)** $\frac{1}{4}$

*Consider the following for the next **two (02)** items that follow :*

The perimeter of a triangle ABC is 6 times the AM of sine of angles of the triangle. Further $BC = \sqrt{3}$ and $CA = 1$.

35. What is the perimeter of the triangle ?

(a) $\sqrt{3} + 1$ **(b)** $\sqrt{3} + 2$
(c) $\sqrt{3} + 3$ **(d)** $2\sqrt{3} + 1$

36. Consider the following statements :

1. ABC is right angled triangle
2. The angles of the triangle are in AP

Which of the statements given above is/are correct ?

(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

*Consider the following for the next **two (02)** items that follow :*

Let $x = \dfrac{\sin^2 A + \sin A + 1}{\sin A}$ where $0 < A \le \dfrac{\pi}{2}$

37. What is the minimum value of x ?
(a) 1 **(b)** 2
(c) 3 **(d)** 4

38. At what value of A does x attain the minimum value ?
(a) $\dfrac{\pi}{6}$ **(b)** $\dfrac{\pi}{4}$
(c) $\dfrac{\pi}{3}$ **(d)** $\dfrac{\pi}{2}$

*Consider the following for the next **two (02)** items that follow :*

In the triangle ABC, $a^2 + b^2 + c^2 = ac + \sqrt{3}\, bc$

39. What is the nature of the triangle ?
(a) Equilateral
(b) Isosceles
(c) Right angled triangle
(d) Scalene but not right angled

40. If $c = 8$, what is the area of the triangle ?
(a) $4\sqrt{3}$ **(b)** $6\sqrt{3}$
(c) $8\sqrt{3}$ **(d)** $12\sqrt{3}$

*Consider the following for the next **two (02)** items that follow :*

Consider the function
$f(x) = |x - 2| + |3 - x| + |4 - x|$, where $x \in R$.

41. At what value of x does the function attain minimum value ?
(a) 2 **(b)** 3
(c) 4 **(d)** 0

42. What is the minimum value of the function ?
(a) 2 **(b)** 3
(c) 4 **(d)** 0

*Consider the following for the next **two (02)** items that follow :*

Consider the sum
$S = 0! + 1! + 2! + 3! + 4! + \ldots + 100!$

43. If the sum S is divided by 8, what is the remainder ?
(a) 0
(b) 1
(c) 2
(d) Cannot be determined

44. If the sum S is divided by 60, what is the remainder ?
(a) 1 **(b)** 3
(c) 17 **(d)** 34

*Consider the following for the next **two (02)** items that follow :*

In a triangle PQR, P is the largest angle and $\cos P = \dfrac{1}{3}$. Further the in-circle of the triangle touches the sides PQ, QR and RP at N, L and M respectively such that the lengths PN, QL and RM are n, $n + 2$, $n + 4$ respectively where n is an integer.

45. What is the value of n ?
(a) 4 **(b)** 6
(c) 8 **(d)** 10

46. What is the length of the smallest side ?
(a) 12 **(b)** 14
(c) 16 **(d)** 18

*Consider the following for the next **two (02)** items that follow :*

Given that
$\sin x + \cos x + \tan x + \cot x + \sec x + \operatorname{cosec} x = 7$

47. The given equation can be reduced to
(a) $\sin^2 2x - 44 \sin 2x + 36 = 0$
(b) $\sin^2 2x + 44 \sin 2x - 36 = 0$
(c) $\sin^2 2x - 22 \sin 2x + 18 = 0$
(d) $\sin^2 2x + 22 \sin 2x - 18 = 0$

48. If $\sin 2x = a - b\sqrt{c}$, where a and b are natural numbers and c is prime number, then what is the value of $a - b + 2c$?
(a) 0 **(b)** 14
(c) 21 **(d)** 28

*Consider the following for the next **two (02)** items that follow :*

A quadratic equation is given by

$(3 + 2\sqrt{2})x^2 - (4 + 2\sqrt{3})x + (8 + 4\sqrt{3}) = 0$

49. What is the HM of the roots of the equation ?
(a) 2 **(b)** 4
(c) $2\sqrt{2}$ **(d)** $2\sqrt{3}$

50. What is the GM of the roots of the equation ?
(a) $\sqrt{2}\left(\sqrt{6} - \sqrt{3} + \sqrt{2} - 1\right)$
(b) $\sqrt{2}\left(\sqrt{6} + \sqrt{3} - \sqrt{2} - 1\right)$
(c) $\left(\sqrt{6} - \sqrt{3} + \sqrt{2} - 1\right)$
(d) $\left(\sqrt{6} + \sqrt{3} + \sqrt{2} - 1\right)$

*Consider the following for the next **two (02)** items that follow :*

Let $\Delta(a, b, c, \alpha) = \begin{vmatrix} a & b & a\alpha + b \\ b & c & b\alpha + c \\ a\alpha + b & b\alpha + c & 0 \end{vmatrix}$

51. If $\Delta(a, b, c, \alpha) = 0$ for every $\alpha > 0$, then which one of the following is correct ?
(a) a, b, c are in AP **(b)** a, b, c are in GP
(c) $a, 2b, c$ are in AP **(d)** $a, 2b, c$ are in GP

52. If $\Delta(7, 4, 2, \alpha) = 0$, then α is a root of which one of the following equations ?
(a) $7x^2 + 4x + 2 = 0$ **(b)** $7x^2 - 4x + 2 = 0$
(c) $7x^2 + 8x + 2 = 0$ **(d)** $7x^2 - 8x + 2 = 0$

*Consider the following for the next **two (02)** items that follow :*

Given that $m(\theta) = \cot^2\theta + n^2\tan^2\theta + 2n$, where n is a fixed positive real number.

53. What is the least value of $m(\theta)$?
(a) n **(b)** $2n$
(c) $3n$ **(d)** $4n$

54. Under what condition does m attain the least value ?
(a) $n = \tan^2\theta$ **(b)** $n = \cot^2\theta$
(c) $n = \sin^2\theta$ **(d)** $n = \cos^2\theta$

*Consider the following for the next **two (02)** items that follow :*

A quadrilateral is formed by the lines $x = 0$, $y = 0$, $x + y = 1$ and $6x + y = 3$.

55. What is the equation of diagonal through origin ?
(a) $3x + y = 0$ **(b)** $2x + 3y = 0$
(c) $3x - 2y = 0$ **(d)** $3x + 2y = 0$

56. What is the equation of other diagonal ?
(a) $x + 2y - 1 = 0$ **(b)** $x - 2y - 1 = 0$
(c) $2x + y + 1 = 0$ **(d)** $2x + y - 1 = 0$

*Consider the following for the next **two (02)** items that follow :*

P(x, y) is any point on the ellipse $x^2 + 4y^2 = 1$. Let *E, F be the foci of the ellipse.*

57. What is PE + PF equal to ?
(a) 1 **(b)** 2
(c) 3 **(d)** 4

58. Consider the following points :
1. $\left(\frac{\sqrt{3}}{2}, 0\right)$
2. $\left(\frac{\sqrt{3}}{2}, \frac{1}{4}\right)$
3. $\left(\frac{\sqrt{3}}{2}, -\frac{1}{4}\right)$

Which of the above points lie on latus rectum of ellipse ?
(a) 1 and 2 only **(b)** 2 and 3 only
(c) 1 and 3 only **(d)** 1, 2 and 3

*Consider the following for the next **two (02)** items that follow :*

The line $y = x$ partitions the circle $(x - a)^2 + y^2 = a^2$ in two segments.

59. What is the area of minor segment ?
(a) $\frac{(\pi - 2)a^2}{4}$ **(b)** $\frac{(\pi - 1)a^2}{4}$
(c) $\frac{(\pi - 2)a^2}{2}$ **(d)** $\frac{(\pi - 1)a^2}{2}$

60. What is the area of major segment ?
(a) $\frac{(3\pi - 2)a^2}{4}$ **(b)** $\frac{(3\pi + 2)a^2}{4}$
(c) $\frac{(3\pi - 2)a^2}{2}$ **(d)** $\frac{(3\pi + 2)a^2}{2}$

*Consider the following for the next **two (02)** items that follow :*

Let A(1, –1, 2) and B(2, 1, –1) be the end points of the diameter of the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz - 1 = 0$.

61. What is $u + v + w$ equal to ?
(a) – 2 **(b)** – 1
(c) 1 **(d)** 2

62. If P(x, y, z) is any point on the sphere, then what is $PA^2 + PB^2$ equal to ?
(a) 15 **(b)** 14
(c) 13 **(d)** 6·5

*Consider the following for the next **two (02)** items that follow :*

Consider two lines whose direction ratios are (2, – 1, 2) and (k, 3, 5). They are inclined at an angle $\frac{\pi}{4}$.

63. What is the value of k ?
(a) 4 **(b)** 2
(c) 1 **(d)** – 1

64. What are the direction ratios of a line which is perpendicular to both the lines ?
(a) (1, 2, 10) **(b)** (– 1, – 2, 10)
(c) (11, 12, – 10) **(d)** (11, 2, – 10)

*Consider the following for the next **two (02)** items that follow :*

Let $\vec{a} = 3\hat{i} + 3\hat{j} + 3\hat{k}$ and $\vec{c} = \hat{j} - \hat{k}$. Let $\vec{b}$ be such that $\vec{a} \cdot \vec{b} = 27$ and $\vec{a} \times \vec{b} = 9\vec{c}$

65. What is $\vec{b}$ equal to ?

(a) $3\hat{i} + 4\hat{j} + 2\hat{k}$ **(b)** $5\hat{i} + 2\hat{j} + 2\hat{k}$
(c) $5\hat{i} - 2\hat{j} + 6\hat{k}$ **(d)** $3\hat{i} + 3\hat{j} + 4\hat{k}$

66. What is the angle between $(\vec{a} + \vec{b})$ and $\vec{c}$

(a) $\frac{\pi}{2}$ **(b)** $\frac{\pi}{3}$
(c) $\frac{\pi}{4}$ **(d)** $\frac{\pi}{6}$

*Consider the following for the next **two (02)** items that follow :*

Let a vector $\vec{a} = 4\hat{i} - 8\hat{j} + \hat{k}$ make angles α, β, γ with the positive directions of x, y, z axes respectively.

67. What is $\cos\alpha$ equal to ?

(a) $\frac{1}{3}$ **(b)** $\frac{4}{9}$
(c) $\frac{5}{9}$ **(d)** $\frac{2}{3}$

68. What is $\cos2\beta + \cos2\gamma$ equal to ?

(a) $-\frac{32}{81}$ **(b)** $-\frac{16}{81}$
(c) $\frac{16}{81}$ **(d)** $\frac{32}{81}$

*Consider the following for the next **two (02)** items that follow :*

The position vectors of two points A and B are $\hat{\imath} - \hat{\jmath}$ and $\hat{\jmath} + \hat{k}$ respectively.

69. Consider the following points :
1. $(-1, -3, 1)$
2. $(-1, 3, 2)$
3. $(-2, 5, 3)$

Which of the above points lie on the line joining A and B ?
(a) 1 and 2 only **(b)** 2 and 3 only
(c) 1 and 3 only **(d)** 1, 2 and 3

70. What is the magnitude of $\overrightarrow{AB}$?
(a) 2 **(b)** 3
(c) $\sqrt{6}$ **(d)** $\sqrt{3}$

*Consider the following for the next **three (03)** items that follow :*

Let $f(x) = Pe^x + Qe^{2x} + Re^{3x}$, where P, Q, R are real numbers. Further $f(0) = 6$, $f'(ln\,3) = 282$ and $\int_0^{ln2} f(x)dx = 11$

71. What is the value of Q ?
(a) 1 **(b)** 2
(c) 3 **(d)** 4

72. What is the value of R ?
(a) 1 **(b)** 2
(c) 3 **(d)** 4

73. What is $f'(0)$ equal to ?
(a) 18 **(b)** 16
(c) 15 **(d)** 14

*Consider the following for the next **two (02)** items that follow :*

Suppose E is the differential equation representing family of curves $y^2 = 2cx + 2c\sqrt{c}$ where c is a positive parameter.

74. What is the order of the differential equation ?
(a) 1 **(b)** 2
(c) 3 **(d)** 4

75. What is the degree of the differential equation ?
(a) 2
(b) 3
(c) 4
(d) Degree does not exist

*Consider the following for the next **three (03)** items that follow :*

Let $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$

76. What is $f(0)$ equal to ?
(a) -1 **(b)** 0
(c) 1 **(d)** 2

77. What is $\lim_{x\to0} \frac{f(x)}{x}$ equal to ?

(a) -1 **(b)** 0
(c) 1 **(d)** **2**

78. What is $\lim_{x\to0} \frac{f(x)}{x^2}$ equal to ?

(a) -1 **(b)** 0
(c) 1 **(d)** **2**

*Consider the following for the next **two (02)** items that follow :*

Let $f(x) = \sin[\pi^2]x + \cos[-\pi^2]x$ where [.] is a greatest integer function

79. What is $f\left(\frac{\pi}{2}\right)$ equal to ?

(a) -1 **(b)** 0
(c) 1 **(d)** **2**

80. What is $f\left(\frac{\pi}{4}\right)$ equal to ?

(a) $-\frac{1}{\sqrt{2}}$ **(b)** -1

(c) 1 **(d)** $\frac{1}{\sqrt{2}}$

*Consider the following for the next **three (03)** items that follow :*

Let $I_1 = \int_0^{\pi} \frac{x}{1+\cos^2 x} dx$ and $I_2 = \int_0^{\pi} \frac{x}{1+\sin^2 x} dx$

81. What is the value of $\frac{I_1 + I_2}{I_1 - I_2}$?

(a) 1 **(b)** π
(c) $\pi^2/2$ **(d)** $\frac{\pi+1}{\pi-1}$

82. What is the value of $8I_1^2$?

(a) π **(b)** π^2
(c) π^3 **(d)** π^4

83. What is the value of I_2 ?

(a) $\frac{\pi}{\sqrt{2}}$ **(b)** $\frac{\pi^2}{2\sqrt{2}}$
(c) $\frac{3\pi}{2\sqrt{2}}$ **(d)** $\frac{\pi}{4\sqrt{2}}$

*Consider the following for the next **two (02)** items that follow :*

Let $l = \int_a^b \frac{|x|}{x} dx,\ a < b$

84. What is l equal to when $a < 0 < b$?
(a) $a + b$ **(b)** $a - b$

(c) $b - a$ **(d)** $\frac{(a+b)}{2}$

85. What is l equal to when $a < b < 0$?
(a) $a + b$ **(b)** $a - b$

(c) $b - a$ **(d)** $\frac{(a+b)}{2}$

*Consider the following for the next **three (03)** items that follow :*

Let $f(x) = |lnx|,\ x \neq 1$

86. What is the derivative of $f(x)$ at $x = 0{\cdot}5$?
(a) -2 **(b)** -1
(c) 1 **(d)** 2

87. What is the derivative of $f(x)$ at $x = 2$?

(a) $-\frac{1}{2}$ **(b)** -1

(c) $\frac{1}{2}$ **(d)** 2

88. What is the derivative of $f \circ f(x)$, where $1 < x < 2$?

(a) $\frac{1}{lnx}$ **(b)** $\frac{1}{xlnx}$

(c) $-\frac{1}{lnx}$ **(d)** $-\frac{1}{xlnx}$

*Consider the following for the next **two (02)** items that follow :*

Let $f(x) = \begin{cases} x+6, & x \leq 1 \\ px+q, & 1 < x < 2 \\ 5x, & x \geq 2 \end{cases}$

and $f(x)$ is continuous

89. What is the value of p ?
(a) 2 **(b)** 3
(c) 4 **(d)** 5

90. What is the value of q ?
(a) 2 **(b)** 3
(c) 4 **(d)** 5

91. Consider the following statements :
1. $f(x) = lnx$ is increasing in $(0, \infty)$
2. $g(x) = e^x + e^x$ is decreasing in $(0, \infty)$

Which of the statements given above is/are correct ?
(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

92. What is the derivative of $\sin^2 x$ with respect to $\cos^2 x$?
(a) -1 **(b)** 1
(c) $\sin 2x$ **(d)** $\cos 2x$

93. For what value of m with $m < 0$, is the area bounded by the lines $y = x$, $y = mx$ and $x = 2$ equal to 3 ?

(a) $-\frac{1}{2}$ **(b)** -1

(c) $-\frac{3}{2}$ **(d)** -2

94. What is the derivative of $\text{cosec}(x°)$?

(a) $-\text{cosec}(x°)\cot(x°)$

(b) $-\frac{\pi}{180}\text{cosec}(x°)\cot(x°)$

(c) $\frac{\pi}{180}\text{cosec}(x°)\cot(x°)$

(d) $-\frac{\pi}{180}\text{cosec}(x)\cot(x)$

95. A solution of the differential equation $\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} = 0$ is

(a) $y = x^2/2 + c$ **(b)** $y = 2x + 4$

(c) $y = x^2 + 1$ **(d)** $y = \frac{(x^2 - x)}{2}$

96. If $f(x) = x^2 + 2$ and $g(x) = 2x - 3$, then what is $(fg)(1)$ equal to ?

(a) 3 **(b)** 1

(c) –2 **(d)** –3

97. What is the range of the function $f(x) = x + |x|$ if the domain is the set of real numbers ?

(a) $(0, \infty)$ **(b)** $[0, \infty)$

(c) $(-\infty, \infty)$ **(d)** $[1, \infty)$

98. If $f(x) = x(4x^2 - 3)$, then what is $f(\sin\theta)$ equal to ?

(a) $-\sin3\theta$ **(b)** $-\cos3\theta$

(c) $\sin3\theta$ **(d)** $-\sin4\theta$

99. What is $\lim_{x\to5}\frac{5-x}{|x-5|}$ equal to ?

(a) – 1 **(b)** 0

(c) 1 **(d)** Limit does not exist

100. What is $\lim_{x\to1}\frac{x^9-1}{x^3-1}$ equal to ?

(a) – 1 **(b)** – 3

(c) 3 **(d)** Limit does not exist

101. The mean and variance of five obser-vations are 14 and 13.2 respectively. Three of the five observations are 11, 16 and 20. What are the other two observations ?

(a) 8 and 15 **(b)** 9 and 14

(c) 10 and 13 **(d)** 11 and 12

102. Let A and B be two independent events such that P(A') = 0.7, P(B') = k, P(A ∪ B) = 0.8. What is the value of k ?

(a) $\frac{5}{7}$ **(b)** $\frac{4}{7}$

(c) $\frac{2}{7}$ **(d)** $\frac{1}{7}$

103. A biased coin with the probability of getting head equal to $\frac{1}{4}$ is tossed five times. What is the probability of getting tail in all the first four tosses followed by head ?

(a) $\frac{81}{512}$ **(b)** $\frac{81}{1024}$

(c) $\frac{81}{256}$ **(d)** $\frac{27}{1024}$

104. A coin is biased so that heads comes up thrice as likely as tails. In four independent tosses of the coin, what is probability of getting exactly three heads ?

(a) $\frac{81}{256}$ **(b)** $\frac{27}{64}$

(c) $\frac{27}{256}$ **(d)** $\frac{9}{256}$

105. Let X and Y be two random variables such that X + Y = 100. If X follows Binomial distribution with parameters $n = 100$ and $p = \frac{4}{5}$, what is the variance of Y?

(a) 1 **(b)** $\frac{1}{2}$

(c) **16** **(d)** $\frac{1}{16}$

106. If two lines of regression are $x + 4y + 1 = 0$ and $4x + 9y + 7 = 0$, then what is the value of x when $y = -3$?

(a) –13 **(b)** –5

(c) 5 **(d)** 7

107. The central angles p, q, r and s (in degrees) of four sectors in a Pie Chart satisfy the relation $9p = 3q = 2r = 6s$. What is the value of $4p - q$?

(a) 12 **(b)** 24

(c) 30 **(d)** 36

108. The observations 4, 1, 4, 3, 6, 2, 1, 3, 4, 5, 1, 6 are outputs of 12 dices thrown simultaneously. If m and M are means of lowest 8 observations and highest 4 observations respectively, then what is $(2m + M)$ equal to ?

(a) 10 **(b)** 12

(c) 17 **(d)** 21

109. A bivariate data set contains only two points (–1, 1) and (3, 2). What will be the line of regression of y on x ?

(a) $x - 4y + 5 = 0$ **(b)** $3x + 2y - 1 = 0$

(c) $x + 4y + 1 = 0$ **(d)** $5x - 4y + 1 = 0$

110. A die is thrown 10 times and obtained the following outputs:
1, 2, 1, 1, 2, 1, 4, 6, 5, 4
What will be the mode of data so obtained ?
(a) 6 **(b)** 4
(c) 2 **(d)** 1

111. Consider the following frequency distribution:

x	1	2	3	5
f	4	6	9	7

What is the value of median of the distribution?
(a) 1 **(b)** 2
(c) 3 **(d)** 3-5

112. For data –1, 1, 4, 3, 8, 12, 17, 19, 9, 11; if M is the median of first 5 observations and N is the median of last five observations, then what is the value of 4M – N ?
(a) 7 **(b)** 4
(c) 1 **(d)** 0

113. Let P, Q, R represent mean, median and mode. If for some distribution $5P = 4Q = \frac{R}{2}$, then what is $\frac{P+Q}{2P+0.7R}$ equal to ?
(a) $\frac{1}{12}$ **(b)** $\frac{1}{7}$
(c) $\frac{2}{9}$ **(d)** $\frac{1}{4}$

114. If G is the geometric mean of numbers 1, 2, 2^2, 2^3,. . ., 2^{n-1}, then what is the value of $1 + 2\log_2 G$?
(a) 1 **(b)** 4
(c) n - 1 **(d)** n

115. If H is the harmonic mean of numbers 1, 2, 2^2, 2^3, . . . , 2^{n-1}, then what is n/H equal to ?
(a) $2-\frac{1}{2^{n+1}}$ **(b)** $2-\frac{1}{2^{n-1}}$
(c) $2+\frac{1}{2^{n-1}}$ **(d)** $2-\frac{1}{2^n}$

116. Let P be the median, Q be the mean and R be the mode of observations x_1, x_2, x_3, ... x_n. Let $S=\sum_{i=1}^{n}(2x_i-a)^2$ S takes minimum value, when a is equal to
(a) P **(b)** $\frac{Q}{2}$
(c) 2Q **(d)** R

117. One bag contains 3 white and 2 black balls, another bag contains 2 white and 3 black balls. Two balls are drawn from the first bag and put it into the second bag and then a ball is drawn from the second bag. What is the probability that it is white ?
(a) $\frac{6}{7}$ **(b)** $\frac{33}{70}$
(c) $\frac{3}{10}$ **(d)** $\frac{1}{70}$

118. Three dice are thrown. What is the probability that each face shows only multiples of 3 ?
(a) $\frac{1}{9}$ **(b)** $\frac{1}{18}$
(c) $\frac{1}{27}$ **(d)** $\frac{1}{3}$

119. What is the probability that the month of December has 5 Sundays ?
(a) **1** **(b)** $\frac{1}{4}$
(c) $\frac{3}{7}$ **(d)** $\frac{2}{7}$

120. A natural number n is chosen from the first 50 natural numbers. What is the probability that $n+\frac{50}{n}<50$?
(a) $\frac{23}{25}$ **(b)** $\frac{47}{50}$
(c) $\frac{24}{25}$ **(d)** $\frac{49}{50}$

ANSWER KEY

Q No	Answer Key	Topic	Chapter
1	**a**	Cube root of unity	Complex Numbers
2	**d**	Number of ways	Permutations and Combinations
3	**a**	Binary operation	Sets
4	**b**	Adjoint of a matrix	Matrices
5	**d**	Cube root of unity	Complex Numbers
6	**b**	Properties of matrices	Matrices
7	**c**	Properties of determinants	Determinants
8	**b**	System of equations	Determinants
9	**b**	Properties of determinants	Determinants
10	**b**	Expansion of determinant	Determinants
11	**d**	Roots of Equations	Complex Numbers
12	**b**	Argument	Complex Numbers
13	**b**	Expansion of determinant	Determinants
14	**c**	Geometric Progression	Sequence and Series
15	**a**	Geometric Progression	Sequence and Series
16	**b**	Arithmetic Progression	Sequence and Series
17	**a**	Nature of roots	Quadratic Equations
18	**b**	Suum of n terms	Sequence and Series
19	**b**	Factorial	Permutations and Combinations
20	**c**	Modulus	Complex Numbers
21	**a**	Roots of Equations	Equations
22	**d**	Onto Functions	Relations and Functions
23	**a**	Roots of Equations	Quadratic Equations
24	**d**	N^{th} term	Binomial Theorem
25	**c**	Binomial Expansion	Binomial Theorem
26	**a**	Arithmetic Progression	Sequence and Series
27	**c**	Combinations	Permutations and Combinations
28	**d**	Combinations	Permutations and Combinations
29	**c**	Number of permutations	Permutations and Combinations
30	**b**	Number of ways	Permutations and Combinations
31	**a**	Trigonometric Identities	Trigonometry
32	**b**	Trigonometric Identities	Trigonometry
33	**a**	Height and Distance	Trigonometry
34	**c**	Height and Distance	Trigonometry
35	**c**	Triangle	Trigonometry
36	**c**	Triangle	Trigonometry
37	**c**	Arithmetic and Geometric Progression	Trigonometry
38	**d**	Minimum Value	Trigonometry
39	**c**	Triangle property	Trigonometry

Q No	Answer Key	Topic	Chapter
40	**c**	Area of triangle	Trigonometry
41	**b**	Extreme values	Continuity and Differentiability
42	**a**	Extreme values	Continuity and Differentiability
43	**c**	Factorial	Permutations and Combinations
44	**d**	Factorial	Permutations and Combinations
45	**c**	Triangle	Trigonometry
46	**a**	Triangle	Trigonometry
47	**a**	Trignometric Relation	Trigonometry
48	**d**	Trignometric Relation	Trigonometry
49	**b**	Harmonic Mean	Sequence and Series
50	**a**	Geometric Mean	Sequence and Series
51	**b**	Expansion of determinant	Determinants
52	**c**	Properties of determinants	Determinants
53	**d**	Trigonometric expressions	Trigonometry
54	**b**	Trigonometric expressions	Trigonometry
55	**c**	Equation of a line	Straight lines
56	**d**	Equation of a line	Straight lines
57	**b**	Ellipse	Conic Section
58	**d**	Ellipse	Conic Section
59	**a**	Circle	Conic Section
60	**b**	Circle	Conic Section
61	**a**	Sphere	3D Geometry
62	**c**	Sphere	3D Geometry
63	**a**	Direction ratios	Three Diimensional Geomtery
64	**d**	Direction ratios	Three Diimensional Geomtery
65	**b**	Product of two vectors	Vector Algebra
66	**a**	Product of two vectors	Vector Algebra
67	**b**	Direction cosines	3D Geometry
68	**a**	Direction cosines	3D Geometry
69	**b**	Line	3D Geometry
70	**c**	Line	3D Geometry
71	**b**	Definite integral	Calculus
72	**c**	Definite integral	Calculus
73	**d**	Differentiation	Calculus
74	**a**	Order and degree	Differential equations
75	**b**	Order and degree	Differential equations
76	**b**	Evaluation of limits	Limits
77	**b**	Evaluation of limits	Limits
78	**a**	Evaluation of limits	Limits
79	**b**	Trigonometric functions	Trigonometry
80	**d**	Trigonometric Functions	Trigonometry
81	**c**	Definite Integral	Calculus

Q No	Answer Key	Topic	Chapter
82	**d**	Definite Integral	Calculus
83	**b**	Definite Integral	Calculus
84	**a**	Definite Integral	Calculus
85	**c**	Definite Integral	Calculus
86	**a**	Differentiation	Calculus
87	**c**	Differentiation	Calculus
88	**d**	Differentiation	Calculus
89	**b**	Continuity	Calculus
90	**c**	Continuity	Calculus
91	**a**	Increasing-decreasing functions	Calculus
92	**a**	Differentiation	Calculus
93	**a**	General Equation of a line	Straight Lines
94	**b**	Differentiation	Calculus
95	**a**	Variable separable	Differential Equations
96	**d**	Operations on functions	Functions
97	**b**	Range	Functions
98	**a**	Value of a function	Functions
99	**d**	Evaluation of limits	Limits
100	**c**	Evaluation of limits	Limits
101	**c**	Mean and variance	Statistics
102	**c**	Independent events	Probability
103	**b**	Independent events	Probability
104	**b**	Independent events	Probability
105	**c**	Binomial distribution	Probability
106	**c**	Regression	Statistics
107	**d**	Angles	Trigonometry
108	**a**	Mean	Statistics
109	**a**	Regression	Statistics
110	**a**	Mode	Statistics
111	**c**	Median	Statistics
112	**d**	Median	Statistics
113	**d**	Mean, median, mode	Statistics
114	**d**	Geometric mean	Sequence and Series
115	**b**	Harmonic mean	Sequence and Series
116	**c**	Derivative	Continuity and Differentiability
117	**b**	Total Probability	Probability
118	**c**	Probability	Probability
119	**c**	Probability	Probability
120	**b**	Probability	Probability

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MATHEMATICS

I

SOLVED PAPER

2023

ANSWERS WITH EXPLANATION

1. Option (a) is correct.

Explanations: We have,

$$\left|\frac{1-\omega}{\omega+\omega^2}\right| = \left|\frac{1-\omega}{-1}\right| = |-1+\omega|$$

$$= \left|-1+\left(\frac{-1+1\sqrt{3}}{2}\right)\right|$$

$$= \sqrt{\left(\frac{-3}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{3}$$

2. Option (d) is correct.

Explanations: For number to be divisible by 6, the number should be divisible by 2 and 3 both.

Now, number is divisible by 2 if units place digit is 0, 2, or 4:

Also, sum of all digits = 0 + 1 + 2 +3 + 4 + 5 = 15

Case I: If units digit is 0; then no. of ways = 5! = 120

Case II: If units digit is either 2 or 4, then no. of ways = 2 × 4! × 4 = 192

So, total number of 6 digit number formed

= 120 + 192 = 312

3. Option (Bonus) is correct.

Explanations: To covert 1011 decimal number, we have,

Divisible by 2	Quotient	Remainder	Binary Bit
1011 ÷ 2	505	1	1
505 ÷ 2	252	1	1
252 ÷ 2	126	0	0
126 ÷ 2	63	0	0
63 ÷ 2	31	1	1
31 ÷ 2	15	1	1
15 ÷ 2	7	1	1
7 ÷ 2	3	1	1
3 ÷ 2	1	1	1
1 ÷ 2	0	1	1

1011 = (1111110011)

4. Option (b) is correct.

Explanations: $|2 \text{ adj } (3A)| = 2^3 |\text{adj } (3A)|$ (i)

Now, $|3A| = 3^3 |A| = 3^3.4 = 3^3.2^2$

$|\text{adj } (3A)| = |3A|^{3-1} = |3A|^2 = |3^3.2^2|^2 = 3^6.2^4$

from (i), we have

$|2 \text{ adj } (3A)| = 2^3.2^4.3^6 = 2^7.3^6 = 2^\alpha.3^\beta$

$\Rightarrow \quad \alpha = 7$ and $\beta = 6$

$\therefore \alpha + \beta = 7 + 6 = 13$

5. Option (d) is correct.

Explanations: We have,

$x^2 - x + 1$

$$x = \frac{1 \pm \sqrt{3}i}{2} \Rightarrow x = -\omega \text{ or } -\omega^2$$

So, $\alpha = -\omega$ and $\beta = -\omega^2$

$$\left|\frac{\alpha^{1\omega}+\beta^{1\omega}}{\alpha^{1\omega}-\beta^{1\omega}}\right| = \left|\frac{\omega^{1\omega}+\omega^{2\omega}}{\omega^{1\omega}-\omega^{2\omega}}\right|$$

$$= \left|\frac{1+\omega^{1\omega}}{1-\omega^{1\omega}}\right| = \left|\frac{1+\omega}{1-\omega}\right|$$

$$\left|\frac{\pm^{100}+\beta^{100}}{\pm^{100}-\beta^{100}}\right| = \left|\frac{(-\omega)^{100}+\left(-\omega^2\right)^{100}}{(-\omega)^{100}-\left(-\omega^2\right)^{100}}\right|$$

$$= \left|\frac{\omega^{100}+\left(1+\omega^{100}\right)}{\omega^{100}+\left(1-\omega^{100}\right)}\right| = \left|\frac{1+\omega^{100}}{1-\omega^{100}}\right| = \left|\frac{1+\omega^{3\times33}\omega}{1-\omega^{3\times33}\omega}\right|$$

$$= \left|\frac{1+\omega}{1-\omega}\right| = \left|\frac{1+\left(\frac{-1-\sqrt{3}i}{2}\right)}{1+\left(\frac{-1+\sqrt{3}i}{2}\right)}\right|$$

$$= \left|\frac{1+\sqrt{3}i}{3+\sqrt{3}i}\right| = \frac{\sqrt{1+3}}{\sqrt{9+3}} = \frac{1}{\sqrt{3}}$$

6. Option (b) is correct.

Explanations: When A and B be symmetric matrices then (AB – BA) is skew symmetric.

7. Option (d) is correct.

Explanations:

$$\because \begin{bmatrix}3 & 5\\ 7 & 3\end{bmatrix}\begin{bmatrix}K\\ 2K\end{bmatrix}=\begin{bmatrix}7 & 3\\ 3 & 5\end{bmatrix}\begin{bmatrix}K\\ 2K\end{bmatrix}$$

$$\begin{bmatrix}13K\\ 13K\end{bmatrix}=\begin{bmatrix}13K\\ 13K\end{bmatrix}$$

$$\begin{bmatrix}3 & 5\\ 7 & 3\end{bmatrix}\neq\begin{bmatrix}7 & 3\\ 3 & 5\end{bmatrix}$$

So, both statements are wrong.

8. Option (b) is correct.

Explanations: We have,

$x + 2y + z = 4$

$2x + 4y + 2z = 8$

$\Rightarrow 2(x + 2y + z) = 8$

$\Rightarrow x + 2y + z = 4$

and $3x + 6y + 3z = 10$

$\Rightarrow 3(x + 2y + z) = 10$

$$\Rightarrow x + 2y + z = \frac{10}{3}$$

So, the linear equations have infinity many solutions.

9. Option (b) is correct.

Explanations: We know that if X_1 and X_2 are solution of system of equations $AX = B$, $B = 0$ then $aX_1 + bX_2$ is also solution iff $a + b = 1$

10. Option (b) is correct.

Explanations: $\begin{vmatrix}0 & x-a & x-b\\ 0 & 0 & x-c\\ x+b & x+c & 1\end{vmatrix}=0$

$\Rightarrow 0 - (x - a)(0 - (x - c)(x + b)) + (x - b)(0 - 0) = 0$

$\Rightarrow (x - a)\,(x + b)\,(x - c) = 0$

$\Rightarrow x = a, x = -b$ or $x = c$

Sum of roots $= a - b + c$

11. Option (d) is correct.

Explanations: $2 - i\sqrt{3}$ is a root of $x^2 + ax + b$.

So, $2 + i\sqrt{3}$ is also the root of $x^2 + ax + b$.

Sum of roots = 4

$-a = 4 \Rightarrow a = -4$

Product of roots = 4 + 3 = 7

$\Rightarrow b = 7$

So, $a + b = -4 + 7 = 3$

12. Option (b) is correct.

Explanations: We have,

$$z=\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\times\frac{1+i\sqrt{3}}{1+i\sqrt{3}}=\frac{1-3+2\sqrt{3}\,i}{1+3}$$

$$=\frac{-2+2\sqrt{3i}}{4}=\frac{-1}{2}+\frac{\sqrt{3}}{2}i$$

$$\text{Now, } \tan\theta=\left(\frac{\frac{\sqrt{3}}{2}}{\frac{-1}{2}}\right)=-\sqrt{3}$$

$$\Rightarrow \theta=\tan^{-1}\left(-\sqrt{3}\right)=\pi-\frac{\pi}{3}=\frac{2\pi}{3}$$

13. Option (b) is correct.

Explanations: We have,

$2b = a + c$ (i) (a, b, c in AP)

$$\text{Let } \Delta=\begin{vmatrix}x+1 & x+2 & x+3\\ x+2 & x+3 & x+4\\ x+a & x+b & x+c\end{vmatrix}$$

$$=\begin{vmatrix}x+1 & 1 & 2\\ x+2 & 1 & 2\\ x+a & b-a & c-a\end{vmatrix}=\begin{vmatrix}x+1 & 1 & 2\\ 1 & 0 & 0\\ x+a & b-a & c-a\end{vmatrix}$$

$(x + 1)(0 - 0) - 1(c - a - 0) + 2(b - a - 0)$

$= a - c + 2b - 2a$

$= -a - c + a + c$ [Using (i)]

$= 0$

14. Option (c) is correct.

Explanations: Since, $\log_x a, a^x, \log_b x$ are in G.P

$$\therefore \left(a^x\right)^2=\left(\log_x a\right)\left(\log_b x\right)$$

$$\Rightarrow a^{2x}=\frac{\log a}{\log x}\frac{\log x}{\log b}=\log_b a$$

Taking log both sides, we get

$2x \log_a = \log (\log_b{}^a)$

$$x=\frac{1}{2}\log_a(\log_b{}^a)$$

15. Option (a) is correct.

Explanations: $2^{1/c}, 2^{b/ac}, 2^{1/a}$ are in G.P

$2^{2b/ac} = 2^{1/c}.2^{1/a} = 2^{2b/ac} = 2^{1/c+1/a}$

$$=\frac{2b}{ac}=\frac{1}{c}+\frac{1}{a}=2b=a+c$$

Hence, a, b, c are in A.P

16. Option (b) is correct.

Explanations: We have,

$$a_n=0=\frac{5}{2}+(n-1)\left(\frac{-7}{12}\right)$$

$$\Rightarrow n-1=\frac{30}{7}\Rightarrow n=\frac{37}{7}$$

So, largest negative term will be for integer $n = 6$

17. Option (a) is correct.

Explanations: We have,

$f(x) = x^2 - 4x + x$ has real roots

$D > 0 = (4)^2 - 4k, 1 > 0 = 16 - 4k > 0$

$k < 4$ (i)

Now, roots of above equation are lying in the internal (0, 5).

$f(0) > 0 = k > 0$ (ii)

and $f(5) > 0 = 25 - 20 + k > 0 = k > -5$ (iii)

from (i), (ii), and (iii) we have,

$k = (0, 4)$

Possible integral values of l are 1, 2 and 3 i.e. 3 is number.

18. Option (b) is correct.

Explanations: We have

$a = x, S_n = 0$

$$\Rightarrow \frac{n}{2}\left[2a+(n-1)d\right]=0 \Rightarrow 2x+(n-1)d=0$$

$$\Rightarrow d=\left(\frac{-2x}{n-1}\right)$$

$$=\frac{m+n}{2}\left[2x+(m+n-1)d\right]-0$$

$$=\frac{m+n}{2}\left[2x+md-2x\right]$$

$$=\left(\frac{m+n}{2}\right)m\left(\frac{-2x}{n-1}\right)$$

$$=\frac{mx(m+n)}{1-n}$$

19. Option (b) is correct.

Explanations:

(1) as 5! = 120

and 5! + 1 = 121 has 1 at unit place.

so, 25! + 1 also has 1 at units place.

25! +1 is not divisible by 26.

(2) 6! = 720

6! + 1 = 721, which is divisible by 7.

So, only (2) is true.

20. Option (c) is correct.

Explanations: Let $x = x + iy$

then $\frac{z-1}{z+1}=\frac{x+iy-1}{x+iy+1}$

$$=\frac{(x-1)+iy}{(x+1)+iy}\times\frac{(x+1)-iy}{(x+1)-iy}$$

$$=\frac{x^2+x+ixy-x-1+iy+ixy+iy-i^2y^2}{(x+1)^2-i^2y^2}$$

$$=\frac{x^2+y^2-1+2iy}{x^2+1+2x+y^2} \qquad (\because i^2=-1)$$

If $\frac{z-1}{z+1}$ is purely imaginary number, then

$$\mathrm{Re}\left(\frac{z-1}{z+1}\right)=0$$

$\Rightarrow x^2 + y^2 = 0$

$\Rightarrow x^2 + y^2 = 1 \Rightarrow |z|^2 = 1$ or $|z| = 1$

Thus the value of $|z| = 1$

21. Option (b) is correct.

Explanations: We have, $|x - 4| + |x - 7| = 15$

There are two cases arise.

Case I: When $x < 4$

$-x + 4 - x + 1 = 15 \Rightarrow n = -2$

Case II: When $x \geq 7$

So, only 2 option possible.

22. Option (d) is correct.

Explanations: $f(x)$ is onto

$3x + 5 = 0 \Rightarrow x = -\frac{5}{3}$

So, A = $\{x = R-(-5/3)\}$

Let, $y = 2x + 3/3x + 5 \Rightarrow 3xy + 5y = 2x + 3$

$$= x = 3 - \frac{5y}{3y} - 2$$

$3y - 2 = 0 = y = \frac{2}{3}$

B = $\{y = R - (2 - 3)\}$

23. Option (a) is correct.

Explanations: We have,

$\alpha + \beta = 0$ (i)

$\alpha^2 + \beta^2 = 2$

$(\alpha + \beta)^2 - 2\alpha\beta = 2$

$2\alpha\beta = -2$ [Using (i)]

$\alpha\beta = -1$

Now, $(\alpha - \beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta = 2 - 2(-1) = 4$

$\alpha - \beta = +2$ (ii)

Solving (i) and (ii), we get

$\alpha = 1$ and $\alpha + \beta = +1$

So, only (1) is sufficient to find x.

24. Option (d) is correct.

Explanations: We have,

$T_{5+1} = 5600$

$8_{C5}(x^{-8/3})^{8-5} = (x^2\log_{10}x)^5 = 5600$

$56.x^{-8}.x^{10}(\log_{10}x)^5 = 5600$

$x^2(\log_{10}x)^5 = (10)^2.(\log_{10}10)^5$

So, $x = 10$

25. Option (c) is correct.

Explanations: We have,

$(3x-y)^4(x+3y)^4 = [(3x-y)(x+3y)]^4$

$= (3x^2 + 9xy - xy - 3y^2)^4$

$= (3x^2 + 8xy - 3y^2)^4$

Here, $r = 3$ and $n = 4$

Required number of terms $= {}^{n+r-1}C_{r-1}$

$= {}^{4+3-1}C_{3-1}$

$= {}^6C_2 = 15$

26. Option (a) is correct.

Explanations: Let $P = 1 - 3d$, $q = a - d$, $r = a + d$

Then,

$P + S = 8$

$a - 3d + a + 3d = 8 \Rightarrow a = 4$

Also,

$qr = 15$

$= a^2 - d^2 = 15$

$= d^2 = 16 - 15$

$d = +1$

If $d = +1$ and $a = 4$, then

Largest number = 7 and smallest number = 1

Required difference = 7 – 1 = 6

27. Option (c) is correct.

Explanations:

Both statements are true.

28. Option (d) is correct.

Explanations:

Selection of 2 parallel lines from m lines $= {}^mC_2$

Selection of 2 parallel lines from n lines $= {}^nC_2$

No. of parallelograms formed $= {}^mC_2.{}^nC_2$

$= 60 = {}^mC_2.{}^nC_2$

$= {}^5C_2 \times {}^4C_2 = {}^mC_2.{}^nC_2$

$\therefore\ m = 5$ and $n = 4$

So, $m + n = 5 + 4 = 9$

29. Option (c) is correct.

Explanations: No. of permutations of the word PERMUTATIONS = 12!/2! (T accurs twice)

No. of permutations of the word COMBINATIONS =12!/2! 2! 2!

(As 0, I, M occurs twice)

y = 12!/2!. 1/4

$= 4y = x$

30. Option (b) is correct.

Explanations: Total 5 digit numbers that can be formed using 0, 1, 2, 4 and 5 without repetition $= 4 \times 4! = 96$

No. of 5 digit numbers greater that 50000 $= 1 \times 4! = 24$

(Ten thousand should be filled by 5 only)

Required percentage = 24/96 × 100 = 25%

(31-32.)

We have

$\sin^2\beta = \sin\alpha\cos\alpha$ (i)

and $2\tan\gamma = \sin\alpha + \cos\alpha$ (ii)

31. Option (a) is correct.

Explanations:

Now, $\cos2\beta = 1 - 2\sin^2\beta$

$= 1 - 2\sin\alpha\cos\alpha$

$= (\sin\alpha - \cos\alpha)^2$

32. Option (b) is correct.

Explanations:

$$\cos 2\gamma = \frac{1-\tan^2\gamma}{1+\tan^2\gamma}$$

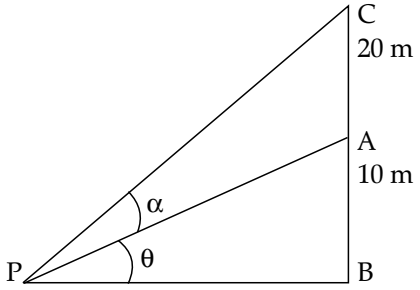
$$\Rightarrow \sec 2\gamma = \frac{1+\tan^2\gamma}{1-\tan^2\gamma} = \frac{1+\left(\frac{\sin\alpha+\cos\alpha}{2}\right)^2}{1-\left(\frac{\sin\alpha+\cos\alpha}{2}\right)^2}$$

$$= \frac{5+2\sin\alpha\cos\alpha}{3-2\sin\alpha\cos\alpha}$$

$$\Rightarrow \sec 2\gamma = \frac{5+\sin 2\alpha}{3-\sin 2\alpha}$$

(33-34).

From the given question, figure should be as follows:



Let AB be the pillar and α be the angle formed by flagstaff.

33. Option (a) is correct.

Explanations: It is given that,

$$\tan\theta = \frac{AB}{AP} = \frac{10}{x}$$

$$\tan(\theta+\alpha) = \frac{30}{x}$$

$$\Rightarrow \frac{\tan\theta+\tan\alpha}{1-\tan\theta\tan\alpha} = \frac{30}{x}$$

$$\Rightarrow \frac{\frac{10}{x}+\frac{1}{2}}{1-\left(\frac{10}{x}\right)\left(\frac{1}{2}\right)} = \frac{30}{x}$$

$$\Rightarrow \frac{20+x}{2x-10}=\frac{30}{x}$$
$$\Rightarrow 20x+x^2-60x+300=0$$
$$\Rightarrow x^2-40x+300=0$$
$$\Rightarrow (x-30)(x-10)=0$$
$$\Rightarrow x=30 \text{ or } x=10$$

Ratio of two values of $x = 1 : 3$
And $x \neq 20$ m
So, only (1) is correct.

34. Option (c) is correct.

Explanations:

Now, $\tan\theta = \frac{10}{30}$ or $\tan\theta = \frac{10}{10}$

$$\tan\theta = \frac{1}{3} \text{ or } 1$$

(35-36).

Let A, B, C be the angle of ΔABC
Now,

$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}=k$$

$\sin A = ak$, $\sin B = bk$ and $\sin c = ck$
It is given that,

$$a+b+c=6\times\left(\frac{\sin A+\sin B+\sin C}{3}\right)$$
$$\Rightarrow 2k=1\Rightarrow k=\frac{1}{2}$$
$$\text{So, } \frac{\sin A}{a}=k\Rightarrow \sin A=BC\frac{1}{2}=\frac{\sqrt{3}}{2}\Rightarrow A=\frac{\pi}{3}$$
$$\frac{\sin B}{b}=k\Rightarrow \sin B=AC\frac{1}{2}=\frac{1}{2}\Rightarrow B=\frac{\pi}{6}$$
$$C=\pi-\left(\frac{\pi}{3}+\frac{\pi}{6}\right)=\frac{\pi}{2}$$

35. Option (c) is correct.

Explanations: Perimeter of triangle
$= \sqrt{3} + 1 + 2 = 3 + \sqrt{3}$

36. Option (c) is correct.

Explanations: $C = \pi/3 = 1/2\ (\pi/2 + \pi/6)$
$C = 1/2\ (A+B)$
A, C, B are in A.P
Both (1) and (2) are true.

37. Option (c) is correct.

Explanations: We have,

$$x=\frac{\sin^2 A+\sin A+1}{\sin A}$$
$$=\sin A+1+\frac{1}{\sin A}$$

Now, $\sin A+\frac{1}{\sin A}\geq 2 \quad (\because AM > GM)$
$$\Rightarrow \sin A+\frac{1}{\sin A}+1\geq 3$$
$$\Rightarrow x\geq 3$$

Minimum value of $x = 3$

38. Option (d) is correct.

Explanations: Now, $x = 3$
$\sin^2 A + \sin A + 1 = 3 \sin A$
$\sin A - 2 \sin A + 1 = 0$
$(\sin A - 1)^2 = 0$
$\sin A = 1 = A = \pi/2$

39. Option (c) is correct.

Explanations: We know that
$a^2 = b^2 + c^2 - 2bc \cos A$
$b^2 = c^2 + a^2 - 2ca \cos B$
$c^2 = a^2 + b^2 - 2ab \cos C$
Adding above equations, we get
$a^2 + b^2 + c^2 = 2a^2 + 2b^2 + 2c^2$
$= 2bc\ c\cos A = 2ca\ c\cos B - 2ab\ c\cos C$
$a^2 + b^2 + c^2 = 2ab\ c\cos C + 2bc\ c\cos A + 2ac\ c\cos B$
Now, it is given that,
$a^2 + b^2 + c^2 = ac + \sqrt{3}\ bc$
$2ab\ c\cos C + 2bc\ c\cos A + 2ac\ c\cos B = ac + \sqrt{3}\ bc$
On comparing, we get
ABC is right angled triangle.

40. Option (c) is correct.

Explanations:
Now, area of ABC = 1/2 × AC × BC
$= 1/2 \times 4\sqrt{3} \times 4$

$$\left(\frac{1}{2}=\frac{BC}{8}=BC=4 \text{ and } \frac{\sqrt{3}}{2}=\frac{AC}{8}=AC=4\sqrt{3}\right)$$
$$=8\sqrt{3}$$

(41-42).

We have,
$f(x) = |x-2| + |3-x| + |4-x|$

$$f(x)=|x-2|+|3-x|+|4-x|$$
$$\Rightarrow f(x)=\begin{cases}-x+2+3-x+4-x, x\in(-\infty, 2)\\ x+2+3-x+4-x, x\in[2, 3)\\ x+2+3-x+4-x, x\in[3, 4)\\ x+2+3-x+4-x, x\geq 4\end{cases}$$
$$\Rightarrow f(x)=\begin{cases}9-3x, x<2\\ 5-x, x\in[2, 3)\\ x-1, x\in[3, 4)\\ 3x-g, x\geq 4\end{cases}$$

$$\Rightarrow f^1(x)=\begin{cases}-3, x<2\\ -1,\ x\in(2,3)\\ 1, x\in(3,4)\\ 3, x\geq 4\end{cases}$$

41. Option (b) is correct.

Explanations: Since sign changes from negative to positive a $x = 3$

$f(x)$ is minimum at $x = 3$

42. Option (a) is correct.

Explanations: Minimum value of $f(x) = f(3)$

$= |3 - 2| + |3 - 3| + |4 - 3|$

$= 1 + 0 + 1 = 2$

43. Option (c) is correct.

Explanations: Given, $s = 0! + 1! + 2! + ... + 100!$

From 41 onwards every terms has 4×2, which is divisible by 8.

Remaining sum $= 0! + 1! + 2! + 3!$

$= 1 + 1 + 2 + 6 = 10$

Now, remainder when 10 is divisible by 8 is 2

so, required remainder = 2

44. Option (d) is correct.

Explanations: Similarly from 5! onwards every terms has 10, which is divisible by 60

Remainder $= 0! + 1! + 2! + 3! + 4!$

$= 1 + 1 + 2 + 6 + 24 = 34$

45. Option (c) is correct.

Explanations: We have

PN = PM (Tangents from an external point)

PN = PM = n

Similarly, QL = QN = $n + 2$

and, RM = RL = $n + 4$

So, sides of triangle are,

PQ = $2n + 2$, QR = $2n + 6$, PR = $2n + 4$

Now, cos P = 1/3

$$\Rightarrow \frac{(PQ)^2+(PR)^2-(QR)^2}{2\,PQ\cdot PR}=\frac{1}{3}$$

$$\Rightarrow \frac{(2n+2)^2+(2n+4)^2-(2n+6)^2}{2\cdot(2n+2)^2(2n+4)}=\frac{1}{3}$$

$$\Rightarrow \frac{4\left((n+1)^2+(n+2)^2-(n+3)^2\right)}{4(n+1)(2n+4)}=\frac{1}{3}$$

$$\Rightarrow \frac{n^2+1+2n+n^2+4+4n-n^2-n-6n}{2n^2+6n+4}=\frac{1}{3}$$

$n = 8$, or $n = -2$

46. Option (a) is correct.

Explanations: Length of smallest side

$= 2n + 2 = 18$

47. Option (a) is correct.

Explanations: We have,

$$\sin x+\cos x+\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}+\frac{1}{\cos x}+\frac{1}{\sin x}=7$$

$$\sin x+\cos x+\frac{1}{\sin x.\cos x}+\frac{\sin x+\cos x}{\sin x.\cos x}=7$$

$$\Rightarrow (\sin x+\cos x)\left(1+\frac{2}{\sin x}\right)=7-\frac{2}{\sin 2x}$$

Squaring both sides, we get,

$$\Rightarrow (1+\sin 2x)\left(1+\frac{2}{\sin x}\right)^2=7-\frac{2}{\sin 2x}$$

$\Rightarrow \sin^2 2x - 44 \sin 2x + 36 = 0$

48. Option (d) is correct.

Explanations: $\sin^2 2x - 44 \sin 2x + 36 = 0$

$a = 22$, $b = 8$ and $c = 7$

So, $a - b + 2c = 22 - 8 + 14 = 28$

49. Option (b) is correct.

Explanations: Let a and b are the roots of the given equation

$$\therefore \alpha+\beta=\frac{4+2\sqrt{3}}{3+2\sqrt{2}} \text{ and } \alpha\beta=\frac{8+4\sqrt{3}}{3+2\sqrt{2}}$$

$$\text{HM of } \alpha \text{ and } \beta=\frac{2\alpha\beta}{\alpha+\beta}$$

$$=\frac{2\cdot\left(8+4\sqrt{3}\right)}{4+2\sqrt{3}}\times\frac{4-2\sqrt{3}}{4-2\sqrt{3}}$$

$$=\frac{2\left(32-16\sqrt{3}+16\sqrt{3}-24\right)}{16-12}=\frac{16}{4}=4$$

50. Option (a) is correct.

Explanations: GM of α and $\beta = \sqrt{\alpha\beta}$

$$=\sqrt{\frac{8+4\sqrt{3}}{3+2\sqrt{2}}}=\sqrt{\frac{2\left(4+2\sqrt{3}\right)}{3+2\sqrt{2}}}$$

$$=\sqrt{2}\left(\sqrt{\frac{\left(\sqrt{3}+1\right)^2}{\left(\sqrt{2}+1\right)^2}}\right.$$

$$=\sqrt{2}\left(\frac{\sqrt{3}+1}{\sqrt{2}+1}\times\frac{\sqrt{2}-1}{\sqrt{2}-1}\right)$$

$$=\sqrt{2}\left(\sqrt{6}-\sqrt{3}+\sqrt{2}-1\right)$$

51. Option (b) is correct.

Explanations:

$$\therefore \begin{vmatrix} a & b & a\alpha+b\\ b & c & b\alpha+c\\ a\alpha+b & b\alpha+c & 0\end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix} 0 & b & a\alpha+b \\ 0 & c & b\alpha+c \\ a\alpha^2+2b\alpha+c & b\alpha+c & 0 \end{vmatrix} = 0$$

$= 0 - 0 + (a\alpha^2 + 2b\alpha + c)(b^2\alpha + bc - ac\alpha - bc)$
$= 0$
$= b^2\alpha + ac\alpha = 0$ or $b^2 - ac = 0 = b^2 = ac$
So, a, b, c are in G.P

52. Option (c) is correct.

Explanations: $(7, 4, 2, \alpha) = 0$
$7\alpha^2 + 8\alpha + 2 = 0$
So, α is root of the equation, $7x^2 + 8x + 2 = 0$

53. Option (d) is correct.

Explanations: We have,
$m(0) = \cot\theta^2 + n^2\tan^2\theta + 2n$
$= (\cot\theta + n\tan\theta)^2$
$m(\theta) > 0$

Now, $\dfrac{\cot\theta + n\tan\theta}{2} \geq \sqrt{n}$

$= (\cot\theta + n\tan\theta)^2 > 4n$
$\therefore$ Minimum value of $m(\theta) = 4n$

54. Option (b) is correct.

Explanations: $(\cot\theta + n\tan\theta)^2 - 4x = 0$
$\Rightarrow (\cot\theta - n\tan\theta)^2 - 4x = 0$
$\Rightarrow \cot\theta = n\tan\theta$
$\Rightarrow x = \cot^2\theta$

55. Option (c) is correct.

Explanations: Equation of line of the quadrilateral is, $x = 0$, $y = 0$, $x + y = 1$ and $6x + y = 3$
Point of intersection of these lines are

$$A\left(\frac{1}{2}, 0\right), B(0,0), C(0,1), D\left(\frac{2}{5}, \frac{3}{5}\right)$$

So, equation of diagonal passes through B is ,

$$BD = y - 0 = \frac{3/5}{2/5}(x - 0)$$

$2y = 3x \Rightarrow 3x - 2y = 0$

56. Option (d) is correct.

Explanations: Equation of diagonal AC is

$$y - 0 = \frac{1-0}{0-1/2}\left(x - \frac{1}{2}\right)$$

$$\Rightarrow y = -2\left(x - \frac{1}{2}\right) \Rightarrow y = -2x + 1 \Rightarrow 2x + y - 1 = 0$$

57. Option (b) is correct.

Explanations: The given ellipse is,

$$\frac{x^2}{1} + \frac{y^2}{\left(\frac{1}{2}\right)^2} = 1$$

As we know, sum of distances of any point P from two is,
PE + PE = $2a = 2$

58. Option (d) is correct.

Explanations: Equation of latus return of ellipse is $x = 2\sqrt{3}/2$
So, points 1, 2 and 3 will be on it.

59. Option (a) is correct.

Explanations: Given equation of circle is

$$(x - a)^2 + y^2 = a^2$$

Now, $y = x$ intersect it 2 parts
Point of intersection of line and circle is, (0, 0) and (a, a)

So, required area $= \int_0^a \sqrt{a^2 - (x-a)^2}\,dx - \int_0^a x\,dx$

$$= \left(\frac{x-a}{2}\sqrt{2ax - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x-a}{a}\right) - \left(\frac{x^2}{2}\right)\right)_0^a$$

$$= \frac{a^2}{2}\sin^{-1}(0) - \frac{a^2}{2}\sin^{-1}(-1) - \frac{a^2}{2}$$

$$= \frac{a^2}{2} \times 0 + \frac{a^2}{2} \times \frac{\pi}{2} - \frac{a^2}{2}$$

$$= \frac{a^2}{4}(\pi - 2)$$

60. Option (b) is correct.

Explanations: Area of major segment
$= \pi r^2$ – Area of minor segment

$$= \pi a^2 - \frac{\pi a^2}{4} - \frac{a^2}{2}$$

$$= \frac{3\pi a^2}{4} + \frac{a^2}{2} = \frac{a^2}{4}(3\pi + 2)$$

61. Option (a) is correct.

Explanations: End points of diameter are A(1, –1, 2) and B(2, 1, –1)

Centre of sphere $= \left(\frac{3}{2}, 0, \frac{1}{2}\right)$

and radius $= \sqrt{\left(\frac{3}{2} - 1\right)^2 + (0+1)^2 + \left(\frac{1}{2} - 2\right)^2}$

$$= \sqrt{\frac{1}{4} + 1 + \frac{9}{4}} = \sqrt{\frac{14}{4}} = \sqrt{\frac{7}{2}}$$

Equation of space is

$$\Rightarrow x^2 + y^2 + z^2 + \frac{9}{4} + \frac{1}{4} - 3x - z = \frac{7}{2}$$

$\Rightarrow x^2 + y^2 + z^2 + 9/4 + 1/4 - 3x - z = 7/2$
$\Rightarrow x^2 + y^2 + z^2 - 3x - z - 1 = 0$

So, from given equation of sphere we have
$2a = -3, 2v = 0$ and $2w = -1$
$\Rightarrow a = -3/2, v = 0, w = -1/2$
So, $u + v + w = 4/2 = -2$

62. Option (b) is correct.

Explanations:
$PA^2 + PB^2 = AB^2$
$= (2-1)^2 + (1+1)^2 + (-1-2)^2$
$= 1 + 4 + 9 = 14$

63. Option (a) is correct.

Explanations: dr1 (2, –1, 2) and $(k, 3, 5)$ indicated at $\pi/4$

$$\therefore \cos\frac{\pi}{4} = \frac{2k-3+10}{\sqrt{4+1+4}\sqrt{k^2+9+25}}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{2k+7}{3\sqrt{k^2+34}}$$

$\Rightarrow 9(k^2 + 34) = 2(2k + 7)^2$
$\Rightarrow 9k^2 + 306 = 2(4k^2 + 49 + 28k)$
$\Rightarrow 9k^2 + 306 - 8k^2 - 98 - 56k = 0$
$\Rightarrow k^2 - 56k + 209 = 0$
$\Rightarrow k^2 - 52k - 4k + 208 = 0$
$\Rightarrow (k-52)(k-4) = 0 \Rightarrow k = 52$ or $k = 4$

64. Option (d) is correct.

Explanations: Let the drs of line perpendicular to given lines be (a, b, c)
Then, $2a - b + 2c = 0$
and $4a + 3b + 5c = 0$

$$\Rightarrow \frac{a}{-5-6} = \frac{b}{8-10} = \frac{c}{6-4}$$

$$\Rightarrow \frac{a}{-11} = \frac{b}{-2} = \frac{c}{10}$$

So, (11, 2, –10) as the required drs.

65. Option (b) is correct.

Explanations: Let $\vec{b} = ai = bj = ck$

Then, $\vec{a} \cdot \vec{b} = 27$

$\Rightarrow 3a + 3b + 3c = 27 \Rightarrow a + b + c = 9$ (i)
Also,

$$\Rightarrow \begin{vmatrix} i & j & k \\ 3 & 3 & 3 \\ a & b & c \end{vmatrix} = 9(j-k)$$

$\Rightarrow i(3c - 3b) - j(3c - 3a) + k(3b - 3a) = 9(j - k)$
$\Rightarrow 3c - 3b = 0, 3c - 3A = 9, 3b - 3a = -9$
$\Rightarrow c = b, a - c = 3, a - b = 3 \Rightarrow c = a - 3, b = a - 3$
From (i), we have
$a + a - 3 + a - 3 = 9$
$\Rightarrow 3a = 15 \Rightarrow a = 5$
$b = 5 - 3 = 2 = c$
So, $b = 5i + 2j + 2k$

66. Option (a) is correct.

Explanations: Now, $a + b = 8i + 5j + 5k$
$c = j - k$
$(a+b).c = |a + b||c| \cos\theta$,
where θ is the required angle

$$\Rightarrow 0 = \sqrt{8^2+5^2+5^2}\sqrt{1+1}\cos\theta$$

$$\Rightarrow \cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$$

67. Option (b) is correct.

Explanations: We have, $a = 4i - 8j + k$

$$\therefore \cos\alpha = \frac{4}{\sqrt{4^2+8^2+1^2}} = \frac{4}{9}$$

68. Option (a) is correct.

Explanations: Also, $\cos\beta = 8/9$ and $\cos\gamma = 1/9$
Now, $\cos 2\beta + \cos 2\gamma = 2\cos^2\beta - 1 + 2\cos^2\gamma - 1$

$$= 2\left(\frac{64}{81} + \frac{1}{81}\right) - 2 = \frac{-32}{81}$$

69. Option (b) is correct.

Explanations: We have, A = (1, –1, 0) and B = (0, 1, 1)
Equation of line AB is,

$$\frac{x-1}{0-1} = \frac{y+1}{1+1} = \frac{z-0}{1-0}$$

$$\frac{1-x}{1} = \frac{y+1}{2} = \frac{z}{1}$$

Now, only (2) and (3) satisfy this equation.

70. Option (c) is correct.

Explanations:
We have, $A = \hat{i} + \hat{j} + 0.\hat{k}$ and $B = oi + j + k$
$AB = (0-1)i + (1+1)i + (1-0) = i + 2j + k$

$$\left|\overline{AB}\right| = \sqrt{1^2+2^2+1} = \sqrt{6}$$

(71-73).

We have,
$f(x) = Pe^x + Qe^{2x} + Re^{3x}$
It is given that, $f(0) = 6$
$P + Q + R = 6$ (i)

$$\int_0^{\ln 2} f(x)\,dx = 11$$

$$\Rightarrow \left(Pe^x + \frac{Qe^{2x}}{2} + \frac{Re^{3x}}{3}\right)_0^{\ln 2} = 11$$

71. Option (b) is correct.

72. Option (c) is correct.

73. Option (d) is correct.

$f(0) = P + 2Q + 3R = 1 + 4 + 9 = 14$

(74-76).

We have,

$y^2 = 2c\,x + 2c$

$= y^4 + 4y^2(y)^2x^2 - 4y^3(y)x - 4y^3.(y)^3 = 0$

74. Option (a) is correct.

75. Option (b) is correct.

76. Option (b) is correct.

Explanations: We have,

$$f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$$

$$\therefore f(0) = \begin{vmatrix} \cos 0 & 0 & 1 \\ 2\sin 0 & 0 & 0 \\ \tan 0 & 0 & 1 \end{vmatrix} = 0$$

77. Option (b) is correct.

Explanations:

$$\lim_{x\to 0}\frac{f(x)}{x} = \lim_{x\to 0}\begin{vmatrix} \cos x & x & 1 \\ 2\sin x/x & x & 2 \\ \tan x & x & 1 \end{vmatrix} = 0$$

78. Option (a) is correct.

Explanations:

$$\lim_{x\to 0}\frac{f(x)}{x^2} = \lim_{x\to 0}\begin{vmatrix} \cos x & 1 & 1 \\ 2\sin x/x & 1 & 2 \\ \tan x & 1 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 0 & 1 & 1 \end{vmatrix} = -1$$

79. Option (b) is correct.

Explanations: We have,

$f(x) = \sin[\pi^2]x + \cos[-\pi^2]x$

$= \sin 9x + \cos(-10x)$

$= \sin 9x + \cos 10x$

$= 1 + (-1) = 0$

80. Option (d) is correct.

Explanations:

$$f\left(\frac{\pi}{4}\right) = \sin\frac{9\pi}{4} + \cos\frac{5\pi}{2}$$

$$= \sin\left(2\pi + \frac{\pi}{4}\right) + 0 = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

81. Option (Bonus) is correct.

Explanations: Since $I_1 = I_2 = \dfrac{\pi^2}{2\sqrt{2}}$

So, 81 and 83 is bonus.

82. Option (d) is correct.

Explanations: $8I^2 = 8\left[\dfrac{\pi^2}{2}\sqrt{2}\right]^2 = \dfrac{8\pi^2}{4} = \pi^2$

83. Option (Bonus) is correct.

84. Option (a) is correct.

Explanations: Now, $a < 0 < b$

$l = b - (-a) = a + b$

85. Option (c) is correct.

Explanations: Now, $a < b < 0$

$l = -(-b) + (-a) = b - a$

86. Option (a) is correct.

Explanations: $f'(0.5) = -1/0.5 = -2$

87. Option (c) is correct.

Explanations: $F'(2) = 1/2$

88. Option (d) is correct.

Explanations:

$$\begin{cases} ln(-ln\,x) & ,x < 0.1 \\ -ln(-ln\,x) & ,x \in (0.1, 1) \\ -ln(-ln\,x) & ,x \in (1, 2) \end{cases}$$

$$\therefore \frac{d}{dx}\left(fof(x) = \frac{-1}{\ln x}\cdot\frac{1}{x} = \frac{-1}{x\ln x}\right.$$

89. Option (b) is correct.

Explanations: $f(x)$ is continuous

$7 = P + q$ (i)

$10 = 2P + q$ (ii)

Solving (i) and (ii) we get

$P = 3$

90. Option (c) is correct.

Explanations: Also, $p = 3, q = 4$

91. Option (a) is correct.

Explanations: Only (1) is true

92. Option (a) is correct.

Explanations:

$$\Rightarrow \frac{du}{dx} = 2\sin x\cos x = \sin 2x$$

$$\text{and } \frac{dv}{dx} = 2\cos x(-\sin x) = -\sin 2x$$

$$\therefore \frac{du}{dv} = \frac{du}{dx}\Big/\frac{dv}{dx} = \frac{\sin 2x}{-\sin 2x} = -1$$

93. Option (a) is correct

Let the equation of line segments of ABC are given A = (2, 2), B(2, 2 m) and C(0, 0)

Since area of ABC = 3

$= |1/2(0 + 2(2m - 0) + 2(0 - 2)| = 3$

$= 4m - 4 = \pm 6$

$m = 5/2$ or $m = -1/2$

$\because m < 0$

$\therefore m = -1/2$

94. Option (b) is correct.

Explanations:

$$x^\circ = \frac{\pi x}{180} \text{ radians}$$

$$\therefore \frac{d}{dx}(\text{cosec } x^\circ) = \frac{d}{dx}\left(\text{cosec } \frac{\pi x}{180}\right)$$

$$= \frac{-\pi}{180}\text{cosec } x^\circ \cdot \cot x^\circ$$

95. Option (a) is correct.

Explanations:

$$\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{dy}{dx} - x\right) = 0$$

$$\Rightarrow \frac{dy}{dx} = 0 \text{ or } \frac{dy}{dx} = x$$

$$\Rightarrow y = \text{C(constant) or } y = \frac{x^2}{2} + c$$

96. Option (d) is correct.

Explanations: $f(x) = x^2 + 2$, $g(x) = 2x - 3$

$f(1) = 1 + 2 = 3$ and $g(1) = -1$

$(fg)(1) = 3$

97. Option (b) is correct.

Explanations: We have

$$\Rightarrow f(x) = \begin{cases} 0 & , x < 0 \\ 2x & , x \geq 0 \end{cases}$$

Range of $f(x) = [0, \infty)$

98. Option (a) is correct.

Explanations:

$f(x) = x(4x^2 - 3)$

$f(\sin\theta) = \sin\theta(4\sin^2\theta - 3)$

$= 4\sin 3\theta - 3\sin\theta = -\sin 3\theta$

99. Option (d) is correct.

Explanations:

$$\lim_{x \to 5} \frac{5 - x}{|x - 5|}$$

LHL = 1 and RHL = –1

So, limit at $x = 5$ does not exist.

100. Option (c) is correct.

Explanations: We have

$$\lim_{x \to 1} \frac{x^9 - 1}{x^3 - 1} = \lim x - 1\left(\frac{x^9 - 1}{x - 1} \times \frac{x - 1}{x^3 - 1}\right)$$

$$= \frac{9 \cdot (1)^8}{3 \cdot (1)^2} = 3$$

101. Option (c) is correct.

Explanations: Let the other two observations be x and y.

$$\therefore \text{ Mean} = \frac{11 + 16 + 20 + x + y}{5}$$

$= 14 \times 5 = 47 + x + y$

$= x + y = 23$ (i)

Now, variance $= \frac{1}{5}[(11 - 14)^2 + (16 - 14)^2$

$+ (20 - 14)^2 + (x - 14) + (y - 14)^2]$

$= 13.2 \times 5 = 9 + 4 + 36 + (x - 14)^2 + (y - 14)^2$

$= x^2 + y^2 - 28(x + y) + 2x.196 + 49 = 66$

$= x^2 + y^2 - 28(23) = -375$

$= x^2 + y^2 = 269$ (ii)

$x = 13, y = 10$

102. Option (c) is correct.

Explanations: Since A and B are independent events

$0.8 = (1 - 0.7) + (1 - k) - (1 - 0.7)(1 - k)$

$0.8 = 0.3 + 1 - k - 0.3(1 - k)$

$0.5 = 1 - k - 0.3 + 0.3k$

$0.1k = 0.2 = k = 2/7$

103. Option (b) is correct

Explanations: P (getting head) $= \frac{1}{4}$

P(getting tail) $= 1 - \frac{1}{4} = \frac{3}{4}$

Now, required probability

$$\left(\frac{3}{4}\right)^4 = \frac{81}{1024}$$

104. Option (b) is correct

Explanations: We have

$p(\text{H}) = \frac{3}{4}$ and $p(\text{T}) = \frac{1}{4}$

Required probability $= 4 \times (3/4)^3\ 1/4 = 27/64$

105. Option (c) is correct.

Explanations: We have, $n = 100$, P $= \frac{4}{5}$, $q = \frac{1}{5}$

For random variable x

$V(x) = npq = 80 \times \frac{1}{5} = 16$

Now, $y = 100 - x$

and, var $(y) = v(100 - x) = 0 + (-1)^2 v(x) = 16$

106. Option (c) is correct.

Explanations: We have

$x + 4y + 1 = 0$ (i)

$4x + 9y + 7 = 0$ (ii)

From (i), $bxy = -4$

From (ii), $byx = -1$

$r^2 = 4$ which is not possible as $0 < r^2 < 1$

$4x + 9y + 7 = 0$ is a line of regression x on y.

$4x = -(9y + 7)$

$x = -(9y + 7)/4 = -(-27 + 7)/4 = 5$

107. Option (d) is correct.

Explanations: Sum of all angles = 360°

$= p + q + r + s = 360°$

$$= \frac{k}{9} + \frac{k}{3} + \frac{k}{2} + \frac{k}{6} = 360^{\circ}$$

So, $4p - q = 144 - 108 = 36°$

108. Option (a) is correct

Explanations:

m = mean of 8 lowest observations

$$= 1 + 1 + 1 + 2 + 3 + 3 + 4 + \frac{4}{8}$$

$$= m = \frac{19}{8}$$

Now, M = Mean of highest 4 observation

$$M = 6 + 6 + 5 + \frac{4}{4} = \frac{21}{4}$$

$$\therefore \quad 2m + M = \frac{19}{4} + \frac{21}{4} = \frac{40}{4} = 10$$

109. Option (a) is correct

Explanations: We have,

$x = -1 + 3 = 2, y = 1 + 2 = 3$

$xy = -1 + 6 = 5, x^2 = 1 + 9 = 10$

Now, $x = \frac{2}{2} = 1$ and $y = \frac{3}{2}$

So, line of regression y on x is,

$y - y = b_{yx}(x - x)$

$$y - \frac{3}{2} = \frac{1}{4}(x - 1) = x - 4y + 5 = 0$$

110. Option (a) is correct

Explanations: Mode = 1

111. Option (c) is correct

Explanations:

x	f	cf
1	4	4
2	6	10
3	9	19
5	7	26
	26	

Now, N/2 = 13

Median = 3 as cf just greater than lies for $x = 3$.

112. Option (d) is correct

Explanations: Arranging observations of first five and last five observations, we get

Set-I: –1, 1, 3, 4, 8

Set-II: 9, 11, 12, 17, 19

M = Median of set I = 3

N = Median of set II = 12

$4M - N = 4 \times 3 - 12 = 12 - 12 = 0$

113. Option (d) is correct

Explanations: Let $P = \frac{k}{5}$, $Q = \frac{k}{4}$ and $R = 2k$

$$P + Q/2P + 0.7R = \frac{\left(\frac{9k}{20}\right)}{\left(\frac{9k}{5}\right)} = \frac{1}{4}$$

114. Option (d) is correct

Explanations: G.M. of $1, 2, 2^2 \ldots 2^{n-1}$

$$= \sqrt[n]{1.2.2^2 - 2^3 \ldots 2^{n-1}}$$

$$= \sqrt[n]{2^{1+2+3+\ldots+n-1}}$$

$$= \sqrt[n]{2^{(n-1)n/2}}$$

$$= G = 2^{(n-1)/2}$$

Now, $1 + 2\log_2 G = 1 + 2(n - 1/2)$

$= 1 + n - 1 = n$

115. Option (b) is correct

Explanations: $HM = \dfrac{n}{1 + \frac{1}{2} = \frac{1}{2^2} + \ldots \frac{1}{2^{n-1}}}$

$$\Rightarrow \frac{n}{H} = 1 + \frac{1}{2} = \frac{1}{2^2} + \ldots \frac{1}{2^{n-1}}$$

$$= \frac{1\left(1 - \frac{1}{2^n}\right)}{1 - \frac{1}{2}} = 2\left(1 - \frac{1}{2^n}\right)$$

$$\Rightarrow \frac{n}{H} - 2 - \frac{1}{2^{n-1}}$$

116. Option (c) is correct

Explanations: Let $P = (2x_i - a)^2$

On differentiating, we get

$$\frac{dp}{da} = 8x_i - 4a$$

$$\frac{dp}{da} = 0$$

$$8x_i = 4a$$

$$a = 2x_i$$

$$a = \frac{2x_i}{n} = 2Q$$

117. Option (Bonus) is correct

Explanations:

Let E1 : Two balls are white

E2 : Two balls are black

E3 : One ball is white & other is black

A: A white ball is from by B

Probability

$$= \frac{{}^3C_2}{{}^5C_2} + \frac{4}{7} + \frac{{}^2C_2}{{}^5C_2} + \frac{2}{7} + \frac{{}^3C_1 \times {}^2C_1}{{}^5C_2} \times \frac{3}{7} = \frac{33}{70}$$

118. Option (c) is correct

Explanations: Total number of ways

$= 6 \times 6 \times 6$

Out of these, multiple of 3 shows $2 \times 2 \times 2$ times

Required probability $= \frac{8}{216} = \frac{1}{27}$

119. Option (c) is correct

Explanations: There are 31 days in december

i.e., $31 = \frac{28}{7} + 3$

Now, on these 3 days, one can be sunday

Required probability $= \frac{3}{7}$

120. Option (b) is correct

Explanations: $n + \frac{50}{n} < 50$

So, n can be 2, 3, ..., 48

Favourable cases = 47

Total cases = 50

Required probability $= \frac{47}{50}$